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END SEM - Mobile Robotics

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## ENDSEM - MOBILE ROBOTICS

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Q1.

(a)  $F_e = 0$   $e \rightarrow$  epipole of 1<sup>st</sup> camera seen in second image  
 $F \rightarrow$  Fundamental Matrix

We Know

$$F = K^{-T} [T_x] R K^{-1} + K^{-T} [T_x(Rp')] K^{-1}$$

~~$R \rightarrow$  rotation matrix~~

$$F = K^{-T} [T_x] R K^{-1} e$$

In the proof of Fundamental Matrix we got that

$$p^T [T_x(Rp')] = 0$$

where  $p^T$  and  $p'$  are normalized coordinates.

$$(K^{-1}p)^T \cdot [T_x R K^{-1} p'] = 0$$

$$p_i^T K^{-T} \cdot [T_x(R K^{-1} p')] = 0 \quad \text{---(1)}$$

It resulted in

$$p_i^T F \cdot p' = 0 \quad \text{---(2)}$$

 $p_i$ ,  $p'$  are original image coordinates.

∴ By comparing eq (1) and (2)  
we get

$$F \cdot p' = K^{-1} \cdot [T_x(R K^{-1} p')] = 0$$

Now putting  $p' = e$  i.e. epipole of 1<sup>st</sup> camera seen in 2<sup>nd</sup> Image.

CL(V) extended (a)

$$F \cdot e = K^{-1} \cdot [T \times (R K^{-1} e)] = 0$$

Here we can see  $(R K^{-1} e) \parallel \text{oo}'$  i.e.  
 $(R K^{-1} e) \parallel T$

$$\therefore [T \times (R K^{-1} e)] = 0$$

$$F \cdot e = K^{-1} \cdot [0] = 0$$

$\therefore$  Hence proved  $\boxed{Fe = 0}$

(b) Fundamental Matrix between  $I_2$  and  $I_1$ ,  
is  $F^T$ .

It is different because.

$$\text{we know } F = K^{-T} [T_x] R K'^{-1}$$

When we want  $I_2$  and  $I_1$  Fundamental Matrix  
then Rotation Matrix (R) changes and  $K'$  and  
 ~~$K$~~  get ~~inter~~ changed

(c) The difference between Fundamental matrix and Essential matrix is that

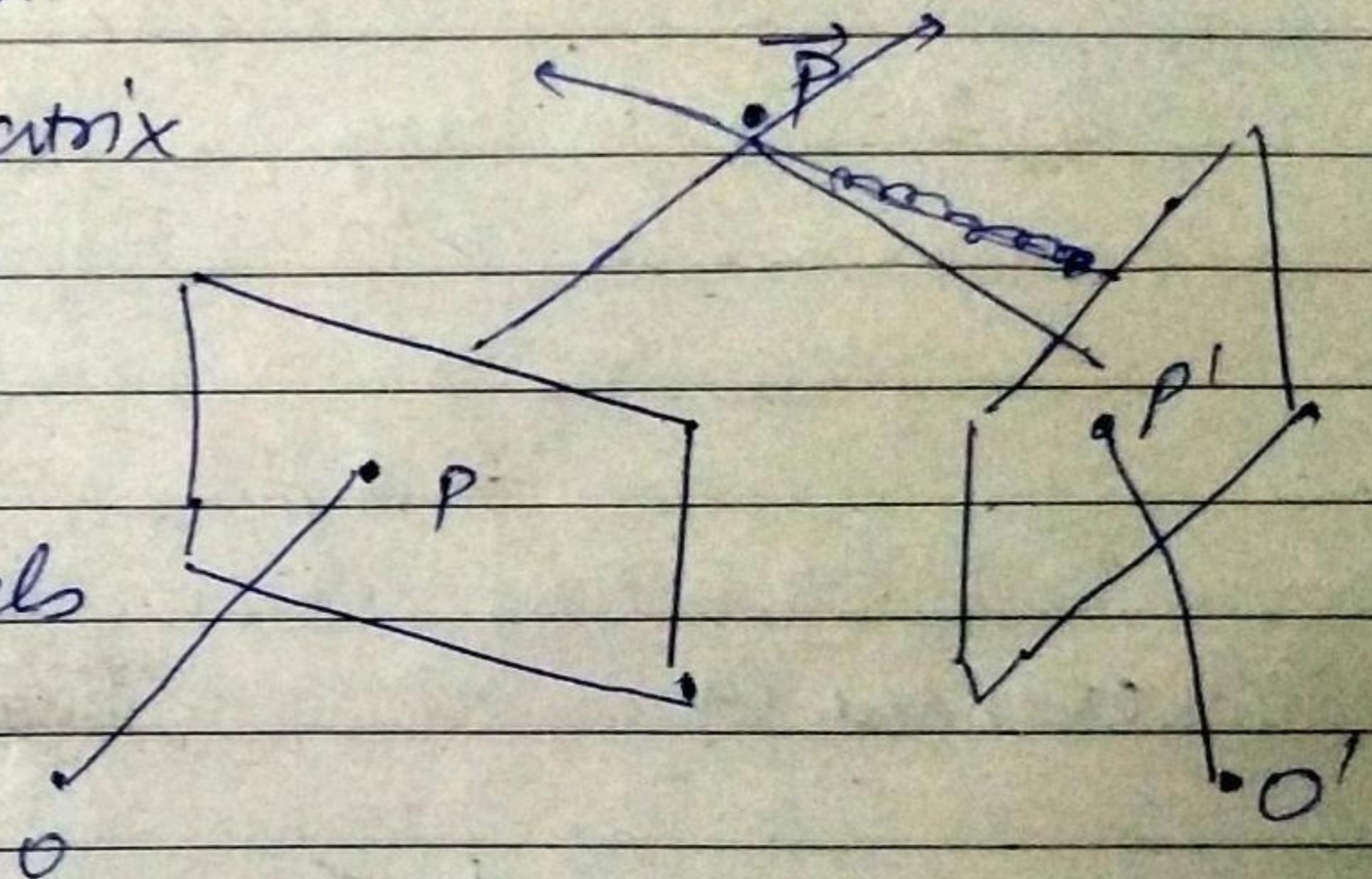
- \* Fundamental Matrix relates two corresponding pixels in the Original Image. Have 9 Degree of freedom. whereas
- \* Essential Matrix relates two corresponding pixels in the Normalized Image. Have 5 Degree of Freedom.

