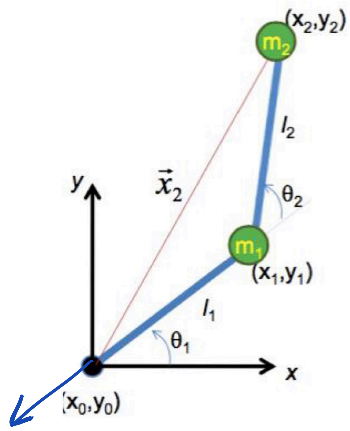
Kathia Coronado 12/8/2020

Homework 5

① For two link robot arm with

$m_1 = m_2 = 0.5$ Kg

$l_1 = l_2 = 3000$ mm



ⓐ Write the dynamical model of the system in the presence of the joints' friction and an external force applied to the tip of the robot.

$$\tau_{Ftip} = J^T F_{tip}$$

$$\tau_{friction} = B\dot{q}$$

$$\tau = M\ddot{q} + C\dot{q} + g + B\dot{q}$$

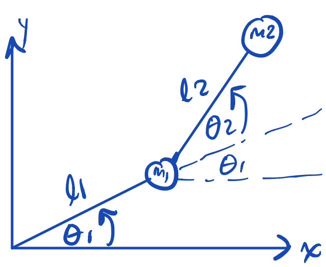
$$\tau_{fric} = B\dot{q} = B\begin{bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{bmatrix}$$

$$F = \begin{bmatrix}F_x \\ F_y\end{bmatrix}$$

Step 1 Calculate Lagrangian

$$L = K - P = (K_1 + K_2) - (P_1 - P_2)$$

$$K = \frac{1}{2}mv^2$$



$$K_1 = \frac{1}{2} m_1 \vec{v}_1^{\,2}$$

$$v_1^2 = \vec{v}_1 \cdot \vec{v}_1 = \vec{v}_1^T \vec{v}_1 \Rightarrow \vec{v}_1 = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} \Rightarrow \vec{v}_1^T = [\dot{x}_1 \ \dot{y}_1]$$

$$v_1^2 = \dot{x}_1^2 + \dot{y}_1^2$$

$$x_1 = l_1 \cos\theta_1 \Rightarrow \dot{x}_1 = -l_1 \dot{\theta}_1 \sin\theta_1$$

$$y_1 = l_1 \sin\theta_1 \Rightarrow \dot{y}_1 = l_1 \dot{\theta}_1 \cos\theta_1$$

$$v_1^2 = (-l_1\dot{\theta}_1 \sin\theta_1)^2 + (l_1\dot{\theta}_1\cos\theta_1)^2$$

$$= l_1^2\dot{\theta}_1^2 \sin^2\theta_1 + l_1^2\dot{\theta}_1^2\cos^2\theta_1 = l_1^2\dot{\theta}_1^2(\underbrace{\sin^2\theta_1 + \cos^2\theta_1}_{1})$$

$$= l_1^2\dot{\theta}_1^2$$

$$K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2$$

$$K_2 = \frac{1}{2} m_2 \vec{v}_2^{\,2}$$

$$\vec{v}_2^{\,2} = \vec{v}_2 \cdot \vec{v}_2 = \vec{v}_2^T \vec{v}_2 \Rightarrow v_2 = J\dot{q} = J\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$x_2 = l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2)$$

$$y_2 = l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2)$$

$$\dot{x}_2 = -l_1\dot{\theta}_1\sin\theta_1 - l_2(\dot{\theta}_1+\dot{\theta}_2)\sin(\theta_1+\theta_2)$$

$$\dot{y}_2 = l_1\dot{\theta}_1\cos\theta_1 + l_2(\dot{\theta}_1+\dot{\theta}_2)\cos(\theta_1+\theta_2)$$

$$\vec{v}_2 = \begin{bmatrix} \dot{x}_2 \\ \dot{y}_1 \end{bmatrix} = J_v \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -l_1\dot{\theta}_1\sin\theta_1 - l_2(\dot{\theta}_1+\dot{\theta}_2)\sin(\theta_1+\theta_2) \\ l_1\dot{\theta}_1\cos\theta_1 + l_2(\dot{\theta}_1+\dot{\theta}_2)\cos(\theta_1+\theta_2) \end{bmatrix}$$

