chap 3. dynamic programming.

* The general Method (Dynamic programming)

Dynamic Programming is optimization technique.

It is improvement over devide & conquer method & greedy algorithms.

In devide and conquer problem (subproblem) may be overlap to each other.

Devide and conquer is top down technique.

Dynamic programming is bottom up approach.

It applies on smallest & simplest subinstances. The main solution can be obtained by continue the process till original problem.

* Multistage Graph:

As the name suggested multistage it is a directed graph containing one sink and source node with source and destination node having n^{th} stage.

In multistage graph we having 2 technique:

1) forward approach.
2) Backward approach.

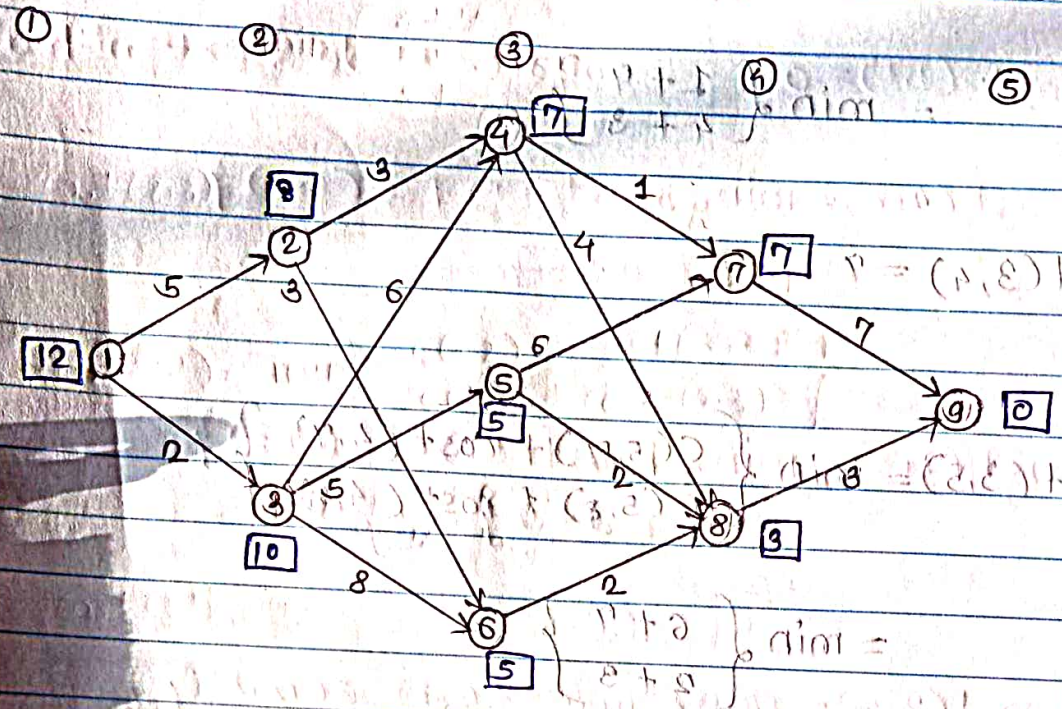
In forward approach we are started the traversal from sink node to source node with minimum weight.

In backword approach we are started the traversing from sink node to source node to destination.

The initial stage and find stage of the directed graph must be contain single node. i.e source and destination respectively, otherwise it is not multistage graph.

* Application of Multistage Graph:
  1) Project Management.
  2) Resource Allocation.
  3) find out deadlock in the project.
  4) finding exception of the project.

Q1 find out minimum cost path from source to destination using forward approach method.



forward approach :

$$cost(i,j) = \min\{ c(j,l) + cost(i+1, l) \}$$

$cost(5,9) = 0$

$cost(4,7) = \min\{ c(7,9) + cost(5,9) \}$
$= \{7+0\}$
$cost(4,7) = 7$

$cost(4,8) = \min\{ c(8,9) + cost(5,9) \}$
$= \{3+0\}$
$cost(4,8) = 3$

$$cost(3,4) = \min \begin{Bmatrix} c(4,7) + cost(4,7) \\ c(4,8) + cost(4,8) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 1+7 \\ 4+3 \end{Bmatrix}$$

$$cost(3,4) = 7$$

$$cost(3,5) = \min \begin{Bmatrix} c(5,7) + cost(4,7) \\ c(5,8) + cost(4,8) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 6+7 \\ 2+3 \end{Bmatrix}$$

$$cost(3,5) = 5$$

$$cost(3,6) = \min \begin{Bmatrix} c(6,7) + cost(4,7) \\ c(6,8) + cost(4,8) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 6+7 \\ 2+3 \end{Bmatrix}$$

$$cost(3,6) = 5$$

$$cost(2,2) = \min \begin{Bmatrix} c(2,4) + cost(3,4) \\ c(2,6) + cost(3,6) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 3+7 \\ 3+5 \end{Bmatrix}$$

$$cost(2,2) = 8$$

$$\text{cost}(2,3) = \min \begin{Bmatrix} c(3,4) + \text{cost}(3,4) \\ c(3,5) + \text{cost}(3,5) \\ c(3,6) + \text{cost}(3,6) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 4+7 \\ 5+5 \\ 8+5 \end{Bmatrix}$$

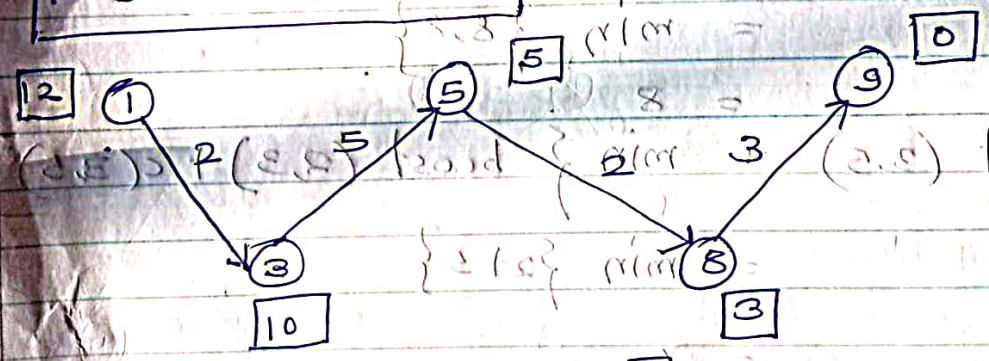
$$\boxed{\text{cost}(2,3) = 10}$$

$$\text{cost}(1,1) = \min \begin{Bmatrix} c(1,2) + \text{cost}(2,7) \\ c(1,3) + \text{cost}(2,3) \end{Bmatrix}$$

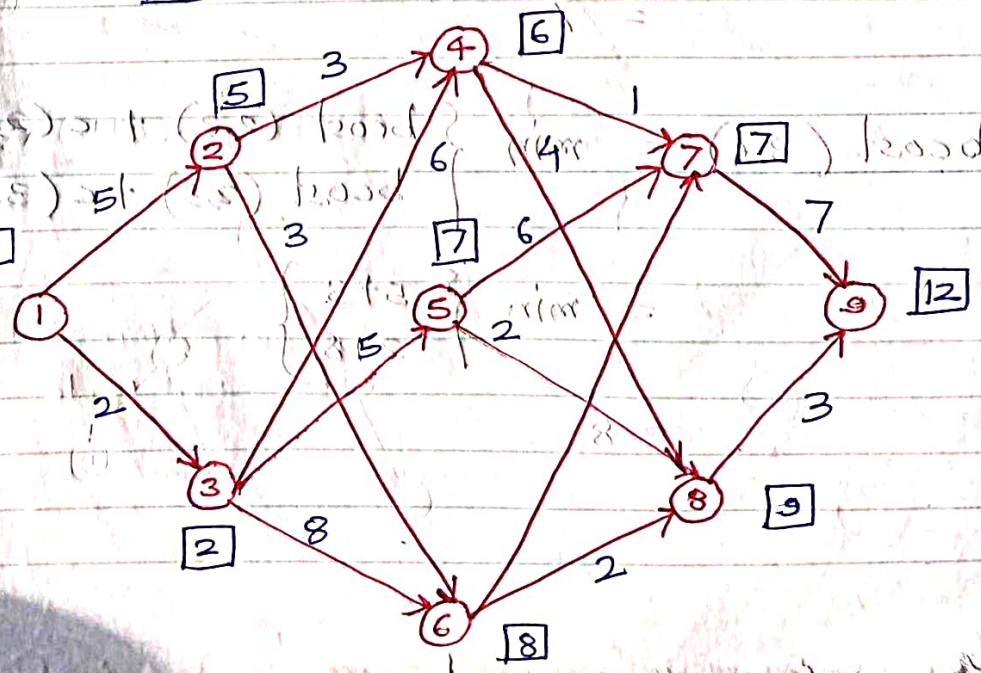
$$= \min \begin{Bmatrix} 5+2 \\ 2+10 \end{Bmatrix}$$

$$\boxed{\text{cost}(1,1) = 12}$$

$$\boxed{1-3-5-8-9 = 12}$$



Q



Backward Approach

$$bcost(i,j) = \min\{bcost(i-1,1) + c(l,j)\}$$

$$bcost(\underset{i}{1},\underset{j}{1}) = 0 \quad \because \text{source node}$$

$$bcost(2,2) = \min\{bcost(1,1) + c(1,2)\}$$
$$= \min\{0+5\}$$
$$= 5$$

$$bcost(2,3) = \min\{bcost(1,1) + c(1,3)\}$$
$$= \min\{0+2\}$$
$$= 2$$

$$bcost(3,4) = \min\left\{\begin{matrix} bcost(2,2) + c(2,4) \\ bcost(2,3) + c(3,4) \end{matrix}\right\}$$
$$= \min\{(5+3) \quad (2+6)\}$$
$$= \min\{8,8\}$$
$$= 8$$

$$bcost(3,5) = \min\{bcost(2,3) + c(3,5)\}$$
$$= \min\{2+5\}$$
$$= 7$$

$$bcost(3,6) = \min\left\{\begin{matrix} bcost(2,2) + c(2,6) \\ bcost(2,3) + c(3,6) \end{matrix}\right\}$$
$$= \min\left\{\begin{matrix} 5+3 \\ 2+8 \end{matrix}\right\}$$
$$= 8$$

$$bcost(4,7) = \min \begin{Bmatrix} bcost(3,4) + c(4,7) \\ bcost(3,5) + c(5,7) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 8 + 1 \\ 7 + 6 \end{Bmatrix}$$

$$= 9$$

$$bcost(4,8) = \min \begin{Bmatrix} bcost(3,4) + c(4,8) \\ bcost(3,5) + c(5,8) \end{Bmatrix}$$

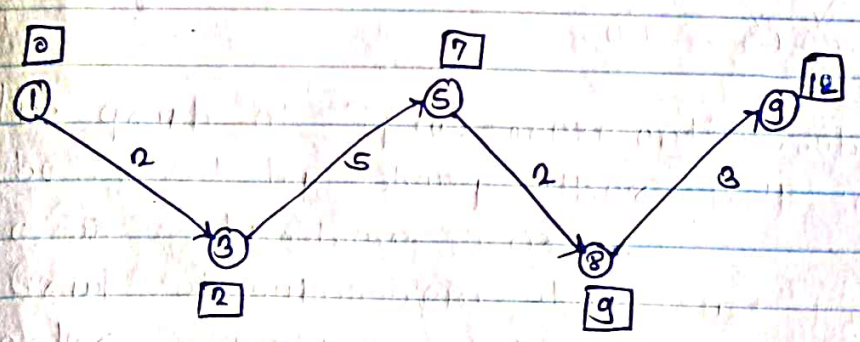
$$= \min \begin{Bmatrix} 6+4 \\ 7+2 \end{Bmatrix}$$

$$= 9$$

$$bcost(5,9) = \min \begin{Bmatrix} bcost(4,7) + c(7,9) \\ bcost(4,8) + c(8,9) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 9+7 \\ 9+3 \end{Bmatrix}$$

$$= 12$$



* All Pairs Shortest Path:

① let G(V,E) is a directed graph
∀,
V = set of Variables
E = Edges

To calculate the length of shortest path between each pair of node.