Eg Fundamental Matrix

$$\boxed{P^T F P' = 0}$$

Here  $P$  and  $P'$  are original Image Pixels

$$K^T P \parallel \bar{P}$$



For Essential Matrix

~~$$P \in \mathbb{R}^{3 \times n}, K^{-1} E K P$$~~

~~$$P \in \mathbb{R}^{3 \times n}, [R | t] P$$~~

$$\boxed{P_1^T E P_1' = 0}$$

Here  $P_1 \parallel \bar{P}'$

\*  $F = (K^{-1})^T E K^{-1}$  OR  $K^T F K = E$

### Q3 Camera Calibration

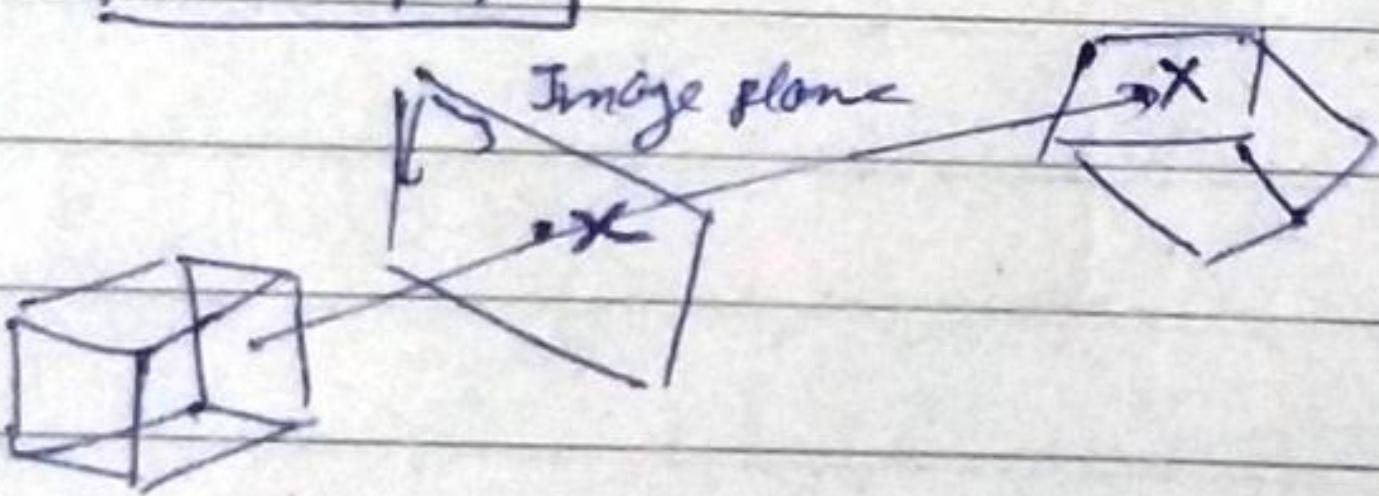
(a) DLT Algorithm proof :-

1) DLT Algorithm is used for Estimating the pose of a camera given coordinates of object point and coordinates (mug) of those object points in an image.

DLT maps any object point  $x$  to image point  $x_i$

$$x = KR [I_3 | -x_o] x$$

$$x = Px$$



$$\begin{aligned} x_{3 \times 1} &= \underbrace{K}_{3 \times 3} \underbrace{R}_{3 \times 3} \underbrace{[I_3 | -x_o]}_{3 \times 3} \underbrace{x}_{4 \times 1} \\ &= P \underbrace{x}_{4 \times 1} \end{aligned}$$

$K \rightarrow$  Intrinsic parameters of camera

$x_o \rightarrow$  origin of camera

$R \rightarrow$  Rotation matrix

$K \rightarrow c, s, m, X_H, Y_H \rightarrow$  5 parameters

$x_o \rightarrow$  3 translation parameters

$R \rightarrow$  3 Rotation parameters.

