Probabilitat i Estadística 1

Problemes Tema 7. Intervalos de confianza

1. In order to ensure efficient usage of a server, it is necessary to estimate the mean number of concurrent users (that at a given moment can be assumed to be normally distributed). According to records, the average number of concurrent users at 100 randomly selected times is 37.7, with a standard deviation $\sigma = 9.2$. Construct a 90% confidence interval for the expectation of the number of concurrent users.

   Resultat: $\mathrm{IC}_{1-\alpha}(\mu) \equiv [36.18673, 39.21327]$

2. Installation of a certain hardware takes random time with a standard deviation of 5 minutes and can be assumed to be normally distributed.

   (a) A computer technician installs this hardware on 64 different computers, with the average installation time of 42 minutes. Compute a 95% confidence interval for the population mean installation time.

   (b) Suppose that the population mean installation time is 40 minutes. A technician installs the hardware on your PC. What is the probability that the installation time will be within the interval computed in the previous point?

   Resultat: (a) $\mathrm{IC}_{1-\alpha}(\mu) \equiv [40.775, 43.225]$. (b) 0.1789471

3. Salaries of entry-level computer engineers have Normal distribution with unknown mean and variance. Three randomly selected computer engineers have salaries (in \$1000s):

   $$30, 50, 70$$

   Construct a 90% confidence interval for the average salary of an entry-level computer engineer.

   Resultat: $\mathrm{IC}_{1-\alpha}(\mu) \equiv [16.28291, 83.71709]$

4. We have to accept or reject a large shipment of items. For quality control purposes, we collect a sample of 200 items and find 24 defective items in it. Construct a 96% confidence interval for the proportion of defective items in the whole shipment.

   Resultat: $\mathrm{IC}_{1-\alpha}(p) \equiv [9.7\%, 14.3\%]$

5. An electronic parts factory produces resistors. Statistical analysis of the output suggests that resistances follow an approximately Normal distribution with a standard deviation of 0.2 ohms. A sample of 52 resistors has the average resistance of 0.62 ohms.

   (a) Based on these data, construct a 95% confidence interval for the population mean resistance.

   (b) If the actual population mean resistance is exactly 0.6 ohms, what is the probability that an average of 52 resistances is 0.62 ohms or higher?

   Resultat: (a) $\mathrm{IC}_{95\%}(\mu) \equiv [0.56563938, 0.67436062]$. (b) 0.2354208

6. A sample of 250 items from lot A contains 10 defective items, and a sample of 300 items from lot B is found to contain 18 defective items. Construct a 98% confidence interval for the difference of proportions of defective items.

   Resultat: $\mathrm{IC}_{1-\alpha}(p_A - p_B) \equiv [-0.063, 0.023]$.

7. A news agency publishes results of a recent poll. It reports that candidate A leads candidate B by 10% because 45% of the poll participants supported Ms. A whereas only 35% supported Mr. B. What margin of error should be reported for each of the listed estimates, 10%, 35%, and 45%? Notice that 900 people participated in the poll, and the reported margins of error typically correspond to 95% confidence intervals.

   Resultat: $\mathrm{IC}_{1-\alpha}(p_A) \equiv [0.417, 0.483]$, $\mathrm{IC}_{1-\alpha}(p_B) \equiv [0.319, 0.381]$, $\mathrm{IC}_{1-\alpha}(p_A - p_B) \equiv [0.042, 0.158]$

8. If $[A, B]$ is a $(1-\alpha)100\%$ confidence interval for the population variance (with $a \geq 0$), prove that $[\sqrt{A}, \sqrt{B}]$ is a $(1-\alpha)100\%$ confidence interval for the population standard deviation.