CS 1332 Lecture Notes

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Contents

1. Arrays

Arrays allow you to store data in *contiguous space in memory*

Note 1.1

Pros:
- Arrays are flexible in what they can store (primitives, reference types, etc)
- *Constant time access* when the index is known
  - Accessing when index is not known (searching) -> O(n)

Cons:
- If you run out of space you need to resize the array which is O(n)

2. ArrayLists

ArrayLists are backed by arrays, and are **contiguous**, which means you cannot have null spaces between data elements. This causes us to need to shift data to fill up the null spaces after remove operations.

2.1. ArrayList Big O

Theorem 2.1.1

Adding

Adding to Front: **O(n)** -> need to shift elements over to make space to add

Adding to Back: **amortized O(1)***. There is no need to shift but its amortized because every n operations, you need to perform an O(n) operation by resizing

- Amortized: when an "expensive" operation occurs infrequently so we can "average" it over the runtimes

Removing

Removing from the Front: O(n) -> must shift elements to fill the empty space

Removing from the back: O(1) -> simple set to null

Adding and Removing at a Given Index: O(n) -> shift data around the index

Accessing at a given index: O(1) -> arraylist backed by array

2.2. Pros and Cons

Note 2.2.1

Pros
- Data elements are stored contiguously
- **Dynamic Memory** - even though we resize the backing array behind the scenes, we consider ArrayLists to be dynamic

Note 2.2.2

Cons
- Cannot store primitives
- Still needs O(n) operations for resizing

3. LinkedList

3.1. Singly Linked List

3.1.1. SLL Big O/Methods

Theorem 3.1.1.1

Adding

Adding to Front: O(1)

- Create new node, point next to head, and set the new node to be the new head
  - If list is empty, the head is null which actually works out anyways without edge case (?)

Adding to Back O(n)

- Need to traverse to last node by iterating until curr.next is null (since we need access to last node). Set last mode w/data's next value to the new node.
- If head is null, point head to new node

Theorem 3.1.1.2

Removing

Removing from Front: O(1)

- Save data from head node, then set head = head.next

Removing from Back: O(n)

- Need to traverse until curr.next.next is null, then set curr.next to null
- If size is zero, throw exception
- If size is 1, set head to null

3.2. Tail Pointer

Having a **tail pointer** makes *adding to back easier*, since you can just set the tails next reference to the new node and update tail. So adding to back is now **O(1)**

3.3. Doubly Linked List

Generally doubly linked lists always have both a head and tail pointer, and contain a reference to previous node.

Note 3.3.1

For a DLL of size 0, both the head and tail point to null. For a DLL of size 1, both the head and tail point to the same node.

3.3.1. DLL Big O/Methods

Theorem 3.3.1.1

Adding

Adding to the Front: O(1)

- Set the new nodes next to head, and set the head's previous to new node. Then set head to the new node.
- When size = 0, set head and tail to new node

Adding to the Back: O(1)

- Set the tail's next to the new node, and the new nodes previous to the tail. Then set tail to new node.
- When size = 0, set head and tail to new nodes

Theorem 3.3.1.2

Removing

Removing from the Back: O(1)

- Set tail to tail's previous, then set tail next to null.
- When size = 0, set head and tail to null

Removing from the Front: O(1)

- Set head to head's next, then set head.previous to null.
- Size = 0 -> exception
- When size = 1, set head and tail to null.

Having **doubly linked lists** makes *removing from back easier*, since to remove you need to go to the node before the last one, and you need to reset tail. So you can set the second to last node.next to null and reset the tail to the second to last node. So it becomes **O(1)**

3.4. Circularly Singly Linked List

The last node in the list points back to the head

Note 3.4.1

For CSLL, we can't use curr == null to check if we've reached the end of the list. Instead, we must use curr == head to terminate the loop

3.4.1. CSLL Big O/Methods

Theorem 3.4.1.1

Adding

Adding to the Front: O(1)

- Create a new, empty node. Connect the new node's next to head's next. Set head's next to the new node. Put the data from head into the new node. Put the data we want to add into the head node.

