

Assignment 02 - CSCE 440/840

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- For Questions 1-4, use Table Particulate Matter. Atmospheric particulate matter are microscopic matter suspended in the air. In particular, particular matter with a mean diameter of $2.5 \mu m$ (PM 2.5) or less causes many health problems because it can easily get into the lungs. In the United States the EPA set a limit of $35 \mu g/m^3$. daily average. Hence many weather stations are monitoring the concentration of particle matter with PM 2.5 or less. Table Particulate Matter shows a set of four weather stations, where SN is the station identification number, T is time in days and PM is the particulate matter per day in g/m^3 . Show all the calculation steps.

- Find the Lagrange interpolating polynomial for the 4th station. Use the Lagrange interpolating polynomial to estimate the PM 2.5 of the 4th weather station at $T = 17$.

$$\begin{aligned}
 L_0(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} & L_1(x) &= \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\
 &= \frac{(x-14)(x-21)(x-28)}{(7-14)(7-21)(7-28)} & &= \frac{(x-7)(x-21)(x-28)}{(14-7)(14-21)(14-28)} \\
 &= \frac{1}{-2058}(x-14)(x-21)(x-28) & &= \frac{1}{686}(x-7)(x-21)(x-28) \\
 L_2(x) &= \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} & L_3(x) &= \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_2)(x_3-x_1)} \\
 &= \frac{(x-7)(x-14)(x-28)}{(21-7)(21-14)(21-28)} & &= \frac{(x-7)(x-14)(x-21)}{(28-7)(28-14)(28-21)} \\
 &= \frac{1}{-686}(x-7)(x-14)(x-28) & &= \frac{1}{2058}(x-7)(x-14)(x-21)
 \end{aligned}$$

$$\begin{aligned}
 P_3(x) &= L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2) + L_3(x)f(x_3) \\
 &= \frac{32}{-2058}(x-14)(x-21)(x-28) \\
 &\quad + \frac{34}{686}(x-7)(x-21)(x-28) \\
 &\quad + \frac{36}{-686}(x-7)(x-14)(x-28) \\
 &\quad + \frac{35}{2058}(x-7)(x-14)(x-21) \\
 &= \frac{-x^3 + 42x^2 - 343x + 22638}{686} \\
 P_3(17) &= \frac{-(17)^3 + 42(17)^2 - 343(17) + 22638}{686} \\
 &= \frac{12016}{343} \approx \mathbf{35.032}
 \end{aligned}$$

(b) Use Neville's Method to estimate the PM 2.5 of the 4th weather station at T = 12.

i	x_i	$f(x_i)$
0	7	32
1	14	34
2	21	36
3	28	35

$$f(12) \approx P_0(12) = f(x_0) = 32$$

$$f(12) \approx P_1(12) = f(x_1) = 34$$

$$f(12) \approx P_2(12) = f(x_2) = 36$$

$$f(12) \approx P_3(12) = f(x_3) = 35$$

$$\begin{aligned} f(12) \approx P_{0,1}(12) &= \frac{(12 - x_1)P_0(12) - (12 - x_0)P_1(12)}{x_0 - x_1} \\ &= \frac{(12 - 14)32 - (12 - 7)34}{7 - 14} \approx 33.429 \end{aligned}$$

$$\begin{aligned} f(12) \approx P_{1,2}(12) &= \frac{(12 - x_2)P_1(12) - (12 - x_1)P_2(12)}{x_1 - x_2} \\ &= \frac{(12 - 21)34 - (12 - 14)36}{14 - 21} \approx 33.429 \end{aligned}$$

$$\begin{aligned} f(12) \approx P_{2,3}(12) &= \frac{(12 - x_3)P_2(12) - (12 - x_2)P_3(12)}{x_2 - x_3} \\ &= \frac{(12 - 28)36 - (12 - 21)35}{21 - 28} \approx 38.143 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$
0	7	32	
1	14	34	33.429
2	21	36	33.429
3	28	35	38.143

$$\begin{aligned} f(12) \approx P_{0,1,2}(12) &= \frac{(12 - x_2)P_{0,1}(12) - (12 - x_0)P_{1,2}(12)}{x_0 - x_2} \\ &= \frac{(12 - 21)33.429 - (12 - 7)33.429}{7 - 21} \approx 33.429 \end{aligned}$$

$$\begin{aligned} f(12) \approx P_{0,1,2}(12) &= \frac{(12 - x_3)P_{1,2}(12) - (12 - x_1)P_{2,3}(12)}{x_1 - x_3} \\ &= \frac{(12 - 28)33.429 - (12 - 14)38.143}{14 - 28} \approx 32.756 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$
0	7	32		
1	14	34	33.429	
2	21	36	33.429	33.429
3	28	35	38.143	32.756

