Assignment 02 - CSCE 440/840

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1. For Questions 1-4, use Table Particulate Matter. Atmospheric particulate matter are microscopic matter suspended in the air. In particular, particular matter with a mean diameter of 2.5 μm (PM 2.5) or less causes many health problems because it can easily get into the lungs. In the United States the EPA set a limit of 35 $\mu g/m^3$. daily average. Hence many weather stations are monitoring the concentration of particle matter with PM 2.5 or less. Table Particulate Matter shows a set of four weather stations, where SN is the station identification number, T is time in days and PM is the particulate matter per day in g/m^3. Show all the calculation steps.

   (a) Find the Lagrange interpolating polynomial for the 4th station. Use the Lagrange interpolating polynomial to estimate the PM 2.5 of the 4th weather station at T = 17.

$$
\begin{aligned}
L_0(x) &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \\
&= \frac{(x-14)(x-21)(x-28)}{(7-14)(7-21)(7-28)} \\
&= \frac{1}{-2058}(x-14)(x-21)(x-28)
\end{aligned}
\qquad
\begin{aligned}
L_1(x) &= \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \\
&= \frac{(x-7)(x-21)(x-28)}{(14-7)(14-21)(14-28)} \\
&= \frac{1}{686}(x-7)(x-21)(x-28)
\end{aligned}
$$

$$
\begin{aligned}
L_2(x) &= \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \\
&= \frac{(x-7)(x-14)(x-28)}{(21-7)(21-14)(21-28)} \\
&= \frac{1}{-686}(x-7)(x-14)(x-28)
\end{aligned}
\qquad
\begin{aligned}
L_3(x) &= \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_2)(x_3-x_2)} \\
&= \frac{(x-7)(x-14)(x-21)}{(28-7)(28-14)(28-21)} \\
&= \frac{1}{2058}(x-7)(x-14)(x-21)
\end{aligned}
$$

$$
\begin{aligned}
P_3(x) =& L_0(x)f(x_0) + L_1(x)f(x_1) + L_2(x)f(x_2) + L_3(x)f(x_3) \\
=& \frac{32}{-2058}(x-14)(x-21)(x-28) \\
& + \frac{34}{686}(x-7)(x-21)(x-28) \\
& + \frac{36}{-686}(x-7)(x-14)(x-28) \\
& + \frac{35}{2058}(x-7)(x-14)(x-21) \\
=& \frac{-x^3+42x^2-343x+22638}{686} \\
P_3(17) =& \frac{-(17)^3+42(17)^2-343(17)+22638}{686} \\
=& \frac{12016}{343} \approx \mathbf{35.032}
\end{aligned}
$$

(b) Use Neville's Method to estimate the PM 2.5 of the 4th weather station at T = 12.

i	x_i	$f(x_i)$
0	7	32
1	14	34
2	21	36
3	28	35

$$f(12) \approx P_0(12) = f(x_0) = 32$$
$$f(12) \approx P_1(12) = f(x_1) = 34$$
$$f(12) \approx P_2(12) = f(x_2) = 36$$
$$f(12) \approx P_3(12) = f(x_3) = 35$$

$$\begin{aligned} f(12) \approx P_{0,1}(12) &= \frac{(12-x_1)P_0(12)-(12-x_0)P_1(12)}{x_0-x_1} \\ &= \frac{(12-14)32-(12-7)34}{7-14} \approx 33.429 \\ f(12) \approx P_{1,2}(12) &= \frac{(12-x_2)P_1(12)-(12-x_1)P_2(12)}{x_1-x_2} \\ &= \frac{(12-21)34-(12-14)36}{14-21} \approx 33.429 \\ f(12) \approx P_{2,3}(12) &= \frac{(12-x_3)P_2(12)-(12-x_2)P_3(12)}{x_2-x_3} \\ &= \frac{(12-28)36-(12-21)35}{21-28} \approx 38.143 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$
0	7	32	
1	14	34	33.429
2	21	36	33.429
3	28	35	38.143

$$\begin{aligned} f(12) \approx P_{0,1,2}(12) &= \frac{(12-x_2)P_{0,1}(12)-(12-x_0)P_{1,2}(12)}{x_0-x_2} \\ &= \frac{(12-21)33.429-(12-7)33.429}{7-21} \approx 33.429 \\ f(12) \approx P_{0,1,2}(12) &= \frac{(12-x_3)P_{1,2}(12)-(12-x_1)P_{2,3}(12)}{x_1-x_3} \\ &= \frac{(12-28)33.429-(12-14)38.143}{14-28} \approx 32.756 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$
0	7	32		
1	14	34	33.429	
2	21	36	33.429	33.429
3	28	35	38.143	32.756

$$\begin{aligned} f(12) \approx P_{0,1,2,3}(12) &= \frac{(12-x_3)P_{0,1,2}(12)-(12-x_0)P_{1,2,3}(12)}{x_0-x_3} \\ &= \frac{(12-28)33.429-(12-7)32.756}{7-28} \approx 33.269 \end{aligned}$$

i	x_i	P_i	$P_{i,i-1}$	$P_{i,i-1,i-2}$	$P_{i,\dots,i-3}$
0	7	32			
1	14	34	33.429		
2	21	36	33.429	33.429	
3	28	35	38.143	32.756	33.269

