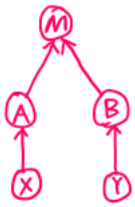
***NAME***: GUO YUEWEN
***Matric Number:*** A0225161R
***Question1:***
(1)

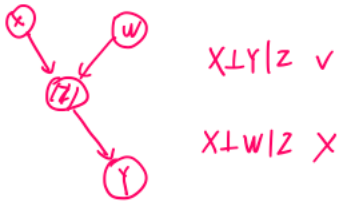



Given Jane and John are not married, which is the status of M, then X and Y become dependent, the days of Jane and John born are dependent.

(2)

i.False

ii.False, the following graph contradicts it.

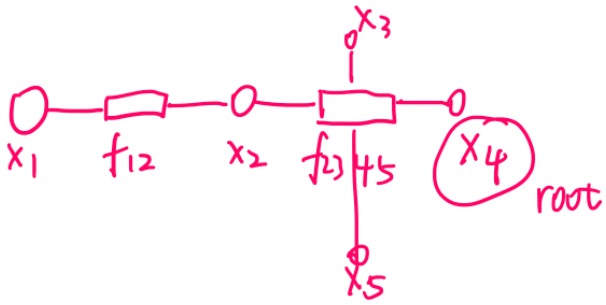



iii. True

iv. True. By Global markov property, there are no path from {X,Y,Z} to {A,B,C} given {D,E,F}.

***Question2:***
(1)
Typo: all f(x) below should be p, eg, f(x1) = p(x1), f(x2|x1) = p(x2|x1)

$$P(X) = f(X_1)\, f(X_2|X_1)\, f(X_3|X_2)\, f(X_4|X_1,X_3,X_5)\, f(X_5|X_2,X_3)$$



factors:

$$f_{12} = f(x_1) f(x_2 | x_1)$$

$$f_{2345} = f(x_3 | x_2) f(x_4 | x_2, x_3, x_5) f(x_5 | x_2, x_3)$$

$$\max_{x_1}(x_2) = \max_{x_1} \mu_{f_{12} \to x_2} = \max_{x_1} f(x_1) f(x_2 | x_1)$$

| $x_1$ | $x_2$ | $f(x_1) f(x_2 \mid x_1)$ |
|---|---|---|
| 0 | 0 | 0.8a |
| 0 | 1 | 0.2a |
| 1 | 0 | c b |
| 1 | 1 | d b |

| $x_2$ | $x_3$ | $x_5$ | $P(x_2, x_3, x_5)$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | $0.3 \times 0.7 \times (0.8 \times 0.4 + 0.2 \times 0.6)$ | 0.105 |
| 0 | 0 | 1 | $0.7 \times 0.7 \times (0.8 \times 0.4 + 0.2 \times 0.6)$ | 0.245 |
| 0 | 1 | 0 | $0.1 \times 0.3 \times (0.8 \times 0.4 + 0.2 \times 0.6)$ | 0.015 |
| 0 | 1 | 1 | $0.9 \times 0.3 \times (0.8 \times 0.4 + 0.2 \times 0.6)$ | 0.135 |
| 1 | 0 | 0 | $0.5 \times 0.6 \times (0.2 \times 0.4 + 0.7 \times 0.4)$ | 0.108 |
| 1 | 0 | 1 | $0.5 \times 0.6 \times (0.2 \times 0.4 + 0.7 \times 0.4)$ | 0.108 |
| 1 | 1 | 0 | $0.8 \times 0.4 \times (0.2 \times 0.4 + 0.7 \times 0.4)$ | 0.1152 |
| 1 | 1 | 1 | $0.2 \times 0.4 \times (0.2 \times 0.4 + 0.7 \times 0.4)$ | 0.0288 |

$$m^{max}_{x_2}(x_4) = \max_{x_2} \mu_{f_{2345} \to x_4}$$

$$= \max_{x_2} f(x_3|x_2) f(x_4|x_2,x_3,x_5) f(x_5|x_2,x_3) \max_{x_1} f(x_1) f(x_2|x_1)$$

| $x_4$ | $x_2$ | $x_3$ | $x_5$ | $p(x_4\|x_2,x_3,x_5)$ | $f(x_1\|x_4)$ | $f(x_5\|x_2,x_3)$ | $m^{max}_{x_1}(x_2)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0.2 | 0.7 | 0.3 | |
| 0 | 0 | 0 | 1 | 0.3 | 0.7 | 0.7 | |
| 0 | 0 | 1 | 0 | 0.6 | 0.3 | 0.1 | |
| 0 | 0 | 1 | 1 | 0.9 | 0.3 | 0.9 | |
| 0 | 1 | 0 | 0 | 0.2 | 0.6 | 0.5 | |
| 0 | 1 | 0 | 1 | 0.3 | 0.6 | 0.5 | |
| 0 | 1 | 1 | 0 | 0.2 | 0.4 | 0.8 | |
| 0 | 1 | 1 | 1 | 0.6 | 0.4 | 0.2 | |
| 1 | 0 | 0 | 0 | 0.8 | 0.7 | 0.3 | |
| 1 | 0 | 0 | 1 | 0.7 | 0.7 | 0.7 | 0.89 |
| 1 | 0 | 1 | 0 | 0.4 | 0.3 | 0.1 | |
| 1 | 0 | 1 | 1 | 0.1 | 0.3 | 0.9 | |
| 1 | 1 | 0 | 0 | 0.8 | 0.6 | 0.5 | |
| 1 | 1 | 0 | 1 | 0.7 | 0.6 | 0.5 | |
| 1 | 1 | 1 | 0 | 0.8 | 0.4 | 0.8 | |
| 1 | 1 | 1 | 1 | 0.4 | 0.4 | 0.2 | |

maximum

$p = 0.10976$

$0.7 \times 0.7 \times 0.7 \times 0.89 = 0.10976$

$a = \frac{0.10976}{0.7^3 \times 0.8} = 0.4$

$b = 1 - a = 0.6$

| $x_4$ | $x_2$ | $x_3$ | $x_5$ | $p(x_4\|x_2,x_3,x_5)$ | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0.2 | | | |
| 0 | 0 | 0 | 1 | 0.3 | | | |
| 0 | 0 | 1 | 0 | 0.6 | | | |
| 0 | 0 | 1 | 1 | 0.9 | | | |
| 0 | 1 | 0 | 0 | 0.2 | | | |
| 0 | 1 | 0 | 1 | 0.3 | | | |
| 0 | 1 | 1 | 0 | 0.2 | | | |
| 0 | 1 | 1 | 1 | 0.6 | | | |
| 1 | 0 | 0 | 0 | 0.8 | | | |
| 1 | 0 | 0 | 1 | 0.7 | | | |
| 1 | 0 | 1 | 0 | 0.4 | 0.3 | 0.1 | cb |
| 1 | 0 | 1 | 1 | 0.1 | | | |
| 1 | 1 | 0 | 0 | 0.8 | | | |
| 1 | 1 | 0 | 1 | 0.7 | | | |
| 1 | 1 | 1 | 0 | 0.8 | | | |
| 1 | 1 | 1 | 1 | 0.4 | | | |

minimal

$P = 0.00216$

$0.4 \times 0.3 \times 0.1 \times cb = 0.00216$

$cb = 0.18$

$c = 0.18 / 0.6 = 0.3$

$d = 1 - c = 0.7$

$$= \sum_{x_1} P(x_1, x_2, x_3, x_5)$$

$$P(x_2, x_3, x_5) = \sum_{x_1} P(x_5|x_2,x_3) P(x_3|x_2) P(x_2|x_1) P(x_1)$$

$=$