Pricing European Call Options on Stocks using the Black-Scholes Equation

By: Kasvina Tirumal

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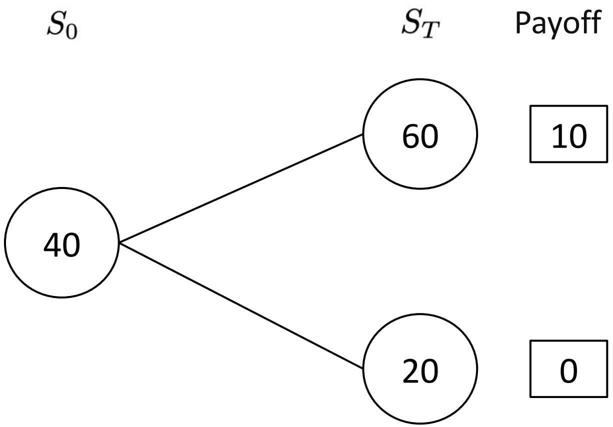
European Call Options

- Gives the buyer **the right to buy** a security at strike price K, and can only be exercised on a specified expiration date, T.
- Underlying security can be any form of financial asset - stocks and bonds.
- Payoff of a European call option:

$$\max(\boxed{S_T} - K, 0)$$

Security price at time of expiration T

Example: Consider call option whose underlying stock has initial price $40, and strike price $50.



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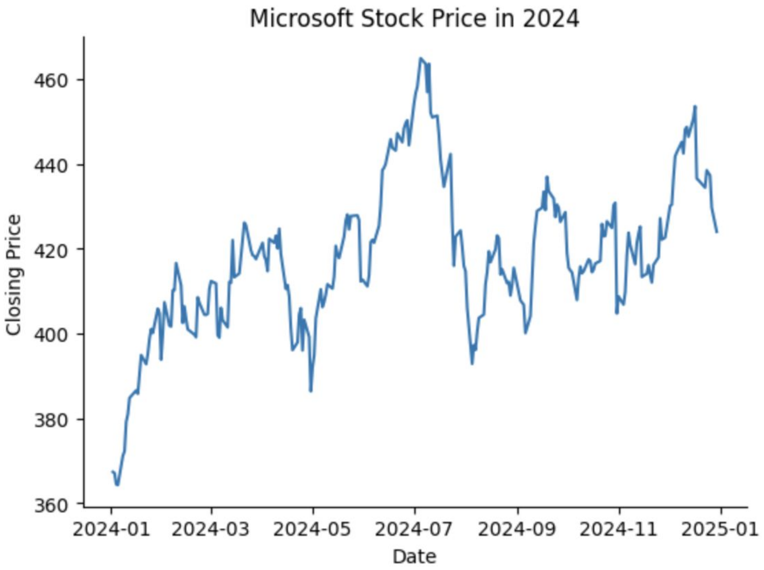
Stocks

- A security that represents ownership within the issuing corporation.
- Prices fluctuates daily.
- Stock price movement can be modeled as a **stochastic process** known as Geometric Brownian Motion (GBM).



Geometric Brownian Motion

- A stochastic process is said to follow a Geometric Brownian Motion (GBM), if it satisfies the following stochastic differential equation:

$$d\boxed{S_t} = \boxed{\mu} S_t\, dt + \boxed{\sigma} S_t\, d\boxed{W_t}$$

Brownian motion

Stock price process

Percentage drift

Percentage volatility

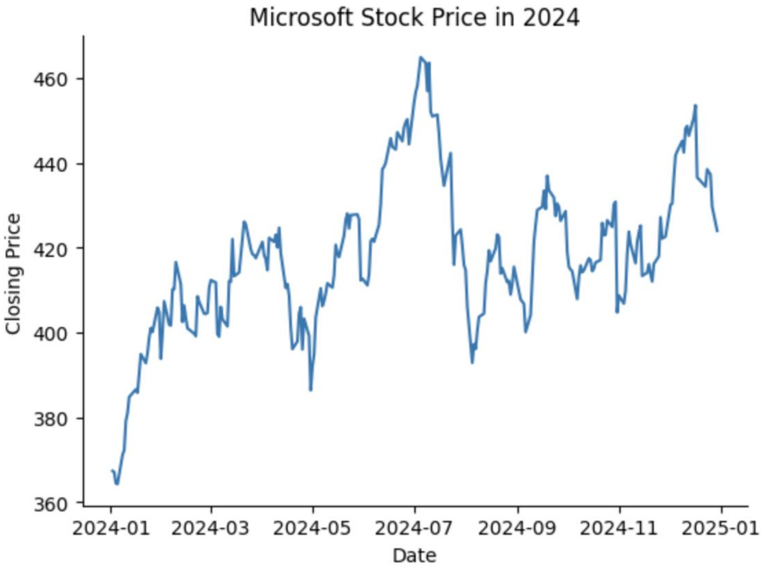
- We can transform the stock price process using the natural logarithm, $Z = \ln(S_t)$, and using **Itô's Lemma**, we can rewrite the differential equation as follows:

$$dZ_t = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t$$

- This implies that dZ_t follows the following normal distribution:

$$Z_{t+\Delta t} - Z_t \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t, \sigma^2 \Delta t\right)$$

Geometric Brownian Motion



$$Z_{t+\triangle t} - Z_t \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)\triangle t, \sigma^2 \triangle t\right)$$

- Using the above general form, we can model daily stock price movements in a given trading year as follows:

$$Z_{\frac{i+1}{251}} - Z_{\frac{i}{251}} \sim N\left(\mu - \frac{1}{2}\left(\frac{\sigma^2}{251}\right), \frac{\sigma^2}{251}\right)$$

- As a result, we can estimate the drift (expected return) and historical volatility of any stock within a given time period.

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Black-Scholes Model

- Models that dynamics of a financial market containing financial derivatives (eg. options, contracts).

$$dB(t) = rB(t)dt,$$
$$dS(t) = \alpha S(t)dt + \sigma S(t)d\bar{W}(t)$$

$$\boxed{\max(S_T - K, 0)}$$

- Given the final payoff of an option, we can solve this system of partial differential equations, to obtain the **Black-Scholes Equation**.

Black-Scholes Equation

- Theoretical estimate of the price of European call options.
- Shows that European call options have a unique price given the risk of the underlying security and its expected return.
- The price of a European Call Option, F, with strike price K and time of maturity T is given by the formula:

Cumulative distribution function for N[0,1]

$$F = S \cdot \boxed{N[d_1(t,s)]} - e^{r(T-t)} \cdot K \cdot N[d_2(t,s)]$$

$$d_1(t,s) = \frac{1}{\boxed{\sigma} \cdot \sqrt{T-t}} \cdot \left[ln\left(\frac{S}{K}\right) + \left(\boxed{r} + \frac{1}{2}\sigma^2\right) \cdot \boxed{(T-t)} \right]$$

Volatility

Interest rate

Time to expiration

$$d_2(t,s) = d_1(t,s) - \sigma \cdot \sqrt{T-t}$$

Let's get pricing!

Methodology

1. **Import** call option data for stocks of the top five S&P 500 companies from Yahoo Finance, which includes: Apple, Nvidia, Microsoft, Amazon, and Meta.
2. Estimate **historical volatility** of each stock using 2024 data and the following distribution:

$$Z_{\frac{i+1}{251}} - Z_{\frac{i}{251}} \sim N\left(\mu - \frac{1}{2}\left(\frac{\sigma^2}{251}\right), \frac{\sigma^2}{251}\right)$$

3. Use the computed historical volatility as the **volatility input**.
4. Choose the current return rate of a 6-month treasury bond as the **interest rate**.
5. **Estimate call option prices** using the Black-Scholes equation.
6. **Compare** the market price to estimated price.

