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BFO-DOLCE mappings: an experiment

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Abstract. add abstract
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1. Introduction

**** direi che si tratta di un mapping fatto in OntoCommons con un'idea pluralistica e che qui riportiamo quali sono stati i challenges incontrati e come li abbiamo affrontati oltre poi a dire qualche cosa su alcune differenze trovate tra bfo e dolce / parlerei fin dall'inizio del fatto che abbiamo cercato di usare dei theorem provers / model generator (con successo parziale)

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OntoCommons project focuses on a network of formal ontologies, called Ontology Commons EcoSystem (OCES), to foster the use of ontology in the NMBP areas.

The idea is to have a pluralistic approach where different ontologies, representing different points of view, coexist in a given network, where they are (partially) aligned via mappings. This network of ontologies does not exist today. The pluralistic approach involves ontologies of all levels: top-, middle- and domain-ontologies. Among top-level ontologies, OntoCommons considers as starting points BFO, DOLCE, and EMMO.

We report on the BFO-DOLCE mapping, in particular we consider the most recent Common Logic (CL) versions of these two ontologies collected from the groups that developed them, see Sect. 2 for more details. The choice to use CL is the result of several considerations including that CL is a consolidated standard (https://www.iso.org/standard/66249.html) and that the visibility and stability of standards increase users' trust in the OCES system.

The aims of this paper are: (*i*) show what are the theoretical and practical challenges one faces in aligning big and complex (more than 100 FOL axioms) ontologies; (*ii*) make evident what are the most delicate choices one is confronted with during the alignment process; and (*iii*) provide an analysis of the main differences/similarities between BFO and DOLCE.

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Table 1
Primitive relations of BFO.

$x ::_t u$	x is an instance of u at time t
$\mathtt{EX}(x,t)$	x exists at time t
$\mathtt{cP}(x,y,t)$	x is a continuant part of y at time t
$\mathtt{oP}(x,y)$	x is an occurrent part of y
$\mathtt{mP}(x,y)$	x is a member part of y
$\mathtt{tmP}(x,y)$	x is a temporal part of y
$\mathtt{STREG}(x,y)$	x occupies spatiotemporal region y
$\mathtt{SREG}(x,y,t)$	x occupies spatial region y at time t
$\mathtt{TREG}(x,y)$	x occupies temporal region y
$\mathtt{TPROJ}(x,y)$	x temporally projects onto y
$\mathtt{SPROJ}(x,y,t)$	x spatially projects onto y at time t
$\mathtt{OCCIN}(x,y)$	x occurs in y
$\mathtt{LOC}(x,y,t)$	x is located in y at time t
$\mathtt{SDEP}(x,y)$	x specifically depends on y
$\mathtt{CONCR}(x,y)$	x concretizes y
$\mathtt{GDEP}(x,y,t)$	x generically depends on y at time t
$\mathtt{PTC}(x,y,t)$	x participates in y at time t
$\mathtt{REAL}(x,y)$	x realizes y
$\mathtt{HIST}(x,y)$	x is the history of y
$\mathtt{PREC}(x,y)$	x precedes y
$\mathtt{FINST}(x,y)$	x is the first instant of y
$\mathtt{LINST}(x,y)$	x is the last instant of y
$\mathtt{MBAS}(x,y,t)$	x is the material basis of y at time t

2. BFO and DOLCE

2.1. *CL version of* BFO *2020 released the 12 Nov 2021*

With $\mathfrak{B}$ we indicate the logical theory consisting of all the axioms in the CL-version of BFO 2020 / 12 November 2021 (named here BFO-CL) available from GitHub[1] in Prover9 syntax with four differences: (*i*) inverse relations in BFO-CL (e.g., hasContinuantPart is the inverse of continuantPartOf) are captured in $\mathfrak{B}$ by inverting the order of arguments (except for the temporal argument); (*ii*) the 'AtSomeTime' and 'AtAllTimes' relations introduced in BFO-CL, but never used in other axioms of BFO-CL, are not considered in $\mathfrak{B}$[2]; (*iii*) relations introduced in BFO-CL with 'if and only if' clauses are here introduced via syntactic definitions; and (*iv*) some syntactic definitions are added to improve the readability of formulas.

Since the names of primitives tend to be long, to have more compact formulas, we adopt here the predicates listed in Table 1 while for the universals we adopt the individual constants in Table 2. The taxonomy of $\mathfrak{B}$ is depicted in Figure 1 (vertical lines represent ISA relationships, when solid they indicate a partition). Notice that all the universals in the taxonomy in Figure 1 are 'rigid' (in the sense that their instances cannot migrate to another universal) except `obj`, `fobj`, and `objagg`.

The whole theory $\mathfrak{B}$ is available in prover9 syntax at XXXX. Due to its length, we do not write it in

[1] https://github.com/BFO-ontology/BFO-2020/tree/master/21838-2/prover9.
[2] Inverse, 'AtSomeTime', and 'AtAllTimes' relations are used and useful in the OWL version of BFO.

FC: PAR non è una costante. Assieme a "entity(.)" e a "UNI(.)" è un predicato. Inserire tutti e tre in tabella 1 oppure aggiungere UNI e entity nella 2?

CM: aggiungere nelle tabelle le primitive che si trovano nel file CL; FC: adesso, le uniche non menzionate sono entità e universale, mancavano inerenza [che è definita, quindi ok] e occorrenti

FC: todo

Table 2
Categories of BFO.

`cfbnd`	Continuant Fiat Boundary
`cnt`	Continuant
`disp`	Disposition
`fln`	Fiat Line
`fobj`	Fiat Object
`fpt`	Fiat Point
`fsf`	Fiat Surface
`fnt`	Function
`gdcnt`	Generically Dep. Continuant
`hist`	History
`idcnt`	Independent Continuant
`imen`	Immaterial Entity
`mten`	Material Entity
`obj`	Object
`objagg`	Object Aggregate
`occ`	Occurrent
PAR	Particular
`pbnd`	Process Boundary
`proc`	Process
`qlt`	Quality
`rqlt`	Relational Quality
`rlzen`	Realizable Entity
`role`	Role
`sdcnt`	Specifically Dep. Continuant
`site`	Site
`sreg`	Spatial Region
`sreg0`	0d Spatial Region
`sreg1`	1d Spatial Region
`sreg2`	2d Spatial Region
`sreg3`	3d Spatial Region
`treg`	Temporal Region
`treg0`	0d Temporal Region
`treg1`	1d Temporal Region
`streg`	Spatiotemporal Region
`tinst`	Temporal Instant
`tint`	Temporal Interval

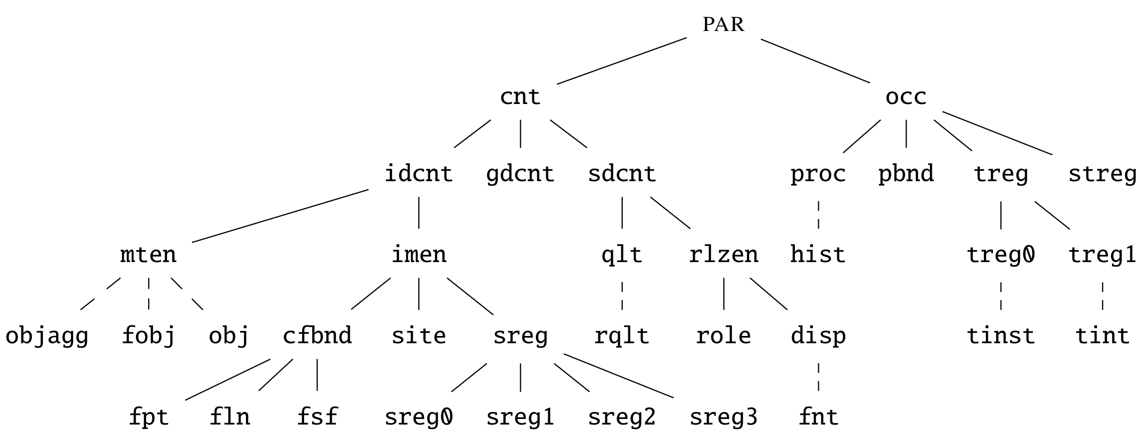

Fig. 1. Taxonomy of BFO (vertical lines represent ISA relationships, when solid they indicate a partition).

full in this paper, instead, we will list in the text only the axioms relevant to the discussion whenever we will need them. We start by reporting some definitions and axioms below that introduce predicates commonly used in $\mathfrak{B}$, such as inheritance (a_b8), the predicates derived from mereological theories, such as overlapping and stric parts (a_b4), (a_b5), (a_b7), subsumption between universals (d_b3), atemporalized instantiation (a_b3), and the class of BFO times (a_b1).

CM: forse conviene mettere qui gli assiomi che poi andiamo a considerare nei teoremi invece che scrivere i teoremi per cui basta scrivere $\mathfrak{D}_b \vdash (\mathbf{a_b1})$ or $\mathfrak{D}_b \nvdash (\mathbf{a_b2})$ o dirlo a parole

CM: vedi quali delle seguenti def servono e metti forse qui tutti gli assiomi di bfo che ci servono

$\mathbf{d_b1}$ $\mathrm{TM}(x) := x ::_x \mathtt{treg}$

$\mathbf{d_b2}$ $x :: u := \exists t (x ::_t u)$

$\mathbf{d_b3}$ $\mathtt{ISA}(x,y) := \forall zt (z ::_t x \rightarrow z ::_t y)$

$\mathbf{d_b4}$ $\mathtt{cO}(x,y,t) := \exists z(\mathtt{cP}(z,x,t) \land \mathtt{cP}(z,y,t))$
$\mathbf{d_b5}$ $\mathtt{oO}(x,y) := \exists z(\mathtt{oP}(z,x) \land \mathtt{oP}(z,y))$
$\mathbf{d_b6}$ $\mathtt{tmO}(x,y) := \exists z(\mathtt{tmP}(z,x) \land \mathtt{tmP}(z,y))$
$\mathbf{d_b7}$ $\mathtt{tmPP}(x,y) := \mathtt{tmP}(x,y) \land x \neq y$
$\mathbf{d_b8}$ $\mathtt{INH}(x,y) := \mathtt{SDEP}(x,y) \land x{::}\mathtt{sdcnt} \land y{::}\mathtt{idcnt} \land \neg(y{::}\mathtt{sreg})$ [tht-1]

2.2. *Extension of* DOLCE *ISO-CL version*

With $\mathfrak{D}$ we indicate the logical theory consisting of all the axioms in the CL-version of DOLCE available from XXXX in Prover9 syntax. With respect to the original version of DOLCE introduced in **?** (here named DOLCE-D18) and following the DOLCE-ISO version , $\mathfrak{D}$ has two main simplifications:

(1) Modality operators are not available in the language of Common Logic.
Formal consequences: DOLCE-D18 is a first order *modal* theory relying on the modal logic QS5 with constant domain, without modal operators the intended models of DOLCE change;
(2) The mereological fusion operator is not available in the language of Common Logic.
Formal consequences: given a property expressible in the theory, DOLCE-D18 assumes the existence of the mereological sum of all the entities that satisfy this property, this kind of entity cannot be ensured to exist in logics without the fusion operator.

The first simplification dramatically weakens several notions of dependence that are strongly grounded on modality. To partially overcome this problem, $\mathfrak{D}$ has the additional primitive of temporary existential dependence ($\mathtt{EXD}$) that, however, differs from the existential dependence in DOLCE-D18 in several aspects.

Concerning the second simplification, to partially overcome the lack of the fusion operation, $\mathfrak{D}$ considers just the axiom of strong supplementation for $\mathtt{P}$ (parthood simpliciter) and $\mathtt{tP}$ (temporary parthood). Therefore, $\mathfrak{D}$ obtains a theory based on extensional mereologies (EM), rather than general extensional mereologies (GEM) as in the case of DOLCE-D18. Notice $\mathfrak{D}$ does not include axioms guaranteeing the closure of the domain under binary sums and products. The user is free to add the sum and product operators whenever needed. Additionally, some important notions defined in DOLCE-D18 by using mereological fusion are introduced in $\mathfrak{D}$ as primitives; in particular, this applies to $\mathtt{TLC}(x,t)$, read as "x is (exactly) located at time t", and $\mathtt{SLC}(x,s,t)$, "at time t, x is (exactly) located at space s".

Following DOLCE-ISO, two additional simplifications are adopted in $\mathfrak{D}$:

– only direct qualities are considered ($\mathtt{DQT}$);
– (Ad56), (Ad57), (Ad63), and (Ad64) in DOLCE-D18 are instantiated only by temporal locations (TL) and time intervals (T) and by spatial locations (SL) and space regions (S), i.e., all the quality leaves explicitly introduced in DOLCE-D18.

Table 3 lists the primitive relations of $\mathfrak{D}$, Table 4 lists the categories of $\mathfrak{D}$, and Figure 2 shows the taxonomy of $\mathfrak{D}$ (vertical lines represent ISA relationships, when solid they indicate a partition).

