ECON 899b: Problem Set 1

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Overview: For this assignment, the goal is to apply different methods of optimization for discrete choice models. In particular, we use the public use microdata on mortgages to study the log-likelihood of the loan being pre-paid within the first-year.

Findings: The attached code ("main_program.jl" and "helper_functions.jl") produces the results described below. In the first step, I wrote functions to evaluate log-likelihood, the score of the log-likelihood function (approximation of the FOC), and the Hessian matrix (approximation of SOC) given some β vector. Specifically, $\beta_0 = -1$ and all other β values in the vector are 0. Then, I use the numerical derivative method in Julia to calculate the FOC and SOC of the log-likelihood function at the same β values. The comparison between the score and Hessian, and the numerical derivatives, are summarized in Tables 1, 3, 4. It appears that the approximations and numerical derivatives are very similar and/or identical.

Then, I wrote a Newton algorithm that solves for maximum likelihood, and compared the resulting coefficients to that of the BFGS (Quasi-Newton) and Simplex algorithms. Using the initial guess, the Newton algorithm converged in 37 iterations and approximately 12 seconds. The results of the algorithm are summarized in Table 2 below.

The implementation of the BFGS algorithm was slightly more complicated. Without providing a gradient function, the algorithm took a very long time to converge. When I input the score of the log-likelihood as the gradient and used the resulting β values from the Newton algorithm as the initial guess, the algorithm converged in 3 iterations and approximately 1 second. I used an educated guess for the initial value, because the simple initial values result in nonsensical results.

Similar to the BFGS algorithm, I was only able to get the Simplex algorithm to converge when I used the Newton algorithm results as the initial values. When I attempted to run the optimization without a refined initial guess, the optimization package returned a "failed line search" message. With an educated guess, the Simplex algorithm converged in 694 iterations, and approximately 140 seconds. Comparing the β results from all 3 algorithms, the coefficients appear to be very similar.

*I collaborated with Anya Tarascina and Claire Kim on this assignment.

Table 1: Comparison of Log Likelihood Score and Numerical First Derivative

Score of Log Likelihood	Numerical First Derivative
−556.32	−556.32
−2835.5	−2835.5
−666.47	−666.47
−1992.44	−1992.44
−2537.75	−2537.75
−4381.48	−4381.48
−1980.73	−1980.73
−10436.78	−10436.78
−9360.45	−9360.45
−39586.27	−39586.27
−39185.51	−39185.51
−2111.58	−2111.58
−940.03	−940.03
−1045.49	−1045.49
−1126.38	−1126.38
−1031.63	−1031.63

Table 2: Comparison of Coefficients

β_{newton}	β_{bfgs}	$\beta_{simplex}$
0.89	0.89	0.89
0.52	0.52	0.52
0.5	0.5	0.5
0.08	0.08	0.08
0.6	0.6	0.6
−0.9	−0.9	−0.9
−0.1	−0.1	−0.1
0.08	0.08	0.08
0.21	0.21	0.21
−0.48	−0.48	−0.48
0.0	0.0	0.0
0.63	0.63	0.63
1.0	1.0	1.0
0.62	0.62	0.62
0.24	0.24	0.24
0.07	0.07	0.07

Table 3: Hessian Matrix for Log Likelihood Function

−880.4	−0.0	−10.1	−404.2	−421.3	−686.3	−332.0	−1720.7	−1655.1	−6608.0	−6563.0	−390.1	−163.8	−189.3	−211.6	−170.1
−0.0	−1816.2	−210.1	−712.5	−859.5	−1515.9	−692.6	−3555.2	−3244.0	−13367.3	−13260.3	−746.2	−360.8	−392.8	−380.8	−339.0
−10.1	−210.1	−855.7	−235.2	−238.3	−409.1	−192.0	−989.2	−877.3	−3472.3	−3448.4	38.8	−77.7	−130.5	−114.5	−97.2
−404.2	−712.5	−235.2	−1550.5	−824.9	−1158.3	−596.2	−3035.1	−3077.9	−11335.2	−11372.4	−590.5	−256.3	−345.1	−351.5	−228.1
−421.3	−859.5	−238.3	−824.9	−967.0	−1478.0	−700.8	−3622.3	−3394.8	−13681.8	−13590.3	−788.6	−375.8	−390.7	−386.2	−337.6
−686.3	−1515.9	−409.1	−1158.3	−1478.0	−2677.0	−1190.9	−6153.7	−5527.5	−23043.4	−22841.3	−1476.4	−657.7	−661.1	−642.5	−570.3
−332.0	−692.6	−192.0	−596.2	−700.8	−1190.9	−633.0	−2837.2	−2588.6	−10605.8	−10519.4	−667.3	−298.9	−307.6	−300.6	−262.4
−1720.7	−3555.2	−989.2	−3035.1	−3622.3	−6153.7	−2837.2	−14925.4	−13444.9	−55598.2	−55158.9	−3253.8	−1563.4	−1594.9	−1564.1	−1353.3
−1655.1	−3244.0	−877.3	−3077.9	−3394.8	−5527.5	−2588.6	−13444.9	−12906.6	−50808.4	−50436.9	−2877.7	−1404.3	−1458.9	−1435.2	−1228.6
−6608.0	−13367.3	−3472.3	−11335.2	−13681.8	−23043.4	−10605.8	−55598.2	−50808.4	−211160.9	−209177.6	−12006.5	−5906.9	−5995.0	−5889.2	−5087.2
−6563.0	−13260.3	−3448.4	−11372.4	−13590.3	−22841.3	−10519.4	−55158.9	−50436.9	−209177.6	−208012.6	−11907.2	−5837.5	−5948.3	−5859.1	−5058.2
−390.1	−746.2	38.8	−590.5	−788.6	−1476.4	−667.3	−3253.8	−2877.7	−12006.5	−11907.2	−1680.6	−352.0	−367.2	−362.3	−319.4
−163.8	−360.8	−77.7	−256.3	−375.8	−657.7	−298.9	−1563.4	−1404.3	−5906.9	−5837.5	−352.0	−800.3	−0.0	−0.0	−0.0
−189.3	−392.8	−130.5	−345.1	−390.7	−661.1	−307.6	−1594.9	−1458.9	−5995.0	−5948.3	−367.2	−0.0	−815.6	−0.0	−0.0
−211.6	−380.8	−114.5	−351.5	−386.2	−642.5	−300.6	−1564.1	−1435.2	−5889.2	−5859.1	−362.3	−0.0	−0.0	−800.1	−0.0
−170.1	−339.0	−97.2	−228.1	−337.6	−570.3	−262.4	−1353.3	−1228.6	−5087.2	−5058.2	−319.4	−0.0	−0.0	−0.0	−695.1