Adding to the Back: O(1)

- Same steps as add to front, but now set head = head.next

Removing

In general, removing cannot be optimized to be O(1) unless removing from front/edge cases

Removing from Front: O(1)

- Save data from head to return
- Copy data from head's next into head
- Set head's next to head.next.next
- If size = 1, just set to null

Removing from Back: O(n)

- Need to iterate to the end of the array and set the 2nd to last node to point to head (?)

4. Stacks

Definition 4.1

A **stack** is a last in, first out (LIFO) linear data structure, meaning that additions and removals happen on the same side of the structure.

The main operations for stacks include:
- **push(data)** - adds the data to the "top" of the stack
- **pop()** - removes the data at the top of the stack and returns it
- **peek()** - returns data for the top of the list without removing

4.1. SLL-Based Stack

- Does not need a tail pointer

Note 4.1.1

An SLL based stack uses the *front of the SLL as the top of the stack*. Thus, push simply becomes addToFront and pop becomes removeFromFront, both of which are **O(1) operations**

4.2. Array-Based Stack

- Requires a size variable along with the array

Note 4.2.1

In this case, the top of the stack is the back of the array. So we push by adding data to **arr[size]** and pop by removing the value at **arr[size-1]**, both of which are **O(1)** operations.

5. Queues

Definition 5.1

A **queue** is a first in, first out abstract data type. Thus, queue and dequeue operations occur at *opposite* ends of the structure

The main operations for queues include:
- **enqueue(data)** - adds data to the "back" of the queue
- **dequeue()** - removes the data from the front of the queue
- **peek** - returns the data at the front without removing it

5.1. SLL Backed Queue

Note 5.1.1

The SLL-backed queue requires a *tail pointer* in order to get O(1) operations.

The "front" of the queue is the front of the list where data is dequeued from, while the "back" of the queue is the back of the list where data is enqueued

enqueue(data) -> **addToBack(data)**, and **dequeue()** -> **removeFromFront()**

5.2. Array Backed Queue

Note 5.2.1

Array backed queues require a size variable but also a front variable, because *the array behaves circularly.*

arr[front] is the front of the queue, and **arr[(front + size) % arr.length]** is the first empty index at the "back"

For **enqueue**
- Put the element at arr[(front+size) % arr.length] then size++

For **dequeue**
- Remove the element at arr[front], increment front and decrement size
- In this case, front = (front + 1) % arr.length when you increment so that *front never goes out of bounds*

5.3. Dequeue

Note 5.3.1

In Deques (double ended queues), we can add and remove from either side of the deque

The main operations include:
- **addFirst(data)**
- **addLast(data)**
- **removeFirst()**
- **removeLast()**

5.3.1. DLL Backed Queue

Note 5.3.1.1

The DLL backed queue requires a tail.

addFirst(data) -> **addToFront(data)**: O(1)

addLast(data) -> **addToBack(data)**: O(1)

removeFirst() -> **removeFromFront()**: O(1)

removeLast() -> **removeFromBack()**: O(1)

5.3.2. Array Backed Deque

Note 5.3.2.1

Uses a front variable and a size variably (*circular again*)

Important Indices
- arr[(front - 1) % capacity] = addFirst()
- arr[front] = removeFirst()
- arr[(front + size) % capacity] = addLast()
- arr[(front + size - 1) % capacity] = removeLast()

6. Trees

Trees are the first **non linear** data structure that we look at.