The whole theory $\mathfrak{D}$ is available at XXXX. Throughout the paper, we will write only the axioms of $\mathfrak{D}$ actually needed for discussion.

2.3. *Notation*

We use expressions (ax), (tx) and (dx) to label axioms, theorems, and syntactic definitions, respectively. More specifically,

add sito web

CM: cita deliverable

CM: non so se vogliamo fare riferimento alla versione iso o meno [FC: stiamo parlando della versione di D.Porello ecc. o un'altra cosa?]

CM: non sono sicuro farei riferimento agli assiomi di dolce-d18 qui [FC: non capisco questo punto: 1-dopo aver letto la versione di DOLCE in Applied Ontology 2022 e quella di D.Porello et al., non vedo come questo punto sia corretto, a meno che non faccia riferimento al mantenimento degli assiomi Ad53 e Ad61. 2-Io voto per non citare assiomi di D18, magari qualcosa tipo "Axioms that quantified over taxonomy categories are not considered."]

TODO

Table 3

Primitive relations of DOLCE.

$\mathtt{DQT}(x,y)$	x is a direct quality of y
$\mathtt{EXD}(x,y,t)$	x is existentially dependent on y at time t
$\mathtt{K}(x,y,t)$	x constitutes y at time t
$\mathtt{P}(x,y)$	x is part of y
$\mathtt{PC}(x,y,t)$	x participates in y at time t
$\mathtt{QL}(x,y)$	x is the immediate quale of y
$\mathtt{SLC}(x,y,t)$	x is (exactly) located at space y at time t
$\mathtt{TLC}(x,t)$	x is (exactly) located at time t
$\mathtt{tP}(x,y,t)$	x is part of y at time t
$\mathtt{tQL}(x,y,t)$	x is the temporary quale of y at time t

Table 4

Categories of DOLCE.

AB	Abstract	PED	Physical Endurant
ACC	Accomplishment	POB	Physical Object
ACH	Achievement	PQ	Physical Quality
APO	Agentive Physical Object	PR	Physical Region
AQ	Abstract Quality	PRO	Process
AR	Abstract Region	Q	Quality
AS	Arbitrary Sum	R	Region
ASO	Agentive Social Object	S	Space Region
ED	Endurant	SAG	Social Agent
EV	Event	SC	Society
F	Feature	SOB	Social Object
M	Amount Of Matter	SL	Spatial Location
MOB	Mental Object	ST	State
NAPO	Non-agentive Physical Object	STV	Stative
NASO	Non-agentive Social Object	T	Time Interval
NPED	Non-physical Endurant	TQ	Temporal Quality
NPOB	Non-physical Object	TL	Temporal Location
PD	Perdurant	TR	Temporal Region

- $(\mathrm{a_b}x)$, $(\mathrm{t_b}x)$, $(\mathrm{d_b}x)$ are used for axioms, theorems, and definitions of $\mathfrak{B}$;
- $(\mathrm{a_d}x)$, $(\mathrm{t_d}x)$, $(\mathrm{d_d}x)$ are used for axioms, theorems, and definitions of $\mathfrak{D}$;
- $(\mathrm{d_{db}}x)$ define the mapping from $\mathfrak{D}$ to $\mathfrak{B}$, and
- $(\mathrm{d_{bd}}x)$ define the mapping from $\mathfrak{B}$ to $\mathfrak{D}$.

3. General strategy for the alignment of TLOs

In this section we introduce the assumptions, the general strategy, and the related considerations that we follow to develop the alignment and verify its “correctness”.

Once the CL-versions of the two ontologies have been collected:

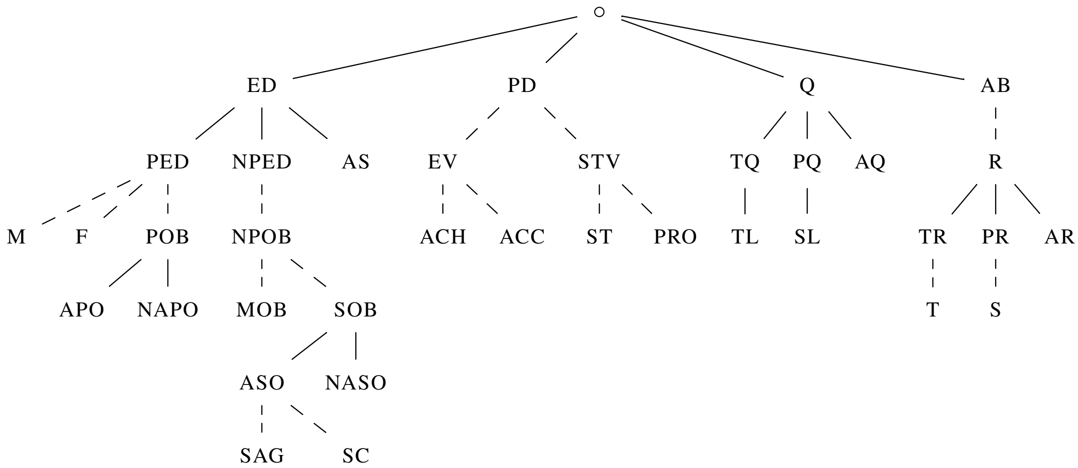

Fig. 2. Taxonomy of DOLCE (vertical lines represent ISA relationships, when solid they indicate a partition).

(1) we analyze the CL-axioms together with the available documentation[3] to ensure proper understanding of the intended interpretations of the primitives of the two ontologies;
(2) we establish and document some methodological choices presented below;
(3) we introduce formal mappings from one ontology to the other (and vice versa);
(4) we test, with the help of theorem provers, what the mappings do and do not preserve.

CM: citare alemeno il documento di bfo che abbiamo considerato [FC: vedi se ti va bene la nota, non so a quale documento di BFO tu hai fatto riferimento]

We start from three assumptions:

– (**M1**) The informal correspondences between the presentations of the notions in the two ontologies resulting from the analysis of their documentation and the examples therein should be used to suggest class and relation mappings. For instance, BFO *occurrents* (*continuants*) are described and used in a way that is similar to DOLCE *perdurants* (*endurants*); at first sight, the BFO relation *continuantPartOf* is applied coherently to the use of the DOLCE relation *temporary parthood*, etc.
– (**M2**) The ideal goal of the mapping is to have the whole domain of quantification of the *source* ontology (namely, $\mathfrak{D}$ in the case analyzed in Sect. 5) included in the domain of quantification of the *target* ontology (namely, $\mathfrak{B}$ in Sect. 5). In other terms, the purpose is to maximize the coverage of the entities of one ontology that are modeled in the other. Given this scenario, the study of which axioms of the target ontology hold or not after optimizing the mappings from point (M1) can highlight the differences between the ontological commitments of the two ontologies, at least as formalized in CL, and what are the entities of the source ontology that are problematic to model in, or even incompatible with, the target ontology.
– (**M3**) Only mappings that can be formalized in FOL are to be considered. This means that we explore the connection between the entities of the source ontology that can be mapped into the domain of the target ontology. Meta-modeling techniques that require the application of abstraction processes or set-theoretical (second-order) constructions are not considered. These techniques usually require enriching the domain of the source ontology with additional entities. This change raises concerns about the actual correspondence between the ontological commitments of the source ontology and that of the theory with the enriched domain. Additionally, the use of FOL allows us to

[3]In our case, **?** and **?** for BFO and DOLCE, respectively.

make use of existing theorem provers to check which axioms of the target ontology hold true in the mapping.

CM: questo ultimo commento non mi convince, abbiamo cercato di restare a FOL perché: (1) possiamo sperare di farci aiutare dai TPs; (2) sfruttiamo l'idea classica di definitional extension (invece che passare a teoremi di rappresentazione che comunque avrebbero senso) [FC: non so seguirti sul punto (2) (teoremi di rappresentazione = ?, in questo contesto), né su M3 originale. PEr punto (1) ho messo frase sui TP]

Assumptions (M2) and (M3) are interrelated and must be seen from the perspective of interactive development of mappings across ontologies where the analytical step (M1) is a prerequisite. In Sect. 5.3 and Sect. 6.3, we list some alternative mappings that modify (by restricting or expanding the mappings proposed in, respectively, Sect. 5.1 and Sect. 6.1) the way some notions of the target ontology are seen in terms of the ones of the source ontology. These alternative mappings are proposed to solve some misalignments raised by the work in Sect. 5 and Sect. 6. We will also mention alternative (definitely more complex) constructions, which extend the domain of the source ontology to match the domain of the target ontology (possibly allowing to define primitives of the target ontology via these new entities).

CM: qui cosa volevamo dire?

In order to clarify some technical background on assumption (M3), we discuss two ways to extend a theory via definitions. The goal is to have formulas in the source ontology that match (or best approximate) the concepts in the target ontology. Here we take $\mathfrak{D}$ as the source ontology and $\mathfrak{B}$ as the target ontology. Analogous considerations hold for the other direction.

The first approach, which we adopt, is to extend theory $\mathfrak{D}$ with a set of syntactic definitions—the mappings in Sect. 5.1—, which inject $\mathfrak{B}$ primitives into $\mathfrak{D}$, i.e., these definitions capture the way $\mathfrak{B}$ primitives can be 'understood' in the DOLCE perspective, in terms of $\mathfrak{D}$ primitives. Syntactic definitions are labelled with the symbol $:=$ and are treated as 'parametric macros': we syntactically substitute the *definendum* with the *definiens* by suitably matching the parameters. For instance, suppose we aim to define the $\mathfrak{B}$ primitive of parthood on occurrents via formula $\mathtt{oP}(x,y) := \phi(x,y)$, where ϕ is an expression in the language of $\mathfrak{D}$ containing x and y. In this case x and y are treated as parameters. The formula $\forall zw(\mathtt{oP}(z,w) \wedge \mathtt{oP}(w,z) \rightarrow z = w)$ in the language of $\mathfrak{B}$ becomes formula $\forall zw(\phi(z,w) \wedge \phi(w,z) \rightarrow z = w)$ in the language of $\mathfrak{D}$. Similarly, $\mathfrak{B}$-formula $\mathtt{oP}(\mathtt{proc\#1}, \mathtt{proc\#2})$ becomes $\mathfrak{D}$-formula $\phi(\mathtt{proc\#1}, \mathtt{proc\#2})$.

On the other hand, one could decide to include these as new formulas in the source ontology, obtaining a theory that is an 'extension by definitions'. This is obtained by using biconditionals ('if and only if') rather than syntactic definitions as described above. E.g., in the previous example one would add the entire formula $\forall xy(\mathtt{oP}(x,y) \leftrightarrow \phi(x,y))$ directly into the source theory. The two approaches are quite similar. The advantage of using syntactic definitions instead of extensions by definitions is that the vocabulary of the theory in which we introduce the syntactic definition does not change. Using the 'if and only if' clause automatically adds the defined predicates to the ontology language (e.g., we would add the $\mathfrak{B}$-predicate $\mathtt{oP}$ into the $\mathfrak{D}$ vocabulary when using the biconditional in the previous example). In Sect. 4.1 we will see that this latter approach may add new ontological commitments with important consequences, in particular on the view of universals.

Once $\mathfrak{D}$ has been extended with these syntactitc definitions (and possibly further syntactic definitions already present in $\mathfrak{B}$)—we dub $\mathfrak{D}_\mathrm{b}$ such extension—it is possible to check how much of the theory $\mathfrak{B}$ is preserved. This is done in Sect. 5.2, where each axiom in $\mathfrak{B}$ expressible[4] in $\mathfrak{D}_\mathrm{b}$ is proved or disproved in $\mathfrak{D}_\mathrm{b}$. The analysis in Sect. 5.3 of the results obtained in Sect. 5.2 allows, modulo the introduced mapping, highlighting and understanding similarities and divergences between the two top-levels. The mapping in the other direction, i.e., the extension $\mathfrak{B}_\mathrm{d}$ with the syntactic definitions of $\mathfrak{D}$-primitives in terms of $\mathfrak{B}$-primitives, is formalized in Sect. 6. The combined analysis of the mappings DOLCE-into-BFO and BFO-into-DOLCE points out genuine ontological differences as well as problematic aspects of the adopted

[4]We will see in Sect. 5.1 that this mapping is only partial.

mapping technique. This analysis allows for refining the mappings in an iterative process to solve, as far as possible, false cases of agreement and disagreement.

3.1. Use of theorems provers and model builders

We use two theorem provers for first-order logic: Prover9[5] and Vampire[6], as well as the model builder Mace4[7]. We used both Vampire and Prover9, since Vampire is generally faster, but was sometimes outperformed by Prover9 in our testing. Furthermore, Prover9 proofs are shorter since the Prover9 syntax does not require explicit quantification of variables.

