BOSTON UNIVERSITY

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PS 211: Introduction to Experimental Design

Fall 2025 · Section C1

Lecture 12: Independent-Sample *t* Tests (Continued), *t* Tests in R

Updates & Reminders

- Class time today was the mid-semester survey deadline.
  - Unfortunately, only 35 students filled it out -> no bonus points *sad face*
- Homework 3 is due Friday at midnight.
- By the end of class today, you will know everything you need to complete it!

Recap: Types of *t* Tests

There are 3 types of *t* tests, used in different research scenarios:

1. **Single-sample *t* test** – Compare a sample mean to a population mean when the population SD is unknown.
2. **Paired-sample *t* test** – Compare two samples when every participant is in both samples (within-subjects design).
3. **Independent-samples *t* test** – Compare two samples when participants are in only one group (between-subjects design).

Which Test Should I Use?

Research question: Are the means of my sample(s) different from each other or from a known population mean?

- **Single-sample, comparing scores to population with known mean and known SD** → use a *z test*
- **Single-sample, comparing scores to population with known mean and unknown SD** → use a *single-sample t test*
- **Within-groups design** → use a *paired-samples t test*
- **Between-groups design** → use an *independent-samples t test*

Review: Independent-Samples *t* Tests

- Compares two means from **independent groups.**
- Each group has different people that each experience **only one** level of the independent variable.
- Example: Comparing quiz scores of students who took the quiz with music vs. students who took the quiz in silence.
  - The scores are independent because no student is in both groups. There are no paired scores.

Paired- vs. Independent-Samples *t* Tests

Paired-Samples *t* Test

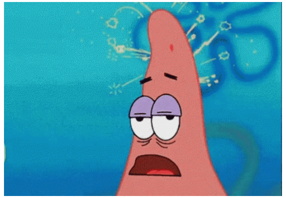
1. Compute difference scores for each participant.
2. Compute mean of these difference scores.
3. Determine probability of observing this mean difference under null hypothesis

> Here, our null distribution is a distribution of mean differences.

Independent-Samples *t* Test

1. Compute mean scores for each group.
2. Compute difference between these group means.
3. Determine probability of observing this mean difference under null hypothesis.

> Here, our null distribution is a distribution of differences between independent group means.



Paired- vs. Independent-Samples *t* Tests

How do we create the distribution of differences between independent group means?

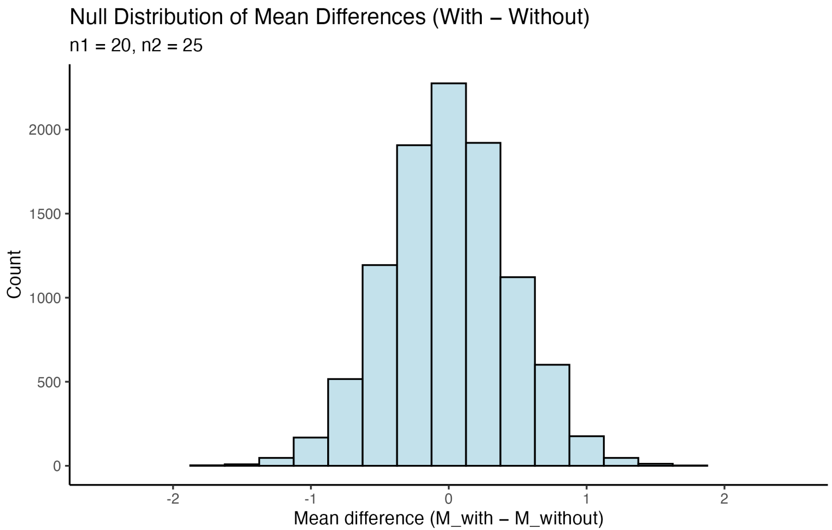
1. Randomly sample n_1 participants for Group 1 and n_2 participants for Group 2 from the population.

2. Compute the mean for each group.

3. Compute the difference between the two group means ($M_1 - M_2$).

4. Repeat many times to build a distribution of mean differences between independent groups.

5. Use this distribution to determine the probability of observing a mean difference as extreme as the one in our sample, assuming the null hypothesis is true.



