ARE GRID CELLS HEXAGONAL FOR PERFORMANCE OR BY CONVENIENCE?

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PROBLEM AND MOTIVATION

This research investigates whether the hexagonal structure of grid cells offers any performance advantages or if it sim- ply represents a biologically convenient arrangement.

Understanding the optimal structure for grid cells in spatial memory tasks is essential for advancing models of spatial cognition.
This study aims not only to evaluate the effectiveness of hexagonal versus square grid cells, but also to deepen our understanding of how grid cell structures influence memory storage and recall in spatial tasks.

BACKGROUND

Grid cells are neurons in the entorhinal cortex of mammals that fire at regular spatial intervals helping the animal to keep track of where it is.
Place cells are neurons in the hippocampus that fire when an animal is in a specific location, forming a neural map of its environment.
They are **closely linked together**. While place cells activate in specific locations, grid cells fire in a grid-like pattern across the environment, representing space in a regular, hexagonal lattice. Grid cells are thought to supply spatial input to place cells, helping them form accurate and stable representations of specific locations. Together, they enable spatial navigation and memory by working as part of the brain's internal GPS system.

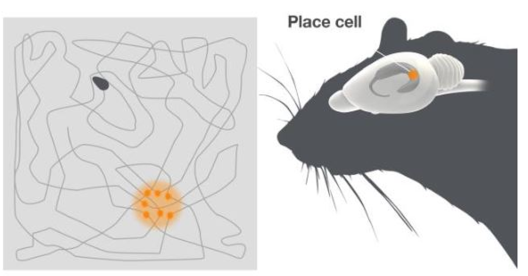

Figure 1: Place-cell in a mouse.

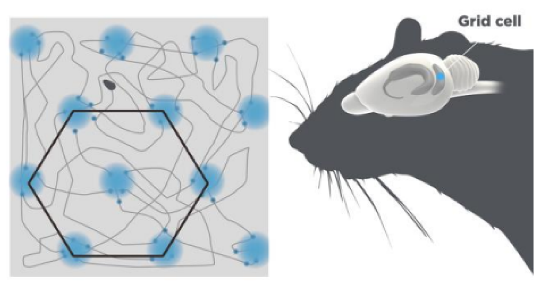

Figure 2: Grid-cell in a mouse.

[1]
The memory cliff issue in Content Addressable Memory (CAM) networks (like Hopfield networks) occurs when the network is able to store and retrieve a small set of patterns with high accuracy, but surpassing its storage limit causes a drastic failure where even adding a single extra pattern leads to the loss of all stored information.

METHOD

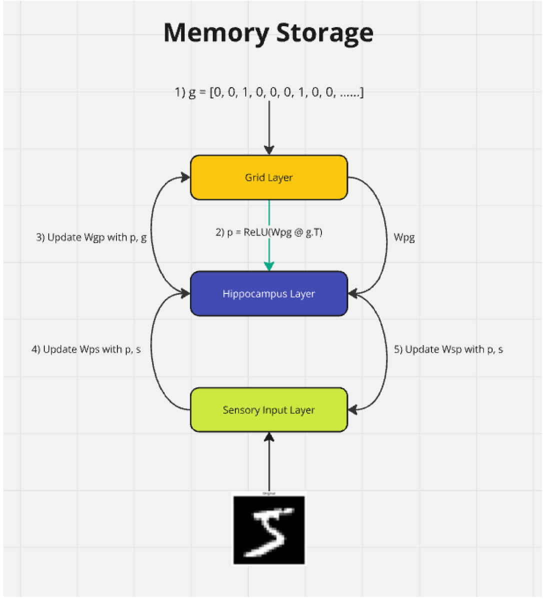

Figure 7: Memory storage process in the Vector-HaSH framework.

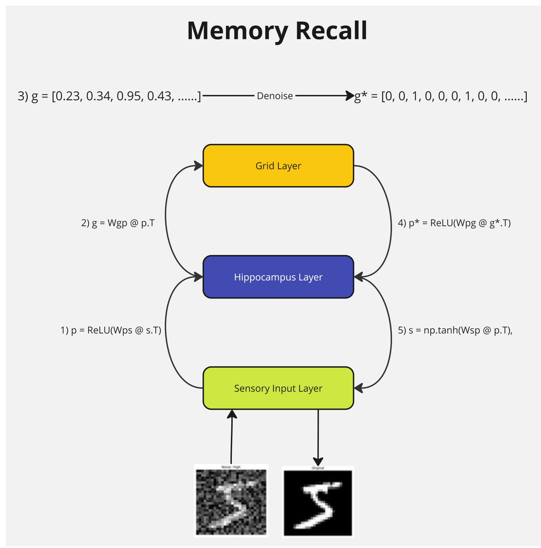

Figure 8: Memory recall in Vector-HaSH.

