Kathia Coronado

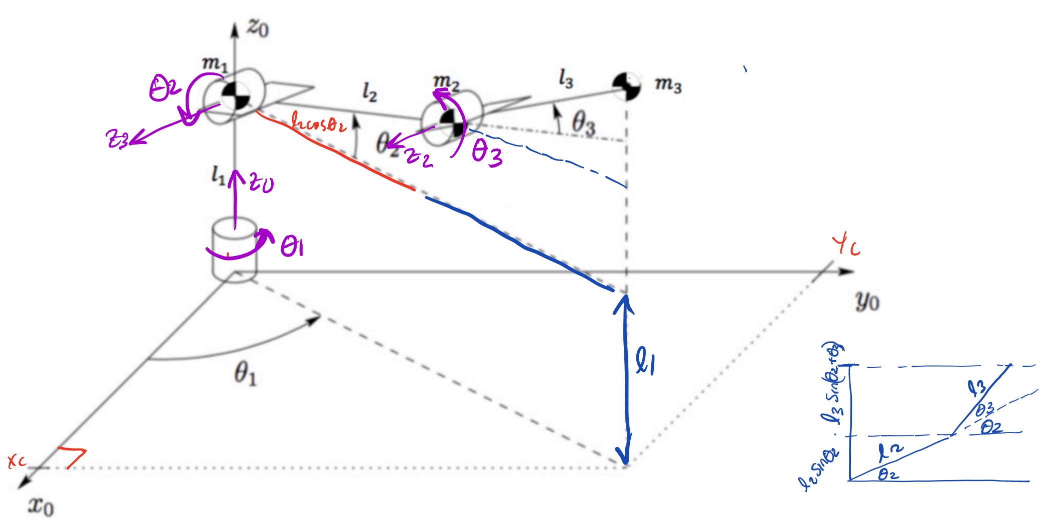
HW #3

Robot Dynamics

11/10/2020

① 3 Link arm Robot - Dynamic Modelling (Pt Masses)

3 link RRR elbow manipulator (3DoF)



ⓐ Form the dynamical model of the Robot symbolically in the compact form

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$$

Step 1 $L = K - P$ $K_i = \frac{1}{2} m v^2$ $P = mgh$

Forward Kinematics

$m_1$: $O_1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$

$m_2$: $O_2 = \begin{bmatrix} (l_2 \cos\theta_2)\cos\theta_1 \\ (l_2\cos\theta_2)\sin\theta_1 \\ l_1 + l_2\sin\theta_2 \end{bmatrix}$

$$m_3 : O_3 = \begin{bmatrix} [l_2\cos\theta_2 + l_3\cos(\theta_2+\theta_3)]\cos\theta_1 \\ [l_2\cos\theta_2 + l_3\cos(\theta_2+\theta_3)]\sin\theta_1 \\ l_1 + l_2\sin\theta_2 + l_3\sin(\theta_2+\theta_3) \end{bmatrix}$$

## velocity kinematics

$$m_1: \quad v_1 = \frac{d}{dt} O_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$m_2: \quad v_2 = \frac{d}{dt} O_2 = \begin{bmatrix} -l_2 \cos(\theta_2)\sin(\theta_1)\,\dot\theta_1 - l_2\cos(\theta_1)\sin(\theta_2)\,\dot\theta_2 \\ l_2\cos(\theta_1)\cos(\theta_2)\,\dot\theta_1 - l_2\sin(\theta_1)\sin(\theta_2)\,\dot\theta_2 \\ l_2\cos(\theta_2)\,\dot\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2\left[\dot\theta_1 s\theta_1 c\theta_2 + \dot\theta_2 c\theta_1 s\theta_2\right] \\ l_2\left[\dot\theta_1 c\theta_1 c\theta_2 + \dot\theta_2 s\theta_1 s\theta_2\right] \\ l_2 \dot\theta_2 c\theta_2 \end{bmatrix}$$

$$m_3: \quad v_3 = \frac{d}{dt} O_3 = \begin{bmatrix} -\cos(\theta_1)(l_3\sin(\theta_2+\theta_3)(\dot\theta_2+\dot\theta_3) + l_2\sin(\theta_2)\,\dot\theta_2 - \sin(\theta_1)(l_2\cos\theta_2 + l_3\cos(\theta_2+\theta_3))\,\dot\theta_1 \\ \cos(\theta_1)\,\dot\theta_1 (l_2\cos(\theta_2)) + l_3\cos(\theta_2+\theta_3) - \sin(\theta_1)(l_3\sin(\theta_2+\theta_3)(\dot\theta_2+\dot\theta_3) + l_2\sin(\theta_2)\,\dot\theta_2 \\ l_3\cos(\theta_2+\theta_3)(\dot\theta_2+\dot\theta_3) + l_2\cos(\theta_2)\,\dot\theta_2 \end{bmatrix}$$