Our methodology is as follows. We translate all axioms and definitions of $\mathfrak{D}$ and $\mathfrak{B}$ in the syntax adopted by the provers (Vampire uses tptp, while Prover9 has its own syntax). To prove a theorem of the mappings from DOLCE to BFO, we add to the translation of $\mathfrak{D}$ the translation of the mappings in Sect.5.1 (i.e., we translate the whole $\mathfrak{D}_b$) and we submit the translation of the theorem to the provers. Similarly in the case of the mappings from BFO to DOLCE. The provers are not able to automatically prove several theorems. We provide manual proofs for all non automatically proved theorems and we verify (some of) them by introducing intermediate lemmas until the thesis was proved by the provers.

To formally verify counterexamples for the non-theorems of the mappings, we produce counterexamples by hand using the syntax of the provers in the following way: we write axioms enforcing the existence of the exact number of constants required by the counterexample, as well as the relations between them. To these axioms we added the axioms of $\mathfrak{D}$ (or $\mathfrak{B}$) taxonomy, to simplify our work, since the inclusions between the classes of the taxonomy allow us to only work with taxonomical leaves, while the disjunction axioms between the classes allow us to skip some axioms stating that individual constants with different names are different. In this way we obtain, for each counterexample, a theory that exactly describes the counterexample syntactically.

CM: leggera modifica [FC: ok]

Afterward, we run Mace4 using all these axioms as inputs, to verify that the set of axioms is consistent and obtain explicit semantic descriptions of all counterexamples. Then, for each axiom of $\mathfrak{D}$ (or $\mathfrak{B}$), we successfully verify that the axiom can be proved from the theory, thus proving that the (unique) model of the theory is a model of $\mathfrak{D}$ ($\mathfrak{B}$). Lastly, we check that in the model the theorem that we want to disprove is actually false, but, at this point, this is trivial.

Unfortunately, the obvious way of obtaining counterexamples with Mace4, that is, asking Mace4 to directly find a model of all the axioms of $\mathfrak{D}$ ($\mathfrak{B}$) with the addition of the negation of the theorem to be disproven, cannot be implemented, as it requires an excessive computational effort.

All the proofs and models can be found in **?**.

CM: mettere documento, il del o un rapporto

4. Preliminary considerations

Before formally introducing the mappings and the subsequent process of axiom verification, we add a few preliminary remarks about two problems related to technical choices made in $\mathfrak{B}$ and $\mathfrak{D}$, and to the different level at which these theories investigate some classes.

[5]https://www.cs.unm.edu/~mccune/prover9/
[6]https://vprover.github.io/
[7]https://www.cs.unm.edu/~mccune/prover9/

4.1. Representation of categories

In $\mathfrak{D}$ the categories in Table 4 are represented by means of FOL unary predicates. These categories are assumed to correspond to rigid properties, that is, an entity cannot change its taxonomic classification. In other terms, if an entity x belongs to a category, it must belong to that category from the time it starts to exist to the time it ceases to exist (if any). For instance, to state that an entity x is an endurant, one writes $\mathrm{ED}(x)$.

In $\mathfrak{B}$, universals are in the domain of quantification and a temporary *instance-of* primitive relation (written ::) is introduced to represent when a particular is an instance of a universal at a given time. For instance, the fact that "at time t, x is a continuant" is formally represented as $x ::_t \mathtt{cnt}$, where `cnt` corresponds to the universal *being a continuant*. All the universals considered in Table 2 are non-empty and rigid through time with the exception of `obj`, `objagg`, and `fobj`. For instance, a material entity could belong to `obj` for some time, and then to `objagg` (or `fobj`) and change again later, provided at each point in time it is classified by one of these three universals.

These different representational choices raise an initial problem because, according to (M3), one should individuate to which kind of DOLCE entities the BFO universals correspond (Sect. 5.3 adds further considerations on this point). We consider only the $\mathfrak{B}$-universals present in the taxonomy, and start from the subset that can be defined in $\mathfrak{D}$. For each universal $\mathtt{u}$ we introduce a syntactic definition of form: $x ::_t \mathtt{u} := \phi(x,t)$. Note that here the 'parameters' are x and t, not $\mathtt{u}$, which is a constant. This means that, first, we do not define the notion of instance-of but we define only the instantiation of a given $\mathtt{u}$. That is, in general, the definitions of $x ::_t \mathtt{u}_1$ and $x ::_t \mathtt{u}_2$ could be different. Second, as discussed earlier, $\mathtt{u}$ will not appear in $\phi(x,t)$ and thus will not become an individual in the domain of $\mathfrak{D}$. If we were to use the biconditional conditions, writing $x ::_t \mathtt{u} \leftrightarrow \phi(x,t)$ in the theory, both the predicate :: and the individual constant $\mathtt{u}$ would be added to the vocabulary of $\mathfrak{D}$, and this would imply to have some universal in the domain of $\mathfrak{D}$. One alternative way to avoid this problem is to substitute $\mathtt{U}(x,t) := \phi(x,t)$ for $x ::_t \mathtt{u} \leftrightarrow \phi(x,t)$, i.e., by mapping the needed BFO categories to predicates and the instance-of relation to (logical) predication. We adopt the first strategy because it is closer to the formalization in $\mathfrak{B}$, and it avoids to include universals in the domain of $\mathfrak{D}$ as well as the relation instance-of among the primitives of $\mathfrak{D}$.

In this perspective, $\mathfrak{B}$ axioms quantifying over universals are disregarded by the mappings. Weaker versions of these axioms are introduced by considering the (finite) set $\{\mathtt{u}_1, \ldots, \mathtt{u}_n\}$ of those universals explicitly used to generate the mappings. For instance, ($\mathrm{a_b}3$) can be substituted by $\exists xt(x ::_t \mathtt{u}_1) \wedge \ldots \wedge \exists xt(x ::_t \mathtt{u}_n)$. CM: still some of these axioms are not present in sect. 3.2

$\mathbf{a_b3}$ $\mathrm{UNI}(u) \rightarrow \exists xt(x ::_t u)$ [mbf-1]

4.2. Different ontological focus and resolution

The informal categorical distinctions in DOLCE and BFO are quite similar. The presentations of BFO occurrents and DOLCE perdurants are very close and this conclusion is further supported by the fact that both the ontologies introduce a primitive of parthood simpliciter for these entities (this parthood relation is called `P` in $\mathfrak{D}$ and `oP` in $\mathfrak{B}$). Analogously, we notice a similarity between BFO continuants and DOLCE endurants, given that both ontologies conceive the temporary parthood relation as a primitive (`tP` in $\mathfrak{D}$ and `cP` in $\mathfrak{B}$). Things are different for temporal/spatial regions and qualities. At first sight, these categories appear to be similar. Formally, they are classified in different ways in the two ontologies. $\mathfrak{B}$ classifies qualities (and, more generally, specific dependent continuants) and spatial regions under continuants while temporal regions are classified under occurrents. In $\mathfrak{D}$, temporal regions, spatial regions, and qualities are neither endurants nor perdurants.[8] In particular, temporal regions and spatial regions are abstract entities in $\mathfrak{D}$. This difference—mainly grounded on the way the distinction between endurants/continuants and perdurants/occurents is characterized—complicates the comparison and needs to be taken into account in defining the mappings (see for instance the definition ($\mathrm{d_{db}}1$) in $\mathfrak{D}_\mathrm{b}$ of the primitive EX (*existsAt*) of $\mathfrak{B}$).

How the most general categories (endurants/continuants on the one hand, perdurants/occurrents on the other hand) are specialized diverges considerably in the two ontologies:

On endurants vs. continuants. In DOLCE perdurants are specialized mainly on the basis of two notions, both extensively discussed in the linguistic and philosophical literature: *homeomericity* (roughly, the parts of a perdurant of a certain kind are also of the same kind) and *cumulativity* (roughly, the mereological sum of perdurants all of which are of a certain kind is of the same kind, too); in BFO processes are distinguished from process boundaries mainly based on the dimensionality of their temporal locations and on the fact that they are temporal proper parts of other occurrents.

On perdurants vs. occurrents. The spatial and material dimensions of this kind of entities play a central role in both DOLCE and BFO (but see Sect. 5.3 where subtle differences are discussed). DOLCE presents a finer taxonomy mainly aimed to cover the notions of agentivity and sociality. BFO is driven by the distinction between fiat vs. bona fide entities, explicitly introduces the notion of aggregate, and relies on the dimensionality of the spatial (rather than temporal) location to distinguish sites vs continuant fiat boundaries.

On qualities. The branch of the taxonomy for qualities in DOLCE is detached from that of endurants/-continuants, differently from BFO, and is motivated by the *comparability* principle: it makes sense to compare the color of a rose and the color of a vase, it does not make sense to compare the color of a rose and the weight of a vase. Qualities are clustered in terms of maximal comparability, e.g., colors, weights, lengths, shapes, etc. form different quality classes. Spaces of regions have a structure and are associated accordingly, e.g., for colors, the space has regions for red, blue, green, etc. Instead, BFO distinguishes classes of specifically dependent continuants according to the existence, the nature of, and the participants in their realization processes.

These different ways of classifying entities are not incompatible *per se*. For instance, one could consider cumulativity, agentivity, comparability in BFO, and dimensionality of temporal/spatial regions and realization processes in DOLCE. But, clearly, this approach requires extending BFO and DOLCE, a choice that should be pondered carefully. This kind of extension is not exploited in this deliverable. The consequence is that some primitive notions and categories of the target ontology cannot be defined in terms of those in the source ontology. This leads to developing a mapping that focuses on the general notions and categories of the two ontologies.

5. The mapping from DOLCE to BFO

This section presents the technical results of the establishment of a mapping from DOLCE to BFO, that is, it shows how to define in $\mathfrak{D}$ the categories and primitive relations of $\mathfrak{B}$. As discussed earlier, the mapping does not cover all categories and relations.

[8] We will resume this issue in more detail below including a discussion of spatiotemporal regions.

The first part of this section, Sect. 5.1, reports the elements covered by the mapping and the conceptual and formal reasons for the limitation. This part makes technically clear the theoretical and formal barriers to complete coverage of the ontology when this kind of mapping is exploited. It ends with the list of primitive relations and the list of categories of $\mathfrak{B}$ which are covered by the mappings followed by the corresponding syntactic definitions in $\mathfrak{D}$.

The second part of this section, Sect. 5.2, verifies whether the axioms of $\mathfrak{B}$ hold in $\mathfrak{D}$ when extended with the given syntactic definitions. The axioms are clustered according to the relations or categories they characterize. This part is essentially a list of theorems and their proofs. The proofs have been tested using state of the art theorem provers.

The third and final part, Sect. 5.3, provides an in-depth analysis of the achieved results, the limitations and possible alternative strategies.

CM: da qui in poi controllare l'uso di $\mathfrak{D}$ vs. DOLCE e lo stesso per bfo [FC: ho cominciato a controllare, ma dopo alcuni controlli mi sono reso conto che 1-l'ordine di grandezza è circa 300/400 controlli da effettuare 2-risulta un po' pesante 3-la corrente lettura non viene ostacolata, anzi, forse è meglio così.]

5.1. Mappings

In this section we establish how the BFO notions can be mapped to DOLCE: according to our previous discussion and point (M3) of Sect. 3, we introduce syntactic definitions of $\mathfrak{B}$ primitives in terms of $\mathfrak{D}$ primitives. Unfortunately, not all the $\mathfrak{B}$ primitives can be defined in $\mathfrak{D}$ in this way. We clarify the main reasons for this limitation when it happens.

First of all, $\mathfrak{D}$ cannot (without suitable extensions) capture the distinctions grounded on the dimensionality of the instances of a given category. This implies that:

- (i) all the subcategories of `sreg` and of `cfbnd` are ruled out;
- (ii) the distinction between `site` and `cfbnd` cannot be captured (sites are three-dimensional while continuant fiat boundaries are two-, one-, or zero-dimensional);
- (iii) the distinction between `tint` and `tinst` can be only roughly characterized.

Concerning (ii) we introduce a definition only for the disjunction of `site` and `cfbnd` (written `site`∪`cfbnd`), see $(\mathrm{d_{db}}12)$. `site`∪`cfbnd` is clearly not among the categories in the BFO taxonomy and it might not be acceptable as a BFO universal. However, it is used in the mapping with a purely technical role: $x ::_t$ `site`∪`cfbnd` is a shortcut for $x ::_t$ `site` $\vee\, x ::_t$ `cfbnd`. Concerning (iii) we identify temporal instants (intervals) with $\mathfrak{D}$ atomic (non-atomic) time intervals, see $(\mathrm{d_{db}}9)$ and $(\mathrm{d_{db}}10)$. Admittedly, this is a very rough characterization. First, temporal atoms can have the same dimensionality of the times they are part of. Second, in BFO the (finite) sum of zero-dimensional temporal regions is still zero-dimensional while the sum of atomic times is always not atomic, i.e., according to $(\mathrm{d_{db}}9)$ and $(\mathrm{d_{db}}10)$, it is an interval. Third, BFO time intervals are convex while DOLCE non-atomic time intervals are not necessarily convex. Thus, $(\mathrm{d_{db}}10)$ seems to approximate the category `treg1` better than `tint`. At the same time, if all the non-atomic entities are instances of `treg1`, then `treg0` and `tinst` collapse. For these reasons, we omit `treg1` and `treg0` and consider only the rough definitions of `tinst` and `tint`. Note that, in BFO, the convexity of intervals is characterized by means of the primitive of precedence (PREC). To define `PREC` in DOLCE one should extend the theory with an order relation defined on time intervals. Similarly for `FINST` (firstInstantOf) and `LINST` (lastInstantOf). We then need to rule out `PREC`, `FINST`, and `LINST`.

