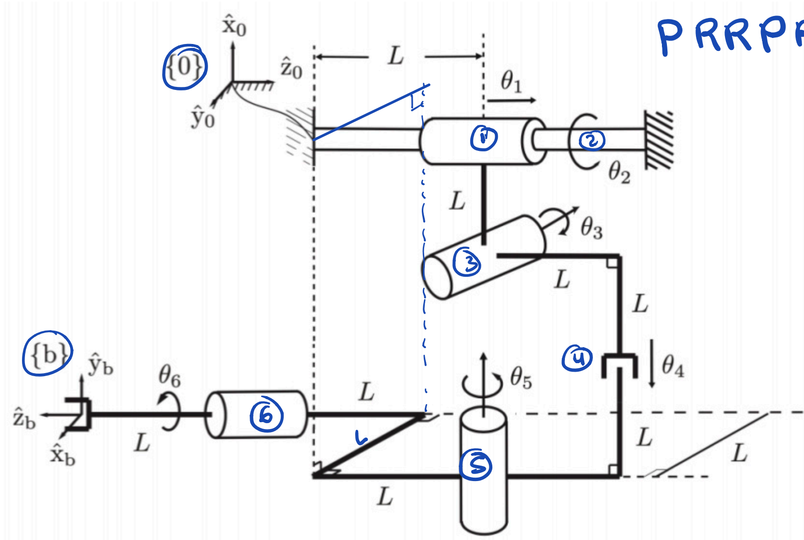
Kathia Coronado — Robot Dynamics

Midterm

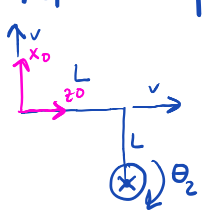


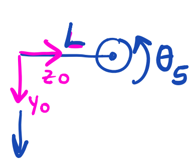
PRRPRR

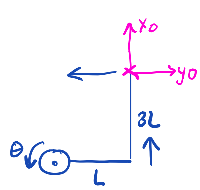
① Derive Fk (T_{0n}) using POE $L=100$

$$M=\begin{bmatrix}0 & 1 & 0 & -3L\\ 1 & 0 & 0 & -L\\ 0 & 0 & -1 & -2L\\ 0 & 0 & 0 & 1\end{bmatrix}\quad R_6^0=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}\quad P_e=\begin{bmatrix}-3L\\ -L\\ -2\end{bmatrix}=\begin{bmatrix}-3\\ -1\\ -2\end{bmatrix}$$

	Slist	ω	v	
P	1	(0,0,0)	(0,0,1)	← Prismatic
R	2	(0,0,1)	(0,0,0)	← Passes thru axis
R	3	(0,-1,0)	(L,0,L)	
P	4	(0,0,0)	(-1,0,0)	← Prismatic
R	5	(1,0,0)	(0,L,0)	
R	6	(0,0,-1)	(L,-3L,0)	







$$SLIST = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & L & -1 & 0 & L \\ 0 & 0 & 0 & 0 & L & -3L \\ 1 & 0 & L & 0 & 0 & 0 \end{bmatrix} \qquad \text{Thetalist} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}$$

Matlab → I have T calculate interms of symbolic $\theta_1\ \theta_2\ \theta_3\ \theta_4\ \theta_5\ \theta_6$.

② Solve for FK in the home position.
$\theta_1=\theta_2=\theta_3=\theta_4=\theta_5=\theta_6=0$)

Matlab →

$$\begin{array}{cc} & \begin{array}{cccc} & & & P_e \downarrow \end{array} \\ T_6^0 = & \begin{bmatrix} 0 & 1 & 0 & -300 \\ 1 & 0 & 0 & -100 \\ 0 & 0 & -1 & -200 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\boxed{P_e = \begin{bmatrix} -300 \\ -100 \\ -200 \end{bmatrix} \quad @\ \theta = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}$$

③ If we have vector [10 10 10] mm in ee frame {6}
calculate same vector in base frame {0}
when at home position.

