

Question 2:

You are viewing the following graph from a research paper. Unfortunately, from the graph, the (x, y) values at only a few points can be observed. You need to obtain the (x, y) values at many other points. Hence you can do the following: you extract the image from the paper, and open it through MATLAB which provides a function called `imread`. This function gives you the (x, y) coordinates of any spatial location pointed by your mouse. However, the coordinate system of the graph and that of MATLAB will be different. Describe a procedure to convert from MATLAB's coordinate system to the coordinate system of the graph. This will help you obtain the (x, y) coordinates in the coordinate system of the graph. Support your answer with suitable equations. There is no need to write any code for this.

Solution:

We have observed 3 points in the graph as well as in Matlab, those coordinates are given below.

	Graph	Matlab
1	(5,625)	(649,1365)
2	(0,550)	(570,187)
3	(-10,580)	(414,666)

We observed graph is not rotated either sheared in matlab, hence we performed below operation, although after checking these points with scaling and translation both simultaneously we observed that, The 3 equations for c_x and t_x results in different values while solving any 2, No solution for this, hence assumption contradicts (rotation and scaling).

Handwritten derivation showing the transformation from graph coordinates to MATLAB coordinates using scaling and translation.

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} c_x & 0 & t_x \\ 0 & c_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Plugging above point yields:-

$$\begin{bmatrix} 649 \\ 1365 \\ 1 \end{bmatrix} = \begin{bmatrix} c_x & 0 & t_x \\ 0 & c_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 625 \\ 1 \end{bmatrix}$$

Solving the equations:

$$\begin{aligned} 5c_x + t_x &= 649, & t_x &= 570 \\ 625c_y + t_y &= 1365, & 550c_y + t_y &= 187 \\ -10c_x + t_x &= 414 \\ 580c_y + t_y &= 666 \end{aligned}$$

c_x, c_y are scaling in x and y directions.

Hence, it is clear that only translation and scaling is not enough, there might be need of rotation and even shearing too, so better to go for **affine transformation** and equations can be formed

from the below matrix product since we can derive the solution through the no of unknown variables and the no. of equations that we got as shown below.

$$\begin{bmatrix} X_2 \\ Y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ 1 \end{bmatrix}$$

Plugging above points again.

$$\begin{aligned} 5a + 625b + t_x &= 649 \\ 5c + 625d + t_y &= 1365 \\ 550b + t_x &= 570 \\ 550d + t_y &= 187 \\ -10a + 580b + t_x &= 414 \\ -10c + 580d + t_y &= 616 \end{aligned}$$