

Supervised Projective Learning with Orthogonal Completeness

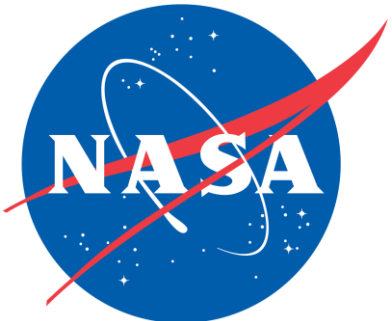
Tyler J Grear

Code 610.1 Intern

Global Modeling and Assimilation Office

NASA Goddard Space Flight Center

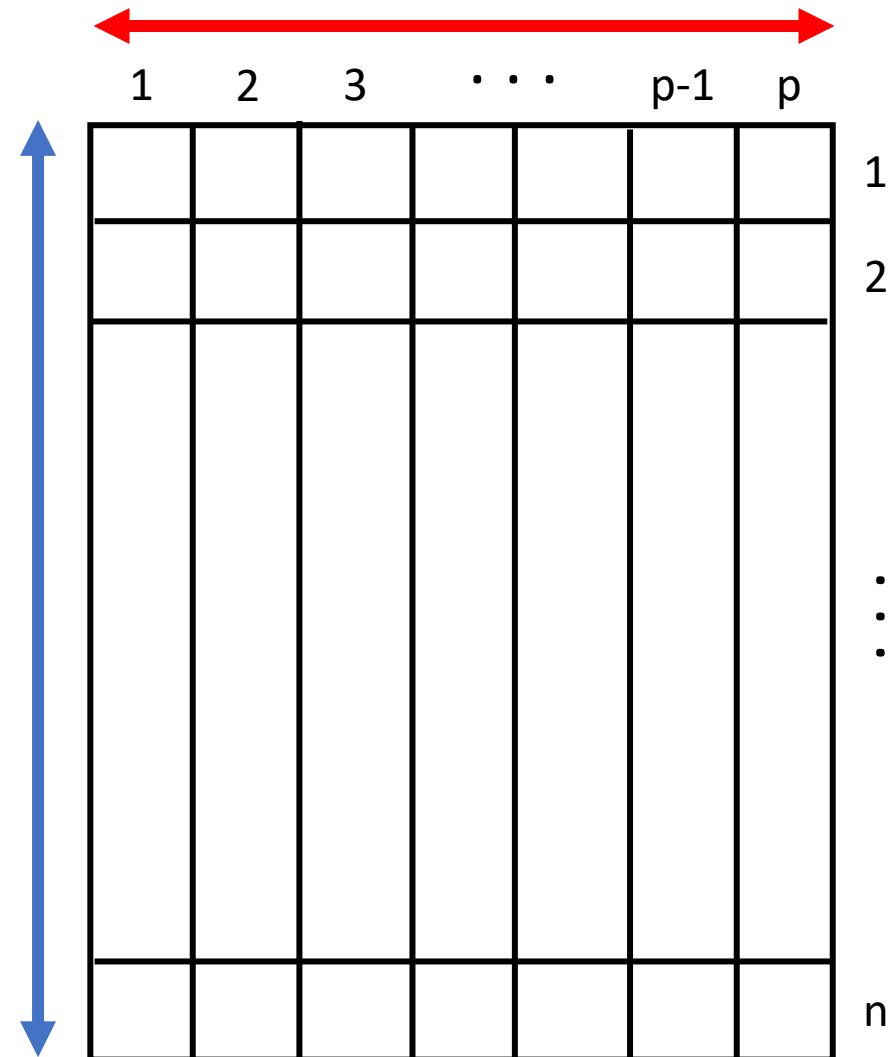
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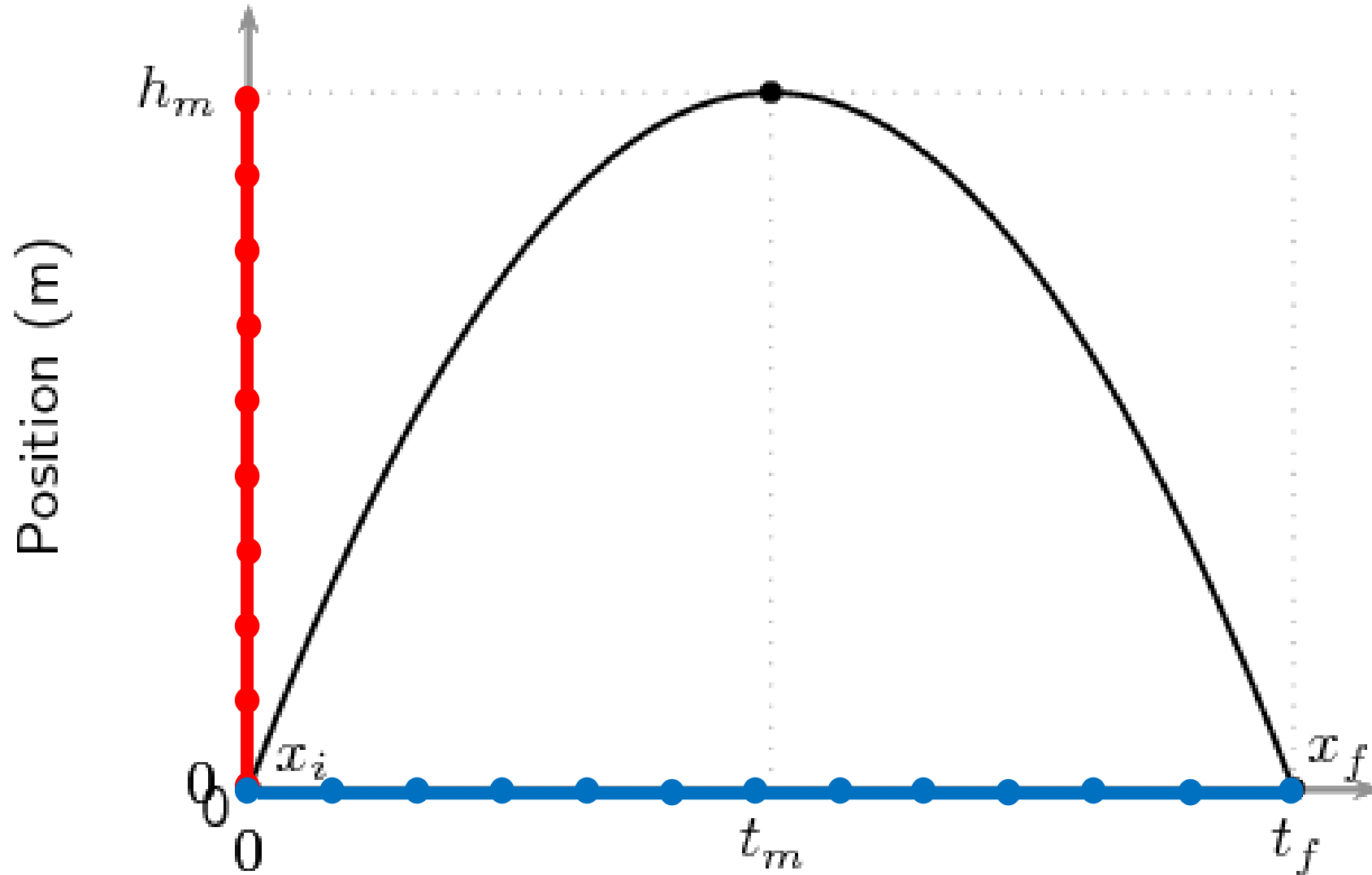
Dimension reduction (DR)

- Dimensions:
 - Features, independent variables, degrees of freedom (df), predictors, etc.