$$P_6 = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix} \qquad T_6^0 = \begin{bmatrix} 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_0 = T_6^0 P_6 = \begin{bmatrix} 0 & 1 & 0 & -300 \\ 1 & 0 & 0 & -100 \\ 0 & 0 & -1 & -200 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} -290 \\ -90 \\ -210 \\ 1 \end{bmatrix}$$

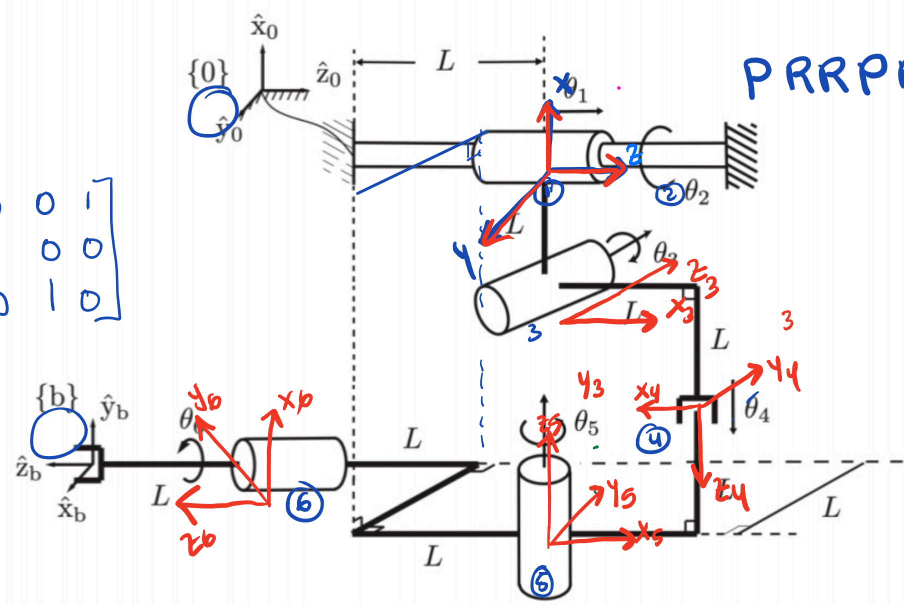
$$\boxed{P_0 = \begin{bmatrix} -290 \\ -90 \\ -210 \end{bmatrix}} \qquad [-290 \ -90 \ -210]$$

④ Derive 6×6 geometric jacobian of the arm. Then, calculate jacobian for home position

$J_v = z_{i-1} \times (P_e - P_{i-1})$

$J_v = z_{i-1}$ for Prismatic

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



PRRPRR

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad z_1 = R_1^0 z_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_3^2 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 \\ s\theta_3 & c\theta_3 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -c\theta_3 & s\theta_3 \\ 0 & -s\theta_3 & -c\theta_3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_4^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$R_5^4 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -c\theta_5 & -s\theta_5 & 0 \\ -s\theta_5 & c\theta_5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_2^0 = R_1^0 R_2^1 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_3^0 = R_2^0 R_3^2 = \begin{bmatrix} 0 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 & c\theta_2 s\theta_3 + c\theta_3 s\theta_2 \\ 0 & -c\theta_2 s\theta_3 - c\theta_3 s\theta_2 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_4^0 = R_3^0 R_4^3 = \begin{bmatrix} 0 & c\theta_2 s\theta_3 + c\theta_3 s\theta_2 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 \\ 0 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 & -c\theta_2 s\theta_3 - c\theta_3 s\theta_2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_5^0 = R_4^0 R_5^4 = \begin{bmatrix} -s\theta_5(c\theta_2 s\theta_3 + c\theta_3 s\theta_2) & c\theta_5(c\theta_2 s\theta_3 + c\theta_3 s\theta_2) & 0 \\ s\theta_5(c\theta_2 c\theta_3 - s\theta_2 s\theta_3) & -c\theta_5(c\theta_2 c\theta_3 - s\theta_2 s\theta_3) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$z_1 = R_1^0 z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_2 = R_2^0 z_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad z_3 = R_3^0 z_0 = \begin{bmatrix} c\theta_2 s\theta_3 + c\theta_3 s\theta_2 \\ s\theta_2 s\theta_3 - c\theta_2 c\theta_3 \\ 0 \end{bmatrix}$$

