Kathryn Bowers

Supervisor - Olivier Restif

Modelling virus evolution in age-structured populations

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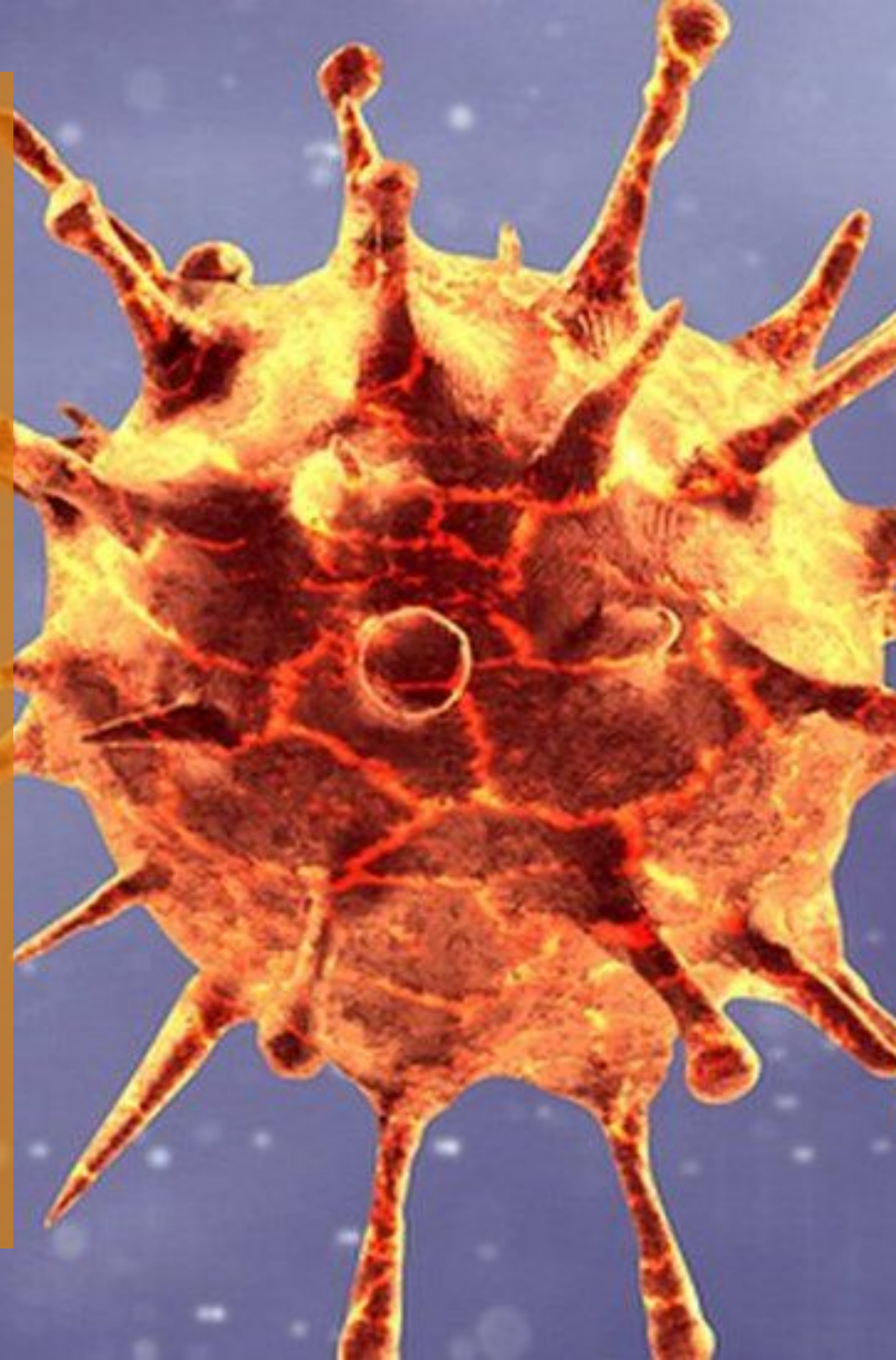
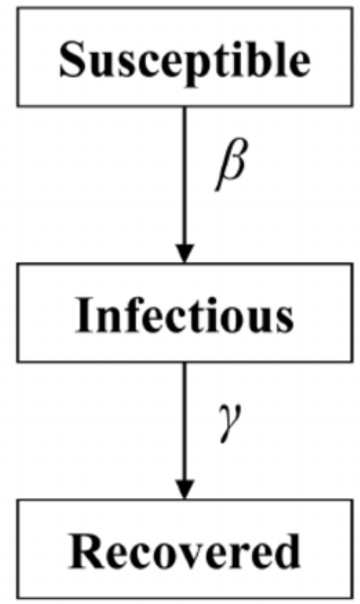