So we require to use

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = P \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}$$

Normalising the points

$$\begin{bmatrix} u/w \\ v/w \\ 1 \end{bmatrix} = P \begin{bmatrix} \cancel{u/w} \\ \cancel{v/w} \\ \cancel{w} \\ 1 \end{bmatrix} \begin{bmatrix} x/t \\ y/t \\ z/t \\ 1 \end{bmatrix} \quad - \textcircled{1}$$

Let  $P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$

$\therefore$  Rewriting eq<sup>n</sup> ①

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} \quad - \textcircled{2}$$

Solving eq<sup>n</sup> ② we get

$$x = \frac{p_{11}x + p_{12}y + p_{13}z + p_{14}}{p_{31}x + p_{32}y + p_{33}z + p_{34}} \quad - \textcircled{II}$$

$$y = \frac{p_{21}x + p_{22}y + p_{23}z + p_{24}}{p_{31}x + p_{32}y + p_{33}z + p_{34}} \quad - \textcircled{III}$$

So for uncalibrated camera we have 11 unknowns  
and DLT is used

So we see from any particular  $x_i, X_i$  corresponding points we get 2 eqn i.e. eq II, III.

So to solve for 11 parameters we require minimum  
 $\{(II_2) = 55\}$  6 points.

So let we have 6 points  $(x_1, x_2, x_3, x_4, x_5, x_6)$   
 $(X_1, X_2, X_3, X_4, X_5, X_6)$  respectively correspondence

$$\therefore x_i = \underset{3 \times 4}{P} X_i = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} X_i$$

Let  $A = \begin{bmatrix} P_{11} \\ P_{12} \\ P_{13} \\ P_{14} \end{bmatrix}, B = \begin{bmatrix} P_{21} \\ P_{22} \\ P_{23} \\ P_{24} \end{bmatrix}, C = \begin{bmatrix} P_{31} \\ P_{32} \\ P_{33} \\ P_{34} \end{bmatrix}$

$$\Rightarrow \therefore x_i = \begin{bmatrix} A^T \\ B^T \\ C^T \end{bmatrix} X_i$$

$\Rightarrow$ 

Multiplying RHS

 $\Rightarrow$ 

$$x_i = \begin{bmatrix} A^T x_i \\ B^T x_i \\ C^T x_i \end{bmatrix}$$

as  $x_i$  is a homogenous coordinates  
Finding Euclidean coordinates

 $\Rightarrow \therefore$ 

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} A^T x_i \\ B^T x_i \\ C^T x_i \end{bmatrix}$$

Comparing LHS = RHS

 $\Rightarrow$ 

$$x_i = \frac{u_i}{w_i} = \frac{A^T x_i}{C^T x_i} \quad | \quad y_i = \frac{v_i}{w_i} = \frac{B^T x_i}{C^T x_i}$$

$$\Rightarrow \therefore x_i C^T x_i - A^T x_i = 0 \quad | \quad y_i C^T x_i - B^T x_i = 0 \quad \text{---(3)}$$

$\Rightarrow$  This leads to system of equation linear  
in  $A, B, C$ .

 $\Rightarrow$  we rewrite equation (3) as $\Rightarrow$ 

$$-x_i^T A + x_i x_i^T C = 0$$

$$-x_i^T B + y_i x_i^T C = 0$$

} we are writing  
like this because  
to match it with  
system of linear  
equation where  
coefficients are known  
and  $A, B, C$  are unknown

Now let  $p$  be a vector such that

$$p = (pk) = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \text{vec}(PT)$$

$\Downarrow$        $12 \times 1$

rows of  $P$  as  
column-vectors  
one below the other

$\Rightarrow$  So now we can rewrite system of equation derived in eq<sup>n</sup> (3) as

$$-x_i^T A + x_i x_i^T C = 0 \rightarrow a_{xi}^T p = 0$$

$$-x_i^T B + y_i x_i^T C = 0 \rightarrow a_{yi}^T p = 0$$

$$\Rightarrow a_{xi}^T \begin{bmatrix} A \\ B \\ C \end{bmatrix} = -x_i^T A + x_i x_i^T C$$

And

$$a_{yi}^T \begin{bmatrix} A \\ B \\ C \end{bmatrix} = -x_i^T B + y_i x_i^T C$$

$\Rightarrow$  Comparing LHS and RHS above

$$a_{xi}^T = (-x_i^T, 0^T, x_i x_i^T)$$

$$a_{yi}^T = (0^T, -x_i^T, y_i x_i^T)$$

$\Rightarrow$ 

i.e.

$$\mathbf{a}_{x_i}^T = (-x_i, -y_i, -z_i, -1, 0, 0, 0, 0, x_i x_i, y_i y_i, z_i z_i, n_i)$$

$$\mathbf{a}_{y_i}^T = (0, 0, 0, 0, -x_i, -y_i, -z_i, -1, y_i x_i, y_i y_i, y_i z_i, y_i)$$

\* This is the ~~post~~ step which will fail if all the corresponding points lie on a plane.

In the next step we will be collecting everything together.

i.e. for all  $(n_1, n_2, \dots, n_e)$  number of points

$$\left[ \begin{array}{c} \mathbf{a}_{n_1}^T \\ \mathbf{a}_{y_1}^T \\ \mathbf{a}_{z_1}^T \\ \vdots \\ \mathbf{a}_{n_e}^T \end{array} \right] \quad P = M_{24 \times 12} \quad P = 0_{12 \times 1}$$

$$\text{So } M = \left[ \begin{array}{ccccccccc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ -x_1 & -y_1 & -z_1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -x_1 & -y_1 & -z_1 & -1 & y_1 x_1 & y_1 y_1 & y_1 z_1 \\ \vdots & \vdots \end{array} \right]$$

If all points lie on a plane.

$$\text{e.g. } z = 0$$

then column 3, 8, 10 will become 0

So only 9 columns will remain non-zero which will lead to rank deficiency.