$$\vec{v}_2^{\,2} = \dot{x}_2^2 + \dot{y}_2^2 = l_1^2\dot{\theta}^2 + l_2^2(\dot{\theta}_1+\dot{\theta}_2)^2 + 2l_1l_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)c\theta_2$$

$$= 2(l_1\dot{\theta}_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2))^2$$

$$K_2 = m_2(l_1\dot{\theta}_1\sin(\theta_1) + l_2\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2))^2$$

$$K = K_1 + K_2 = m_2(l_1\dot{\theta}_1\sin(\theta_1) + l_2\sin(\theta_1+\theta_2)(\dot{\theta}_1+\dot{\theta}_2))^2 + \frac{l_1^2 m_1\dot{\theta}_1^2}{2}$$

$$P_1 = m_1 g l_1 \sin\theta_1$$
$$P_2 = m_2 g l_1 \sin\theta_1 + m_2 g l_2 \sin(\theta_1+\theta_2)$$
$$P = P_1 + P_2 = (m_1+m_2) g l_1 \sin\theta_1 + m_2 g l_2 \sin(\theta_1+\theta_2)$$

$$L = K - P$$
$$L = \frac{1}{2}(m_1+m_2) l_1^2 \dot\theta_1^2 + \frac{1}{2} m_2 l_2^2 (\dot\theta_1+\dot\theta_2)^2 + m_1 l_1 l_2 (\dot\theta_1^2 + \dot\theta_1\dot\theta_2) c_2$$
$$-(-m_1+m_2) g l_1 s_1 - m_2 g l_2 s_{12}$$

Step2
$$\tau_1 = \frac{d}{dt}\frac{\partial L}{\partial \theta_1} - \frac{\partial L}{\theta_1}$$
$$\tau_1 = [(m_1+m_2) l_1^2 + m_2 l_2^2 + 2 m_2 l_1 l_2 \cos\theta_2]\ddot\theta_1$$
$$+[m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2]\ddot\theta + [-m_2 l_1 l_2 (2\dot\theta_1\dot\theta_2 + \dot\theta_2^2)\sin\theta_2]$$
$$+[(m_1+m_2) g l_1 \cos\theta_1 + m_2 g l_2 \cos(\theta_1+\theta_2)]$$
$$\tau_2 = \frac{d}{dt}\frac{\partial L}{\partial \theta_2} - \frac{\partial L}{\theta_2}$$
$$\tau_2 = [m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2]\ddot\theta_1 + m_2 l_2^2 \ddot\theta_2$$
$$+ m_2 l_1 l_2 \theta_1^2 \sin\theta_2 + m_2 g l_2 \cos(\theta_1+\theta_2)$$

$$\tau = M(q)\ddot q + C(q,\dot q)\dot q + G(q) + J^T F_{tip} + B\dot q$$
$$\tau = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \underbrace{\begin{bmatrix}(m_1+m_2) l_1^2 + m_2 l_2^2 + 2 m_2 l_1 l_2 \cos\theta_2 & m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2 \\ m_2 l_2^2 + m_2 l_1 l_2 \cos\theta & m_2 l_2^2\end{bmatrix}}_{M(q)} \begin{bmatrix}\ddot\theta_1\\ \ddot\theta_2\end{bmatrix}$$

$C(q,\dot q)$

$$+ \begin{bmatrix}-m_2 l_1 l_2 (2\dot\theta_1\dot\theta_2 + \dot\theta_2)\sin\theta_2 \\ m_2 l_1 l_2 \dot\theta_1^2 \sin\theta_2\end{bmatrix}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\end{bmatrix}$$

$G(q)$

$$+\begin{bmatrix}(m_1+m_2)gl_1\cos\theta_1+m_2gl_2\cos(\theta_1+\theta_2)\\ m_2gl_2\cos(\theta_1+\theta_2)\end{bmatrix}$$

