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Conclusion :-

Q2) ⇒ Inverting sign of the observed non zero / maximum point gives us the translation b/w the two images.

(a)

Ans ⇒ The procedure for predicting translation b/w two images uses Fourier transform properties described in the paper in 3 steps :-

Image f_1 be translated to f_2.

$$f_2(x, y) = \cancel{f_1(x - x_0, y - y_0)}$$
$$f_1(x - x_0, y - y_0).$$

Step-1 :- Obtain fourier transform of the two images F_1 & F_2.

$$\boxed{F_2(u, v) = e^{j2\pi(ux_0 + vy_0)} \cdot F_1(u, v)}$$

(using Fourier shift theorem).

Step-2 :- Cross power spectrum of the two images :-

$$\frac{F_1(u, v) \cdot F_2^*(u, v)}{|F_1(u, v) F_2(u, v)|} = e^{j2\pi(ux_0 + vy_0)}$$

Step-3 :- Inverse fourier transform of the cross-power spectrum :-

$$\boxed{F^{-1}\left(e^{j2\pi(ux_0 + vy_0)}\right) = \delta(x + x_0, y + y_0)}$$

⇒ By taking Inverse fourier transform of the representation in frequency domain, will have

f(x) that was

impulse at $(-x_0, -y_0)$ and approximately 0 every where $(-x_0, y_0)$ is the displacement that needed to optimally register the two image. which will give us the translation b/w two image.

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Time Complexity :-

$\Rightarrow$ Step 1 + Step 2 + Step 3

Step-1 :- Obtaining fourier transform ~~the~~ of two Image. can be done by using FFT.

FFT of two Images $O(MN \log(MN))$ given size $M \times N$.

but we got $N \times N$ $\Rightarrow O(N^2 \log N^2)$

$+ O(N^2 \log N^2)$

$\simeq O(N^2 \log N)$ — (1)

Step-2 :- Point wise multiply

$= O(N^2)$. — (2) {Note these are asymptotic so, Ignoring constant}.

Step-3 :- again FFT for Inverse fourier:- (Inverse. FFT).

$O(NN \log_2(N^2)) = O(N^2 \log N)$ — (3)

(1) + (2) + (3)

$= O(N^2 \log N)$.

$\Rightarrow$ for Pixel wise Image Compression ~~looping~~ Iterating over different values of the translation and perform :-

1) Image wraping :- $O(N^2)$ obtaing Intensity by visiting each pixel in Image.

2) MSSD :- ~~we find the mean~~

$O(N^2) \Rightarrow$ Subtracting transformed Image & reference Image.

(Substracting each intensity pixel in the $N \times N$ ~~2~~ array).

Iterating over each pixel. $= O(N^2(N^2 + N^2)) = O(N^4)$

and the finding minimum $= O(N^2)$

$\Rightarrow$ Total time $= O(N^4)$.

hence speed using FFT $= \dfrac{O(N^4)}{O(N^2 \log N)} = O\left(\dfrac{N^2}{\log N}\right)$.

2) For Rotation :-

$$f_2(x, y) = f_1(x\cos\theta_0 + y\sin\theta_0 - x_0, -x\sin\theta_0 + y\cos\theta_0 - y_0).$$

1. Relation b/w their fourier transform :-

$$F_2(u, v) = e^{j2\pi(ux_0 + vy_0)} \cdot F_1(u\cos\theta_0 + v\sin\theta_0, -u\sin\theta_0 + v\cos\theta_0).$$

2. Relation b/w the magnitude of F_1 (M_1) & F_2 (M_2).

$\boxed{\xi = u}$

$$M_2(u, v) = M_1(u\cos\theta_0 + v\sin\theta_0, -u\sin\theta_0 + v\cos\theta_0).$$

$\Rightarrow$ changed here from paper.

$\boxed{\eta = v}$

$$\boxed{M_1(\rho, \theta) = M_2(\rho, \theta - \theta_0)}$$ (polar form)

hence using phase correction $\{f_2(x,y) = f_1(x - x_0, y - y_0)\}$ from first part (just for reference). we can estimate θ_0.

$\Rightarrow$ This method rotation even if there is translation too..