$$z_4 = R_4^0 z_0 = \begin{bmatrix} s\theta_2 s\theta_3 - c\theta_2 c\theta_3 \\ -c\theta_2 s\theta_3 - c\theta_3 s\theta_2 \\ 0 \end{bmatrix} \qquad z_5 = R_5^0 z_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$J_\omega = \begin{bmatrix} z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}$$

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$\underset{\substack{@\text{home} \\ \theta=0}}{J} = \begin{bmatrix} 0 & 100 & 300 & -1 & 0 & 0 \\ 0 & -300 & 0 & 0 & 300 & 0 \\ 1 & 0 & -200 & 0 & -100 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5 Find singularites of the arm.

singularity happens if $\det(J_v) = 0$

But J_v is not squre so

$\det(J J^T) = 0$ indicates a singularit

Matlab → singularity occurs @ home position

$$\theta = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad sing\_J = \begin{bmatrix} 0 & 100 & 300 & -1 & 0 & 0 \\ 0 & -300 & 0 & 0 & 300 & 0 \\ 1 & 0 & -200 & 0 & -100 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\det(sing\_J) = 0$

*⑥ Inverse Velocity: For ee at home position to have a linear velocity [10 0 10] mm/sec wrt to base frame {0}. Solve for joint velocities. How many solutions exists?

$$\dot{\vec{q}} = J_v^+ \dot{\vec{P}}$$

$$J_{v\_home} = \begin{bmatrix} 0 & 100 & 300 & -1 & 0 & 0 \\ 0 & -300 & 0 & 0 & 300 & 0 \\ 1 & 0 & -200 & 0 & -100 & 0 \end{bmatrix} \qquad P = \begin{bmatrix} 10 \\ 0 \\ 10 \end{bmatrix}$$

$$J_v^+ = \mathrm{pinv}(J_{v\_home}) = \begin{bmatrix} 0.0013 & 0.0005 & 0.0019 \\ -0.0199 & -0.0100 & -0.0299 \\ 0.0100 & 0.0033 & 0.0100 \\ -0.0009 & -0.0004 & -0.0013 \\ -0.0199 & -0.0066 & -0.0299 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\dot{\vec{q}} = J_v^+ \dot{\vec{P}} = \begin{bmatrix} 0.0013 & 0.0005 & 0.0019 \\ -0.0199 & -0.0100 & -0.0299 \\ 0.0100 & 0.0033 & 0.0100 \\ -0.0009 & -0.0004 & -0.0013 \\ -0.0199 & -0.0066 & -0.0299 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 10 \end{bmatrix} = \boxed{\begin{bmatrix} 0.0319 \\ -0.4986 \\ 0.1995 \\ -0.0219 \\ -0.4986 \\ 0 \end{bmatrix}}$$

Just 1 solution for the joint velocities exists when the end effector is at the home position and has a linear velocity of [10 0 10] mm/s wrt to the base frame.

(7) Inverse Position Kinematics. For end effector to be in the position [-350, 50, -250] mm wrt base frame{0}
Solve for joint variables using numerical inverse Kinematics. How many solutions exist.

$$P_d = \begin{bmatrix} -350 \\ 50 \\ -250 \end{bmatrix} \qquad q_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad q = J^{-1}$$

did this on on matlab.
The loop code I wrote takes a really long time to run. I think it Might be my computer
The first Iteration of my code out put

Iter 1

$$q_1 = \begin{bmatrix} 0.9706 \\ -0.2503 \\ -0.0306 \\ 1.0214 \\ 1.4929 \\ 1.000 \end{bmatrix}$$

