Grau en Enginyeria Informàtica
Facultat d'Informàtica de Barcelona
Universitat Politècnica de Catalunya

MATEMÀTIQUES 1

Part I: Teoria de grafs

Respostes a alguns exercicis

Curs 2022-2023(2)

Aquest document conté les respostes a alguns dels problemes de la segona part de l'assignatura Matemàtiques 1. Aprofitem per fer constar i agrair la tasca del becari docent Gabriel Bernardino en la redacció de les solucions.

Us ho agraïrem si ens comuniqueu qualsevol errada que detecteu.

Anna de Mier
Montserrat Maureso
Dept. Matemàtiques

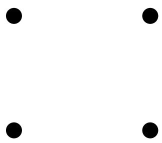
Respostes

Conceptes bàsics de grafs

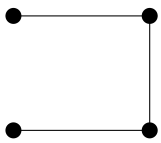
1.1

1)

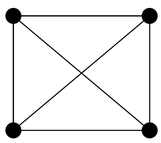
N_4



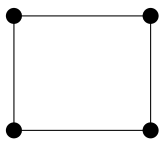
T_4



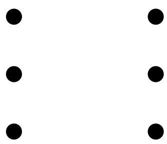
K_4



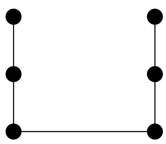
C_4



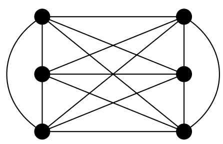
N_6



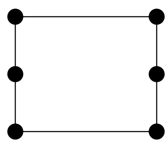
T_6



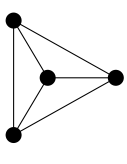
K_6



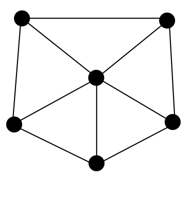
C_6



W_4



W_6



2)

$$M_A(N_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad M_A(K_5) = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

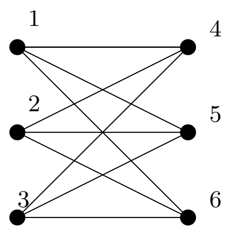
$$M_A(T_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad M_A(C_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$M_A(W_5) = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

3) Per a $n \geq 3$ ($n \geq 4$ en el cas del graf W_n):
$N_n = (V,E) : \; |V| = n, \; |E| = 0, \delta(N_n) = 0, \; \Delta(N_n) = 0$
$K_n = (V,E) : \; |V| = n, \; |E| = \binom{n}{2} = \frac{n(n-1)}{2}, \; \delta(K_n) = n-1, \; \Delta(K_n) = n-1$
$T_n = (V,E) : \; |V| = n, \; |E| = n-1, \; \delta(T_n) = 1, \; \Delta(T_n) = 2$
$C_n = (V,E) : \; |V| = n, \; |E| = n, \; \delta(C_n) = 2, \; \Delta(C_n) = 2$
$W_n = (V,E) : \; |V| = n, \; |E| = 2n-2, \; \delta(W_n) = 3, \; \Delta(W_n) = n-1$

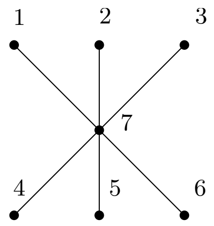
1.2

- Solució d' 1. i 2.



1	2	3	4	5	6
4	4	4	1	1	1
5	5	5	2	2	2
6	6	6	3	3	3

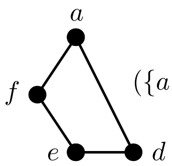
- Solució de 3. i 4.



