Kathia Coronado

Robot Dynamics HW#4

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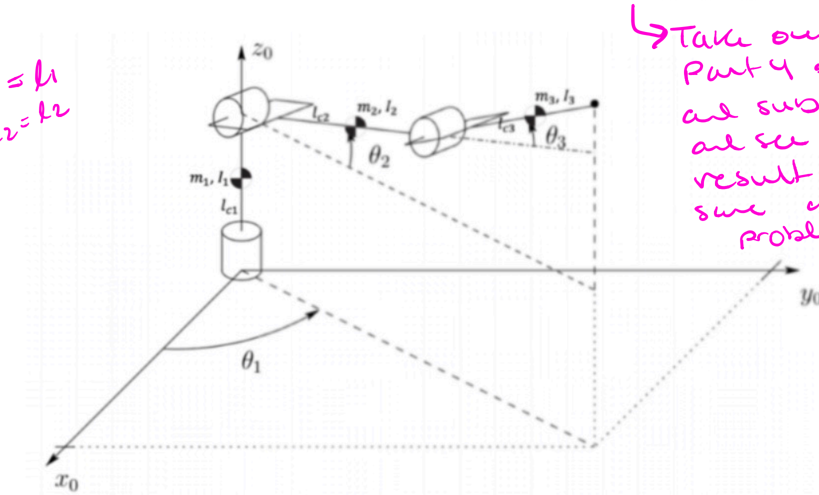
Three-Link Arm Robot – Dynamic Modeling (Lagrange's Method vs. Newton's Method)

For the 3-link RRR elbow manipulator (3-DOF) shown below, let l_{c1}, l_{c2}, and l_{c3} be the distances of the centers of mass of the three links from the respective joint axes and l_1, l_2, and l_3 be the length of the three links. Also let m_1 , m_2, and m_3 be the masses of the three links. Finally, let I_1, I_2, and I_3 be the moments of inertia *relative to the centers of mass* of the three links, respectively. For this problem,

1) Symbolically, derive the total kinetic energy of the robot and form the 3-by-3 Inertia Matrix $D(q)$. **(25 pts.)**
2) Using Christoffel Symbols, symbolically derive the 3-by-3 Coriolis/Centripetal Coupling Matrix $C(q, \dot{q})$. **(15 pts.)**
3) Symbolically, derive the total potential energy of the robot and form the 3-by-1 gravity term $g(q)$. **(5 pts.)**
4) Form the dynamical model of the robot in the compact form $\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$. **(5 pts.)**
5) Rewrite the dynamic model using the Newton's method. Show your work step by step and finally put the model in the compact form. The result must be the same as what you got in Part (4). **(35 pts.)**
6) In the derived model from either part (4) or part (5) (they must be the same), consider $l_{c1} = l_1$, $l_{c2} = l_2$, and $l_{c3} = l_3$. Also, consider $I_1 = I_2 = I_3 = 0$. Then, rewrite the dynamical model. Now, consider the robot in its home position i.e. $q_i = \theta_i = 0$ with the following physical parameters: $l_1 = l_2 = l_3 = 0.3\,m$, $m_1 = m_2 = m_3 = 0.5\,kg, g = 9.8$. Solve the dynamical model with these parameters and show that the result is the same as the result for Problem 1-b in HW3? **(15 pts.)**

