[redacted]

KATE O'NEILL CSU11001 HOMEWORK III

Q1

(a) student number: 21365768

$$v = \begin{pmatrix} 2 \\ 3 \\ 5 \\ 6 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 1 & 3 & 0 & -1 & 2 \\ -1 & 1 & -1 & 0 & 3 \\ 2 & 2 & 3 & 3 & 5 \\ 1 & 1 & 1 & 2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 0 & -1 & 2 \\ 0 & 4 & -1 & -1 & 5 \\ 0 & -4 & 3 & 5 & 1 \\ 0 & -2 & 1 & 3 & 4 \end{pmatrix} \begin{array}{l} \\ R2 \to R2 + R1 \\ R3 \to R3 - 2R1 \\ R4 \to R4 - R1 \end{array}$$

$$\begin{pmatrix} 1 & 3 & 0 & -1 & 2 \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{5}{4} \\ 0 & -4 & 3 & 5 & 1 \\ 0 & -2 & 1 & 3 & 4 \end{pmatrix} \begin{array}{l} \\ R2 \to \frac{1}{4}R2 \\ \\ \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & \frac{3}{4} & -\frac{1}{4} & -\frac{7}{4} \\ 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{5}{4} \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & \frac{1}{2} & \frac{5}{2} & \frac{13}{2} \end{pmatrix} \begin{array}{l} R1 \to R1 - 3R2 \\ \\ R3 \to R3 + 4R2 \\ R4 \to R4 + 2R2 \end{array}$$

$$\begin{pmatrix} 1 & 0 & \frac{3}{4} & -\frac{1}{4} & -\frac{7}{4} \\ 0 & 1 & \frac{1}{4} & -\frac{1}{4} & \frac{5}{4} \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & \frac{1}{2} & \frac{5}{2} & \frac{13}{2} \end{pmatrix} \begin{array}{l} \\ \\ R3 \to \frac{1}{2}R3 \\ \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{7}{4} & -4 \\ 0 & 1 & 0 & \frac{1}{4} & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & \frac{3}{2} & 5 \end{pmatrix} \begin{array}{l} R1 \to R1 - \frac{3}{4}R3 \\ R2 \to R2 + \frac{1}{4}R3 \\ \\ R3 \to R3 - \frac{1}{2}R3 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & -\frac{7}{4} & -4 \\ 0 & 1 & 0 & \frac{1}{4} & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & \frac{10}{3} \end{pmatrix} \begin{array}{l} \\ \\ \\ R4 \to \frac{2}{3}R4 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{11}{6} \\ 0 & 1 & 0 & 0 & \frac{7}{6} \\ 0 & 0 & 1 & 0 & -\frac{11}{3} \\ 0 & 0 & 0 & 1 & \frac{10}{3} \end{pmatrix} \begin{array}{l} R1 \to R1 + \frac{7}{4}R4 \\ R2 \to R2 - \frac{1}{4}R4 \\ R3 \to R3 - 2R4 \\ \\ \end{array}$$

(c) The reduced row echelon form of the ~~augmented~~ matrix has a leading one in every row.

Q2

(a) 23 5 7 6 8

matrix $\rightarrow \begin{pmatrix} 2 & 3 & 5 \\ 1 & 6 & 7 \end{pmatrix}$

(b)

$$\begin{pmatrix} 2 & 3 & 5 \\ 1 & 6 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3/2 & 5/2 \\ 1 & 6 & 7 \end{pmatrix} R1 \rightarrow \tfrac{1}{2} R1$$

$$\begin{pmatrix} 1 & 3/2 & 5/2 \\ 0 & 9/2 & 9/2 \end{pmatrix} R2 \rightarrow R2 \;\; R1$$

$$\begin{pmatrix} 1 & 3/2 & 5/2 \\ 0 & 1 & 1 \end{pmatrix} R2 \rightarrow 2/9\, R2$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} R1 \rightarrow R1 - 3/2\, R2$$

Since the leading ones appear in the first and second columns, the column space is spanned by the vectors

$$\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 6 \end{pmatrix} \right\}$$

(c) reduced row echelon form: $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

for the null Space the reduced row echelon form of the matrix corresponds to the equations

$$x_1 + x_3 = 0$$
$$x_2 + x_3 = 0$$

$x_3 = 4$

$x_1 = -4$

$x_2 = -4$