PAGE NO.:
DATE / /

Q3) Prove the convolution theorem in 2-D FT.

Ans $f_1(x, y)$ & $f_2(x, y)$ be two descrete ~~FT~~ functions defined for same domain.

{domain size $M \times N$} $(f_1 * f_2)(x, y) = \sum_{l=0}^{M-1} \sum_{m=0}^{N-1} f_1(l, m) f_2(x-l, y-m)$

$(f_1 * f_2)(x+M, y+N)$

Periodic with M as x axis, and N as y axis.

$= \sum_{l=0}^{M-1} \sum_{m=0}^{N-1} f_1(l, m) f_2(x+M-l, y+N-m)$

$= \sum_{l=0}^{M-1} \sum_{m=0}^{N-1} f_1(l, m) f_2(x-l, y-m)$.

$= (f_1 * f_2)(x, y)$

Hence periodic with in domain, therefore the steps of convolution first we zero pad f_1 & f_2 to $2M \times 2N$ but consider only the central $M \times N$ output of the convolution, the remaining values outside $M \times N$ are part of other period.

$DFT(f(x,y)) = F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \ldots$

$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \exp\left(-2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$

IDFT of the signal f is F in 2D:-

$IDFT(F(u, v)) = f(x, y)$

$= \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} (u, v) \exp\left(2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$

~~$DFT(f_1 * f_2) = F_1 . F_2$~~

2D-DFT of signal f_1 & f_2 are $F_1(u, v)$ and $F_2(u, v)$ respectively.

PAGE NO.:
DATE / /

~~IDFT~~

$$DFT(f_1 * f_2) = F_1 \cdot F_2$$

↳ we prove this is true by showing that
$$IDFT(F_1 \cdot F_2) = f_1 * f_2.$$

$$\Rightarrow IDFT(F_1 \cdot F_2) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} (F_1(u,v) \cdot F_2(u,v)) \cdot \exp\left(2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$$

$$= \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \left[\left(\sum_{p=0}^{M-1} \sum_{q=0}^{N-1} f_1(p,q) \exp\left(-2\pi i \left(\frac{up}{M} + \frac{vq}{N}\right)\right)\right)\right] \cdot F_2(u,v) \exp\left(2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$$

Simplying the eqn in between removing upper limits.

$$= \frac{1}{MN} \sum_{u,v} \left(\sum_{p,q} f_1(p,q) \exp\left(-2\pi i \left(\frac{up}{M} + \frac{vq}{N}\right)\right)\right) \cdot F_2(u,v) \exp\left(2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$$

$$= \frac{1}{MN} \sum_{p,q} \sum_{u,v} f_1(p,q) \exp\left(-2\pi i \left(\frac{up}{M} + \frac{vq}{N}\right)\right) \cdot F_2(u,v) \exp\left(2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$$

{Interchanging order of summation}.

$$= \frac{1}{MN} \sum_{p,q} f_1(p,q) \cdot \sum_{u,v} \exp\left(-2\pi i \left(\frac{up}{M} + \frac{vq}{N}\right)\right) \cdot F_2(u,v) \exp\left(2\pi i \left(\frac{ux}{M} + \frac{vy}{N}\right)\right)$$

$$= \sum_{p,q} f_1(p,q) \left(\frac{1}{MN} \sum_{u,v} F_2(u,v) \exp\left(2\pi i \left(\frac{u(x-p)}{M} + \frac{v(y-q)}{N}\right)\right)\right)$$

$$= \sum_{p,q} f_1(p,q) f_2(x-p, y-q)$$

$\boxed{IDFT = f_1 * f_2}$ ∴ Hence proved