$\Rightarrow M\hat{p} = 0$   
 This system is called Homogeneous system  
 and could be found using SVD i.e. finding  
 Null space of matrix  $M$ .

$\Rightarrow$  Due to redundant observation ( $M\hat{p} \neq 0$ )  
 $M\hat{p} = w$  - (5)

So we have to find  $p$  which minimizes  $w \cdot \text{or}$   
 $w^T w$ .

$$\text{Let } Q = w w^T$$

$$\hat{p} = \underset{P}{\arg \min} w^T w$$

From (5)

$$\hat{p} = \arg \min p^T M^T M p$$

So we find this using SVD on  $M$

$$M = U \begin{matrix} S \\ 2 \times 12 \end{matrix} V^T = \sum_{i=1}^{12} s_i v_i v_i^T$$

corresponding to least eigen value

Eigen singular vector  $v_{12}$  in  $V$  is the vector  
 which minimizes singular value. So we select  $v_{12}$   
 as singular vector.

$$\text{So } p = v_{12} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_{34} \end{bmatrix}$$

which we can arrange as

$$P_{34} = \begin{bmatrix} p_{11} & -p_{14} \\ -p_{34} & \end{bmatrix}$$

⇒ Deriving Intrinsic and Extrinsic parameters.

$$P_{34} = [KR \mid -KRX_0] = [H \mid h]$$

$$H = KR \quad h = -KRX_0$$

We have  $H$  and  $h$  now

$$\text{so } \boxed{X_0 = -H^{-1}h}$$

For finding  $K$  and  $R$

we can use  $g^u$   $H = KR$

Decompose  $H^u$  using QR Decomposition matrix.

as  $K \rightarrow$  triangular matrix

$R \rightarrow$  Rotation Matrix.

∴ Hence DTF proof.

Coding Question :

(1)  $f(x) = e^{-ax} \sin nx + b$

parameters  $(a, b)$

Jacobian =  $\begin{bmatrix} \frac{\partial f}{\partial a} & \frac{\partial f}{\partial b} \\ -x e^{-ax} \sin nx & 1 \end{bmatrix}$

Q(2)

(A) Let  $C_1$  and  $C_2$  be two camera frames

Let there be a point  $X_i$  in 3D whose homogeneous coordinates w.r.t.  $C_1$  and  $C_2$  be  $x_{ic_1}, x_{ic_2}$  respectively.

And they form image point for  $x_{ic_1}, x_{ic_2}$  at  $x_{ic_1'}, x_{ic_2'}$  respectively.

$$\therefore x_{ic_1'} = K x_{ic_1} \quad \text{--- (1)}$$

$$x_{ic_2'} = K x_{ic_2} \quad \text{--- (2)}$$

We know  $T$  be the transformation matrix from  $C_1$  to  $C_2$  in pure rotation

$$\therefore T = [R \ 0]$$

putting in eqns

$$x_{ic_2} = T x_{ic_1} = [R \ 0] x_{ic_1} = R x_{ic_1} \quad \text{--- (3)}$$

from eqn (1), (2), (3)

$$x_{ic_2'} = K x_{ic_2} \Rightarrow x_{ic_2'} = K R x_{ic_1}$$

$$\text{As } X_{ic_1} = K^{-1} x_{ic_1}$$

$$x_{ic_2'} = K R K^{-1} x_{ic_1}$$

Putting  $H = K R K^{-1}$

$$\boxed{x_{i_2} = H x_{i_1}}$$

$H$  is the Homography Matrix.

This homography matrix does not hold for translation as in that case epipolar geometry is involved.

In case of Translation Matrix

$$T = [R \ t]$$

∴ putting in eqn's derived above we get

$$x_{i_2} = [R \ t] x_{i_1} = Rx_{i_1} + t$$

$$\boxed{x_{i_2} = H x_{i_1} + Kt}$$

is no longer a Homography eqn.

(b)

Three ways to relate pixels are:

(i) Homography:

In this formation, frames  $I_1$  and  $I_2$  are specified by rotation  $R$  from  $I_1$  to  $I_2$ .

Relation given is

$$\underline{x_{i2} = (K_2 R K_1^{-1}) x_{i1}}$$

$K_1$  and  $K_2$  are intrinsic parameters.

### (2) Epipolar geometry:

Here we have matrix called Fundamental Matrix

Relection i.e.  $\underline{x_{i1}^T F x_{i2} = 0}$

where  $F = K_1^{-T} [T_x] R K_2^{-1}$

$K_1 - K_2 \rightarrow$  intrinsic camera calibration matrix for Camera 1 and 2.

$[T_x]$  is a skew-symmetric matrix

(3) Stereo :- representation of translation b/w  $I_1$  &  $I_2$

$$[T_x] R = E \text{ i.e. } \text{essential matrix}$$

(3) Stereo:-

When  $I_1$  and  $I_2$  are separated by translation i.e.  $R = 0$ ,  $t = (T, 0, 0)$  we have 11' epipolar lines and epipoles lie at  $\infty$ .