- Samples:
 - Observations, frames, measurements, time, etc.



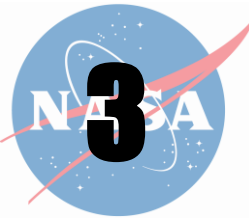
Dimension reduction (DR): a projectile example



Supervised Projective Learning with Orthogonal Completeness

- ▶ **Supervised:** class labels associated with data
- ▶ **Projective:** develop projection operators (linear operators)
- ▶ **Learning:** optimization formalized as a recurrent neural network (RNN)
- ▶ **Orthogonal Completeness:** procured basis sets are constrained to satisfy the completeness theorem such that:

$$\langle j, s | i, s \rangle = \delta_{ji} \quad \text{and} \quad \sum_{i=1}^p |i, s\rangle \langle i, s| = I$$

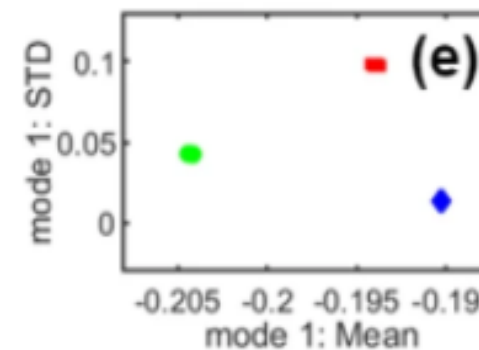
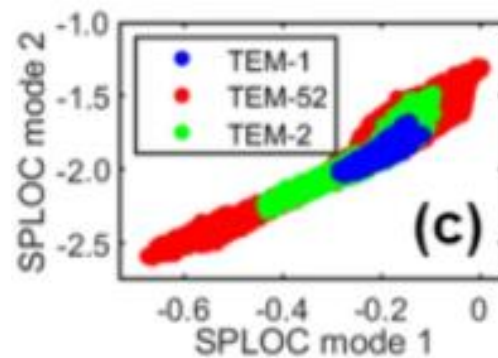
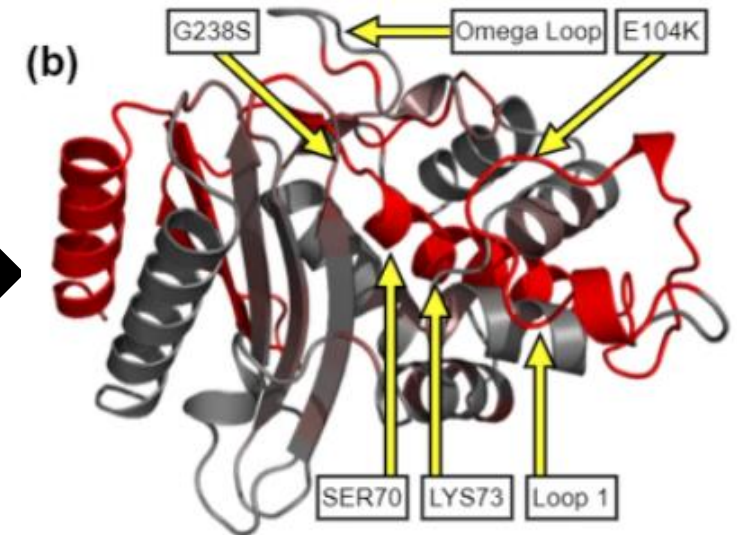
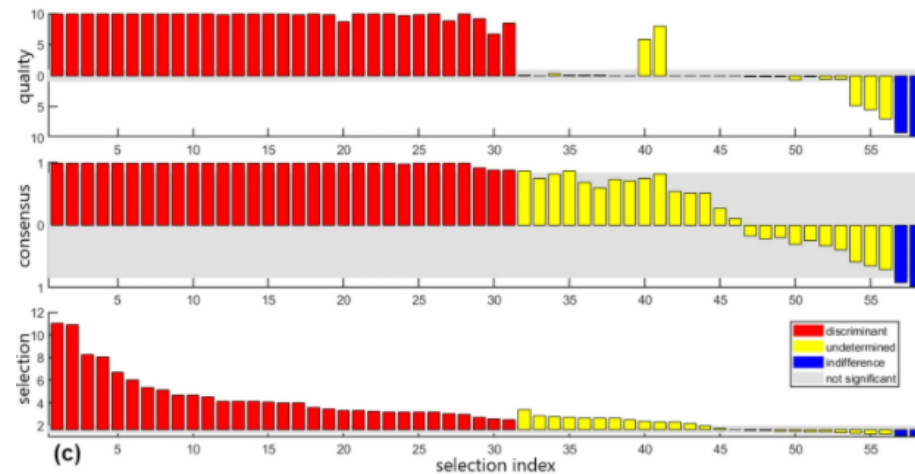
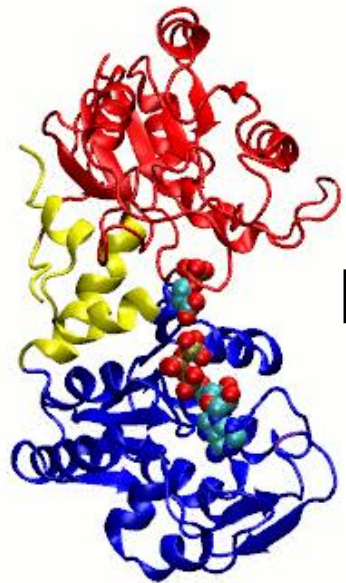


Molecular function recognition by supervised projection pursuit machine learning

Tyler Grear, Chris Avery, John Patterson & Donald J. Jacobs 

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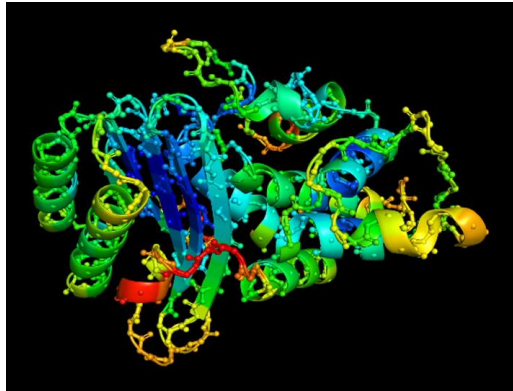
Example: Essential Dynamics (PCA)

Empirical orthogonal function (EOFs)

Basis vectors

Each of these “modes” of motion are mapped to a single eigenvector from PCA.

