CS 335
Graphics and Multimedia

Matrix Algebra Tutorial

Properties of Vector Cross Product

Cross product of parallel vectors

$$V_1 \times V_2 = V_2 \times V_1 = \vec{0} \qquad \text{iff } V_1 \text{ parallel to } V_2$$

Anti-commutative

$$V_1 \times V_2 = -\left(V_2 \times V_1\right)$$

Not associative

$$V_1 \times \left(V_2 \times V_3\right) \neq \left(V_1 \times V_2\right) \times V_3$$

Distributive with respect to vector addition

$$V_1 \times \left(V_2 + V_3\right) = \left(V_1 \times V_2\right) + \left(V_1 \times V_3\right)$$

Basic Definitions

- *Scalar* – a number
  - e.g. 3.1, 2.9, 10.7e5
- *Vector* – a column (or a row) of scalars

$$V = [u, v, s, t], \quad V = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}, \quad V = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

- *Matrix* – an array of numbers (or a collection of vectors)

$$M = \begin{bmatrix} 3.2 & 9.4 \\ 5 & 7.1 \end{bmatrix}, \qquad N = \begin{bmatrix} a & b & c & d \\ e & f & g & i \\ x & y & z & w \end{bmatrix}$$

Vector Math

- Add

$$V_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, V_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

$$V_1 + V_2 = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix}$$

- Scale

$$aV = \begin{bmatrix} ax \\ ay \\ az \end{bmatrix}$$

- Transpose

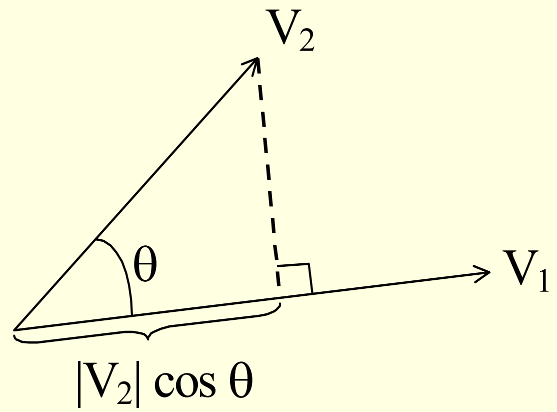
$$V^T = \begin{bmatrix} x \\ y \\ z \end{bmatrix}^T = \begin{bmatrix} x & y & z \end{bmatrix}$$

Vector Math

- Dot product

$$V_1 \cdot V_2 = |V_1||V_2| \cos\theta$$

where θ is the angle between the two vectors



$$V_1 \cdot V_2 = V_{1x}V_{2x} + V_{1y}V_{2y} + V_{1z}V_{2z}$$

$$V_1 \cdot V_2 = V_2 \cdot V_1 = 0 \quad \text{if and only if } V_1 \perp V_2$$

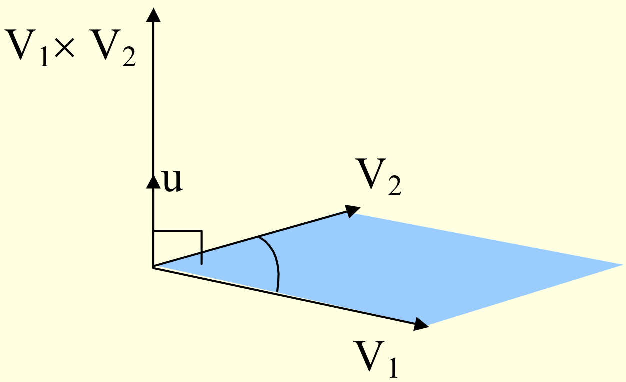
$$V_1 \cdot (V_2 + V_3) = V_1 \cdot V_2 + V_1 \cdot V_3$$

Vector Product (cross product)

Multiplication of two vectors to produce another *vector*.

$$V_1 \times V_2 = u|V_1||V_2|\sin\theta$$

Where u is a unit vector that is perpendicular to both V_1 and V_2



(Vector Notation)

$$V_1 \times V_2 = \begin{pmatrix} V_{1y}V_{2z} - V_{1z}V_{2y} \\ V_{1z}V_{2x} - V_{1x}V_{2z} \\ V_{1x}V_{2y} - V_{1y}V_{2x} \end{pmatrix}$$

cross product gives a vector in a direction perpendicular to the two original vectors (u) and with a magnitude equal to the area of the shaded parallelogram

$$V_1 \times V_2 = (V_{1y}V_{2z} - V_{1z}V_{2y}, V_{1z}V_{2x} - V_{1x}V_{2z}, V_{1x}V_{2y} - V_{1y}V_{2x})$$

Nice applet demo at:
http://physics.syr.edu/courses/java-suite/crosspro.html

Matrix Algebra

Watch the dimensions!!

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, N = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

- Matrix + Matrix (add)

$$M + N = \begin{bmatrix} a+x & b+y \\ c+z & d+w \end{bmatrix}$$

- Transpose

$$M^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

- Scale

$$sM = \begin{bmatrix} sa & sb \\ sc & sd \end{bmatrix}$$

- Matrix * Matrix (multiplication)

$$MN = \begin{bmatrix} ax+bz & ay+bw \\ cx+dz & cy+dw \end{bmatrix}$$

Matrix Inverse

- M's inverse, denoted as M^{-1}, is a matrix such that

$$MM^{-1} = M^{-1}M = I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & & \cdots & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix Algebra

- Matrix * Vector

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad V = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$MV = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Take-home Exercise

$$A=\begin{bmatrix} a & b \\ 0 & c \\ 1 & 0 \end{bmatrix}, M=\begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 8 \\ 4 & 1 & 2 \end{bmatrix}, N=\begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}, V=\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

compute:

$MN=$

$(AM)N=$ $\qquad$ $(A^T M)N=$

$A^T(MN)=$

$NV=,$ $\qquad$ $V^T N=$

$VV^T=$ $\qquad$ $V^T V=$