② There will be node start with 1 & till n.

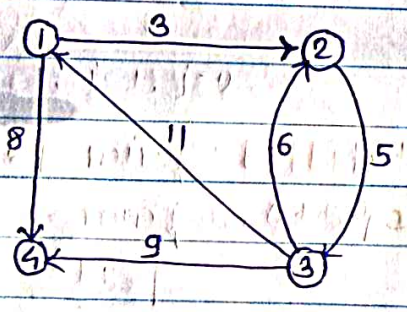
③ Construct matrix L by following formula.

$L[i][j] = 0$ if $i = j$

$L[i][j] = \infty$ if there is no edge exist between i & j

$L[i][j] = w$ weight is present in path.

eg:



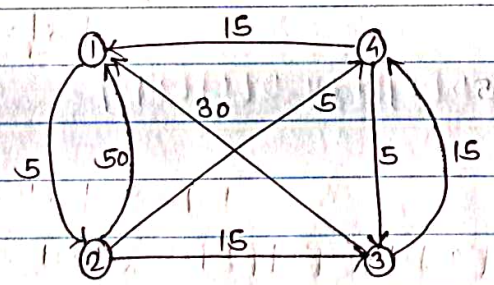
$$L = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 8 \\ 2 & \infty & 0 & 5 & \infty \\ 3 & 11 & 6 & 0 & 9 \\ 4 & \infty & \infty & \infty & 0 \end{array}$$

all pairs shortest path L[][]

```
{
  D=L
  for (k=1 to n)
  {
    for (i=1 to n)
    {
      for (j=1 to n)
        D[i][j]=min{D[i][j], D[i][k]+D[k][j]};
    }
  }
}
```

example:

1) find the shortest path between all pairs of nodes in following graph.



→ Initially,

$$L = \begin{bmatrix} 0 & 5 & \infty & \infty \\ 50 & 0 & 15 & 5 \\ 30 & \infty & 0 & 15 \\ 15 & \infty & 5 & 0 \end{bmatrix}$$

Initialize the matrix D=[ ]
i.e.

$$D = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 5 & \infty & \infty \\ 50 & 0 & 15 & 5 \\ 30 & \infty & 0 & 15 \\ 15 & \infty & 5 & 0 \end{bmatrix} \end{array}$$

To find all pairs shortest path we require 4 iterations i.e- D_1, D_2, D_3, D_4 and D_4 is out all pair shortest path
Now find D_1 by using matrix D.

$D_1[i][j] = \min\{D[i][j], D[i][k] + D[k][j]\}$

$k=1, i=1, j=1$

$D_1[1][1] = \min\{D[1][1], D[1][1]+D[1][1]\}$
$= \min\{0, 0+0\}$
$= \boxed{0}$

$k=1, i=1, j=2$

$D_1[1][2] = \min\{D[1][2], D[1][1] + D[1][2]\}$
$= \min\{5, 0+5\}$
$= \boxed{5}$

$k=1, i=1, j=3$

$D_1[1][3] = \min\{D[1][3], D[1][1]+D[1][3]\}$
$= \min\{\infty, 0+\infty\}$
$= \boxed{\infty}$

$k=1, i=1, j=4$

$D_1[1][4] = \min\{D[1][4], D[1][1] + D[1][4]\}$
$= \min\{\infty, 0+\infty\}$
$= \boxed{\infty}$

Now, i is increament upto 4,

$k=1,\ i=2,\ j=1$

$D_1[2][1] = \min\{D[2][1],\ D[2][1] + D[1][1]\}$

$= \min\{50,\ 50+0\}$

$= \boxed{50}$

$k=1,\ i=2,\ j=2$

$D_1[2][2] = \min\{D[2][2],\ D[2][1] + D[1][2]\}$

$= \min\{0,\ 50+5\}$

$= \boxed{0}$

$k=1,\ i=2,\ j=3$

$D_1[2][3] = \min\{D[2][3],\ D[2][1] + D[1][3]\}$

$= \min\{15,\ 50+\infty\}$

$= \boxed{15}$

$D_1[2][4] = \min\{D[2][4],\ D[2][1] + D[1][4]\}$

$= \min\{5,\ 50+\infty\}$

$= \boxed{5}$

~~Now~~

$k=1,\ i=3,\ j=1$

$D_1[3][1] = \min\{D[3][1],\ D[3][1] + D[1][1]\}$

$= \min\{30,\ 30+0\}$

$= \boxed{30}$

$k=1,\ i=3,\ j=2$

$D_1[3][2] = \min\{D[3][2],\ D[3][1] + D[1][2]\}$

$= \min\{\infty,\ 30+5\}$

$= \boxed{35}$

$K=1,\ i=3,\ j=3$

$D_1[1][3] = \min\{D[1][3],\ D[1][$

$D_1[3][3] = \min\{D[3][3],\ D[3][1] + D[1][3]\}$

$= \min\{0,\ 30+\infty\}$

$= \boxed{0}$

$K=1,\ i=3,\ j=4$

$D_1[3][4] = \min\{D[3][4],\ D[3][1] + D[1][4]\}$

$= \min\{15,\ 30+\infty\}$

$= \boxed{15}$

$K=1,\ i=4,\ j=1$

$D_1[4][1] = \min\{D[4][1],\ D[4][1] + D[1][1]\}$

$D_1[4][1] = \min\{15,\ 15+0\}$

$= \boxed{15}$

$K=1,\ i=4,\ j=2$

$D_1[4][2] = \min\{D[4][2],\ D[4][1] + D[1][2]\}$

$= \min\{\infty,\ 15+5\}$

$= \boxed{20}$

$K=1,\ i=4,\ j=3$

$D_1[4][3] = \min\{D[4][3],\ D[4][1] + D[1][3]\}$

$= \min\{5,\ 15+\infty\}$

$= \boxed{5}$

$K=1,\ i=4,\ j=4$

$D_1[4][4] = \min\{D[4][4],\ D[4][1] + D[1][4]\}$

$= \min\{0,\ 15+\infty\}$

$= \boxed{0}$

$$D_1 = \begin{bmatrix} 0 & 5 & \infty & \infty \\ 50 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix}$$

$k=2,\ i=1,\ j=1$

$D_2[1][1] = \min\{D_1[1][1],\ D_1[1][2] + D_1[2][1]\}$

$= \min\{0, 5+50\}$

$= \boxed{0}$

$k=2,\ i=1,\ j=2$

$D_2[1][2] = \min\{D_1[1][2],\ D_1[1][2] + D_1[2][2]\}$

$= \min\{5, 5+0\}$

$= \boxed{5}$

$k=2,\ i=1,\ j=3$

$D_2[1][3] = \min\{D_1[1][3],\ D_1[1][2] + D_1[2][3]\}$

$= \min\{\infty, 5+15\}$

$= \boxed{20}$

$k=2,\ i=1,\ j=4$

$D_2[1][4] = \min\{D_1[1][4],\ D_1[1][2] + D_1[2][4]\}$

$= \min\{\infty, 5+5\}$

$= \boxed{10}$

$k=2,\ i=2,\ j=1$

$D_2[2][1] = \min\{D_1[2][1],\ D_1[2][2] + D_1[2][1]\}$

$= \min\{50, 0+50\}$

$= \boxed{50}$

$k=2,\ i=2,\ j=2$

$D_2[2][2] = \min\{D_1[2][2],\ D_1[2][2] + D_1[2][2]\}$

$= \{0,\ 0+0\}$

$= \boxed{0}$

$k=2,\ i=2,\ j=3$

$D_2[2][3] = \min\{D_1[2][3],\ D_1[2][2] + D_1[2][3]\}$

$= \min\{15,\ 0+15\}$

$= \boxed{15}$

$k=2,\ i=2,\ j=4$

$D_2[2][4] = \min\{D_1[2][4],\ D_1[2][2] + D_1[2][4]\}$

$= \min\{5,\ 0+5\}$

$= \boxed{5}$

$k=2,\ i=3,\ j=1$

$D_2[3][1] = \min\{D_1[3][1],\ D_1[3][2] + D_1[2][1]\}$

$= \min\{30,\ 35+50\}$

$= \boxed{30}$

$k=2,\ i=3,\ j=2$

$D_2[3][2] = \min\{D_1[3][2],\ D_1[3][2] + D_1[2][2]\}$

$= \min\{35,\ 0+35\}$

$= \boxed{35}$

$k=2,\ i=3,\ j=3$

$D_2[3][3] = \min\{D_1[3][3],\ D_1[3][2] + D_1[2][3]\}$

$= \min\{0,\ 35+15\}$

$= \boxed{0}$

$k=2,\ i=3,\ j=4$

$D_2[3][4] = \min\{D_1[3][4],\ D_1[3][2] + D_1[2][4]\}$

$= \min\{15,\ 35+5\}$

$= \boxed{15}$

$K=2,\ i=4,\ j=1$

$D_2[4][1] = \min\{D_1[4][1],\ D_1[4][2] + D_1[2][1]\}$

$= \min\{15,\ 20+50\}$

$= \boxed{15}$

~~D2~~ $K=2,\ i=4,\ j=2$

$D_2[4][2] = \min\{D_1[4][2],\ D_1[4][2] + D_1[2][2]\}$

$= \min\{20,\ 20+0\}$

$= \boxed{20}$

$K=2,\ i=4,\ j=3$

$D_2[4][3] = \min\{D_1[4][3],\ D_1[4][2] + D_1[2][3]\}$

$= \min\{5,\ 20+15\}$

$= \boxed{5}$

$K=2,\ i=4,\ j=4$

$D_2[4][4] = \min\{D_1[4][4],\ D_1[4][2] + D_1[2][4]\}$

$= \min\{0,\ 20+5\}$

$= \boxed{0}$

$$D_2 = \begin{bmatrix} 0 & 5 & 20 & 10 \\ 50 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix}$$

$K=3,\ i=1,\ j=1$

$D_3[1][1] = \min\{D_2[1][1],\ D_2[1][3] + D_2[3][1]\}$

$= \min\{0,\ 20+30\}$

$= \boxed{0}$

$k=3$, $i=1$, $j=2$

$D_3[1][2] = \min\{D_2[1][2], D_2[1][3]+D_2[3][2]\}$

$= \min\{5, 20+35\}$

$= \boxed{5}$

$k=3$, $i=1$, $j=3$

$D_3[1][3] = \min\{D_2[1][3], D_2[1][3]+D_2[3][3]\}$

$= \min\{20, 20+20\}$

$= \boxed{20}$

$k=3$, $i=1$, $j=4$

$D_3[1][4] = \min\{D_2[1][4], D_2[1][3]+D_2[3][4]\}$

$= \min\{10, 20+15\}$

$= \boxed{10}$

$k=3$, $i=2$, $j=1$

$D_3[2][1] = \min\{D_2[2][1], D_2[1][3]+D_2[3][1]\}$

$= \min\{50, 0+30\}$

$= \boxed{30}$

$k=3$, $i=2$, $j=2$

$D_3[2][2] = \min\{D_2[2][2], D_2[1][3]+D_2[3][2]\}$

$= \min\{0, 0+35\}$

$= \boxed{0}$

$k=3$, $i=2$, $j=3$

$D_3[2][3] = \min\{D_2[2][3], D_2[1][3]+D_2[3][3]\}$

$= \min\{15, 20+0\}$

$= \boxed{15}$

$k=3$, $i=2$, $j=4$

$D_3[2][4] = \min\{D_2[2][4], D_2[1][3]+D_2[3][4]\}$

$= \min\{5, 10+15\}$

$= \boxed{5}$

$k=3,\ i=3,\ j=1$

$D_3[3][1] = \min\{D_2[3][1],\ D_2[3][3] + D_2[3][1]\}$

$= \min\{30,\ 0+30\}$

$= \boxed{30}$

$k=3,\ i=3,\ j=2$

$D_3[3][2] = \min\{D_2[3][2],\ D_2[3][3] + D_2[3][2]\}$

$= \min\{35,\ 0+35\}$

$= \boxed{35}$

$k=3,\ i=3,\ j=3$

$D_3[3][3] = \min\{D_2[3][3],\ D_2[3][3] + D_2[3][3]\}$

$= \min\{0,\ 0+0\}$

$= \boxed{0}$

$k=3,\ i=3,\ j=4$

$D_3[3][4] = \min\{D_2[3][4],\ D_2[3][3] + D_2[3][4]\}$

$= \min\{15,\ 0+15\}$

$= \boxed{15}$

$k=3,\ i=4,\ j=1$

$D_3[4][1] = \min\{D_2[4][1],\ D_2[4][3] + D_2[3][1]\}$

$= \min\{15,\ 5+30\}$

$= \boxed{15}$

$k=3,\ i=4,\ j=2$

$D_3[4][2] = \min\{D_2[4][2],\ D_2[4][3] + D_2[3][2]\}$

$= \min\{20,\ 5+35\}$

$= \boxed{20}$

$k=3,\ i=4,\ j=3$

$D_3[4][3] = \min\{D_2[4][3],\ D_2[4][3] + D_2[3][3]\}$

$= \min\{5,\ 5+0\}$

$= \boxed{5}$

$k=3,\ i=4,\ j=4$

$D_3[4][4] = \min\{D_2[4][4],\ D_2[4][3] + D_2[3][4]\}$

$= \min\{0,\ 5+15\}$

$= \boxed{0}$

$$D_3 = \begin{bmatrix} 0 & 5 & 20 & 10 \\ 45 & 0 & 15 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix}$$