$f(12) \approx \mathbf{33.269}$

(c) Use Newton's Divided Differences Method to find the interpolating polynomial for the 4th weather station. Use the Newton interpolating polynomial to estimate the PM 2.5 of the 4th weather station at T= 10.

i	x_i	$f[x_i]$
0	7	32
1	14	34
2	21	36
3	28	35

$$f[x_0] = 32$$
$$f[x_1] = 34$$
$$f[x_2] = 36$$
$$f[x_3] = 35$$

$$f[x_0, x_1, ..., x_i] = \frac{f[x_1, ..., x_i] - f[x_0, ..., x_{i-1}]}{x_i - x_0}$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$
0	7	32	
			.2857
1	14	34	
			.2857
2	21	36	
			$-.1429$
3	28	35	

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{34 - 32}{14 - 7} \approx .2857$$
$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{36 - 34}{21 - 14} \approx .2857$$
$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{35 - 36}{28 - 21} \approx -.1429$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, ..., x_{i+2}]$
0	7	32		
			.2857	
1	14	34		0
			.2857	
2	21	36		$-.03061$
			$-.1429$	
3	28	35		

$$\begin{aligned} f[x_0, ..., x_2] &= \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \\ &= \frac{.2857 - .2857}{21 - 7} = 0 \\ f[x_1, ..., x_3] &= \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} \\ &= \frac{-.1429 - .2857}{28 - 14} \approx -.03061 \end{aligned}$$

$$f[x_0, ..., x_3] = \frac{f[x_1, ..., x_3] - f[x_0, ..., x_2]}{x_3 - x_0} = \frac{-.03061 - 0}{28 - 7} \approx -0.00145$$

i	x_i	$f[x_i]$	$f[x_i, x_{i+1}]$	$f[x_i, ..., x_{i+2}]$	$f(x_i, ..., x_{i+3})$
0	7	32			
			.2857		
1	14	34		0	
			.2857		$-.001458$
2	21	36		$-.03061$	
			$-.1429$		
3	28	35			

$$\begin{aligned} P_i(x) =& f(x_0) + (x - x_0)f[x_0, x_1] + ... + (x - x_0)...(x - x_{i-1})f[x_0, ..., x_i] \\ P_3(x) =& 32 + .2857(x - 7) - .001458(x - 7)(x - 14)(x - 21) \\ =& -0.001458x^3 + 0.061236x^2 - 0.500162x + 33.000664 \\ P_3(10) =& \mathbf{32.6646} \end{aligned}$$

(d) Find the cubic spline interpolation for the 5th weather station using natural cubic spline algorithm.

$$f(x) = \begin{array}{ll} .0048x^3 - .072x^2 + .64x + 26 & x \in [5, 10] \\ -.016x^3 + .552x^2 - 5.6x + 46.8 & x \in (10, 15] \\ .0112x^3 - .672x^2 + 12.76x - 45 & x \in (15, 20] \end{array}$$

2. Write a program to find the Lagrange interpolating polynomials for each of the weather stations. Use the Lagrange interpolating polynomials to estimate the PM 2.5 for each of the weather stations at T = 17.

Output

```
Weather Station 1 PM 2.5 at T = 17:
         P_9(17) = 30.8568
Weather Station 2 PM 2.5 at T = 17:
         P_9(17) = 33.2584
Weather Station 3 PM 2.5 at T = 17:
         P_4(17) = 36.6764
Weather Station 4 PM 2.5 at T = 17:
         P_4(17) = 35.0321
Weather Station 5 PM 2.5 at T = 17:
         P_4(17) = 33.136
Weather Station 6 PM 2.5 at T = 17:
         P_4(17) = 38.6764
```

