Kathria Coronado

RBE 501 Fall 2020

Modern Robotics: Mech, Planing + control

HW 1 - 3.6, 3.16, 3.17, 3.27, 3.28, 3.37, 4.2, 4.12

① 3.6 Given $R = Rot(\hat{x}, \frac{\pi}{2}) Rot(\hat{z}, \pi)$ Find unit vector $\hat{\omega}$ and θ such that $R = e^{[\hat{\omega}]\theta}$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ 0 & \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{bmatrix} \begin{bmatrix} \cos(\pi) & -\sin(\pi) & 0 \\ \sin(\pi) & \cos(\pi) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$tr\,R = r_{11} + r_{22} + r_{33} = 1 + 2\cos\theta$$
$$= -1 + 0 + 0 = -1$$

If $tr\,R = -1$ then $\theta = \pi$

$$1 + 2\cos\theta = -1 \rightarrow 2\cos\theta = -2 \rightarrow \cos\theta = -1$$

$$\theta = \cos^{-1}(-1) = \pi \Rightarrow \boxed{\theta = \pi} \checkmark$$

$$\hat{\omega} = \frac{1}{\sqrt{2(1+r_{33})}} \begin{bmatrix} r_{13} \\ r_{23} \\ 1+r_{33} \end{bmatrix} = \frac{1}{\sqrt{2(1+0)}} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\boxed{\hat{\omega} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}} \qquad \hat{\omega} = \frac{1}{\sqrt{2}}(-\hat{y} + \hat{z})$$

② **3.16** In terms of frame {s}, frame

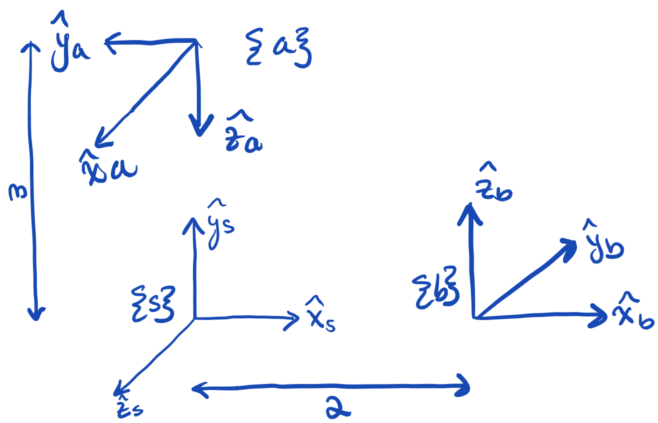
{a}: $\hat{x}_a \rightarrow (0,0,1)$ $\hat{y}_a \rightarrow (-1,0,0)$ Origin {a} $= (3,0,0)$ in {s}

{b}: $\hat{x}_b \rightarrow (1,0,0)$ $\hat{y}_b \rightarrow (0,0,-1)$ origin {b} $= (0,2,0)$ in {s}

ⓐ draw diagram {a} and {b} relative to {s}

$\hat{z}_a = (0,-1,0)$

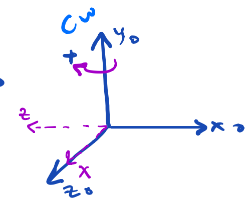
$\hat{z}_b = (0,-1,0)$



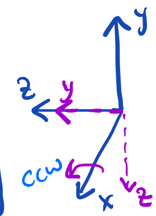
ⓑ write rotation matrices R_{sa} and R_{sb} and Transformation matrices T_{sa} and T_{sb}

cw (+)
ccw (−)

$R_{sa} \rightarrow$ ① rotate $\{s\}$ by $90°$ in y_s axis



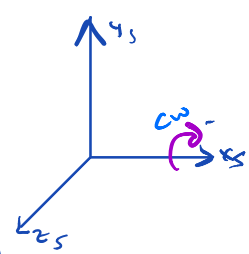
② rotate $\{s\}$ by $90°$ about z_s axis



$$R_{sa} = \begin{array}{c} z \\ \begin{bmatrix} \cos(90) & -\sin(90) & 0 \\ \sin(90) & \cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} y \\ \begin{bmatrix} \cos(90) & 0 & \sin(90) \\ 0 & 1 & 0 \\ -\sin(90) & 0 & \cos(90) \end{bmatrix} \end{array}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & +1 & 0 \\ +1 & 0 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = R_{sa}}$$

