Question 4:

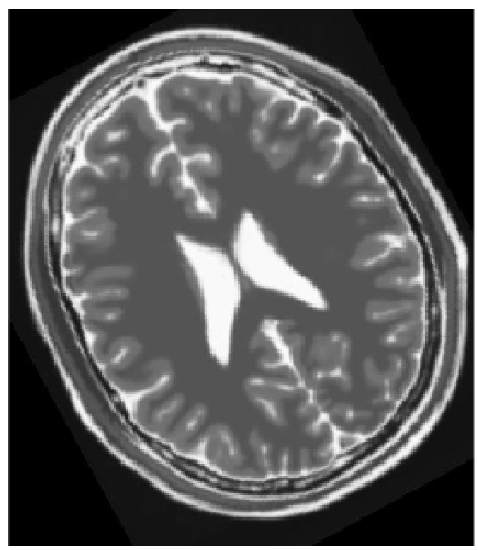
Read in the images T1.jpg and T2.jpg from the homework folder using the MATLAB function "imread" and cast them as a double array. Let us call these images as J1 and J2. These are magnetic resonance images of a portion of the human brain, acquired with different settings of the MRI machine. They both represent the same anatomical structures and are perfectly aligned (i.e. any pixel at location (x, y) in both images represents the exact same physical entity). We are going to perform a simulation experiment for image alignment in a setting where the image intensities of physically corresponding pixels are different. To this end, do as follows:

A) Write a piece of MATLAB code to rotate the second image by $\theta = 28.5$ degrees anti-clockwise. You can use the "imrotate" function in MATLAB to implement the rotation using any interpolation method. Note that the rotation is performed implicitly about the centroid of the image. While doing so, assign a value of 0 to unoccupied pixels. Let us denote the rotated version of J2 as J3.

Solution:

Code attached adding a snippet of rotated images. Used Bilinear interpolation

```
%%% PART - A

% Reading Images
T1 = imread('./HW1/T1.jpg');
T2 = imread('./HW1/T2.jpg');

% Casting Images to double
J1 = im2double(T1);
J2 = im2double(T2);


% Rotation of J3 image with Bilinear Interpolation
theta = 28.5;
J3 = imrotate(J2, theta, "bilinear", "crop");

% Assigning 0 value to unoccupied pixels
J3(isnan(J3)) = 0;

% Saving the Image
imshow(J3)
imwrite(J3, 'J3.jpg' ,'jpg')
```



B) Our job will now be to align J3 with J1 keeping J1 fixed. To this end, we will do a brute-force search over θ ranging from -45 to +45 degrees in steps of 1 degree. For each θ, apply the rotation to J3 to create an intermediate image J4, and compute the measures of dependence between J1 and J4.

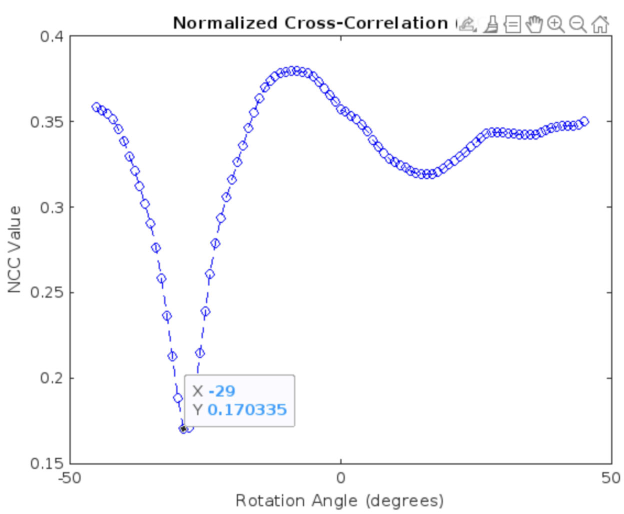
Solution:

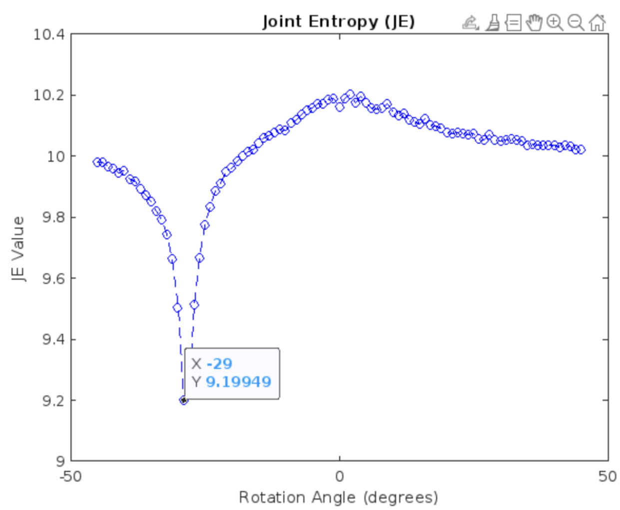
Adding Code Snippet here and the way in which NCC, JE and QMI are calculated. Their corresponding formulas are used in the code accordingly and also computed the values of the 3 measures.

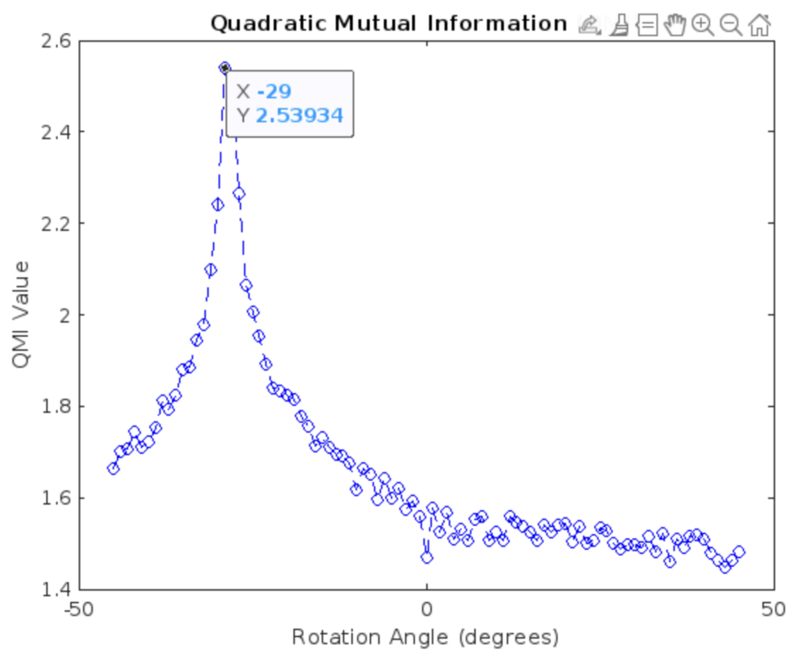
```
%%% PART - B

% Range of angles for search
angles = -45:1:45;

% Initialization to store NCC, JE, and QMI values
nccValues = zeros(size(angles));
jeValues  = zeros(size(angles));
qmiValues = zeros(size(angles));

% Computing NCC, JE, and QMI values
for i = 1:length(angles)

    % Rotating J3 image accordingly
    angle = angles(i);
    J4 = imrotate(J3, angle, 'bilinear', 'crop');
    J4(isnan(J4)) = 0;

    % Normalizing J4 and J1
    J4_norm = (J4 - mean(J4(:))) / std(J4(:));
    J1_norm = (J1 - mean(J1(:))) / std(J1(:));

    % NCC calculation
    nccValues(i) = sum(J1_norm(:) .* J4_norm(:)) / numel(J1);

    % JE Calculation
    jointHist = histcounts2(J1_norm(:), J4_norm(:), 256);

    jointProb = jointHist / sum(jointHist(:));
    jeValues(i) = -sum(jointProb(jointProb > 0) .* log2(jointProb(jointProb > 0)));

    % QMI Calculation
    qmiValues(i) = computeQMI(J1, J4);
end
```

```
    mutual_info = 0;
    for i = 1:num_bins
        for j = 1:num_bins
            p_ij = p_joint(i, j);
            p_i = hist_image1(i);
            p_j = hist_image2(j);

            if p_ij > 0 && p_i > 0 && p_j > 0
                mutual_info = mutual_info + p_ij * log2(p_ij / (p_i * p_j));
            end
        end
    end

    qmi = mutual_info;
end
```

C) Plot separate graphs of the values of NCC, JE, QMI versus θ and include them in the report PDF.







