BOSTON UNIVERSITY

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PS 211: Introduction to Experimental Design

Fall 2025 · Section C1

Lecture 7: Hypothesis Testing with Z Tests

Updates and Reminders

- Exam 1 grades will be posted by the end of the day.
- People did well!
- If you would like to go over your exam, please come to office hours next week.
- My office hours tomorrow (Wednesday) are canceled due to conference travel.
- The next exam is somewhat soon, on **Thursday, October 16**.
- Homework 2 will be posted by the end of the week and is due on **Friday, October 10**.

Standard Normal Distribution Tables

How can we convert z scores to percentiles?

- Recall, in a normal distribution:
  - z = 0 is the mean (50th percentile)
  - z = 1 is 1 standard deviation above the mean (~84th percentile)
  - z = -1 is 1 standard deviation below the mean (~16th percentile)
- What if a z score isn't a whole number?
- We can use a computer program to find the percentile (like we just did!), or:
  - We use a **standard normal table** to find the percentile.

Using the Standard Normal (Z) Table

- Z scores are one way to locate a point in the normal curve.
- A standard normal distribution (or z) table shows the percentile associated with each z score.
- Usually only positive z values are shown.
- Negative z values are mirror images (the curve is symmetric).
  - Can compute by subtracting percentile from 1.

Standard Normal Probabilities

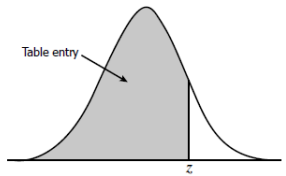


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177

Looking Up a Z Score with the Standard Normal Table

Steps:

1. Convert raw score to z score.

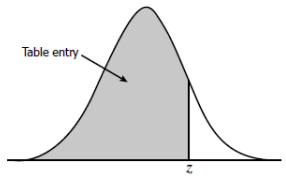
Remember how to do this?

2. Use the *row* for the first two digits and the *column* for the second decimal place.

3. Use the *sign* to determine if you need to subtract from 1.

4. Multiply by 100 to get the percentile.

Standard Normal Probabilities



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Z Table Practice:

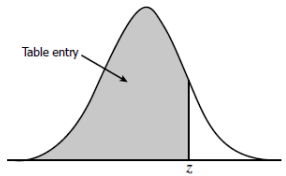
What percentile is z = 0.67? BEFORE looking at the table, ask yourself: Should this be above or below 50%?

1. Find row 0.6 and column 0.07 → 0.7486
2. z is positive, so percentile = 0.7486 = **74.86%**

What percentile is z = -.4?

1. Find row 0.4 and column 0.0 -> .6554
2. z is negative, so percentile = 1 - .6554 = .3446 = **34.46%**

Standard Normal Probabilities



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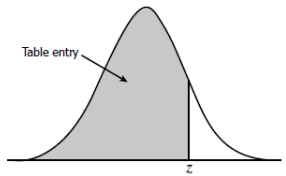
Z Table Practice (More!):

You want to know how well you did on Exam 1 relative to your classmates. You scored a 28 out of 30. The class mean was 27 with a standard deviation of 4.

Approximately what proportion of the class scored lower than you?

1. Compute z: $z = (28 - 27)/4 = 0.25$
2. Find row 0.2 and column 0.05 → 0.5987
3. z is positive, so percentile = 0.5987 = **59.87%**.
4. You did better than about 60% of the class.
5. That means about 60% of the class scored lower than you.

Standard Normal Probabilities



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You can also use R to find percentiles!

```r
# Find percentile for z = 0.25
pnorm(0.25)
# [1] 0.5987063
```

From Z Scores to Hypothesis Testing

- Hypothesis tests ask: **How extreme is a score?**
- When we start testing hypotheses, we want to know if a score is extreme enough to be considered unusual.
- We can use *z* scores to determine this.
- We will more precisely define what we mean by "extreme" and "unusual" soon.

Building an intuition about "extreme" scores

Building an intuition about "extreme" scores

Z Statistics for Distribution of *Means*

If we want to think about the *z* statistic for a **group**, we need to change a few things:

1. We use means instead of raw scores.
2. We calculate the mean and standard error for the distribution of means.
3. Then we calculate a *z* statistic for the sample mean.

When might this be useful?

Answer: All the time!

We often want to know if a sample is different from a population.

Example: Dating Profiles

Researchers are studying online dating profile ratings.

- They have a sample of 30 profiles from Rhode Island (RI).
- RI sample (n=30): $M = 2.84$
- U.S. population: $\mu = 2.5$, $SD = 0.833$
- **Is the RI sample mean different from the U.S. mean?**

What is the standard error for the distribution of means?

Answer: $SE = SD/\sqrt{n} = 0.833/\sqrt{30} \approx 0.152$

What does this standard error tell us?

Answer: The average sample mean computed from samples of size $n = 30$ will be about 0.152 away from the population mean (μ) of 2.5.

Example: Dating Profiles (Continued)

That's all for today!

Next time: more on **hypothesis testing**