$R_{sb} \rightarrow$ ① Rotate -90 about x_s axis



$$R_{sb} = \begin{array}{c} x \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(+90) & -\sin(+90) \\ 0 & \sin(+90) & \cos(+90) \end{bmatrix} \end{array} = \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = R_{sb}}$$

$$\boxed{T_{sa} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

(c) how do you calculate T_{sb}^{-1} without using matrix inverse?

$$T_{sb}^{-1} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R^T & -R^Tp \\ 0 & 1 \end{bmatrix}$$

$$R_{sb}^T = R_{sb}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$-R^Tp = -\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$\boxed{T_{sb}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

(d) How do you calculate Tab.

$$T_{ab} = T_{as} T_{sb} = T_{sa}^{-1} T_{sb}$$

$$T_{sa}^{-1} = \begin{bmatrix} R^T & -R^Tp \\ 0 & 1 \end{bmatrix}$$

$$R_{sa}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \qquad -R^Tp = -\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$$T_{as} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{ab} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

(e) Let $T = T_{sb}$ is a transformation operator consisting of $Rot(\hat{x}, -90)\ trans(\hat{y}, 2)$. Calc $T_1 = T_{sa} T$. Does T_1 correspond to rot + trans about $\hat{x}_s$ and $\hat{y}_s$ (T_{sa} in $\{s\}$) or $\hat{x}_a$ and $\hat{y}_a$ (T_{sa} in $\{b\}$) calculate $T_2 = T T_{sa}$. Does T_2 correspond with $\{s\}$ or $\{b\}$ Transformation of T_{sa}?

$$T = T_{sb} = Rot(\hat{x}, -90)\, trans(\hat{y}, 2) \qquad T_{sa} T_{sb}$$

$$T_1 = T_{sa} T$$

$$= \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

T_1 corresponds to the rotation and translation about $\hat{x}_a$ and $\hat{y}_a$. This is because for T_1 we post (or right) multiply T_{sa} by T. With post multiplication the transformation occurs in the coordinates of the second subscript, in this case $\{a\}$ which is the body-fixed frame of T_{sa}

$$T_2 = T T_{sa}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & -1 & 0 & 3 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

T_2 corresponds to the rotation and translation about $\hat{x}_s$ and $\hat{y}_s$, which is the world fixed transformation of T_{sa}. This is because we premultiplied T_{sa} by T. With premultiplication the transformation occurs in the coordinates of the 1st subscript, in this case $\{s\}$.

(f) Use T_{sb} to change representation of point $p_b=(1,2,3)$ in b coordinates to {s} coordinates

$$\begin{bmatrix} p_s \\ 1 \end{bmatrix} = T_{sb} \begin{bmatrix} p_b \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix} \qquad \boxed{p_s = (1,-1,2)}$$

(g) $p_s = (1,2,3)$ in {s} coordinates. Calc $p' = T_{sb} p_s$ and $p'' = T_{sb}^{-1} p_s$

$$\begin{bmatrix} p' \\ 1 \end{bmatrix} = T_{sb} \begin{bmatrix} p_s \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix} \qquad \boxed{p' = (1,-1,2)}$$

The result p' should be interpreted as as moving the location of the point without changing the reference frame of representation

$$\begin{bmatrix} p'' \\ 1 \end{bmatrix} = T_{sb}^{-1} \begin{bmatrix} p_s \\ 1 \end{bmatrix} = T_{bs} \begin{bmatrix} p_s \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} \qquad \boxed{p'' = (1,3,0)}$$

The result p'' should be interpreted as changing coordinates from {s} to {b} frame without moving point p

(h) A twist $\mathcal{V}$ is represented in $\{s\}$ as $\mathcal{V}_s = (3,2,1,-1,-2,-3)$

Find $\mathcal{V}_a$ in frame $\{a\}$.

$$\mathcal{V}_a = Ad_{T_{as}} \mathcal{V}_s$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ -1 \\ -2 \\ -3 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ -3 \\ -2 \\ -9 \\ 1 \\ -1 \end{bmatrix}}$$

$$Ad_{T_{as}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

(i) calculate matrix logarithm $[S]\theta$ of T_{sa}.