$J^T\vec{F}_{tip}$

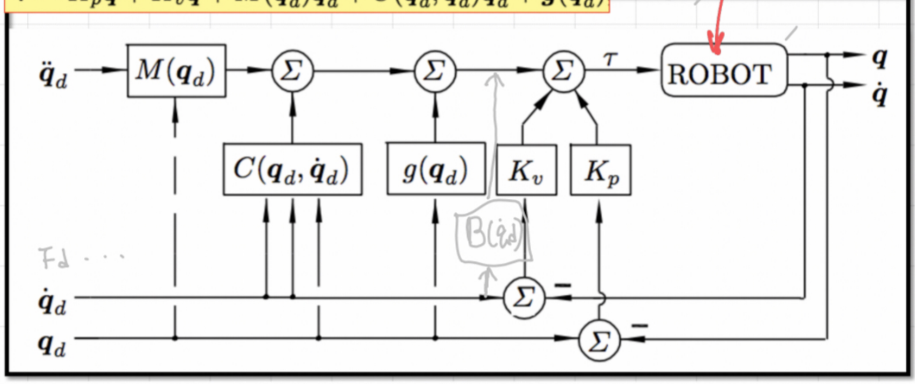
$$+\begin{bmatrix}-l_2s_{12}-l_1s_1 & l_2c_{12}+l_1c_1\\ -l_2s_{12} & l_2c_{12}\end{bmatrix}\begin{bmatrix}F_x\\ F_y\end{bmatrix}+B_{viscous}\begin{bmatrix}\dot\theta_1\\ \dot\theta_2\end{bmatrix}$$

$$\tau=M(q)\ddot q+C(q,\dot q)\dot q+G(q)+J^TF_{tip}+B_{viscous}\dot q$$

$\tilde q=e \quad \dot{\tilde q}=\dot e$ L12 P.13

$\tau=M\ddot q+C\dot q+g$

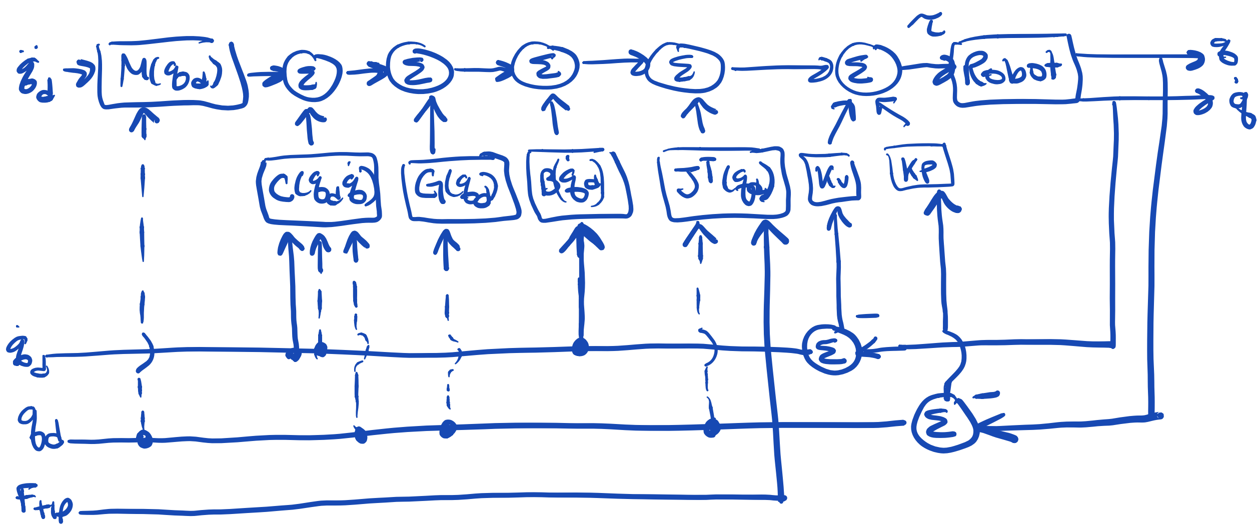
$$\tau=\overbrace{K_p\tilde{q}+K_v\dot{\tilde{q}}}^{PD}+\overbrace{M(q_d)\ddot{q}_d+C(q_d,\dot{q}_d)\dot{q}_d+g(q_d)}^{\text{feed forward}}+B(\dot q_d)$$



