Assignment 02 - CSCE 440/840

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1. For Questions 1-4, use Table Particulate Matter. Atmospheric particulate matter are microscopic matter suspended in the air. In particular, particular matter with a mean diameter of 2.5 μm (PM 2.5) or less causes many health problems because it can easily get into the lungs. In the United States the EPA set a limit of 35 $\mu g/m^3$. daily average. Hence many weather stations are monitoring the concentration of particle matter with PM 2.5 or less. Table Particulate Matter shows a set of four weather stations, where SN is the station identification number, T is time in days and PM is the particulate matter per day in g/m^3. Show all the calculation steps.

   (a) Find the Lagrange interpolating polynomial for the 4th station. Use the Lagrange interpolating polynomial to estimate the PM 2.5 of the 4th weather station at T = 17.

$$
\begin{aligned}
L_0(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \\
&= \frac{(x-14)(x-21)(x-28)}{(7-14)(7-21)(7-28)} \\
&= \frac{1}{-2058}(x-14)(x-21)(x-28)
\end{aligned}
$$

$$
\begin{aligned}
L_1(x) &= \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\
&= \frac{(x-7)(x-21)(x-28)}{(14-7)(14-21)(14-28)} \\
&= \frac{1}{686}(x-7)(x-21)(x-28)
\end{aligned}
$$

$$
\begin{aligned}
L_2(x) &= \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \\
&= \frac{(x-7)(x-14)(x-28)}{(21-7)(21-14)(21-28)} \\
&= \frac{1}{-686}(x-7)(x-14)(x-28)
\end{aligned}
$$

$$
\begin{aligned}
L_3(x) &= \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_2)(x_3-x_2)} \\
&= \frac{(x-7)(x-14)(x-21)}{(28-7)(28-14)(28-21)} \\
&= \frac{1}{2058}(x-7)(x-14)(x-21)
\end{aligned}
$$

$$
\begin{aligned}
P_3(x) =& L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2) + L_3(x)f(x_3) \\
=& \frac{32}{-2058}(x-14)(x-21)(x-28) \\
&+ \frac{34}{686}(x-7)(x-21)(x-28) \\
&+ \frac{36}{-686}(x-7)(x-14)(x-28) \\
&+ \frac{35}{2058}(x-7)(x-14)(x-21) \\
=& \frac{-x^3 + 42x^2 - 343x + 22638}{686} \\
P_3(17) =& \frac{-(17)^3 + 42(17)^2 - 343(17) + 22638}{686} \\
=& \frac{12016}{343} \approx \mathbf{35.032}
\end{aligned}
$$

(b) Use Neville's Method to estimate the PM 2.5 of the 4th weather station at T = 12.

i	x_i	$f(x_i)$
0	7	32
1	14	34
2	21	36
3	28	35

$$f(12) \approx P_0(12) = f(x_0) = 32$$
$$f(12) \approx P_1(12) = f(x_1) = 34$$
$$f(12) \approx P_2(12) = f(x_2) = 36$$
$$f(12) \approx P_3(12) = f(x_3) = 35$$

$$\begin{aligned} f(12) \approx P_{0,1}(12) &= \frac{(12 - x_1)P_0(12) - (12 - x_0)P_1(12)}{x_0 - x_1} \\ &= \frac{(12 - 14)32 - (12 - 7)34}{7 - 14} \approx 33.429 \\ f(12) \approx P_{1,2}(12) &= \frac{(12 - x_2)P_1(12) - (12 - x_1)P_2(12)}{x_1 - x_2} \\ &= \frac{(12 - 21)34 - (12 - 14)36}{14 - 21} \approx 33.429 \\ f(12) \approx P_{2,3}(12) &= \frac{(12 - x_3)P_2(12) - (12 - x_2)P_3(12)}{x_2 - x_3} \\ &= \frac{(12 - 28)36 - (12 - 21)35}{21 - 28} \approx 38.143 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$
0	7	32	
1	14	34	33.429
2	21	36	33.429
3	28	35	38.143

