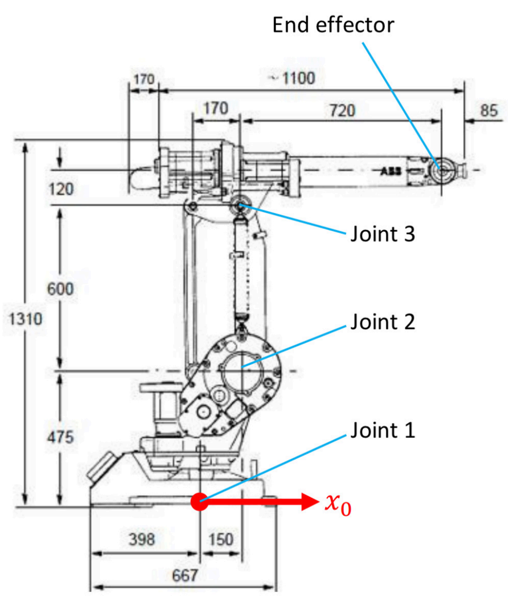
Kathia Coronado

Robot Dynamics

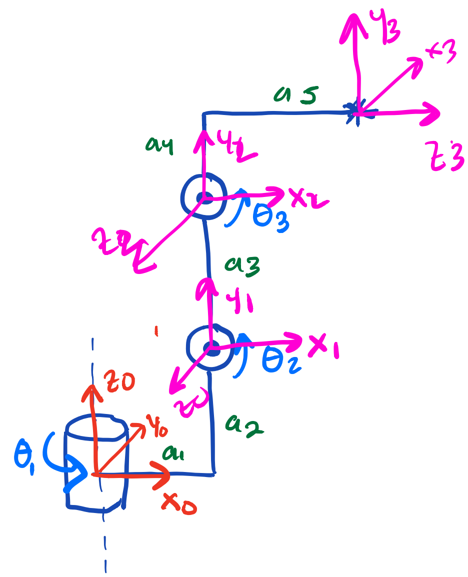
HW #2

10/13/2020

① Consider robot as 3 DOF. Disregard 85 mm dim



The robot in its home position



$a_1 = 150$

$a_2 = 475$

$a_3 = 600$

$a_4 = 120$

$a_5 = 720$

ⓐ Develope Jacobian $n=3$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \underset{6\times 3}{\begin{bmatrix} R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_1^0) & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_2^0) \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}}$$

$$R_0^0 = R_{00} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad d_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

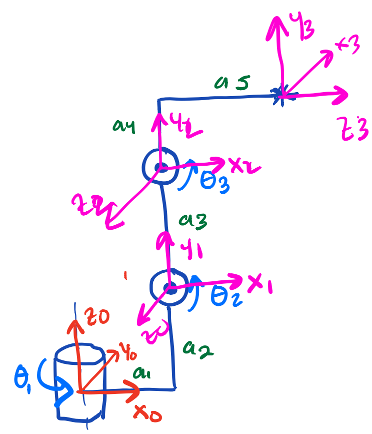
$M =$

$$R_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = R_2^0 \qquad d_1^0 = \begin{bmatrix} a_1 \\ 0 \\ a_2 \end{bmatrix} = \begin{bmatrix} 150 \\ 0 \\ 475 \end{bmatrix} \qquad d_2^0 = \begin{bmatrix} a_1 \\ 0 \\ a_2+a_3 \end{bmatrix} = \begin{bmatrix} 150 \\ 0 \\ 1075 \end{bmatrix}$$

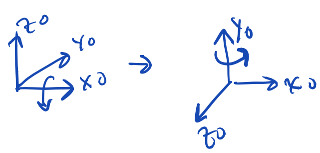
$$R_3^0 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad d_3^0 = \begin{bmatrix} a_1+a_5 \\ 0 \\ a_2+a_3+a_4 \end{bmatrix} = \begin{bmatrix} 870 \\ 0 \\ 1195 \end{bmatrix}$$

$$J = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 870 \\ 0 \\ 1195 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 720 \\ 0 \\ 720 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 720 \\ 0 \\ 120 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \end{bmatrix} = \boxed{J = \begin{bmatrix} 0 & -720 & -120 \\ 870 & 0 & 0 \\ 0 & 720 & 720 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix}}$$

$$M = \begin{bmatrix} 0 & 0 & 1 & a_1+a_5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & a_2+a_3+a_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 870 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1195 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



i	ω	v
1	(0,0,1)	(0,0,0)
2	(0,-1,0)	$(a_2, 0, -a_1)$
3	(0,-1,0)	$(a_2+a_3, 0, -a_1)$.



