ECON 899b: Problem Set 1

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Overview: For this assignment, the goal is to apply different methods of optimization for discrete choice models. In particular, we use the public use microdata on mortgages to study the log-likelihood of the loan being pre-paid within the first-year.

Findings: The attached code ("main_program.jl" and "helper_functions.jl") produces the results described below. In the first step, I wrote functions to evaluate log-likelihood, the score of the log-likelihood function (approximation of the FOC), and the Hessian matrix (approximation of SOC) given some β vector. Specifically, $\beta_0 = -1$ and all other β values in the vector are 0. Then, I use the numerical derivative method in Julia to calculate the FOC and SOC of the log-likelihood function at the same β values. The comparison between the score and Hessian, and the numerical derivatives, are summarized in Tables 1, 3, 4. It appears that the approximations and numerical derivatives are very similar and/or identical.

Then, I wrote a Newton algorithm that solves for maximum likelihood, and compared the resulting coefficients to that of the BFGS (Quasi-Newton) and Simplex algorithms. Using the initial guess, the Newton algorithm converged in 37 iterations and approximately 12 seconds. The results of the algorithm are summarized in Table 2 below.

The implementation of the BFGS algorithm was slightly more complicated. Without providing a gradient function, the algorithm took a very long time to converge. When I input the score of the log-likelihood as the gradient and used the resulting β values from the Newton algorithm as the initial guess, the algorithm converged in 3 iterations and approximately 1 second. I used an educated guess for the initial value, because the simple initial values result in nonsensical results.

Similar to the BFGS algorithm, I was only able to get the Simplex algorithm to converge when I used the Newton algorithm results as the initial values. When I attempted to run the optimization without a refined initial guess, the optimization package returned a "failed line search" message. With an educated guess, the Simplex algorithm converged in 694 iterations, and approximately 140 seconds. Comparing the β results from all 3 algorithms, the coefficients appear to be very similar.

*I collaborated with Anya Tarascina and Claire Kim on this assignment.

Table 1: Comparison of Log Likelihood Score and Numerical First Derivative

Score of Log Likelihood	Numerical First Derivative
−2605.91	−2605.91
−556.32	−556.32
−1156.86	−1156.86
−222.82	−222.82
−933.04	−933.04
−1215.13	−1215.13
−2109.63	−2109.63
−948.07	−948.07
−5049.88	−5049.88
−4534.79	−4534.79
−19401.9	−19401.9
−19164.66	−19164.66
−918.86	−918.86
−351.75	−351.75
−466.69	−466.69
−582.47	−582.47
−546.41	−546.41

Table 2: Comparison of Coefficients

β_{newton}	β_{bfgs}	$\beta_{simplex}$
-6.06	-6.06	-6.06
0.87	0.87	0.87
0.53	0.53	0.53
0.6	0.6	0.6
0.16	0.16	0.16
0.87	0.87	0.87
-0.06	-0.06	-0.06
0.22	0.22	0.22
1.01	1.01	1.01
0.34	0.34	0.34
-0.28	-0.28	-0.28
0.19	0.19	0.19
0.76	0.76	0.76
1.15	1.15	1.15
0.77	0.77	0.77
0.38	0.38	0.38
0.24	0.24	0.24

Table 3: Hessian Matrix for Log Likelihood Function

−3224.6	−880.4	−1428.4	−387.6	−1305.7	−1546.8	−2619.4	−1210.7	−6304.6	−5761.3	−23783.2	−23599.2	−1405.0	−664.4	−681.9	−674.4	−583.0
−880.4	−880.4	−0.0	−10.1	−404.2	−421.3	−686.3	−332.0	−1720.7	−1655.1	−6608.0	−6563.0	−390.1	−163.8	−189.3	−211.6	−170.1
−1428.4	−0.0	−1428.4	−165.2	−560.3	−676.0	−1192.1	−544.7	−2796.0	−2551.2	−10512.7	−10428.5	−586.9	−283.7	−308.9	−299.4	−266.6
−387.6	−10.1	−165.2	−715.6	−185.7	−187.9	−325.0	−152.3	−783.9	−694.0	−2739.2	−2721.0	43.9	−59.8	−104.7	−92.4	−77.3
−1305.7	−404.2	−560.3	−185.7	−1305.7	−693.5	−973.1	−501.4	−2556.3	−2592.7	−9553.5	−9586.9	−502.0	−214.5	−291.0	−299.4	−192.9
−1546.8	−421.3	−676.0	−187.9	−693.5	−806.4	−1231.5	−585.3	−3024.7	−2841.4	−11435.2	−11359.5	−660.5	−312.1	−326.3	−325.5	−283.0
−2619.4	−686.3	−1192.1	−325.0	−973.1	−1231.5	−2224.7	−992.5	−5125.9	−4620.8	−19218.6	−19050.9	−1228.3	−545.1	−551.4	−540.1	−477.4
−1210.7	−332.0	−544.7	−152.3	−501.4	−585.3	−992.5	−528.6	−2369.9	−2169.3	−8869.9	−8798.6	−557.5	−248.1	−257.3	−253.3	−220.4
−6304.6	−1720.7	−2796.0	−783.9	−2556.3	−3024.7	−5125.9	−2369.9	−12464.4	−11264.9	−46482.7	−46118.4	−2718.9	−1297.8	−1333.2	−1318.9	−1134.6
−5761.3	−1655.1	−2551.2	−694.0	−2592.7	−2841.4	−4620.8	−2169.3	−11264.9	−10834.7	−42612.4	−42303.3	−2419.5	−1169.8	−1223.7	−1213.9	−1033.6
−23783.2	−6608.0	−10512.7	−2739.2	−9553.5	−11435.2	−19218.6	−8869.9	−46482.7	−42612.4	−176722.1	−175070.8	−10062.1	−4908.2	−5017.5	−4970.1	−4271.6
−23599.2	−6563.0	−10428.5	−2721.0	−9586.9	−11359.5	−19050.9	−8798.6	−46118.4	−42303.3	−175070.8	−174101.2	−9978.7	−4851.0	−4978.9	−4944.8	−4247.1
−1405.0	−390.1	−586.9	43.9	−502.0	−660.5	−1228.3	−557.5	−2718.9	−2419.5	−10062.1	−9978.7	−1405.0	−293.0	−306.9	−305.7	−268.4
−664.4	−163.8	−283.7	−59.8	−214.5	−312.1	−545.1	−248.1	−1297.8	−1169.8	−4908.2	−4851.0	−293.0	−664.4	−0.0	−0.0	−0.0
−681.9	−189.3	−308.9	−104.7	−291.0	−326.3	−551.4	−257.3	−1333.2	−1223.7	−5017.5	−4978.9	−306.9	−0.0	−681.9	−0.0	−0.0
−674.4	−211.6	−299.4	−92.4	−299.4	−325.5	−540.1	−253.3	−1318.9	−1213.9	−4970.1	−4944.8	−305.7	−0.0	−0.0	−674.4	−0.0
−583.0	−170.1	−266.6	−77.3	−192.9	−283.0	−477.4	−220.4	−1134.6	−1033.6	−4271.6	−4247.1	−268.4	−0.0	−0.0	−0.0	−583.0

