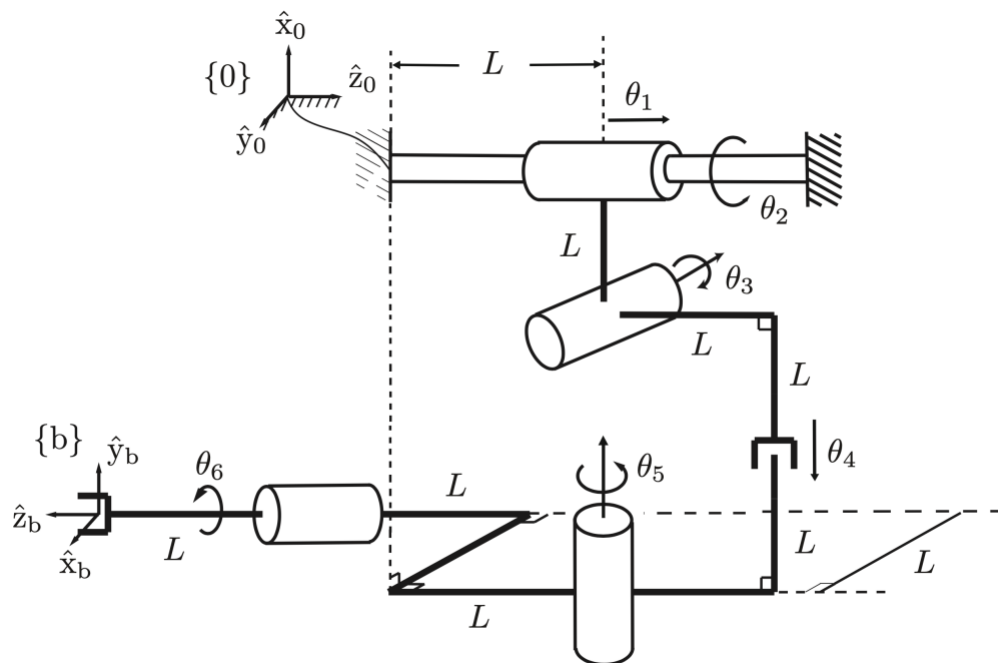




For the exam, you can use MATLAB. However, what you submit should be in the form of a PDF file. For this purpose, use the MATLAB PDF publisher. Create a folder and include the original code(s) along with the PDF file under the folder and then ZIP and submit.

Problem

The spatial PRRPRR open chain, shown in the figure (Figure 4.25 of Modern Robotics) is shown in its zero position, with space/base and end-effector frames chosen as indicated.



- 1) Derive the forward kinematics (T_{0n}) using Product of Exponentials approach. (40 Pt.)

For the following questions, consider $L = 100 \text{ mm}$:

- 2) Using the transformation matrix derived from part 1), solve the forward kinematics for the home position. In other words, calculate the position of the end-effector {b} with respect to the base frame {0} when the arm is in its home position (Remember, in the home position, all joint variables are zero!). (5 Pt.)
- 3) If we have the vector $[10, 10, 10] \text{ mm}$ in the end-effector frame {b}, calculate the same vector in the base frame {0} when all joint variables are zero (the home position). Show your work. (5 Pt.)

- 4) Derive the 6×6 geometric Jacobian of the arm. Then, calculate the Jacobian for the home position. **(10 Pt.)**
- 5) Find Singularities of the arm. If there is no singularity, show the proof. **(10 Pt.)**
- 6) Inverse Velocity: For the end-effector, in the home position, to have linear velocity of $[10, 0, 10] \text{ mm/sec}$ with respect to the base frame $\{0\}$, solve for the required joint velocities. In addition, discuss, with either mathematical proof OR intuitive reasoning, how many solutions exist for this problem? **(10 Pt.)**
- 7) Inverse Position Kinematics: For the end-effector to be in the position of $[-350, 50, -250] \text{ mm}$ with respect to the base frame $\{0\}$, solve for the required joint variables using numerical inverse kinematics approach. In addition, discuss, with either mathematical proof OR intuitive reasoning, how many solutions exist for this problem. **(20 Pt.)**

Extra Credit (20 Pt)

Derive the 6×6 analytical Jacobian of the arm considering XYZ Euler angles. Then, calculate the analytical Jacobian for the home position.

* I will hold an office hour from 10am to 11am on Sunday 10/18/2020, and will stay longer if need be. Please join the office hour if you have any questions or need clarifications in regard to the midterm exam.

Good Luck!