For given  $x_{i1}, x_{i2}$  can be directly related as both of their y-coordinate will be same & we have to search for  $x_{j2}$  on same row as  $x_{i1}$ .

so Mathematically

$$\Rightarrow R = I \quad \& \quad t = (T, 0, 0)$$

$$\Rightarrow E = t \times R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$$\Rightarrow x_{i1}^T K_i^{-T} E K_2^{-1} x_{j2} = 0 \quad [x_{i1}^T F n_{i1} = 0]$$

$$x_{i1}^T K_i^{-1} = [v \ v' \ 1]$$

$$\& \quad K_2^{-1} x_{i2} = (v' \ v' \ 1)^T$$

$$[v \ v' \ 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} v' \\ v' \\ 1 \end{bmatrix} = 0$$

$$TV = TV' \Rightarrow \text{same } y\text{-coordinates}$$

(c)

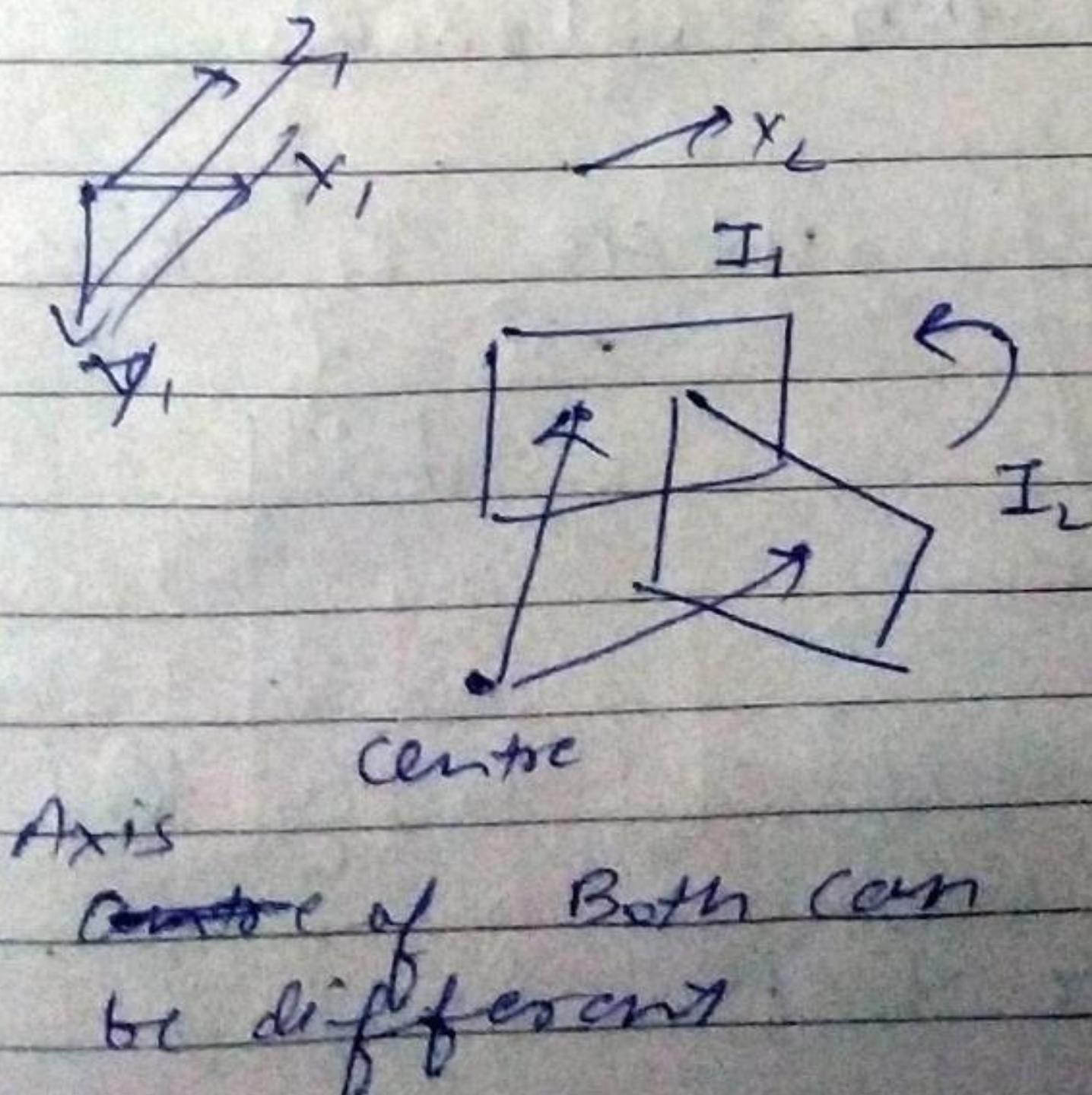
In stereo rectification, we project the two images formed by cameras on plane lie to line between camera centres.

For that we need two homographies one for each input image reprojection.

(ii)  $H_1 \rightarrow$  Rotation Homography is applied to only second frame  $I_2$  such that it becomes parallel to  $I_1$  and this Homography can be found by finding essential matrix  $E$  using 8 point algorithm & then decompose it to get homography  $H$  i.e.  $R$ .

$$\text{So eqn } \boxed{X_{i_2} = R X_{i_1}}$$

Now as  $I_1$  &  $I_2$  are parallel i.e. their axis & image plane are parallel.