Note that here we have **two** ns (n_1 and n_2) because the two groups can have different sample sizes.

What are the characteristics of the distribution of differences between independent group means?

Remember, **normally we do not have to build this distribution by simulating many samples.** Instead, we can compute its characteristics directly.

- Under H_0, the mean of this distribution is 0.
- The standard error (SE) of this distribution is computed using both groups' sample standard deviations and sample sizes.
- We use both groups' data to estimate the *pooled variance*, which is our best estimate of the population variance.
- From the pooled variance, we can compute the standard error of the difference between independent means.

Steps of an Independent-Samples *t* Test

1. Identify populations, distribution, & assumptions.
2. State null and research hypotheses.
3. Determine characteristics of comparison distribution.
4. Determine critical values (cutoffs).
5. Calculate test statistic.
6. Make a decision.

Steps 3-5 are similar to those for paired-samples t tests, but the formulas differ slightly because we are dealing with two independent groups.

3. Determine characteristics of comparison distribution.
4. Determine critical values (cutoffs).
5. Calculate test statistic.

How do we compute the standard error for an independent-samples *t* test?

Five steps to compute the standard error:

1. Compute each sample's variance (s^2).
2. Compute the pooled variance (s^2_{pooled}).
3. Convert the pooled variance from the squared standard deviation (SD^2) to the squared standard error (SE^2).
4. Add the two squared standard errors together to get the variance of the difference between means ($SE^2_{difference}$).
5. Take the square root of the variance of the difference to get the standard error of the difference between means ($SE_{difference}$).

How do we compute the standard error for an independent-samples *t* test?

Step 1: calculate the variance for each sample:

$$s_1^2 = \frac{\sum_{i=1}^{n_1}(X_{1i} - M_1)^2}{n_1 - 1}$$

$$s_2^2 = \frac{\sum_{i=1}^{n_2}(X_{2i} - M_2)^2}{n_2 - 1}$$

Why do we divide by (n – 1) instead of n?

A. To correct for bias in estimating the population variance from a sample.

B. To account for the fact that we are working with two samples.

C. To account for different numbers of participants in each group.

D. There is no particular reason; it's just a convention.

How do we compute the standard error for an independent-samples *t* test?

Step 2: Compute the pooled variance:

- We have to weight our estimates because the sample sizes are different.
- To do this, we compute the degrees of freedom (df) for each sample:
  - $df_1 = n_1 - 1$
  - $df_2 = n_2 - 1$
- Then, we compute the pooled variance as a weighted average of the two sample variances:

$$s^2_{pooled} = \frac{df_1}{df_{total}} s^2_1 + \frac{df_2}{df_{total}} s^2_2$$

How do we compute the standard error for an independent-samples *t* test?

Step 2: Compute the pooled variance:

$$s^2_{pooled} = \frac{df_1}{df_{total}} s^2_1 + \frac{df_2}{df_{total}} s^2_2$$

If our first sample has 10 participants and our second sample has 15 participants, which variance will have more weight in the pooled variance calculation?

A. They will have equal weight.

B. The variance from the first sample ($n_1 = 10$).

C. The variance from the second sample ($n_2 = 15$).

D. We cannot tell without knowing the variances.

How do we compute the standard error for an independent-samples *t* test?

Step 3: Convert the pooled variance from the squared standard deviation (SD^2) to the squared standard error (SE^2).

- We now have our best estimate of the population variance (s^2pooled).
- Remember, variance = s^2.
- Remember, $SE = \frac{s}{\sqrt{n}}$.
- That means that $SE^2 = \frac{s^2}{n}$.
- So, for each sample, we convert the pooled variance to the squared standard error by dividing by the respective sample size.

How do we compute the standard error for an independent-samples *t* test?

Step 4: Add the squared standard errors (SE^2) together.

- We ultimately want the standard error of the *difference* between means.
- The squared standard error of the difference between means is the sum of the two squared standard errors.
  - This is a mathematical property of variances that is beyond the scope of this course.

$$SE^2_{difference} = SE^2_1 + SE^2_2$$

How do we compute the standard error for an independent-samples *t* test?