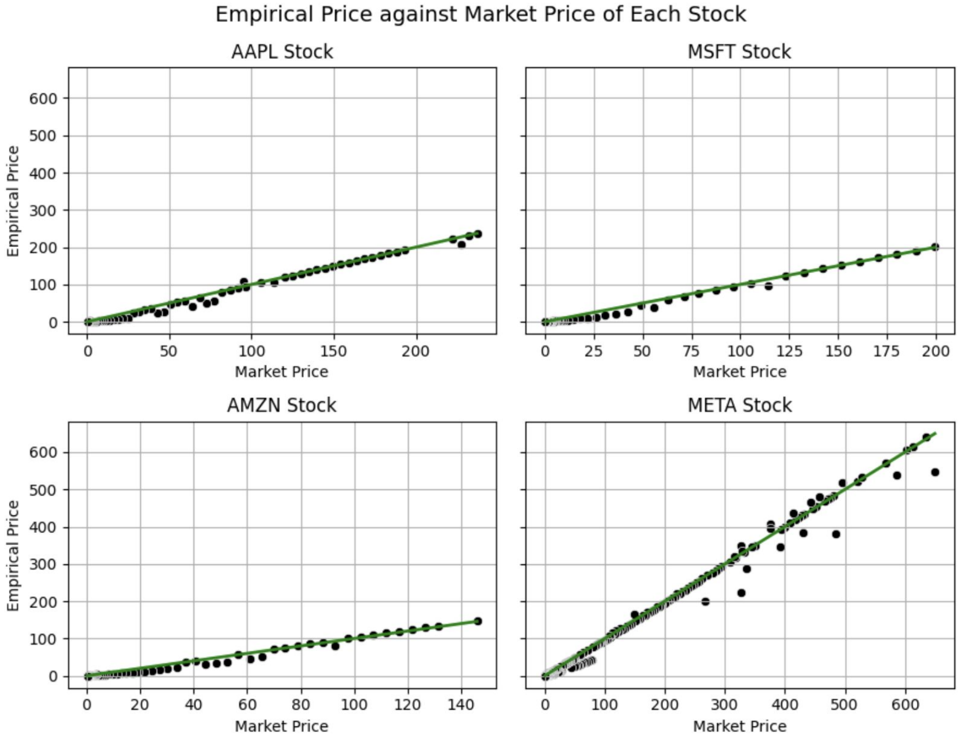
Results

r = 0.0434

Estimated Price

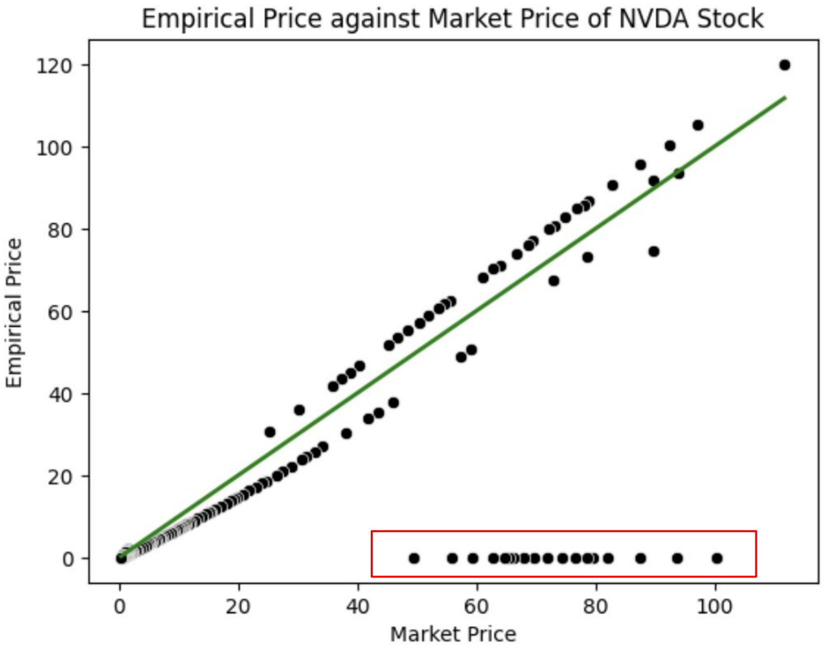
	Stock_Abbv	Strike	Bid	Ask	Implied Volatility	Trade Date	Months to Exp	Market Price	Stock Price	Historical Volatility	Empirical Price
411	META	535	161.65	163.70	0.4904	2025-02-12	7	162.675	668.200012	0.356621	162.253682
131	NVDA	135	11.55	11.65	0.5392	2025-03-03	6	11.600	108.760002	0.519978	8.252041
310	AMZN	205	24.35	24.60	0.3849	2025-03-03	6	24.475	196.589996	0.280979	13.771835
19	AAPL	125	119.80	120.65	0.7022	2025-02-12	7	120.225	241.574387	0.223086	119.666438
201	NVDA	2170	72.85	76.05	4.3241	2024-06-06	15	74.450	123.510773	0.519978	0.000035