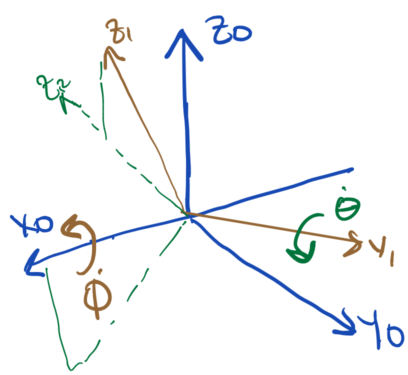
only on solution exist for this problem

⑧ Extra Credit Derive 6x6 analytical Jacobian of the arm considering XYZ Euler Angles. Then calculate Analytical Jacobian for home position.

$$\alpha = [\phi, \theta, \psi]^T \quad \phi \to x,\ \theta \to y,\ \psi \to z$$

$$\omega_x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \dot{\phi}$$

$$\omega_y = Rot(x,\phi) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ c\phi \\ s\phi \end{bmatrix} \dot{\theta}$$



$$\omega_z = Rot(x,\phi)\, Rot(y,\theta) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} s\theta \\ -c\theta s\phi \\ c\phi c\theta \end{bmatrix} \dot{\psi}$$

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & s\theta \\ 0 & c\phi & -c\theta s\phi \\ 0 & s\phi & c\phi c\theta \end{bmatrix}}_{B(\alpha)\ xyz} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} = \begin{pmatrix} atan2(-r_{23}, r_{33}) \\ atan2(r_{13}, \sqrt{r_{11}^2 + r_{12}^2}) \\ atan2(-r_{12}, r_{11}) \end{pmatrix} \qquad T = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\vec{d}} \\ \vec{\omega} \end{bmatrix} = J\dot{\vec{q}} \Rightarrow \begin{bmatrix} \dot{\vec{d}} \\ B(\alpha)\dot{\vec{\alpha}} \end{bmatrix} = J\dot{\vec{q}} \Rightarrow T(\alpha) \begin{bmatrix} \dot{\vec{d}} \\ \dot{\vec{\alpha}} \end{bmatrix} = J\dot{\vec{q}}$$

$$\Rightarrow \underset{6\times1}{\begin{bmatrix} \dot{\vec{d}} \\ B(\alpha)\dot{\vec{\alpha}} \end{bmatrix}} = \underbrace{\begin{bmatrix} \boxed{I}_{3\times3} & \boxed{0}_{3\times3} \\ \boxed{0}_{3\times3} & \boxed{B(\alpha)}_{3\times3} \end{bmatrix}}_{T(\alpha)\ 6\times6} \underset{6\times1}{\begin{bmatrix} \dot{\vec{d}} \\ \dot{\vec{\alpha}} \end{bmatrix}} = T(\alpha) \begin{bmatrix} \dot{\vec{d}} \\ \dot{\vec{\alpha}} \end{bmatrix}$$

$$T(\alpha)\underbrace{\begin{bmatrix}\dot{\vec{d}}\\ \dot{\vec{\alpha}}\end{bmatrix}}_{J_a\dot{\vec{q}}} = J\dot{\vec{q}} \Rightarrow T(\alpha)\,J_a\dot{\vec{q}} = J\dot{\vec{q}}$$

$$\Rightarrow J = T(\alpha)J_a$$

$$\underbrace{J_a = T^{-1}(\alpha)\,J}_{\begin{bmatrix}J_v\\ J_\alpha\end{bmatrix}} \leftarrow \text{geometric}$$

I solved for Ja symbolically using matlab with the equation above.

Look at Matlab output

@ home position all joint variables equal zero

$\Theta = [0\ 0\ 0\ 0\ 0\ 0]$

$$T(\alpha)_{home} = \begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&-1&0\\0&0&0&0&0&-1\end{bmatrix}$$

$$\boxed{J_{a\,home} = \begin{bmatrix}0&100&300&-1&0&0\\0&-300&0&0&300&0\\1&0&-200&0&-100&0\\0&0&0&0&-1&0\\0&0&0&1&0&0\\-1&-1&0&0&0&0\end{bmatrix}}$$