We implemented the Vector-HaSH framework[3] from scratch, **focusing on two distinct grid cell structures: square and hexagonal**. By comparing the performance of these struc- tures in storing and recalling memories, we aim to clarify the relative advantages of each.
Our evaluation includes a range of test scenarios, such as image datasets like MNIST, Fashion-MNIST and CIFAR-100, as well as path simulations including straight lines, Brownian motion, and Lévy walks, which mimic the foraging patterns of wild animals.
We used the mean cosine similarity and the standard deviation to measure the performance of these two structures.

$$
\begin{array}{cc}
g\_y \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 & \underset{g\_x}{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}
\end{array}
$$

Figure 9: Square grid representation

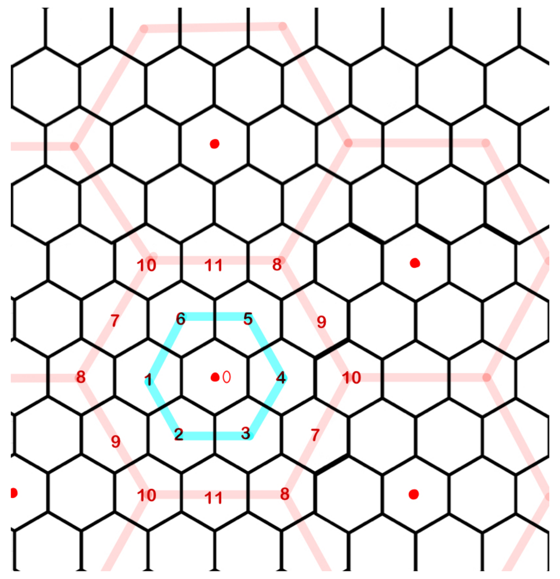

Figure 10: Hexagonal grid represenation

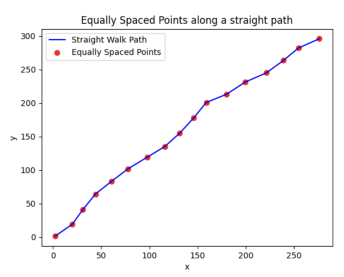

Figure 11: Straight path

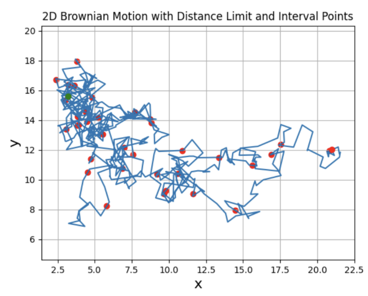

Figure 12: Brownian walk

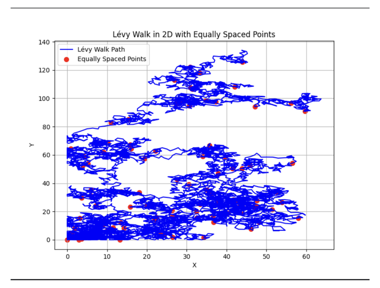

Figure 13: Lévy walk

GRAPHICAL RESULTS

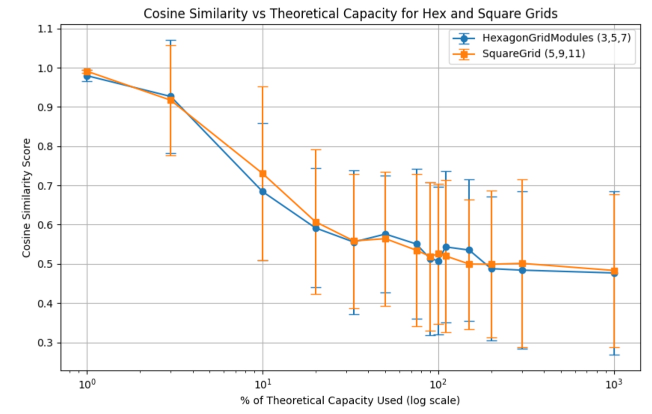

Figure 3: Performance of the 2 structures on Fashion-MNIST

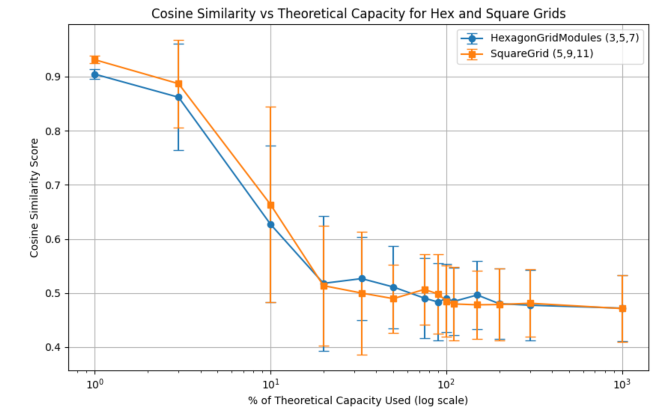

Figure 4: Performance of the 2 structures on MNIST

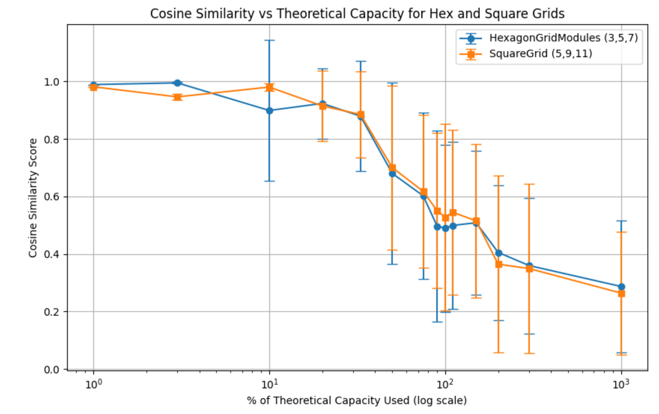

Figure 5: Performance of the 2 structures on CIFAR-100

RELATED WORKS

Our work is significantly inspired by two experimental models that leveraged the biological phenomenon of grid cells:
The Memory Scaffold with Heteroassociation (MESH) model [2] by Sharma et al. (2022)
and the
Vector Hippocampal Scaffolded Heteroassociative Memory (Vector-HaSH) model [1] by Chandra et al. (2023).