$k=4,\ i=1,\ j=1$

$D_4[1][1] = \min\{D_3[1][1],\ D_3[1][4] + D_3[4][1]\}$

$= \min\{0,\ 10+45\}$

$= \boxed{0}$

$k=4,\ i=1,\ j=2$

$D_4[1][2] = \min\{D_3[1][2],\ D_3[1][4] + D_3[4][2]\}$

$= \min\{5,\ 10+35\}$

$= \boxed{5}$

$k=4,\ i=1,\ j=3$

$D_4[1][3] = \min\{D_3[1][3],\ D_3[1][4] + D_3[4][3]\}$

$= \min\{20,\ 10+5\}$

$= \boxed{15}$

$k=4,\ i=1,\ j=4$

$D_4[1][4] = \min\{D_3[1][4],\ D_3[1][4] + D_3[4][4]\}$

$= \min\{10,\ 10+0\}$

$= \boxed{10}$

$k=4,\ i=2,\ j=1$

$D_4[2][1] = \min\{D_3[2][1],\ D_3[2][4] + D_3[4][1]\}$

$= \min\{30,\ 5+15\}$

$= \boxed{20}$

$k=4,\ i=2,\ j=2$

$D_4[2][2] = \min\{D_3[2][2],\ D_3[2][4] + D_3[4][2]\}$

$= \min\{0,\ 5+20\}$

$= \boxed{0}$

$k=4,\ i=2,\ j=3$

$D_4[2][3] = \min\{D_3[2][3],\ D_3[2][4] + D_3[4][3]\}$

$= \min\{15,\ 5+5\}$

$= \boxed{10}$

$k=4,\ i=2,\ j=4$

$D_4[2][4] = \min\{D_3[2][4],\ D_3[2][4] + D_3[4][4]\}$

$= \min\{5,\ 5+0\}$

$= \boxed{5}$

$k=4,\ i=3,\ j=1$

$D_4[3][1] = \min\{D_3[3][1],\ D_3[3][4] + D_3[4][1]\}$

$= \min\{30,\ 15+15\}$

$= \boxed{30}$

$k=4,\ i=3,\ j=2$

$D_4[3][2] = \min\{D_3[3][2],\ D_3[3][4] + D_3[4][2]\}$

$= \min\{35,\ 15+20\}$

$= \boxed{35}$

$k=4,\ i=3,\ j=3$

$D_4[3][3] = \min\{D[3][3],\ D_3[3][4] + D_3[4][2]\}$

$= \min\{0,\ 15+20\}$

$= \boxed{0}$

$= \boxed{5}$

$k=4,\ i=3,\ j=4$

$D_4[3][4] = \min\{D_3[3][4],\ D_3[3][4] + D_3[4][4]\}$

$= \min\{15,\ 15+0\}$

$= \boxed{15}$

$k=4,\ i=4,\ j=1$

$D_4[4][1] = \min\{D_3[4][1],\ D_3[4][4] + D_3[4][1]\}$

$= \min\{15,\ 0+15\}$

$= \boxed{15}$

$k=4,\ i=4,\ j=2$

$D_4[4][2] = \min\{D_3[4][2],\ D_3[4][4] + D_3[4][2]\}$

$= \min\{20,\ 0+20\}$

$= \boxed{20}$

$k=4,\ i=4,\ j=3$

$D_4[4][3] = \min\{D_3[4][3],\ D_3[4][4] + D_3[4][3]\}$

$= \min\{5,\ 0+5\}$

$= \boxed{5}$

$k=4,\ i=4,\ j=4$

$D_4[4][4] = \min\{D_3[4][4],\ D_3[4][4] + D_3[4][4]\}$

$= \min\{0,\ 0+0\}$

$= \boxed{0}$

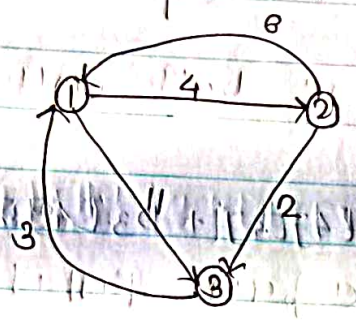
Now,

$$D_4 = \begin{bmatrix} 0 & 5 & 15 & 10 \\ 20 & 0 & 10 & 5 \\ 30 & 35 & 0 & 15 \\ 15 & 20 & 5 & 0 \end{bmatrix}$$

Hence, this is final solution and all pair shortest path.

example:

Solve all pair shortest path problem for given problem.



→ Now we have to find matrix L, which is 3X3

$$L = \begin{bmatrix} 0 & 4 & 11 \\ 6 & 0 & 2 \\ 3 & \infty & 0 \end{bmatrix}$$

Now initiate matrix,

$D = L$

hence,

$$D = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 0 & 4 & 11 \\ 6 & 0 & 2 \\ 3 & \infty & 0 \end{bmatrix}$$

Now, $k=1, i=1, j=1$

$D_1[1][1] = \min\{D[1][1], D[1][1] + D[1][1]\}$

$= \min\{0, 0+0\}$

$= \boxed{0}$

$k=1, i=1, j=2$

$D_1[1][2] = \min\{D[1][2], D[1][1] + D[1][2]\}$

$= \min\{4, 0+4\}$

$= \boxed{4}$

$k=1,\ i=1,\ j=3$

$D_1[1][3] = \min\{D[1][3],\ D[1][1] + D[1][3]\}$

$= \min\{11,\ 0+11\}$

$= \boxed{11}$

$k=1,\ i=2,\ j=1$

$D_1[2][1] = \min\{D[2][1],\ D[2][1] + D[1][1]\}$

$= \min\{6,\ 6+0\}$

$= \boxed{6}$

$k=1,\ i=2,\ j=2$

$D_1[2][2] = \min\{D[2][2],\ D[2][1] + D[1][2]\}$

$= \min\{0,\ 6+4\}$

$= \boxed{0}$

$k=1,\ i=2,\ j=3$

$D_1[2][3] = \min\{D[2][3],\ D[2][1] + D[1][3]\}$

$= \min\{2,\ 6+11\}$

$= \boxed{2}$

$k=1,\ i=3,\ j=1$

$D_1[3][1] = \min\{D[3][1],\ D[3][1] + D[1][1]\}$

$= \min\{3,\ 3+0\}$

$= \boxed{3}$

$k=1,\ i=3,\ j=2$

$D_1[3][2] = \min\{D[3][2],\ D[3][1] + D[1][2]\}$

$= \min\{\infty,\ 3+4\}$

$= \boxed{7}$

$k=1,\ i=3,\ j=3$

$D_1[3][3] = \min\{D[3][3],\ D[3][1] + D[1][3]\}$

$= \min\{0,\ 3+11\}$

$= \boxed{0}$

$$D_1 = \begin{bmatrix} 0 & 4 & 11 \\ 6 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$$

Now find D_2 by using D_1

$K=2, i=1, j=1$

$D_2[1][1] = \min\{D_1[1][1], D_1[1][2] + D_1[2][1]\}$

$= \min\{0, 4+6\}$

$= \boxed{0}$

$K=2, i=1, j=2$

$D_2[1][2] = \min\{D_1[1][2], D_1[1][2] + D_1[2][2]\}$

$= \min\{4, 4+0\}$

$= \boxed{4}$

$K=2, i=1, j=3$

$D_2[1][3] = \min\{D_1[1][3], D_1[1][2] + D_1[2][3]\}$

$= \min\{11, 4+2\}$

$= \boxed{6}$

$K=2, i=2, j=1$

$D_2[2][1] = \min\{D_1[2][1], D_1[2][2] + D_1[2][1]\}$

$= \min\{6, 0+6\}$

$= \boxed{6}$

$K=2, i=2, j=2$

$D_2[2][2] = \min\{D_1[2][2], D_1[2][2] + D_1[2][2]\}$

$= \min\{0, 0+0\}$

$= \boxed{0}$

$K=2, i=2, j=3$

$D_2[2][3] = \min\{D_1[2][3], D_1[2][2] + D_1[2][3]\}$

$= \min\{2, 0+2\}$

$= \boxed{2}$

$k=2,\ i=3,\ j=1$

$D_2[3][1] = \min\{D_1[3][1], D_1[3][2] + D_1[2][1]\}$

$= \min\{3, 7+6\}$

$= \boxed{3}$

$k=2,\ i=3,\ j=2$

$D_2[3][2] = \min\{D_1[3][2], D_1[3][2] + D_1[2][2]\}$

$= \min\{7, 7+0\}$

$= \boxed{7}$

$k=2,\ i=3,\ j=3$

$D_2[3][3] = \min\{D_1[3][3], D_1[3][2] + D_1[2][3]\}$

$= \min\{0, 7+2\}$

$= \boxed{0}$

$$D_2 = \begin{bmatrix} 0 & 4 & 6 \\ 6 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$$

Now find D_3 from D_2,

$k=3,\ i=1,\ j=1$

$D_3[1][1] = \min\{D_2[1][1], D_2[1][3] + D_2[3][1]\}$

$= \min\{0, 6+6\}$

$= \boxed{0}$

$k=3,\ i=1,\ j=2$

$D_3[1][2] = \min\{D_2[1][2], D_2[1][3] + D_2[3][2]\}$

$= \min\{4, 6+0\}$

$= \boxed{4}$

$k=3,\ i=1,\ j=3$

$D_3[1][3] = \min\{D_2[1][3], D_2[1][2] + D_2[3][2]\}$

$= \min\{6, 0+7\} = \boxed{6}$

$K=3,\ i=2,\ j=1$

$D_3[2][1] = \min\{D_2[2][1],\ D_2[2][3] + D_2[3][1]\}$

$= \min\{6, 2+3\}$

$= \boxed{5}$

$K=3,\ i=2,\ j=2$

$D_3[2][2] = \min\{D_2[2][2],\ D_2[2][3] + D_2[3][2]\}$

$= \min\{0, 2+7\}$

$= \boxed{0}$

$K=3,\ i=2,\ j=3$

$D_3[2][3] = \min\{D_2[2][3],\ D_2[2][3] + D_2[3][3]\}$

$= \min\{2, 2+0\}$

$= \boxed{2}$

$K=3,\ i=3,\ j=1$

$D_3[3][1] = \min\{D_2[3][1],\ D_2[3][3] + D_2[3][1]\}$

$= \min\{3, 0+3\}$

$= \boxed{3}$

$K=3,\ i=3,\ j=2$

$D_3[3][2] = \min\{D_2[3][2],\ D_2[3][3] + D_2[3][2]\}$

$= \min\{7, 0+7\}$

$= \boxed{7}$

$K=3,\ i=3,\ j=3$

$D_3[3][3] = \min\{D_2[3][3],\ D_2[3][3] + D_2[3][3]\}$

$= \min\{0, 0+0\}$

$= \boxed{0}$

$$D_3 = \begin{bmatrix} 0 & 4 & 6 \\ 5 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$$

..... final solution and all pair shortest path....