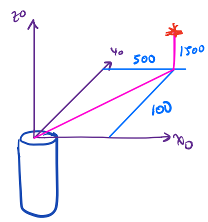
Ⓑ Calculate joint velocities of robot leading to ee <u>linear velocity</u> (J_v) of $\dot{\vec{x}} = [5\ 5\ 10]^T$ mm/sec.

Consider ee of the robot in position $[500\ 100\ 1500]^T$ mm

$\dot{\vec{x}} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} 500 \\ 100 \\ 1500 \end{bmatrix}$ we want to calc $\dot{q}$

$$\dot{P} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{6\times1} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}_{6\times n} \dot{q}_{n\times1}$$

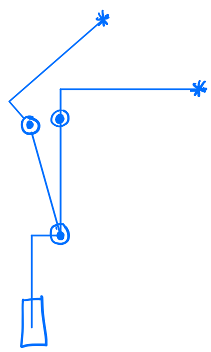
$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = J_v \dot{q} \qquad \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J_\omega \dot{q}$$

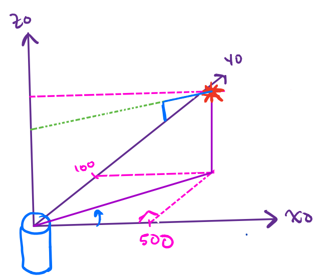


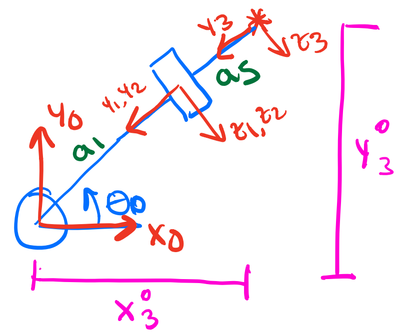
$\dot{q} = J_v^{-1} \dot{P}$

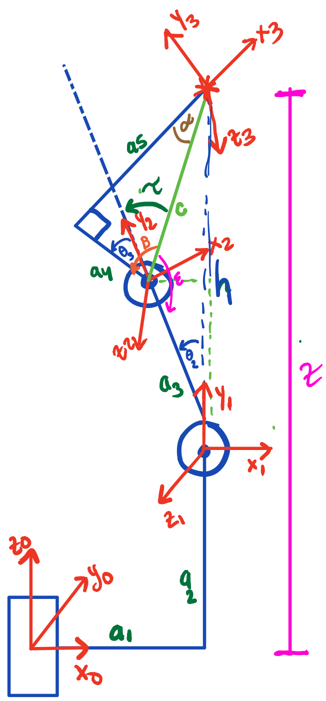
$\dot{P} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}^T$

J_v is a function of joint variables and configuration need to calculate J. Need inverse kinematics









$h = z - a_2 = 1025$ $\text{atan2}\left(\frac{y_3^0}{x_3^0}\right) = 0.1974$

$C = \sqrt{a_5^2 + a_4^2} = 729.9315$

$\beta = \cos^{-1}\left(\frac{a_4^2 + c^2 - a_5^2}{2(a_4 c)}\right) = 1.4056$

$\alpha = \pi - \beta = 178.5944$

$\varepsilon = \cos^{-1}\left(\frac{c^2 + a_3^2 - h^2}{2(c\, a_3)}\right) = 1.7520$

$\tau = 180 - \varepsilon = 1.3896$

$\theta_3 = \beta - \tau = 0.0160$

$\theta_2 = \cos^{-1}\left(\frac{a_3^2 + h^2 - c^2}{2(a_3 h)}\right) = 0.7761$

$$\theta = \begin{bmatrix} 0.1974 \\ 0.7761 \\ 0.0161 \end{bmatrix} \qquad P_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad P_e = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 500 \\ 100 \\ 1500 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} a_1 \\ 0 \\ a_2 \end{bmatrix} \qquad z_1 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} a_1 - a_3 \sin\theta_2 \\ 0 \\ a_2 + a_3 \cos\theta_3 \end{bmatrix} = 1000 * \begin{bmatrix} -0.2703 \\ 0 \\ 1.0749 \end{bmatrix} = \begin{bmatrix} -270.3 \\ 0 \\ 1074.9 \end{bmatrix} \qquad z_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$J_v = \hat{z}_{i-1} \times (P_e - P_{i-1})$$

$$J_v = \begin{bmatrix} \hat{z}_0 \times (P_e - P_0) & z_1 \times (P_e - P_1) & z_2 \times (P_e - P_0) \end{bmatrix}$$

