# ECON 899: Problem Set 5

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Overview: For this assignment, the goal is to use the Krusell-Smith (1998) algorithm to approximate an equilibrium when there is aggregate uncertainty (as defined in Aiyagari (1994)). To solve this problem efficiently, we use interpolation and function minimization.

## Krusell-Smith Algorithm

Below is a brief sketch of the algorithm used to solve for law of motion for the approximate mean of aggregate capital (as an approximation of the law of motion for the wealth distribution).

- 1. Choose a moment to approximate: mean of aggregate capital  $\bar{K}$
- 2. Guess a functional form for the law of motion for  $\bar{K}$ : suppose it follows a log-linear form and there are separate equations for each aggregate state z

$$\log \bar{K}'_{i} = \begin{cases} a_{0} + a_{1} \log \bar{K} & \text{if } z = z_{g} \\ b_{0} + b_{1} \log \bar{K} & \text{if } z = z_{b} \end{cases}$$

- 3. Draw a sequence of aggregate economy shocks z of length T, and for each aggregate economy state, draw idiosyncratic shocks  $\epsilon$  for a total of N simulated agents. The z and  $\epsilon$  transition probabilities are related, and must satisfy certain conditions (specified in handout).
- 4. Given an initial guess for the law of motion for  $\bar{K}$  coefficients, use interpolation and function minimization to solve the consumer's problem through value function iteration.
- 5. Using the policy function generated in step 4, simulate a panel of k and K using the sequence of z and  $\epsilon$  shocks drawn in step 3.
- 6. Run two regressions to estimate the coefficients for the law of motion for  $\bar{K}$ , one for each aggregate state (good and bad economies). Get the  $R^2$  value to check the goodness of fit.
- 7. If the existing guess and estimated coefficients are sufficiently close, then the algorithm is done. If not, use the estimated coefficients to update the existing guess by adjustment parameter  $\lambda$  and repeat steps 4 7 until convergence.

<sup>\*</sup>I collaborated with Anya Tarascina and Claire Kim on this assignment.

## Results

Using the initial guesses  $a_0 = b_0 = 0.999$ ,  $a_1 = 0.095$  and  $b_1 = 0.085$ , my estimates for the coefficients of the low of motion for  $\bar{K}$  are:

$\overline{a_0}$	$a_1$	$b_0$	$b_1$	$R^2$
0.093	0.963	0.084	0.965	0.994

The code for this program are attached with this submission. The "model\_and\_functions.jl" file contains functions and structures for the following four sections:

- 1. set up strucs and functions to initialize (including draw shocks)
- 2. functions for value function iteration
- 3. functions for simulating capital path
- 4. functions for estimating regression and checking for convergence

The "main\_program.jl" program calls the functions declared in "model\_and\_functions.jl" to run the Krusell-Smith algorithm. My main program completed in around 15 minutes, after 9 iterations of the outer while loop.