22/10/21

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Q1 (a) $x = 8$ $y = 6$ $z = 7$

$2(8) + 1(6) - 2(7) = b_1 = 8$

$2(8) + 3(6) + 1(7) = b_2 = 41$

$3(8) + 2(6) + 2(7) = b_3 = 50$

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(b) $\begin{cases} 2x + y - 2z = 8 \\ 2x + 3y + z = 41 \\ 3x + 2y + 2z = 50 \end{cases}$

Q2 (a) To start, I will convert the system into augmented matrix form:

$$\begin{pmatrix} 2 & 1 & -2 & 8 \\ 2 & 3 & 1 & 41 \\ 3 & 2 & 2 & 50 \end{pmatrix} \begin{matrix} \Rightarrow R_1 \\ \Rightarrow R_2 \\ \Rightarrow R_3 \end{matrix}$$

then, using the steps provided in Dr. Huggard's slides on Gaussian elimination, I will compute the above matrix into reduced-row echelon form:

$$\begin{pmatrix} 2 & 1 & -2 & 8 \\ 2 & 3 & 1 & 41 \\ 3 & 2 & 2 & 50 \end{pmatrix} \cdot \tfrac{1}{2}$$

multiplying R_1 by a constant to get a leading entry of 1

$$= \begin{pmatrix} 1 & \tfrac{1}{2} & -1 & 4 \\ 2 & 3 & 1 & 41 \\ 3 & 2 & 2 & 50 \end{pmatrix} \begin{matrix} \\ + (-2)R_1 \\ + (-3)R_1 \end{matrix}$$

adding multiples of R_1 to R_2 and R_3 so that all entries below the leading 1 are zero.

$$= \begin{pmatrix} 1 & \tfrac{1}{2} & -1 & 4 \\ 0 & 2 & 3 & 33 \\ 0 & \tfrac{1}{2} & 5 & 38 \end{pmatrix} \cdot (\tfrac{1}{2})$$

multiplying R_2 by a constant to get a leading entry of 1

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$$\left(\begin{array}{cccc} 1 & \frac{1}{2} & -1 & 4 \\ 0 & 1 & \frac{3}{2} & \frac{33}{2} \\ 0 & \frac{1}{2} & 5 & 38 \end{array}\right) \begin{array}{l} +\left(-\frac{1}{2}\right)R_2 \\ \\ +\left(-\frac{1}{2}\right)R_2 \end{array}$$

adding multiples of R_2 to R_1 and R_3 so that all entries above and below the leading 1 are zero.

$$\left(\begin{array}{cccc} 1 & 0 & -\frac{7}{4} & -\frac{17}{4} \\ 0 & 1 & \frac{3}{2} & \frac{33}{2} \\ 0 & 0 & \frac{17}{4} & \frac{119}{4} \end{array}\right) \cdot \frac{4}{17}$$

multiplying R_3 by a constant to get a leading entry of 1

$$\left(\begin{array}{cccc} 1 & 0 & -\frac{7}{4} & -\frac{17}{4} \\ 0 & 1 & \frac{3}{2} & \frac{33}{2} \\ 0 & 0 & 1 & 7 \end{array}\right) \begin{array}{l} +\frac{7}{4}R_3 \\ +\left(-\frac{3}{2}\right)R_3 \\ \\ \end{array}$$

adding multiples of R_3 to R_1 and R_2 so that all entries above the leading 1 are zero.

$$\left(\begin{array}{cccc} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 7 \end{array}\right)$$

now that the matrix is in reduced row echelon form, we have solved the system!

$x = 8$, $y = 6$, $z = 7$

(b) We can check that the solution above is correct by substiting the values into the equations:

$$\begin{cases} 2(8) + 6 - 2(7) = 8 \\ 2(8) + 3(6) + 7 = 41 \\ 3(8) + 2(6) + 2(7) = 50 \end{cases}$$

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$$\Rightarrow \begin{cases} 8 = 8 \\ 41 = 41 \\ 50 = 50 \end{cases}$$

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Q4 (a) $2x + y - 2z = 8$

$2x + 3y + z = 41$

$2y + 3z = 41 - 8 = 33$

(b) Same as in Q2(a), I will convert the system into augmented matrix form:

$$\begin{pmatrix} 2 & 1 & -2 & 8 \\ 2 & 3 & 1 & 41 \\ 0 & 2 & 3 & 33 \end{pmatrix} \begin{matrix} \Rightarrow R_1 \\ \Rightarrow R_2 \\ \Rightarrow R_3 \end{matrix}$$

now, I will compute the above matrix into reduced echelon form:

$$\begin{pmatrix} 2 & 1 & -2 & 8 \\ 2 & 3 & 1 & 41 \\ 0 & 2 & 3 & 33 \end{pmatrix} \cdot \tfrac{1}{2}$$

multiplying R_1 by a constant to get a leading entry of 1

$$\begin{pmatrix} 1 & \tfrac{1}{2} & -1 & 4 \\ 2 & 3 & 1 & 41 \\ 0 & 2 & 3 & 33 \end{pmatrix} + (-2)R_1$$

adding a multiple of R_1 to R_2 so that all entries below the leading 1 are zero.

$$\begin{pmatrix} 1 & \tfrac{1}{2} & -1 & 4 \\ 0 & 2 & 3 & 33 \\ 0 & 2 & 3 & 33 \end{pmatrix} \cdot \tfrac{1}{2}$$

multiplying R_2 by a constant to get a leading entry of 1

$$\begin{pmatrix} 1 & \tfrac{1}{2} & -1 & 4 \\ 0 & 1 & \tfrac{3}{2} & \tfrac{33}{2} \\ 0 & 2 & 3 & 33 \end{pmatrix} \begin{matrix} + (-\tfrac{1}{2})R_2 \\ \\ + (-2)R_2 \end{matrix}$$

adding multiples of R_2 to R_1 and R_3 so that all entries above and below the leading 1 are zero.

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$$\begin{pmatrix} 1 & 0 & -7/4 & -17/4 \\ 0 & 1 & 3/2 & 33/2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

✓

now that the matrix is in reduced row echelon form, we have solved the system

As R_3 consists only of zeros, we can determine that there exists an infinite amount of solutions to this system.

One can find this infinite solution relationship by introducing a parameter

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