SB: perchè rough? si può toglierlo?

As anticipated earlier, following point (M1) of Sect. 3 the category of specifically dependent continuants (`sdcnt`) seem to correspond to the DOLCE category of qualities. However, as observed in Sect. 4.2, BFO and DOLCE distinguish the subclasses of these categories based on different criteria. In BFO, the subcategories of `sdcnt` are mainly characterized by means of the primitives `REAL` (realizes) and `MBAS` (materialBasisOf) for which, however, the characterization is missing.[9] As a consequence, REAL, MBAS, and the subcategories of sdcnt are not included in the mapping.

A similar situation holds for the subcategories of material entities (mten). In particular, objects aggregates are intimately connected to the memberOf (mP) relation. None of these notions can be defined without an extension of DOLCE. It follows that mP and the subcategories of mten are not covered by the mapping.

Other cases are more subtle. The primitive historyOf (HIST) seems to be naturally defined in $\mathfrak{D}$ as:

$$\mathtt{HIST}(x,y) := \forall z(\mathtt{P}(z,x) \leftrightarrow \forall t(\mathtt{PRE}(z,t) \rightarrow \mathtt{PC}(y,z,t))$$

However the effectiveness of this definition strongly depends on the existential assumptions on perdurants. For instance, if y just participates in a perdurant p but only during a part t of the temporal extension of p and the temporal part of p during t does not exist—and DOLCE does not commit to the existence of all the temporal slices of a perdurant—then y would not have an history. These technical difficulties, and the fact that histories seem to have a marginal role in the BFO ontology, suggest to leave out HIST and hist.

Spatiotemporal regions are not explicitly present in DOLCE. One could introduce them among the regions together with the correspondent qualities. Links to the spatial and temporal projections (corresponding to the BFO TPROJ and SPROJ) would be also necessary. Alternatively, violating (M2), spatiotemporal regions could be built (at the semantic level, for instance) as sets of couples ⟨time interval, space region⟩. These options could be investigated but, since they are technical extensions, we do not consider them here. However, it seems that spatiotemporal regions do not add expressive power so the problem might be less controversial from the technical viewpoint. Sect. **??** delineates how one can avoid spatiotemporal regions by rewriting BFO axioms concerning spatiotemporal regions/locations in terms of DOLCE time intervals/TLC and space regions/SLC.

FC: M3?

FC: sezione vecchia da decidere cosa fare

Finally, following the discussion in Sect. 4.1, instance-of (::) is not directly defined. We define in $\mathfrak{D}$ the instantiation of the needed universals.

Summing up, among the BFO notions, in the following we introduce:

(i) syntactic definitions for the primitive relations: EX, cP, oP, tmP, SREG, TREG, OCCIN, LOC, SDEP, CONCR, GDEP, PTC; and

(ii) syntactic definitions with form $x ::_t \mathtt{u}$ for each category $\mathtt{u} \in \{$cnt, occ, idcnt, gdcnt, sdcnt, mten, imen, site∪cfbnd, sreg, proc, pbnd, treg, tinst, tint$\}$.

Below we list these syntactic definitions together with a short informal description. A deeper analysis about these definitions and their impact on the preservation of the axioms of BFO is done in Sect. 5.3.

$\mathbf{d_{db}1}$ $\mathtt{EX}(x,t) := \mathtt{PRE}(x,t) \vee (\mathrm{T}(x) \wedge \mathtt{P}(t,x)) \vee (\mathrm{AB}(x) \wedge \neg\mathrm{T}(x) \wedge \mathrm{T}(t))$.
EX extends PRE: for EDs, PDs, and Qs, EX and PRE coincide but EX applies also to ABs. Time intervals EX-exist at every subinterval of themselves while the other abstracts entities EX-exist at every time. This extension tries to match the fact that, in BFO, both temporal regions and spatial regions (which seem very close to DOLCE time intervals and space regions, respectively, see ($\mathrm{d_{db}}$7), ($\mathrm{d_{db}}$13)) are in time.

[9] Some necessary conditions can be collected for REAL but these are not enough to characterize the relation.

$\mathbf{d_{db}2}$ $\mathtt{oP}(x,y) := \mathtt{P}(x,y) \wedge ((\mathrm{PD}(x) \wedge \mathrm{PD}(y)) \vee (\mathrm{T}(x) \wedge \mathrm{T}(y)))$

oP restricts P that originally applied to all ABs. In this way we try to preserve the fact that in BFO oP is defined only on occurrents (that include temporal regions while spatial regions are classified under continuants).

$\mathbf{d_{db}3}$ $\mathtt{tmP}(x,y) := \mathtt{oP}(x,y) \wedge \forall z(\mathtt{oP}(z,y) \wedge \forall t(\mathtt{EX}(z,t) \rightarrow \mathtt{EX}(x,t)) \rightarrow \mathtt{oP}(z,x))$

This is the classical definition of temporal part/slice, i.e., x is the maximal part of y during the temporal extension of x.

$\mathbf{d_{db}4}$ $\mathtt{TREG}(x,t) := \mathrm{PD}(x) \wedge \mathtt{TLC}(x,t)$

TREG is the restriction of TLC to PDs (in $\mathfrak{D}$ TLC is defined for all the entities present in time, i.e., also for EDs and Qs).

$\mathbf{d_{db}5}$ $x ::_t \mathtt{pbnd} := \mathrm{PD}(x) \wedge \exists y(\mathtt{tmPP}(x,y)) \wedge \mathtt{TLC}(x,t) \wedge \mathtt{AT}(t)$

A process boundary is a temporally atomic perdurant that is a temporal proper part of at least another perdurant. As said, the atomicity of the temporal location of a perdurant is an approximation of its instantaneity.

$\mathbf{d_{db}6}$ $x ::_t \mathtt{proc} := \mathrm{PD}(x) \wedge \mathtt{PRE}(x,t) \wedge \neg(x ::_t \mathtt{pbnd})$

Processes are perdurants that are not process boundaries (all the perdurants are present at some times ($\mathrm{t_d}1$)).

$\mathbf{d_{db}7}$ $x ::_t \mathtt{treg} := \mathrm{T}(x) \wedge \mathtt{EX}(x,t)$

Temporal regions coincide with $\mathfrak{D}$ time intervals.

$\mathbf{d_{db}8}$ $x ::_t \mathtt{occ} := x ::_t \mathtt{pbnd} \vee x ::_t \mathtt{proc} \vee x ::_t \mathtt{treg}$

As said before, spatiotemporal regions are ruled out (but see Sect. **??**). FC: sezione vecchia?

$\mathbf{d_{db}9}$ $x ::_t \mathtt{tinst} := x ::_t \mathtt{treg} \wedge \mathtt{AT}(x)$

Temporal instants are atomic time intervals.

$\mathbf{d_{db}10}$ $x ::_t \mathtt{tint} := x ::_t \mathtt{treg} \wedge \neg\mathtt{AT}(x)$

Temporal intervals are non-atomic time intervals (in BFO they are self-connected but this property cannot be defined in DOLCE).

$\mathbf{d_{db}11}$ $x ::_t \mathtt{mten} := \mathrm{ED}(x) \wedge \mathtt{PRE}(x,t) \wedge \exists u(\mathtt{PRE}(x,u) \wedge \neg\mathtt{AT}(u)) \wedge$
$\forall u(\mathtt{PRE}(x,u) \rightarrow \exists ysr(\mathrm{M}(y) \wedge \mathtt{SLC}(x,s,u) \wedge \mathtt{SLC}(y,r,u) \wedge \mathtt{P}(r,s)))$

Material entities are non instantaneous (to match $x :: \mathtt{mten} \rightarrow \exists t(t ::_t \mathtt{treg1} \wedge \mathtt{EX}(x,t))$, BFO [zuw-1] axiom) endurants that during their whole life are (at least partially) spatially co-localized with an amount of matter.

$\mathbf{d_{db}12}$ $x ::_t (\mathtt{site} \cup \mathtt{cfbnd}) := \mathrm{F}(x) \wedge \mathtt{PRE}(x,t) \wedge \exists s(\mathtt{SLC}(x,s,t)) \wedge$
$\forall u(\mathtt{PRE}(x,u) \rightarrow \neg\exists ysr(\mathrm{M}(y) \wedge \mathtt{SLC}(x,s,u) \wedge \mathtt{SLC}(y,r,u) \wedge \mathtt{P}(r,s)))$

Both sites and continuant fiat boundaries are $\mathfrak{D}$ features localized in space that during their whole life are never (partially) spatially co-localized with an amount of matter.

$\mathbf{d_{db}13}$ $x ::_t \mathtt{sreg} := \mathrm{S}(x) \wedge \mathtt{EX}(x,t)$

Spatial regions coincide with $\mathfrak{D}$ space regions.

$\mathbf{d_{db}14}$ $x ::_t \mathtt{imen} := x ::_t (\mathtt{site} \cup \mathtt{cfbnd}) \vee x ::_t \mathtt{sreg}$

$\mathbf{d_{db}15}$ $x ::_t \mathtt{idcnt} := x ::_t \mathtt{mten} \vee x ::_t \mathtt{imen}$

$\mathbf{d_{db}16}$ $x ::_t \mathtt{sdcnt} := \mathrm{Q}(x) \wedge \mathtt{PRE}(x,t) \wedge \exists y(y ::_t \mathtt{idcnt} \wedge \neg\mathrm{S}(y) \wedge \mathtt{DQT}(x,y))$

Specifically dependent continuants are $\mathfrak{D}$ qualities inhering (in the sense of DQT) in an independent continuant (as defined in ($\mathrm{d_{db}}15$)) that is not a spatial region.

$\mathbf{d_{db}17}$ $\mathtt{CONCR}(x,y,t) := x ::_t \mathtt{sdcnt} \wedge \mathrm{NPED}(y) \wedge \neg\exists s(\mathtt{SLC}(y,s,t)) \wedge \mathtt{EXD}(y,x,t)$

Concretization is a form of $\mathfrak{D}$ existential dependence (EXD) between a non-physical endurant y that does not have a spatial localization and a specifically dependent continuant (as defined in ($\mathrm{d_{db}}$16)) x.

$\mathbf{d_{db}18}$ $x ::_t \mathtt{gdcnt} := \mathtt{PRE}(x,t) \wedge \forall u(\mathtt{PRE}(x,u) \rightarrow \exists y(\mathtt{CONCR}(y,x,u)))$

Generic dependent continuants are non-physical endurants (from ($\mathrm{d_{db}}$17)) that are concretized during their whole life.

$\mathbf{d_{db}19}$ $x ::_t \mathtt{cnt} := x ::_t \mathtt{idcnt} \vee x ::_t \mathtt{sdcnt} \vee x ::_t \mathtt{gdcnt}$

$\mathbf{d_{db}20}$ $\mathtt{cP}(x,y,t) := x ::_t \mathtt{cnt} \wedge y ::_t \mathtt{cnt} \wedge (\mathtt{tP}(x,y,t) \vee \mathtt{P}(x,y) \vee \exists zu(\mathtt{DQT}(x,z) \wedge \mathtt{DQT}(y,u) \wedge \mathtt{tP}(z,u,t)))$

According to ($\mathrm{d_{db}}$19), ($\mathrm{d_{db}}$15), ($\mathrm{d_{db}}$16), and ($\mathrm{d_{db}}$18), continuants include EDs, Ss, and Qs. For EDs, cP coincides with tP. For PDs, cP coincides with P. For Qs, in DOLCE, there is not a parthood relation but we can use the parthood relation among the endurants these qualities inhere in (according to DOLCE each quality inheres in a single entity, $\mathtt{DQT}(x,y) \wedge \mathtt{DQT}(x,z) \rightarrow y = z$).[10]

$\mathbf{d_{db}21}$ $\mathtt{SREG}(x,s,t) := x ::_t \mathtt{idcnt} \wedge \neg \mathrm{S}(x) \wedge \mathtt{SLC}(x,s,t)$

SREG is the resctriction of SLC to independent continuants (as defined in ($\mathrm{d_{db}}$15)) that are not spatial regions.

$\mathbf{d_{db}22}$ $\mathtt{PTC}(x,y,t) := x ::_t \mathtt{cnt} \wedge \neg \mathrm{S}(x) \wedge y ::_t \mathtt{proc} \wedge \mathtt{PC}(x,y,t)$

PTC is the resctriction of PC to continuants that are not spatial regions and to processes (as defined above).