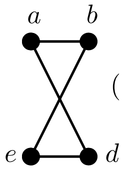
1	2	3	4	5	6	7
7	7	7	7	7	7	1
						2
						3
						4
						5
						6

1.4 1) $\frac{r \cdot n}{2}$; 2) $r \cdot s$;

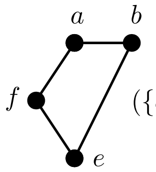
1.5



$(\{a, d, e, f\}, \{ad, de, ef, fa\})$



$(\{a, b, e, d\}, \{ab, be, ed, da\})$



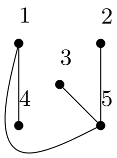
$(\{a, b, e, f\}, \{ab, be, ef, fa\})$

1.6

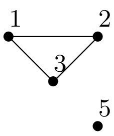
1) 5; 4. 2) 4; 2. 3) 5; 5. 4) 9; 8.

1.7

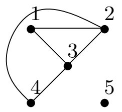
G^c: $A = \{14, 15, 25, 35\}$; $M_A = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$;



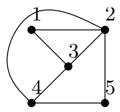
$G - 4$: $A = \{12, 13, 23\}$; $M_A(G-4) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$;



$G-45$: $\quad A=\{12,13,23,24,34\}$; $\quad M_A(G-45)=\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$;



$G+25$: $\quad A=\{12,13,23,24,25,34,45\}$; $\quad M_A(G+25)=\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$;

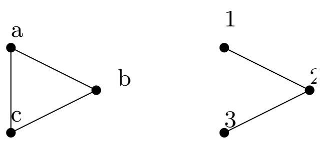


1.8

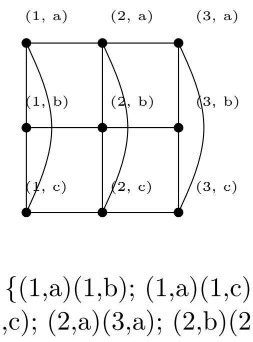
- $G^c=(V,A)$: $|V|=n$; $|A|=\binom{n}{2}-m=\frac{n(n-1)}{2}-m$.
- $G-v=(V,A)$: $|V|=n-1$; $|A|=m-g(u)$.
- $G-a=(V,A)$: $|V|=n$; $|A|=m-1$.

1.10

1) $K_3\cup T_3$ $\qquad A=\{ab,ac,bc,12,23\}$



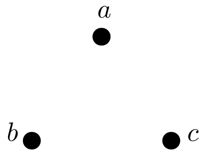
2) $T_3\times K_3$



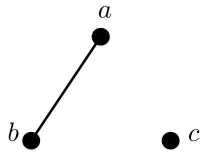
$A=\{$(1,a)(1,b); (1,a)(1,c); (1,a)(2,a); (1,b)(1,c); (1, b)(2,b); (1,c)(2,c); (2,a)(2,b); (2,a)(2,c); (2,a)(3,a); (2,b)(2,c); (2,b)(3,b); (2,c)(3,c); (3,a)(3,b); (3,a)(3,c); (3,b)(3,c)$\}$

1.11 Ordre $|V_1||V_2|$, $g_{G_1\times G_2}(u_1,u_2)=g_{G_1}(u_1)+g_{G_2}(u_2)$ i mida $|V_1||A_2|+|V_2||A_1|$.

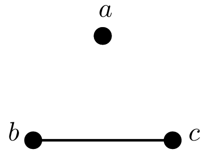
1.13



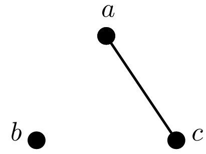
$(V, \emptyset)$



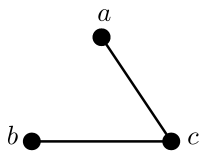
$(V, \{ab\})$



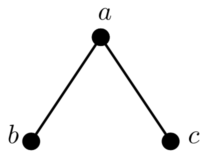
$(V, \{bc\})$



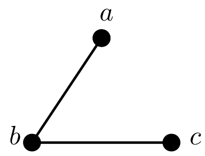
$(V, \{ca\})$



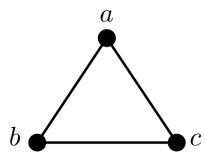
$(V, \{bc, ca\})$



$(V, \{ca, ab\})$



$(V, \{ab, bc\})$

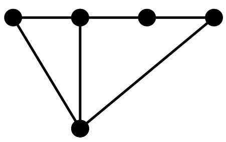


$(V, \{ab, bc, ca\})$

1.14 21; $2^{21} = 2097152$.

