Computational Approach to Estimating AR(1) Process with SMM:

1. Housekeeping:
   - Initialize the algorithm. Set appropriate parameters (true values of ρ and σ, T, H).
   - Simulate data under "true" parameters. Set a seed. Using true values of ρ and σ, simulate a time series of data of length T. Calculate moments from the time series. These are your "data" moments.
   - Draw and store a set of shocks that are of H vectors of length T. Just like the Krusell-Smith problem set, we want to make sure we are using the *same* shocks when we simulate our model.
2. 1st Step Estimate of Parameters: The parameters that minimize your objective function J are your first step estimates.
   - Set the weight matrix to be the identity matrix.
   - Define an objective function J to minimize. This will be a function of your parameters that compares simulated moments to the "data" moments computed in (1).
   - Calculate standard errors.
3. 2nd Step Estimate of Parameters:
   - Calculate optimal weight matrix using Newey-West.
   - Minimize the same objective function defined in (2) using the new weight matrix.
   - Calculate standard errors. Confirm that the standard are smaller under the optimal weight matrix.

Algorithm 1 Estimate AR(1) Model via SMM

procedure SMM
    call GetMoments(ρ_0, σ_0, T, H = 1)
    return $m(\rho_0, \sigma_0)$

    $W = I$
    call MinObjFunc($W, \tilde{\rho}, \tilde{\sigma}, m(\rho_0, \sigma_0)$)   ▷ $\{\tilde{\rho}, \tilde{\sigma}\}$ is some initial guess
    return $\hat{\rho}_1, \hat{\sigma}_1$ ▷ First Step Estimator

    call ComputeSE($\hat{\rho}_1, \hat{\sigma}_1$)
    return $diag(\Sigma_1(\hat{\rho}_1, \hat{\sigma}_1))$ ▷ See equation (8)

    call NeweyWest($m(\rho_0, \sigma_0), m(\hat{\rho}_1, \hat{\sigma}_1)$)
    return S

    $W^* = S^{-1}$
    call MinObjFunc($W^*, \tilde{\rho}, \tilde{\sigma}, m(\rho_0, \sigma_0)$)
    return $\hat{\rho}_2, \hat{\sigma}_2$ ▷ Second Step Estimator

    call ComputeSE($\hat{\rho}_2, \hat{\sigma}_2$)
    return $diag(\Sigma_2(\hat{\rho}_2, \hat{\sigma}_2))$

end procedure

function GETMOMENTS(ρ, σ, T, H)
  - Simulate H AR(1) processes of length T.
  - Compute appropriate moments for each of the T simulations.
  - Average across H individual moments: $m(\rho, \sigma) = 1/H \sum_H m_h(\rho, \sigma)$
  return $m(\rho, \sigma)$
end function

function MINOBJFUNC($W, \hat{\rho}, \hat{\sigma}, m(\rho_0, \sigma_0)$)
  minimize OBJFUNC($W, \hat{\rho}, \hat{\sigma}, m(\rho_0, \sigma_0)$)
  return $\{\hat{\rho}, \hat{\sigma}\}$
end function

function OBJFUNC($W, \hat{\rho}, \hat{\sigma}, m(\rho_0, \sigma_0)$)
  GETMOMENTS($\hat{\rho}, \hat{\sigma}, T, H$) **return** $m(\hat{\rho}, \hat{\sigma})$
  $J = [m(\hat{\rho}, \hat{\sigma}) - m(\rho_0, \sigma_0)]' W [m(\hat{\rho}, \hat{\sigma}) - m(\rho_0, \sigma_0)]$
  return J
end function

function COMPUTESE($\hat{\rho}, \hat{\sigma}$)
  $\varepsilon = 1e^{-10}$
  GETMOMENTS($\hat{\rho}, \hat{\sigma}, T, H$) **return** $m(\hat{\rho}, \hat{\sigma})$
  GETMOMENTS($\hat{\rho} - \varepsilon, \hat{\sigma}, T, H$) **return** $m(\hat{\rho} - \varepsilon, \hat{\sigma})$
  GETMOMENTS($\hat{\rho}, \hat{\sigma} - \varepsilon, T, H$) **return** $m(\hat{\rho}, \hat{\sigma} - \varepsilon)$
  $\nabla_\rho = [m(\hat{\rho} - \varepsilon, \hat{\sigma}) - m(\hat{\rho}, \hat{\sigma})]/\varepsilon$ and $\nabla_\sigma = [m(\hat{\rho}, \hat{\sigma} - \varepsilon) - m(\hat{\rho}, \hat{\sigma})]/\varepsilon$
  $\nabla_b = [\nabla_\rho, \nabla_\sigma]$
  return $diag(\Sigma(\hat{\rho}, \hat{\sigma}))$ ▷ See equation (8)
end function

function NEWEYWEST($m(\rho_0, \sigma_0), m(\hat{\rho}, \hat{\sigma})$)
  See `HelpfulFunction.jl`
  return S
end function