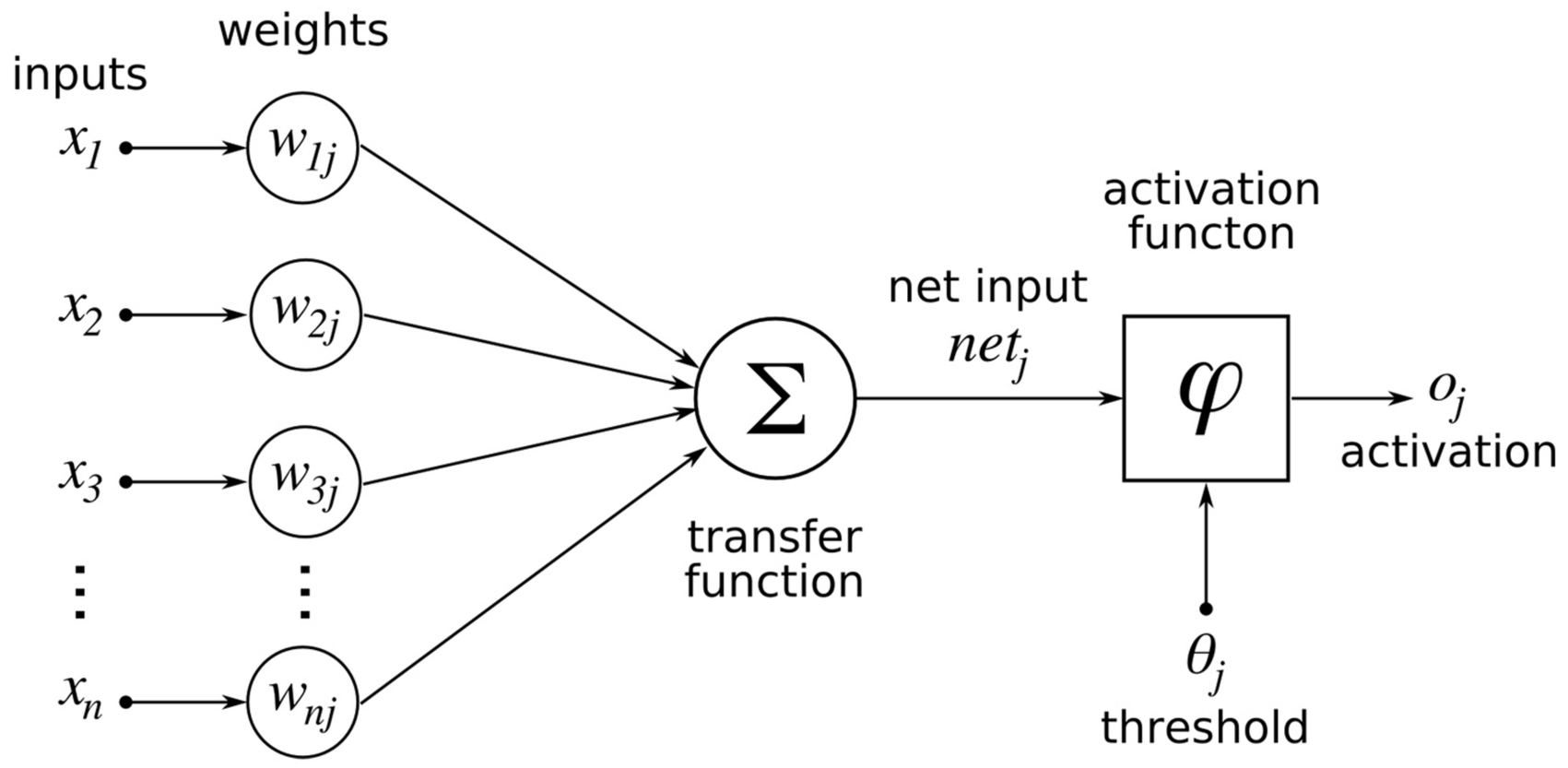
Back Propagation algorithm



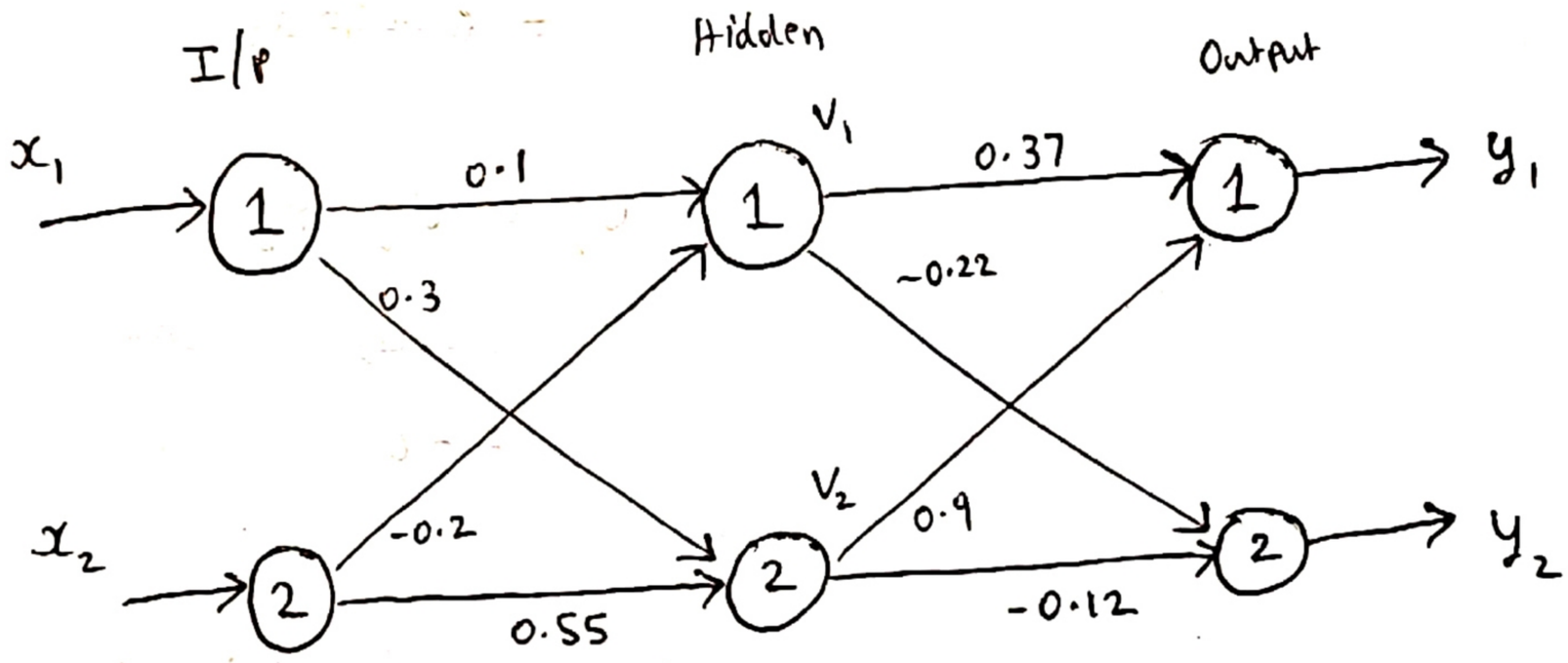
Q. Solve using Back-propagation algorithm.

Pattern	x_1	x_2	d_1	d_2
1	0.5	-0.5	0.9	0.1
2	-0.5	0.5	0.1	0.9

$$W_{ij} = \begin{bmatrix} 0.37 & 0.9 \\ -0.22 & -0.12 \end{bmatrix}$$

$$W_{jk} = \begin{bmatrix} 0.1 & -0.2 \\ 0.3 & 0.55 \end{bmatrix}$$

Soln. We can represent this in a 2 layer network as



From our earlier derivation, we have

$$\delta_i = y_i (1 - y_i)(d_i - y_i)$$

$$\delta_{Hj} = V_j (1 - V_j) \sum_{i=1}^{n} \delta_i W_{ij}$$

$$W_{ij}(t+1) = W_{ij}(t) + \delta_i V_j$$

$$W_{jk}(t+1) = W_{jk}(t) + \delta_{Hj} x_k$$

To find δ we need e (error), which can be found by finding y.