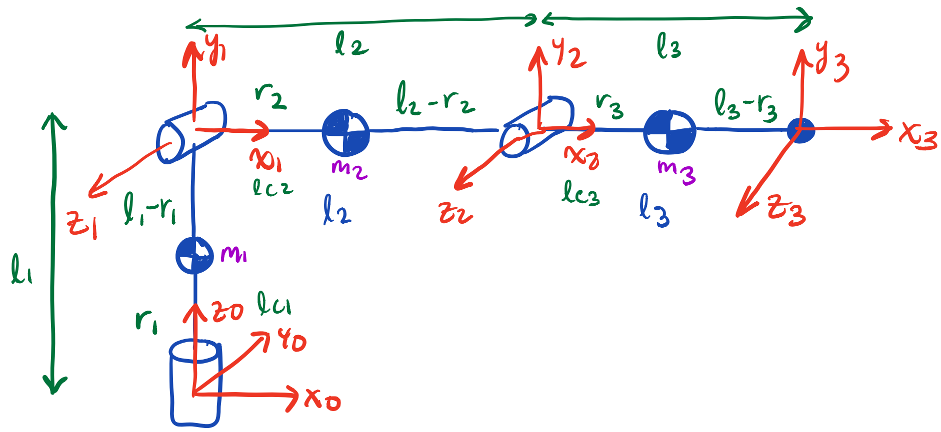
- $l_{c1} = l_1$
- $l_{c2} = l_2$



↳ Take out part from Part 4 ~~or~~ or 5 and sub vals and see what get result should be same as 1b in problem 3

① Derive the total kinetic energy of the robot and form 3x3 inertia Matrix $D(q)$.

home position



$$Slist = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & l_1 & l_1 \\ 0 & 0 & 0 \\ 0 & 0 & -l_2 \end{bmatrix}$$

$$M_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{01} = T\{1\} M_{01}$$

$$M_{02} = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{02} = T\{1\} * T\{2\} M_{02}$$

$$M_{03} = \begin{bmatrix} 1 & 0 & 0 & l_2 + l_3 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{03} = T\{1\} * T\{2\} * T\{3\} M_{03}$$

$$T_{1c1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -l_1+r_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T^0_{c1} = T^0_1 * T^1_{c1}$$

$$T_{2c2} = \begin{bmatrix} 1 & 0 & 0 & -l_2+r_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T^0_{c2} = T^0_2 T^2_{c2}$$

$$T_{3c3} = \begin{bmatrix} 1 & 0 & 0 & -l_3+r_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T^0_{c3} = T^0_3 T^3_{c2}$$

$$K_v = \frac{1}{2} \dot{q}^T \left[\sum m_i J_{v_i}^T J_{v_i} \right] \dot{q} \implies D_v = \sum m_i J_{v_i}^T J_{v_i}$$

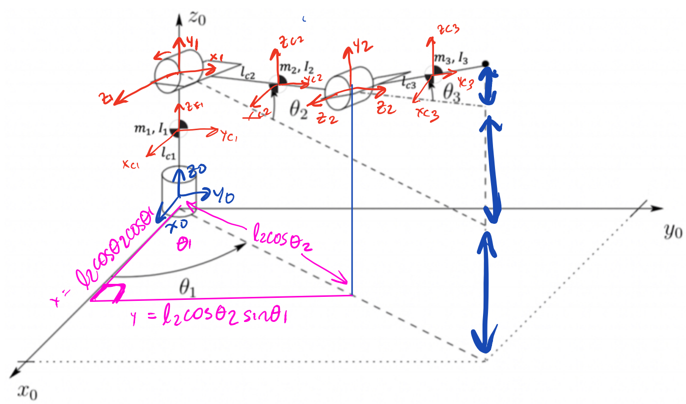
$$K_\omega = \frac{1}{2} \dot{q}^T \left[\sum J_{\omega_i}^T R_i I_i R_i^T J_{\omega_i} \right] \dot{q} \implies D_\omega = \sum J_{\omega_i}^T R_i I_i R_i^T J_{\omega_i}$$

Then solve for Jacobian at all of the masses ⇒ MatLAB

(2) Use Christoffel symbols, derive coriolis | centripetal coupling matrix $C(q, \dot{q})$

