School of Computing
National University of Singapore
CS5340: Uncertainty Modeling in AI
Semester 1, AY 2020/21

Online Quiz 2 (11 November 2020)

Total duration: 1 hour 15 minutes (**Start: 7.30pm, End: 8.45pm**)
(1 hour to answer the questions, 15 minutes to take and upload photos/scans of the answers)

Answer ALL questions.

Question 1:

Table 1.1 shows nine observations $\{\mathbf{x}_1, \dots, \mathbf{x}_9\}$ of 2-dimensional features $[x, y]$, where each observation is generated from an image of 1-out-of-3 handwritten alphabets. We further assume the sampling of each image is fully independent, and the observations given the alphabet follow a bivariate Gaussian distribution (see Equation 1). Figure 1.1 shows a plot of the nine 2-dimensional features in Table 1.1. Given a new observation $\mathbf{x}_{\text{Test}} = [14.65, 11.00]$, find the probability distribution of the alphabet on its corresponding image. **Explain and show all your workings clearly.**

$\mathbf{x}_n$	$[x, y]$
$\mathbf{x}_1$	[3.83, 14.48]
$\mathbf{x}_2$	[0.31, 2.06]
$\mathbf{x}_3$	[13.62, 8.89]
$\mathbf{x}_4$	[5.74, 1.35]
$\mathbf{x}_5$	[4.02, 15.69]
$\mathbf{x}_6$	[11.82, 9.88]
$\mathbf{x}_7$	[12.39, 10.8]
$\mathbf{x}_8$	[1.64, 15.22]
$\mathbf{x}_9$	[1.84, 0.68]

Table 1.1

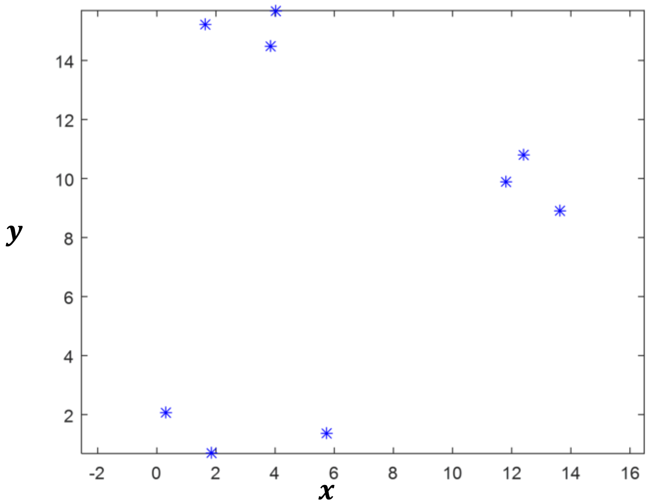

Figure 1.1

Useful Equations:

1. $p(\mathbf{x}) = (2\pi)^{-1} \det(\Sigma)^{-0.5} \exp\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\}$,

2. $\det\begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{pmatrix} = \sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}$,

3. $\begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{pmatrix}^{-1} = \frac{1}{\sigma_{11}\sigma_{22}-\sigma_{12}\sigma_{21}}\begin{pmatrix}\sigma_{22} & -\sigma_{12}\\ -\sigma_{21} & \sigma_{11}\end{pmatrix}$.

(20 marks)

Question 2:

Figure 2.1 shows a three time-step Hidden Markov Model (HMM) with binary-state latent $Z_t \in \{0,1\}$ and observed $X_t \in \{0,1\}$ random variables. The local conditional and prior probabilities of the HMM are shown in Table 2.1. Using variational inference, find the approximate posterior distribution of $p(Z_1, Z_2, Z_3 \mid X_1, X_2, X_3)$ using the mean-field approximation, i.e. $q(Z_1, Z_2, Z_3) = \prod_{t=1}^{t=3} q_t(Z_t)$ in one iteration. Assume the initial value of $q_2(Z_2 = 0) = 0.5$, and $X_1 = 0, X_2 = 1, X_3 = 0$. **Explain and show all your workings clearly.**

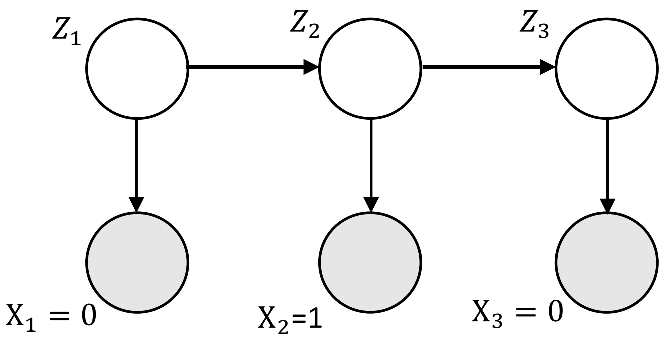

Figure 2.1

z_1	$p(z_1)$
0	0.2
1	0.8

z_2	z_3	$p(z_3 \mid z_2)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

z_1	z_2	$p(z_2 \mid z_1)$
0	0	0.3
0	1	0.7
1	0	0.9
1	1	0.1

x_t	z_t	$p(x_t \mid z_t)$
0	0	0.6
0	1	0.7
1	0	0.4
1	1	0.3

Table 2.1

(***20 marks***)