Table 4: Numerical Second Derivative

−880.3	0.0	−10.1	−404.2	−421.3	−686.3	−332.0	−1720.7	−1655.1	−6608.0	−6563.0	−390.1	−163.8	−189.3	−211.5	−170.0
0.0	−1816.2	−210.1	−712.5	−859.5	−1515.9	−692.6	−3555.2	−3244.0	−13367.3	−13260.3	−746.2	−360.8	−392.8	−380.8	−339.0
−10.1	−210.1	−855.4	−235.2	−238.4	−409.0	−192.0	−989.2	−877.3	−3472.3	−3448.4	38.8	−77.7	−130.5	−114.6	−97.2
−404.2	−712.5	−235.2	−1550.4	−824.9	−1158.3	−596.2	−3035.1	−3077.9	−11335.2	−11372.4	−590.5	−256.3	−345.1	−351.5	−228.0
−421.3	−859.5	−238.4	−824.9	−966.7	−1478.0	−700.8	−3622.3	−3394.8	−13681.8	−13590.3	−788.6	−375.8	−390.7	−386.2	−337.6
−686.3	−1515.9	−409.0	−1158.3	−1478.0	−2676.7	−1191.0	−6153.7	−5527.5	−23043.4	−22841.3	−1476.4	−657.8	−661.1	−642.5	−570.3
−332.0	−692.6	−192.0	−596.2	−700.8	−1191.0	−632.8	−2837.2	−2588.6	−10605.8	−10519.4	−667.3	−298.9	−307.6	−300.6	−262.4
−1720.7	−3555.2	−989.2	−3035.1	−3622.3	−6153.7	−2837.2	−14925.2	−13444.8	−55598.2	−55158.9	−3253.8	−1563.4	−1594.9	−1564.1	−1353.3
−1655.1	−3244.0	−877.3	−3077.9	−3394.8	−5527.5	−2588.6	−13444.8	−12906.4	−50808.4	−50436.9	−2877.7	−1404.3	−1458.9	−1435.2	−1228.6
−6608.0	−13367.3	−3472.3	−11335.2	−13681.8	−23043.4	−10605.8	−55598.2	−50808.4	−211160.6	−209177.6	−12006.6	−5906.8	−5995.0	−5889.2	−5087.2
−6563.0	−13260.3	−3448.4	−11372.4	−13590.3	−22841.3	−10519.4	−55158.9	−50436.9	−209177.6	−208012.2	−11907.2	−5837.5	−5948.3	−5859.1	−5058.2
−390.1	−746.2	38.8	−590.5	−788.6	−1476.4	−667.3	−3253.8	−2877.7	−12006.6	−11907.2	−1680.5	−352.0	−367.2	−362.3	−319.3
−163.8	−360.8	−77.7	−256.3	−375.8	−657.8	−298.9	−1563.4	−1404.3	−5906.8	−5837.5	−352.0	−800.3	−0.0	0.0	−0.0
−189.3	−392.8	−130.5	−345.1	−390.7	−661.1	−307.6	−1594.9	−1458.9	−5995.0	−5948.3	−367.2	−0.0	−815.6	0.0	−0.0
−211.5	−380.8	−114.6	−351.5	−386.2	−642.5	−300.6	−1564.1	−1435.2	−5889.2	−5859.1	−362.3	0.0	0.0	−800.0	0.0
−170.0	−339.0	−97.2	−228.0	−337.6	−570.3	−262.4	−1353.3	−1228.6	−5087.2	−5058.2	−319.3	−0.0	−0.0	0.0	−695.0