Motion Control

(1b) Draw Block diagram representing PD control Plus feed forward to execute motion in Problem 2 of HW2.

$$\tau = \underbrace{K_p \tilde{q} + K_v \dot{\tilde{q}}}_{PD} + \underbrace{M(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + g(q_d)}_{\text{feed forward}} + B(\dot{q}_d) + J^T F_{tip}$$



(1c) Write out corresponding controller (control law for the robot) (torque input for robot. in expanded form

$\tau = M\ddot{q} + C\dot{q} + g + B\dot{q} + J^T F_{tip}$

$u = M[\ddot{q}_d + K_v \dot{e} + K_p e] + C\dot{q} + g + B\dot{q} + J^T F_{tip}$

$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + J^T \vec{F} + B_{vis}\dot{q} + K_p \tilde{q}$
$+ K_v \dot{\tilde{q}}$

$$= \begin{bmatrix} (m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos\theta_2 & m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2 \\ m_2 l_2^2 + m_2 l_1 l_2 \cos\theta & m_2 l_2^2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} -m_2 l_1 l_2 (2\theta_1 \dot{\theta}_2 + \dot{\theta}_2^2) \sin\theta_2 \\ m_2 l_1 l_2 \dot{\theta}_1^2 \sin\theta_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$+ \begin{bmatrix} (m_1+m_2) g l_1 \cos\theta_1 + m_2 g l_2 \cos(\theta_1+\theta_2) \\ m_2 g l_2 \cos(\theta_1+\theta_2) \end{bmatrix}$$

$$+ \begin{bmatrix} -l_2 s_{12} - l_1 s_1 & l_2 c_{12} + l_1 c_1 \\ -l_2 s_{12} & l_2 c_{12} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} + B_{viscou} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

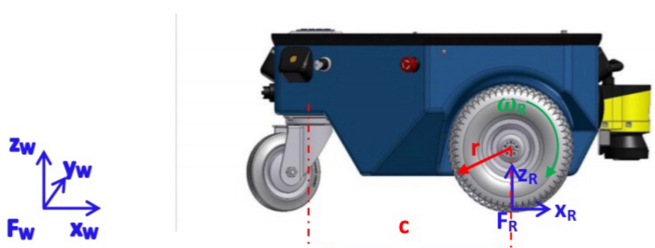
$$+ K_p \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + K_v \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

② Mobile Wheeled Robot

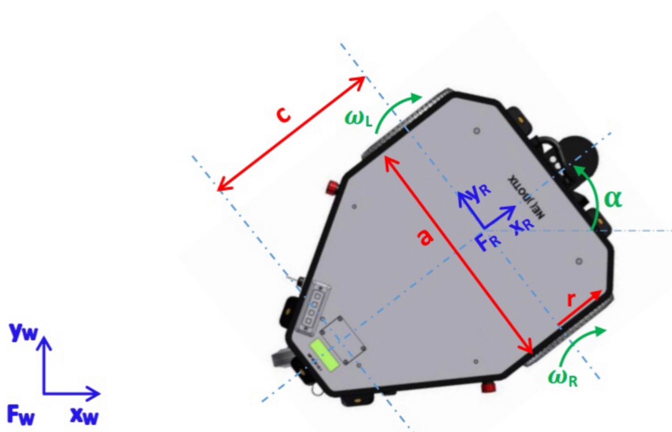
Robot has mobile base with 2 indep driven wheels with a wheel base of a. and angular velocity ω_R and ω_L both positive, Robot then drives in forward x_R direction.

