Algorithms EECS 336 **Problem Set #6**
Date: Tuesday, February 19, 2019 **Due:** Tuesday, February 26, 2019

Submit your solution on Canvas.
Do not discuss these problems with other students. You should solve these problems on your own.

Problem 1. A hedge fund Forecast336 is designing a machine learning algorithm that predicts time series $X_1, \ldots, X_n$. It is known that all numbers X_i must belong to the set $\{0, 1, \ldots, M\}$ and $X_1 \leq X_2 \leq \cdots \leq X_n$. The prediction algorithm consists of two functions: The first function finds an initial guess $Y_1, \ldots, Y_n$. This guess may not satisfy the constraints imposed on the series $X_1, \ldots, X_n$. Specifically, the sequence $Y_1, \ldots, Y_n$ may be non-monotone. The second function transforms $Y_1, \ldots Y_n$ into a **non-decreasing** sequence $X_1, \ldots, X_n$ that is close to $Y_1, \ldots Y_n$.

In this exercise, your task is to design a dynamic programming algorithm for the second function. Your algorithm receives an array $Y_1, \ldots Y_n$ and a parameter M. Each element Y_i belongs to the set $\{0, 1, \ldots, M\}$. Your algorithm needs to find a sequence $X_1, \ldots, X_n$ such that

a. Each X_i is in the set $\{0, 1, \ldots, M\}$.

b. The sequence $X_1, \ldots, X_m$ is non-decreasing i.e., $X_1 \leq X_2 \cdots \leq X_n$.

The algorithm needs to minimize the following objective:

$$\text{cost}(X) = \sum_{i=1}^{n} |X_i - Y_i|^2.$$

To design your algorithm, please do the following.

1. Define a subproblem or several subproblems.
2. Describe the base cases.
3. Write a recurrence relation.
4. Prove that your recurrence relation is correct.
5. Give an algorithm for finding a solution to your subproblem.
6. Give an algorithm for finding the optimal solution to the original problem.
7. Analyze the running time of your algorithm. To get full credit for this problem, the running time of your algorithm should be $O(nM)$ or less. You will get most credit if the running time of your algorithm is $O(nM^2)$.

Remark: In this exercise, your algorithm needs to output the cost of the solution $\text{cost}(X)$. You do not need to explain how to find the actual vector X using backtracking.

Problem 2. We ask you to implement your algorithms for Problem 1 in function

- `int FindMonotonePrediction (const std::vector<int>& y, int M)`

`FindMonotonePrediction` should return the cost of the optimal vector X for a given y and M.

Instructions for the programming assignment. Download files

- `student_code_6.h` – this file should contain your solution.
- `problem_solver_6.cpp` – this is the main file in the project (don't edit this file!).
- `test_framework.h` – this is a library responsible for reading and writing data files (don't edit this file!)
- `problem_set_6.in` – this file contains test problems for your algorithm (don't edit this file!)

Remark: These files will posted online on Wednesday, February 20.

Place all files in a new folder/directory. Write your code in function `FindMonotonePrediction`. Also, write your name in the function `GetStudentName`. Both functions are located in file `student_code_6.h`. Compile and run your code. To compile your code do the following.

- If you use GNU C++ compiler, type
  `g++ -std=c++11 problem_solver_6.cpp -o problem_solver_6`
- If you use CLang compiler, type
  `clang++ -std=c++11 problem_solver_6.cpp -o problem_solver_6`
- If you use Microsoft Visual C++ compiler, start `Developer Command Prompt` and type
  `cl /EHsc problem_solver_6.cpp`

Your compiler should be compatible with `C++11`. If you work in TLab, you need to start developer tools first: Type

- `scl enable devtoolset-4 bash`

Once you compile your code, start your program. Type `./problem_solver_6` on Unix or Mac and `problem_solver_6.exe` on Windows. Make sure that the executable is located in the same folder as file `problem_set_6.in`. Your program will generate `solution_6.dat` that contains solutions to the problems from file `problem_set_6.in`. If your code works correctly, you will get the following message:

- Problem set 6. Your algorithm solved all test problems correctly. Congratulations!
- Don't forget to submit your source code and file `solution_6.dat` via Canvas.

If your code makes a mistake, you may get a message like this:

- `Problem set 6. Mistake in problem #15. Correct answer: 4. Your answer: 12.`

Finally, when your code is ready, submit files `student_code_6.h` and `solution_6.dat` via Canvas. Make sure that you are submitting the latest versions.

Remark: If you want to debug your code, please, type `./problem_solver_6 15` on Unix or Mac and `problem_solver_6.exe 15` on Windows. This command will call your function only on one problem – the problem #15 and thus let you debug your code on the problem where your program erred. Note that this command will not generate or update `solution_6.dat`. So before submitting your solution, you need to run your program without any command line arguments.