$$= \left[\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} 500 \\ 100 \\ 1500 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \left(\begin{bmatrix} 500 \\ 100 \\ 1500 \end{bmatrix} - \begin{bmatrix} 150 \\ 0 \\ 475 \end{bmatrix} \right) \quad \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \times \left(\begin{bmatrix} 500 \\ 100 \\ 1500 \end{bmatrix} - \begin{bmatrix} -270.3 \\ 0 \\ 1074.9 \end{bmatrix} \right) \right]$$

$$= \begin{bmatrix} -100 & -1025.0 & -425.1 \\ 500 & 0 & 0 \\ 0 & 350 & 770.3 \end{bmatrix}$$

$$\dot{q} = J_v^+ \dot{x}$$

$$J_v^+ = \text{pinv}(J_v) = \begin{bmatrix} 0 & 0.0020 & 0 \\ -0.0012 & -0.0002 & -0.0007 \\ 0.0005 & 0.0001 & 0.0016 \end{bmatrix}$$

matlab

$$\boxed{\dot{q} = J_v^+ \dot{x} = \begin{bmatrix} 0.0100 \\ -0.0138 \\ 0.0193 \end{bmatrix}}$$

1c) Derive and discuss singularities of the robot

singularity: $\det(J_v) = 0$

rank of $J_v < 3$

$$J_v = \begin{bmatrix} -100 & -1025 & -425.1 \\ 500 & 0 & 0 \\ 0 & 350 & 770.3 \end{bmatrix}$$

Solve for when $\det(J_v)=0$

```
det_Jv = det(Jv
sin_eqn = det_Jv == 0
[sin_th1, sin_th2, sin_th3]
    = solve(sin_eqn, [th1, th2, th3])
```

Matlab Code $\longrightarrow$

I solved for where singularities occured. singularity will occur when the joint position is $[0, 0, -2.3622]^T$

To prove this is a singularity I took the determinant of J when joint position is $[0, 0, -2.3622]^T$

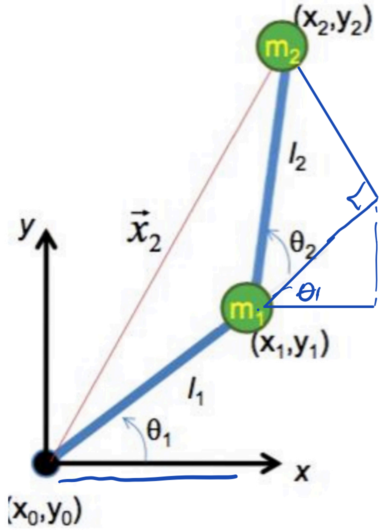
I found that $\det(J_v) = 0$

$\therefore$ It is a singularity

1D Solve inverse kinematics using numerical approach for end effector

Matlab code $\longrightarrow$

② 2 link arm robot $l_1 = l_2 = 300m$



ⓐ Develope a trajectory.

$t_0 = 0$ sec $\quad t_f = 5$ sec

$\dot{q}_0 = \dot{\vec{x}}_0 = 0 \quad \dot{q}_F = \dot{\vec{x}}_f = 0$

$q_0 = \vec{x}_{20} = (300, 450)$ mm $\quad q_f = \vec{x}_{2f} = (-300, 450)$

(elbow Down) $\quad$ elbow up

Plot PvT, VvT, AvT

ee resultant P, V, A

I am given $q_0, \dot{q}, q_f, \dot{q}_F$ ∴ cubi

$$x_1 = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) = 300$$
$$y_1 = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) = 500$$
$$x_2 = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) = -300$$
$$y_2 = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) = 450$$

→ matlab

get two solutions

<u>Solution 1</u>

$$\theta_0 = q_0 = \begin{bmatrix} 1.430626 \\ -0.895665 \end{bmatrix}$$

$$\theta_f = q_f = \begin{bmatrix} 2.606631 \\ -0.895665 \end{bmatrix}$$

<u>Solution 2</u>

$$\theta_0 = q_0 = \begin{bmatrix} 0.534961 \\ 0.895665 \end{bmatrix}$$

$$\theta_f = q_f = \begin{bmatrix} 1.710967 \\ 0.895665 \end{bmatrix}$$

<u>Cubic polynomials</u>

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$
$$q_0(t) = q(t_0) = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 = \begin{bmatrix} 300 \\ 450 \end{bmatrix}$$
$$q_f(t) = q(t_f) = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 = \begin{bmatrix} -300 \\ 450 \end{bmatrix}$$
$$\dot{q}_0(t) = \dot{q}(t_0) = a_1 + 2a_2 t_0 + 3a_3 t_0^2 = 0$$

$$\dot{q}_f(t) = \dot{q}(t_f) = a_1 + 2a_2 t_f + 3a_3 t_f^2 = 0$$
$$\ddot{q}_0(t) = \ddot{q}(t_0) = 2a_2 + 6a_3 t_0$$
$$\ddot{q}_f(t) = \ddot{q}(t_0) = 2a_2 + 6a_3 t_f$$

