④ Given :- $g_1 = f_1 + h_2 \times f_2$ Known Parameters

$g_2 = h_1 \times f_1 + f_2$ g_1, g_2, h_1, h_2

a) Find eq for f_1 and f_2

sol:-

$g_1 = f_1 + h_2 \times f_2 \Rightarrow f_2 = \frac{(g_1 - f_1)}{h_2} \rightarrow ①$

$g_2 = h_1 \times f_1 + f_2 \Rightarrow g_2 = h_1 \times f_1 + \frac{(g_1 - f_1)}{h_2}$ ($\because$ from ①)

$\Rightarrow g_2 \times h_2 = h_1 h_2 f_1 + g_1 - f_1$

$\Rightarrow f_1 = h_1 h_2 f_1 + g_1 - g_2 h_2$

$\Rightarrow f_1 (1 - h_1 h_2) = g_1 - g_2 h_2$

$$\boxed{\therefore f_1 = \frac{g_1 - g_2 h_2}{(1 - h_1 h_2)}}$$

From ①, $f_2 = \frac{(g_1 - f_1)}{h_2} = \frac{g_1 - \left[\frac{g_1 - g_2 h_2}{(1 - h_1 h_2)}\right]}{h_2}$

$\Rightarrow f_2 = \frac{g_1 - g_1 h_1 h_2 - g_1 + g_2 h_2}{h_2 (1 - h_1 h_2)} = \frac{h_2 (g_2 - g_1 h_1)}{h_2 (1 - h_1 h_2)}$

$$\Rightarrow \boxed{f_2 = \frac{g_2 - g_1 h_1}{(1 - h_1 h_2)}}$$

b) The first observation is that, the solution of f_1 and f_2 becomes undefined when $(1-h_1 h_2)=0$ $\Rightarrow h_1 * h_2 = 1$. We know that h_1 and h_2 are low-pass filter kernels or blur kernels and so through fourier transforms we know that for lower frequencies $h_1 \times h_2$ becomes 1 ~~bits~~ like $h_1(0) = 1 = h_2(0)$ and resulting no definite solution. So first issue is reconstruction at higher frequencies through kernels (h_1, h_2) is fine ~~bg~~ but for lower frequencies it is not suitable.

Also, another point is these formulas provided do not account for noise in the images, which might significantly affect the resuts. Since noises & uncertainities are inevitable in real-life scenarios. To mitigate above issues we could add small ε value to address issues like

$$f_1 = \frac{g_2 - h_1 \times g_1}{(1-h_1 h_2)+\varepsilon} \quad \text{and} \quad f_2 = \frac{g_1 - h_2 g_2}{(1-h_1 h_2)+\varepsilon}$$

so issues could be mitigated. though the resultant image is bit incorreclty reconstructed and look with some noise.