CSCE 222: Discrete Structures for Computing
Section 502
Spring 2018

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Homework 1

Due: 28 January 2017 (Sunday) before 11:59 p.m. on gradescope (`gradescope.com`).
You must show your work in order to recieve credit.

Aggie Honor Statement: On my honor as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment.

Checklist: Did you...
1. abide by the Aggie Honor Code?
2. solve all problems?
3. start a new page for each problem?
4. show your work clearly?
5. type your solution?
6. submit a PDF to gradescope?

Problem 1.

Let p, q, r be the propositions

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

Write these propositions using p, q, r and logical connectives:

1. Berries are ripe along the trail, but grizzly bears have not been seen in the area.
2. Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
3. If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
4. It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
5. For hiking on the trail be safe, it is necessary but not sufficient that the berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
6. Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Solution.

1. $r \wedge \neg p$
2. $\neg p \wedge q \wedge r$

p	q	r	$\neg p$	$\neg p \wedge q \wedge r$
T	T	T	F	F
T	T	F	F	F
T	F	F	F	F
T	F	T	F	F
F	F	F	T	F
F	F	T	T	F
F	T	T	T	T
F	T	F	F	F

3. $r \rightarrow (q \leftrightarrow \neg p)$
4. $\neg q \wedge \neg p \wedge r$
5. $q \rightarrow (\neg r \wedge \neg p)$
6. $(p \wedge r) \rightarrow \neg q$

p	q	r	$p \wedge r$	$\neg q$	$(p \wedge r) \rightarrow \neg q$
T	T	T	T	F	F
T	T	F	F	F	T
T	F	F	F	T	T
T	F	T	T	T	T
F	F	F	F	T	T
F	F	T	F	T	T
F	T	T	F	F	T
F	T	F	F	F	T

Problem 2.
Four friends have been identified as suspects for unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said “Carlos did it.” Bob said “I did not do it.” Carlos said “Diana did it.” Diana said “Carlos is a liar”.

1. If exactly one of the four is telling the truth, who did it? Explain your reasoning.
2. If exactly one of the four is lying, who did it? Explain your reasoning.

Solution.

1. If Alice is telling the truth and everyone else is lying, then Carlos and Bob are both possible suspects. If Bob is telling the truth, then none of them are the culprit. If Carlos is telling the truth, then either Diana or Bob is the suspect. If Diana is telling the truth, then there is only one suspect, Bob. By analyzing the possibilities if one person is telling the truth, and determining that Diana was being honest by eliminating stories with anything but one culprit, I can conclude that Bob did it.
2. If Alice is lying, then there are no suspects. If Bob is lying, then either he did it, or Carlos did. If Carlos is lying, then Carlos is the only suspect. If Diana is lying, then Carlos and Diana are both suspects. By process of elimination I can conclude that if only one of them is lying, it must be Carlos and he is the one that did it.

Problem 3.
The following specification is taken from the specification of a telephone system:

> If the directory database is opened, then the monitor is put in a closed state, if the system is not in its initial state.

Find an equivalent, easier to understand, specification that involves only disjunctions and negations.

Solution.

Problem 4.
Use logical equivalences to determine the correct equivalence for each expression:
Answer Bank:

T	p	q	$p \wedge q$	$p \wedge \neg q$	$\neg p \wedge q$	$\neg p \wedge \neg q$	$p \oplus q$
F	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg p \vee q$	$p \vee \neg q$	$p \vee q$	$p \leftrightarrow q$

1. $p \vee (\neg(p \rightarrow p) \vee (\neg p \wedge (\neg p \vee (p \wedge (p \vee p))))) \equiv$ ______
2. $(((p \vee p) \vee p) \rightarrow \neg p) \rightarrow (\neg(\neg(p \wedge \neg p) \vee \neg(p \wedge \neg(p \rightarrow p))) \wedge \neg(p \rightarrow (p \wedge \neg p))) \equiv$ ______
3. $p \rightarrow \neg(((p \wedge p) \leftrightarrow \neg p) \oplus ((p \vee (p \leftrightarrow p)) \leftrightarrow p)) \equiv$ ______
4. $\neg\neg(\neg\neg(p \vee \neg\neg\neg q) \wedge \neg q) \rightarrow \neg\neg\neg\neg\neg\neg((\neg(q \wedge \neg(q \rightarrow q)) \wedge \neg\neg q) \wedge \neg p) \equiv$ ______
5. $\neg\neg q \wedge ((q \rightarrow \neg p) \wedge \neg((p \rightarrow ((q \rightarrow \neg q) \wedge p)) \rightarrow \neg p)) \equiv$ ______
6. $(\neg\neg\neg((p \rightarrow (q \wedge \neg q)) \wedge \neg(p \leftrightarrow (\neg q \vee q))) \vee (\neg(\neg p \vee \neg q) \vee \neg(\neg q \vee \neg\neg\neg q))) \equiv$ ______
7. $(\neg\neg q \wedge (\neg\neg\neg p \wedge \neg\neg p)) \wedge \neg\neg\neg(\neg(\neg\neg q \vee \neg p) \wedge ((p \rightarrow \neg\neg p) \vee \neg\neg p)) \equiv$ ______

Solution.
Example

$$
\begin{aligned}
& (\neg((p \vee p) \rightarrow p) \vee \neg\neg\neg\neg p) & \\
\equiv & (\neg((p \vee p) \rightarrow p) \vee \neg\neg p) & \text{double negation} \\
\equiv & (\neg((p \vee p) \rightarrow p) \vee p) & \text{double negation} \\
\equiv & (\neg(p \rightarrow p) \vee p) & \text{idempotent} \\
\equiv & (\neg(\neg p \vee p) \vee p) & \text{definition of implication} \\
\equiv & (\neg T \vee p) & \text{negation (tautology)} \\
\equiv & (F \vee p) & \\
\equiv & p & \text{identity}
\end{aligned}
$$

Problem 5.
A set of logical operators is called **functionally complete** if every compound proposition is logically equivalent to a compound proposition involving only these logical operators. For example, the set $\{\neg, \wedge, \vee\}$ is functionally complete. Furthermore, due to De Morgan's laws, each of the sets $\{\neg, \wedge\}$ and $\{\neg, \vee\}$ are also functionally complete. The singleton set $\{\downarrow\}$ (NOR) is functionally complete, and $\downarrow$ is therefore called a universal operator since all statements in propositional logic can be formed using only $\downarrow$. The same is true of $\{|\}$ (NAND). $\{\downarrow\}$ and $\{|\}$ are the only 1-element functionally complete operator sets, but there are several 2-element functionally complete operator sets, of which $\{\neg, \wedge\}$ and $\{\neg, \vee\}$ are just two examples.

1. Show that $\{\neg, \wedge, \vee\}$ is a functionally complete set of logical operators.
   *Hint: every truth table over n variables can be expressed in **disjunctive normal form** (a disjunction of conjunctions that specifies when the compound proposition is true).*
2. Show that $\{\neg, \wedge\}$ is a functionally complete set of logical operators.
   (Give a logical equivalence for disjunction that involves only negation and conjunction, then construct a truth table to show that the equivalence is valid.)
3. Show that $\{|\}$ is a functionally complete set of logical operators.
   (Give logical equivalences for negation and conjunction that involve only NAND, then construct a truth table to show that the equivalences are valid.)
4. Express the following propositions using only NAND.
   You must show your work. If you cannot show your work, find another way to solve it.
   (a) $p \vee q$
   (b) $p \rightarrow q$
   (c) $p \oplus q$