EJERCICIO 03.

- $$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \nabla^2 u = \frac{u_{i,j}^{\ell+1} - 2u_{i,j}^{\ell} + u_{i,j}^{\ell-1}}{(\Delta t)^2} = \alpha^2 \nabla^2 u(\rho,\phi)$$

$$\Rightarrow u_{i,j}^{\ell+1} - 2u_{i,j}^{\ell} + u_{i,j}^{\ell-1} = \frac{\alpha^2(\Delta t)^2}{(\Delta\rho)^2}\left[u_{i+1,j}^{\ell} - 2u_{i,j}^{\ell} + u_{i-1,j}^{\ell} + \frac{\Delta\rho}{\rho[i]}\left[u_{i,j}^{\ell} - u_{i-1,j}^{\ell}\right] + \frac{\lambda^2}{\rho[i]^2}\left[u_{i,j+1}^{\ell} - 2u_{i,j}^{\ell} + u_{i,j-1}^{\ell}\right]\right]$$

$$\Rightarrow \boxed{u_{i,j}^{\ell+1} = \nu^2\left[u_{i+1,j}^{\ell} - 2u_{i,j}^{\ell} + u_{i-1,j}^{\ell} + \frac{\Delta\rho}{\rho[i]}\left(u_{i,j}^{\ell} - u_{i-1,j}^{\ell}\right) + \left(\frac{\lambda}{\rho[i]}\right)^2\left(u_{i,j+1}^{\ell} - 2u_{i,j}^{\ell} + u_{i,j-1}^{\ell}\right)\right] + 2u_{i,j}^{\ell} - u_{i,j}^{\ell-1}}$$

USE:

* diferencias finitas: [redacted] ▪ $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

* operador de Laplace en coordenadas cilíndricas (4.12) : $\nabla^2 u(\rho,\phi) = \dfrac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta\rho)^2} + \dfrac{1}{\rho[i]}\left(\dfrac{u_{i,j} - u_{i-1,j}}{\Delta\rho}\right) + \dfrac{1}{\rho[i]^2}\left(\dfrac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta\phi)^2}\right)$

* $\lambda := \dfrac{\Delta\rho}{\Delta\phi}$

* $\nu := \dfrac{\alpha\Delta t}{\Delta\rho}$