ELSEVIER

European Journal of Operational Research 108 (1998) 165–169

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

Theory and Methodology

Aggregating individual judgments and priorities with the Analytic Hierarchy Process

Ernest Forman [a,*], Kirti Peniwati [b,1]

[a] *Management Science Department, George Washington University, Washington, DC 20052, USA*

[b] *Operations Management and Decision Sciences, Institute for Management Education and Development, Jakarta, Indonesia*

Received 3 October 1995; accepted 6 April 1997

Abstract

The Analytic Hierarchy Process (AHP) is often used in group settings where group members either engage in discussion to achieve a consensus or express their own preferences. Individual judgments can be aggregated in different ways. Two of the methods that have been found to be most useful are the aggregation of individual judgments (AIJ) and the aggregation of individual priorities (AIP). We propose that the choice of method depends on whether the group is assumed to act together as a unit or as separate individuals and explain why AIJ is appropriate for the former while AIP is appropriate for the latter. We also address the relationships between the choice of method, the applicability of the Pareto principle, and the use of arithmetic or geometric means in aggregation. Finally, we discuss Ramanathan and Ganesh's method to derive priorities for individual decision-makers that can be used when aggregate group preferences of individuals whose judgments are not all equally weighted. We conclude that while this method can be useful, it is applicable only in special circumstances. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Analytic Hierarchy Process; Aggregating individual judgments; Aggregating individual priorities; Geometric mean

1. Introduction

The Analytic Hierarchy Process (AHP) of Saaty (1980) is one of the most popular and powerful techniques for decision making in use today. AHP is generally used to derive priorities based on sets of pairwise comparisons. The AHP is built on a human being's intrinsic ability to structure his perceptions or his ideas hierarchically, compare pairs of similar things against a given criterion or a common property, and judge the intensity of the importance of one thing over the other. The AHP then synthesizes all the judgments, using the framework given by the hierarchy, to obtain the overall priority of the elements. There are several possible ways to aggregate information when more than one (perhaps many) individuals participate in a decision process,

* Corresponding author. Fax: +1 202 994 4930; e-mail: forman@gwis2.circ.gwu.edu.

[1] E-mail: LEMBAGA.PPM36@graha.Sprint.com.

0377-2217/98/$19.00 © 1998 Elsevier Science B.V. All rights reserved.
PII S0377-2217(97)00244-0

including: (1) aggregating the individual judgments for each set of pairwise comparisons into an 'aggregate hierarchy'; (2) synthesizing each of the individual's hierarchies and aggregating the resulting priorities; and (3) aggregating the individual's derived priorities in each node in the hierarchy. In any case, the relative importance of the decision-makers may either be assumed to be equal, or else incorporated in the aggregation process. We will focus on the first two of these methods here and refer to them as aggregating individual judgments (AIJ) and aggregating individual priorities (AIP). While technically the AHP can deal with the third method, it is less meaningful and not commonly used.

Three fundamental questions need to be addressed interdependently to meaningfully obtain group preference from individual information with the AHP. First, whether the group is assumed to be a synergistic unit or simply a collection of individuals. The response to this question determines whether to use the AIJ or AIP method. Second, what mathematical procedure should be used to aggregate individual judgments? The answer is dependent on the response to the first question. Third, if the individuals are not weighted equally, how to obtain their weights and how to incorporate them in the aggregation process.

Both the geometric mean and the arithmetic mean are appropriate procedures for ratio scales. Ramanathan and Ganesh (1994) observe that employing AIJ violates the Pareto principle of social choice theory. Insisting that the principle should apply, they have proposed that a weighted AIP be used instead. We view AIJ and AIP as two circumstances that are philosophically different. We consider when the Pareto principle is relevant and under what circumstances AIJ or AIP should be used. We also provide reasons for using the geometric rather than the arithmetic mean.