We can map the PCA motions to the exact atoms taking part as shown by the coloring here.



=



+



+



What is projection pursuit?

Randomly generated unit vectors are iterated through a high-dimensional space while an objective function is optimized to identify interesting univariate projections.

PCA

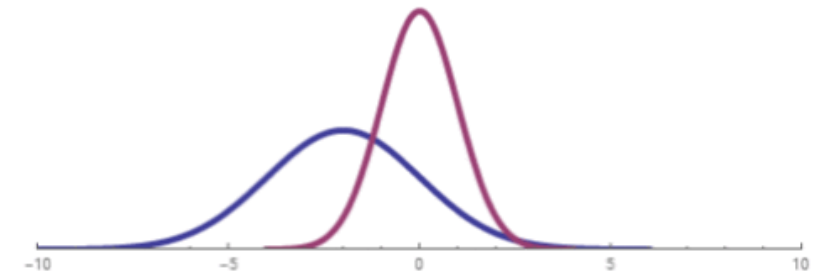
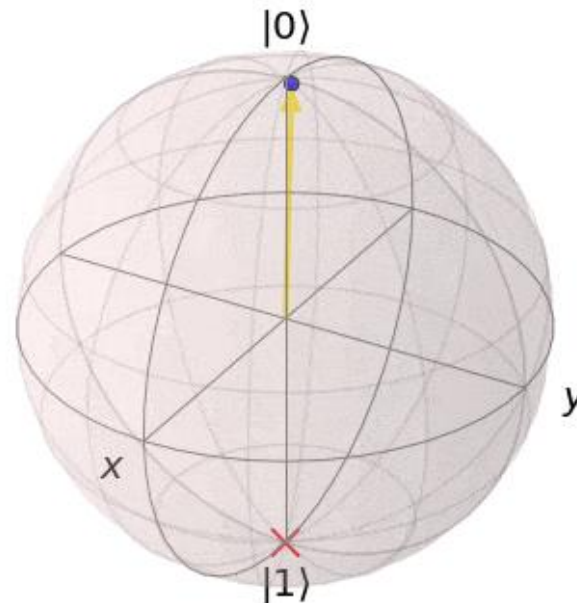
$$f(x) = \max[\text{var}(\mathbf{X}\mathbf{r})]$$

PLS

$$f(x) = \max[\text{cov}(\mathbf{t}, \mathbf{u})]^2$$

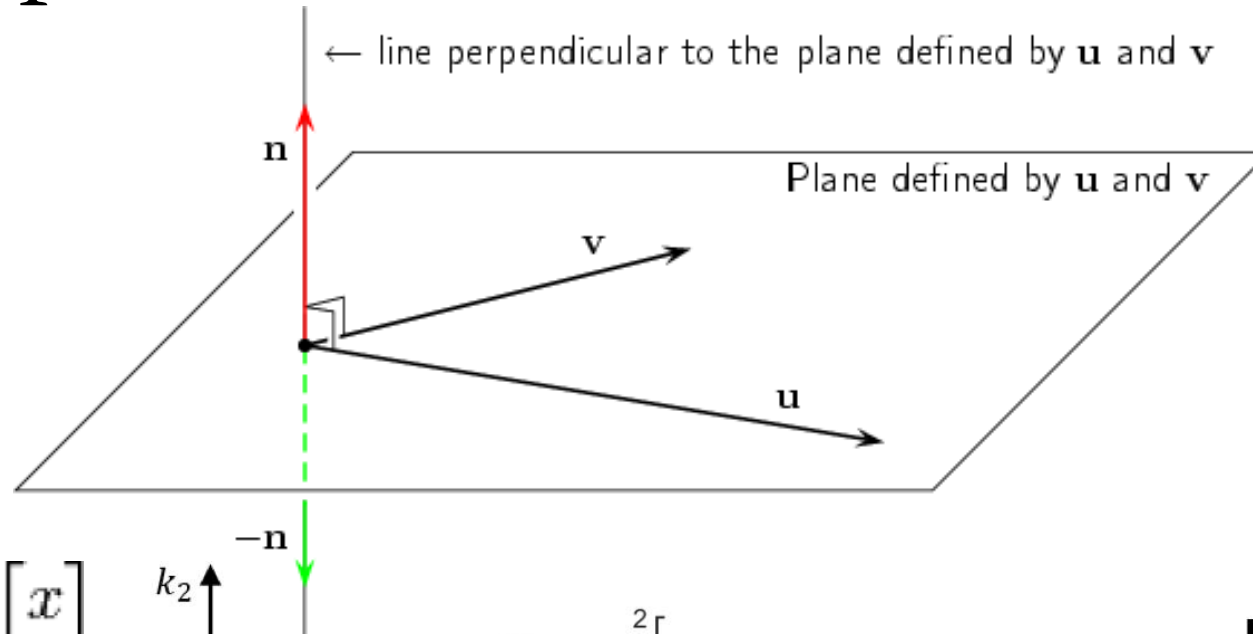
ICA

$$f(x) = \max|\text{kurt}(x)|$$



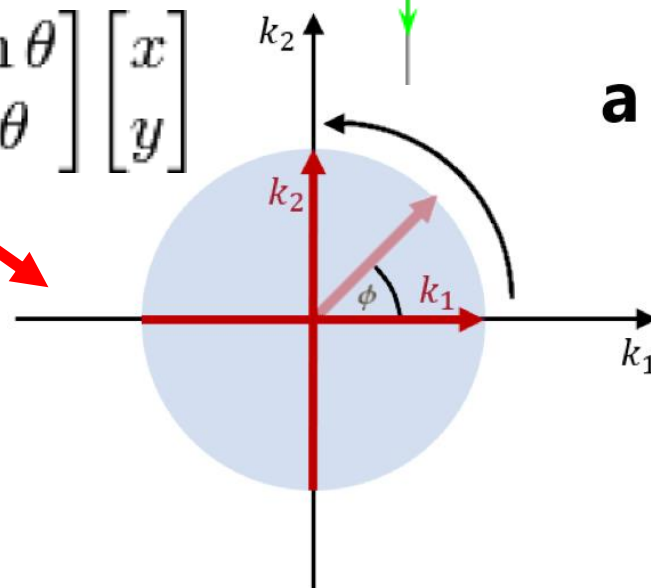
Friedman and Tukey (1974)