$\mathbf{d_{db}23}$ $\mathtt{OCCIN}(x,y) := \mathrm{PD}(x) \wedge \exists t(y ::_t \mathtt{mten} \vee y ::_t \mathtt{site} \cup \mathtt{cfbnd}) \wedge$
$\forall t(\mathtt{PRE}(x,t) \rightarrow \exists sr(\mathtt{SLC}(x,s,t) \wedge \mathtt{SLC}(y,r,t) \wedge \mathtt{P}(s,r)))$

$\mathtt{OCCIN}(x,y)$ holds when the spatial location of the perdurant x is always included in the one of the material entity/site/continuant fiat boundary y (this definition closely corresponds to the one in the documentation of BFO).

$\mathbf{d_{db}24}$ $\mathtt{LOC}(x,y,t) := x ::_t \mathtt{idcnt} \wedge \neg \mathrm{S}(x) \wedge y ::_t \mathtt{idcnt} \wedge \neg \mathrm{S}(y) \wedge \exists sr(\mathtt{SLC}(x,s,t) \wedge \mathtt{SLC}(y,r,t) \wedge \mathtt{P}(s,r))$

At time t, and independent continuant (that is not a space region) is located in another independent continuant (that is not a space region) when the spatial location (at t) of the first continuant is included in the spatial location (at t) of the second continuant (this definition closely corresponds to the one in the documentation of BFO).

$\mathbf{d_{db}25}$ $\mathtt{SDEP}(x,y) := x :: \mathtt{sdcnt} \wedge \mathtt{DQT}(x,y)$

SDEP is the restriction of DQT to specifically dependent continuants (as defined in ($\mathrm{d_{db}}$16).

$\mathbf{d_{db}26}$ $\mathtt{GDEP}(x,y,t) := x ::_t \mathtt{gdcnt} \wedge y ::_t \mathtt{idcnt} \wedge \neg \mathrm{S}(y) \wedge \exists zs(\mathtt{P}(s,t) \wedge \mathtt{DQT}(z,y) \wedge \mathtt{CONCR}(z,x,s))$

At time t, the generic dependent continuant x generically depends on the independent continuant (that is not a spatial region) y, when it is concretized (during a part s of t) by a specifically dependent continuant z inhering in y.

$\mathbf{d_{db}27}$ $\mathrm{PAR}(x) := \exists t(\mathtt{EX}(x,t))$

Particulars are entities that EX-exist in time (to match $\mathtt{EX}(x,t) \rightarrow \mathrm{PAR}(x) \wedge \mathrm{TM}(t)$ BFO-[oap-1] and $\mathrm{PAR}(x) \rightarrow \exists t(\mathtt{EX}(x,t))$ BFO-[nmq-1]).

[10]Note that this definitions allows qualities of different kind to be one part of the other (for instance the color of an object is part of the temperature of a bigger object). To avoid these cases one could assume that every specialization of $\mathfrak{D}$ has a (finite) set of leaf-types of qualities and require, in the last disjunct in ($\mathrm{d_{db}}$20), x and y to be instances of the same leaf-type. We leave the implementation and the study of this new definition for future work.

$\mathbf{d_{db}28}$ $\textsc{uni}(x) \coloneqq \neg\textsc{par}(x)$

Universals are non-particulars (to match $\textsc{par}(x) \vee \textsc{uni}(x)$ BFO-[eto-1] and $\textsc{par}(x) \rightarrow \neg\textsc{uni}(x)$ [qkp-1]).

5.2. *Preservation of the original* BFO *axioms*

As discussed in Sect. 4.2 (and made explicit in Sect. 5.1) some BFO-axioms involve relations or categories that have not been defined in the mappings. In what follows, when possible and relevant we consider approximations of these axioms as expressible in the $\mathfrak{D}$ language, otherwise we set them aside.

In the rest of this section, $\mathfrak{D}_\mathsf{b}$ indicates the 'extension' of $\mathfrak{D}$ with the mappings introduced in Sect. 5.1 together with the syntactic definitions introduced in Sect. 2.1, i.e., formally $\mathfrak{D}_\mathsf{b} = \mathfrak{D} \cup \{(\mathrm{d_{db}}1)\text{-}(\mathrm{d_{db}}28)\} \cup \{(\mathrm{d_b}1), (\mathrm{d_b}4)\text{-}(\mathrm{d_b}8)\}$|. FC. check references

Note that ($\mathrm{d_b}2$) and ($\mathrm{d_b}3$) are not included in $\mathfrak{D}_\mathsf{b}$ because they require universals in the domain of quantification. Moreover, ($\mathrm{d_b}2$) could be avoided by including the existential quantifier on time. Alternatively, for each predicate $\mathtt{u}$ representing a BFO-category, one could introduce the syntactic definition $x :: \mathtt{u} \coloneqq \exists t(x ::_t \mathtt{u})$. Definition ($\mathrm{d_b}3$) is also not needed since it does not occur in the BFO axioms. This definition will be used in the mapping from BFO to $\mathfrak{D}$, see Sect. 6.1.

Several proofs have been verified by using theorems provers, they are reported in Sect. **??**|. FC: frase da modificare

5.3. *Analysis*

We begin this analysis by considering how the entities in the domain of $\mathfrak{D}$ are classified from the point of view of BFO. ($\mathrm{t_{db}}1$) shows that endurants, perdurants, and qualities are all BFO particulars (in the sense of the BFO notion of particular, PAR). The case of abstracts is slightly more complex. ($\mathrm{t_{db}}2$) shows that if there exists at least a time interval then all the abstracts are BFO particulars too. This is due to the definition of EX ($\mathrm{d_{db}}1$) that assures that abstracts exist at every time. However, while in DOLCE the existence of endurants, perdurants, and qualities requires them to have a temporal location (and then to be present in time), see ($\mathrm{a_d}6$) and ($\mathrm{t_d}1$), this does not hold for abstracts, i.e., it is possible to have models of DOLCE that contain only abstracts and no time intervals. In this case we would not have BFO particulars, only universals ($\mathrm{t_{db}}3$).

$\mathbf{a_d1}$ $(\textsc{ed}(x) \vee \textsc{pd}(x) \vee \textsc{q}(x)) \rightarrow \exists t(\mathtt{TLC}(x,t))$

$\mathbf{t_d1}$ $\mathfrak{D} \vdash (\textsc{ed}(x) \vee \textsc{pd}(x) \vee \textsc{q}(x)) \rightarrow \exists t(\mathtt{PRE}(x,t))$

$\mathbf{t_{db}1}$ $\mathfrak{D}_\mathsf{b} \vdash (\textsc{ed}(x) \vee \textsc{pd}(x) \vee \textsc{q}(x)) \rightarrow \textsc{par}(x)$

$\mathbf{t_{db}2}$ $\mathfrak{D}_\mathsf{b} \vdash \exists u(\textsc{t}(u)) \rightarrow \forall x(\textsc{par}(x))$

$\mathbf{t_{db}3}$ $\mathfrak{D}_\mathsf{b} \vdash \neg\exists u(\textsc{t}(u)) \rightarrow \forall x(\textsc{uni}(x))$

($\mathrm{t_d}1$) seems to conform to both (M1) and (M2), by corroborating the idea that DOLCE particulars roughly correspond to BFO particulars and that all the particulars of DOLCE are 'imported' into the ones of BFO. However, ($\mathrm{t_{db}}3$) introduces some doubts about the nature of abstracts. The definition of EX ($\mathrm{d_{db}}1$), starting from the fact that temporal and spatial regions are particulars in BFO, extended this idea to all regions (and to all abstracts), i.e., there is a presupposition that all the regions have a uniform nature, they are all particulars in this case (even though, in BFO, temporal and spatiotemporal regions are occurrents while spatial regions are continuants). However, other options are possible. One could consider, for instance, the following two variants of ($\mathrm{d_{db}}1$):

– $\mathtt{EX}(x,t) \coloneqq \mathtt{PRE}(x,t) \vee (\textsc{t}(x) \wedge \mathtt{P}(t,x)) \vee (\textsc{s}(x) \wedge \textsc{t}(t))$

– $\mathtt{EX}(x,t) := \mathtt{PRE}(x,t)$

In the first case time intervals and space regions become BFO particulars as before (matching the fact that temporal and spatial regions are particulars in BFO) but, according to ($d_{db}27$) and ($d_{db}28$), the rest of abstracts would be included as universals. This possibility is interesting because the examples for DOLCE (spaces of) regions are colors, weights, shapes, etc., while, in BFO, *being red*, *being blue*, *being round*, etc., are universals. In this perspective, one can also presuppose that the $\mathtt{P}$ relation holding between regions (in a given space) represents a sort of (intensional) ISA relation, while $\mathtt{tQL}$—or, maybe better, the composition of $\mathtt{DQT}$ and $\mathtt{tQL}$—represents instantiation (at a time). An option that deserves a detailed analysis even though it introduces differentiation between time intervals and space regions on the one side, and the remaining regions on the other side.

The second option excludes all abstracts, including time intervals and space regions, from particulars. Even in this case one could see $\mathtt{P}$ as a sort of ISA relation and $\mathtt{TLC}$ and $\mathtt{SLC}$ as sorts of instantiation relations (here note that the (a_d2) and (a_d3) go in the direction of considering $\mathtt{TLC}$ and $\mathtt{SLC}$ as compositions of $\mathtt{DQT}$ and $\mathtt{tQL}$ as suggested in the previous case also).

In both these cases, the fact that (some) regions become universals is grounded on ($d_{db}27$) and ($d_{db}28$). The latter try to match (a_b4) and (a_b5), i.e., the fact that the entities in the domain of BFO are partitioned into universals and particulars. One could, however, decide to split (a_b4) and assume that regions are neither BFO particulars nor BFO universals. They would form a kind of entity that BFO does not consider. Also this option deserves a deeper analysis.

$\mathbf{a_d2}$ $\mathrm{PD}(x) \rightarrow (\mathtt{TLC}(x,t) \leftrightarrow \exists y(\mathtt{DQT}(y,x) \wedge \mathtt{QL}(t,y) \wedge \mathrm{T}(t)))$

$\mathbf{a_d3}$ $\mathrm{PED}(x) \rightarrow (\mathtt{SLC}(x,s,t) \leftrightarrow \exists y(\mathtt{DQT}(y,x) \wedge \mathrm{SL}(y) \wedge \mathtt{tQL}(s,y,t)))$

$\mathbf{a_b4}$ $\mathrm{PAR}(x) \vee \mathrm{UNI}(x)$ [eto-1]

$\mathbf{a_b5}$ $\mathrm{PAR}(x) \rightarrow \neg\mathrm{UNI}(x)$ [qkp-1]

Moving to finer correspondences, ($t_{db}4$) shows that perdurants are the union of processes and process boundaries. However, ($t_{db}5$), ($t_{db}6$), ($t_{db}7$), ($t_{db}8$), and ($t_{db}9$) make evident that the distinction between processes and processes boundaries behaves quite differently from the original one in BFO.

CM: qui bisogna dire che questi sono tutti assiomi di bfo, non credo serva introdurre gli assiomi di bfo esplicitamente

$\mathbf{a_b6}$ $x ::_t \mathtt{proc} \wedge \mathtt{oP}(x,y) \rightarrow \exists t'(y ::_{t'} \mathtt{proc})$

$\mathbf{a_b7}$ $x ::_t \mathtt{proc} \rightarrow \exists y t'(y ::_{t'} \mathtt{pbnd} \wedge \mathtt{oP}(y,x))$

$\mathbf{a_b8}$ $x ::_t \mathtt{pbnd} \wedge \mathtt{oP}(y,x) \rightarrow \exists t'(y ::_{t'} \mathtt{pbnd})$

$\mathbf{a_b9}$ $\exists t(x ::_t \mathtt{pbnd}) \leftrightarrow \exists y(\mathtt{tmP}(x,y) \wedge \exists t(y ::_t \mathtt{proc})) \wedge \exists t(\mathtt{TREG}(x,t) \wedge t ::_t \mathtt{tinst})$

$\mathbf{a_b10}$ $\exists t(x ::_t \mathtt{proc}) \wedge \mathtt{TREG}(x,t) \rightarrow \exists t'(t' ::_{t'} \mathtt{tint} \wedge \mathtt{tmP}(t',t))$

$\mathbf{t_{db}4}$ $\mathfrak{D}_b \vdash \mathrm{PD}(x) \leftrightarrow (\exists t(x ::_t \mathtt{proc}) \vee \exists t(x ::_t \mathtt{pbnd}))$

$\mathbf{t_{db}5}$ $\mathfrak{D}_b \nvdash$ (a_b6)

$\mathbf{t_{db}6}$ $\mathfrak{D}_b \nvdash$ (a_b7)

$\mathbf{t_{db}7}$ $\mathfrak{D}_b \nvdash$ (a_b8)

$\mathbf{t_{db}8}$ $\mathfrak{D}_b \nvdash$ (a_b9)

$\mathbf{t_{db}9}$ $\mathfrak{D}_b \nvdash$ (a_b10)