2. The group: A new 'individual' or a collection of independent individuals

There are two basic ways to aggregate individual preferences into a group preference, depending on whether the group wants to act together as a unit or as separate individuals. An example of the former is a group of department heads meeting to set corporate policy. An example of the latter is a group consisting of representative constituencies with stakes in welfare reform, such as taxpayers, those on welfare, politicians, etc. One needs to decide which situation is applicable in order to determine the proper procedure for AIP. Ramanathan and Ganesh do not distinguish between these situations, considering only the latter.

3. Aggregating individual judgments (AIJ)

When individuals are willing to, or must relinquish their own preferences (values, objectives) for the good of the organization, they act in concert and pool their judgments in such a way that the group becomes a new 'individual' and behaves like one. There is a synergistic aggregation of individual judgments. Individual identities are lost with every stage of aggregation and a synthesis of the hierarchy produces the group's priorities. Because we are not concerned with individual priorities, and because each individual may not even make judgments for every cluster of the hierarchy, there is no synthesis for each individual – individual priorities are irrelevant or non-existent. Thus, the Pareto principle, which Ramanathan and Ganesh claim is violated, is irrelevant. Furthermore, since the group becomes a new 'individual' and behaves like one, the reciprocity requirement for the judgments must be satisfied and the geometric mean rather than an arithmetic mean must be used as will be shown below. Note that while individual identities are lost when synthesizing the hierarchy, they are maintained for each cluster of elements where an individual provides judgments. Inconsistencies in an individual's set of judgments can be examined and the group can ask an individual to consider revising one or more judgments if the inconsistency is deemed to be too high. The group could also decide to exclude an individual's judgments from the geometric average for a cluster, based on the inconsistency ratio.

4. Aggregating individual priorities

When individuals are each acting in his or her own right, with different value systems, we are concerned about each individual's resulting alternative priorities. An aggregation of each individual's resulting priorities can be computed using either a geometric or arithmetic mean. Neither method will violate the Pareto principle (as will be shown below). Ramanathan and Ganesh erroneously conclude that the arithmetic mean must be used.

5. The Pareto principle and AHP

The Pareto (unanimity, agreement) principle essentially says that given two alternatives A and B, if each member of a group of individuals prefers A to B, then the group must prefer A to B. The principle has been formulated and applied in the social sciences in the AIP context described above. The AIJ approach on the other hand, is a synergistic aggregation of individual judgments when individuals are willing to, or must out of necessity, relinquish their own preferences for the good of the organization. This new perception of a group of individuals was not possible with ordinal measurement methods prior to the AHP. With AIJ the individuals first work together to agree on a common hierarchy before they can work on aggregating their judgments. The agreement on a common hierarchy is the first step in 'merging' the different individuals into a new 'individual' representing the group. The next 'merging' process occurs at the judgment level. Even at this level, the 'merging process' occurs step by step from the most general at the higher level to the more specific at the lower level of the hierarchy. In other words, after agreeing to the general structure of the hierarchy, the group 'merges' further by agreeing on the relative importance of the criteria. Once this process is done, the previous individual judgments with respect to the relative importance of the criteria become irrelevant, the same way their original hierarchies do as soon as a common hierarchy is agreed upon. Consequently there is no synthesis for each individual and the Pareto principle is inapplicable.