I calculate the kinetic energy on matlab
please look at code

$K = K_1 + K_2 + K_3$

$$P = mgh \qquad \begin{aligned} P_1 &= m_1 g l_1 \\ P_2 &= m_2 g (l_1 + l_2 \sin\theta_2) \\ P_3 &= m_3 g (l_1 + l_2 \sin\theta_2 + l_3 \sin(\theta_2+\theta_3)) \end{aligned}$$

$$P = P_1 + P_2 + P_3$$
$$= g m_2 (l_1 + l_2\sin\theta_2) + g m_3 (l_1 + l_2\sin\theta_2 + l_3\sin(\theta_2+\theta_3)) + m_1 g l_1$$

$L = K - P$

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right)$$

see Matlab for $\tau$ calculations

ⓑ $l_1 = l_2 = l_3 = 0.3\,m$

$m_1 = m_2 = m_3 = 0.5\,kg$

$g = 9.8$

solve numerically for dynamics when robot is at home

$q_i = \theta_i = 0$

substituting these givens into tau gives me

$$\tau = \frac{147}{25} = \underline{3675}$$

② 3 link arm Robot – Dynamic Modeling (Lagrange Method)

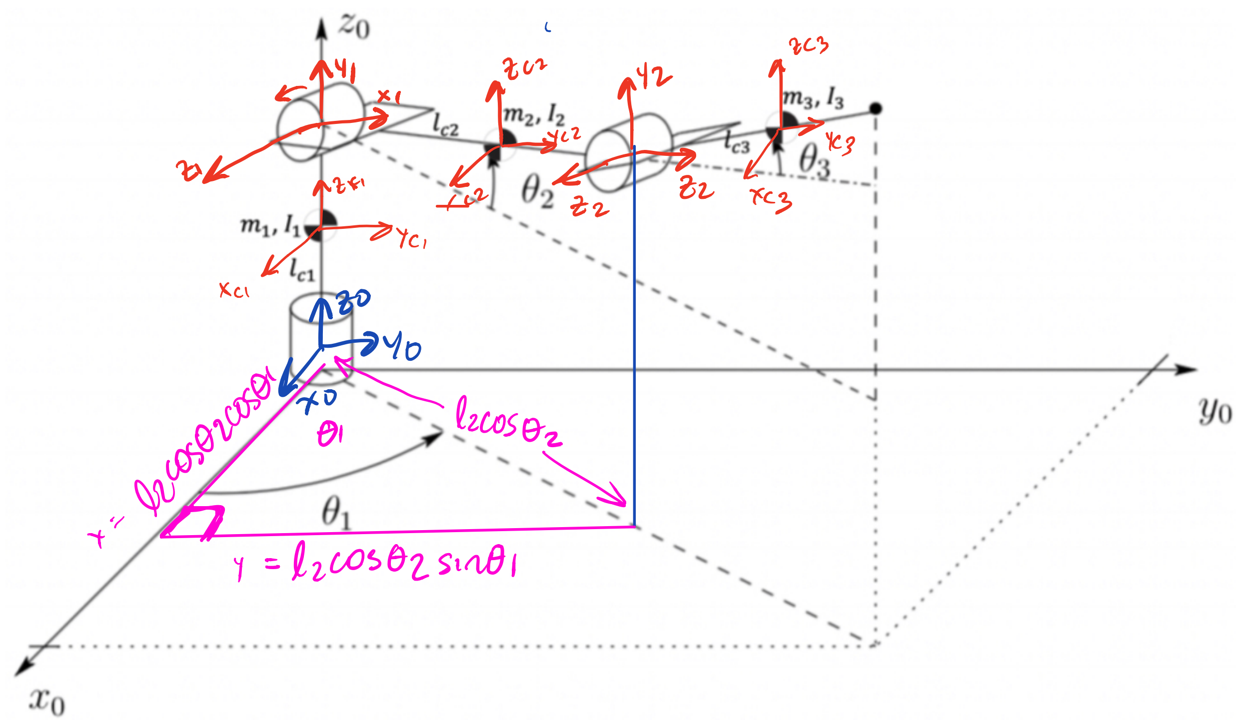
RRR robot elbow manip 3 DOF

$l_{c1}, l_{c2}$ $l_{c3}$ = distance of centers of mass of the 3 links from joint axes $l_1$ $l_2$ $l_3$.

