Assignment 02 - CSCE 440/840

Katie Gerot: 79862841

October, 9 2019

1. For Questions 1-4, use Table Particulate Matter. Atmospheric particulate matter are microscopic matter suspended in the air. In particular, particular matter with a mean diameter of 2.5 μm (PM 2.5) or less causes many health problems because it can easily get into the lungs. In the United States the EPA set a limit of 35 $\mu g/m^3$. daily average. Hence many weather stations are monitoring the concentration of particle matter with PM 2.5 or less. Table Particulate Matter shows a set of four weather stations, where SN is the station identification number, T is time in days and PM is the particulate matter per day in g/m^3. Show all the calculation steps.

   (a) Find the Lagrange interpolating polynomial for the 4th station. Use the Lagrange interpolating polynomial to estimate the PM 2.5 of the 4th weather station at T = 17.

$$\begin{aligned} L_0(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \\ &= \frac{(x-14)(x-21)(x-28)}{(7-14)(7-21)(7-28)} \\ &= \frac{1}{-2058}(x-14)(x-21)(x-28) \end{aligned}$$

$$\begin{aligned} L_1(x) &= \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\ &= \frac{(x-7)(x-21)(x-28)}{(14-7)(14-21)(14-28)} \\ &= \frac{1}{686}(x-7)(x-21)(x-28) \end{aligned}$$

$$\begin{aligned} L_2(x) &= \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \\ &= \frac{(x-7)(x-14)(x-28)}{(21-7)(21-14)(21-28)} \\ &= \frac{1}{-686}(x-7)(x-14)(x-28) \end{aligned}$$

$$\begin{aligned} L_3(x) &= \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_2)(x_3-x_2)} \\ &= \frac{(x-7)(x-14)(x-21)}{(28-7)(28-14)(28-21)} \\ &= \frac{1}{2058}(x-7)(x-14)(x-21) \end{aligned}$$

$$\begin{aligned} P_3(x) =& L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2) + L_3(x)f(x_3) \\ =& \frac{32}{-2058}(x-14)(x-21)(x-28) \\ &+ \frac{34}{686}(x-7)(x-21)(x-28) \\ &+ \frac{36}{-686}(x-7)(x-14)(x-28) \\ &+ \frac{35}{2058}(x-7)(x-14)(x-21) \\ =& \frac{-x^3+42x^2-343x+22638}{686} \\ P_3(17) =& \frac{-(17)^3+42(17)^2-343(17)+22638}{686} \\ =& \frac{12016}{343} \approx \mathbf{35.032} \end{aligned}$$

(b) Use Neville's Method to estimate the PM 2.5 of the 4th weather station at T = 12.

i	x_i	$f(x_i)$
0	7	32
1	14	34
2	21	36
3	28	35

$$f(12) \approx P_0(12) = f(x_0) = 32$$
$$f(12) \approx P_1(12) = f(x_1) = 34$$
$$f(12) \approx P_2(12) = f(x_2) = 36$$
$$f(12) \approx P_3(12) = f(x_3) = 35$$

$$\begin{aligned} f(12) \approx P_{0,1}(12) &= \frac{(12 - x_1)P_0(12) - (12 - x_0)P_1(12)}{x_0 - x_1} \\ &= \frac{(12 - 14)32 - (12 - 7)34}{7 - 14} \approx 33.429 \\ f(12) \approx P_{1,2}(12) &= \frac{(12 - x_2)P_1(12) - (12 - x_1)P_2(12)}{x_1 - x_2} \\ &= \frac{(12 - 21)34 - (12 - 14)36}{14 - 21} \approx 33.429 \\ f(12) \approx P_{2,3}(12) &= \frac{(12 - x_3)P_2(12) - (12 - x_2)P_3(12)}{x_2 - x_3} \\ &= \frac{(12 - 28)36 - (12 - 21)35}{21 - 28} \approx 38.143 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$
0	7	32	
1	14	34	33.429
2	21	36	33.429
3	28	35	38.143