<u>Solution 1</u>

@ <u>$t=0$</u> $\quad \theta_0 = q_0 = \begin{bmatrix} 1.430626 \\ -0.895665 \end{bmatrix}, \dot{q} = 0$

@ $t=5$ $\quad \theta_f = q_f = \begin{bmatrix} 2.606631 \\ -0.895665 \end{bmatrix}, \dot{q} = 0$

$$q_0 = a_0 + \cancel{a_1 t_0} + \cancel{a_2 t_0^2} + \cancel{a_3 t_0^3} = \begin{bmatrix} 1.430626 \\ -0.895665 \end{bmatrix}$$
$$\Rightarrow \boxed{a_0 = \begin{bmatrix} 1.430626 \\ -0.895665 \end{bmatrix}}$$

$$\dot{q}_0 = a_1 + \cancel{2a_2 t_0} + \cancel{3a_3 t_0^2} = 0$$
$$\Rightarrow \boxed{a_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

$$q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$$
$$\begin{bmatrix} 2.606631 \\ -0.895665 \end{bmatrix} = \begin{bmatrix} 1.430626 \\ -0.895665 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}(5) + a_2(5)^2 + a_3(5)^3$$

$$\dot{q}_F = \dot{q}(t_f) = a_1 + 2a_2 t_f + 3a_3 t_f^2$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + 2a_2(5) + 3a_3(5^2)$$

2 eqn 2 unknown

$$2.606631 = 1.430626 + 0(5) + a_{2_x}(5)^2 + a_{3_x}(5)^3$$
$$-0.895665 = -0.895665 + 0(5) + a_{2_y}(5)^2 + a_{3_y}(5)^3$$

$$0 = 0 + 2a_{2_x}(5) + 3a_{3_x}(5^2)$$
$$0 = 0 + 2a_{2_y}(5) + 3a_{3_y}(5^2)$$

$$\boxed{a_2 = \begin{bmatrix} 0.1410 \\ -0.2150 \end{bmatrix}}$$
$$\boxed{a_3 = \begin{bmatrix} -0.0188 \\ 0.0287 \end{bmatrix}}$$

run code for graph

Look @ matlab

(2b)

<u>Matlab code</u>

run code for graph.

Contents

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- 2a part 2
- Problem 2b


```
% Kathia coronado : Robot Dynamics : 10/13/2020
% HW  2
```

PROBLEM 1a : develope jacobian matrixx

```
%%{
th1=pi/2;
th2=pi/2;
th3=0;
R00= [1 0 0; 0 1 0; 0 0 1];
R00v= R00*[0;0;1];
d00= [0;0;0];
R01= [1 0 0;0 0 -1;0 1 0];
R01v= R01*[0;0;1];
R02v=R01v;
d01= [a1;0;a2];
d02=[a1;0;a2+a3];
d03=[a1+a5;0; a2+a3+a4];
J= [cross(R00v,(d03-d00)) cross(R01v,(d03-d01)) cross(R02v,(d03-d02)); R00v R01v R02v];
%}
%{
%using POE
syms th1 th2 th3
a1=150;a2=475;a3=600;a4=120;a5=720;
%M= [0 0 1 a1+a5;1 0 0 0; 0 1 0 a2+a3+Ba4;0 0 0 1];
M= [1 0 0 a1+a5;0 1 0 0; 0 0 1 a2+a3+a4;0 0 0 1];
Slist= [[0;0;1;0;0;0],[0;-1;0;a2;0;-a1],[0;-1;0;a2+a3;0;-a2]];
thetalist= [th1, th2,th3];
I = eye(3);
T ={};
for i = 1 : length(thetalist)
 %w = vector_2_skew(Slist(1:3,i));
 w= VecToso3(Slist(1:3,i));
 v = Slist(4:6,i);
 theta = thetalist(i);
 R = I + sin(theta)* w + (1-cos(theta))*w^2;
 star = (I * theta+(1-cos(theta))*w+(theta-sin(theta))*w^2)*v;
 T{i} = [R star; 0 0 0 1];
end
T= T{1}*T{2}*T{3}*M;
P=T(1:4,4);
Jv=jacobian(P,[th1, th2, th3]);

J=subs(Jv,[th1,th2,th3],[0, 0,0])

%}
```

```
Unrecognized function or variable 'a1'.

Error in HW2 (line 14)
d01= [a1;0;a2];
```

