(Q 4')

Ans (a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ Seperable ? [If it can be represented in form of outproduct

$U = [U_1, U_2, U_3]$ $V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} [U_1, U_2 U_3] \text{ (outer product)}$$

$$= \begin{bmatrix} v_1 U_1 & v_1 U_2 & v_1 U_3 \\ v_2 U_1 & v_2 U_2 & v_2 U_3 \\ v_3 U_1 & v_3 U_2 & v_3 U_3 \end{bmatrix}$$

$v_1 U_1 = 0 \Rightarrow$ $v_1 = 0$ or $U_1 = 0$ or $v_1 = 0$ & $U_1 = 0$

$v_1 U_2 = 1$, $v_2 U_1 = 1$ so, neither of them can be 0.

So, Laplacian is not seperable filter.

(b) $L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$f(n) = [f(n-1, y)\ f(n, y)\ f(n+1, y)]$

$f(y) = \begin{bmatrix} f(n, y-1) \\ f(n, y) \\ f(n, y+1) \end{bmatrix}$.

$\Rightarrow (L * (f(n) * f(y)))$

$= f(n, y-1) + f(n, y+1) - 4f(n, y) + f(n-1, y) + f(n+1, y)$.

$\hookrightarrow [1\ -2\ 1] * f(n) = f(n-1, y) - 2f(n, y) + f(n+1, y)$

$\hookrightarrow [1\ -2\ 1] * f(y)^T = f(n, y-1) - 2f(n, y) + f(n, y+1)$ — (1)

$\hookrightarrow [1\ -2\ 1] * (f(n) + f(y)^T) =$ — (3)

$f(n-1, y) - 4f(n, y) + f(n, y+1) + f(n+1, y) + f(n, y-1)$

(1) + (3) $\Rightarrow$

$= [1, 1, -4, 1, 1] * [f(n-1, y), f(n, y-1), f(n, y), f(n+1, y), f(n, y+1)]$

$= (L * (f(n) * f(y)))$.

Hence the laplacian can only be implemented entirely using 1D convolution.

* is convolution not matrix multiplication.