BFO presupposes that processes and process boundaries have a different temporal nature, and constrains their relationship, which is quite complex, via a set of axioms. Vice versa ($d_{db}9$) and ($d_{db}10$)—together with ($d_{db}6$) and ($d_{db}7$)—reduce this difference to atomicity and to the existence of bigger, with respect to temporal proper part, perdurants. Looking at the counterexamples used in the proof of the previous theorems, this different behavior about processes and process boundaries seems mainly due to the poor approximation of the notions of temporal instant and temporal interval provided by ($\mathrm{d_{db}}9$) and ($\mathrm{d_{db}}10$) together with the low commitment of DOLCE on the existence of perdurants. For instance, in DOLCE it is possible to have three temporally co-localized perdurants p_1, p_2, and p_{12} all with atomic temporal locations and such that the latter is the sum of the firsts, i.e., $\mathtt{SUM}(p_{12}, p_1, p_2)$. There can also be two additional perdurants p_3 and p_{123} such that the temporal location of p_3 does not overlap the one of p_1 and also $\mathtt{SUM}(p_{123}, p_{12}, p_3)$ holds. In this scenario, according to ($\mathrm{d_{db}}6$) and ($\mathrm{d_{db}}7$), p_1 is a process because it has an atomic temporal location but it is not a temporal proper part of anything, p_1 is part of p_{12}, but p_{12} is a process boundary (against ($\mathrm{t_{db}}5$) and ($\mathrm{t_{db}}7$)) because it has an atomic temporal location and it is a proper temporal part of p_{123}. For another example, in DOLCE it is possible to have perdurants that are not proper temporal parts of any other perdurant and such that all their parts are temporally co-localized with them (possibly with an atomic temporal location) and in their turn are not proper temporal part of any other perdurant (e.g., discard p_3 and p_{123} from the previous example). According to ($\mathrm{d_{db}}6$) and ($\mathrm{d_{db}}7$), these perdurants as well as all their parts are classified as processes against ($\mathrm{t_{db}}6$), ($\mathrm{t_{db}}8$), and ($\mathrm{t_{db}}9$). These examples may look 'exotic', yet it is not easy to rule out them. One could get rid of some of these counterexamples by requiring that processes have a non-atomic temporal location. In this case, we need to remove ($\mathrm{t_{db}}4$) also because it is possible to have perdurants with an atomic temporal location that however are not temporal proper part of any other perdurant.

The different 'temporal behavior' of DOLCE perdurants vs. BFO occurrents is highlighted also by ($\mathrm{t_{db}}15$), which is relative to ($\mathrm{a_b}11$) (meaning that the thesis of ($\mathrm{t_{db}}15$) is a modified version of ($\mathrm{a_b}11$), which does not use the predicate $\mathtt{STREG}$). In BFO, occurrent parthood between processes or process boundaries is equivalent to occurrent parthood between the corresponding spatiotemporal regions, therefore different processes/process boundaries cannot be spatiotemporally co-localized. DOLCE makes the opposite choice and allows for spatiotemporally co-localized perdurants. A similar situation arises between DOLCE endurants and BFO continuants as highlighted by ($\mathrm{t_{db}}10$) and ($\mathrm{t_{db}}11$). In DOLCE, an amount of matter constituting a statue at a given time t is different from the statue and it is not a $\mathtt{tP}$-part of the statue at t (and vice versa) even though the two endurants are spatially co-localized at t (and possibly during their whole life), see ($\mathrm{t_d}2$) and ($\mathrm{a_d}4$). Similarly, in DOLCE, a sum of bricks is different from a wall even though they can mereologically coincide at a given time. These differences are among the most important we found. In DOLCE, space and time are not fundamental for the identity of entities. This gives room for stratification of co-localized entities, which are further distinguished on the basis of their 'modal' properties. In comparison, BFO embraces a 'reductionist' perspective by assuming that a given spatial or spatiotemporal region cannot be the location of different continuants or occurrents.

CM: non sono sicuro su identities [FC: magari così?]

CM: nel teorema ($\mathrm{t_{db}}15$) manca un riferimento al mapping che riguarda lo spazio tempo, da capire se vogliamo tenere questo o invece toglierlo

$\mathbf{a_d4}$ $\mathtt{K}(x,y,t) \to \forall s(\mathtt{SLC}(x,s,t) \leftrightarrow \mathtt{SLC}(y,s,t))$

$\mathbf{t_d2}$ $\mathtt{K}(x,y,t) \to x \neq y$

$\mathbf{a_b11}$ $\exists t(x ::_t \mathtt{proc} \lor x ::_t \mathtt{pbnd}) \land \exists t(y ::_t \mathtt{proc} \lor y ::_t \mathtt{pbnd}) \to$
$(\mathtt{oP}(x,y) \leftrightarrow \exists rs(\mathtt{STREG}(x,r) \land \mathtt{STREG}(y,s) \land \mathtt{oP}(r,s)))$

$\mathbf{a_b12}$ $x ::_t \mathtt{mten} \land y ::_t \mathtt{mten} \land \mathtt{SREG}(x,r,t) \land \mathtt{SREG}(y,r,t) \to \mathtt{cP}(x,y,t) \land \mathtt{cP}(y,x,t)$

$\mathbf{a_b13}$ $\exists t(x ::_t \mathtt{idcnt} \land y ::_t \mathtt{idcnt} \land \neg(x ::_t \mathtt{objagg}) \land \neg(y ::_t \mathtt{objagg}) \land \mathtt{cP}(x,y,t) \land \mathtt{cP}(y,x,t)) \to x = y$

$\mathbf{a_b14}$ $\mathtt{cP}(x,y,t) \land x \neq y \to \exists z(\mathtt{cP}(z,y,t) \land z \neq y \land \neg\mathtt{cO}(z,x,t))$

$\mathbf{a_b15}$ $\mathtt{cO}(x,y,t) \to \exists z(\forall w(\mathtt{cP}(w,z,t) \leftrightarrow \mathtt{cP}(w,x,t) \land \mathtt{cP}(w,y,t)))$

$\mathbf{a_b16}$ $\mathtt{oO}(x,y) \to \exists z(\forall w(\mathtt{oP}(w,z) \leftrightarrow \mathtt{oP}(w,x) \land \mathtt{oP}(w,y)))$

$\mathbf{t_{db}10}$ $\mathfrak{D}_\mathrm{b} \nvdash (\mathrm{a_b}12)$

$\mathbf{t_{db}11}$ $\mathfrak{D}_\mathrm{b} \nvdash (\mathrm{a_b}13)$

$\mathbf{t_{db}12}$ $\mathfrak{D}_\mathrm{b} \nvdash (\mathrm{a_b}14)$

$\mathbf{t_{db}13}$ $\mathfrak{D}_b \nvDash (a_b 15)$

$\mathbf{t_{db}14}$ $\mathfrak{D}_b \nvDash (a_b 16)$

$\mathbf{t_{db}15}$ $\mathfrak{D}_b \cup \{(d_{db}\mathbf{??})\} \nvDash \exists t(x ::_t \mathtt{proc} \vee x ::_t \mathtt{pbnd}) \wedge \exists t(y ::_t \mathtt{proc} \vee y ::_t \mathtt{pbnd}) \rightarrow$
$\mathtt{oP}(x,y) \leftrightarrow \forall t(\mathtt{EX}(x,t) \rightarrow \exists rs(\mathtt{SREG}_0(x,r,t) \wedge \mathtt{SREG}_0(y,s,t) \wedge \mathtt{cP}(r,s,t)))$

FC: ricordarsi sezione STREG

While endurants and occurrents, as well as tP and cP, seem intuitively to correspond, (t_{db}11) shows a relevant difference. There are other differences: (t_{db}12) shows that the cP (as defined in (d_{db}20)) does not satisfy the supplementation axiom, and (t_{db}13) that the existence of the product is not guaranteed. Note that the existence of products cannot be inferred for oP (t_{db}14) either. According to (d_{db}20) and the definitions of the subclasses of cnt, for independent continuants that are not spatial regions, cP reduces to tP. However, for independent continuants that are not object aggregates, the original cP is antisymmetric, but is not so the relation defined by (d_{db}20), that in this case it is equivalent to tP. As we have seen, this fact has an important impact on the endurant/continuant entities accepted by the two theories. One could recover the antisymmetry by changing the definition of cP. This amounts to break the correspondence between tP and cP for independent continuants by 'injecting' antisymmetry into (d_{db}20), e.g.,

CM: forse portare la nota nel testo e aggiungere i teoremi/assiomi necessari [FC: fatto]

– $\mathtt{cP}(x,y,t) := x ::_t \mathtt{cnt} \wedge y ::_t \mathtt{cnt} \wedge ((\mathtt{tP}(x,y,t) \wedge (\neg \mathtt{tP}(y,x,t) \vee x = y)) \vee \mathtt{P}(x,y) \vee \exists zu(\mathtt{DQT}(x,z) \wedge \mathtt{DQT}(y,u) \wedge \mathtt{tP}(z,u,t)))$

This new mapping has the advantage of (*i*) recovering the antisymmetry as in the original BFO; and of (*ii*) making explicit one difference between cP and tP. However, we can have endurants that in DOLCE are linked by tP that are still imported under idcnt but are not linked by the newly defined cP. For instance, in the previous example, the (sum of the) bricks and the wall would not be cP-related. In these cases (t_{db}10) still holds. To solve this problem one could explicitly specialize the definition of cP to take into account these cases, e.g.,

– $\mathtt{cP}(x,y,t) := x ::_t \mathtt{cnt} \wedge y ::_t \mathtt{cnt} \wedge$
$((\mathtt{tP}(x,y,t) \wedge (\neg \mathtt{tP}(y,x,t) \vee x = y)) \vee$
$\exists sr(\mathtt{SLC}(x,s,t) \wedge \mathtt{SLC}(y,r,t) \wedge \mathtt{P}(s,r)) \vee$
$\mathtt{P}(x,y) \vee \exists zu(\mathtt{DQT}(x,z) \wedge \mathtt{DQT}(y,u) \wedge \mathtt{tP}(z,u,t)))$

However, in this way the antisymmetry of cP is lost once more because the bricks and the wall would cP-coincide at t even though they are different. One could then change the definition of SREG stating that the bricks (or the wall) have no spatial location at t, or that at t they have different spatial locations. Similarly in the case of the statue and the clay. However, this option seems too strong to pursue.[11]

These examples suggest following a different approach possibly more respectful of the original commitments of the two ontologies. Let us go back to the classical example of the statue and the clay, or to the one of the brick and the wall. There are two different endurants that are spatially coincident at least at a given time. One can think that (a_b13) and (a_b12) aim at ruling out these kinds of examples: it is not a matter of avoiding spatial coincidence or parthood between the brick and the wall, their goal, one could argue, is to rule out one of these two entities, i.e., to rule out from the domain of quantification, either the brick or the wall.[12] One can then adopt a strategy that 'filters out' some of the DOLCE endurants. Setting this filter is not trivial. For the case of the statue and the clay, one can rely on the fact that constitution is

CM: forse si può fare riferimento agli ai teoremi o vice versa si possono introdurre gli assiomi e semplicemente dire che questi non valgono in dolce+def

[11] We have a similar problem with the class of occurrents as highlighted by (t_{db}15).

[12] Clearly, this is in contrast with (M2) and with the mappings that, vice versa, allow to import all the endurants of DOLCE into BFO.

a sort of order relation in DOLCE. One can maintain the substratum (the clay) in the domain and discard the statue due to the fact that it is the substratum that constitutes the statue, not vice versa. However, in the case of the bricks and the wall we cannot use tP to do the same, because tP is not antisymmetric, differently than constitution. One needs to find a different relation or some other principle to rule out one of these. But this is not all. Let us assume that we do not import the statue into BFO. Does it mean that BFO cannot talk about statues or that it relies on a different representation? After all, it classifies the clay under amount of matter as well as under statue. One would need to translate the DOLCE claims about the statue into BFO claims about the amount of clay (since it is the only element left in the domain of BFO). The consequences of this approach deserve more investigation. It is however important to note that this first analytical step allows us to understand the differences and to individuate the main problems that a mapping needs to address.

