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Inductive proof of the tetrahedral number formula

A tetrahedral number of size n is the sum of the triangular numbers from 1 to n.

Givens

1. Let $g(n)$ be the nth triangular number.
   $g(n) = \frac{n(n+1)}{2}$
2. Let $F(n)$ be the nth tetrahedral number.
   $F(n) = g(1) + g(2) + g(3) + \cdots + g(n)$
3. Assume:
   $F(n) = \frac{n(n+1)(n+2)}{6}$
4. Let $F(n)$ be the nth tetrahedral number.
   $F(n) = g(1) + g(2) + g(3) + \cdots + g(n)$

Base Case

1. $F(1) = \frac{1(1+1)(1+2)}{6} = 1$
2. $g(1) = \frac{1(1+1)}{2} = 1$
3. $F(1) = g(1) = 1$

Inductive Step

1. $F(n+1) = g(1) + g(2) + g(3) + \cdots + g(n) + g(n+1)$
2. $F(n+1) = F(n) + g(n+1)$
3. $F(n+1) = \frac{n(n+1)(n+2)}{6} + \frac{(n+1)(n+2)}{2}$
4. $F(n+1) = \frac{n(n+1)(n+2)}{6} + \frac{3(n+1)(n+2)}{6}$
5. $F(n+1) = \frac{n(n+1)(n+2)+3(n+1)(n+2)}{6}$
6. $F(n+1) = \frac{(n+1)(n+2)(n+3)}{6}$

$$\therefore F(n) = \frac{n(n+1)(n+2)}{6}$$