341	AMZN	360	0.27	0.30	0.3333	2025-03-03	6	0.285	196.589996	0.280979	0.024821
37	AAPL	220	34.85	35.05	0.3406	2025-02-28	6	34.950	241.574387	0.223086	31.129842
139	NVDA	144	9.00	9.15	0.5311	2025-03-03	6	9.075	108.760002	0.519978	6.388282
109	NVDA	98	28.75	28.95	0.6006	2025-03-03	6	28.850	108.760002	0.519978	22.200111
330	AMZN	305	1.26	1.30	0.3200	2025-02-27	6	1.280	212.279999	0.280979	0.876350

Results

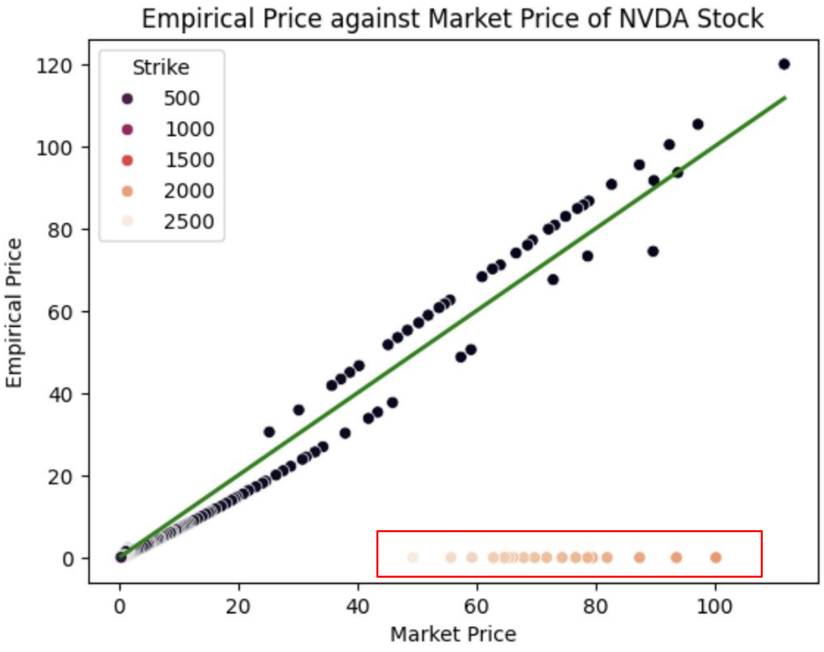


Empirical Price against Market Price of Each Stock
AAPL Stock
MSFT Stock
AMZN Stock
META Stock
Empirical Price
Market Price

Results



Empirical Price against Market Price of NVDA Stock
Empirical Price
Market Price
0
20
40
60
80
100
120

