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Q1 $n = 10x$

$n^2 = 100x^2$

$n^2 = 25(4x^2)$

Q2 (a) Let P be the statement : " $n^2 + 4n + 5$ is odd "

Let Q be the statement : " n is even "

The Statement we have been given is in the form $P \rightarrow Q$. It's contrapositive is $\neg Q \rightarrow \neg P$ is:

" If n is not even, then $(n^2 + 4n + 5)$ is not odd."

More simply, the contrapositive says:

" If n is odd, then $(n^2 + 4n + 5)$ is even.

We will now prove that the statement

" If n is odd, then $(n^2 + 4n + 5)$ is even" is true

If n is odd, then $n = 2k + 1$ for some integer.

from this we get

$$n^2+4n+5 = (2k+1)^2+4(2k+1)+5$$
$$= 4k^2+4k+1+8k+4+5$$
$$= 4k^2+12k+10$$
$$= 2(2k^2+6k+5)$$

- Hence, n^2+4n+5 is a multiple of 2 and so it is even.

This proves that if n is odd, then n^2+4n+5 is even.
Therefore $\neg Q \to \neg P$ is true and hence the equivalent statement $P \to Q$ is also true.
That is: It's true that if n^2+4n+5 is odd, then n is even.

(b) Let P be the statement "n^2+4n+5 is odd" and let Q be the statement "n is even"

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The statement we want to prove is $P \Leftrightarrow Q$.
We will first show that $P \to Q$ is true.
We do this by assuming that P is true and showing that it follows that Q is true.

Then we show that $Q \to P$ is true. We do this by assuming that Q is true and showing that it follows that P is true.

We start by showing $P \to Q$ is true:

Let n be an integer such that n^2+4n+5 is odd.
Then $n^2+4n+5 = 2k+1$ for some integer k. ~~Hence $n = 2k+4$.~~ So

$$n^2+4n+5 = 2k+1$$

$$(2k+1)^2+4(2k+1)+5$$

$$= 4k^2+4k+1+8k+4+5$$

$$= 4k^2+12k+10$$

$$= 2(2k^2+6k+5)$$

We can conclude that n^2+4n+5 is even
i.e. we have shown that $P \to Q$ is true.

Next we show that $Q \to P$ is true. We do this by assuming Q is true and showing that it follows that P is true.
Let n be an integer such that n^2+4n+5 is even. Then $n^2+4n+5 = 2k$ for some integer k.

$n^2 + 4n + 5 = 2k$

$(2k)^2 + 4(2k) + 5$

$= 4k^2 + 8k + 5$

Hence, $n^2 + 4n + 5$ is not even given that n is odd

We can conclude that $P \to Q$ is not true