Note 6.1

Characteristics of Trees

1. No cycles, so you can't reach a node using itself
2. Highly recursive
3. Usually implemented with linked lists rather than arrays (arrays get messy)

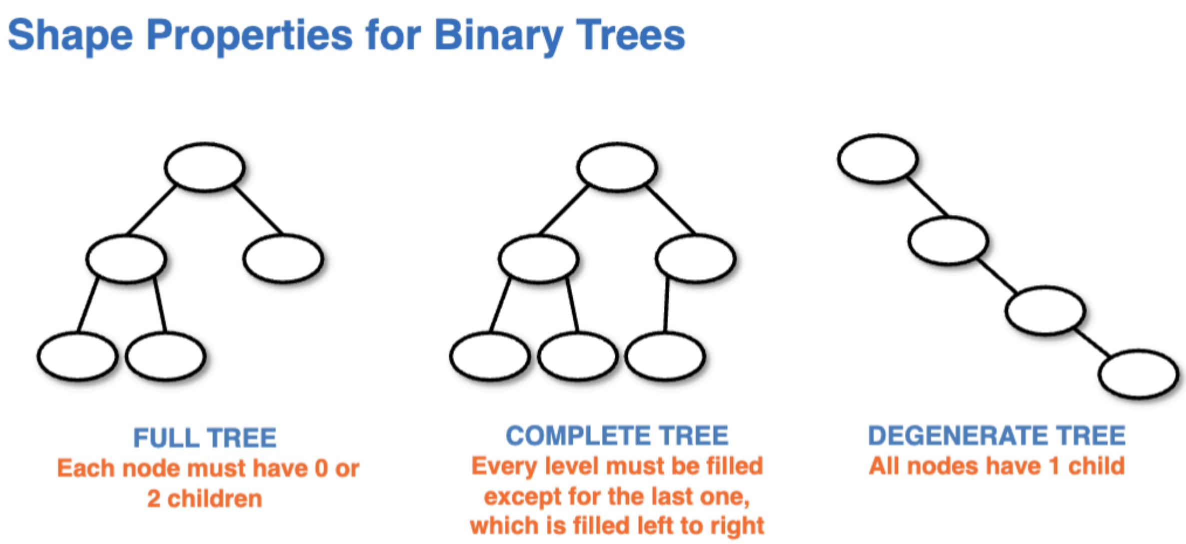
A tree is **full** if each node has exactly 0 or 2 kids, it cannot have just 1.

A tree is **complete** if each level except the last has the maximum number of nodes (2^n w/ root has n = 0)

- The last level must be filled left to right, but doesn't need to be full

A node is **balanced** if its children heights differ by only 0 or 1 (height of missing children = −1). Thus, a tree is **balanced** if all of its nodes are balanced.

- Leaf nodes are always balanced



Theorem 6.1

Important Terminology

Root: the head/entry point of the tree

Children: what the nodes point towards (can have multiple children, each node can only have one parent though)

External/Leaf Nodes: nodes without children

Internal Nodes: nodes with children

Definition 6.1

The **depth** of a node is the distance of it from the root, or how many nodes away it is(?)

The **height** of a node is the distance of a node from the furthest leaf node

- $\text{height} = 1 + \max(\text{height}(\text{children}))$

6.1. Binary Trees

Note 6.1.1

Characteristics of Binary Trees

- Shape: each node can have at most 2 children
- Children are labeled left and right (left precedes right)

Theorem 6.1.1

Iterating through a binary tree

```
public void traverse(Node node) {
  if (node != null) {
    traverse(node.left);
    traverse(node.right);
  }
}
```

6.2. Binary Search Trees

Definition 6.2.1

Binary Search Trees are Binary Trees with the given rule: any child node to its left must be less than that node, and any child note to the right must be greater than it. This applies for all children nodes, not just direct children.

The motivation behind BST's are the **binary search algorithm**, because we are able to split the search space in half each operation, making searching **O(logn)**.

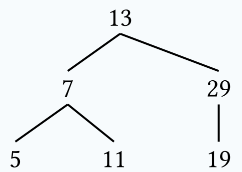
6.2.1. Pre Order Traversal

Definition 6.2.1.1

In **preorder traversal**, we look at each node and then traverse left, then right. If you draw a "glove" around the tree, you would print a node every time you touch the left side of the node

```
preorder (Node node):
  if node is not null:
    look at the data in node
    recurse left
    recurse right
```

Example 6.2.1.1



In this case, our traversal would print *13 7 5 11 29 19*

6.2.2. Post Order Traversal

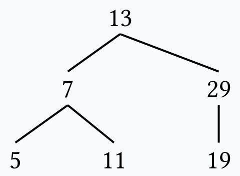
```
ppreorder (Node node):
  if node is not null:
    recurse left
```

```
    recurse right
    look at the data in node
```

Definition 6.2.2.1

In **postorder traversal** we first recurse left and right before looking at the data in the node. So if you wrap around the tree like a glove, every time you reach the right side of a node, you print.