Treat castor as omniwheel with distance c to axis passing thru the driven wheel centers. radius is r

Neobotix Mobile Wheeled Robot MM-500

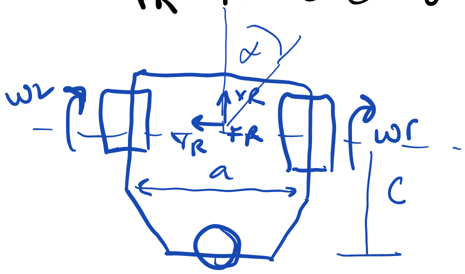


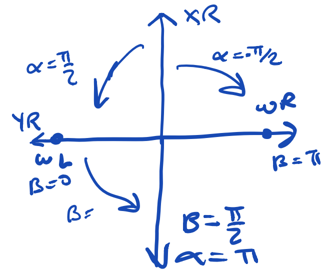
Lateral view



Top View

(a) constraint equations wrt to robot fixed frame F_R for each of 3 wheels on base (6 total)





Fixed Standard Wheel

$$\boxed{R}\quad [\sin(\alpha+\beta) \quad -\cos(\alpha+\beta) \quad -l\cos\beta] \begin{bmatrix} v_x^R \\ v_y^R \\ \omega^R \end{bmatrix} - r\dot{\phi} = 0$$

$$\boxed{S}\quad [\cos(\alpha+\beta) \quad \sin(\alpha+\beta) \quad l\sin\beta] \begin{bmatrix} v_x^R \\ v_y^R \\ \omega^R \end{bmatrix} = 0$$

Left $\alpha = \frac{-\pi}{2}$ $\beta = 0$ $l = \frac{a}{2}$

$\alpha + \beta = -\pi/2,\ \beta = 0$

$$\boxed{R}\quad \left[\sin\frac{\pi}{2} \quad -\cos\frac{\pi}{2} \quad \frac{-a}{2}\cos 0\right] \begin{bmatrix} v_x^R \\ v_y^R \\ \omega^R \end{bmatrix} - r\dot{\phi} = 0$$

$$\boxed{[1 \quad 0 \quad \tfrac{-a}{2}] \begin{bmatrix} v_x^R \\ v_y^R \\ \omega^R \end{bmatrix} - r\omega_R = 0}$$

$$\boxed{S}\quad \left[\cos\frac{\pi}{2} \quad \sin\frac{\pi}{2} \quad \frac{a}{2}\sin 0\right] \begin{bmatrix} v_x^R \\ v_y^R \\ \omega_R \end{bmatrix} = 0$$

$$\boxed{[0 \quad 1 \quad 0] \begin{bmatrix} v_{xR} \\ v_{yR} \\ \omega_R \end{bmatrix} = 0}$$

Omni wheel $\gamma = 0$ $\alpha = \pi$ $\beta = \frac{\pi}{2}$ $l = c$

$\alpha + \beta = 3\frac{\pi}{2}$ $\beta = \frac{\pi}{2}$ $\cancel{\omega_c} = 0$

$$\boxed{R}\quad \boxed{[-1 \quad 0 \quad 0] \begin{bmatrix} v_x^R \\ v_y^R \\ \omega_R \end{bmatrix} - \cancel{r\omega_c}^{\,0} = 0}$$

$$(S)\quad \boxed{\begin{bmatrix} 0 & -1 & c \end{bmatrix}\begin{bmatrix} V_x R \\ V_y R \\ \omega R \end{bmatrix} = 0}$$

<u>right</u> $\alpha = \frac{-\pi}{2}$ $\beta = \pi$ $l = \frac{a}{2}$

$$\boxed{R}\quad \begin{bmatrix} \sin\frac{\pi}{2} & -\cos\frac{\pi}{2} & -\frac{a}{2}\cos\pi \end{bmatrix}\begin{bmatrix} V_x^R \\ V_y^R \\ \omega R \end{bmatrix} - r\omega_R = 0$$

$$\boxed{\begin{bmatrix} 1 & 0 & \frac{a}{2} \end{bmatrix}\begin{bmatrix} V_x^R \\ V_y^R \\ \omega^R \end{bmatrix} - r\omega_R = 0}$$

$$\boxed{S}\quad \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} & \frac{a}{2}\sin\pi \end{bmatrix}\begin{bmatrix} V_x^R \\ V_y^R \\ \omega R \end{bmatrix} = 0$$

$$\boxed{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} V_x^R \\ V_y^R \\ \omega^R \end{bmatrix} = 0}$$