$$h_j = \sum_{k=1}^{n} W_{jk} x_k \qquad V_j = \frac{1}{1 + e^{-h_j}}$$

$$s_i = \sum_{j=1}^{n} W_{ij} V_j \qquad y_i = \frac{1}{1 + e^{-s_i}}$$

Iteration 1, Pattern 1

$$h_1 = w_{11}x_1 + w_{12}x_2 = M_{11}x_1 + M_{12}x_2 = 0.15$$

$$h_2 = w_{21}x_1 + w_{22}x_2 = M_{21}x_1 + M_{22}x_2 = -0.125$$

$$V_1 = \frac{1}{1+e^{-h_1}} = 0.53743$$

$$V_2 = \frac{1}{1+e^{-h_2}} = 0.46879$$

$$\delta_1 = W_{11}V_1 + W_{12}V_2 = 0.62076$$

$$\delta_2 = W_{21}V_1 + W_{22}V_2 = -0.174489$$

$$y_1 = \frac{1}{1+e^{-\delta_1}} = 0.65039$$

$$y_2 = \frac{1}{1+e^{-\delta_2}} = 0.45649$$

$$e_1 = d_1 - y_1 = 0.24961$$

$$e_2 = d_2 - y_2 = -0.35649$$

$$\delta_1 = y_1(1-y_1)e_1 = 0.05676$$

$$\delta_2 = y_2(1-y_2)e_2 = -0.08845$$

$$\delta_{H1} = V_1(1-V_1)(\delta_1 W_{11} + \delta_2 W_{21}) = 0.010075$$

$$\delta_{H2} = V_2(1-V_2)(\delta_1 W_{12} + \delta_2 W_{22}) = 0.015364$$

Now, update the weights

$$W_{11}(t+1) = W_{11}(t) + \delta_1 V_1 = 0.40050$$

$$W_{12}(t+1) = W_{12}(t) + \delta_1 V_2 = 0.92661$$

$$W_{21}(t+1) = W_{21}(t) + \delta_2 V_1 = -0.25773$$

$$W_{22}(t+1) = W_{22}(t) + \delta_2 V_2 = -0.16164$$

$$M_{11}(t+1) = M_{11}(t) + \delta_{H_1} x_1 = 0.10504$$

$$M_{12}(t+1) = M_{12}(t) + \delta_{H_1} x_2 = -0.20504$$

$$M_{21}(t+1) = M_{21}(t) + \delta_{H_2} x_1 = 0.307682$$

$$M_{22}(t+1) = M_{22}(t) + \delta_{H_2} x_2 = 0.542318$$

Iteration 1, Pattern 2

$$h_1 = M_{11}x_1 + M_{12}x_2$$
$$= -0.15504$$

$$h_2 = M_{21}x_1 + M_{22}x_2$$
$$= 0.117318$$

$$V_1 = \frac{1}{1+e^{-h_1}}$$
$$= 0.461318$$

$$V_2 = \frac{1}{1+e^{-h_2}}$$
$$= 0.529596$$

$$S_1 = W_{11}V_1 + W_{12}V_2$$
$$= 0.675208$$

$$S_2 = W_{21}V_1 + W_{22}V_2$$
$$= -0.20888$$

$$y_1 = \frac{1}{1+e^{-S_1}}$$
$$= 0.662668$$

$$y_2 = \frac{1}{1+e^{-S_2}}$$
$$= 0.44797$$

$$e_1 = d_1 - y_1$$
$$= -0.562668$$

$$e_2 = d_2 - y_2$$
$$= 0.45203098$$

$$\delta_1 = y_1(1-y_1)e_1$$
$$= -0.125778$$

$$\delta_{H1} = V_1(1-V_1)(\delta_1 W_{11} + \delta_2 W_{21})$$
$$= -0.0200012$$

$$\delta_2 = y_2(1-y_2)e_2$$
$$= 0.111784$$

$$\delta_{H2} = V_2(1-V_2)(\delta_1 W_{12} + \delta_2 W_{22})$$
$$= -0.033533$$

Now, updated weights at the end of 1st iteration are

$W_{11}(t+1) = 0.342479$ $\quad$ $M_{11}(t+1) = 0.115038$

$W_{12} = 0.860033$ $\quad$ $M_{12} = -0.215038$

$W_{21} = -0.2159661$ $\quad$ $M_{21} = 0.324485$

$W_{22} = -0.10229633$ $\quad$ $M_{22} = 0.5255$

1st iteration ends here!