$$\begin{aligned} f(12) \approx P_{0,1,2,3}(12) &= \frac{(12 - x_3)P_{0,1,2}(12) - (12 - x_0)P_{1,2,3}(12)}{x_0 - x_3} \\ &= \frac{(12 - 28)33.429 - (12 - 7)32.756}{7 - 28} \approx 33.269 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$	$P_{i,\dots,i-3}$
0	7	32			
1	14	34	33.429		
2	21	36	33.429	33.429	
3	28	35	38.143	32.756	33.269

$$f(12) \approx \mathbf{33.269}$$

- (c) Use Newton's Divided Differences Method to find the interpolating polynomial for the 4th weather station. Use the Newton interpolating polynomial to estimate the PM 2.5 of the 4th weather station at T= 10.

i	x_i	$f[x_i]$	
0	7	32	$f[x_0] = 32$
1	14	34	$f[x_1] = 34$
2	21	36	$f[x_2] = 36$
3	28	35	$f[x_3] = 35$

$$f[x_0, x_1, \dots, x_i] = \frac{f[x_1, \dots, x_i] - f[x_0, \dots, x_{i-1}]}{x_i - x_0}$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$
0	7	32	
			.2857
1	14	34	
			.2857
2	21	36	
			-.1429
3	28	35	

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{34 - 32}{14 - 7} \approx .2857$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{36 - 34}{21 - 14} \approx .2857$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{35 - 36}{28 - 21} \approx -.1429$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, \dots, x_{i+2}]$
0	7	32		
			.2857	
1	14	34		0
			.2857	
2	21	36		-.03061
			-.1429	
3	28	35		

$$f[x_0, \dots, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{.2857 - .2857}{21 - 7} = 0$$

$$f[x_1, \dots, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \frac{-.1429 - .2857}{28 - 14} \approx -.03061$$

$$f[x_0, \dots, x_3] = \frac{f[x_1, \dots, x_3] - f[x_0, \dots, x_2]}{x_3 - x_0} = \frac{-.03061 - 0}{28 - 7} \approx -0.00145$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, \dots, x_{i+2}]$	$f(x_i, \dots, x_{i+3})$
0	7	32			
			.2857		
1	14	34		0	
			.2857		-.001458
2	21	36		-.03061	
			-.1429		
3	28	35			

$$P_i(x) = f(x_0) + (x - x_0)f[x_0, x_1] + \dots + (x - x_0)\dots(x - x_{i-1})f[x_0, \dots, x_i]$$

$$P_3(x) = 32 + .2857(x - 7) - .001458(x - 7)(x - 14)(x - 21)$$

$$= -0.001458x^3 + 0.061236x^2 - 0.500162x + 33.000664$$

$$P_3(10) = \mathbf{32.6646}$$

(d) Find the cubic spline interpolation for the 5th weather station using natural cubic spline algorithm.

$$f(x) = \begin{cases} .0048x^3 - .072x^2 + .64x + 26 & x \in [5, 10] \\ -.016x^3 + .552x^2 - 5.6x + 46.8 & x \in (10, 15] \\ .0112x^3 - .672x^2 + 12.76x - 45 & x \in (15, 20] \end{cases}$$

2. Write a program to find the Lagrange interpolating polynomials for each of the weather stations. Use the Lagrange interpolating polynomials to estimate the PM 2.5 for each of the weather stations at $T = 17$.

Output

```
Weather Station 1 PM 2.5 at T = 17:
    P_9(17) = 30.8568
Weather Station 2 PM 2.5 at T = 17:
    P_9(17) = 33.2584
Weather Station 3 PM 2.5 at T = 17:
    P_4(17) = 36.6764
Weather Station 4 PM 2.5 at T = 17:
    P_4(17) = 35.0321
Weather Station 5 PM 2.5 at T = 17:
    P_4(17) = 33.136
Weather Station 6 PM 2.5 at T = 17:
    P_4(17) = 38.6764
```

Source Code at <https://git.io/JeW5f>

3. Write a program that implements Neville's Method and estimate the PM 2.5 for each of the weather stations at $T = 12$.