(2b) combined Position

$$J_1\begin{bmatrix} V_x R \\ V_y R \\ \omega R \end{bmatrix} - J_2\begin{bmatrix} \omega_L \\ \omega R \\ \cancel{\omega R}\ 0 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 0 & -a/2 \\ 1 & 0 & a/2 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} V_x^R \\ V_y^R \\ \omega R \end{bmatrix} - \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}\begin{bmatrix} \omega_L \\ \omega R \\ 0 \end{bmatrix} = 0$$

$$J_1 = \begin{bmatrix} 1 & 0 & -a/2 \\ 1 & 0 & a/2 \\ -1 & 0 & 0 \end{bmatrix} \qquad J_2 = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$$

$R + S$

$$\begin{bmatrix} 1 & 0 & -a/2 \\ 1 & 0 & a/2 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ \cancel{0} & \cancel{1} & \cancel{0} \\ 0 & -1 & c \end{bmatrix} \begin{bmatrix} v_x^R \\ v_y^R \\ \omega_R \end{bmatrix} = \begin{bmatrix} r\omega_L \\ r\omega_R \\ r\cancel{\omega_L} \end{bmatrix}$$

relaxtn

$$\begin{bmatrix} 1 & 0 & -a/2 \\ 1 & 0 & a/2 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & c \end{bmatrix} \begin{bmatrix} v_x^R \\ v_y^R \\ \omega_R \end{bmatrix} = \begin{bmatrix} r\omega_L \\ r\omega_R \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

remove

omniwheel

consn

$$\Rightarrow \begin{bmatrix} 1 & 0 & -a/2 \\ 1 & 0 & a/2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_x^R \\ v_y^R \\ \omega^R \end{bmatrix} = \begin{bmatrix} r\omega_L \\ r\omega_R \\ 0 \end{bmatrix}$$

$R_1^0 \quad R_0^1$

2C Mobile Kinematics

$$\begin{array}{ccc} \dot{\xi}_w & T_w^R & \dot{\xi}_R \end{array}$$

$$\begin{bmatrix} v_x^w \\ v_y^w \\ \omega^w \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x^R \\ v_y^R \\ \omega^R \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -a/2 \\ 1 & 0 & a/2 \\ 0 & 1 & 0 \end{bmatrix}^T \begin{bmatrix} r\omega_L \\ r\omega_R \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ -1/a & -1/a & 0 \end{bmatrix} \begin{bmatrix} r\omega_L \\ r\omega_R \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \\ -1/a & -1/2 & 0 \end{bmatrix} \begin{bmatrix} r\omega_L \\ r\omega_R \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cos\alpha}{2} & \frac{\cos\alpha}{2} & -\sin\alpha \\ \frac{\sin\alpha}{2} & \frac{\sin\alpha}{2} & \cos\alpha \\ -1/a & 1/a & 0 \end{bmatrix} \begin{bmatrix} r\omega_L \\ r\omega_R \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} V_x^w \\ V_y^w \\ w^w \end{bmatrix} = \begin{bmatrix} \frac{\cos\alpha}{2} & \cos\alpha/2 \\ \frac{\sin\alpha}{2} & \sin\alpha/2 \\ -1/2 & 1/a \end{bmatrix} \begin{bmatrix} r\omega_L \\ r\omega_R \end{bmatrix}}$$

2d

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & x_r \\ \sin\alpha & \cos\alpha & 0 & y_r \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ ← frame btwn world frame w/ Robot frm

$T_T^R \rightarrow FK$

$$T_T^w = T_R^w \, T_T^R = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & x_r \\ \sin\alpha & \cos\alpha & 0 & y_r \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_T^R$$

(e) <u>Combined velocity kinematics</u> solve for full velocity kinematics that reps 6 DOF velocity linear and angular in robot tip frame F_T wrt to F_W as a function of drive wheels + joint angular velocity