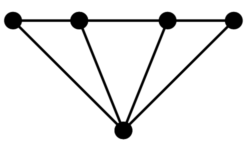
1.15

1)



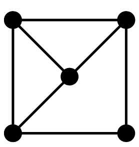
2) No existeix.

3)



4) No existeix.

5)

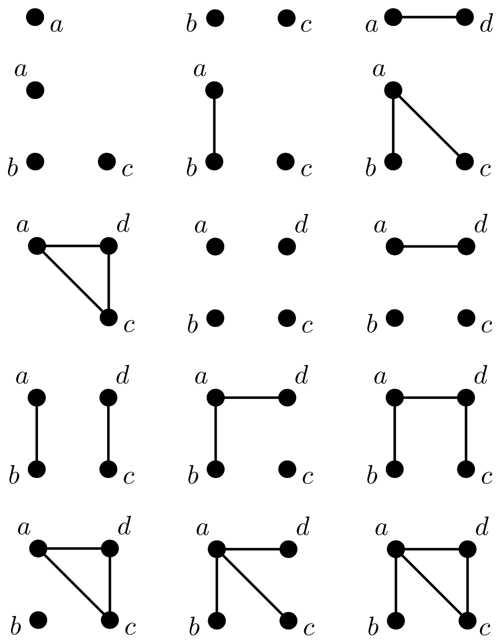


6) No existeix.

1.20 4 l'Aran i 4 la parella.

1.21

1.22



1.23

- $G_1 \cong G_2$
- $G_3 \cong G_4$
- $G_5 \cong G_6$
- G_7
- $G_8 \cong G_9 \cong G_{10}$
- G_{11}
- G_{12}
- G_{13}

1.25 2.

Recorreguts, connexió i distància

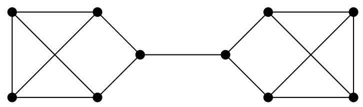
2.1 G_1: Camí de longitud 9: 1 2 3 4 5 10 7 9 6 8. No hi ha camins de longitud 11 ja que té ordre 10. Cicles: 1 2 3 4 5 1; 1 2 3 8 10 5 1; 1 6 8 10 7 9 4 5 1; 1 2 3 4 9 7 10 8 6 1.
G_2: 1 2 3 4 5 10 6 7 8 9. No hi ha camins de longitud 11 ja que té ordre 11. Cicles: 1 2 3 4 5 1; 5 10 6 11 9 4 5; 2 3 4 5 10 9 8 7 2; 5 1 2 3 4 9 11 6 10 5.

2.4 1) $\langle\{a,b,d,e,f,g,i,j\}\rangle \bigcup \langle\{c,h\}\rangle$. 2) $\langle\{a,b,d,e,g,h,j,m\}\rangle \bigcup \langle\{c,f,i,k\}\rangle \bigcup \langle\{l\}\rangle$.

2.9

- G_1. Vèrtexs de tall: 4. Arestes pont: cap.
- G_2. Vèrtexs de tall: $3, 6$. Arestes pont: 36.
- G_3. Vèrtexs de tall: cap. Arestes pont: cap.

2.11 $n = 10$



2.14 1)

v	a	b	d	e	f	g	i	j
d(a,v)	0	2	1	1	1	3	3	3
d(b,v)	2	0	1	2	2	1	1	1

2)

v	a	b	d	e	g	h	j	m
d(a,v)	0	1	2	2	2	2	1	3
d(b,v)	1	0	1	1	1	1	1	2

2.15 1) 1. 2) $D(G_1) = 2$, $D(G_2) = 4$. 3) 2. 4) $\lfloor n/2 \rfloor$. 5) 2. 6) $n-1$.