Problem 1b: joint velocities

```
%geometric
%%{
a1=150;a2=475;a3=600;a4=120;a5=720;
x= transpose([500 100 1500]);
x3= x(1);
y3= x(2);
z3= x(3);
xdot=transpose([5 5 10]);
h= z3-a2;
c = sqrt(a5^2+a4^2);
B= acos((a4^2+c^2-a5^2)/(2*a4*c));
alpha=180-B;
ep= acos((c^2+a3^2-h^2)/(2*c*a3));
tao= pi- ep;
the3=B-tao;
the2=acos((a3^2 + h^2 -c^2)/(2*a3*h));
the1= atan2(y3,x3);
the1, the2, the3
P0= transpose([0 0 0]);
P1= transpose([a1 0 a2]);
P2= transpose([a1-a3*sin(the2) 0 a2+a3*cos(the3)]);
Pe= x;
z0= transpose([0 0 1]);
z1= transpose([0 -1 0]);
z2= transpose([0 -1 0]);
Jv= [cross(z0,(Pe-P0)) cross(z1,(Pe-P1)) cross(z2,(Pe-P2))];
Jv_inv= pinv(Jv);
qdot= pinv(Jv)*xdot
%}
```

Problem 1c: proof of singularities

```
%syms a1 a2 a3 a4 a5 th1 th2 th2
syms  th1 th2 th3
a1=150;a2=475;a3=600;a4=120;a5=720;
M= [1 0 0 a1+a5;0 1 0 0; 0 0 1 a2+a3+a4;0 0 0 1];
Slist= [[0;0;1;0;0;0],[0;-1;0;a2;0;-a1],[0;-1;0;a2+a3;0;-a2]];
thetalist= [th1, th2,th3];

I= eye(3);
T={};
for i = 1 : length(thetalist)
 %w = vector_2_skew(Slist(1:3,i));
 w= VecToso3(Slist(1:3,i));
 v = Slist(4:6,i);
 theta = thetalist(i);
 R = I + sin(theta)* w + (1-cos(theta))*w^2;
 star = (I * theta+(1-cos(theta))*w+(theta-sin(theta))*w^2)*v;
 T{i} = [R star; 0 0 0 1];
end
T= T{1}*T{2}*T{3}*M;
p=T(1:4,4);
```

```
Jv=jacobian(p,[th1, th2, th3]);
Jv= Jv(1:3,1:3)

%J=subs(Jv,[th1,th2,th3],[0, 0,0])
det_Jv= det(Jv)
sin_eqn = det_Jv == 0;
[sin_th1, sin_th2, sin_th3] = solve(sin_eqn, [th1, th2, th3]);
singularity = double ((- (log(5078665)*1i)/2) -double(log(- 1603/5078665 - 1584i/5078665)*1i))

J=subs(Jv,[th1,th2,th3],[0, 0,singularity])
det(J)
det_j = -6.6899e+07+6.6899e+07+-5.1450e+07+3.9571e+07+1.1879e+07 % this is the result and it is equal to zero
disp ( 'joint position [0,0,-2.3622] is a singularity because the determinant is zero and the Jv is full rank')
```

Problem 1d : solve inverse kinematics using numerial approach

```
syms  th1 th2 th3
a1=150;a2=475;a3=600;a4=120;a5=720;
M= [1 0 0 a1+a5;0 1 0 0; 0 0 1 a2+a3+a4;0 0 0 1];
Slist= [[0;0;1;0;0;0],[0;-1;0;a2;0;-a1],[0;-1;0;a2+a3;0;-a2]];
thetalist= [th1, th2,th3];

I= eye(3);
T={};
for i = 1 : length(thetalist)
 %w = vector_2_skew(Slist(1:3,i));
 w= VecToso3(Slist(1:3,i));
 v = Slist(4:6,i);
 theta = thetalist(i);
 R = I + sin(theta)* w + (1-cos(theta))*w^2;
 star = (I * theta+(1-cos(theta))*w+(theta-sin(theta))*w^2)*v;
 T{i} = [R star; 0 0 0 1];
end
T= T{1}*T{2}*T{3}*M;
p=T(1:4,4)
Jv=jacobian(p,[th1, th2, th3]);
Jv= Jv(1:3,1:3);

J=subs(Jv,[th1,th2,th3],[0, 0,0])
det_Jv= det(Jv);
sin_eqn = det_Jv == 0;
[sin_th1, sin_th2, sin_th3] = solve(sin_eqn, [th1, th2, th3]);
singularity = double (- (log(5078665)*1i)/2) -double(log(- 1603/5078665 - 1584i/5078665)*1i);

%%{
q0 = [0; 0; 0];
pd = [500;100;1500];
qq = q0;
num_Jv= J(1:3,:)
%num_Jv = subs(Jv,[th1,th2,th3],qq);
Jv_inv= pinv(num_Jv)
disp(num_Jv)
pi = subs(p, [th1 th2 th3], qq');
pi= pi(1:3);
%error = double(abs(norm(pd - pi)))
error = double(vpa(abs(norm(pd - pi))));

%%{
while error > 1
 %num_Jv = subs(Jv,[th1,th2,th3],qq);
 Jv_inv= pinv(num_Jv)
```

```
 delta_q = Jv_inv * (pd - pi(1:3));
 disp AA
 qq = qq + delta_q;
 disp qq
 disp BB
 pi = subs(p, [th1 th2 th3], qq')

 disp CC
 %double(pi)
 %vpa(pi)
 disp DD
 %error = double(abs(norm(pd - pi)))
 double(pi)
 error = double((abs(norm(pd - pi(1:3)))))
 disp EE
end
%}
double (qq)
```