Source Code at `https://git.io/JeW5f`

3. Write a program that implements Neville's Method and estimate the PM 2.5 for each of the weather stations at T = 12.

Output

```
30
33  38.2500
35  37.6667  37.3333
27  27.0000  27.0000  27.0000
29  27.0000  27.0000  27.0000  27.0000
32  26.7500  27.0000  27.0000  27.0000  27.0000
35  25.0000  27.5000  27.0000  27.0000  27.0000  27.0000
37  30.0000  20.0000  29.5455  27.0000  27.0000  27.0000  27.0000
39  27.6667  33.3333  10.6667  33.5909  27.0000  27.0000  27.0000  27.0000
Weather Station 1 PM 2.5 at T = 12 ~= 27
36
35  31.0000
30  27.0000  25.2857
28  27.0000  27.0000  27.1905
34  29.2000  27.9429  27.6286  27.5034
32  38.0000  30.4571  28.7810  28.2871  27.9932
36  27.2000  44.1714  31.6000  29.3850  28.7494  28.3533
37  30.5000  24.3714  52.9714  33.1265  30.0866  29.2588  28.7470
40  29.2000  32.5429  20.2857  62.3102  34.6625  30.7403  29.7146  29.0926
Weather Station 2 PM 2.5 at T = 12 ~= 29.0926
42
36  36.8571
38  35.7143  36.3673
40  35.7143  35.7143  36.1808
Weather Station 3 PM 2.5 at T = 12 ~= 36.1808
32
34  33.4286
36  33.4286  33.4286
35  37.2857  32.8776  33.2974
Weather Station 4 PM 2.5 at T = 12 ~= 33.2974
28
30  30.8000
33  31.2000  31.0800
31  34.2000  31.8000  31.4160
Weather Station 5 PM 2.5 at T = 12 ~= 31.416
30
37  34.0000
42  34.8571  34.2449
44  39.1429  33.9388  34.1866
Weather Station 6 PM 2.5 at T = 12 ~= 34.1866
```

Source Code at `https://git.io/JeWb4`

4. Write a program that implements Newtons Divided Differences Method and estimate the PM 2.5 for each of the weather stations at T = 7.

Output

```
30
33   0.7500
35   0.6667  -0.0119
27  -2.0000  -0.3810  -0.0335
29   0.6667   0.3810   0.0762   0.0078
32   0.7500   0.0119  -0.0335  -0.0078  -0.0009
35   1.0000   0.0357   0.0024   0.0026   0.0006   0.0001
37   0.5000  -0.0714  -0.0097  -0.0009  -0.0002  -0.0000  -0.0000
39   0.6667   0.0238   0.0095   0.0014   0.0001   0.0000   0.0000   0.0000
Weather Station 1 PM 2.5 at T = 7 ~= 36.8172
36
35  -0.5000
30  -1.0000  -0.0714
28  -1.0000   0.0000   0.0079
34   1.2000   0.3143   0.0262   0.0013
32  -1.0000  -0.3143  -0.0698  -0.0069  -0.0005
36   0.8000   0.2571   0.0476   0.0084   0.0008   0.0001
37   0.5000  -0.0429  -0.0333  -0.0058  -0.0009  -0.0001  -0.0000
40   0.6000   0.0143   0.0048   0.0027   0.0004   0.0001   0.0000   0.0000
Weather Station 2 PM 2.5 at T = 7 ~= 36.2572
42
36  -0.8571
38   0.2857   0.0816
40   0.2857   0.0000  -0.0039
Weather Station 3 PM 2.5 at T = 7 ~= 40.3499
32
34   0.2857
36   0.2857   0.0000
35  -0.1429  -0.0306  -0.0015
Weather Station 4 PM 2.5 at T = 7 ~= 32
28
30   0.4000
33   0.6000   0.0200
31  -0.4000  -0.1000  -0.0080
Weather Station 5 PM 2.5 at T = 7 ~= 28.296
30
37   1.0000
42   0.7143  -0.0204
44   0.2857  -0.0306  -0.0005
Weather Station 6 PM 2.5 at T = 7 ~= 28.895
```

Source Code at `https://git.io/JeWNQ`

5. Write a program to implements Hermite Interpolation using Divided Differences to find the Hermite polynomial $H_{11}(x)$ for the data in Table 2 and approximate the value when x= 0.75.

Output

```
Wasn't able to produce output
```

Source Code at `https://git.io/JeWNh`

Table 1: Particulate Matter

SN	T	PM
1	1	30
1	5	12
1	8	35
1	12	27
1	15	29
1	19	32
1	22	35
1	26	37
2	2	36
2	4	35
2	9	30
2	11	28
2	16	34
2	18	32
2	23	36
2	25	37
2	30	40
3	6	42
3	13	36
3	20	38
3	27	40
4	7	32
4	14	34
4	21	36
4	28	35
5	5	28
5	10	30
5	15	33
5	20	31
6	8	30
6	15	37
6	22	42
6	29	44