PROBLEMA 01.

1. $f(x) = x^2$

- $$f'(x) = \lim_{h \to 0} \frac{-(x+2h)^2 + 4(x+h)^2 - 3x^2}{2h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{-\cancel{x^2} - 4hx - \cancel{4h^2} + \cancel{4x^2} + 8hx + \cancel{4h^2} - \cancel{3x^2}}{2h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\overset{2}{\cancel{4}}\cancel{h}x}{\cancel{2h}} = \lim_{h \to 0} 2x = \boxed{2x. = f'(x)}$$

- $$f''(x) = \lim_{h \to 0} \frac{(x+h)^2 - 2x^2 + (x-h)^2}{h^2}$$

$$\Rightarrow f''(x) = \lim_{h \to 0} \frac{\cancel{x^2} + \cancel{2hx} + h^2 - \cancel{2x^2} + \cancel{x^2} - \cancel{2hx} + h^2}{h^2}$$

$$\Rightarrow f''(x) = \lim_{h \to 0} \frac{2\cancel{h^2}}{\cancel{h^2}} = \lim_{h \to 0} 2 = \boxed{2. = f''(x)}$$

2. $f(x) = \sin(x).$

- $$f'(x) = \lim_{h\to 0} \frac{-\sin(x+2h) + 4\sin(x+h) - 3\sin(x)}{2h}$$

$$\Rightarrow \quad f'(x) = \lim_{h\to 0} \frac{-\sin(x)\cos(2h) - \cos(x)\sin(2h) + 4\sin(x)\cos(h) + 4\cos(x)\sin(h) - 3\sin(x)}{2h}$$

$$\Rightarrow \quad f'(x) = \lim_{h\to 0} \frac{\sin(x)\left[-\cos(2h) + 4\cos(h) - 3\right] + \cos(x)\left[-\sin(2h) + 4\sin(h)\right]}{2h.}$$

$$\Rightarrow \quad f'(x) = \lim_{h\to 0} \sin(x)\left\{\cancel{\frac{1-\cos(2h)}{2h}}^{\,0} - \cancel{\frac{2\left[1-\cos(h)\right]}{h}}^{\,0}\right\}$$

$$+ \lim_{h\to 0} \cos(x)\left\{-\frac{\sin(2h)}{2h}^{\,-1} + \frac{2\sin(h)}{h}^{\,2}\right\}$$

$$\Rightarrow \quad f'(x) = \cos(x)\,[2-1] = \boxed{\cos(x) = f'(x)}$$

- $f''(x) = \lim_{h \to 0} \dfrac{\sin(x+h) - 2\sin(x) + \sin(x-h)}{h^2}$

$$\Rightarrow f''(x) = \lim_{h \to 0} \frac{\sin(x)\cos(h) + \cancel{\cos(x)\sin(h)} - 2\sin(x) + \sin(x)\cos(h) - \cancel{\cos(x)\sin(h)}}{h^2}$$

$$\Rightarrow f''(x) = \lim_{h \to 0} -2\sin(x) \overbrace{\left[\frac{1-\cos(h)}{h^2}\right]}^{\frac{1}{2}} = \boxed{-\sin(x) = f''(x)}$$