D) Determine the optimal rotation between J3 and J1 using each of these three measures. What do you observe from the plots with regard to estimating the rotation?
Solution:

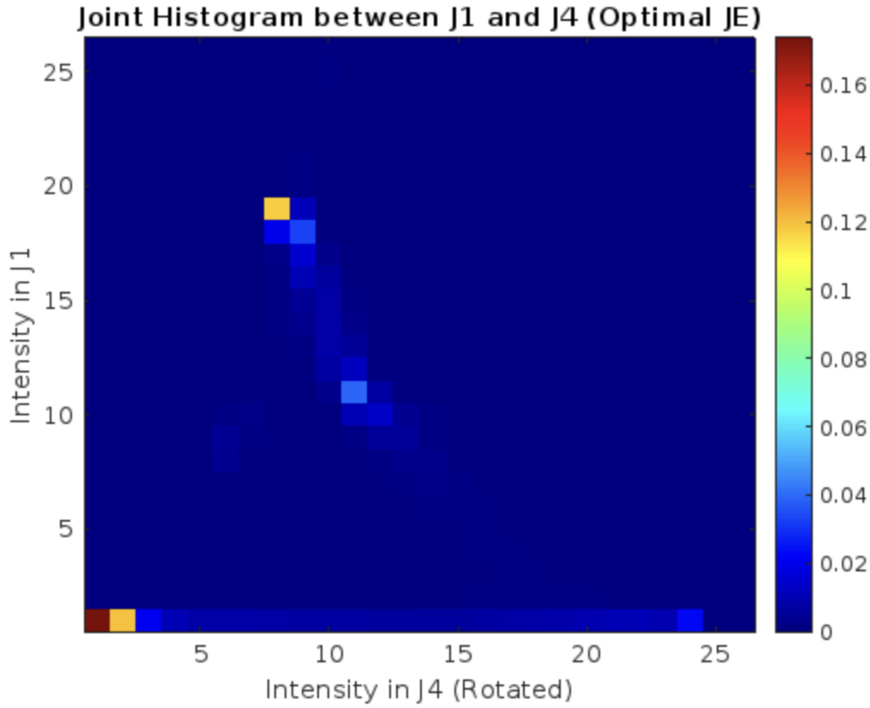
```
Best NCC angle: -29.00 degrees (NCC value: 0.1703)
Best JE angle: -29.00 degrees (JE value: 9.1995)
Best QMI angle: -29.00 degrees (QMI value: 2.5393)
```

We observe from the plots that all the three measurements imply 29 degrees anti-clockwise rotation as the best possible solution for Image alignment as intensities have least possible entropy there. Mutual Information is maximum at 29 degrees and is minimum for JE and NCC. We see the usage of all 3 measures through this exercise successfully.

E) For the optimal rotation using JE, plot the joint histogram between J1 and J4 using the imagesc function in MATLAB along with colorbar. Include it in the report PDF.
Solution:

The optimal rotation degrees obtained from JE used to rotate image to form J4 is 29 degrees

In this case we tried with different bins, but as asked we decided with bin-width = 10 and since intensity ranges from 1 to 255. We took the ceil(255/10) = 26 bins. The graph that is plotted implies the joint histogram with the asked configuration between J1 and optimal_JE(J4). Note that the histogram pictorization differs with varied bin size



F) We have studied NCC and JE in class. What is the intuition regarding QMI?
Solution:

Quantitative Mutual Information (QMI) measures the extent to which two random variables (here being as pixel intensities in images) exhibit statistical dependence beyond what would be expected if they were independent. It quantifies the dissimilarity between their joint distribution and the product of their marginal distributions. When I1 and I2 are statistically independent, their joint distribution aligns closely with the product of their marginals, resulting in a low QMI value. Conversely, when I1 and I2 are dependent, deviations from this expected behavior increase the QMI score. Thus, QMI captures both the strength and nature of their statistical relationship, making it useful for assessing how closely two variables are related and distinguishing between independence and dependence. Like QMI provides a refined understanding of the statistical relationship between two random variables. It evaluates the joint distribution of I1 and I2 in comparison to the hypothetical distribution expected if they were independent. When I1 and I2 are independent, their joint distribution closely matches the product of their marginal distributions, resulting in a low QMI. However, if their joint distribution significantly deviates from this expected form, the QMI value rises, indicating the presence of dependence. QMI's sensitivity to joint and marginal interactions makes it valuable for capturing nuanced dependencies that might not be evident through other measures.