<u>Matlab</u>

(3) Derive total potential energy. gravity term $g(q)$



Direction of gravity is in the z direction

$$P_{c1} = m_1 g r_1$$

$$P_{c2} = m_2 g\,(l_1 + r_2 \sin(q_2))$$

$$P_{c3} = m_3 g\,(l_1 + r_3 \sin(q_2+q_3) + l_2 \sin(q_2))$$

$$P = P_{c1} + P_{c2} + P_{c3}$$

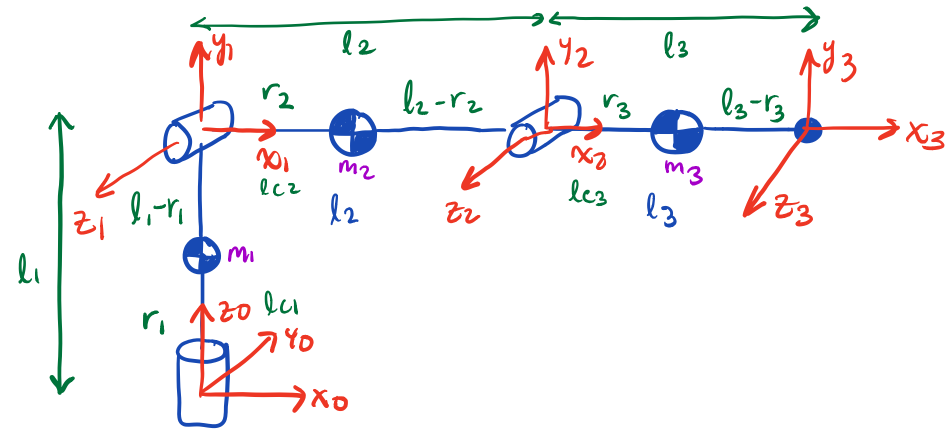
$$g_1 = \frac{\partial P}{\partial q_1} = 0 \qquad g_2 = \frac{\partial P}{\partial q_2} \qquad g_3 = \frac{\partial P}{\partial q_3}$$

$$G = \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} 0 \\ g m_3 (r_3 \cos(q_2+q_3) + l_2 \cos(q_2)) + g m_2 r_2 \cos(q_2) \\ g m_3 r_3 \cos(q_2+q_3) \end{bmatrix}$$

④ Form the dynamical model of Robot in compact form $\tau = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)$

Matlab $\Longrightarrow$

⑤ Rewrite Dynamical model using Newton's method.



$$R_{01} = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix} \quad R_{02} = \begin{bmatrix} c_1c_2 & -c_1s_2 & s_1 \\ c_2s_1 & -s_1s_2 & -c_1 \\ s_2 & c_2 & 0 \end{bmatrix} \quad R_{03} = \begin{bmatrix} c_{23}c_1 & -s_{23}c_1 & s_1 \\ c_{23}s_1 & -s_{23}s_1 & -c_1 \\ s_{23} & c_{23} & 0 \end{bmatrix}$$

$$R_{12} = R_{01}R_{02} \qquad R_{23} = R_{02}R_{03}$$

$$z_0 = k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_1 = R_{01}\hat{k} = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \qquad z_2 = R_{02}\hat{k} = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

$$b_1 = R_{01}^T z_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$b_2 = R_{02}^T z_1 = \begin{bmatrix} 0 \\ 0 \\ \cos^2 q_1 + \sin^2 q_1 \end{bmatrix} \qquad b_3 = R_{03}^T z_2 = \begin{bmatrix} 0 \\ 0 \\ \cos^2 q_1 + \sin^2 q_2 \end{bmatrix}$$

$$\omega_1 = (R_{01}^T \omega_0) + (b_1 \dot{q}_1)$$
$$\omega_2 = (R_{12}^T \omega_1) + (b_2 \dot{q}_2)$$
$$\omega_3 = (R_{23}^T \omega_2) + (b_3 \dot{q}_3)$$

$$\alpha_1 = (R_{01}^T \alpha_0) + (b_1 \dot{q}_1) + \omega_1 \times (b_1 \dot{q}_1)$$
$$\alpha_2 = (R_{12}^T \alpha_1) + (b_2 \dot{q}_2) + \omega_2 \times (b_2 \dot{q}_2)$$
$$\alpha_3 = (R_{23}^T \alpha_2) + (b_3 \dot{q}_3) + \omega_3 \times (b_3 \dot{q}_3)$$