Output

```

30
33 38.2500
35 37.6667 37.3333
27 27.0000 27.0000 27.0000
29 27.0000 27.0000 27.0000 27.0000
32 26.7500 27.0000 27.0000 27.0000 27.0000
35 25.0000 27.5000 27.0000 27.0000 27.0000 27.0000
37 30.0000 20.0000 29.5455 27.0000 27.0000 27.0000 27.0000
39 27.6667 33.3333 10.6667 33.5909 27.0000 27.0000 27.0000 27.0000
Weather Station 1 PM 2.5 at T = 12 ~= 27
36
35 31.0000
30 27.0000 25.2857
28 27.0000 27.0000 27.1905
34 29.2000 27.9429 27.6286 27.5034
32 38.0000 30.4571 28.7810 28.2871 27.9932
36 27.2000 44.1714 31.6000 29.3850 28.7494 28.3533
37 30.5000 24.3714 52.9714 33.1265 30.0866 29.2588 28.7470
40 29.2000 32.5429 20.2857 62.3102 34.6625 30.7403 29.7146 29.0926
Weather Station 2 PM 2.5 at T = 12 ~= 29.0926
42
36 36.8571
38 35.7143 36.3673
40 35.7143 35.7143 36.1808
Weather Station 3 PM 2.5 at T = 12 ~= 36.1808
32
34 33.4286
36 33.4286 33.4286
35 37.2857 32.8776 33.2974
Weather Station 4 PM 2.5 at T = 12 ~= 33.2974
28
30 30.8000
33 31.2000 31.0800
31 34.2000 31.8000 31.4160
Weather Station 5 PM 2.5 at T = 12 ~= 31.416
30
37 34.0000
42 34.8571 34.2449
44 39.1429 33.9388 34.1866
Weather Station 6 PM 2.5 at T = 12 ~= 34.1866

```

Source Code at <https://git.io/JeWb4>

4. Write a program that implements Newtons Divided Differences Method and estimate the PM 2.5 for each of the weather stations at $T = 7$.

Output

```

30
33  0.7500
35  0.6667  -0.0119
27 -2.0000  -0.3810  -0.0335
29  0.6667   0.3810   0.0762   0.0078
32  0.7500   0.0119  -0.0335  -0.0078  -0.0009
35  1.0000   0.0357   0.0024   0.0026   0.0006   0.0001
37  0.5000  -0.0714  -0.0097  -0.0009  -0.0002  -0.0000  -0.0000
39  0.6667   0.0238   0.0095   0.0014   0.0001   0.0000   0.0000   0.0000
Weather Station 1 PM 2.5 at T = 7 ~ = 36.8172
36
35 -0.5000
30 -1.0000  -0.0714
28 -1.0000   0.0000   0.0079
34  1.2000   0.3143   0.0262   0.0013
32 -1.0000  -0.3143  -0.0698  -0.0069  -0.0005
36  0.8000   0.2571   0.0476   0.0084   0.0008   0.0001
37  0.5000  -0.0429  -0.0333  -0.0058  -0.0009  -0.0001  -0.0000
40  0.6000   0.0143   0.0048   0.0027   0.0004   0.0001   0.0000   0.0000
Weather Station 2 PM 2.5 at T = 7 ~ = 36.2572
42
36 -0.8571
38  0.2857   0.0816
40  0.2857   0.0000  -0.0039
Weather Station 3 PM 2.5 at T = 7 ~ = 40.3499
32
34  0.2857
36  0.2857   0.0000
35 -0.1429  -0.0306  -0.0015
Weather Station 4 PM 2.5 at T = 7 ~ = 32
28
30  0.4000gg
33  0.6000   0.0200
31 -0.4000  -0.1000  -0.0080
Weather Station 5 PM 2.5 at T = 7 ~ = 28.296
30
37  1.0000
42  0.7143  -0.0204
44  0.2857  -0.0306  -0.0005
Weather Station 6 PM 2.5 at T = 7 ~ = 28.895

```

Source Code at <https://git.io/JeWNQ>

5. Write a program to implements Hermite Interpolation using Divided Differences to find the Hermite polynomial $H_{11}(x)$ for the data in Table 2 and approximate the value when $x= 0.75$.

Output

Wasn't able to produce output

Source Code at <https://git.io/JeWNh>

Table 1: Particulate Matter

SN	T	PM
1	1	30
1	5	12
1	8	35
1	12	27
1	15	29
1	19	32
1	22	35
1	26	37
2	2	36
2	4	35
2	9	30
2	11	28
2	16	34
2	18	32
2	23	36
2	25	37
2	30	40
3	6	42
3	13	36
3	20	38
3	27	40
4	7	32
4	14	34
4	21	36
4	28	35
5	5	28
5	10	30
5	15	33
5	20	31
6	8	30
6	15	37
6	22	42
6	29	44