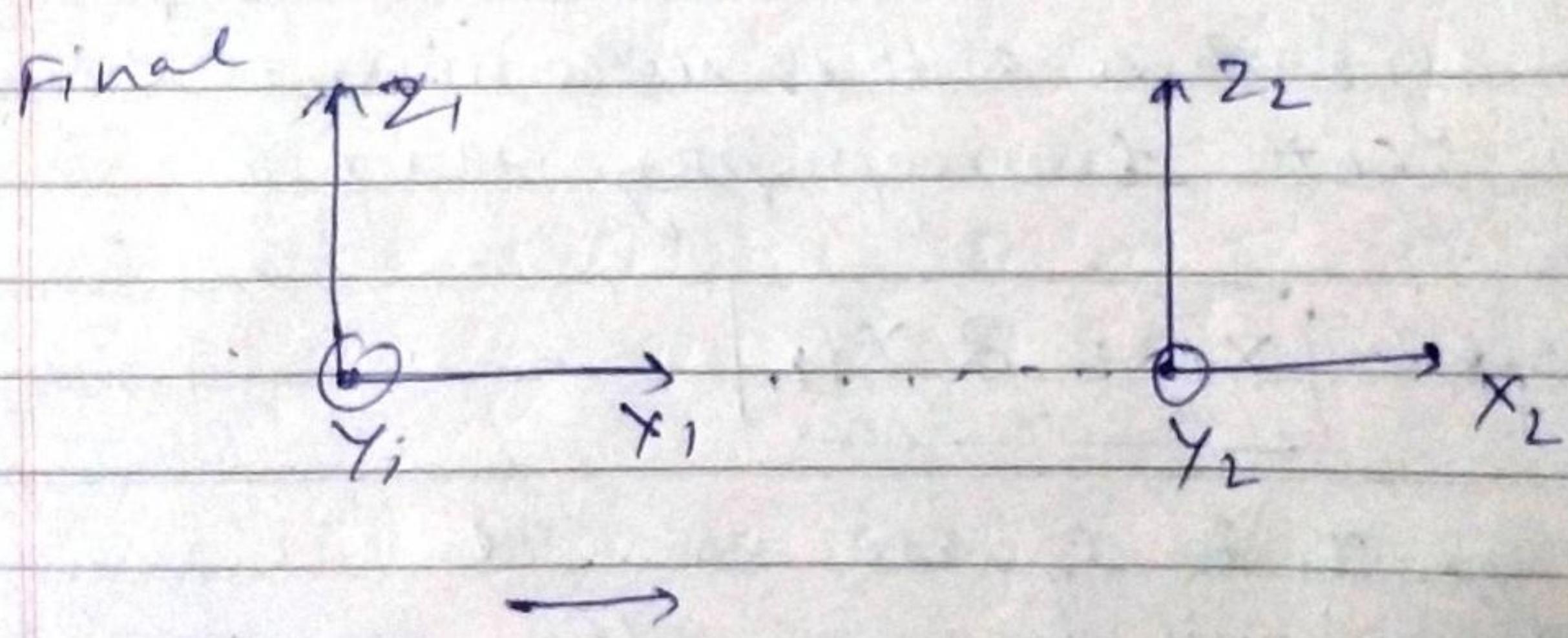
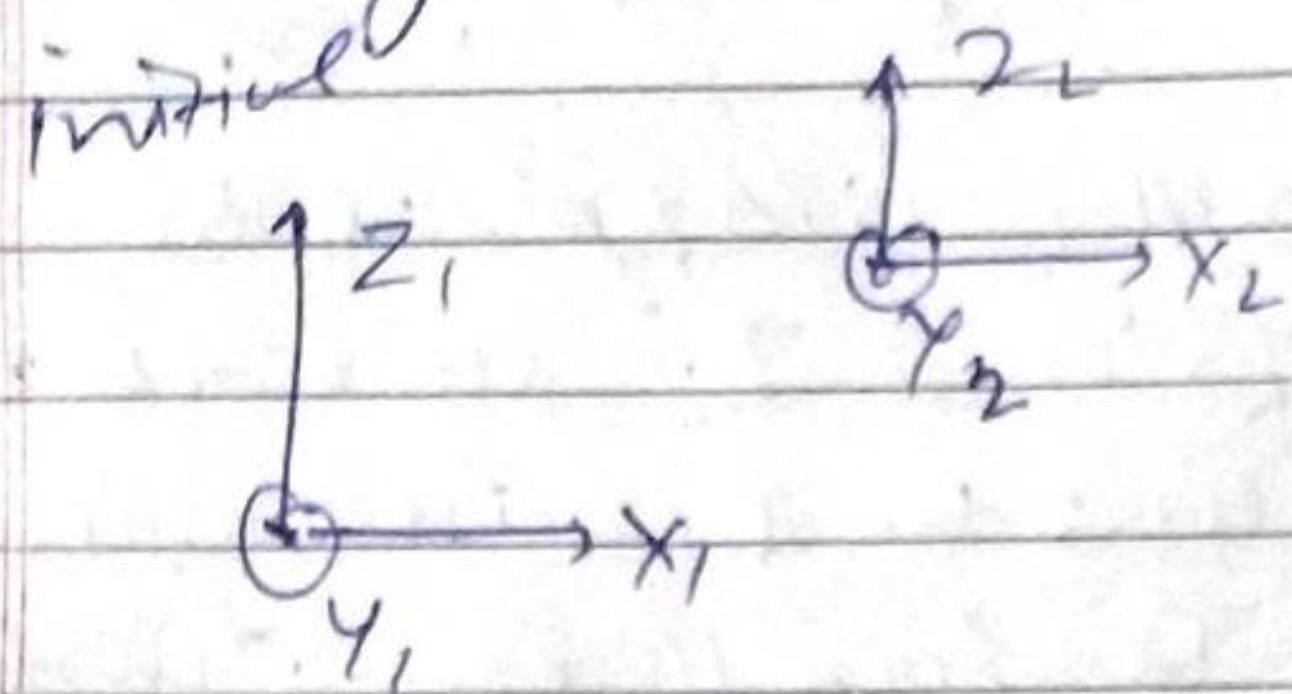


