CST 112-2 : CALCULUS
END SEMESTER EXAMINATION
SEMESTER I - 2022
Department of Computer Science and Informatics
Faculty of Applied Sciences
Uva Wellassa University

INSTRUCTIONS

1. Time : 2 hours (start at 6.00 a.m.)
2. You have to turn off your mobile phones during the exam period.
3. You cannot keep any unauthorized materials with you during the exam period. Calculators are allowed.
4. You cannot use the internet to find the answers throughout the exam period.
5. The exam will stop at 8.00 a.m. After that 15 minutes will be given to scan your answers, make a pdf, and upload it to the VLE. When you are uploading your answers to the VLE please rename the file by your registration number.

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1. (a) Evaluate the following limits
   i. $\lim_{x\to 5} \frac{2x^2-50}{x-5}$
   ii. $\lim_{x\to 2} \frac{2^{x+8}-1024}{4^x-16}$
   iii. $\lim_{x\to 0} \frac{\sqrt{x^2+5}-\sqrt{5}}{\sqrt{\sin^2 x+20}-\sqrt{20}}$
   iv. $\lim_{x\to 0} \frac{e^x-1}{x}$

   (b) Let $f(x) = x^3 \sin \frac{1}{\sqrt[3]{x^2+2}}$. Prove that,

   $$\lim_{x\to 0} f(x) = 0$$

2. (a) Differentiate with respect to x

   $$y = \cos^{-1}\left(\frac{2\cos x + 1}{\cos x + 2}\right)$$

   (b) If $y = e^{3x}\cos 2x$ then prove that,

   $$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$$

   show that,

   $$\left(\frac{d^2y}{dx^2}\right)_{x=0} = 5$$

   (c) Show that the equation of normal at any point on the curve,

   $$x = 3\cos\theta - \cos^3\theta$$
   $$y = 3\sin\theta - \sin^3\theta$$

   is $4\left(y\cos^3\theta - x\sin^3\theta\right) = 3\sin(4\theta)$.

3. (a) Sketch the graph of,

$$y = \frac{9\left(x^2 + 2x - 4\right)}{(x+2)^3}; (x \neq -2)$$

(b) Water is dripping out at a steady rate of $1cm^3$ per second $(1cm^3s^{-1})$ through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4cm, find the rate of decrease of slant height, where the vertical angle of the conical vessel is $\left(\frac{\pi}{6}\right)$.

4. (a) By a suitable substitution evaluate the following integral

$$I = \int \sqrt{\cos x} \quad \sin^5 x \quad dx$$

(b) Using integration by parts obtain,

$$I = \int \frac{xe^x}{(1+x)^2} \quad dx$$

(c) Evaluate the following definite integral

$$I = \int_0^{\frac{\pi}{4}} \ln|1 + \tan\theta| \quad d\theta$$

5. (a) Find the area of the region enclosed by the following curves, $y = x^2$, $\quad y = x + 6$, $\quad x = 0$ and $x = 5$

(b) i. The curve $y = \sqrt{x}$ with $0 < x < 4$ is rotated about the x axis. Find the volume of the revolution.

ii. Determine the volume of the solid obtained by rotating the region bounded by $y = 2x^2$ and $y = x^3$ about the x-axis