2. Normal Forms

a) Since A does not appear on the right-hand side of any functional dependencies of R, A must be a part of every key. We have one FD with A on the left, and the FD is $A \rightarrow EH$, which means that neither E nor H are a part of the keys of R. The two other functional dependencies of R have B and CDE on the left side, meaning that all the keys must contain either B or CD (since we know that no keys contain E). Therefore, the two keys are AB and ACD.

b) In order for R to satisfy BCNF, for all $X \rightarrow A$ in F+, either $A \in X$ or X contains a key for R. To show that this is not true, we must show that there is some element in F+ where both of these properties do not hold. The functional dependency $E \rightarrow F$ is a part of the closure F+ because of the following derivation:

1. $CDE \rightarrow BGF$
2. $CDE \rightarrow B$
3. $CDE \rightarrow CDF$ (because of 2)
4. $E \rightarrow F$ (because CD is on both sides)
   $E \rightarrow F$ does not satisfy the rules for BCNF, as F is not $\in$ E and E is not a key (AB and ACD are the only keys).

In order for R to satisfy 3NF, for all $X \rightarrow A$ in F+, either $A \in X$, X contains a key for R, or A is a part of some key for R. Using the same example above, $E \rightarrow F$, we can show that there is some element of F+ that does not satisfy these three properties. We already showed above that E is not a key of R and that F is not $\in$ E. As the only keys are AB and ACD, F is not a part of any keys of R. Therefore, none of these properties hold for $E \rightarrow F$, proving that R is neither in BCNF nor 3NF.

c) $B \rightarrow C$
$B \rightarrow D$
$B \rightarrow F$
$A \rightarrow E$
$A \rightarrow H$
$CDE \rightarrow B$
$CDE \rightarrow G$
$CDE \rightarrow F$

We can remove $CDE \rightarrow F$ because $CDE \rightarrow B$, and $B \rightarrow F$. Therefore, our minimal cover is $\{B\rightarrow C, B\rightarrow D, B\rightarrow F, A\rightarrow E, A\rightarrow H, CDE\rightarrow B, CDE\rightarrow G\}$. We can first decompose on $B\rightarrow CDF$, which gives us BCDF and ABEGH. We still need to continue, so we decompose on $A\rightarrow EH$, leaving us with BCDF, AEH, and ABG. Unfortunately, we have not preserved all the dependencies as $CDE\rightarrow B$ and $CDE\rightarrow G$ were not preserved. Therefore, we have to add BCDEG to our decomposition to get the final decomposition of BCDF, AEH, ABG, BCDEG.