$$r_{1c1} = l_{c1} \hat{k} = r_1 \hat{k} \qquad r_2 c_2 = l_{c2} \hat{i} = r_2 \hat{i} \qquad r_3 c_3 = l_{c3} \hat{i} = r_3 \hat{i}$$
$$r_2 c_1 = (l_{c1} - l_1)\hat{k} = (r_1 - l_1)\hat{k} \qquad r_3 c_2 = (l_{c2} - l_2)\hat{i} = (r_2 - l_2)\hat{i} \qquad r_4 c_3 = (l_{c3} - l_3)\hat{i} = (r_3 - l_3)\hat{i}$$
$$r_{12} = l_1 \hat{k} \qquad r_{23} = l_2 \hat{i} \qquad r_{34} = l_3 \hat{i}$$

$$a_{c1} = R_1^{0\,T} a_{e0} + \dot{\omega}_1 \times r_{1c1} + \omega_1 \times (\omega_1 \times r_{1c1})$$
$$a_{e1} = R_1^{0\,T} a_{e0} + \dot{\omega}_1 \times r_{12} + \omega_1 \times (\omega_1 \times r_{12})$$

$$a_{c2} = R_2^{1\,T} a_{e1} + \dot{\omega}_2 \times r_{2c2} + \omega_2 \times (\omega_2 \times r_{2c2})$$
$$a_{e2} = R_2^1 a_{e1} + \dot{\omega}_2 \times r_{23} + \omega_2 \times (\omega_2 \times r_{23})$$

$$a_{c3} = R_3^{2\,T} a_{e2} + \dot{\omega}_3 \times r_{3c3} + \omega_3 \times (\omega_3 \times r_{3c3})$$
$$a_{e3} = R_3^2 a_{e2} + \dot{\omega}_3 \times r_{34} + \omega_3 \times (\omega_3 \times r_{34})$$

$$g_n = -R_n^{0\,T} g \hat{k}$$

$$g_1 = -R_1^{0\,T} g \hat{k}$$
$$g_2 = -R_2^{0\,T} g \hat{k}$$
$$g_3 = -R_3^{0\,T} g \hat{k}$$

$$f_i = R_{i+1}^i f_{i+1} + m_i a_{ci} - m_i g_i$$
$$\tau_i = R_{i+1}^i \tau_{i+1} - f_i \times r_{ici} + (R_{i+1}^i f_{i+1}) \times r_{i+1,ci} + I_i \alpha_i + \omega_i \times (I_i \omega_i)$$

$$f_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \tau_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_{34} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$f_3 = R^3_4 f_4 + m_3 a_{c3} - m_3 g_3$$
$$\tau_3 = R^3_4 \tau_4 - f_3 \times r_{3C3} + (R^3_4 f_4) \times r_{4c3} + I_3 \alpha_3 + \omega_3 \times (I_3 \omega_3)$$
$$f_2 = R^1_2 f_3 + m_2 a_{c2} - m_2 g_2$$
$$\tau_2 = R^1_2 \tau_3 - f_2 \times r_{2c2} + (R^1_2 f_3) \times r_{3c2} + I_2 \alpha_2 + \omega_2 \times (I_2 \omega_2)$$
$$f_1 = R^0_1 f_2 + m_1 a_{c1} - m_1 g_1$$
$$\tau_1 = R^0_1 \tau_2 - f_1 \times r_{1c1} + (R^0_1 f_2) \times r_{2c1} + I_1 \alpha_1 + \omega_1 \times (I_1 \omega_1)$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \qquad (\tau_{lagm} - \tau_{new}) = 0 \ ✓$$

⑥ Substitute vals to τ

$l_{c1} = l_1 = l_{c2} = l_2 = l_{c3} = l_3 = 0.3$

$m_1 = m_2 = m_3 = 0.5$

$g = 9.8$

$I_1 = I_2 = I_3 = 0$

matlab $\quad \tau = \begin{bmatrix} 0 \\ 4.41 \\ 1.47 \end{bmatrix}$