Projection pursuit in SPLOC

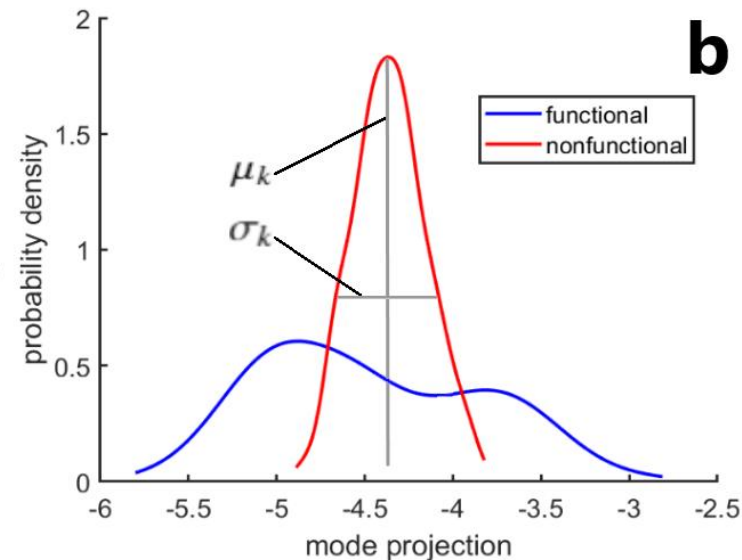


Rotation matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



a



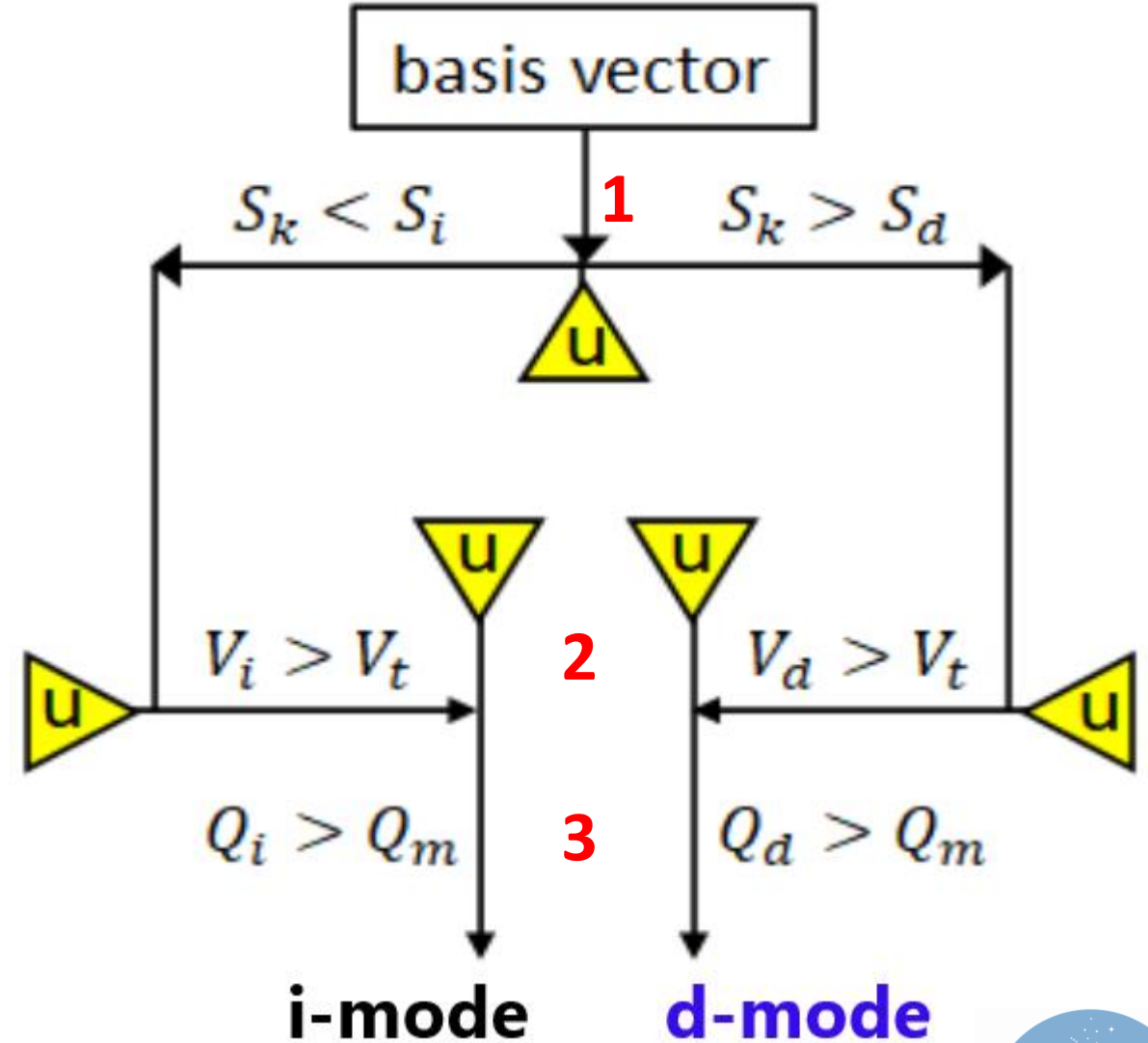
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The decision triad