$$\begin{aligned} f(12) \approx P_{0,1,2}(12) &= \frac{(12 - x_2)P_{0,1}(12) - (12 - x_0)P_{1,2}(12)}{x_0 - x_2} \\ &= \frac{(12 - 21)33.429 - (12 - 7)33.429}{7 - 21} \approx 33.429 \\ f(12) \approx P_{0,1,2}(12) &= \frac{(12 - x_3)P_{1,2}(12) - (12 - x_1)P_{2,3}(12)}{x_1 - x_3} \\ &= \frac{(12 - 28)33.429 - (12 - 14)38.143}{14 - 28} \approx 32.756 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$
0	7	32		
1	14	34	33.429	
2	21	36	33.429	33.429
3	28	35	38.143	32.756

$$\begin{aligned} f(12) \approx P_{0,1,2,3}(12) &= \frac{(12 - x_3)P_{0,1,2}(12) - (12 - x_0)P_{1,2,3}(12)}{x_0 - x_3} \\ &= \frac{(12 - 28)33.429 - (12 - 7)32.756}{7 - 28} \approx 33.269 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$	$P_{i,\ldots,i-3}$
0	7	32			
1	14	34	33.429		
2	21	36	33.429	33.429	
3	28	35	38.143	32.756	33.269

$f(12) \approx \mathbf{33.269}$

(c) Use Newton's Divided Differences Method to find the interpolating polynomial for the 4th weather station. Use the Newton interpolating polynomial to estimate the PM 2.5 of the 4th weather station at T= 10.

i	x_i	$f[x_i]$
0	7	32
1	14	34
2	21	36
3	28	35

$$f[x_0] = 32$$
$$f[x_1] = 34$$
$$f[x_2] = 36$$
$$f[x_3] = 35$$

$$f[x_0, x_1, ..., x_i] = \frac{f[x_1, ..., x_i] - f[x_0, ..., x_{i-1}]}{x_i - x_0}$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$
0	7	32	
			.2857
1	14	34	
			.2857
2	21	36	
			−.1429
3	28	35	

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{34 - 32}{14 - 7} \approx .2857$$
$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{36 - 34}{21 - 14} \approx .2857$$
$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{35 - 36}{28 - 21} \approx -.1429$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, ..., x_{i+2}]$
0	7	32		
			.2857	
1	14	34		0
			.2857	
2	21	36		−.03061
			−.1429	
3	28	35		

$$\begin{aligned} f[x_0, ..., x_2] &= \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \\ &= \frac{.2857 - .2857}{21 - 7} = 0 \\ f[x_1, ..., x_3] &= \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} \\ &= \frac{-.1429 - .2857}{28 - 14} \approx -.03061 \end{aligned}$$

$$f[x_0, ..., x_3] = \frac{f[x_1, ..., x_3] - f[x_0, ..., x_2]}{x_3 - x_0} = \frac{-.03061 - 0}{28 - 7} \approx -0.00145$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, ..., x_{i+2}]$	$f(x_i, ..., x_{i+3})$
0	7	32			
			.2857		
1	14	34		0	
			.2857		−.001458
2	21	36		−.03061	
			−.1429		
3	28	35			

$$\begin{aligned} P_i(x) =& f(x_0) + (x - x_0)f[x_0, x_1] + ... + (x - x_0)...(x - x_{i-1})f[x_0, ..., x_i] \\ P_3(x) =& 32 + .2857(x - 7) - .001458(x - 7)(x - 14)(x - 21) \\ =& -0.001458x^3 + 0.061236x^2 - 0.500162x + 33.000664 \\ P_3(10) =& \mathbf{32.6646} \end{aligned}$$

(d) Find the cubic spline interpolation for the 5th weather station using natural cubic spline algorithm.

2. Write a program to find the Lagrange interpolating polynomials for each of the weather stations. Use the Lagrange interpolating polynomials to estimate the PM 2.5 for each of the weather stations at T = 17.

Output

```
Weather Station 1 PM 2.5 at T = 17:
         P_9(17) = 30.8568
Weather Station 2 PM 2.5 at T = 17:
         P_9(17) = 33.2584
Weather Station 3 PM 2.5 at T = 17:
         P_4(17) = 36.6764
Weather Station 4 PM 2.5 at T = 17:
         P_4(17) = 35.0321
Weather Station 5 PM 2.5 at T = 17:
         P_4(17) = 33.136
Weather Station 6 PM 2.5 at T = 17:
         P_4(17) = 38.6764
```