Those two models are the first artificial associative memory models not to suffer from the memory cliff problem, that is, when more memo- ries are added to them than their capacity, they fail gracefully, instead of loosing all their memories, the quality of the memories slowly degrades.

What sets our work apart is that, while we use the Vector-HaSH model as the foundation for our logic and implementation, we created two different structures—square grid cells and isotropic hexagonal grid cells—to enable a concrete comparison. The original Vector- HaSH model only used anisotropic hexagonal grid cells and highlighted its innovation over predecessors like the MESH model and Hopfield networks.

RESULTS

The performance metrics were nearly identical between the two grid cell structures, across the many different tests.
The comparable performance between hexagonal and square grid cells suggests that, for the task of memory recall, the choice of grid structure may not provide a distinct computational advantage. This observation supports the hypothesis that the brain's use of hexagonal grid cells could be attributed to biological simplicity rather than functional superiority.

In the context of computational models for memory recall, the biological benefits of hexagonal tiling do not necessarily translate into a computational advantage.

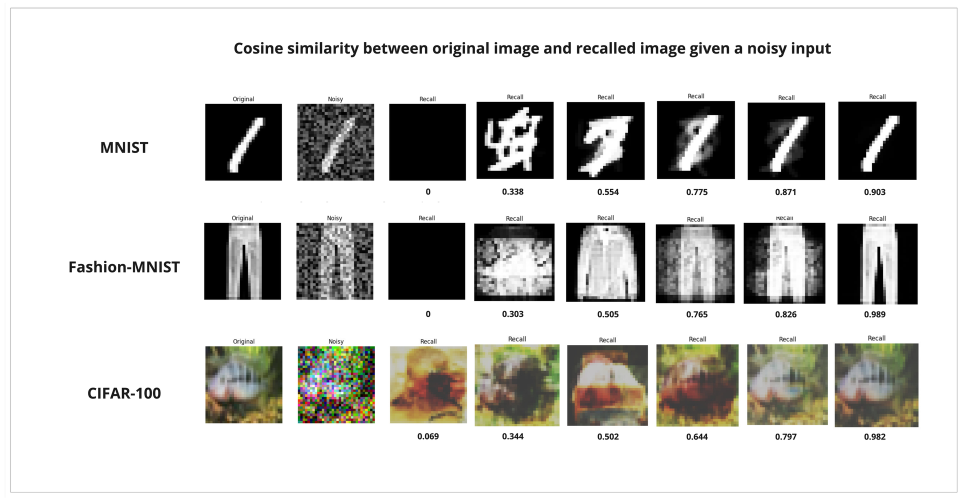

Figure 6: Different cosine similarity values, for good and bad recalls.

REFERENCES

[1] Nobel Prize Outreach AB. Advanced information. https://www.nobelprize.org/prizes/medicine/2014/advanced-information/, 2024. Accessed: Thu. 12 Sep 2024.

[2] Sugandha Sharma, Sarthak Chandra, and Ila R. Fiete. Content addressable memory without catastrophic forgetting by heteroassociation with a fixed scaffold. *arXiv*, 2022.

[3] Sarthak Chandra, Sugandha Sharma, Rishdev Chaudhuri, and Ila Fiete. High-capacity flexible hippocampal associative and episodic memory enabled by prestructured “spatial” representations. *bioRxiv*, 2023.