$$\begin{aligned} f(12) \approx P_{0,1,2}(12) &= \frac{(12 - x_2)P_{0,1}(12) - (12 - x_0)P_{1,2}(12)}{x_0 - x_2} \\ &= \frac{(12 - 21)33.429 - (12 - 7)33.429}{7 - 21} \approx 33.429 \\ f(12) \approx P_{0,1,2}(12) &= \frac{(12 - x_3)P_{1,2}(12) - (12 - x_1)P_{2,3}(12)}{x_1 - x_3} \\ &= \frac{(12 - 28)33.429 - (12 - 14)38.143}{14 - 28} \approx 32.756 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$
0	7	32		
1	14	34	33.429	
2	21	36	33.429	33.429
3	28	35	38.143	32.756

$$\begin{aligned} f(12) \approx P_{0,1,2,3}(12) &= \frac{(12 - x_3)P_{0,1,2}(12) - (12 - x_0)P_{1,2,3}(12)}{x_0 - x_3} \\ &= \frac{(12 - 28)33.429 - (12 - 7)32.756}{7 - 28} \approx 33.269 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$	$P_{i,\ldots,i-3}$
0	7	32			
1	14	34	33.429		
2	21	36	33.429	33.429	
3	28	35	38.143	32.756	33.269

$$f(12) \approx \mathbf{33.269}$$

(c) Use Newton's Divided Differences Method to find the interpolating polynomial for the 4th weather station. Use the Newton interpolating polynomial to estimate the PM 2.5 of the 4th weather station at T= 10.

i	x_i	$f[x_i]$
0	7	32
1	14	34
2	21	36
3	28	35

$$f[x_0] = 32$$
$$f[x_1] = 34$$
$$f[x_2] = 36$$
$$f[x_3] = 35$$

$$f[x_0, x_1, ..., x_i] = \frac{f[x_1, ..., x_i] - f[x_0, ..., x_{i-1}]}{x_i - x_0}$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$
0	7	32	
			.2857
1	14	34	
			.2857
2	21	36	
			−.1429
3	28	35	

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{34 - 32}{14 - 7} \approx .2857$$
$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{36 - 34}{21 - 14} \approx .2857$$
$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{35 - 36}{28 - 21} \approx -.1429$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, ..., x_{i+2}]$
0	7	32		
			.2857	
1	14	34		0
			.2857	
2	21	36		−.03061
			−.1429	
3	28	35		

$$\begin{aligned} f[x_0, ..., x_2] =& \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \\ =& \frac{.2857 - .2857}{21 - 7} = 0 \\ f[x_1, ..., x_3] =& \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} \\ =& \frac{-.1429 - .2857}{28 - 14} \approx -.03061 \end{aligned}$$

$$f[x_0, ..., x_3] = \frac{f[x_1, ..., x_3] - f[x_0, ..., x_2]}{x_3 - x_0} = \frac{-.03061 - 0}{28 - 7} \approx -0.00145$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, ..., x_{i+2}]$	$f(x_i, ..., x_{i+3})$
0	7	32			
			.2857		
1	14	34		0	
			.2857		−.001458
2	21	36		−.03061	
			−.1429		
3	28	35			

$$\begin{aligned} P_i(x) =& f(x_0) + (x - x_0)f[x_0, x_1] + ... + (x - x_0)...(x - x_{i-1})f[x_0, ..., x_i] \\ P_3(x) =& 32 + .2857(x - 7) - .001458(x - 7)(x - 14)(x - 21) \\ =& -0.001458x^3 + 0.061236x^2 - 0.500162x + 33.000664 \\ P_3(10) =& \mathbf{32.6646} \end{aligned}$$

(d) Find the cubic spline interpolation for the 5th weather station using natural cubic spline algorithm.

2. Write a program to find the Lagrange interpolating polynomials for each of the weather stations. Use the Lagrange interpolating polynomials to estimate the PM 2.5 for each of the weather stations at T = 17.