Extract S and θ. Find the $\{q, s, h\}$ representation of the screw axis. redraw fixed frame $\{s\}$ and draw S

$$T_{SA} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_{SA} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \quad p = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\operatorname{tr} R = 1 + 2\cos\theta \rightarrow \quad \theta = \cos^{-1}\left(\frac{\operatorname{tr} R - 1}{2}\right) = \cos^{-1}\left(\frac{-1}{2}\right)$$

$$\operatorname{tr} R = 0$$

$$\boxed{\theta = 2.094 \text{ rads } (120^\circ)}$$

$$[\hat{\omega}] = \theta \frac{1}{2\sin\theta}\left(R_{SA} - R_{SA}^T\right)$$

$$= 2.094 \frac{1}{2\sin(2.094)} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1.2092 & -1.2092 \\ 1.2092 & 0 & -1.2092 \\ 1.2092 & 1.2092 & 0 \end{bmatrix}$$

$$\text{matLog} = \begin{bmatrix} [\hat{\omega}] & \left(I - \frac{[\hat{\omega}]}{2} + \left(\frac{1}{\theta} - \frac{\cos\frac{\theta}{2}}{2}\right) \times \frac{[\omega]^2}{\theta}\right) p \\ 0 & 0 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 0 & -1.2092 & -1.2092 & 2.2092 \\ 1.2092 & 0 & -1.2092 & -2.2092 \\ 1.2092 & 1.2092 & 0 & -1.4184 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$[S]\theta = \begin{bmatrix} [\omega]\theta & v\theta \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2.5325 & -2.5325 & -2.0944 \\ 2.5325 & 0 & -2.5325 & -4.1888 \\ 2.5325 & 2.5325 & 0 & -6.2832 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S\theta = \begin{bmatrix} 2.5325 \\ -2.5325 \\ 2.5325 \\ -2.0944 \\ -4.1888 \\ -6.2832 \end{bmatrix} \qquad \boxed{S = \begin{bmatrix} 1.2092 \\ -1.2092 \\ 1.2092 \\ -1 \\ -2 \\ -3 \end{bmatrix}}$$

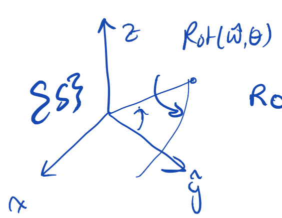
1) Calculate matrix exponential corresponding to the exponential coordinates of rigid body motion

$$S\theta = (\underbrace{0,1,2}_{\omega\theta}, \underbrace{3,0,0}_{v\theta}) \qquad [S\theta] = \begin{bmatrix} 0 & -2 & 1 & 3 \\ 2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{T = MatrixExp6(S\theta) = \begin{bmatrix} -0.6173 & -0.7037 & 0.3518 & 1.0555 \\ 0.7037 & -0.2938 & 0.6469 & 1.9407 \\ -0.3518 & 0.6469 & 0.6765 & -0.9704 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

ω is a unit vector

$\omega_x = 0\hat{x}$
$\omega_y = 1\hat{y}$
$\omega_z = 2\hat{z}$



$$Rot(\hat{\omega},\theta) = e^{[\hat{\omega}]\theta} = I + \sin\theta[\omega] + (1-\cos\theta)[\hat{\omega}]^2$$

③ 3.17

{a} : Pixel frame {c} : Camera

{b}: end effector {d} workpiece

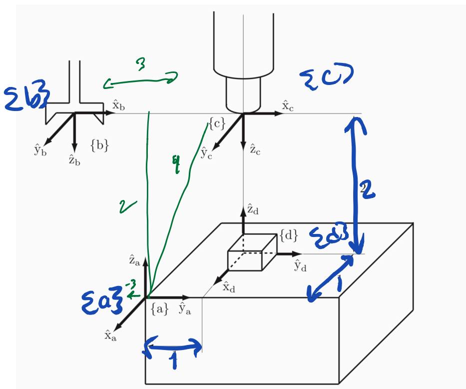


Figure 3.23: Four reference frames defined in a robot's workspace.