* Optimal Binary Search Tree (OBST):

Two Possible binary search trees :

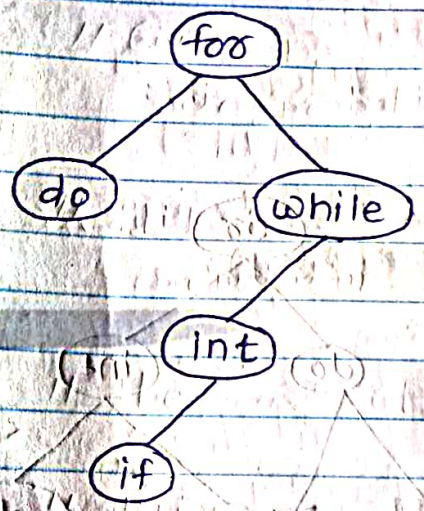


fig = (a)

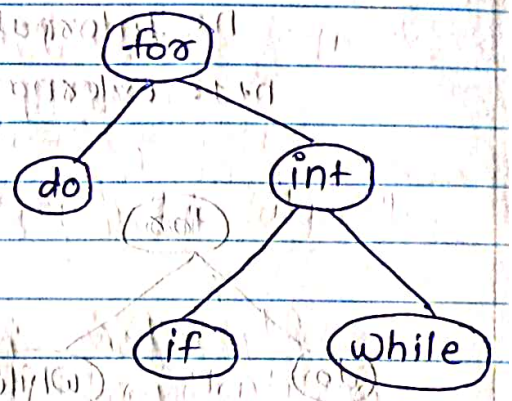


fig = (b)

Two different binary search trees for same identifier set ~~of~~ to have difference performance charactoristics

① No. of comparisions for fig (a)

$$\frac{1+2+2+3+4}{5} = \frac{12}{5} = \underline{\underline{2.4}}$$

② No. of comparisions for fig (b)

$$\frac{1+2+2+3+3}{5} = \frac{11}{5} = \underline{\underline{2.2}}$$

Hence, fig (b) is better than (a)

Cost function for binary search tree:

If binary search tree represents n internal nodes ie identifiers then there will be exactly,

n = Internal Nodes

n+1 = External Nodes

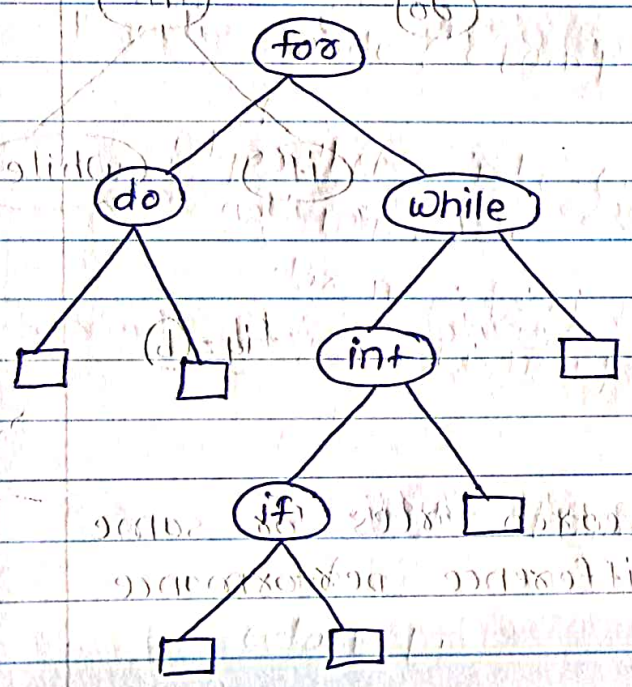


fig (a)

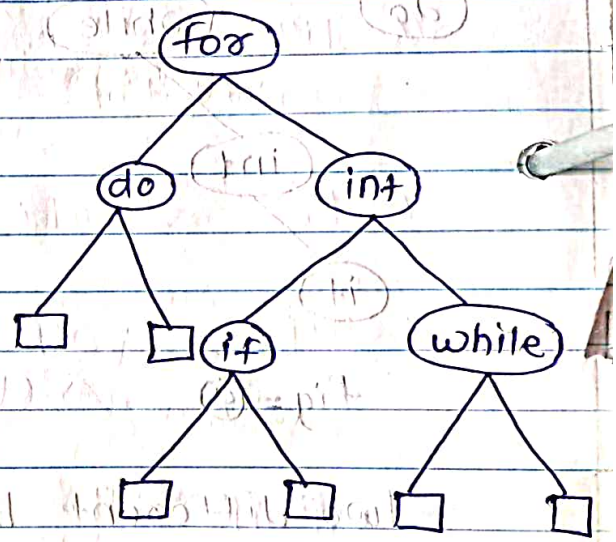


fig (b)

Algorithm for finding minimum cost binary search Tree:

Algorithm OBST(P, q, n)

// Given n distinct identifiers $q_1 < q_2 < \dots < q_n$ & their probabilities.

// P[i], $1 \le i \le n$ & q[i], $0 \le i \le n$, this algorithm computation.

// the cost c[i, j] of OBST tij for identifiers.

// q_{i+j} ..... q_j. It also computes r[i, j], the root of tij.

// w[i, j] is the is the weight of tij.

```
{
    for i=0 to n-1 do
    {
        // Initialize
        w[i,i] = q[i] ; r[i,j]=0 ; c[i,i]=0 ;
        // optimal trees with one node
        w[i, i+1] = q[i] + q[i+1] + p[i+1];
        r[i, i+1] = i+1 ;
        c[i, i+1] = q[i] + q[i+1] + p[i+1];
    }
    w[n,n] = q[n] ; r[n,n]=0 ; c[n,n]=0 ;
        for m=2 to n do
        for i=0 to n-m do
        {
```

```
    j = i+m;
    w[i,j] = w[i,j-1] + p[j] + q[j];
    k = find (c, r, i, j);
    // A value of l in the range r[i,j-1] ≤ t
    // ≤ r[i+1,j] that minimizes
        c[i, l-1] + c[l,j];
    c[i,j] = w[i,j] + c[i, k-1] + c[k,j];
    r[i,j] = k;
  }
  write (c[0,n], w[0,n], r[0,n]);
}
Algorithm find (c, r, i, j)
{
  min = ∞;
  for m = r[i,j-1] to r[i+1,j] do
    if (c[i,m-1] + c[m,j]) < min then
    {
      min = c[i,m-1] + c[m,j]; t = m;
    }
  return t;
}
```

* Optimal Binary Search Tree

Let $p(i)$ be the probability with we can search a_i

$q(i)$ probability of searching element x, such that

$$\boxed{a_i < x < a_{i+1}} \quad \&$$

$$\boxed{0 \leq i \leq n}$$

In other words

$p(i)$ = Probability for successful search

$q(i)$ = Probability for unsuccessful search

$$\boxed{\sum_{1 \leq i \leq n} p(i) + \sum_{1 \leq i \leq n} q(i)}$$

We have to obtain a tree with minimum cost called optimal binary search tree (OBST)

$T_{ij} = (a_{i+1}, a_{i+2}, \ldots, a_n)$

c_{ij} = Cost of T_{ij}

w_{ij} = Weight of each T_{ij}

r_{ij} = root values of T_{ij}

T_{0n} = Final tree which is obtained.

T_{00} = Empty tree

$T_{i,i+1}$ = Single root tree that has element a_{i+1}

Process of Creating OBST

1) Let T_{0n} be the OBST for elements $a_1 < a_2 < \dots < a_n$

Let L & R be its left subtree & Right subtree respectively

2) Suppose that root of T_{0n} is a_k for K

Elements in left subtree L are $(a_1, a_2, \dots a_{k-1})$

Elements in right subtree R are $(a_{k+1}, a_{k+2}, \dots a_n)$

3) The cost of computing T_{0n} can be given as

$$C(T_{0n}) = c(L) + c(R) + p_1 + \dots + p_n + q_1 + q_2 + \dots + q_n$$

$$\boxed{C(T_{0n}) = c(L) + c(R) + W}$$

where

$$W = p_1 + p_2 + \dots + p_n + q_1 + q_2 + \dots q_n$$

p_i = Probability for successful search

q_i = Probability for unsuccessful search.

④ i] Calculate w_{ij} :=

initially $i=j$

ie $i=j=0$ then $w_{00} = q_0$

$i=j=1$ then $w_{11} = q_1$ & so on

$i \neq j$ then,

$w_{ij} = w_{ij-1} + p_j + q_j$

ii] C_{ij} :=

Initially i & j are equal hence,

$C_{ij} = 0$

$i=j=0$ then $C_{00}=0$

$i=j=1$ then $C_{11}=0$

When difference between i & j is 1.

$C_{ij} = q_i + q_j + p_j$

When difference between i & j is more than 1 then,

$$C_{ij} = \min_{i<k\leq j} \left\{ C_{i,k-1} + C_{kj} \right\} + w_{ij}$$

iii) r_{ij} :=

$i=j=0$

$r_{ij}=0$ initially

$r_{00}=0$

$i=j=1$ then $r_{11}=0$

When difference between i & j is 1 then,

$$\boxed{r_{ij}=i+1}$$

when,

$|i-j|>1$ then

$$\boxed{r_{ij}=k}$$

* example :

Construct OBST for n=4 = n = { do, if, int, while }

$P(1:4) = (4,2,1,1)$

$q(0:4) = (2,3,1,1,1)$ construct OBST.

A,

P(1:4) is the values for finding of identifier in the given binary tree.

q(0:4) is the values for unsuccessful search or identifiers not found in that search.

Given,

$P_1 = 4$ $q_0 = 2$

$P_2 = 2$ $q_1 = 3$

$P_3 = 1$ $q_2 = 1$

$P_4 = 1$ $q_3 = 1$

$q_4 = 1$

By using this values of P and q we have to find the values of ① w_{ij} ② C_{ij} ③ r_{ij}

First we calculate w_{ij}

Initially, $i=j=0$ $w_{00} = q_0 = 2$

$i=j=1$ $w_{11} = q_1 = 3$

$i=j=2$ $w_{22} = q_2 = 1$

$i=j=3$ $w_{33} = q_3 = 1$

$i=j=4$ $w_{44} = q_4 = 1$

if $i \neq j$

$w_{ij} = w_{ij-1} + p_j + q_j$

1) $i=0 \quad j=1$

$w_{01} = w_{00} + p_1 + q_1 = 2+4+3 = 9$

$\underline{\underline{w_{01} = 9}}$

2) $i=0 \quad j=2$

$w_{02} = w_{01} + p_2 + q_2 = 9+2+1 = 9+3 = 12$

$\underline{\underline{w_{02} = 12}}$

3) $i=0 \quad j=3$

$w_{03} = w_{02} + p_3 + q_3 = 12+1+1 = 14$

$\underline{\underline{w_{03} = 14}}$

4) $i=0 \quad j=4$

$w_{04} = w_{03} + p_4 + q_4 = 14+1+1 = 16$

$\underline{\underline{w_{04} = 16}}$

5) $i=1 \quad j=2$

$w_{12} = w_{11} + p_2 + q_2 = 3+2+1 = 6$

$\underline{\underline{w_{12} = 6}}$

6) $i=1 \quad j=3$

$w_{13} = w_{12} + p_3 + q_3 = 6+1+1 = 8$

$\underline{\underline{w_{13} = 8}}$

7) $i=1, \; j=4$

$w_{14} = w_{13} + p_4 + q_4 = 8+1+1 = 10$

$\underline{\underline{w_{14} = 10}}$

8) $i=2,\ j=3$

$w_{23} = w_{22} + p_3 + q_3 = 1+1+1 = 3$

$\underline{\underline{w_{23} = 3}}$

9) $i=2,\ j=4$

$w_{24} = w_{23} + p_4 + q_4 = 3+1+1 = 5$

$\underline{\underline{w_{24} = 5}}$

10) $i=3,\ j=4$

$w_{34} = w_{33} + p_4 + q_4 = 1+1+1$

$\underline{\underline{w_{34} = 3}}$

$i \rightarrow$

$j-1 \downarrow$

	0	1	2	3	4
0	$w_{00}=$ 2	$w_{11}=$ 3	$w_{22}=$ 1	$w_{33}=$ 1	$w_{44}=$ 1
1	$w_{01}=$ 9	$w_{12}=$ 6	$w_{23}=$ 3	$w_{34}=$ 3	
2	$w_{02}=$ 12	$w_{13}=$ 8	$w_{24}=$ 5		
3	$w_{03}=$ 14	$w_{14}=$ 10			
4	$w_{04}=$ 16				

