ESERCIZIO 10

- $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial y^2}$; $(x,y) \in \Omega$; $t \in [0,T]$.

$$\Rightarrow \frac{u_{i,j}^{l+1} - u_{i,j}^{l}}{\Delta t} + u_{i,j}^{l}\left[\frac{u_{i+1,j}^{l} - u_{i-1,j}^{l}}{2\Delta x}\right] + u_{i,j}^{l}\left[\frac{u_{i,j+1}^{l} - u_{i,j-1}^{l}}{2\Delta y}\right]$$

$$= \nu\left[\frac{u_{i+1,j}^{l} - 2u_{i,j}^{l} + u_{i-1,j}^{l}}{(\Delta x)^2}\right] + \nu\left[\frac{u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l}}{(\Delta y)^2}\right]$$

$$\Rightarrow u_{i,j}^{l+1} = u_{i,j}^{l} - \frac{\Delta t}{2\Delta x}u_{i,j}^{l}\left[u_{i+1,j}^{l} - u_{i-1,j}^{l}\right] - \frac{\Delta t}{2\Delta y}u_{i,j}^{l}\left[u_{i,j+1}^{l} - u_{i,j-1}^{l}\right]$$

$$+ \frac{\nu\Delta t}{(\Delta x)^2}\left[u_{i+1,j}^{l} - 2u_{i,j}^{l} + u_{i-1,j}^{l}\right] + \frac{\nu\Delta t}{(\Delta y)^2}\left[u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l}\right]$$

$$\Rightarrow u_{i,j}^{l+1} = \left[1 - \frac{\Delta t}{2\Delta x}\left[u_{i+1,j}^{l} - u_{i-1,j}^{l}\right] - \frac{\Delta t}{2\Delta y}\left[u_{i,j+1}^{l} - u_{i,j-1}^{l}\right] - \frac{2\nu\Delta t}{(\Delta x)^2} - \frac{2\nu\Delta t}{(\Delta y)^2}\right]u_{i,j}^{l}$$

$$+ \frac{\nu\Delta t}{(\Delta x)^2}\left[u_{i+1,j}^{l} + u_{i-1,j}^{l}\right] + \frac{\nu\Delta t}{(\Delta y)^2}\left[u_{i,j+1}^{l} + u_{i,j-1}^{l}\right]$$