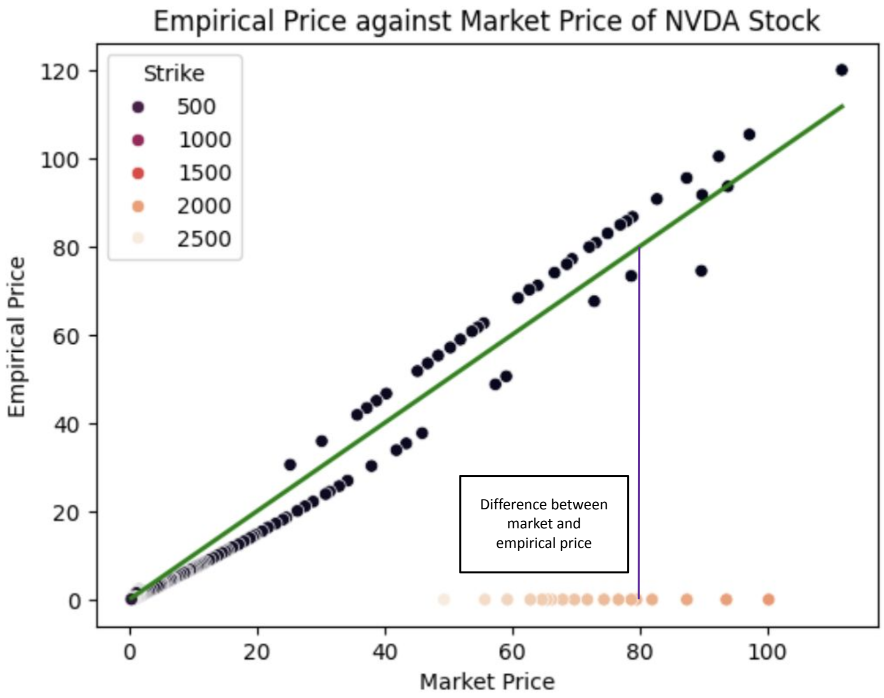


Empirical Price against Market Price of NVDA Stock
Strike
500
1000
1500
2000
2500
Empirical Price
Market Price

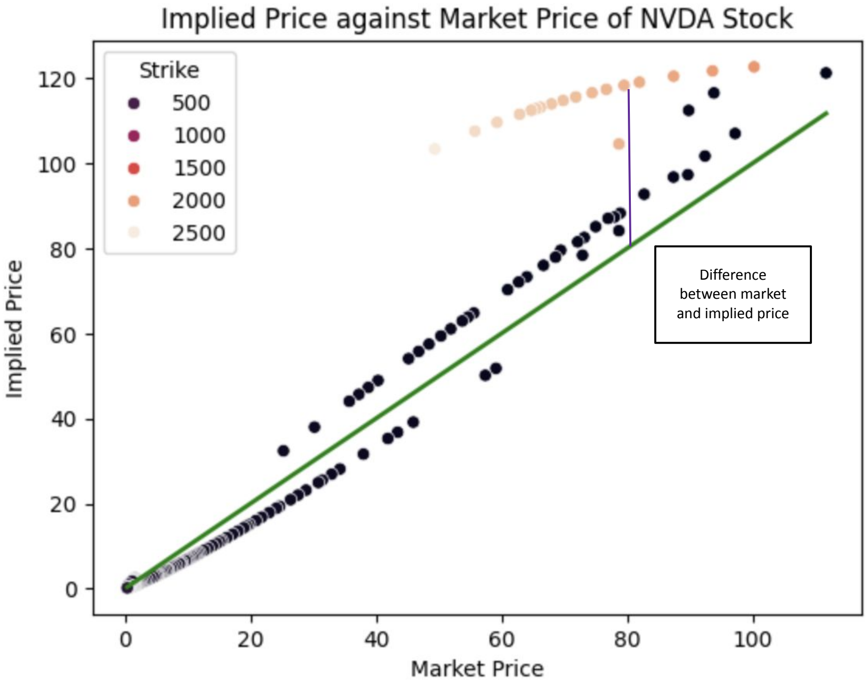
Methodology cont.