$\boxed{\text{Calculate } C_{ij} \text{ \& } r_{ij}}$

Initially i & j are equal

$i=j=0$	$C_{00}=0$	$r_{00}=0$
$i=j=1$	$C_{11}=0$	$r_{11}=0$
$i=j=2$	$C_{22}=0$	$r_{22}=0$
$i=j=3$	$C_{33}=0$	$r_{33}=0$
$i=j=4$	$C_{44}=0$	$r_{44}=0$

* When $|i-j| = 1$

$C_{ij} = q_i + q_j + P_j$ & $r_{ij} = i+1$

1) $i=0,\ j=1;$

$C_{01} = q_0 + q_1 + P_1$

$= 2+3+4$

$C_{01} = 9$

1) $i=0,\ j=1$

$r_{01} = 0+1$

$r_{01} = 1$

2) $i=1,\ j=2;$

$C_{12} = q_1 + q_2 + P_2$

$= 3+1+2$

$C_{12} = 6$

2) $i=1,\ j=2$

$r_{12} = 1+1$

$r_{12} = 2$

3) $i=2,\ j=3;$

$C_{23} = q_2 + q_3 + P_3$

$= 1+1+1$

$C_{23} = 3$

3) $i=2,\ j=3$

$r_{23} = 2+1$

$r_{23} = 3$

4) $i=3,\ j=4;$

$C_{34} = q_3 + q_4 + P_4$

$= 1+1+1$

$C_{34} = 3$

4) $i=3,\ j=4;$

$r_{34} = 3+1$

$r_{34} = 4$

* When $|i-j|>1$ then,

$$\boxed{C_{ij} = \min_{i<k\le j} \left\{ C_{i,k-1} + C_{kj} \right\} + w_{ij}}$$

k = root Node $\boxed{r_{ij} = K}$

$\left.\begin{array}{ll} i=0 & j=2 \\ i=1 & j=3 \\ i=2 & j=4 \end{array}\right\}$ 2 difference $\left.\begin{array}{ll} i=0 & j=3 \\ i=1 & j=4 \end{array}\right\}$ 3 diff^n

$\left.\begin{array}{ll} i=0 & j=4 \end{array}\right\}$ 4 diff^n

We know that,

$\boxed{i<k\le j}$

(i) $i=0 \quad k=1 \quad j=2$

$C_{02} = \{ C_{00} + C_{12} \} + w_{02}$

$= 0+6+12$

$=18$

(ii) $i=0, k=2, j=2$

$C_{02} = \min \{ C_{01} + C_{12} \} + w_{02}$

$= 9+12+0$

$=21$

$C_{02} = \min \left\{ \begin{array}{l} 18 \\ 21 \end{array} \right\}$

$\boxed{C_{02} = 18 \ \& \ r_{02} = 1}$ ✓

(iii) $i=1 \quad k=2/3 \quad j=3$

$$C_{13} = \min \left\{ \begin{array}{l} C_{11} + C_{23} \\ C_{12} + C_{33} \end{array} \right\} \begin{array}{l} + w_{13} \\ + w_{13} \end{array}$$

$$C_{13} = \min \left\{ \begin{array}{l} 0+3+8 \\ 6+0+8 \end{array} \right\} = \min \{ 11, 14 \}$$

$\boxed{C_{13} = 11 \ \& \ r_{13} = 2}$

(IV) $i=2, \quad k=3/4 \quad j=4$

$$C_{24} = \min \begin{Bmatrix} C_{22} + C_{34} + w_{24} \\ C_{23} + C_{44} + w_{24} \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 0+3+5 \\ 3+0+5 \end{Bmatrix}$$

// same value for both $k=3$ & $k=4$

hence select $k=3$ as it is less value than 4

$$\boxed{C_{24} = 8 \quad \& \quad r_{24} = 3}$$

(V) $i=0, \quad k=1/2/3, \quad j=3$

$$C_{03} = \min \begin{Bmatrix} C_{00} + C_{13} + w_{03} \\ C_{01} + C_{23} + w_{03} \\ C_{02} + C_{33} + w_{03} \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 0+11+14 \\ 9+3+14 \\ 18+0+14 \end{Bmatrix}$$

$$\boxed{C_{03} = 25 \quad \& \quad r_{03} = 1}$$

(VI) $i=1, \quad k=2/3/4, \quad j=4$

$$C_{14} = \min \begin{Bmatrix} C_{11} + C_{24} + w_{14} \\ C_{12} + C_{34} + w_{14} \\ C_{13} + C_{44} + w_{14} \end{Bmatrix}$$

$$C_{14} = \min \begin{Bmatrix} 0+8+10 \\ 6+3+10 \\ 11+0+10 \end{Bmatrix}$$

$$\boxed{C_{14} = 18 \quad \& \quad r_{14} = 2}$$

(VII) $i=0, j=k$ & $k=1/2/3/4$

$$C_{04} = \min \begin{Bmatrix} C_{00} + C_{14} + W_{04} \\ C_{01} + C_{24} + W_{04} \\ C_{02} + C_{34} + W_{04} \\ C_{03} + C_{44} + W_{04} \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 0+18+16 \\ 9+8+16 \\ 18+3+16 \\ 25+0+16 \end{Bmatrix} = \min \{34, 33, 37, 41\}$$

$$\boxed{C_{04} = 33 \ \& \ r_{04} = 2}$$

$i \rightarrow$

$j-i \downarrow$

	0	1	2	3	4
0	$C_{00}=0$ $r_{00}=0$	$C_{11}=0$ $r_{11}=0$	$C_{22}=0$ $r_{22}=0$	$C_{33}=0$ $r_{33}=0$	$C_{44}=0$ $r_{44}=0$
1	$C_{01}=9$ $r_{01}=1$	$C_{12}=6$ $r_{12}=2$	$C_{23}=3$ $r_{23}=3$	$C_{34}=3$ $r_{34}=4$	
2	$C_{02}=18$ $r_{02}=1$	$C_{13}=11$ $r_{13}=2$	$C_{24}=8$ $r_{24}=3$		
3	$C_{03}=25$ $r_{03}=1$	$C_{14}=18$ $r_{14}=2$			
4	$C_{04}=33$ $r_{04}=2$				

- Finally we got $r_{04}=2$, Using optimal binary search tree root node will be $r_{04}=2$ ie (if) will be root node

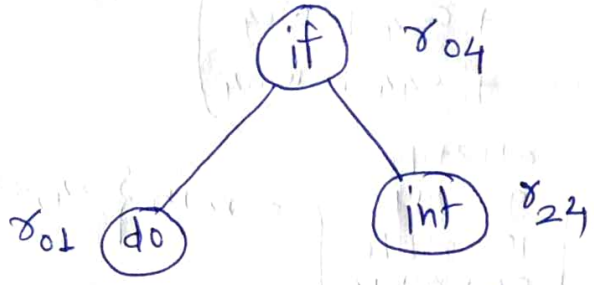
- To calculate the childrens we will be use following formula.

$r_{ij}=k$ then Left Children $= r_{ik-1}$
Right Children $= r_{kj}$

$r_{04}=02$ ie
$i=0, j=4, k=2$

- Left child $r_{ik-1} = r_{01} = 1$ (do)

right child $r_{kj} = r_{24} = 3$ (int)



- To Calculate the children of do.

$r_{01} = k = 1$

ie $i = 0$, $j = 1$, $k = 1$

Left child $r_{ik-1} = r_{00} = 0$ ∵ Empty Node.

Right child $r_{kj} = r_{11} = 0$ ∵ Empty Node

- To Calculate child of r_{24}

$r_{24} = 3$

ie $i = 2$, $j = 4$, $k = 3$

Left child $r_{ik-1} = r_{22} = 0$ ∵ Empty Node

right child. $r_{kj} = r_{34} = 4$ ie. (while)

Hence Final Tree

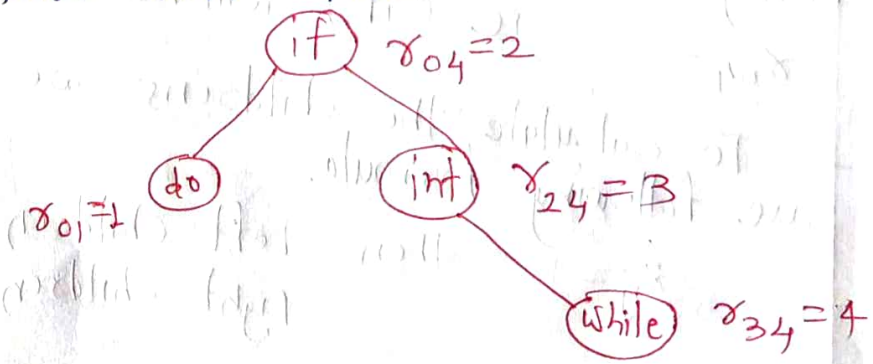
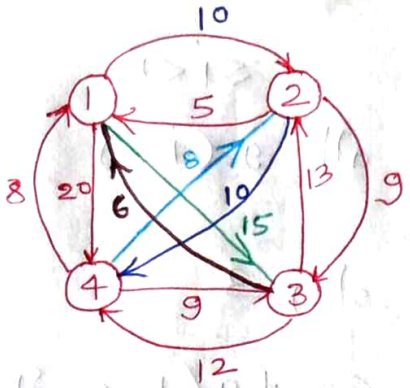


Fig:- Optimal Binary Search Tree



Edge Length matrix

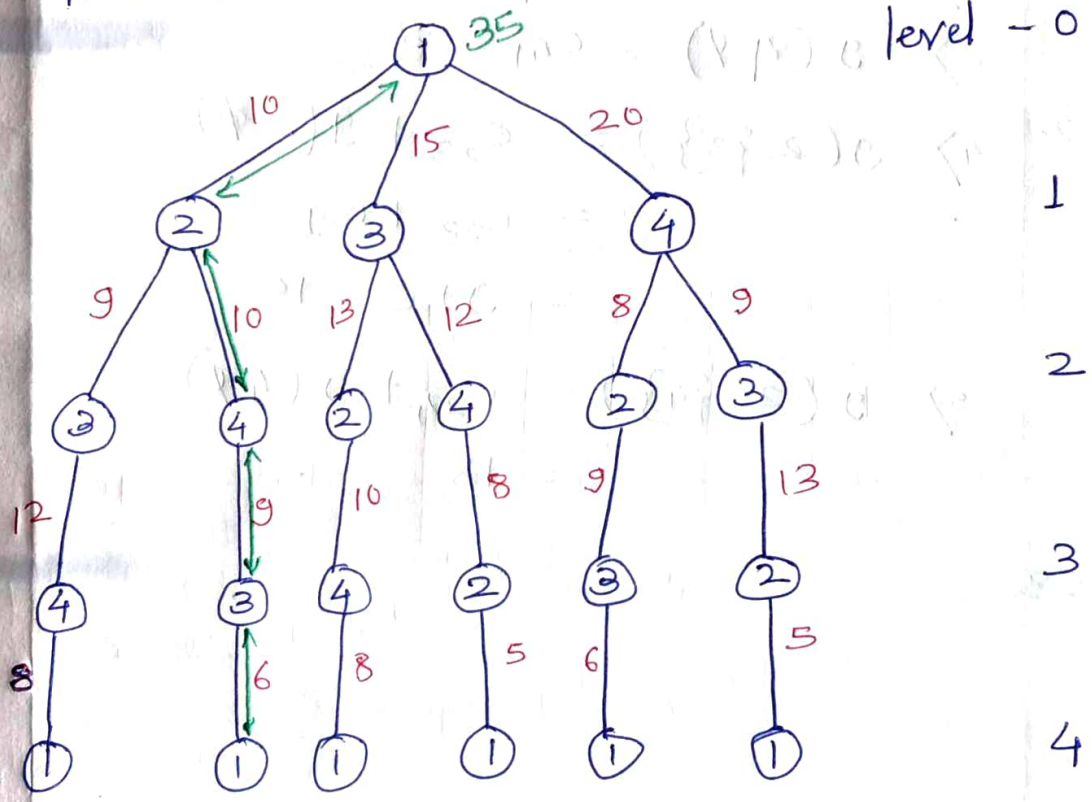
The adjacency matrix for given directed graph

if $V\langle i,j\rangle$ = cost

$V\langle i,i\rangle = 0$

$$A = \begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

By Using brutforce Approach the tree is as follows



Tree:- Using Brutforce Travelling

From the example we got 35 as a shortest path with 1-2-4-3-1

After solving problem it should be same as it

$g(i, \phi) = c_{i1} \qquad 2 \leq i < n$

$|s| = 1$ obtain all $g(i, s)$

then $|s| = |s| + 1$

if $|s| < n-1$ goto step 3

$$\boxed{g(i,s) = \min_{k \in s} \left\{ c_{ik} + g(k, s - \{k\}) \right\}}$$

$$\boxed{g(i, \phi) = c_{i1} \qquad 2 \leq i < n}$$

1) $g(2, \phi) = c_{21} = 5$

2) $g(3, \phi) = c_{31} = 6$

3) $g(4, \phi) = c_{41} = 8$

$i=2$

4) $g(2, \{3\}) = c_{23} + g(3, \phi)$

$= c_{23} + c_{31}$

$= 9 + 6 = 15$

5) $g(2, \{4\}) = c_{24} + g(4, \phi)$

$= 18$

$i=3$

* Travelling Salesperson :