1) Signal-to-noise

2) Statistical significance

3) Quality of clustering



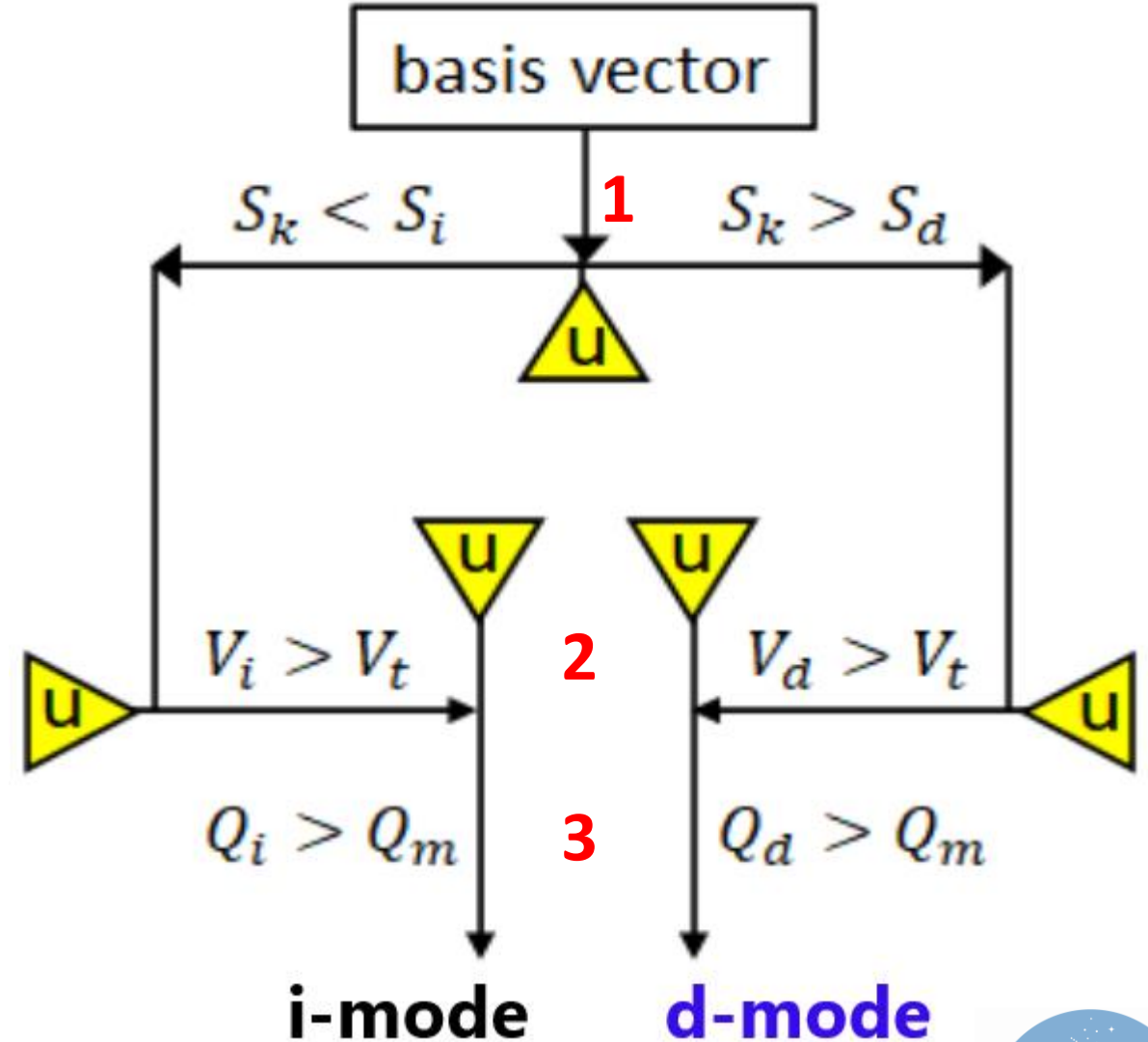
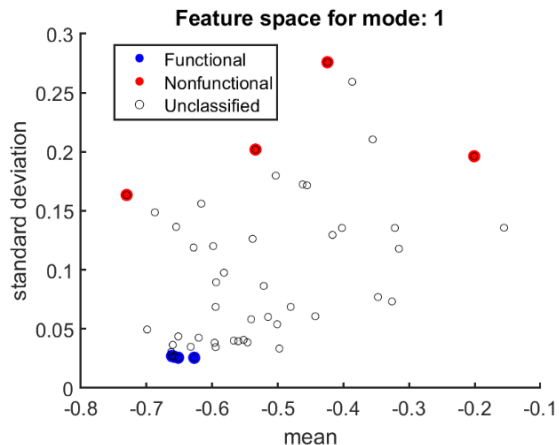
The decision triad

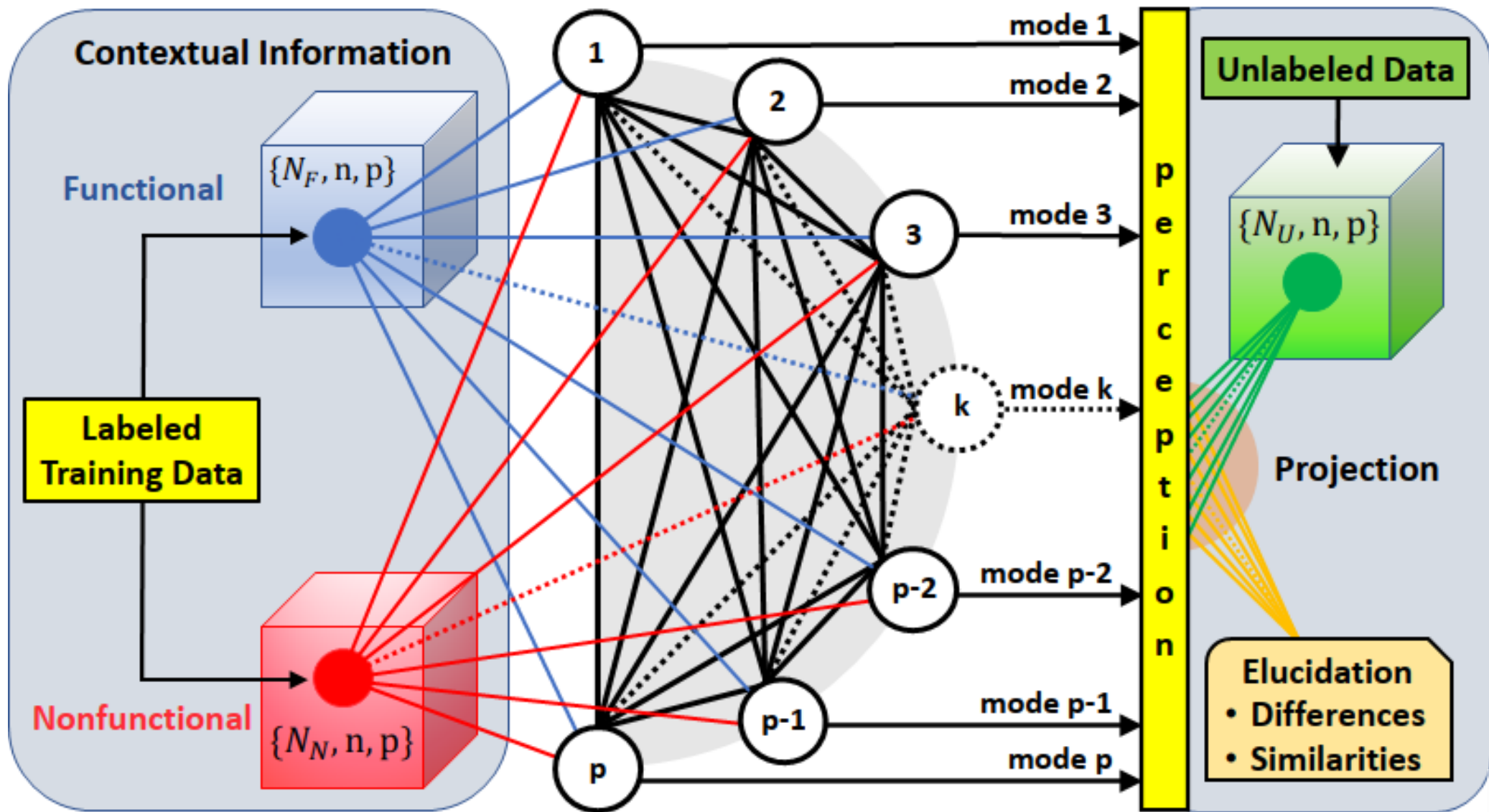
$$1) S_k(\alpha, \beta) = \begin{cases} \sqrt{sb n^2 + rex^2} + 1 & \text{when } > S_d \\ \sqrt{snr^2 + rex^2} + 1 & \text{when } < S_i \\ S_m & \text{otherwise} \end{cases}$$