Output

```
Weather Station 1 PM 2.5 at T = 17:
         P_9(17) = 30.8568
Weather Station 2 PM 2.5 at T = 17:
         P_9(17) = 33.2584
Weather Station 3 PM 2.5 at T = 17:
         P_4(17) = 36.6764
Weather Station 4 PM 2.5 at T = 17:
         P_4(17) = 35.0321
Weather Station 5 PM 2.5 at T = 17:
         P_4(17) = 33.136
Weather Station 6 PM 2.5 at T = 17:
         P_4(17) = 38.6764
```

Source Code at `https://git.io/JeW5f`

3. Write a program that implements Neville's Method and estimate the PM 2.5 for each of the weather stations at T = 12.

Output

```
33         42
35         41    39.7143
27         17  -0.142857   -18.2597
29    30.3333    34.1429         41    49.4657
32       30.5    30.4524    31.1234    32.5343    34.4156
35         30    30.3571    30.4048    30.6614    31.2123     31.975
37       32.5    29.2857    30.1623    30.3182    30.4898    30.7994    31.2226
39         31    33.5714    28.4286    29.9147    30.1995    30.3654    30.5824    30.8568
Weather Station 1 PM 2.5 at T = 17 ~= 30.8568
35       28.5
30         22    14.5714
28         22         22    26.9524
34       35.2    37.0857    38.3429    39.1565
32         33    33.3143    33.7333    34.0626     34.381
36       31.2    32.7429    33.0286    33.3306    33.5618    33.7958
37         33    30.9429    32.5429    32.8204    33.0755    33.2607    33.4469
40       32.2    33.6857    30.7143    32.4122    32.6915    32.9292     33.095    33.2584
Weather Station 2 PM 2.5 at T = 17 ~= 33.2584
36    32.5714
38    37.1429    36.1633
40    37.1429    37.1429    36.6764
Weather Station 3 PM 2.5 at T = 17 ~= 36.6764
34    34.8571
36    34.8571    34.8571
35    36.5714    35.2245    35.0321
Weather Station 4 PM 2.5 at T = 17 ~= 35.0321
30       32.8
33       34.2      34.48
31       32.2       32.8     33.136
Weather Station 5 PM 2.5 at T = 17 ~= 33.136
37         39
42    38.4286    38.6327
44    40.5714    38.7347    38.6764
Weather Station 6 PM 2.5 at T = 17 ~= 38.6764
```

Source Code at `https://git.io/JeWb4`

4. Write a program that implements Neville's Method and estimate the PM 2.5 for each of the weather stations at T = 12.

Output

```
33      38.25
35    37.6667    37.3333
27         27         27         27
29         27         27         27         27
32      26.75         27         27         27         27
```

```
35         25       27.5         27         27         27         27
37         30         20    29.5455         27         27         27         27
39    27.6667    33.3333    10.6667    33.5909         27         27         27         27
Weather Station 1 PM 2.5 at T = 12 ~= 27
35         31
30         27    25.2857
28         27         27    27.1905
34       29.2    27.9429    27.6286    27.5034
32         38    30.4571     28.781    28.2871    27.9932
36       27.2    44.1714       31.6     29.385    28.7494    28.3533
37       30.5    24.3714    52.9714    33.1265    30.0866    29.2588     28.747
40       29.2    32.5429    20.2857    62.3102    34.6625    30.7403    29.7146    29.0926
Weather Station 2 PM 2.5 at T = 12 ~= 29.0926
36    36.8571
38    35.7143    36.3673
40    35.7143    35.7143    36.1808
Weather Station 3 PM 2.5 at T = 12 ~= 36.1808
34    33.4286
36    33.4286    33.4286
35    37.2857    32.8776    33.2974
Weather Station 4 PM 2.5 at T = 12 ~= 33.2974
30       30.8
33       31.2      31.08
31       34.2       31.8     31.416
Weather Station 5 PM 2.5 at T = 12 ~= 31.416
37         34
42    34.8571    34.2449
44    39.1429    33.9388    34.1866
Weather Station 6 PM 2.5 at T = 12 ~= 34.1866
```

Source Code at `https://git.io/JeWb4`

Table 1: Particulate Matter

SN	T	PM
1	1	30
1	5	12
1	8	35
1	12	27
1	15	29
1	19	32
1	22	35
1	26	37
2	2	36
2	4	35
2	9	30
2	11	28
2	16	34
2	18	32
2	23	36
2	25	37
2	30	40
3	6	42
3	13	36
3	20	38
3	27	40
4	7	32
4	14	34
4	21	36
4	28	35
5	5	28
5	10	30
5	15	33
5	20	31
6	8	30
6	15	37
6	22	42
6	29	44