If $G\langle V, E\rangle$ is directed graph & each edge associated with cost c_{ij} for all i, j.

$\therefore$ $C_{ij} \geq 0$ for all i & j

$C_{ij} = \infty$ if $\langle i, j\rangle \notin E$

$|V| = n$ no. of vertices $\forall$ $n > 1$

The tour start & end at node 1 ; every tour consist of edge $\langle 1, k\rangle$ for some $k \in V-1$ & path from vertex k to 1

Initialization :

$g(i, \phi) = C_{i1}$ $2 \leq i < n$

$|S| = 1$ obtain all $g(i, S)$

$|S| = |S| + 1$

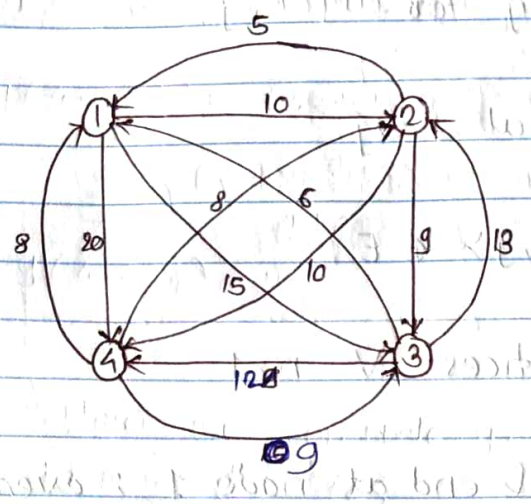
if $|S| < n-1$ goto step 3.

$$\boxed{g(i, S) = \min_{k \in S} \left\{ C_{ik} + g(k, S - \{k\}) \right\}}$$

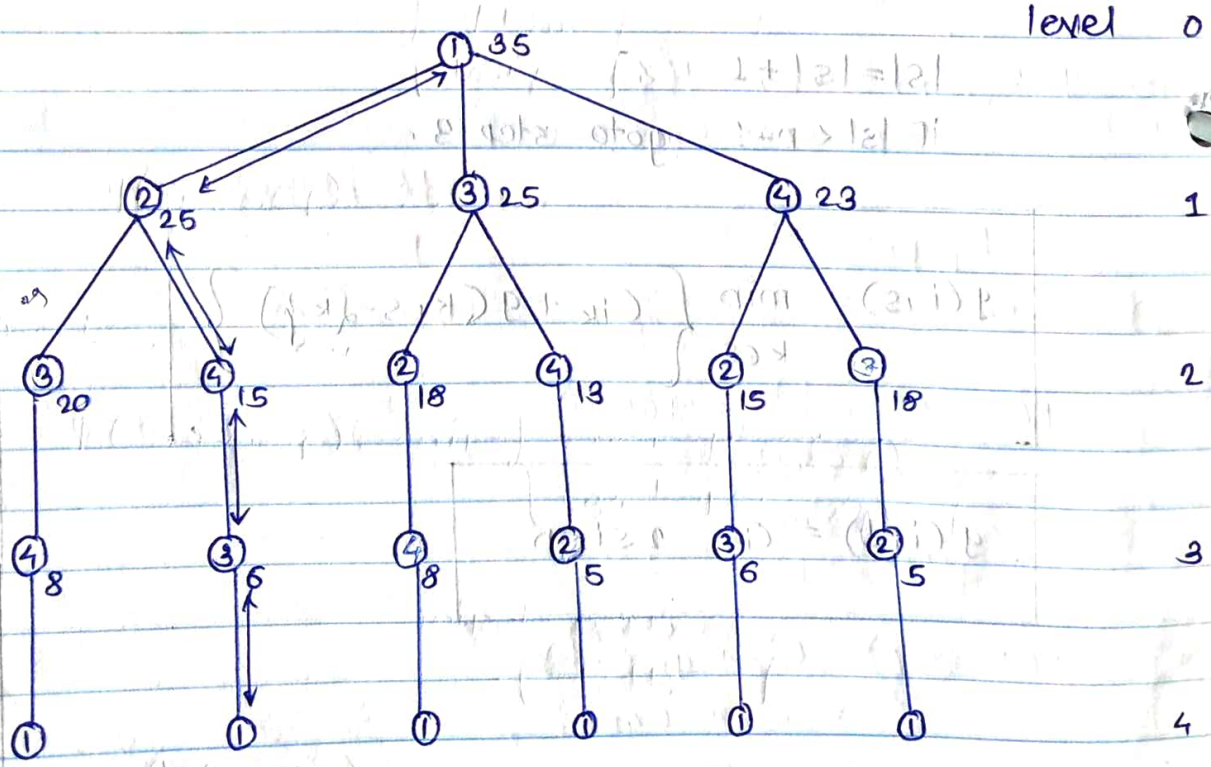
$$\boxed{g(i, \phi) = C_{i1} \quad 2 \leq i \leq n}$$

example:

1) solve by using travelling salesperson problem.



$$A = \begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$



Tree: possibilities of travelling Brutforce.

After level 3 the person has to return to level 4 (which we have assume as vertex 1)

$$g(1,\{2,3,4\}) = \min_{k \in \{2,3,4\}} \{ c_{1k} + g(k,\{2,3,4\} - \{k\}) \}$$

$$g(i,S) = \min_{k \in S} \{ c_{ik} + g(k, S - \{k\}) \}$$

① $g(2,\phi) = c_{21} = \underline{\underline{5}}$

② $g(3,\phi) = c_{31} = \underline{\underline{6}}$

③ $g(4,\phi) = c_{41} = \underline{\underline{8}}$

④ $g(2,\{3\}) = c_{23} + g(3,\phi)$

$= c_{23} + c_{31}$

$\underline{\underline{g(2,\{3\}) = 15}}$

⑤ $g(2,\{4\}) = c_{24} + g(4,\phi)$

$= c_{24} + c_{41}$

$\underline{\underline{g(2,\{4\}) = 18}}$

$i = 3$

⑥ $g(3,\{2\}) = c_{32} + g(2,\phi)$

$= 18 + 5$

$\underline{\underline{g(3,\{2\}) = 18}}$

⑦ $g(3,\{4\}) = c_{34} + g(4,\phi)$

$= c_{34} + c_{41}$

$= 12 + 8$

$\underline{\underline{g(3,\{4\}) = 20}}$

$i=4$

⑧ $g(4,\{2\}) = C_{42} + g(2,\phi)$

$= C_{42} + C_{21}$

$= 8+5$

$\underline{\underline{g(4,\{2\}) = 13}}$

⑨ $g(4,\{3\}) = C_{43} + g(3,\phi)$

$= C_{43} + C_{31}$

$= 9+6$

$\underline{\underline{g(4,\{3\}) = 15}}$

Now we compute $g(i,s)$ with $|s|=2$

$i \neq 1$, $1 \notin s$, $i \notin s$

$i=2$, $s=3,4$

⑩ $g(2,\{3,4\}) = \min \begin{Bmatrix} C_{23} + g(3,\{4\}) \\ C_{24} + g(4,\{3\}) \end{Bmatrix}$

$= \begin{Bmatrix} 9+20 \\ 10+15 \end{Bmatrix}$

$\underline{\underline{g(2,\{3,4\}) = 25}}$

⑪ $i=3$, $s=2,4$

$g(3,\{2,4\}) = \min \begin{Bmatrix} C_{32} + g(2,\{4\}) \\ C_{34} + g(4,\{2\}) \end{Bmatrix}$

$= \min \begin{Bmatrix} 13+18 \\ 12+13 \end{Bmatrix}$

$\underline{\underline{g(3,\{2,4\}) = 25}}$

$i=4,\ S=2,3$

② $g(4,\{2,3\}) = \min\begin{Bmatrix} C_{42} + (2,\{3\}) \\ C_{43} + (3,\{2\}) \end{Bmatrix}$

$= \min\begin{Bmatrix} 8+15 \\ 9+18 \end{Bmatrix}$

$\underline{\underline{g(4,\{2,3\}) = 23}}$

$i=1,\ S=2,3,4$

③ $g(1,\{2,3,4\}) = \min\begin{Bmatrix} C_{12} + g(2,\{3,4\}) \\ C_{13} + g(3,\{2,4\}) \\ C_{14} + g(4,\{2,3\}) \end{Bmatrix}$

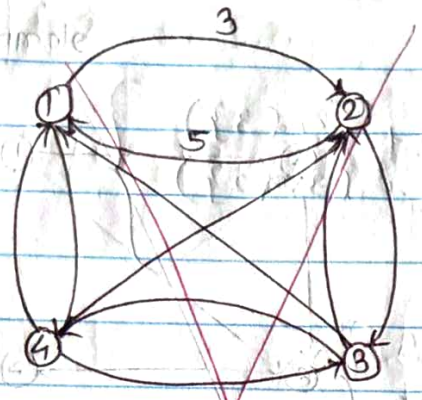
$= \min\begin{Bmatrix} 10+25 \\ 15+25 \\ 20+23 \end{Bmatrix}$

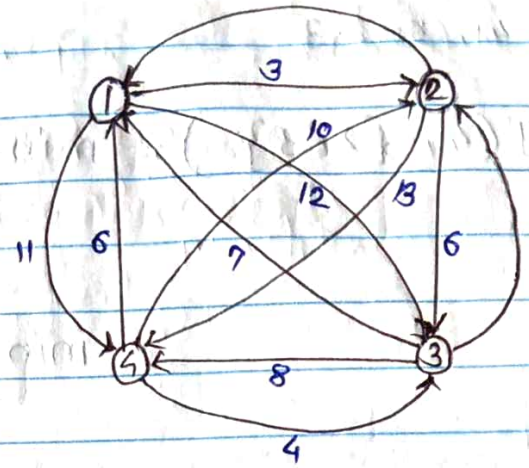
$\underline{\underline{g(1,\{2,3,4\}) = 35}}$

Which is belongs to 1-2-4-3-1

Hence shortest path is 1-2-4-3-1

②





$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 3 & 12 & 11 \\ 5 & 0 & 6 & 13 \\ 7 & 9 & 0 & 8 \\ 6 & 10 & 4 & 0 \end{bmatrix}$$

$$g(i, \phi) = C_{i1} \quad 2 \leq i < n$$

$$g(i, S) = \min_{k \in S} \left\{ C_{ik} + g(k, S - \{k\}) \right\}$$

$$g(2, \phi) = C_{21} = 5$$

$$g(3, \phi) = C_{31} = 7$$

$$g(4, \phi) = C_{41} = 6$$

$$g(1, S) = \min_{k \in S} \left\{ C_{1k} + g(k, S - \{k\}) \right\} \quad \text{——— ②}$$