Step 5: Take the square root of the variance of the difference to get the standard error of the difference between means (SEdifference).

- Now that we have the squared standard error of the difference between means, we can take the square root to get the standard error.

$$SE_{difference} = \sqrt{SE_1^2 + SE_2^2}$$

How do we compute the standard error for an independent-samples *t* test?

Putting it all together:

1. Compute each sample's variance (s^2).

2. Compute the pooled variance (s^2pooled).

3. Convert the pooled variance from the squared standard deviation (SD^2) to the squared standard error (SE^2).

4. Add the two squared standard errors together to get the variance of the difference between means (SE^2difference).

5. Take the square root of the variance of the difference to get the standard error of the difference between means (SEdifference).

$$SE_{difference} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)} \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Are you kidding me?

$$SE_{difference} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Here is the good news:

- In practice, you will almost never have to compute this by hand.
- It's important to understand the steps so that you understand *where* this formula comes from.
- But in practice, you will use statistical software (e.g., R) to compute independent-samples *t* tests for you.

Let's test our intuitions

If we increase the sample sizes (n_1 and n_2) while keeping the sample variances (s_1^2 and s_2^2) constant, what happens to the standard error of the difference between means (SEdifference)?

A. It increases.

B. It decreases.

C. It stays the same.

D. We cannot tell without knowing the sample sizes.

Answer: B. It decreases.

Why is this the case?

Answer: As sample sizes increase, our estimates of the population parameters become more precise, leading to a smaller standard error.

What are the characteristics of the distribution of differences between independent group means?

- Under H_0, the mean of this distribution is 0.
- We now know how to compute the standard error (SE) of this distribution using both groups′ sample standard deviations and sample sizes.

Step 4: Determine Critical Values (Cutoffs)

- We now need to determine where the middle 95% of this comparison distribution lies (since we are using an alpha level of .05 for a two-tailed test).
- We are using a *t* distribution to account for our additional uncertainty due to estimating the population standard deviations from our samples.
- We can find out critical values from a t-table or computer program.
- Here our degrees of freedom (df) is computed as:
  - $df = (n_1 - 1) + (n_2 - 1) = n_1 + n_2 - 2$
- This is because we are estimating the population standard deviation twice (once for each sample).

Step 5: Compute the Test Statistic

- We now need to compute the test statistic (*t*), by subtracting the hypothesized population mean difference (0 under H_0) from the observed mean difference between our two samples, and dividing by the standard error of the difference between means.

$$t = \frac{M_D - \mu_D}{SE}$$

- Here:
  - M_D = observed mean difference between the two samples
  - μ_D = hypothesized population mean difference (0 under H_0)
  - SE = standard error of the difference between means
- This simplifies to:

Step 6: Make a Decision

- We use exactly the same decision rule as before:
  - If the calculated t value falls in the critical region (beyond the critical values), we reject H_0.
  - If the calculated t value does not fall in the critical region, we fail to reject H_0.

Confidence Intervals for Independent-Samples *t* Tests

- We use exactly the same calculation as before:

$$CI = (M_1 - M_2) \pm (t_{crit} \times SE_{difference})$$

Effect Sizes for Independent-Samples *t* Tests

- We use exactly the same calculation as before:

$$d = \frac{(M_1 - M_2)}{s}$$

Where s is the pooled standard deviation:

$$s = \sqrt{s^2_{pooled}}$$

Conducting *t* tests in R

- To conduct an *t* test in R, you can use the `t.test()` function.

- Here's a basic example:

```r
# Sample data
group1 <- c(5, 6, 7, 8, 9)
group2 <- c(10, 11, 12, 13, 14)

# Conducting the t-test
t.test(group1, group2, var.equal = TRUE, paired = FALSE)
```

- This will give you the t-statistic, degrees of freedom, and p-value for the test.

- Let's open R and see what this looks like.

Worksheet time!

- The shared worksheet walks you through the steps of an independent-samples *t* test.
- Work through it with a partner or small group.
- We will go over the solution together on Thursday.

That's all for today!