$$T_{ad} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{cd} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{bc} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{ab} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4) <u>3.27</u> Draw screw axis for $\mathcal{V} = (\underbrace{0,2,2}_{\omega},\underbrace{4,0,0}_{v})$

$q = (0,0,2)$

$$\|w\| = \sqrt{0^2+2^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\mathcal{V} = \begin{bmatrix} \omega \\ v \end{bmatrix} = S\dot{\theta}$$

$$\hat{\omega} = \hat{S} = \frac{\omega}{\|w\|} = \frac{\langle 0,2,2\rangle}{2\sqrt{2}} = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

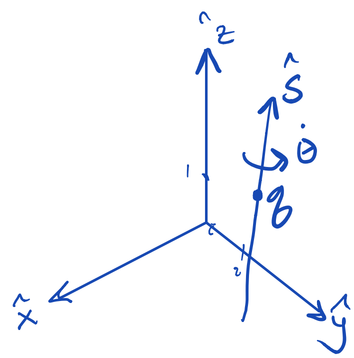
$$h = \hat{\omega}^T v/\dot{\theta} \qquad \dot{\theta} = \|w\| = 2\sqrt{2}$$

$$= \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} (2\sqrt{2}) = 0$$

$r = -q$

If q is equal to $(0,2,1)$

$$-\hat{S}\dot{\theta} \times q = -\begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} 2\sqrt{2} \times \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$



$$v = -\omega \times r + h\hat{S}\dot{\theta} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$$

(5) 3.28 Assume space frame angular velocity is $\omega_s = (1,2,3)$ for a moving body with frame $\{b\}$. Calculate ω_b in $\{b\}$

$$R_{bs} = R_{sb}^{-1}$$

$$R_{sb} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \qquad \omega_b = R_{bs}\,\omega_s = R_{sb}^{-1} \times \omega_s$$

$$R_{bs} = R^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \qquad \omega_b = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

$$\boxed{\omega_b = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}}$$

(6) 3.37 Consider wrist mechanism with 2 revolute joints θ_1 and θ_2. End effector frame given by:

$$R = e^{[\hat{\omega}_1]\theta_1} e^{[\hat{\omega}_2]\theta_2}$$

$\hat{\omega}_1 = (0,0,1)$ $\hat{\omega}_2 = (0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$, is the orientation reachable? for $R = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

$$R = e^{[\hat{\omega}_1]\theta_1} e^{[\hat{\omega}_2]\theta_2}$$

$$= \left(I + \sin\theta_1[\hat{\omega}_1] + (1-\cos\theta_1)[\hat{\omega}_1]^2\right)\left(I + \sin\theta_2[\hat{\omega}_2] + (1-\cos\theta_2)[\hat{\omega}_2]^2\right)$$

$$[\hat{\omega}_1] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad [\hat{\omega}_2] = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

$$e^{[\hat{\omega}_1]\theta_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sin\theta_1 \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (1-\cos\theta_1)\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^2$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e^{[\hat{\omega}_2]\theta_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \sin\theta_2 \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} - \cos\theta_2 \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_2 & \frac{1}{\sqrt{2}}\sin\theta_2 & \frac{1}{\sqrt{2}}\sin\theta_2 \\ -\frac{1}{\sqrt{2}}\sin\theta_2 & \frac{1}{2}(1+\cos\theta_2) & -\frac{1}{2}(1-\cos\theta_2) \\ -\frac{1}{\sqrt{2}}\sin\theta_2 & \frac{1}{2}(-1+\cos\theta_2) & \frac{1}{2}(1+\cos\theta_2) \end{bmatrix}$$

$$e^{[\hat{\omega}_1]\theta_1} e^{[\hat{\omega}_2]\theta_2} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} = \frac{-\sin\theta_2}{\sqrt{2}}$$

$$\theta_2 = 270$$

$$\cos\theta_1 \underbrace{\cos\theta_2}_{-1} + \frac{\sin\theta_1 \overbrace{\sin\theta_2}^{-1}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin\theta_1 = -1 \rightarrow \theta_1 = 270$$