$\boxed{i=2}$

$$g(2, \{3\}) = C_{23} + g(3, \phi)$$
$$= C_{23} + C_{31}$$
$$= 6 + 7$$
$$= 13$$

$g(2,\{4\}) = C_{24} + g(4,\phi)$

$= C_{24} + C_{41}$

$= 13 + 6$

$g(2,\{4\}) = 19$

$\boxed{i=3}$

$g(3,\{2\}) = C_{32} + g(2,\phi)$

$= C_{32} + C_{21}$

$= 9 + 5$

$g(3,\{2\}) = 14$

$g(3,\{4\}) = C_{34} + g(4,\phi)$

$= C_{34} + g_{41}$

$= 8 + 6$

$g(3,\{4\}) = 14$

$\boxed{i=4}$

$g(4,\{2\}) = C_{42} + g(2,\phi)$

$= C_{42} + C_{21}$

$= 10 + 5$

$g(4,\{2\}) = 15$

$g(4,\{3\}) = C_{43} + g(3,\phi)$

$= C_{43} + C_{31}$

$= 4 + 7$

$g(4,\{3\}) = 11$

$\boxed{i=2,\ S=3,4}$

$$g(2,\{3,4\}) = \min \begin{Bmatrix} C_{23} + g(3,\{4\}) \\ C_{24} + g(4,\{3\}) \end{Bmatrix}$$

$$= \begin{Bmatrix} 6+14 \\ 13+11 \end{Bmatrix}$$

$$\underline{\underline{g(2,\{3,4\}) = 20}}$$

$\boxed{i=3,\ S=2,4}$

$$g(3,\{2,4\}) = \min \begin{Bmatrix} C_{32} + g(2,\{4\}) \\ C_{24} + g(4,\{2\}) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 9+19 \\ 8+15 \end{Bmatrix}$$

$$\underline{\underline{g(3,\{2,4\}) = 23}}$$

$\boxed{i=4,\ S=2,3}$

$$g(4,\{2,3\}) = \min \begin{Bmatrix} C_{42} + g(2,\{3\}) \\ C_{43} + g(3,\{2\}) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 10+13 \\ 4+14 \end{Bmatrix}$$

$$\underline{\underline{g(4,\{2,3\}) = 18}}$$

$\boxed{i=1,\ S=2,3,4}$

$$g(1,\{2,3,4\}) = \min \begin{Bmatrix} C_{12} + g(2,\{3,4\}) \\ C_{13} + g(3,\{2,4\}) \\ C_{14} + g(4,\{2,3\}) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 3+20 \\ 12+23 \\ 11+18 \end{Bmatrix}$$

$$\underline{\underline{g(1,\{2,3,4\} = 23}}$$

which is belongs to 1-2-3-4-1

Hence, shortest path is 1-2-3-4-1

③

$$B = \begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 18 \\ 6 & 13 & 0 & 12 \\ 8 & 20 & 9 & 0 \end{bmatrix}$$

$g(i,\phi) = C_{ij}$, $2 \le i < n$

$g(i,S) = \min\{C_{ik} + g(k, S-\{k\})\}$

① $g(2,\phi) = C_{21} = \underline{\underline{5}}$

② $g(3,\phi) = C_{31} = \underline{\underline{6}}$

③ $g(4,\phi) = C_{41} = \underline{\underline{8}}$

$$g(i,S) = \min_{k \in S} \{ C_{ik} + g(k, S-\{k\}) \}$$

$\boxed{i=2}$

$$g(2,\{3\}) = C_{23} + g(3,\phi)$$
$$= C_{23} + C_{31}$$
$$= 9+6$$
$$\underline{\underline{g(2,\{3\}) = 15}}$$

$$g(2,\{4\}) = C_{24} + g(4,\phi)$$
$$= C_{24} + C_{41}$$
$$= 8+8$$
$$\underline{\underline{g(2,\{4\}) = 16}}$$

$\boxed{i=3}$

$$g(3,\{2\}) = C_{32} + g(2,\phi)$$
$$= C_{32} + C_{21}$$
$$= 13+5$$
$$\underline{\underline{g(3,\{2\}) = 18}}$$

$$g(3,\{4\}) = C_{34} + g(4,\phi)$$
$$= C_{34} + C_{41}$$
$$= 12+8$$
$$\underline{\underline{g(3,\{4\}) = 20}}$$

$\boxed{i=4}$

$$g(4,\{2\}) = C_{42} + g(2,\phi)$$
$$= 20+5$$
$$= 25$$
$$\underline{\underline{g(4,\{2\}) = 25}}$$

$$g(4,\{3\}) = C_{43} + g(3,\phi)$$
$$= C_{43} + C_{3}$$
$$= 9+6$$
$$\underline{\underline{g(4,\{3\}) = 15}}$$

$\boxed{i=2,\ S=3,4}$

$$g(2,\{3,4\}) = \min\begin{Bmatrix} C_{23} + g(3,\{4\}) \\ C_{24} + g(4,\{3\}) \end{Bmatrix}$$

$$= \min\begin{Bmatrix} 9+20 \\ 8+15 \end{Bmatrix}$$

$$\underline{\underline{g(2,\{3,4\}) = 23}}$$

$\boxed{i=3,\ S=2,4}$

$$g(3,\{2,4\}) = \min\begin{Bmatrix} C_{32} + g(2,\{4\}) \\ C_{34} + g(4,\{2\}) \end{Bmatrix}$$

$$= \min\begin{Bmatrix} 13+16 \\ 12+25 \end{Bmatrix}$$

$$\underline{\underline{g(3,\{2,4\}) = 29}}$$

$\boxed{i=4,\ S=2,3}$

$$g(4,\{2,3\}) = \min\begin{Bmatrix} C_{42} + g(2,\{3\}) \\ C_{43} + g(3,\{2\}) \end{Bmatrix}$$

$$= \min\begin{Bmatrix} 20+15 \\ 9+18 \end{Bmatrix}$$

$$\underline{\underline{g(4,\{2,3\}) = 27}}$$

$\boxed{i=1,\ S=2,3,4}$

$$g(1,\{2,3,4\}) = \min \begin{Bmatrix} c_{12} + g(2,\{3,4\}) \\ c_{13} + g(3,\{2,4\}) \\ c_{14} + g(4,\{2,3\}) \end{Bmatrix}$$

$$= \min \begin{Bmatrix} 10 + 23 \\ 15 + 29 \\ 20 + 27 \end{Bmatrix}$$

$$\underline{\underline{g(1,\{2,3,4\}) = 33}}$$

shortest path is 1-2-4-3-1

0/1 knapsack Problem:

In a dynamic program as the name suggest 0/1 knapsack Avoid fraction of weights and such objects.

If w_i = Weight,
P_i = Profit,
M = Capacity,
then

① $\sum_{i=1}^{n} w_i x_i \leq M$

② $\sum_{i=1}^{n} P_i x_i$ is maximize

x_i assign the value either 0 or 1 which means objects can completely fill or not fill.

Algorithm:

```
Struct knapsack 0/1
{
    int profit;
    int wt;
} pairs [1,n]
knapsack 0/1 ()
{
    initialize pairs [] to their values
    int m;
    int n;

    S° = {(0,0)}
    for (i=1 to n-1)
    {
```

$S_1^{i-1} = \{ \langle P, W \rangle \mid \langle P - P_i, W - W_i \rangle \in S^{i-1} \& W$

$S^i = \text{Murge \& Purge}(S^{i-1}, S_1^{i-1})$

}

Let (P_x, W_x) be the pair in S^{n-1}

$(P_y, W_y) = (P^1 + P_n, W^1 + W_n)$

Here,

W^1 is largest W in any pair that $W + W_n \leq M$

If $(P_n > P_y)$ then $x_n = 0$
else $x_n = 1$

Traceback $(x_{n-1}, x_{n-2}, \ldots\ldots x_n)$

}

* Procedure :

Initially we are taking bag as (ie knapsack) empty with pairs (P, W) ie (profit & weight)

$(P, W) = (0, 0)$

As all we know, we are taking all the objects & their profits & weights ie addition of previous bag capacity (P, W).

Then we take union of previous two possibilities of two ~~fat~~ further consideration.

ie, possiblities $P_1 = (P_i, W_i)$

$P_2 = (P_j, W_j)$

then

$P_3 = \{(P_i, W_i), (P_j, W_j)\}$

example :

Object	Profit	weight	
x_1	1	3	0
x_2	2	2	1
x_3	5	15	
x_4	3	4	1

Capacity of knapsack $M = 21$

Now first possiblity or initial possiblity should be,

$P_0 = (0,0)$

Profit Weight

$P_1 = (0,0) + (1,3)$
$= (1,3)$

$P_2 = P_0 \cup P_1$
$= \{(0,0), (1,3)\}$

$P_3 = \{(0,0) + (2,2), (1,3) + (2,2)\}$
$= \{(2,2), (3,5)\}$

$P_4 = P_2 \cup P_3$
$= \{(0,0), (1,3), (2,2) (3,5)\}$

$P_5 = \{(0,0) + (5,15), (1,3) + (5,15),$
$\underline{(2,2) + (5,15),} \quad (3,5) + (5,15)\}$

$= \{(5,15), (6,18), (7,17), (8,20)\}$

$P_6 = P_4 \cup P_5$

$= \{(0,0), (1,3), (2,2), (4,4), (5,15), (6,18), (7,17), (8,20)\}$

$P_7 = \{(P_6) + (3,4)\}$

$= \{(3,4), (4,7), (5,6), (7,8), (8,19), (9,22), (10,21), (11,24)\}$

finally,

$P_8 = P_6 \cup P_7$

$P_8 = \{(0,0), (1,3), (2,2), (4,4), (5,15), (6,18), (7,17), (9,19), (3,4), (4,7), (5,6), (7,8), (8,19), (9,22), (10,21), (11,24)\}$

Now from given pairs select such pairs which having weight = 21 ie (10,21)

Hence, profit = 10 with optimal solution (0,1,1,1)

2)

object	Profit	weight
x_1	1	2
x_2	2	3
x_3	5	4

Capacity of bag $= M = 6$

Initially in bag,

$S_0 = (0,0)$

Profit Weight

$S_1 = \{(0,0) + (1,2)\}$
$= \{(1,2)\}$

$S_1' = S_0 \cup S_1$
$= \{(0,0), (1,2)\}$

$S_2 = \{(0,0) + (2,3), (1,2) + (2,3)\}$
$= \{(2,3), (3,5)\}$

$S_2' = S_1' \cup S_2$
$= \{(0,0), (1,2), (2,3), (3,5)\}$

$S_3 = \{(0,0) + (5,4), (1,2) + (5,4)$
$(2,3) + (5,4), (3,5) + (5,4)\}$
$= \{(5,4), (6,6), (7,7) (8,9)\}$

$S_3' = \{(0,0), (1,2), (2,3), (3,5),$
$(5,4), (6,6), (7,7), (8,9)\}$

weight is high weight is high

optimal solution $= (x_1, x_2, x_3)$
$= (1, 0, 1)$

Maximum profit $= 6$.

3)

object	profit	weight	
x_1	2	10	1
x_2	5	15	0
x_3	8	6	1
x_4	1	9	1

capacity = 25.

Initially in bag.