6. Geometric mean or arithmetic mean with AIJ and AIP

In general one must decide in advance whether to represent a group by aggregating their individual judgments (AIJ) or by aggregating their individual (final) priorities (AIP), but not both. Treating the group as a new 'individual' with AIJ requires satisfaction of the reciprocity condition for the judgments. Aczel and Saaty (1983), and in a more general paper Aczel and Roberts (1989), have shown that when aggregating the judgments of n individuals where the reciprocal property is assumed even for a single n-tuple, only the geometric mean satisfies the Pareto principle (unanimity condition) and the homogeneity condition (if all individuals judge a ratio t times as large as another ratio, then the synthesized judgment should also be t times as large). Thus, for AIJ, the geometric mean *must* be used. As for AIP, either an arithmetic or geometric mean can be used to aggregate the individuals' priorities. Most people were taught, and have grown up to feel comfortable with the arithmetic mean, or what is commonly referred to as the mean or average. In general, if one wishes to take an 'average' of measurements possessing only interval scale meaning, an arithmetic average must be used since it is meaningless to multiply interval scale numbers. However, for ratio scale measurements (as we always have with AHP) both arithmetic and geometric averages are meaningful. The aggregation of individual priorities will satisfy the Pareto principle with *either* an arithmetic or geometric average:

If $a_i \geqslant b_i$, $i = 1, 2, \ldots, n$ then $\sum_{i=1}^{n} a_i/n \geqslant \sum_{i=1}^{n} b_i/n$ for an arithmetic mean, and

$$\sqrt[n]{\prod_{i=1}^{n} a_i} \geqslant \sqrt[n]{\prod_{i=1}^{n} b_i} \quad \text{for a geometric mean}$$

provided $a_i \geqslant 0$ and $b_i \geqslant 0$, $i = 1, 2, \ldots, n$.

James and James (1968) provide definitions of arithmetic and geometric means that furnish additional insight into the choice of using an arithmetic or geometric mean for the AIP case:

> The arithmetic average of two numbers is the middle term in an arithmetic progression of three terms including the two given numbers.

Thus, for example, the arithmetic mean of 1 and 9 is 5 since the arithmetic progression from 1 to 9 occurs in two equal *intervals* of 4.

> The geometric average of two numbers is the middle term in a geometric progression of three terms including the two given numbers.

Thus, for example, the geometric mean of 1 and 9 is 3, since the geometric progression from 1 to 9 occurs in two equal *ratios* of 3.

While either an arithmetic or geometric mean can be used for AIP, the geometric mean is more consistent with the meaning of both judgments and priorities in AHP. In particular, judgments in AHP represent ratios of how many times more important (preferable) one factor is than another. Synthesized alternative priorities in AHP are *ratio* scale measures and have meaning such that the ratio of two alternatives' priorities represents how many times more preferable one alternative is than the other.

7. Weighted arithmetic and geometric means

When calculating the geometric average of the judgments (AIJ) or either the arithmetic or geometric average of priorities (AIP) we often assume that the individuals are of equal importance. If, however, group members are not equally important, we can form a weighted geometric mean or weighted arithmetic mean as follows:

Weighted geometric mean of judgments (AIJ):

$$J_g(k,l) = \prod_{i=1}^{n} J_i(k,l)^{w_i},$$

where: $J_g(k,l)$ refers to the group judgement of the relative importance of factors k and l, $J_i(k,l)$ refers to individual i's judgment of the relative importance of factors k and l, w_i is the weight of individual i; $\sum_{i=1}^{n} w_i = 1$; and n the number of decision-makers.

Weighted (Un-normalized) geometric mean of priorities (AIP):

$$P_g(A_j) = \prod_{i=1}^{n} P_i(A_j)^{w_i},$$

where $P_g(A_j)$ refers to the group priority of alternative j, $P_i(A_j)$ to individual i's priority of alternative j, w_i is the weight of individual i; $\sum_{i=1}^{n} w_i = 1$; and n the number of decision-makers.

Weighted arithmetic mean of priorities (AIP):

$$P_g(A_j) = \sum_{i=1}^{n} w_i P_i(A_j),$$

where $P_g(A_j)$ refers to the group priority of alternative j, $P_i(A_j)$ to individual i's priority of alternative j, w_i is the weight of individual i; $\sum_{i=1}^{n} w_i = 1$; and n is the number of decision-makers.