Problem 2a : solve inverse kinematics using numerial approach

```
%%{

l1 = 300;% link lengths
l2 = 300;

syms q10 q20 q1f q2f
eq1q = l1*cos(q10) + l2*cos(q10+q20) ==300; % x of x0
eq2q = l1*sin(q10) + l2*sin(q10+q20) ==450; % y of x0
[q1_0, q2_0]= solve([eq1q eq2q],[q10 q20]); % solve for theta 0

eq3q = l1*cos(q1f) + l2*cos(q1f+q2f) ==-300; % x of xf
eq4q = l1*sin(q1f) + l2*sin(q1f+q2f) ==450 % y of xf
[q1_f, q2_f] = solve([eq3q eq4q],[q1f q2f]); % solve for theta f

q1_0 = real (double(q1_0));
q2_0 = real (double(q2_0));
q1_f = real (double(q1_f));
q2_f = real (double(q2_f));
% display values
fprintf('q1_0 is: %f \n', real(double(q1_0)));
fprintf('q2_0 is: %f \n', real(double(q2_0)));
fprintf('q1_f is: %f \n', real(double(q1_f)));
fprintf('q2_f is: %f \n', real(double(q2_f)));
%SOLUTION 1
a0_x = 1.4306;  a0_y = -0.8957;
a1_x = 0;       a1_y = 0;
a2_x = 0.1410;  a2_y = -0.2150;
a3_x = -0.0188; a3_y = 0.0287;

syms t
eq1_ = a0_x + a1_x * t + a2_x * (t^2) + a3_x * (t^3);
eq2_ = a0_y + a1_y * t + a2_y * (t^2) + a3_y * (t^3);

eq1_d = a1_x + 2 * a2_x * t + 3 * a3_x * (t^2);
eq2_d = a1_y + 2 * a2_y * t + 3 * a3_y * (t^2);

eq1_dd = 2 *a2_x + 6*a3_x*t;
eq2_dd = 2 *a2_y + 6*a3_y*t;

t_t = 0:0.1:5;
```

```
figure
q1__ = subs(eq1_, t, t_t);
plot(t_t, q1__)
title('Jointx: Time-Position Solution 1')
xlabel('Time')
ylabel('Position')

figure
q1_d__ = subs(eq1_d, t, t_t);
plot(t_t, q1_d__)
title('Jointx: Time-Velocity Solution 1')
xlabel('Time')
ylabel('Velocity')

figure
q1_dd__ = subs(eq1_dd, t, t_t);
plot(t_t, q1_dd__)
title('Jointx: Time-Acceleration Solution 1')
xlabel('Time')
ylabel('Acceleration')

figure
q2__ = subs(eq2_, t, t_t);
plot(t_t, q2__)
title('Jointy: Time-Position Solution 1')
xlabel('Time')
ylabel('Position')

figure
q2_d__ = subs(eq2_d, t, t_t);
plot(t_t, q2_d__)
title('Jointy: Time-Velocity Solution 1')
xlabel('Time')
ylabel('Velocity')

figure
q2_dd__ = subs(eq2_dd, t, t_t);
plot(t_t, q2_dd__)
title('Jointy: Time-Acceleration Solution 1')
xlabel('Time')
ylabel('Acceleration')

%SOLUTION 2
a0_x = 0.5350;a0_y = 0.8957;
a1_x = 0;a1_y = 0;
a2_x = 0.0025;a2_y = 0;
a3_x = -0.0188;a3_y = 0;

syms t
eq1_ = a0_x + a1_x * t + a2_x * (t^2) + a3_x * (t^3);
eq2_ = a0_y + a1_y * t + a2_y * (t^2) + a3_y * (t^3);

eq1_d = a1_x + 2 * a2_x * t + 3 * a3_x * (t^2);
eq2_d = a1_y + 2 * a2_y * t + 3 * a3_y * (t^2);

eq1_dd = 2 * a2_x + 6 * a3_x * t;
eq2_dd = 2 * a2_y + 6 * a3_y * t;

t_t = 0:0.1:5;
figure
q1__ = subs(eq1_, t, t_t);
plot(t_t, q1__)
title('Jointx: Time-Position Solution 2')
```

```
xlabel('Time')
ylabel('Position')

figure
q1_d__ = subs(eq1_d, t, t_t);
plot(t_t, q1_d__)
title('Jointx: Time-Velocity Solution 2')
xlabel('Time')
ylabel('Velocity')

figure
q1_dot_dot_set_c = subs(eq1_dd, t, t_t);
plot(t_t, q1_dot_dot_set_c)
title('Jointx: Time-Acceleration Solution 2')
xlabel('Time')
ylabel('Acceleration')

figure
q2__ = subs(eq2_, t, t_t);
plot(t_t, q2__)
title('Jointy: Time-Position Solution 2')
xlabel('Time')
ylabel('Position')

figure
q2_d__ = subs(eq2_d, t, t_t);
plot(t_t, q2_d__)
title('Jointy: Time-Velocity Solution 2')
xlabel('Time')
ylabel('Velocity')

figure
q2_dd__ = subs(eq2_dd, t, t_t);
plot(t_t, q2_dd__)
title('Jointy: Time-Acceleration Solution 2')
xlabel('Time')
ylabel('Acceleration')
```