$\Rightarrow$  Applying rot.  $R$  or we can say rotational homography that relates two camera that suffer from pure rotation w.r.t. each other

$$\boxed{H = R K K^{-1}}$$

## Homography 2

making the axis of ~~I<sub>1</sub>~~ & I<sub>2</sub> colinear  
 i.e. epipoles at  $\infty$  and epipolar lines  
 become ll'. and the transformed  
 frame only be separated by translation  
 along x-axis.



$$\text{translation} = (T, 0, 0)$$

$$\text{Let } R_{\text{rect}} = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \quad \text{where } \mathbf{r}_1^T = K^{-1} \mathbf{e}_2 \\ = \hat{\mathbf{t}} = \frac{\vec{\mathbf{t}}}{\|\vec{\mathbf{t}}\|}$$

$$\therefore \mathbf{r}_2^T = \hat{\mathbf{t}} \cdot [0 \ 0 \ 1]^T = [\hat{t}_x \ \hat{t}_y \ \hat{t}_z]^T$$

$$\text{where } \hat{\mathbf{t}} = [\hat{t}_x \ \hat{t}_y \ \hat{t}_z]$$

$\delta_3^T = n^T \times r_2^T$  as Rrect is orthogonal

$$\therefore \text{Rect } K^{-1} \hat{e}_2 = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix} \quad K^{-1} \hat{e}_2 = \begin{bmatrix} \|K^{-1} \hat{e}_2\|^2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$  image of the point  $[1 \ 0 \ 0]^T =$

$$K[T \ 0] \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = K \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} f_x \\ 0 \\ 0 \\ 0 \end{bmatrix} = f \times \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} =$$

point at  
noo in the  
image

This is precisely what we want i.e.  
epipole at  $\infty$

So

we found homography

$$H_2 = K \text{Rect } K^{-1}$$

that takes  
 $e_2 = [e_{2x} \ e_{2y} \ 1]^T$  to  $[1 \ 0 \ 0]^T$  i.e. pt at  $\infty$

$\therefore$  find  $m_{ii}, m_{ij}$

$$x_{ii} \rightarrow (K \text{Rect } K^{-1}) x_{ii}$$

$$n_{ji} \rightarrow (K \text{Rect } K^{-1}) n_{ji}$$

Rotation  $R_{de Rot}$  corresponds to Homography  $H_1 \& H_2$  as they simply rotate the frame. Here frames are from same camera but from different angle which is definition of Homography. Hence ~~they are called~~

CPM  
→

(a) Variables :-

Robot state  $\Rightarrow u_0, u_1, u_2, \dots, u_n$

where  $u_i = [u_{xi}, u_{yi}, u_{\theta i}]^T$

$u_1, u_2, u_3, \dots, u_n \rightarrow$  controls  
typically  $u_i = [\Delta\phi, T]^T$

$\Delta\phi$  and Move by  $T$

$l_1, l_2, l_3, \dots, l_m \rightarrow$  landmarks

$l_i = [u_{mix}, e_{mix}]^T$

Motion Model

$$\hat{u}_{i+1} = f(\hat{u}_i, e_{i+1}) = \begin{bmatrix} \hat{u}_{xi} \\ \hat{u}_{yi} \\ \hat{\theta}_i \end{bmatrix} + \begin{bmatrix} T \cos(\hat{\theta}_i + \Delta\phi) \\ T \sin(\hat{\theta}_i + \Delta\phi) \\ \Delta\phi \end{bmatrix}$$

↓  
typically from odometry

SAM optimizes cost function

$$\sum_{i=0}^{n+1} \| f(\hat{u}_i, u_{i+1}) - \hat{u}_{i+1} \|^2_2$$

$$+ \sum_{i=1}^n \sum_{k=1}^m \| z_{ik} - \hat{z}_{ik} \|^2_2 \rightarrow \textcircled{4}$$

Linearising odometry & term:-

$$OT = f(\hat{u}_i, u_{i+1}) - \hat{u}_{i+1} = F_i \Delta \hat{u}_i - a_i$$

where  $a_i = f(\hat{u}_i, u_{i+1})$

$a_i$  is the odometry error term and  
 $F_i$  is the  $3 \times 3$  motion Jacobian