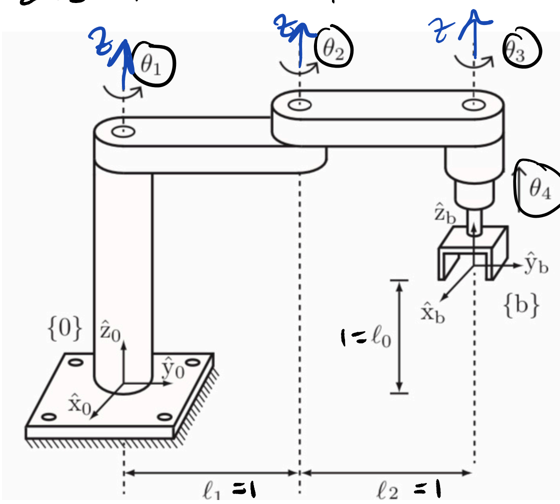
$\theta_1 = 270^\circ$

$\theta_2 = 270^\circ$

The solution (θ_1, θ_2) for $R = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$ exists

Infact, multiple solutions can exist

(7) 4.2 RRRP SCARA robot shown at zero position. Find end effector zero pos config M, screw axis S_i and screw axes B_i in $\{b\}$. for $l_0 = l_1 = l_2 = 1$ and $\theta = (0, \frac{\pi}{2}, \frac{-\pi}{2}, 1)$

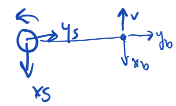


$L = 1$

$$M = \begin{array}{c} \\ x_s \\ y_s \\ z_s \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} x_e & y_e & z_e & p \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} = T(0)$$

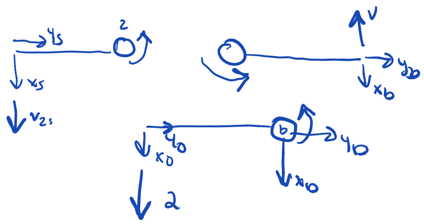
$\boxed{J1}$

$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2L \\ 0 \\ 0 \end{bmatrix}$$



$\boxed{J2}$

$$S_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ L \\ 0 \\ 0 \end{bmatrix} \begin{array}{l} \leftarrow \omega \\ \\ \\ \leftarrow v \\ \\ \\ \end{array} \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -L \\ 0 \\ 0 \end{bmatrix}$$



$\boxed{J3}$

$$S_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2L \\ 0 \\ 0 \end{bmatrix} \quad B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{J4}$

$$S_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad B_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\theta = \begin{bmatrix} 0 \\ \frac{\pi}{2} \\ -\pi/2 \\ 1 \end{bmatrix}$$

$T = \text{FKinBody}(M, B, \theta)$

$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \checkmark$$

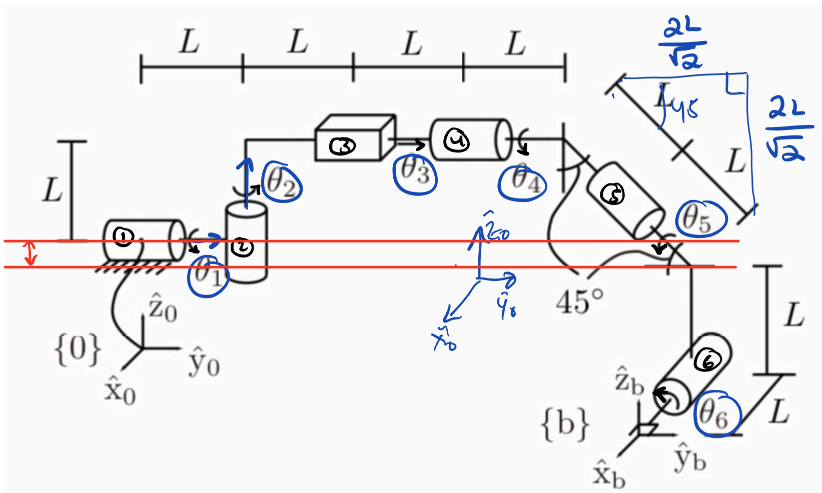
$T = \text{FKinSpace}(M, S, \theta)$

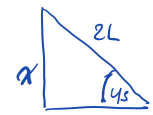
$$= \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \checkmark$$