2a part 2

```
a0_x = 1.4306;a0_y = -0.8957;
a1_x = 0;a1_y = 0;
a2_x = 0.1410;a2_y = -0.2150;
a3_x = -0.0188;a3_y = 0.0287;

syms t
eq1 = a0_x + a1_x * t + a2_x * (t^2) + a3_x * (t^3);
eq2 = a0_y + a1_y * t + a2_y * (t^2) + a3_y * (t^3);

eq1_d = a1_x + 2 * a2_x * t + 3 * a3_x * (t^2);
eq2_d = a1_y + 2 * a2_y * t + 3 * a3_y * (t^2);

eq1_dd = 2 * a2_x + 6 * a3_x * t;
eq2_dd = 2 * a2_y + 6 * a3_y * t;

t_t = 0:0.1:5;
figure
x = subs(eq1, t, t_t);
y = subs(eq2, t, t_t);
plot(x, y)
title('EE position Solution 1')
```

```
xlabel('x')
ylabel('y')

figure
x_dot = subs(eq1_d, t, t_t);
y_dot = subs(eq2_d, t, t_t);
plot(x_dot, y_dot)
title('EE velocity Solution 1')
xlabel('xd')
ylabel('yd')

figure
x_dot_dot = subs(eq1_dd, t, t_t);
y_dot_dot = subs(eq2_dd, t, t_t);
plot(x_dot_dot, y_dot_dot)
title('EE acceleration solution 1')
xlabel('xdd')
ylabel('ydd')


%SOLUTION 2
a0_x = 0.5350;a0_y = 0.8957;
a1_x = 0;a1_y = 0;
a2_x = 0.0025;a2_y = 0;
a3_x = -0.0188;a3_y = 0;

syms t
eq1= a0_x + a1_x * t + a2_x * (t^2) + a3_x * (t^3);
eq2= a0_y + a1_y * t + a2_y * (t^2) + a3_y * (t^3);

eq1_d = a1_x + 2 * a2_x * t + 3 * a3_x * (t^2);
eq2_d = a1_y + 2 * a2_y * t + 3 * a3_y * (t^2);

eq1_dd = 2 * a2_x + 6 * a3_x * t;
eq2_dd = 2 * a2_y + 6 * a3_y * t;

t_t = 0:0.1:5;
figure
x = subs(eq1, t, t_t);
y = subs(eq2, t, t_t);
plot(x, y)
title('EE position Solution 2')
xlabel('x')
ylabel('y')

figure
x_dot = subs(eq1_d, t, t_t);
y_dot = subs(eq2_d, t, t_t);
plot(x_dot, y_dot)
title('EE velocity Solution 2')
xlabel('xd')
ylabel('yd')


figure
x_dd = subs(eq1_dd, t, t_t);
y_dd = subs(eq2_dd, t, t_t);
plot(x_dot_dot, y_dot_dot)
title('EE acceleration solution 2')
xlabel('xdd')
ylabel('ydd')
```

```
%}
```