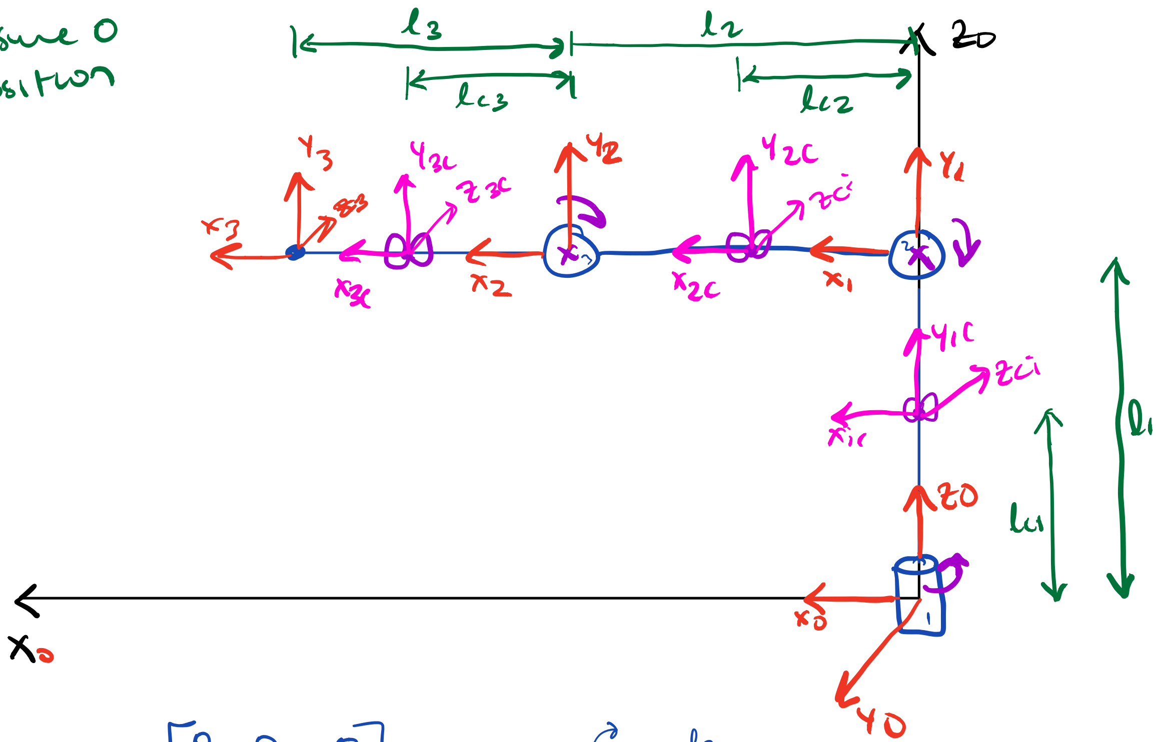
$m_1$ $m_2$ $m_3$ is the mass of 3 links

let $I_1$ $I_2$ $I_3$ be moments of inertia relative to center mass

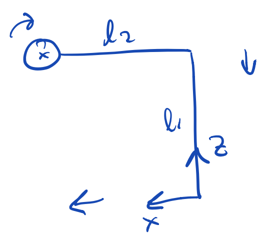
Derive the total KE and form 3x3 Inertia Matrix $D(q)$



Assume 0 position




$$Slist = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & l_1 & l_1 \\ 0 & 0 & 0 \\ 0 & 0 & -l_2 \end{bmatrix}$$




$$M_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_{02} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{03} = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_{01} = T\{1\}\, M01$

$T_{02} = T\{1\}\, T\{2\}\, M02$

$T_{03} = T\{1\}\, T\{2\}\, T\{3\}\, M03$

↑ look Matlab

$$T_{1-c1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_{c1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{2-c2} = \begin{bmatrix} 1 & 0 & 0 & l_{c2}-l_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{3c3} = \begin{bmatrix} 1 & 0 & 0 & l_{c3}-l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{0-c1} = T_{01} \times T_{1-c1}$$
$$T_{0-c2} = T_{02} \times T_{2-c2}$$
$$T_{0-c3} = T_{03} \times T_{3-c3}$$

look @ matlab for ω cal