School of Computing
National University of Singapore
CS5340: Uncertainty Modeling in AI
Semester 1, AY 2020/21

Online Quiz 1 (30 September 2020)

Total duration: 1 hour 15 minutes (**Start: 7.30pm, End: 8.45pm**)
(1 hour to answer the questions, 15 minutes to take and upload photos/scans of the answers)

Answer ALL questions.

Question 1:

a) Assume that the day of the week that females are born on, X, is independent of the day of the week, Y, on which males are born. Assume, however, that the old rhyme is true, and that personality is dependent on the day of the week you're born on. If A represents the female personality type and B the male personality type, then $(A \top X)$ and $(B \top Y)$, but $(A \perp B)$. Whether or not a male and a female are married, M, depends strongly on their personality types, $(M \top A, B)$, but is independent of X and Y if we know A and B. Draw a graphical model that can represent this setting. What can we say about the (graphical) dependency between the days of the week that John and Jane are born on, given that they are not married?

Show all your workings clearly.

Note: $(A \top X)$ is the shorthand for A is dependent on X.

(*8 marks*)

b) Prove that the following relations are true or false:

i. $(X \perp Y)$ & $(Y \perp Z) \Longrightarrow (X \perp Z)$?
ii. $(X \perp Y \mid Z) \Longrightarrow (X \perp Y, W \mid Z)$?
iii. Given that $(A \perp C \mid D, B)$ & $(A \perp B \mid D) \Longrightarrow (A \perp B, C \mid D)$, does $(A \perp B \mid D, C)$ & $(A \perp C \mid D, B) \Longrightarrow (A \perp B, C \mid D)$?
iv. $(X, Y, Z \perp A, B, C \mid D, E, F) \Longrightarrow (X \perp A, B \mid D, E, F)$ & $(X, Y \perp A \mid D, E, F)$ …, i.e. $(any\ subset\ of\ \{X, Y, Z\} \perp any\ subset\ of\ \{A, B, C\} \mid D, E, F)$?

Show all your workings clearly.

(12 *marks*)

Question 2:

Figure 2.1 shows a directed graphical model with five random variables X_1, X_2, X_3, X_4, X_5, where $X_i \in \{0,1\}$. The respective conditional probabilities are shown in Table 2.1, where a, b, c and d are unknown values. Given that the minimal probability 0.00216 occurs at the configuration of $X_1 = 1, X_2 = 0, X_3 = 1, X_4 = 1, X_5 = 0$, and the maximal probability 0.10976 occurs at the configuration of $X_1 = 0, X_2 = 0, X_3 = 0, X_4 = 1, X_5 = 1$, find the **numerical values** of the probability distribution $p(X_2, X_3, X_5)$. **Show all your workings clearly**.

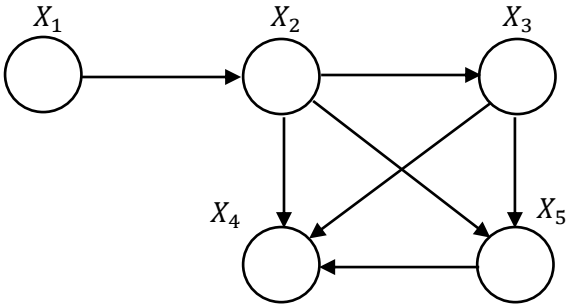

Figure 2.1.

(*20 marks*)

X_2	X_3	X_5	$p(X_5\|X_2, X_3)$
0	0	0	0.3
0	0	1	0.7
0	1	0	0.1
0	1	1	0.9
1	0	0	0.5
1	0	1	0.5
1	1	0	0.8
1	1	1	0.2

X_2	X_3	$p(X_3\|X_2)$
0	0	0.7
0	1	0.3
1	0	0.6
1	1	0.4

X_1	X_2	$p(X_2\|X_1)$
0	0	0.8
0	1	0.2
1	0	c
1	1	d

X_4	X_2	X_3	X_5	$p(X_4\|X_2, X_3, X_5)$
0	0	0	0	0.2
0	0	0	1	0.3
0	0	1	0	0.6
0	0	1	1	0.9
0	1	0	0	0.2
0	1	0	1	0.3
0	1	1	0	0.2
0	1	1	1	0.6
1	0	0	0	0.8
1	0	0	1	0.7
1	0	1	0	0.4
1	0	1	1	0.1
1	1	0	0	0.8
1	1	0	1	0.7
1	1	1	0	0.8
1	1	1	1	0.4

X_1	$p(X_1)$
0	a
1	b

Table 2.1.