$$2) f_d(x) = [1 + \exp(16(x_d - x))]^{-p_d(x)}$$

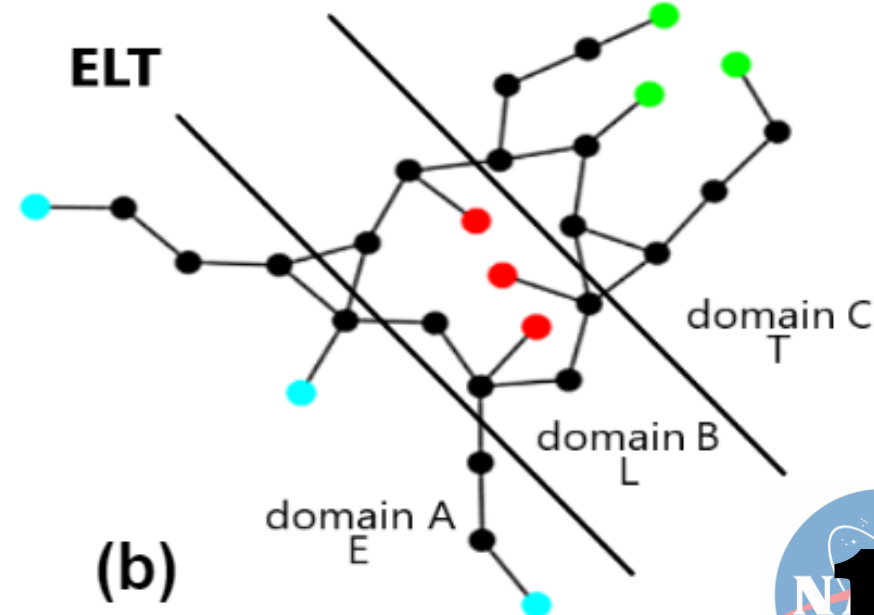
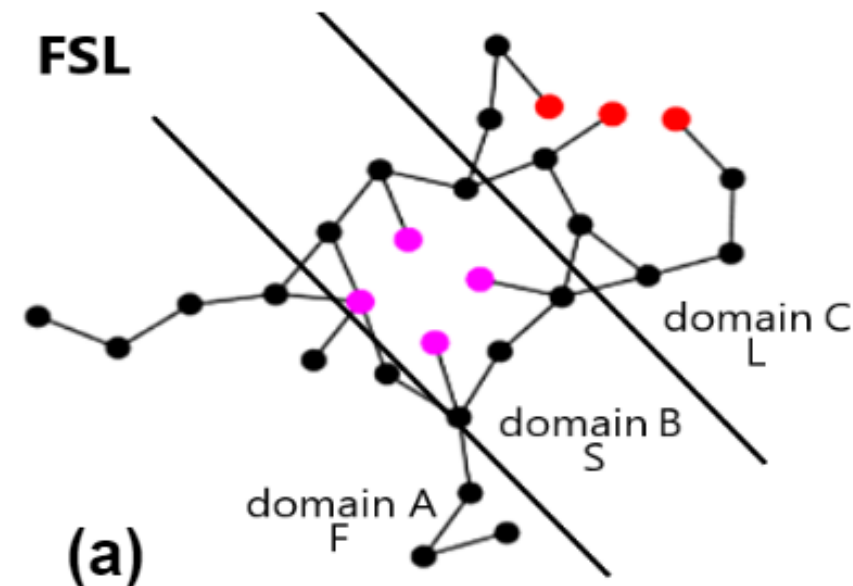
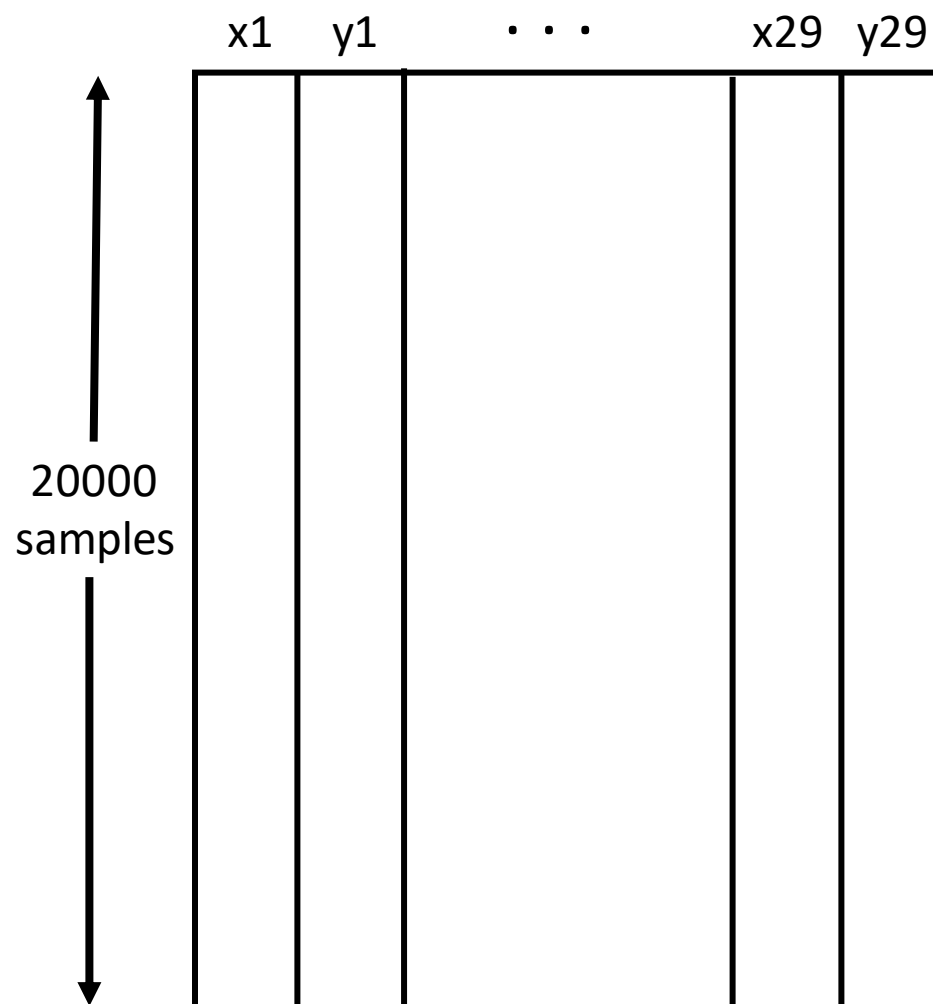
$$f_i(x) = [1 - [1 + \exp(16(x_i - x))]^{-1}]^{p_i(x)}$$