Tip origin

$$O_T^R = \begin{bmatrix} x_T^R \\ y_T^R \\ z_T^R \end{bmatrix} \qquad O_T^w = R_R^w O_T^R + O_R^w = \begin{pmatrix} x_r + x_T^R \cos\alpha - y_T^R \sin\alpha \\ y_r + y_T^R \cos\alpha + x_T^R \sin\alpha \\ z_T^R \end{pmatrix}$$

$$J = \begin{bmatrix} J_{base} & J_{arm} \end{bmatrix} = \begin{bmatrix} \overset{\text{mobile base}}{J_v^{base}} & \overset{\text{arm}}{J_v^{arm}} \\ J_w^{base} & J_w^{arm} \end{bmatrix} = \begin{bmatrix} J_{v1} & J_{v2} & J_{v3} \\ J_{w1} & J_{w2} & J_{w3} \end{bmatrix}$$

2wheels   3joints

$$\frac{d}{dt} O_T^w = \begin{bmatrix} v_x^w + (-\sin\alpha\, x_T^R - \cos\alpha\, y_T^R)\omega^w \\ v_y^w + (\sin\alpha\, y_T^R + \cos\alpha\, x_T^R)\omega^w \\ \cancel{\frac{d}{dt} z_T^R} \end{bmatrix}$$

$$v_x^w + (\sin\alpha\, x_T^R - \cos\alpha\, y_T^R)\omega^w = \left(\frac{\cos\alpha\, r w_L}{2} + \frac{\cos\alpha\, r w_R}{2}\right) + (\sin\alpha\, x_T^R - \cos y_T^R)\left(\frac{-r w_L}{2d} + \frac{r w_R}{2d}\right)$$

$$J_{v_1} = \begin{bmatrix} \frac{r\cos\alpha}{2} + \sin\alpha\, x_T^R \left(\frac{-r}{a}\right) - (\cos\alpha)\, y_T^R \left(\frac{-r}{a}\right) \\ \\ \end{bmatrix}$$

$$J_{v_2} = \begin{bmatrix} \frac{r\cos\alpha}{2} & -\sin\alpha\, x_T^R \left(\frac{r}{a}\right) & -\cos\alpha\, y_T^R \left(\frac{\pi}{\alpha}\right) \\ \\ \end{bmatrix}$$

$$w^w = \begin{pmatrix} 0 \\ 0 \\ -\frac{r w_L}{2} + \frac{r w_R}{a} \end{pmatrix} \qquad J_w = \begin{bmatrix} 0 \\ 0 \\ -\frac{\pi}{a} \end{bmatrix} \qquad J_{w_2} = \begin{bmatrix} 0 \\ 0 \\ \pi \\ a \end{bmatrix}$$

$$base = \begin{bmatrix} J_{v_1} & J_{v_2} \\ J_{\omega_1} & J_{\omega_2} \end{bmatrix}$$

<u>Jarm</u>

$$J_{v_3} = \frac{\partial}{\partial q} O_T^w = \frac{\partial}{\partial q}\left(R_R^w O_T^R + O_R^w\right)$$

$$= R_R^w \frac{\partial}{\partial q} O_T^R = R_R^w J_v^R \qquad J_{\omega_3} = R_R^w z_0^R$$

$$\boxed{\begin{bmatrix} V^w \\ \omega^w \end{bmatrix} = \begin{bmatrix} J_{base} & J_{arm} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \\ \dot{b} \end{bmatrix}}$$

(f)

⑨ Force Propogation

$$\tau = \underset{\uparrow \atop J_{base}}{J^T} F \qquad F = \left(f_x \ f_y \ F_z \ \ n_x \ n_y \ n_z \right)^T$$

$$\tau = \begin{bmatrix} \frac{r\cos\alpha}{2} + \sin\alpha \, x_T^R \left(\frac{-r}{a}\right) - (\cos\alpha)\, y_T^R \left(\frac{-r}{a}\right) \\ \frac{r\cos\alpha}{2} - \sin\alpha \, x_T^R \left(\frac{r}{a}\right) - \cos\alpha \, y_T^R \left(\frac{\pi}{\alpha}\right) \end{bmatrix} \begin{bmatrix} f_x \\ f_y \\ f_z \\ n_x \\ n_y \\ n_z \end{bmatrix}$$