(8) **4.12** RRPRRR spatial open chain shown @ 0 pos

Find end effector zero position config M, the screw axis S_i @ {0} and screw axes B_i @ {B}

$\theta_5 = \pi$ all other joint variables $= 0$. Find T_{0b} and T_{b0}





$\sin\theta = \frac{x}{2L}$

$x = 2L\sin(45)$

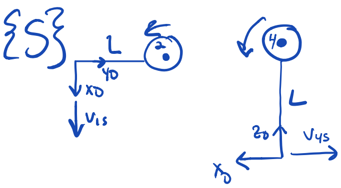
$= \frac{2L}{\sqrt{2}}$

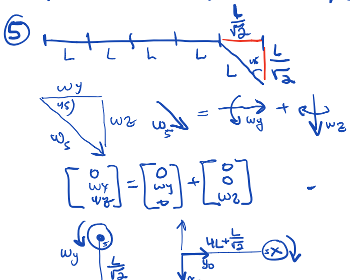
$\frac{2L}{\sqrt{2}} - \cancel{L} + \cancel{L}$

$\frac{2L\sqrt{2}}{\sqrt{2}\sqrt{2}} = \sqrt{2}$

$$M = \begin{array}{c} \\ x_0 \\ y_0 \\ z_0 \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} x_e & y_e & z_e & P_e \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 1 & 0 & 4L + \frac{2L}{\sqrt{2}} \\ 0 & 0 & 1 & -\frac{2L}{\sqrt{2}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

i	ω_i	S_i	B_i
1	$(0,1,0)$	$(0,0,0)$	$-\sqrt{2}L, 0, -L$
2	$(0,0,1)$	$(L,0,0)$	$-(3L+\sqrt{2}L), (L+\sqrt{2}L), 0$
3	$(0,0,0)$	$(0,1,0)$	$0,1,0$
4	$(0,0,0)$	$(-L,0,0)$	$-(L+\sqrt{2}L), 0, -L$
5	$(0,\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}})$	$(-(4L+L\sqrt{2}),0,0)$	$-(L+\sqrt{2}L), 0, -L$
6	$(1,0,0)$	$(0,-(\sqrt{2}L),-(4L+\sqrt{2}L))$	$(0\ 0\ 0)$

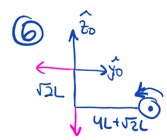




$$\begin{bmatrix} 0 \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_y \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_z \end{bmatrix}$$

$4L + \frac{2L\sqrt{2}}{\cancel{\sqrt{2}\sqrt{2}}}$ $\frac{2L\sqrt{2}}{2}$

$4L + L\sqrt{2}$



$$T_{06\text{-}S} = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & -5.4142 \\ 0 & 0 & 1 & -1.4142 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{06\text{-}B} = \begin{bmatrix} -0.079 & 0 & 0 & 0.0790 \\ 0 & 0.8420 & 0.3815 & -4.0192 \\ 0 & 0.3815 & 0.079 & -1.9537 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{60} = \begin{bmatrix} -1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & -5.4142 \\ 0 & 0 & 1 & 1 & 1.4142 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2