Linearising measurement term:-

$$MT = h(\hat{u}_i, \hat{u}_{mk}) + \frac{\partial h}{\partial u_i} s_{ui} + \frac{\partial h}{\partial u_{mk}} s_{umk} \\ - z_{ik}$$

$$= h(u_i, \hat{u}_{mk}) + H_{ik} \Delta \hat{u}_i + J_{ik} \Delta \hat{u}_{mk} - z_{ik}$$

$$= H_{ik} \hat{s}_{ui} + J_{ik} \hat{s}_{umk} - c_{ik}$$

where  $c_{ik} = h(u_i - \hat{u}_{mk})$  is the measurement  
 and  $\hat{u}_i, \hat{u}_{mk} \rightarrow$  current estimate of  $u_i, u_{mk}$   
 & if it is correct  $c_{ik} \rightarrow 0$

$$\text{so, } MT = H_{ik} \hat{s}_{ui} + J_{ik} \hat{s}_{umk} - c_{ik}$$

Hence, eq  $\textcircled{4}$  becomes:-

$$\sum_{i=1}^{n_1} \| F_i \hat{s}_{ii} - a_i \|_2^2 + \sum_{i=1}^m \sum_{k=1}^{n_2} \| H_{ik} \hat{s}_{ii} + J_{ik} s_{ii} \|^2$$

where  $F_i$  is  $3 \times 3$  motion Jacobian

- (5)

$H_{ik}$  is  $3 \times 3$  landmark Jacobian w.r.t.

poses

$J_{ik}$  is  $3 \times 3$  landmark Jacobian w.r.t. landmarks

rewriting eqn (5)

$$\begin{bmatrix}
 F_1 & 0 & \dots & \dots & \dots & 0 \\
 H_{11} & 0 & 0 & I_{11} & 0 & \\
 H_{12} & 0 & 0 & 0 & I_{12} & 0 \\
 \vdots & & & & & \\
 H_{1m} & 0 & & & I_{1m} & \\
 0 & F_2 & & & & \\
 0 & H_{21} & 0 & I_{21} & 0 & \\
 \vdots & & & & & \\
 0 & H_{2m} & 0 & 0 & I_{2m} & \\
 \vdots & & & & & \\
 0 & F_N & 0 & 0 & 0 & \\
 0 & H_{N1} & J_M & 0 & & \\
 \vdots & & & & & \\
 0 & H_{Nm} & & & I_{Nm} & \\
 \end{bmatrix}
 \begin{bmatrix}
 \hat{s}_{11} \\
 \hat{s}_{22} \\
 \vdots \\
 \hat{s}_{nn} \\
 \end{bmatrix}
 = \begin{bmatrix}
 a_1 \\
 c_1 \\
 \vdots \\
 c_m \\
 \vdots \\
 c_{nm} \\
 \end{bmatrix}$$

$(n+m) \times 1$

$m(n+1) \times (m+1)$

(QW)

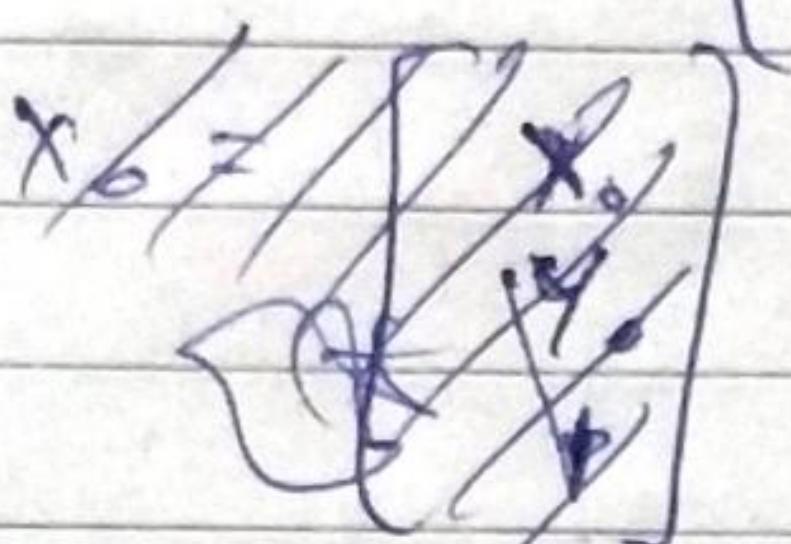
(b)

Given RGB images across trajectory

~~y<sub>0</sub>, f<sub>0</sub>~~

Let the starting point of trajectory will be  $x_0$ .

i.e.  $x_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix}$  or ~~in homogeneous coordinates~~



$$x_0 = f^{-1}x_1$$

Firstly we will take two ~~consecut~~ consecutive trajectory images. And using SIFT for image matching we will find the corresponding points in the two images.

Let Corresponding points be

$$\{x_{i,j}, y_{i,j}\}_{i=1}^n \Leftrightarrow \{x_i^j, y_i^j\}_{i=1}^n$$

So after finding the points we use SVD to find the Fundamental matrix between the two images.

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As we find Fundamental matrix  $\alpha$

so  $|X^T F X| = 0$

using  $F$  matrix and triangulation, we can now get a rough estimation of landmark postures.

So now we know  $F$  and  $K$  we can compute e and qr decomposition, which will give us  $R$  and  $T$  estimations between camera postures. We obtain four potential camera stances. However we can estimate poses when neither camera has the point on the front. We now have an approximate estimate for the camera postures and 3D coordinates.



for the same landmark locations,  
and we will use bundle adjustment  
to specify these coordinates.