The analysis of more refined correspondences, like ($t_{db}16$), ($t_{db}17$), and ($t_{db}18$), shows that temporally extended amounts of matters/physical objects are material entities. Yet, the other direction does not hold: non-physical endurants could be very well material entities. However, ($t_{db}4$), ($t_{db}16$), together with ($t_{db}19$), ($t_{db}20$), and ($t_{db}21$), show that there are endurants and qualities, all of which are particulars (see ($t_{db}1$)), that are neither continuants nor occurrents. This means that the partition of particulars into continuants and occurrents assumed by BFO is not preserved by the mapping, hence, some endurants and qualities are in a sort of ontological 'limbo'.

t_{db}16 $\mathfrak{D}_b \nvdash (\mathrm{M}(x) \vee \mathrm{POB}(x)) \to \exists t(x ::_t \mathtt{cnt})$

t_{db}17 $\mathfrak{D}_b \vdash (\mathrm{M}(x) \vee \mathrm{POB}(x)) \wedge \exists t(\mathtt{TLC}(x,t) \wedge \neg \mathtt{AT}(t)) \to \exists t(x ::_t \mathtt{mten})$

t_{db}18 $\mathfrak{D}_b \nvdash \exists t(x ::_t \mathtt{mten}) \to \mathrm{PED}(x)$

t_{db}19 $\mathfrak{D}_b \nvdash \mathrm{F}(x) \to \exists t(x ::_t \mathtt{cnt})$

t_{db}20 $\mathfrak{D}_b \nvdash \mathrm{Q}(x) \to \exists t(x ::_t \mathtt{cnt})$

t_{db}21 $\mathfrak{D}_b \nvdash \mathrm{NPED}(x) \to \exists t(x ::_t \mathtt{cnt})$

On the other hand, the imported entities do not necessarily cover all the categories of BFO. There are at least two reasons for that, one factual and one structural. (*i*) The factual reason is that some of the DOLCE categories may be empty. For instance, if there are no features, then imen would be empty. Consequently, (a_b3), which states that all universals are non-empty, is not preserved. (*ii*) The structural reason is that, according to the mappings, some categories (as defined in the mappings) are necessarily empty. This is the case of universals (when there is at least a time in the domain of quantification, see ($t_{db}2$)). One solution is to 'force' some types of entities in the ontological 'limbo' to be mapped into BFO universals. We already discussed the import of some regions into universals, similarly one could assume that specific subclasses of non-physical endurant (NPED) are concepts behaving like BFO universals. This approach has several consequences on the set of theorems.

CM: ci sarebbero un sacco di altre osservazioni da fare sui teoremi che non seguono SB: lascierei così almeno per ora

One way to choose among the different mapping approaches is to look at what the mapping optimizes. For instance, strict and restrictive mappings, carefully set to not allow exchanges of entities that would be in the ontological 'limbo' of the target ontology, might be preferred in the context of data transfer, they are safer in this context. Mappings focusing on ontological significance and the maximization of the number of imported entities are instead arguably better for the goals of comparing ontologies, highlighting core differences, and establishing general interoperability results.

6. The mapping from BFO to DOLCE

6.1. Mappings

In this section we introduce syntactic definitions of DOLCE notions in terms of BFO primitives. Analogously to the DOLCE to BFO direction of Sect. 5 and with similar motivations (see the analysis of Sect. 6.3), we present only a partial mapping covering a subset of the categories and primitives of DOLCE.

As discussed in Sect. 4.2, in DOLCE, the agentive and social dimensions play an important role in characterizing the subcategories of physical object (POB) and non-physical endurant (NPED) but those dimensions are not considered in BFO. Similarly, the subcategories of perdurant (PD) rely on the notions of homeomericity and cumulativity, but these notions are beyond the scope of BFO. We then rule out all these subcategories from the mappings. Furthermore, while it is not clear how the distinction between amounts of matter and physical objects can be made, for features we can rely on the BFO categories of site, continuant fiat boundary and fiat object. Thus, within the category of physical endurants (PED) we consider only the category of features (F).

We will see that physical endurants correspond to independent continuants that are not regions, see ($\mathrm{d_{bd}}3$), while the only entities that can be mapped to non-physical endurants are generically dependent continuants, see ($\mathrm{d_{bd}}5$). In DOLCE, arbitrary sums (AS) have necessarily a physical and a non-physical part, see ($\mathrm{a_d}5$). To match this axiom, in BFO we should find continuants that are not spatial regions and are neither independent nor generically dependent continuants. The only option is that of specifically dependent continuants. However, following our guidelines in Sect. 3, these are mapped to DOLCE physical qualities as stated by ($\mathrm{d_{bd}}6$). As a result, AS would be necessarily empty. For this reason, this category is not covered by the mapping.

$\mathbf{a_d5}$ $\mathrm{AS}(x) \rightarrow \exists yzut(\mathtt{tP}(y,x,t) \wedge \mathrm{PED}(y) \wedge \mathtt{tP}(z,x,u) \wedge \mathrm{NPED}(z))$

In Sect. 5.3 we have seen that one of the main differences between DOLCE and BFO concerns the reduction of the parthood relations to spatial and spatiotemporal inclusions. In particular, in BFO there are no spatially co-localized distinct independent continuants (that are not object aggregates) and there are no spatiotemporally co-localized distinct occurrents. We then lack one of the main basis to build a constitution relation among endurants and perdurants. In principle, one could see the relation between a generic dependent continuant and the mereological sum of its carriers at a given time as a form of constitution. This solution is debatable and would model only a very limited notion of constitution, quite different from the general one in DOLCE. Thus, we refrain from introducing K in the mappings.

Concerning qualities, BFO accepts only qualities inhering in independent continuants. From the point of view of DOLCE (considering ($\mathrm{d_{bd}}3$) below), this restricts the mapping to physical qualities only.

In the case of regions, we have just time intervals and space regions but to locate entities in time and space we need to rely only on TLC and SLC because temporal and spatial locations have no correspondent entities in BFO and the QL and tQL relations do not make sense in this setting. Alternatively, as previously discussed, one could import some BFO universals into DOLCE regions and assume that instance-of corresponds to (DQT composed with) tQL. In this case, the ISA relation corresponds to the primitive P (that in DOLCE is defined on regions). Besides the mapping presented in this section, this other alternative is also worth developing. We leave it for future work.

Finally, the categories of *fact* and *set* that in the original taxonomy of DOLCE appear under abstract (AB) have not been explicitly taken into account in DOLCE-CL, we safely ignore them here.

Summing up, among the DOLCE notions, in the following we introduce:

(i) syntactic definitions for the primitive relations: $\mathtt{P}$, $\mathtt{tP}$, $\mathtt{TLC}$, $\mathtt{SLC}$, $\mathtt{PC}$, $\mathtt{DQT}$, $\mathtt{EXD}$; and
(ii) syntactic definitions for the categories: ED, PD, Q, AB, F, PED, NPED, PQ, R, TR, PR, T, and S.

Below we list these syntactic definitions together with short informal descriptions. A deeper analysis about these definitions and their impact on the preservation of the axioms of DOLCE is done in Sect. 6.3.

d$_{\mathtt{bd}}$1 $\mathrm{PD}(x) := x{::}\mathtt{proc} \vee x{::}\mathtt{pbnd}$
Perdurants coincide with the disjunction of processes and process boundaries.

d$_{\mathtt{bd}}$2 $\mathrm{ED}(x) := x{::}\mathtt{cnt} \wedge \neg(x{::}\mathtt{sreg}) \wedge \neg(x{::}\mathtt{sdcnt})$
We rule out from endurants temporal and spatial regions that are mapped, respectively, to time intervals and space regions, see (d$_{\mathtt{bd}}$8) and (d$_{\mathtt{bd}}$10).

d$_{\mathtt{bd}}$3 $\mathrm{PED}(x) := x{::}\mathtt{idcnt} \wedge \neg(x{::}\mathtt{sreg})$
Physical endurants are independent continuants that are not spatial regions.

d$_{\mathtt{bd}}$4 $\mathrm{F}(x) := x{::}\mathtt{site} \vee x{::}\mathtt{cfbnd} \vee x{::}\mathtt{fobj}$
Features are the union of sites, continuant fiat boundaries, and fiat objects.

d$_{\mathtt{bd}}$5 $\mathrm{NPED}(x) := x{::}\mathtt{gdcnt}$
Non-physical endurants coincide with generically dependent continuants.

d$_{\mathtt{bd}}$6 $\mathrm{PQ}(x) := x{::}\mathtt{sdcnt}$
Physical qualities coincide with specifically dependent continuants. Note that relational qualities and realizable entities like roles and dispositions are imported into physical qualities. CM: vedi nota importante tolta

d$_{\mathtt{bd}}$7 $\mathrm{Q}(x) := \mathrm{PQ}(x)$
Only physical qualities exist (BFO has only qualities of independent continuants)

d$_{\mathtt{bd}}$8 $\mathrm{T}(x) := x{::}\mathtt{treg}$
Time intervals coincide with temporal regions. Here one could consider a stronger mapping, i.e., assume that T corresponds to $\mathtt{tint}$. However, following (M2) we try here to import all the spatial regions.

d$_{\mathtt{bd}}$9 $\mathrm{TR}(x) := \mathrm{T}(x)$
Among temporal regions there are only time intervals.

d$_{\mathtt{bd}}$10 $\mathrm{S}(x) := x{::}\mathtt{sreg}$
Space regions coincide with spatial regions.

d$_{\mathtt{bd}}$11 $\mathrm{PR}(x) := \mathrm{S}(x)$
Among physical regions there are only space regions.

d$_{\mathtt{bd}}$12 $\mathrm{R}(x) := \mathrm{TR}(x) \vee \mathrm{PR}(x)$

d$_{\mathtt{bd}}$13 $\mathrm{AB}(x) := \mathrm{R}(x)$
Among abstracts there are only time intervals and space regions.

d$_{\mathtt{bd}}$14 $\mathtt{TLC}(x,t) := (\mathrm{PD}(x) \vee \mathrm{ED}(x) \vee \mathrm{Q}(x)) \wedge \mathrm{T}(t) \wedge \forall u(\mathtt{EX}(x,u) \leftrightarrow \mathtt{tmP}(u,t))$
The temporal location of x is the maximal time at which x exists. In BFO $\mathtt{TREG}$ is defined only on processes and process boundaries).

d$_{\mathtt{bd}}$15 $\mathtt{P}(x,y) := (((\mathrm{PD}(x) \wedge \mathrm{PD}(y)) \vee (\mathrm{T}(x) \wedge \mathrm{T}(y))) \wedge \mathtt{oP}(x,y)) \vee (\mathrm{S}(x) \wedge \mathrm{S}(y) \wedge \forall t(\mathtt{EX}(x,t) \rightarrow \mathtt{cP}(x,y,t)))$
For perdurant and time intervals $\mathtt{P}$ coincides with $\mathtt{oP}$ while for space regions it reduces to constant parthood, i.e., x is a temporary part of y during its whole existence (note that in BFO, all the spatial regions exist at least at a time).

d$_{\mathtt{bd}}$16 $\mathtt{tP}(x,y,t) := \mathrm{ED}(x) \wedge \mathrm{ED}(y) \wedge \mathtt{cP}(x,y,t)$
For endurants $\mathtt{tP}$ coincides with $\mathtt{cP}$.

d$_{bd}$17 $\mathtt{PC}(x,y,t) := \mathrm{ED}(x) \wedge \mathrm{PD}(y) \wedge \exists z(\mathtt{tmP}(y,z) \wedge \mathtt{PTC}(x,z,t))$

PTC is not defined on process boundaries while DOLCE does not impose any constraint on the kind of the process involved in PC. The existential quantification on z aims to mitigate this difference.

d$_{bd}$18 $\mathtt{DQT}(x,y) := \mathtt{INH}(x,y)$

d$_{bd}$19 $\mathtt{EXD}(x,y,t) := (\mathtt{SDEP}(x,y) \wedge \mathtt{EX}(x,t)) \vee \mathtt{GDEP}(x,y,t) \vee \mathtt{PTC}(x,y,t) \vee \mathtt{PTC}(y,x,t)$

EXD is the generalization of SDEP, GDEP, and PTC (PTC is a sort of mutual dependence).

d$_{bd}$20 $\mathtt{SLC}(x,s,t) := (x ::_t \mathtt{idcnt} \wedge \neg(x ::_t \mathtt{sreg}) \wedge \mathtt{SREG}(x,s,t)) \vee$
$(x ::_t \mathtt{gdcnt} \wedge \exists y(\mathtt{GDEP}(x,y,t) \wedge \forall z(\mathtt{GDEP}(x,z,t) \to \mathtt{cP}(z,y,t)) \wedge \mathtt{SREG}(y,s,t))) \vee$
$(\mathrm{Q}(x) \wedge \exists y(\mathtt{INH}(x,y) \wedge \forall z(\mathtt{INH}(x,z) \to \mathtt{cP}(z,y,t)) \wedge \mathtt{SREG}(y,s,t))) \vee$
$(\mathrm{PD}(x) \wedge \exists y(\mathtt{STREG}(x,y) \wedge \mathtt{SPROJ}(y,s,t)))$

For independent continuants that are not spatial regions, SLC coincides with SREG. The spatial location, at t, of a generically dependent continuant x is the spatial region of the maximal entity (if it exists) on which x generically depends on at t. The spatial location of a quality x at t is the spatial region of the maximal (at t) entity (if it exists) in which x inheres. The spatial location of a perdurant is the spatial projection of its spatiotemporal location. (Note that in BFO, no axiom guarantees that a specifically dependent continuant x inheres in a unique continuant, therefore for qualities we define the spatial location only when there is a 'maximal continuant' in which x inheres in. Similarly for generic dependent continuants.)

