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Assignment 3

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Question 6: If F is the continuous Fourier operator, prove that $F(F(F(F(f(t))))) = f(t)$. Hint: Prove that $F(F(f(t))) = f(-t)$ and proceed further from there.

Solution:

Given, $\mathcal{F}$ is continuous Fourier operator.

Need to proof : $\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = f(t)$.

If $\mathcal{F}(\mathcal{F}(f(t))) = f(-t)$

then, $\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t)))) = \mathcal{F}(f(-t))$

$$\Rightarrow \mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = \mathcal{F}(\mathcal{F}(f(-t)))$$
$$= f(-(-t))$$
$$\boxed{\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = f(t)} \quad \text{—— (I)}$$

So we only need to prove $\mathcal{F}(\mathcal{F}(f(t))) = f(-t)$ because if it is true then (I) is a direct implication.

Now, $\mathcal{F}$ is continuous fourier operator

$$\Rightarrow \mathcal{F}(f(t)) = F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi\mu t}\, dt$$

$$\Rightarrow \mathcal{F}(\mathcal{F}(f(s))) = \mathcal{F}(F(t)) = \int_{-\infty}^{\infty} F(t)\, e^{-j2\pi\mu t}\, dt$$
$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(s)\, e^{-j2\pi\mu s}\, ds \right] e^{-j2\pi\mu t}\, dt$$
$$= \int_{-\infty}^{\infty} f(s) \left[\int_{-\infty}^{\infty} e^{-j2\pi\mu(s+t)}\, dt\right] ds$$

(using Fubini's Theorem)

$$= \int_{-\infty}^{\infty} f(s)\, \delta(s+t)\, ds = f(-t)$$

(where δ is Dirac Delta).

replacing s by $t2$ as they are independent from each other, $\Rightarrow \boxed{\mathcal{F}(\mathcal{F}(f(t))) = f(-t)}$ —— (II)

Hence, using (II) we have (I).