Source Code at `https://git.io/JeW5f`

3. Write a program that implements Neville's Method and estimate the PM 2.5 for each of the weather stations at T = 12.

Output

```
33     38.25
35   37.6667   37.3333
27        27        27        27
29        27        27        27        27
32     26.75        27        27        27        27
35        25      27.5        27        27        27        27
37        30        20   29.5455        27        27        27        27
39   27.6667   33.3333   10.6667   33.5909        27        27        27        27
Weather Station 1 PM 2.5 at T = 12 ~= 27
35        31
30        27   25.2857
28        27        27   27.1905
34      29.2   27.9429   27.6286   27.5034
32        38   30.4571    28.781   28.2871   27.9932
36      27.2   44.1714      31.6    29.385   28.7494   28.3533
37      30.5   24.3714   52.9714   33.1265   30.0866   29.2588    28.747
40      29.2   32.5429   20.2857   62.3102   34.6625   30.7403   29.7146   29.0926
Weather Station 2 PM 2.5 at T = 12 ~= 29.0926
36   36.8571
38   35.7143   36.3673
40   35.7143   35.7143   36.1808
Weather Station 3 PM 2.5 at T = 12 ~= 36.1808
34   33.4286
36   33.4286   33.4286
35   37.2857   32.8776   33.2974
Weather Station 4 PM 2.5 at T = 12 ~= 33.2974
30      30.8
33      31.2     31.08
31      34.2      31.8    31.416
Weather Station 5 PM 2.5 at T = 12 ~= 31.416
37        34
42   34.8571   34.2449
44   39.1429   33.9388   34.1866
Weather Station 6 PM 2.5 at T = 12 ~= 34.1866
```

Source Code at `https://git.io/JeWb4`

4. Write a program that implements Newtons Divided Differences Method and estimate the PM 2.5 for each of the weather stations at T = 7.

Output

```
30
33        0.75
35    0.666667   -0.0119048
27          -2    -0.380952   -0.0335498
```

```
29     0.666667     0.380952    0.0761905   0.00783859
32         0.75    0.0119048   -0.0335498  -0.00783859 -0.000870954
35            1    0.0357143   0.00238095   0.00256648  0.000612063  7.06199e-05
37          0.5   -0.0714286  -0.00974026 -0.000865801 -0.000190682  -3.8226e-05 -4.35383e-06
39     0.666667    0.0238095   0.00952381     0.001376  0.000131871  1.53597e-05  2.23274e-06  2.35
Weather Station 1 PM 2.5 at T = 7 ~= 36.8172
36
35         -0.5
30           -1   -0.0714286
28           -1            0   0.00793651
34          1.2     0.314286    0.0261905   0.00130385
32           -1    -0.314286   -0.0698413  -0.00685941 -0.000510204
36          0.8     0.257143     0.047619   0.00839002  0.000802602  6.25146e-05
37          0.5   -0.0428571   -0.0333333  -0.00578231 -0.000885771 -8.03987e-05 -6.21362e-06
40          0.6    0.0142857    0.0047619   0.00272109  0.000447547  6.34914e-05  5.53423e-06  4.19
Weather Station 2 PM 2.5 at T = 7 ~= 36.2572
42
36    -0.857143
38     0.285714    0.0816327
40     0.285714            0  -0.00388727
Weather Station 3 PM 2.5 at T = 7 ~= 40.3499
32
34     0.285714
36     0.285714            0
35    -0.142857   -0.0306122  -0.00145773
Weather Station 4 PM 2.5 at T = 7 ~= 32

28
30          0.4
33          0.6         0.02
31         -0.4         -0.1       -0.008
Weather Station 5 PM 2.5 at T = 7 ~= 28.296
30
37            1
42     0.714286   -0.0204082
44     0.285714   -0.0306122 -0.000485909
Weather Station 6 PM 2.5 at T = 7 ~= 28.895
```

Source Code at `https://git.io/JeWb4`

Table 1: Particulate Matter

SN	T	PM
1	1	30
1	5	12
1	8	35
1	12	27
1	15	29
1	19	32
1	22	35
1	26	37
2	2	36
2	4	35
2	9	30
2	11	28
2	16	34
2	18	32
2	23	36
2	25	37
2	30	40
3	6	42
3	13	36
3	20	38
3	27	40
4	7	32
4	14	34
4	21	36
4	28	35
5	5	28
5	10	30
5	15	33
5	20	31
6	8	30
6	15	37
6	22	42
6	29	44