- Use the implied volatility provided in the dataset to estimate call option prices using the Black-Scholes equation.
- Compare market price to estimated price using implied volatility.

Results



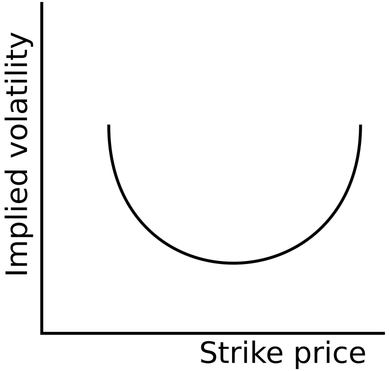
Empirical Price against Market Price of NVDA Stock
Strike
500
1000
1500
2000
2500
Difference between market and empirical price
Empirical Price
Market Price
0
20
40
60
80
100
120



Implied Price against Market Price of NVDA Stock
Strike
500
1000
1500
2000
2500
Difference between market and implied price
Implied Price
Market Price
0
20
40
60
80
100
120

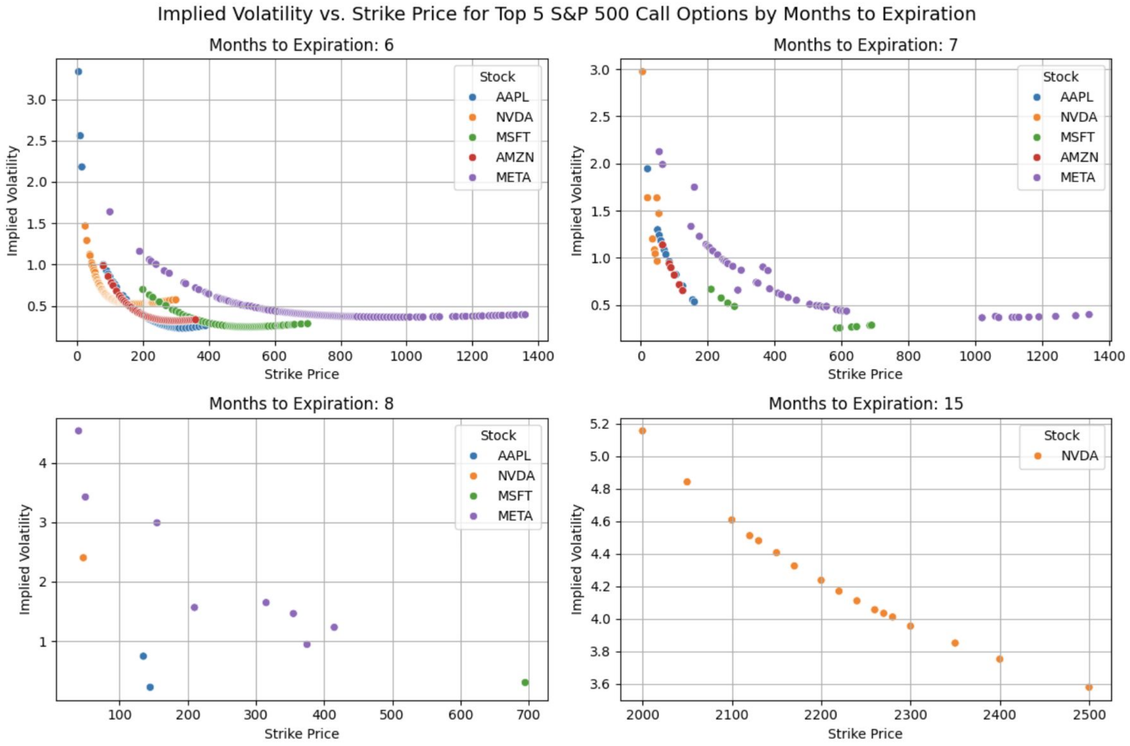
Volatility Smile

- Patterns in implied volatility that arise when pricing financial options.
- Theoretically, the curve takes the following form:



- I was curious whether this pattern can be observed in my dataset.

Results



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Thank you for your time!