2.16 1) $G = W_6$ i u un vèrtex de grau 3. 2) $G = W_7$, u el vèrtex de grau 6. 3) $G = ([4], \{12, 13, 14, 23\})$, $u = 4$.

2.17 1) a) G_1: $e(v) = 2$, $1 \leq v \leq 10$; $r(G_1) = 2$; tots els vèrtexs són centrals. El centre és, doncs, G_1. G_2: $e(1) = e(11) = 4$; $e(v) = 3$, si $2 \leq v \leq 10$; $r(G_2) = 3$; v és vèrtex central si $2 \leq v \leq 10$. El centre és el subgraf induït pel conjunt de vèrtexs $\{v : 2 \leq v \leq 10\}$, que és el mateix que $G_2 - \{1, 11\}$. b) G: $e(4) = 2$, $e(v) = 3$, $v \neq 4$; $r(G) = 2$; l'únic vèrtex central és el 4. Per tant, el centre és un graf trivial, K_1. 2) C_6. 3) T_5.

Grafs eulerians i hamiltonians

3.1 Només és eulerià el graf G_4

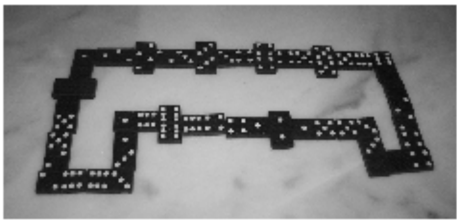
3.2 Tots, llevat del primer dibuix.

3.3 1) 5; 2) 4.

3.4 r i s parells.

3.5 Si els dos components són complets, 4; altrament, 3.

3.7



3.8 2) 2^n; $n2^{n-1}$; Q_n és n-regular. 3) n parell.

3.9 Només són hamiltonians els grafs G_1 i G_2.

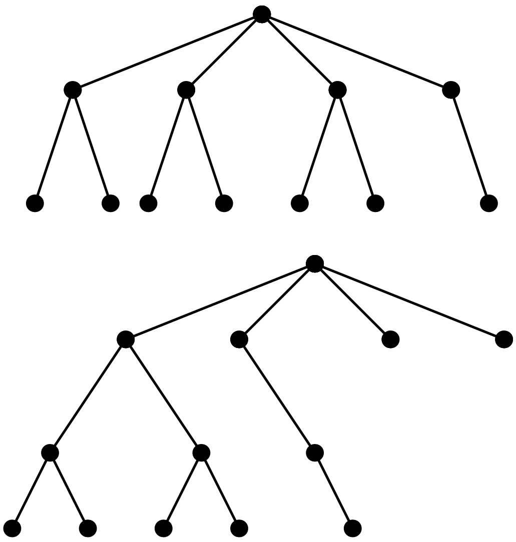
3.12 Dues.

Arbres

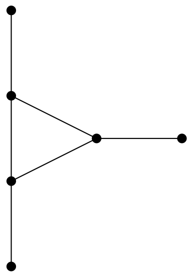
4.3 $n = 18$; ordre de T_2: 36; mida de T_2: 35.

4.4 1 i 3.

4.5 4,3,3,3,2,1,1,1,1,1,1,1.



4.6

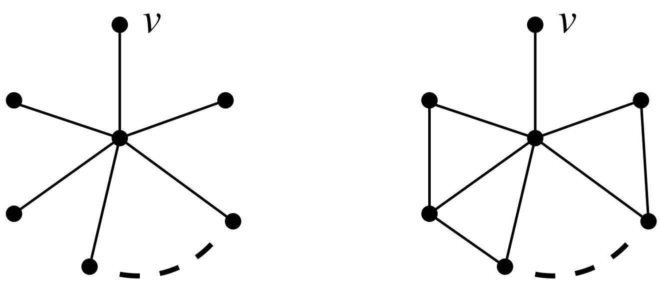


4.12

1) n; 1.