Problem 2b

```
l1 = 300;% link lengths
l2 = 300;

syms q10 q20 q1f q2f
eq1q = l1*cos(q10) + l2*cos(q10+q20) ==300; % x of x0
eq2q = l1*sin(q10) + l2*sin(q10+q20) ==450; % y of x0
[q1_0, q2_0]= solve([eq1q eq2q],[q10 q20]); % solve for theta 0

eq3q = l1*cos(q1f) + l2*cos(q1f+q2f) ==-300; % x of xf
eq4q = l1*sin(q1f) + l2*sin(q1f+q2f) ==450 % y of xf
[q1_f, q2_f] = solve([eq3q eq4q],[q1f q2f]); % solve for theta f

q1_0 = real (double(q1_0));
q2_0 = real (double(q2_0));
q1_f = real (double(q1_f));
q2_f = real (double(q2_f));
% display values
fprintf('q1_0 is: %f \n', real(double(q1_0)));
fprintf('q2_0 is: %f \n', real(double(q2_0)));
fprintf('q1_f is: %f \n', real(double(q1_f)));
fprintf('q2_f is: %f \n', real(double(q2_f)));

%SOLUTION 1
a0_x = 1.4306; a0_y = -0.8957;
a1_x = 0; a1_y = 0;

syms t
eqn1 = a0_x + a1_x * t;
eqn2 = a0_y + a1_y * t;

eqn1_d = a1_x;
eqn2_d = a1_y;

eqn1_dd = 0;
eqn2_dd = 0;

t_t = 0:0.1:5;
figure
q1__ = subs(eqn1, t, t_t);
plot(t_t, q1__)
title('J2: Time-Position x_line Solution 1')
xlabel('Time')
ylabel('Position')

figure
q1_d__ = subs(eqn1_d, t, t_t);
plot(t_t, q1_d__)
title('J2: Time-Velocity x_d_line Solution 1')
xlabel('Time')
ylabel('Velocity')

figure
q1_dd__ = subs(eqn1_dd, t, t_t);
plot(t_t, q1_dd__)
title('J2: Time-Acceleration x_dd_line Solution 1')
xlabel('Time')
```

```
ylabel('Acceleration')

figure
q2__ = subs(eqn2, t, t_t);
plot(t_t, q2__)
title('J2: Time-Position y_line Solution 1')
xlabel('Time')
ylabel('Position')

figure
q2_d__ = subs(eqn2_d, t, t_t);
plot(t_t, q2_d__)
title('J2: Time-Velocity y_d_linear Solution 1')
xlabel('Time')
ylabel('Velocity')

figure
q2_dd__ = subs(eqn2_dd, t, t_t);
plot(t_t, q2_dd__)
title('J2: Time-Acceleration y_dd_line Solution 1')
xlabel('Time')
ylabel('Acceleration')

%SOLUTION 2
a0_x = 0.5350;a0_y = 0.8957;
a1_x = 0; a1_y = 0;

syms t
eqn1 = a0_x + a1_x * t;
eqn2 = a0_y + a1_y * t;

eqn1_d = a1_x;
eqn2_d = a1_y;

eqn1_dd = 0;
eqn2_dd = 0;

t_t = 0:0.1:5;
figure
q1__ = subs(eqn1, t, t_t);
plot(t_t, q1__)
title('J2: Time-Position x_line Solution 2')

figure
q1_d__ = subs(eqn1_d, t, t_t);
plot(t_t, q1_d__)
title('J2: Time-Velocity x_d_line Solution 2')
xlabel('Time')
ylabel('Velocity')

figure
q1_dot_dot_set = subs(eqn1_dd, t, t_t);
plot(t_t, q1_dot_dot_set)
title('J2: Time-Acceleration x_dd_line Solution 2')
xlabel('Time')
ylabel('Acceleration')

figure
q2__ = subs(eqn2, t, t_t);
plot(t_t, q2__)
title('J2: Time-Position y_line Solution 2')
xlabel('Time')
ylabel('Position')
```

```
figure
q2_d__ = subs(eqn2_d, t, t_t);
plot(t_t, q2_d__)
title('J2: Time-Velocity y_d_linear Solution 1')
xlabel('Time')
ylabel('Velocity')

figure
q2_dd__ = subs(eqn2_dd, t, t_t);
plot(t_t, q2_dd__)
title('J2: Time-Acceleration y_dd_line Solution 2')
xlabel('Time')
ylabel('Acceleration')
```

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