3)





The synthetic molecules

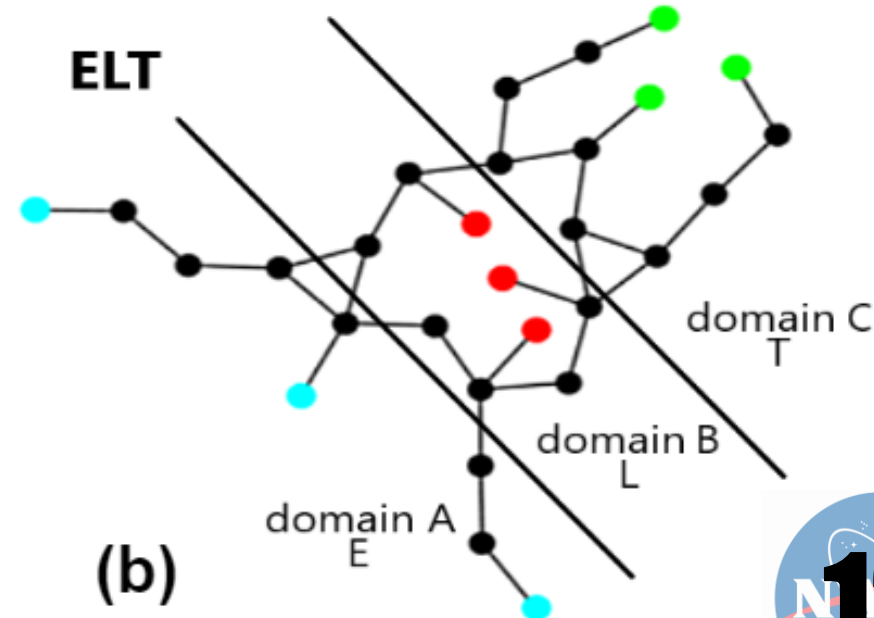
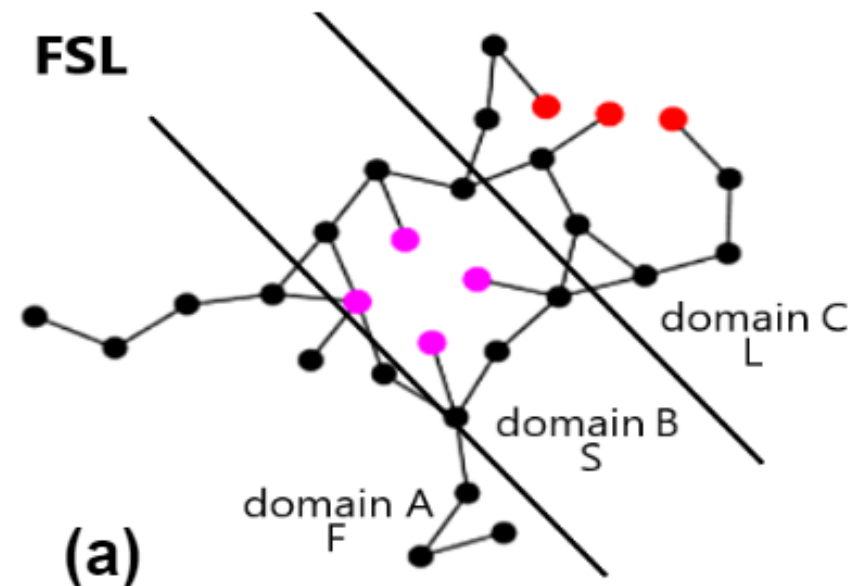


The training data

- Labeled functional:
 - {FLL,FLF}
- Labeled nonfunctional:
 - {FFF,FFL}
- Remaining 20/24 synthetic molecules are not labeled (unknown to machine).