2) $r2^{r-1}$; $\lceil r/2 \rceil$.

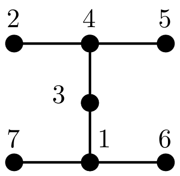
4.13



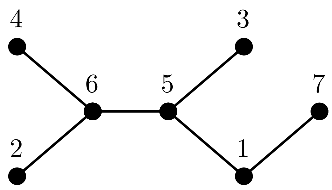
4.14 Dos.

4.16 $(1,1,1,5)$; $(1,1,2,2,2,1)$; $(3,3,1,2,4,4,2,5,5)$.

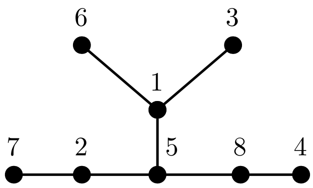
4.17 1)



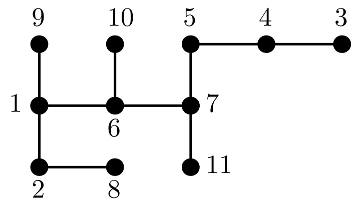
2)



3)



4)



4.18 Els trajectes d'ordre 3.

4.19

1) Els grafs estrella.

2) Els grafs "biestrella": dos vèrtexs u, v de grau almenys 2 adjacents i la resta de vèrtexs són fulles que pengen d'u o de v (o sigui, tant d'u com de v penja almenys 1 fulla!).

Excercicis de repàs i consolidació

A.1

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

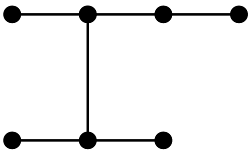
A.9 4 components connexos. 7,14,2,4,6,8,10,12,3,9,15,5.

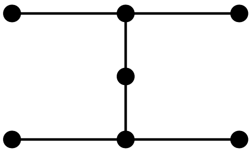
A.19 Sí; no.

A.22 K_1 i T_4.

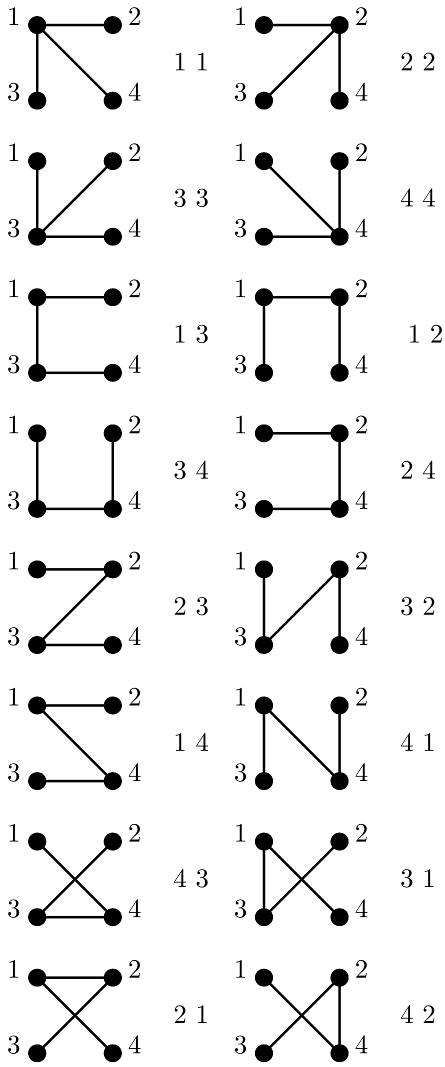
A.23 $k-1$.

A.24 3,3,2,1,1,1,1.





A.29



A.30 Els trajectes d'ordre $n \geq 4$.