Table 4: Numerical Second Derivative

−3224.4	−880.4	−1428.4	−387.6	−1305.7	−1546.8	−2619.4	−1210.7	−6304.6	−5761.3	−23783.2	−23599.2	−1405.0	−664.4	−681.9	−674.4	−582.9
−880.4	−880.4	0.0	−10.1	−404.2	−421.3	−686.3	−332.0	−1720.8	−1655.1	−6608.0	−6563.0	−390.1	−163.8	−189.3	−211.6	−170.1
−1428.4	0.0	−1428.3	−165.2	−560.3	−676.0	−1192.1	−544.7	−2796.0	−2551.2	−10512.7	−10428.5	−586.9	−283.7	−308.9	−299.4	−266.6
−387.6	−10.1	−165.2	−715.6	−185.7	−187.9	−325.0	−152.4	−783.9	−694.0	−2739.2	−2721.0	43.9	−59.8	−104.7	−92.4	−77.3
−1305.7	−404.2	−560.3	−185.7	−1305.6	−693.5	−973.1	−501.4	−2556.4	−2592.7	−9553.5	−9586.9	−502.0	−214.5	−291.0	−299.4	−192.9
−1546.8	−421.3	−676.0	−187.9	−693.5	−806.5	−1231.5	−585.3	−3024.7	−2841.4	−11435.2	−11359.5	−660.5	−312.1	−326.3	−325.5	−283.0
−2619.4	−686.3	−1192.1	−325.0	−973.1	−1231.5	−2224.7	−992.5	−5125.9	−4620.7	−19218.6	−19050.9	−1228.3	−545.1	−551.4	−540.1	−477.4
−1210.7	−332.0	−544.7	−152.4	−501.4	−585.3	−992.5	−528.5	−2369.9	−2169.3	−8869.9	−8798.6	−557.5	−248.1	−257.3	−253.3	−220.4
−6304.6	−1720.8	−2796.0	−783.9	−2556.4	−3024.7	−5125.9	−2369.9	−12464.4	−11264.8	−46482.7	−46118.4	−2719.0	−1297.9	−1333.3	−1318.9	−1134.6
−5761.3	−1655.1	−2551.2	−694.0	−2592.7	−2841.4	−4620.7	−2169.3	−11264.8	−10834.7	−42612.4	−42303.3	−2419.5	−1169.9	−1223.7	−1213.9	−1033.6
−23783.2	−6608.0	−10512.7	−2739.2	−9553.5	−11435.2	−19218.6	−8869.9	−46482.7	−42612.4	−176722.2	−175070.8	−10062.1	−4908.2	−5017.5	−4970.1	−4271.6
−23599.2	−6563.0	−10428.5	−2721.0	−9586.9	−11359.5	−19050.9	−8798.6	−46118.4	−42303.3	−175070.8	−174101.2	−9978.7	−4851.0	−4978.9	−4944.8	−4247.1
−1405.0	−390.1	−586.9	43.9	−502.0	−660.5	−1228.3	−557.5	−2719.0	−2419.5	−10062.1	−9978.7	−1404.9	−293.0	−306.9	−305.7	−268.4
−664.4	−163.8	−283.7	−59.8	−214.5	−312.1	−545.1	−248.1	−1297.9	−1169.9	−4908.2	−4851.0	−293.0	−664.3	0.0	0.0	0.0
−681.9	−189.3	−308.9	−104.7	−291.0	−326.3	−551.4	−257.3	−1333.3	−1223.7	−5017.5	−4978.9	−306.9	0.0	−681.8	0.0	0.0
−674.4	−211.6	−299.4	−92.4	−299.4	−325.5	−540.1	−253.3	−1318.9	−1213.9	−4970.1	−4944.8	−305.7	0.0	0.0	−674.3	0.0
−582.9	−170.1	−266.6	−77.3	−192.9	−283.0	−477.4	−220.4	−1134.6	−1033.6	−4271.6	−4247.1	−268.4	0.0	0.0	0.0	−582.9