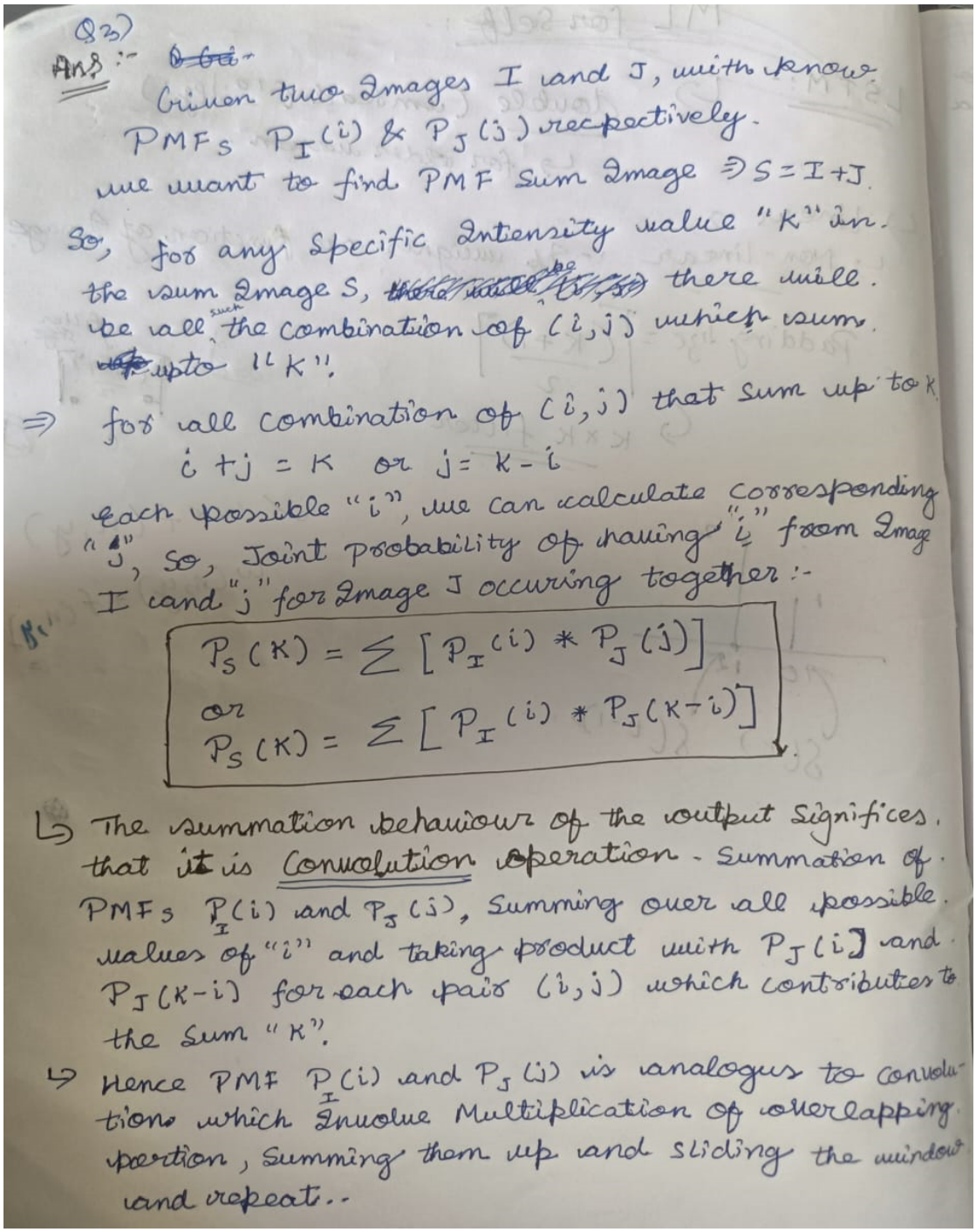
Question 3:

Consider two images I and J whose intensity values (in each location) are randomly drawn from the known probability mass functions (PMFs) pI (i) and pJ (j) respectively. Derive an expression for the PMF of the image I + J. The expression resembles which operation?

Solution:



Q3)

Ans :-

Given two images I and J, with known PMFs $P_I(i)$ & $P_J(j)$ respectively.

We want to find PMF sum image $\Rightarrow S = I + J$.

So, for any specific intensity value "K" in the sum image S, there will be all the combination of (i, j) which sum upto "K".

$\Rightarrow$ for all combination of (i, j) that sum up to K.

$i + j = K$ or $j = K - i$

Each possible "i", we can calculate corresponding "j", so, Joint probability of having "i" from Image I and "j" for image J occuring together :-

$$P_S(K) = \sum [P_I(i) * P_J(j)]$$

or

$$P_S(K) = \sum [P_I(i) * P_J(K-i)]$$

↳ The summation behaviour of the output signifies, that it is Convolution operation - Summation of PMFs $P_I(i)$ and $P_J(j)$, Summing over all possible values of "i" and taking product with $P_J(i)$ and $P_J(K-i)$ for each pair (i, j) which contributes to the sum "K".

↳ Hence PMF $P_I(i)$ and $P_J(j)$ is analogous to convolutions which involve Multiplication of overlapping portion, Summing them up and sliding the window and repeat..