HW 4, 11/10/2020

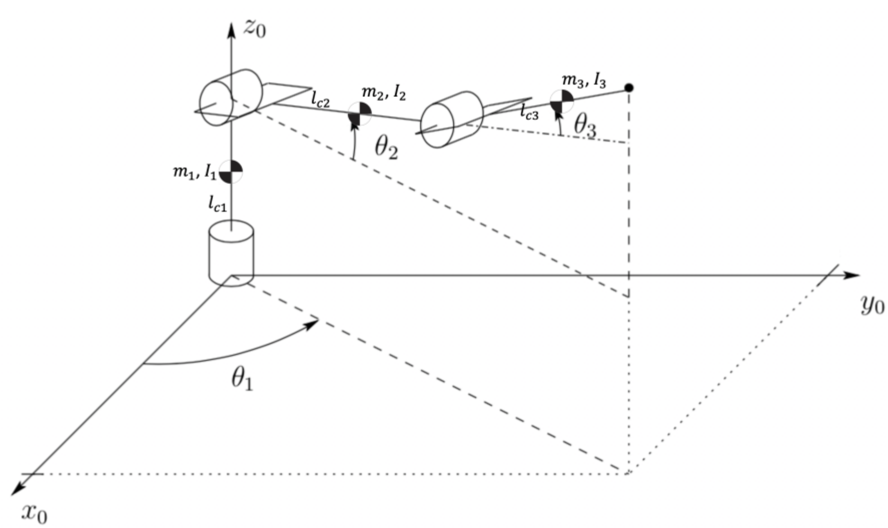
Robot Dynamics

Due: Tuesday, 11/24/2020 @11:59 PM

Three-Link Arm Robot – Dynamic Modeling (Lagrange's Method vs. Newton's Method)

For the 3-link RRR elbow manipulator (3-DOF) shown below, let l_{c1}, l_{c2}, and l_{c3} be the distances of the centers of mass of the three links from the respective joint axes and l_1, l_2, and l_3 be the length of the three links. Also let m_1 , m_2, and m_3 be the masses of the three links. Finally, let I_1, I_2, and I_3 be the moments of inertia *relative to the centers of mass* of the three links, respectively. For this problem,

1) Symbolically, derive the total kinetic energy of the robot and form the 3-by-3 Inertia Matrix $D(q)$. **(25 pts.)**
2) Using Christoffel Symbols, symbolically derive the 3-by-3 Coriolis/Centripetal Coupling Matrix $C(q, \dot{q})$. **(15 pts.)**
3) Symbolically, derive the total potential energy of the robot and form the 3-by-1 gravity term $g(q)$. **(5 pts.)**
4) Form the dynamical model of the robot in the compact form $\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$. **(5 pts.)**
5) Rewrite the dynamic model using the Newton's method. Show your work step by step and finally put the model in the compact form. The result must be the same as what you got in Part (4). **(35 pts.)**
6) In the derived model from either part (4) or part (5) (they must be the same), consider $l_{c1} = l_1$, $l_{c2} = l_2$, and $l_{c3} = l_3$. Also, consider $I_1 = I_2 = I_3 = 0$. Then, rewrite the dynamical model. Now, consider the robot in its home position i.e. $q_i = \theta_i = 0$ with the following physical parameters: $l_1 = l_2 = l_3 = 0.3\,m$, $m_1 = m_2 = m_3 = 0.5\,kg, g = 9.8$. Solve the dynamical model with these parameters and show that the result is the same as the result for Problem 1-b in HW3? **(15 pts.)**



Good Luck!