One could think that these mappings are too restrictive: in DOLCE physical and non-physical endurants are not limited to, respectively, independent continuants and generically dependent continuants (the latter are a quite special kind of entities), and so on. This may suggest to include $x :: \mathtt{idcnt} \wedge \neg(x :: \mathtt{sreg}) \to \mathrm{PED}(x)$ as well as $x :: \mathtt{gdcnt} \to \mathrm{NPED}(x)$, but not the converse formulas. However, we need to remember that we are considering the mapping from BFO to DOLCE. The starting point is the domain of BFO, i.e., we need first of all to try to classify the entities *in the domain of* BFO in terms of the categories of DOLCE. By looking at the mappings it is easy to see that all the particulars[13] of BFO are classified in terms of DOLCE categories except spatiotemporal regions:[14] (*i*) independent continuants which are not spatial regions are mapped to physical endurants; (*ii*) spatial regions to space regions; (*iii*) generically dependent continuants to non-physical endurants; (*iv*) specifically dependent continuants to DOLCE qualities; (*v*) process and process boundaries to perdurants; and (*vi*) temporal regions to time intervals. The fact that DOLCE intuitively accepts, for instance, additional physical or non-physical endurants shows that, in general, DOLCE has a domain larger than the domain of BFO. Our methodological choice (M2) does not allow the mappings to 'enrich' the domain of BFO with new entities. However, by introducing the mapping at the semantic level, i.e., in this case, as an operator translating BFO-structures into a DOLCE-structure, one could enrich the original domain of BFO by set-theoretically building new entities. For instance, this is the strategy followed in the literature to map theories of time based on points to theories based on intervals.[15] The theory based on intervals does not admit points, but points can be built as sets of intervals staying in a given relation that is definable in the theory of intervals. Analogously, one can think of building some entities, e.g., statues as opposed to amounts of clay, starting from the amounts of clay and the specific kinds of universals they instantiate. The investigation of 'semantic mappings' is not discussed here.

FC: non chiaro perché lo proibisce, forse M3? Ma non chiaro comunque

[13]We already discussed the possibility to import universals as regions or non-physical endurants, see Sect. 5.3.

[14]We already discussed the fact that spatiotemporal regions seem superfluous, see Sect. 5.1. However, it is easy to modify the mappings to import spatiotemporal regions under DOLCE regions.

[15]See van Benthem, J., *The Logic of Time*, Springer, 2nd ed., 1991.

6.2. *Preservation of the original* DOLCE *axioms*

As discussed in Sect. 4.2 and made explicit in Sect. 6.1, some DOLCE-axioms involve relations or categories that have not been defined in the mappings. In what follows, when possible and relevant we consider approximations of these axioms as expressible in the BFO language, otherwise we set them aside.

Furthermore (a_b3) states that all the universals are non-empty. From an applicative perspective, this is a strong constraint because it forces the user to model even entities which might not be relevant to the application (and perhaps time consuming to analyze). Note that, once (a_b3) is removed, the existential constraints in $\mathfrak{B}$ are compatible with the instantiation of only some categories. From a technical perspective, (a_b3) requires very large models that make the identification and management of counterexamples very difficult even using dedicated software. For these reasons, in this section we do not consider (a_b3).

In the following, $\mathfrak{B}_d$ indicates the 'extension' of $\mathfrak{B} \setminus \{(a_b3)\}$ with the mappings introduced in Sect. 6.1 together with the syntactic definitions introduced in Sect. 2.2, i.e., $\mathfrak{B}_d = \{\mathfrak{B} \setminus \{(a_b3)\}\} \cup \{(d_{db}1)$-$(d_{db}20)\}$.

Several proofs has been verified by using theorems provers, they are reported in Sect. **??**. FC: frase da modificare

6.3. *Analysis*

The results discussed in Sect. 6.1 show that, with the exception of the spatiotemporal regions, all the particulars of BFO find their place in DOLCE. The possibility to avoid spatiotemporal regions, once one has the temporal and the spatial locations of entities, has been analyzed in Sect. **??**. For importing BFO universals into DOLCE we already suggested some possibilities, e.g. mapping universals to regions or to non-physical endurants. Both alternatives would require to modify the mappings $(d_{bd}5)$ and/or $(d_{bd}12)$ (and eventually $(d_{bd}9)$ and $(d_{bd}11)$). Vice versa, some of the original categories of DOLCE remain necessarily 'empty'. It is the case of AS, AR, TQ, and AQ. Other categories are drastically reduced: physical qualities/regions reduce to spatial locations/space regions and temporal regions reduce to time intervals. It seems that DOLCE accepts a larger variety of entities. This is also confirmed by the possibility to have in DOLCE spatially co-localized, and possibly layered, entities linked by coincidence (mutual tP-parthood at a time) and constitution. Whether this larger variety of entities translates into a higher expressive power needs to be investigated. As shown in the case of spatiotemporal regions, more complex mappings could at least mitigate the gap. FC: ricordarsi di attuare o eliminare FC: idem qui

Let us now take into account the results in Sect. 6.2. Even though the proofs of some theorems are still lacking, some observations can already be made. It results quite evident that the majority of the original axioms of DOLCE are not preserved by the mappings. At first sight, one could take this as evidence of the bad quality of mappings, or of a genuine difference between the two ontologies, or conclude, more drastically, that a comparison of the mappings in the two directions shows that BFO is 'weaker' than DOLCE. A deeper analysis of the reasons underlying the counterexamples of some theorems may suggest a more complex situation with pros and cons on both sides. FC: questo paragrafo dipendono dalla futura struttura del paper

Few, and quite technical, differences between the two theories seem to cause several systematic problems for a comprehensive mapping. In DOLCE, PRE is defined only for the entities that have a temporal extension, see (d_d1), while in BFO EX is a primitive relation.[16] The counterexample for $(t_{bd}1)$ shows that in BFO an entity may exist at two non overlapping temporal regions without existing at any bigger rivedere dopo aver posizionato i controesempi

[16] One could think that TREG corresponds to TLC but actually TREG is defined only for occurents and it is only quite minimally linked to EX.

temporal region. In these situations the condition $\forall u(\mathtt{EX}(x,u) \leftrightarrow \mathtt{tmP}(u,t))$ in the definition of $\mathtt{TLC}(x,t)$, see ($\mathrm{d_{bd}}$14), is never satisfied and the relation PRE is always 'empty'. This problem has a huge impact on the proofs in Sect. 6.2 because PRE is heavily used in the characterization of the primitives of DOLCE. At the same time, it is a quite subtle problem whose origin seems to have a technical, more than ontological, nature; at the end, in both ontologies the entity exists at both times.

$\mathbf{d_d}$1 $\mathtt{PRE}(x,t) := \exists u(\mathtt{TLC}(x,u) \land \mathtt{P}(t,u))$
$\mathbf{a_d}$6 $\mathfrak{B}_{\mathrm{d}} \nvDash (\mathrm{ED}(x) \lor \mathrm{PD}(x) \lor \mathrm{Q}(x)) \rightarrow \exists t(\mathtt{TLC}(x,t))$
$\mathbf{a_d}$7 $\mathfrak{B}_{\mathrm{d}} \nvDash \mathtt{tP}(x,y,t) \land \mathtt{tP}(x,y,u) \land \mathtt{SUM}(s,t,u) \rightarrow \mathtt{tP}(x,y,s)$
$\mathbf{a_d}$8 $\mathfrak{B}_{\mathrm{d}} \nvDash \mathtt{PC}(x,y,t) \land \mathtt{PC}(x,y,u) \land \mathtt{SUM}(s,t,u) \rightarrow \mathtt{PC}(x,y,s)$
$\mathbf{a_d}$9 $\mathfrak{B}_{\mathrm{d}} \nvDash \mathtt{tP}(x,y,t) \land \mathtt{SLC}(x,s,t) \rightarrow \exists r(\mathtt{SLC}(y,r,t))$
$\mathbf{t_{bd}}$1 $\mathfrak{B}_{\mathrm{d}} \nvDash (\mathrm{a_d}6)$
$\mathbf{t_{bd}}$2 $\mathfrak{B}_{\mathrm{d}} \nvDash (\mathrm{a_d}7)$
$\mathbf{t_{bd}}$3 $\mathfrak{B}_{\mathrm{d}} \nvDash (\mathrm{a_d}8)$
$\mathbf{t_{bd}}$4 $\mathfrak{B}_{\mathrm{d}} \nvDash (\mathrm{a_d}9)$

The same problem affects theorems like ($\mathrm{t_{bd}}$2) and ($\mathrm{t_{bd}}$3) where from the holding of a given relation at two times, one wants to infer the holding of the relation at the sum of the times (provided such sum exists). Since most relations do not hold at the times when the related entities do not exist, anytime one of such relations holds between entities that exist at two temporal regions without existing at their sum, a theorem analogous to ($\mathrm{t_{bd}}$2) and ($\mathrm{t_{bd}}$3) is falsified. The BFO relation SREG suffers this problem, i.e., a continuant can occupy spatial regions (even the same spatial region) at two given times without having a spatial location at the sum of these times (see the counterexample for ($\mathrm{t_{bd}}$4)). This fact, together with the definition of SLC ($\mathrm{d_{bd}}$20) make so that many theorems involving SLC are falsified.

CM: questi non sono axioms

anche qui sarà da modificare dopo inserimento controesempi

This problem suggests a different mapping technique. Rather than encapsulating the relation between EX and TLC into the mapping ($\mathrm{d_{bd}}$14), one could follow a 'two steps' procedure: first introduce a direct mapping between EX and PRE and then define TLC by using PRE (as defined in the mapping), e.g.,

- $\mathtt{PRE}(x,t) := (\mathrm{PD}(x) \lor \mathrm{ED}(x) \lor \mathrm{Q}(x)) \land \mathtt{EX}(x,t)$
- $\mathtt{TLC}(x,t) := \forall u(\mathtt{PRE}(x,u) \leftrightarrow \mathtt{P}(u,t))$

In this way, entities that, for instance, exist at two times but not at their mereological sum would still lack a temporal location TLC but this would not prevent them to PRE-exist (just relying on the first mapping). We can then hope to recover some theorems that does not require TLC at the price of possibly losing the original link between PRE and TLC. The study of how this mapping technique can be extended to other primitives and its actual impact are left for future work.

The problems highlighted up to this point have important impacts but are primarily of technical nature. Some genuine ontological differences seem also to exist. First, the defined P, like oP, does not satisfy the supplementation axiom, see ($\mathrm{t_{bd}}$5). Second, the relation INH is not equivalent to the original DQT, in particular it does not satisfy the non-migration principle ($\mathrm{t_{bd}}$6). For relational qualities this can be problematic if, for instance, one wants to distinguish "Mary loves John" from "John loves Mary": in both cases we would have a relational quality inhering in both John and Mary. Third, the reflexivity of the defined tP, like the one of cP, does not hold for all the endurants/continuants. It follows that in BFO one can have continuants for which cP is not defined. Fourth, while in DOLCE all the perdurants have participants at every time they exist ($\mathrm{t_{bd}}$7), in BFO, processes need to have at least a participant but this does not hold for process boundaries. Furthermore, in BFO, continuants do not necessarily participate in processes while in DOLCE this is mandatory: all the endurants participate in a process when they exist ($\mathrm{t_{bd}}8$).

$\mathbf{a_d}$10 $(\mathrm{AB}(x) \vee \mathrm{PD}(x)) \wedge \neg \mathtt{P}(x,y) \rightarrow \exists z(\mathtt{P}(z,x) \wedge \neg \mathtt{O}(z,y))$
$\mathbf{a_d}$11 $\mathtt{DQT}(x,y) \wedge \mathtt{DQT}(x,z) \rightarrow y = z$
$\mathbf{a_d}$12 $\mathrm{PD}(x) \wedge \mathtt{PRE}(x,t) \rightarrow \exists yu(\mathtt{P}(u,t) \wedge \mathtt{PC}(y,x,u))$
$\mathbf{a_d}$13 $\mathrm{ED}(x) \wedge \mathtt{PRE}(x,t) \rightarrow \exists yu(\mathtt{P}(u,t) \wedge \mathtt{PC}(x,y,u))$
$\mathbf{t_{bd}}$5 $\mathfrak{B}_\mathrm{d} \nvDash (\mathrm{a_d}10)$
$\mathbf{t_{bd}}$6 $\mathfrak{B}_\mathrm{d} \nvDash (\mathrm{a_d}11)$
$\mathbf{t_{bd}}$7 $\mathfrak{B}_\mathrm{d} \nvDash (\mathrm{a_d}12)$
$\mathbf{t_{bd}}$8 $\mathfrak{B}_\mathrm{d} \nvDash (\mathrm{a_d}13)$

7. Inferring OWL subclass mappings

metti qualche cosa anche vedendo presentazione per Ontocommons oppure togli, non so [FC: non ho visto bene i mapping owl]

8. Conclusion

add conclusions

Acknowledgements

ontocommons etc.