The solutions are suppose to be the same.
I must have been a mistake
while calculating the screw axes

```
%3.6
%{
R1= [1 0 0 ; 0 cos(pi/2) -sin(pi/2); 0 sin(pi/2) cos(pi/2)];
R2= [cos(pi) -sin(pi) 0; sin(pi) cos(pi) 0; 0 0 1];
R= R1*R2;
trR=trace(R);
theta= acos((trR-1)/2)
x= [R(1,3);R(2,3);1+R(3,3)];
w= 1/sqrt(2*(1+R(3,3)))*x
%}
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%3.16B
%{
R1= [cos(pi/2) -sin(pi/2) 0; sin(pi/2) cos(pi/2) 0 ; 0 0 1];
R2= [cos(-pi/2) 0 sin(-pi/2); 0 1 0; -sin(-pi/2) 0 cos(-pi/2)];
RSA= R1*R2;
RSB= [1 0 0 ; 0 cos(pi/2) -sin(pi/2);0 sin(pi/2) cos(pi/2)];
TSA= RpToTrans(RSA,[3;0;0])
TSB= RpToTrans(RSB,[0;2;0])
%3.16C
TSB_INV=[ RSB^(-1) -RSB^(-1)*[0;2;0]; 0 0 0 1]
%3.16D
TSA_INV=[ RSA^(-1) -RSA^(-1)*[3;0;0]; 0 0 0 1]
TAB= TSA_INV*TSB
%3.16E
T1= TSA*TSB
T2= TSB*TSA
%3.16F
PB= [1; 2; 3;1]
PA = TSB*PB
%3.16G
PS= [1; 2 ; 3; 1]
P1=TSB*PS
P11=TSB_INV*PS
%3.16H
VS=[3;2;1;-1;-2;-3]
ADTAS=Adjoint(TSA_INV)
VA=ADTAS*VS
%3.16I

ML_TSA=MatrixLog6(TSA)
thetai= acos((trace(RSA)-1)/2)
omgmat=thetai*1/(2*sin(thetai))*(RSA-RSA^-1)
MatLog = [omgmat, (eye(3) - omgmat / 2 ...
                       + (1 / theta - cot(theta / 2) / 2) ...
                         * omgmat * omgmat / theta) * p;
               0, 0, 0, 0];
sthi=[omgmat*thetai [-1;-2;-3]*theta; 0 0 0 0]
%3.16j
sth= [0;1;2;3;0;0]
sthm=VecTose3(sth)
T=MatrixExp6(sthm)
%}
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%3.27
%{
v27= [0 ;2 ;2;4;0;0]
vv=[4;0;0]
ww= [0;2;2]
dw =sqrt(0^2+2^2+2^2)
thetad= dw
sh= ww/dw % same as wh
R37= [1/sqrt(2) 0 -1/sqrt(2);
h= transpose(sh)*vv/thetad
q= [0;2;1]
cross(-sh*thetad,q)
r= -q
vvv= cross(-ww,r)+ h*sh*thetad
%}
%3.28
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%{
R28= [0 -1 0;0 0 -1; 1 0 0]
R28_inv= R28^-1
ws= [1;2;3]
wb=R28_inv*ws
%}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%4.2
%{
l=1
l0,l1, l2=l;
thetalist=[0;pi/2;-pi/2;1];
%Blist= [0 0 0 0;0 0 0 0; 1 1 1 0; -2 -1 0 0; 0 0 0 0; 0 0 0 1]
M2=[[1,0,0,0];[0,1,0,l1+l2];[0,0,1,l0];[0,0,0,1]];
Slist = [[0;0;1;0;0;0],[0;0;1;l1;0;0],[0;0;1;l1+l2;0;0],[0;0;0;0;0;1]]
Blist = [[0;0;1;-(2*l);0;0],[0;0;1;-l;0;0],[0;0;1;0;0;0],[0;0;0;0;0;1]];
```

```
T_S = FKinSpace(M2,Slist,thetalist)
T_B = FKinBody(M2,Blist,thetalist)

%}
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%4.12
%%{
L=1;
I=eye(3); % identity matrix
M12 = [ 1 0 0 L; 0 1 0 4*L+((2*L)/sqrt(2));0 0 1 -L*sqrt(2);0 0 0 1];
SLIST =[0 1 0 0 0 0; 0 0 1 L 0 0; 0 0 0 0 1 0; 0 1 0 -L 0 0; 0 1 0 -L 0 0; 0 1/sqrt(2) -1/sqrt(2) -L*(4+sqrt(2)) 0 0; 1 0 0 0 -L*sqrt(2) -(4*L+L*sqrt(2))];
BLIST =[0 1 0 -sqrt(2)*L 0 -L; 0 0 1 -L*(3+sqrt(2)) L*(1+sqrt(2)) 0; 0 0 0 0 1 0; 0 1 0 -L*(1+sqrt(2)) 0 -L;  0 1/sqrt(2) -1/sqrt(2) -L*(1+sqrt(2)) 0 -L; -1 0 0 0 0
thetalist = [0; 0; 0; 0; pi; 0];
thetalist= [0; 0; 0; 0;pi; 0];
ESS=[];
for i=1:6 % loop through the 6 different joints
S=SLIST(:,i); % one of the Screw axes size 6
