Q5) (i)

Ans) given

$f(x, y)$ is real function.
$F(u, v)$ is discreate fourier transfor of $f(x, y)$

$$F^*(u, v) = F(-u, -v)$$

$$F(u, v) = \frac{1}{\sqrt{w_1 w_2}} \sum_{x=0}^{w_1 - 1} \sum_{y=0}^{w_2 - 1} f(x, y) e^{-j\frac{2\pi x u}{w_1}} e^{-j\frac{2\pi y v}{w_2}}$$

$$F^*(u, v) = \frac{1}{\sqrt{w_1 w_2}} \sum_{x=0}^{w_1 - 1} \sum_{y=0}^{w_2 - 1} f^*(x, y) e^{\frac{j 2\pi x u}{w_1}} e^{\frac{j 2\pi y v}{w_2}}$$

Since $f(x, y)$ is real valued $i = 0$

$$f^*(x, y) = f(x, y).$$

$$F^*(u, v) = \frac{1}{\sqrt{w_1 w_2}} \sum_{x=0}^{w_1 - 1} \sum_{y=0}^{w_2 - 1} f(x, y) e^{\frac{-j 2\pi x u}{w_1}} e^{-\frac{j 2\pi y v}{w_2}}$$

~~F(~~

↳ Replacing u with $-\mu$ & v with $-\nu$

$$= F^*(-\mu, -\nu) = \frac{1}{\sqrt{w_1 w_2}} \sum_{x=0}^{w_1 - 1} \sum_{y=0}^{w_2 - 1} f(x, y) e^{-j\frac{2\pi x \mu}{w_1}} e^{-j\frac{2\pi y \nu}{w_2}}$$

$\mu \Rightarrow$ meu
$\nu \Rightarrow$ eta

different the v, v

$$= F(\mu, \nu)$$

altering values back.

$$\boxed{F^*(u, v) = F(-u, -v)}$$

PAGE NO:
DATE / /

(ii) Given :- $f(x,y) = f(-x,-y)$
i.e. $f(x,y)$ is even.

To prove :- $F(U,V)$ is also real & even.

from part (i) $F^*(U,V) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y) e^{j2\pi xU/W_1} e^{j2\pi Vy/W_2}$

Replacing x with $-x$, y with $-y$.
(Limits would get changed).

$F^*(U,V) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=1-W_1}^{0} \sum_{y=1-W_2}^{0} f(-x,-y) e^{-j2\pi xU/W_1} e^{-j2\pi yV/W_2}$

$= \frac{1}{\sqrt{W_1 W_2}} \sum_{x=1-W_1}^{0} \sum_{y=1-W_2}^{0} f(x,y) e^{-j2\pi xU/W_1} e^{-j2\pi yV/W_2}$

using periodicity law for $e^{-j2\pi xU/W_1}$
and for corresponding y & $f(x,y)$

i.e. $e^{-j2\pi(x-W_1)U/W_1} = e^{-j2\pi xU/W_1}$

$f(x-W_1, y-W_2) = f(x,y)$

Also $f(0,0) = f(W_1, W_2)$

$\Rightarrow$ Thus x by $x-W_1$, y by $y-W_2$

$\Rightarrow F^*(U,V) = \frac{1}{\sqrt{W_1 W_2}} \sum_{x=1}^{W_1} \sum_{y=1}^{W_2} f(x,y) e^{-j2\pi xU/W_1} e^{-j2\pi yV/W_2}$

$= \frac{1}{\sqrt{W_1 W_2}} \sum_{x=0}^{W_1-1} \sum_{y=0}^{W_2-1} f(x,y) e^{-j2\pi xU/W_1} e^{-j2\pi yV/W_2}$

$= F(U,V)$

PAGE NO.:

DATE / /

Hence, $F(u,v)$ is real valued.

As $F^*(u,v) = F(u,v)$

and from (i) $F^*(u,v) = F(-u,v)$

we get :- $F(u,v) = F(-u,-v)$

hence $F(u,v)$ is also Even

Hence proved