The question arises as to how to compute the w_i's. Saaty (1994) suggests forming a hierarchy of factors such as expertise, experience, previous performance, persuasive abilities, effort on the problem, etc. to determine the priorities of the decision-makers. But who is to provide judgments for this hierarchy? If it *cannot* be agreed that one person (a supra decision-maker) will provide the judgments, it is possible to ask the same decision-makers who provided judgments for the original hierarchy to provide judgments for this hierarchy as well. If so, we have a meta-problem of how to weight their individual judgments or priorities in the aggregation process to determine the weights for the decision-makers to apply to the aggregation of the original hierarchy. One possibility is to assume equal weights. Ramanathan and Ganesh provide another method, which they call the eigenvector method of weight derivation. They reason that, if $\overline{w} = (w_1, w_2, \ldots, w_n)^t$ is the 'true' (but unknown) weight priority vector for the individual's weights, and if the individual weight priority vectors derived from the judgments from each of the individuals are arranged in a matrix: $\overline{M} = (\overline{m}_1, \overline{m}_2, \ldots, \overline{m}_n)$, then we can aggregate to find the priorities of the n individuals, $\bar{x}$, where $\bar{x} = \overline{M} * \bar{w}$. Then Ramanathan and Ganesh reason that $\bar{x} = \bar{w}$, resulting in the eigenvector equation: $\bar{w} = \overline{M} * \bar{w}$. We observe that this method is attractive but reasonable *only if* the weights for obtaining priorities of the decision-makers are assumed to be the same as the weights to be used to aggregate the decision-makers' judgments/priorities for obtaining the alternative priorities in the original hierarchy. In general, this need not be the case.

8. Summary and conclusions

When several individuals provide judgments with the Analytic Hierarchy Process, one may AIJ or AIP. The choice of method depends on whether the group is assumed to act together as a unit or as separate individuals. In the former case, the geometric average of individual judgments (AIJ) satisfies the reciprocity requirement, implying a synergistic aggregation of individual preferences in such a way that the group becomes a new 'individual' and behaves like one. Individual identities are lost with every stage of aggregation and the Pareto principle is irrelevant. When group members act as individuals (AIP), one may take either a geometric mean (representing an average ratio) or an arithmetic mean (representing an average interval) of their resulting priorities. While the Pareto principle will not be violated in either case, the geometric mean is more consistent with the meaning of both judgments and priorities in AHP.

If the group members are not considered to be of equal importance, a weighted geometric mean can be used with AIJ or weighted geometric or arithmetic mean with AIP. A separate hierarchy can be constructed to derive priorities of the decision-makers. There is great flexibility in determining who makes the judgments for this hierarchy. When the original group members themselves make these judgments, Ramanathan and Ganesh's eigenvector method can be used *provided* the relative importance of the decision-makers in aggregating to obtain decision-maker priorities are assumed to be the same as the priority of the decision-makers in aggregating the priorities of the hierarchy of the original problem.

References

Aczel, J., Roberts, F.S., 1989. On the Possible Merging Functions, Mathematical Social Sciences 17, 205–243.

Aczel, J., Saaty, T.L., 1983. Procedures for Synthesizing Ratio Judgments. Journal of Mathematical Psychology 27, 93–102.

James, J., 1968. Mathematics Dictionary, 3rd ed. D, Van Nostrand Co., Princeton, NJ.

Ramanathan, R., Ganesh, L.S., 1994. Group Preference Aggregation Methods Employed in AHP: An Evaluation and Intrinsic Process for Deriving Members' Weightages. European Journal of Operational Research 79, 249–265.

Saaty, T.L., 1980. The Analytic Hierarchy Process, McGraw-Hill Book Co., NY.

Saaty, T.L., 1994. Fundamentals of Decision Making and Priority Theory with The Analytic Hierarchy Process, RWS Publications, Pittsburgh PA, 204–220.

Stevens, S.S., 1946. On the theory of scales of measurement. Science 103, 677–680.

Stevens, S.S., 1968. Measurement, statistics and the schemapiric view. Science 161 (3844) 849–856.