w=S(1:3);% w vector
v= S(4:6); % velocity vector
wk=[0, -w(3), w(2); w(3), 0, -w(1); -w(2), w(1), 0]; % skew w [w]
%Sk= [wk v; 0 0 0 0]; % skew symmetric screw axis in space frame
theta=thetalist(i); % angle
R = I +(sin(theta)*wk)+(1-cos(theta))*wk^2;%rotation matrix
V= (I*theta+(1-cos(theta))*wk+(theta-sin(theta))*wk^2)*v;
es =[[R;0 0 0], [V;1]]; % rotation matrix for this iteration
ESS=[ESS, es]; % array appends all of the rotation matrices
end
Ts= eye(4); % identity matrix
for i = 1:4:21 %column index. add 4  to go to next rotation matrix
j=i+3; % last column of the matrix
%fprintf('a = %.4f b = %.4f \n', i,j)
Ts= Ts*ESS(1:4,i:j);% multiply all of the rotation matrices
end
T06_S=Ts * M12 % post multiply rotation matrices by zero position matrix M
T60_S=T06_S^-1

ESB=[];
for i=1:6 % for T60
B=BLIST(:,i); % one of the Screw axes size 6
w=B(1:3);% w vector
v= B(4:6); % velocity vector
wk=[0, -w(3), w(2); w(3), 0, -w(1); -w(2), w(1), 0]; % skew w [w]
%Bk= [wk v; 0 0 0 0]; % skew symmetric screw axis in body frame
theta=thetalist(i);%angle
R = I +(sin(theta)*wk)+(1-cos(theta))*wk^2; %rotation matrix
V= (I*theta+(1-cos(theta))*wk+(theta-sin(theta))*wk^2)*v;
es =[[R;0 0 0], [V;1]]; % rotation matrix for this iteration
ESB=[ESB, es];% array appends all of the rotation matrices
end
TB= eye(4); % identity matrix
for i = 1:4:21 %column index. add 4  to go to next rotation matrix
j=i+3; % last column of the matrix
%fprintf('a = %.4f b = %.4f \n', i,j)
TB= TB*ESB(1:4,i:j); % multiply all of the rotation matrices
end
T06_B=M12*TB % pre multiply rotation matrices by zero position matrix M
T60_B=T06_B^-1

T_B = FKinBody(M12,BLIST,thetalist) % check for space frame calculation
T_S = FKinSpace(M12,SLIST,thetalist) % check for body frame calculation
%%}
```

```
T06_S =

   -1.0000   -0.0000         0   -1.0000
    0.0000   -1.0000         0   -5.4142
         0         0    1.0000   -1.4142
         0         0         0    1.0000


T60_S =

   -1.0000    0.0000         0   -1.0000
   -0.0000   -1.0000         0   -5.4142
         0         0    1.0000    1.4142
         0         0         0    1.0000


T06_B =

  -12.6569   -0.0000    0.0000    1.0000
    0.0000   -1.0000    4.8284    5.4142
   -0.0000    4.8284  -10.6569   -1.4142
         0         0         0    1.0000


T60_B =

   -0.0790    0.0000   -0.0000    0.0790
   -0.0000    0.8420    0.3815   -4.0192
```

```
    0.0000    0.3815    0.0790   -1.9537
         0         0         0    1.0000


T_B =

   -0.3480   -0.3588    0.8661    1.0000
    0.3588    0.8026    0.4766    5.4142
   -0.8661    0.4766   -0.1506   -1.4142
         0         0         0    1.0000


T_S =

   -1.0000   -0.0000         0   -1.0000
    0.0000   -1.0000         0   -5.4142
         0         0    1.0000   -1.4142
         0         0         0    1.0000
```

Published with MATLAB® R2020b