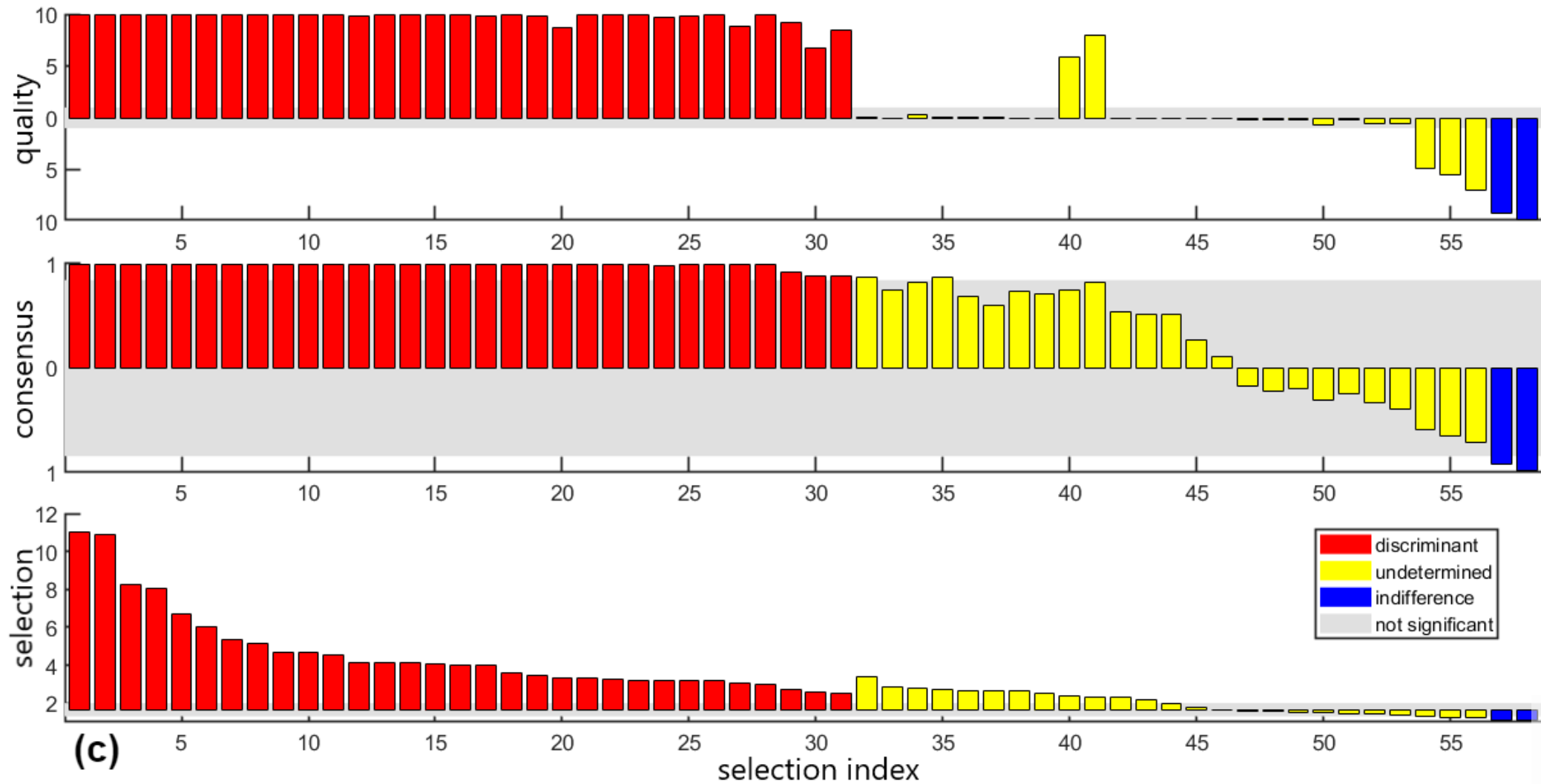
**Training
set**

**Testing
set**

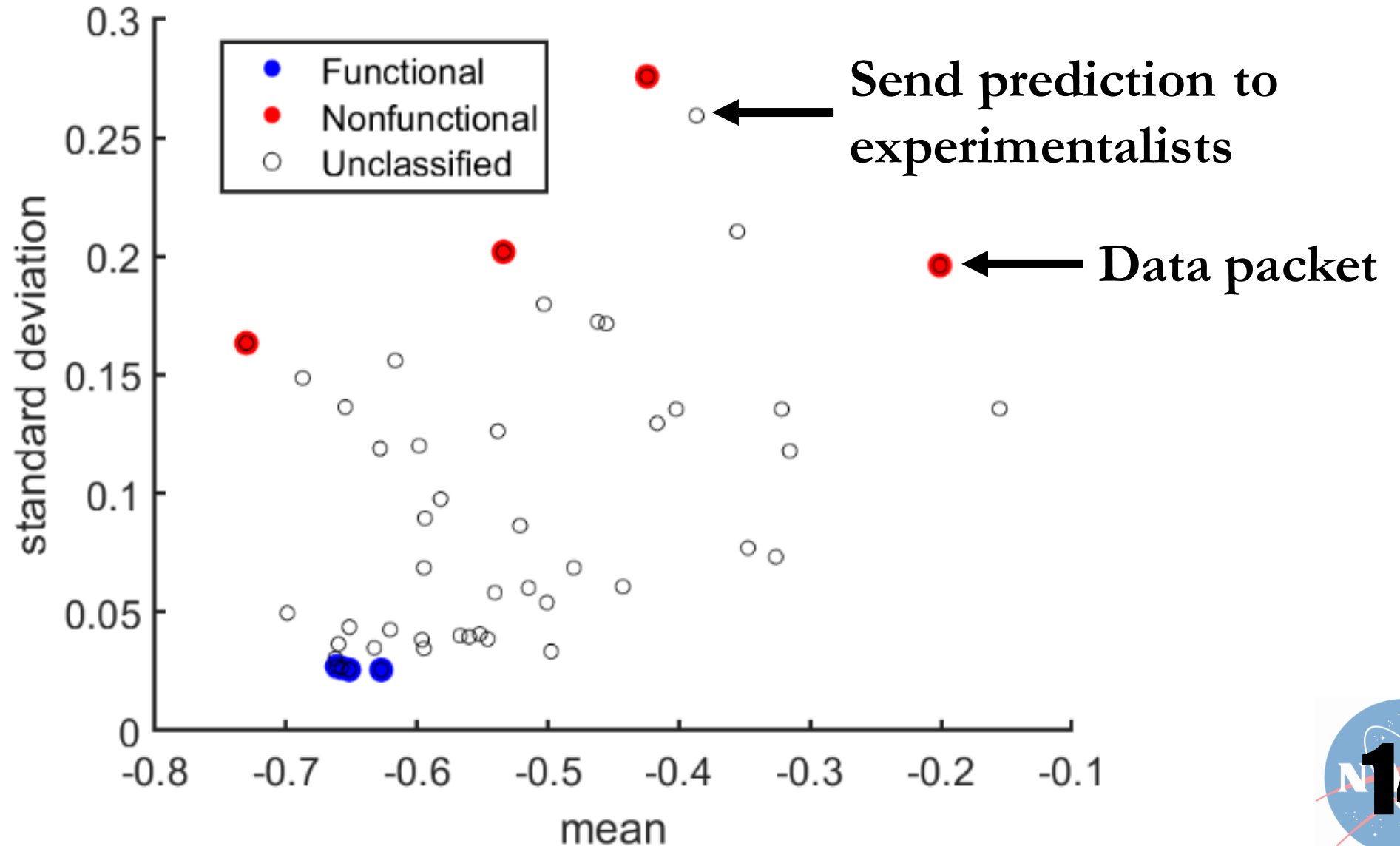


SPLOC basis vector spectrum

Decision triad



Mode feature space plane



Thank you for your time,

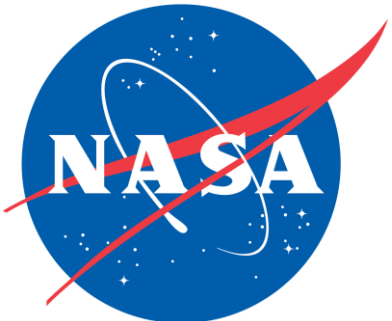
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<https://github.com/BioMolecularPhysicsGroup-UNCC/MachineLearning>