Example 6.2.2.1



In this case, our traversal would print *5, 11, 7, 19, 29, 13*

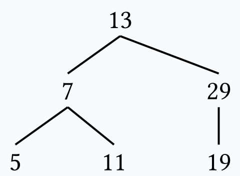
6.2.3. In Order Traversal

Definition 6.2.3.1

In **inorder traversal**, we first recurse left, then look at node, then recurse right. So if you wrap around the tree like a glove, every time you reach the bottom of a node you print it.

```
preorder (Node node):
  if node is not null:
    recurse left
    look at the data in node
    recurse right
```

Example 6.2.3.1



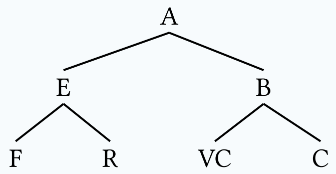
In this case, our traversal would print *5, 7, 11, 13, 19, 29*

6.2.4. Level Order Traversals

Definition 6.2.4.1

Level Order Traversal: print all the nodes at depth 0, then depth 1, then depth 2, etc

Example 6.2.4.1



In this example, we would print *A E B F VC C* in that order

7. Heaps

Definition 7.1

Heaps have the shape property of a binary tree (having 0-2 children), but heaps must also be *complete*.

This property is what makes heaps easy to implement with arrays

7.1. Min Heap

Definition 7.1.1

In **minheap**, the smallest data lives in the root and each child is greater than its parents value. There is no relationship between siblings

Heaps can be illustrated as an array given their characteristics, displayed in level order.

Definition 7.1.2

Given data at index n:

- Left child: $2 * n$
- Right child: $2 * n + 1$
- Parent: $\frac{n}{2}$

Example 7.1.1

Heaps Use Cases

- Not designed for arbitrary searching, mostly designed for accessing root
  - They are no better than just searching an arraylist (O(n))
- Heaps are often used to back priority queues

7.2. Heap Operations

7.2.1. Add Algorithm

- Add to the next spot in the array to maintain completeness
  - this would be index[size]

Note 4.1.1

We do not use **index zero** for heaps

- Up heap starting from the new data to fix order property
  - Compare the data with the parent, and swap the data with the parent as necessary until we read the top or no swap is needed. *This differs for min heap and max heap.*

Note 7.2.1.1

Time complexity of adding a new element is **O(logn)**. While adding is O(1), the upheap process is O(logn).

7.2.2. Remove Algorithm

- Move the last element of the heap to replace the root
- Down heap starting from the root to fix the order property. If two children, compare data with higher priority child.

8. Hashmaps

Definition 8.1

Hashmaps: array backed data structure that allows us to use "custom" or flexible keys instead of using only indices. It uses a *hashing* function to dictate which indexes corresponds to what data in the backing array.

Theorem 8.1

Where to put Key?

```
index = abs (key.hashCode() & arr.length)
```

How do we avoid collisions?

We do this by controlling the size of the backing array. **For hashmaps, it is very bad to let the backing array get full**, because this causes collisions (from the `%` operator). In order to solve this, we resize while there is still space left in the array.

Theorem 8.2

$\text{load factor} = \frac{\text{size}}{\text{capacity}}$

We usually set a maximum allowed load factor in order to avoid collisions. This factor is typically between 0.6 to 0.8, and Java's default is 0.75. After this loadfactor, we resize.

Note 8.1

Small Load Factors has more resizes and fewer collisions, while **Large Load Factors** are more efficient with memory but cause more collisions.

Another way to avoid collisions is to have a *good hashing function*, where different items have different hashes, and a harder goal: similar items have way different hashcodes.