$S_0 = (0,0)$

$S_1 = \{(0,0) + (2,10)\}$
$= \{(2,10)\}$

$S_1' = \{S_0 \cup S_1\}$
$= \{(0,0), (2,10)\}$

$S_2 = \{(0,0)+(5,15), (2,10)+(5,15)\}$
$= \{(5,15), (7,25)\}$

$S_2' = \{S_1 \cup S_1'\}$

$= \{(0,0)(2,10), (5,15), (7,25)\}$

$S_3 = \{(0,0)+(8,6), (2,10)+(8,6)$
$(5,15)+(8,6), (7,25)+(8,6)\}$
$= \{(8,6), (10,16), (13,21), (15,31)\}$

$S_3' = \{S_2 \cup S_3\}$
$= \{(0,0), (2,10), (5,15), (7,25)$
$(8,6), (10,16), (13,21), (15,31)\}$

$S_4 = \{ (0,0) + (1,9),\ (2,10) + (1,9)$
$(5,15) + (1,9),\ (7,25) + (1,9)$
$(8,6) + (1,9),\ \underline{(10,16) + (1,9)}$
$(13,21) + (1,9),\ (15,31) + (1,9) \}$

$S_4 = \{ (1,9),\ (3,19)\ (6,24),\ (8,34)$
$(9,15),\ \underline{(11,25)},\ (14,30),\ (16,40) \}$
$\times \quad \times$

= optimal solution $(x_1\ x_2\ x_3\ x_4) =$
$(1, 0, 1, 1)$

= maximum profit $= 11$, weight is 25.

Reliability Design:

Possibilities of Success:

In a serial connection the reliability of that device multiplication of all the reliability which are assemble for that device.

In a dynamic programming we have to find out the maximum reliability for the given set of devices.

In Reliability design we have to given,

Total system cost and we have to ~~given~~ find out the reliability with respect to the total cost of all supplimentry devices may be equals to total system cost or less than total system cost.

If the exceids the limit of total system cost then we can't assume that as a feasible solution.

Assume 5 magnetic tapes on PC working with reliability of 0.95 each then combinely they give,

$= 0.95 \times 0.95 \times 0.95 \times 0.95 \times 0.95$

$= 0.7737 \approx \underline{\underline{0.77}}$

In a serial connection, It is reduced upto 18% hence our target to keep that relibility as it is or maximize.

* $$u_i = \frac{c + c_i - \sum_{j=1}^{n} c_j}{c_i}$$

Where,

u_i = Maximum available copies or No. of items or devices for each object.

C = Total cost of system in problem

C_i = Cost of current device.

$\sum_{j=1}^{n} C_j$ = Sum of all cost for all devices.

* Now find the reliability:

$$= \left(1 - (1 - r_i)^{u_i}\right)$$

examples :

1) find out the Reliability of the following problem. The total system cost = 175.

Device	D1	D2	D3
Cost	40	25	35
Reliability	0.75	0.85	0.60

Total cost = 40 + 25 + 35
= 100

The total cost after taking a single devices for a design which is less than total system cost, hence which is,

$$U_i = \frac{C + C_i - \sum_{j=1}^{n} C_j}{C_i}$$

$\forall$,

U_i = No. of itemes or devices for each object.

C = Total system cost.

C_i = Cost of current device.

$\sum_{j=1}^{n} C_j$ = sum of all cost for all devices.

1) first object quantity

$$U_1 = \frac{175 + 40 - (40 + 25 + 35)}{40}$$

$$= \frac{115}{40}$$

$= 2.9$

Taking lower bound = 2

2) No. of U_2 objects

$$U_2 = \frac{175 + 25 - 100}{25}$$

$$U_2 = 4$$

3) No. of U_3 objects

$$U_3 = \frac{175 + 35 - 100}{35}$$

$$= \frac{210 - 100}{35}$$

$$= \frac{110}{35}$$

$$= 3.1$$

$$U_3 \approx 3$$

No. of objects are (maximum objects)
(2, 4, 3)

Hence,

$2 \times 40 + 4 \times 25 + 3 \times 35$

$= 80 + 100 + 105$

$= 395$

$175 < 395$ Hence we can't take all object at once time.

1) $D_1 = 1$, $D_2 = 1$, $D_3 = 1$

$40 + 25 + 35$

total cost $= 100$

2) $D_1 = 2$, $D_2 = 1$, $D_3 = 1$

$40 \times 2 + 25 + 35$

total cost $= 140$

3) $D_1=2$, $D_2=2$, $D_3=1$

$2\times 40+2\times 25+35$

total cost $=165$

4) $D_1=2$, $D_2=1$, $D_3=2$

$2\times 40+25+2\times 35$

total cost $=175$

5) $D_1=1$, $D_2=1$, $D_3=3$

$40+25+3\times 35$

total cost $=170$

6) $D_1=1$, $D_2=3$, $D_3=1$

$40+25\times 3+35$

total cost $=150$

7) $D_1=2$, $D_2=3$, $D_3=1$

$2\times 40+3\times 25+35$

total cost $=190$

8) $D_1=1$, $D_2=2$, $D_3=2$

$1\times 40+2\times 25+2\times 35$

total cost $=160$

9) $D_1=1$, $D_2=2$, $D_3=3$

$1\times 40+2\times 25+3\times 35$

total cost $=195$

~~10) $D_1=1$, $D_2=1$, $D_3=3$~~

Now find out the reliability,

$$(1-(1-r_i)^{u_i})$$

1) (1, 1, 1)
2) (2, 1, 1)
3) (2, 2, 1)
4) (2, 1, 2)
5) (1, 3, 1)
6) (1, 2, 2)
~~7) (1, 4, 1)~~
8) (1, 1, 3)
9) (1, 2, 1)
10) (1, 1, 2)

1) $\{1-(1-0.75)^1\} * \{1-(1-0.85)^1\} * \{(1-(1-0.35)\}$

$= 0.75 \times 0.85 \times 0.6$

$= 0.45 \times 0.85$

$= 0.3825$ $\therefore$ $\boxed{38.25\%}$

2) (2, 1, 1)

$= \{1-(1-0.75)^2 * 0.85 \times 0.6$

$= 0.4781$ $\therefore$ $\boxed{47.81\%}$

3) (2, 2, 1)

$= \{1-(1-0.75)^2\} * \{1-(1-0.85)^2\} * \{0.6\}$

$= 0.9775$ $\therefore$ $\boxed{97.75\%}$

4) (2, 1, 2)

$= \{1-(1-0.75)^2\} * \{0.85\} * \{1-(1-0.6)^2\}$

$= 0.6693$ $\therefore$ $\boxed{66.93\%}$

5) (1, 3, 1)

$= \{0.75\} * \{1-(1-0.85)^3\} * \{1-(1-0.6)^1\}$

$= 0.4484$ $\therefore$ $\boxed{44.84\%}$

6) (1,2,2)

$= \{0.75\} * \{1-(1-0.85)^2\} * \{1-(1-0.6)^2\}$

$= 0.6158$ $\therefore$ $\boxed{61.58\ \%}$

7) (1,1,3)

$= \{0.75\} * \{0.85\} * \{1-(1-0.6)^3\}$

$= 0.5967$ $\therefore$ $\boxed{59.67\ \%}$

8) (1,2,1)

$= 0.75 * \{1-(0.85)^2\} * 0.6$

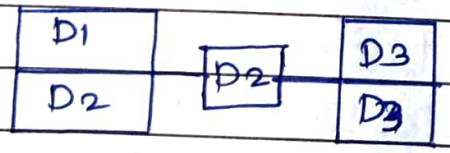
$= 0.4398$ $\therefore$ $\boxed{43.98\ \%}$

9) (1,1,2)

$= 0.75 * 0.85 * \{(1-(1-0.6)^2)\}$

$= .5388$ $\therefore$ $\boxed{53.88\ \%}$

highest reliability is (2,1,2) with 66.93 % reliability ~~or~~ of devices.



2) find out the Reliability of the following problem. The total system cost = 105

Cost	Reliability	Device
30	0.9	2
15	0.8	3
20	0.5	3

Now calculate the possibilities,

1) $D_1 = 1$, $D_2 = 1$, $D_3 = 1$
$30 + 15 + 20 = \underline{\underline{65}}$

2) $D_1 = 1$, $D_2 = 2$, $D_3 = 1$
$30 + 15 \times 2 + 20 = \underline{\underline{80}}$

3) $D_1 = 1$, $D_2 = 1$, $D_3 = 2$
$30 + 15 + 2 \times 20 = \underline{\underline{85}}$

4) $D_1 = 2$, $D_2 = 1$, $D_3 = 2$
$2 \times 30 + \cancel{2 \times} 15 + 2 \times 20 = \underline{\underline{115}}$

5) $D_1 = 2$, $D_2 = 2$, $D_3 = 2$
$30 \times 2 + 15 \times 2 + 2 \times 20 = \underline{\underline{130}}$

6) $D_1 = 2$, $D_2 = 1$, $D_3 = 1$
$2 \times 30 + 15 + 20 = \underline{\underline{95}}$

7) $D_1 = 1$, $D_2 = 1$, $D_3 = 3$
$30 \times 1 + 15 \times 1 + 20 \times 3 = \underline{\underline{105}}$

8) $D_1 = 1$, $D_2 = 3$, $D_3 = 1$
$30 + 3 \times 15 + 20 = \underline{\underline{95}}$

9) $(1,2,2) = D_1 = 1$, $D_2 = 2$, $D_3 = 2$
$30 \times 1 + 15 \times 2 + 20 \times 2 = \underline{\underline{95}}$

1) (1, 1, 1)
2) (1, 2, 1)
3) (1, 2, 2)
4) (2, 1, 2)
5) (2, 2, 2)
6) (2, 1, 1)
7) (1, 1, 3)
8) (1, 3, 1)
9) (1, 2, 2)

1) (1,1,1)

$= 0.9 * 0.8 * 0.5$

$= 0.3600 = \boxed{36.00\%}$

2) (1,2,1)

$0.9 * (1-(1-0.8)^2) * 0.5$

$= 0.4320$

$= \boxed{43.20\%}$

3) (1,1,2)

$= 0.9 * 0.8 * (1-(1-0.5)^2)$

$= 0.54$

$= \boxed{54.00\%}$

4) (2,1,1)

$(1-(1-0.9)^2) * 0.8 * 0.5$

$= 0.5400$

$= \boxed{54.00\%}$

5) (1,1,3)

$= 0.9 * 0.8 * (1-(1-0.5)^3)$

$= 0.6264 = \boxed{62.64\%}$

6) (1,3,1)

$= 0.9 * (1-(1-0.8)^3) * 1$

$= 0.4455 = \boxed{44.55\%}$

7) (1,2,2)

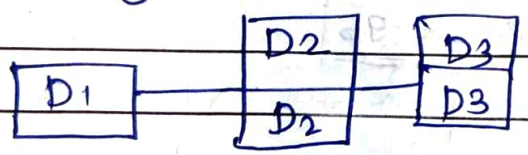
$= 0.9 * (1-(1-0.8)^2) * (1-(1-0.6)^2)$

$= 0.6400$

$= \boxed{64.00\%}$

$(1,2,2) = \boxed{64.00\%}$

The highest reliability is (1,2,2) with 64.00%



3) Solve the following reliability problem with 3 stages, cost = 140

Device	D1	D2	D3
Cost	47	48	45
Reliability	0.8	0.6	0.9

given total cost = 47 + 48 + 45
= 140

$$u_i = \frac{c + c_i - \sum_{j=1}^{n} c_j}{c_i}$$

$\forall$,

u_i = No. of items or devices for each object.

c = Total system cost.

c_i = cost of current device.

$\sum_{j=1}^{n} c_j$ = sum of all cost for all devices.

1) first device quantity,

$$u_1 = \frac{140 + 47 - 140}{47}$$

$u_1 = 1$

2) second device quantity

$$u_2 = \frac{140 + 48 - 140}{48}$$

$u_2 = 1$

3) Third device quantity,

$$u_3 = \frac{140 + 45 - 140}{45}$$

$$u_3 = 1$$

then,

No. of objects are (Maximum objects)

(1, 1, 1)

Hence,

$1 \times 48 + 1 \times 47 + 1 \times 45$

$= 48 + 47 + 45$

$= 140$

find out Reliability

(1, 1, 1)

reliability $= (1 - (1 - r_i)^{u_i})$

$D_1 = (1 - (1 - 0.8)^1)$

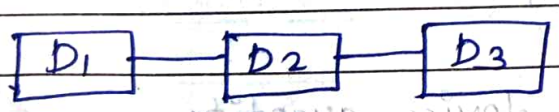
$= 0.8$

$D_2 = 0.6$

$D_3 = 0.9$

$(1, 1, 1) = 0.8 \times 0.6 \times 0.9$

$= 0.432$

then

$\boxed{43.20\ \%}$