

## Computational Approach to solving Consea-Krueger Model:

1. Initialize the algorithm: Set parameters, grid bounds, number of grid points. The states of the economy are age  $j$ , assets  $a$  and productivity  $z$ . Initialize (i.e give an initial guess) asset and labor policy function and value function, stationary distribution ( $\mu$ ), and aggregate variables ( $K$  and  $L$ ). Given the aggregate variables, we can derive the prices ( $w$ ,  $b$  and  $r$ ).
2. Backward Induction: Take as given aggregate variables ( $K$  and  $L$ ) – and thus by extension prices ( $w$ ,  $b$  and  $r$ ). Solve household's decision problem taking as given next age's value function  $V(j+1, a', z')$ . Once done, you should get out value  $V(j, a, z)$ , asset policy function  $a(j, a, z)$ , and labor policy function  $l(j, a, z)$  at each stage of the agent's life ( $j$ ).
  - Note: there is no need to 'iterate until convergence'. Since we are doing backward induction, we only need to solve backward once.
  - This is exercise 1 in the problem set.
3. Stationary Distribution: Take as given policy function solved for in (2). Calculate the stationary asset distribution following the law of motion.
  - Note: again, there is no 'iterate until convergence' here. We assume that when agents are born ( $j = 1$ ) they come in with zero assets and their productivity is drawn from the invariant Markov distribution (Dean gives the probabilities in the problem set).
  - Note: the pset notes that the relative size of each age cohort is decreasing.
  - This is exercise 2 in the problem set.
4. Capital and Labor Market Clearing: Taking as given the policy function solved for in (2) and stationary distribution solved for in (3), calculate new aggregate capital and labor from the formula given in the problem set.
  - Note: This is just one big sum over all the states and corresponding asset decision (from the policy function) and distribution weight

If market clearing condition is met – that is, if Capital and Labor supply (approximately) equal Capital and Labor demand – you are done. Otherwise, 'update'  $K$  and  $L$  from the formula given by Dean, and update the wage, interest rate, and benefits too. Then repeat steps (2) - (4).

- This is exercise 3 in the problem set.

## Pseudo Code to Solve Consea-Krueger Model (steps (2) - (4)):

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**Algorithm 1** Consea-Krueger

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1: procedure MAIN CODE
2:    $K_0 = K_{\text{init}} \ \& \ L_0 = L_{\text{init}}$  ▷ Hint:  $K_{\text{init}} = 3.3 \ \& \ L_{\text{init}} = 0.3$ 
3:   Given  $\{K_0, L_0\}$  solve for  $\{w, r, b\}$ 

4:   convergence flag = 0
5:   while convergence flag = 0 do
6:     call BACKWARDINDUCTION( )
7:     return  $\{V(j, a, z), a'(j, a, z), l(j, a, z)\}$ 

8:     call CALCULATESTATDIST( )
9:     return  $\mu(j, a, z)$ 

10:    call CALCULATEMARKETCLEARING( )
11:    return  $\{K_1, L_1\}$ 

12:    if  $|K_1 - K_0| + |L_1 - L_0| > \epsilon$  then ▷ Changing  $\lambda$  will vary convergence speed
13:       $K_0 \leftarrow \lambda K_1 + (1 - \lambda)K_0$ 
14:       $L_0 \leftarrow \lambda L_1 + (1 - \lambda)L_0$ 
15:    else if  $|K_1 - K_0| + |L_1 - L_0| < \epsilon$  then
16:      return convergence flag = 1
17:    end if

18:  end while
19: end procedure
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**function** BACKWARDINDUCTION( )**for**  $j = j_N : -1 : 1$  **do**

▷ Backward Induction

**for**  $a = 1 : n_a; \quad z = 1 : n_z$  **do**

Solve HH Problem ▷ Separate problem for end of life, retirees, and workers

 $V(j, a, z) = u(c, l) + \beta E_{z'}\{V(j+1, a', z')\}$  ▷ Continuation Value**end for****end for****return**  $\{V(j, a, z), a'(j, a, z), l(j, a, z)\}$ **end function****function** CALCULATESTATDIST( )Define  $\mu(1, 0, z = z^H) = 0.2037 / \sum_{j=0}^{j_n-1} \left(\frac{1}{1+n}\right)^j$ Define  $\mu(1, 0, z = z^L) = 0.7963 / \sum_{j=0}^{j_n-1} \left(\frac{1}{1+n}\right)^j$ Construct  $\Pi_{n_a n_z \times n_a n_z}(j)$ ▷  $\Pi$  depends on agent's ageUpdate  $\mu(j+1, a, z) = \Pi(j)' \mu(j, a, z) \frac{1}{1+n}$ **return**  $\mu(j, a, z)$ **end function****function** CALCULATEMARKETCLEARING( )

Use formulas Dean gives in the problem set.

**return**  $\{K_1, L_1\}$ **end function**

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