Project Overview

- Varying infectivity and severity in different age groups
- Adapt and develop existing epidemic models of COVID-19
- Trade-off between infectivity and lethality
- Rarely favour strains that maximise $\mathscr{R}_0$

Simplest Epidemic Model



$$\frac{dS}{dt} = -\beta SI$$

$$\frac{dI}{dt} = \beta SI - \gamma I$$

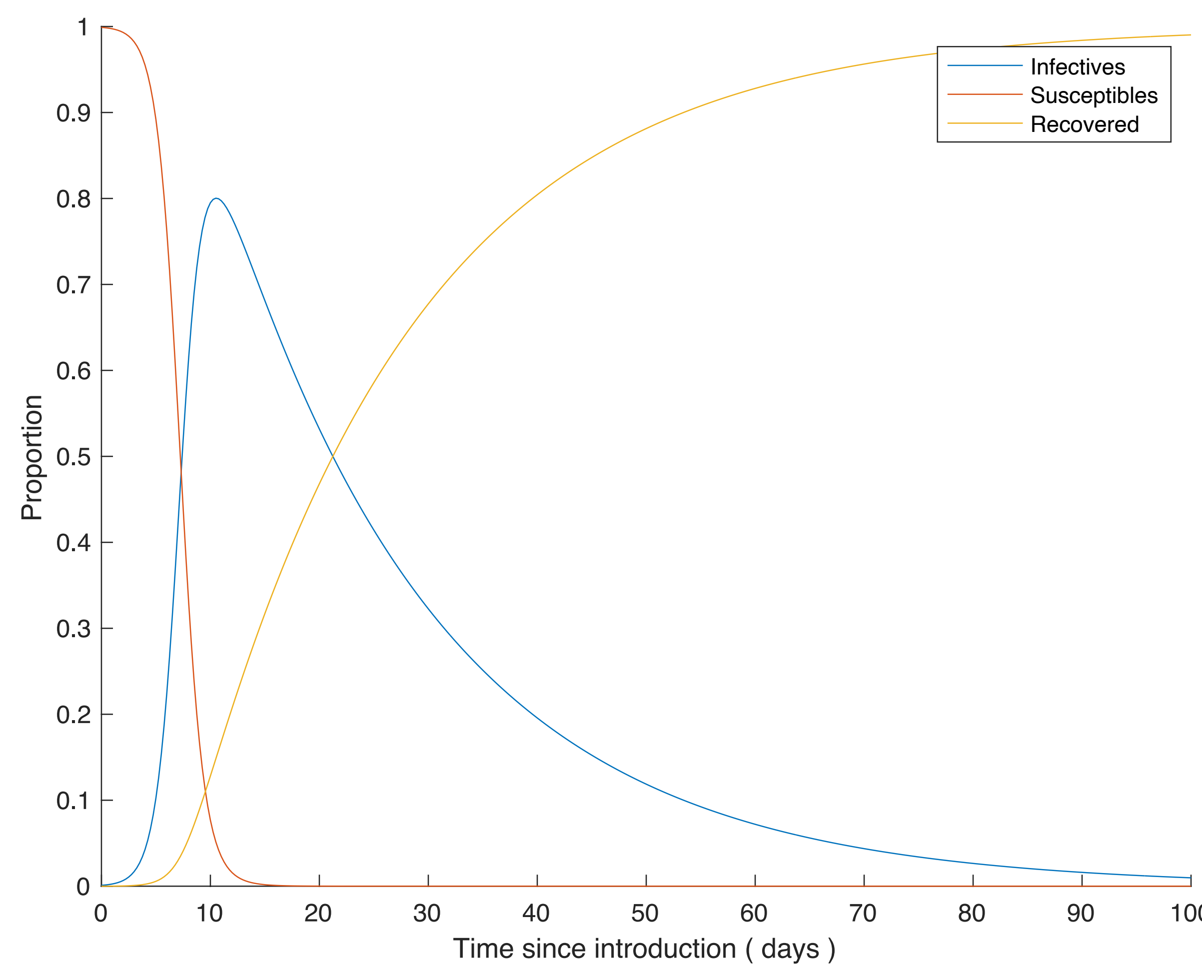
$$\frac{dR}{dt} = \gamma I$$

S(t) Density of susceptible hosts

E(t) Latent stage

I(t) Density of infectious hosts

R(t) Density of recovered hosts

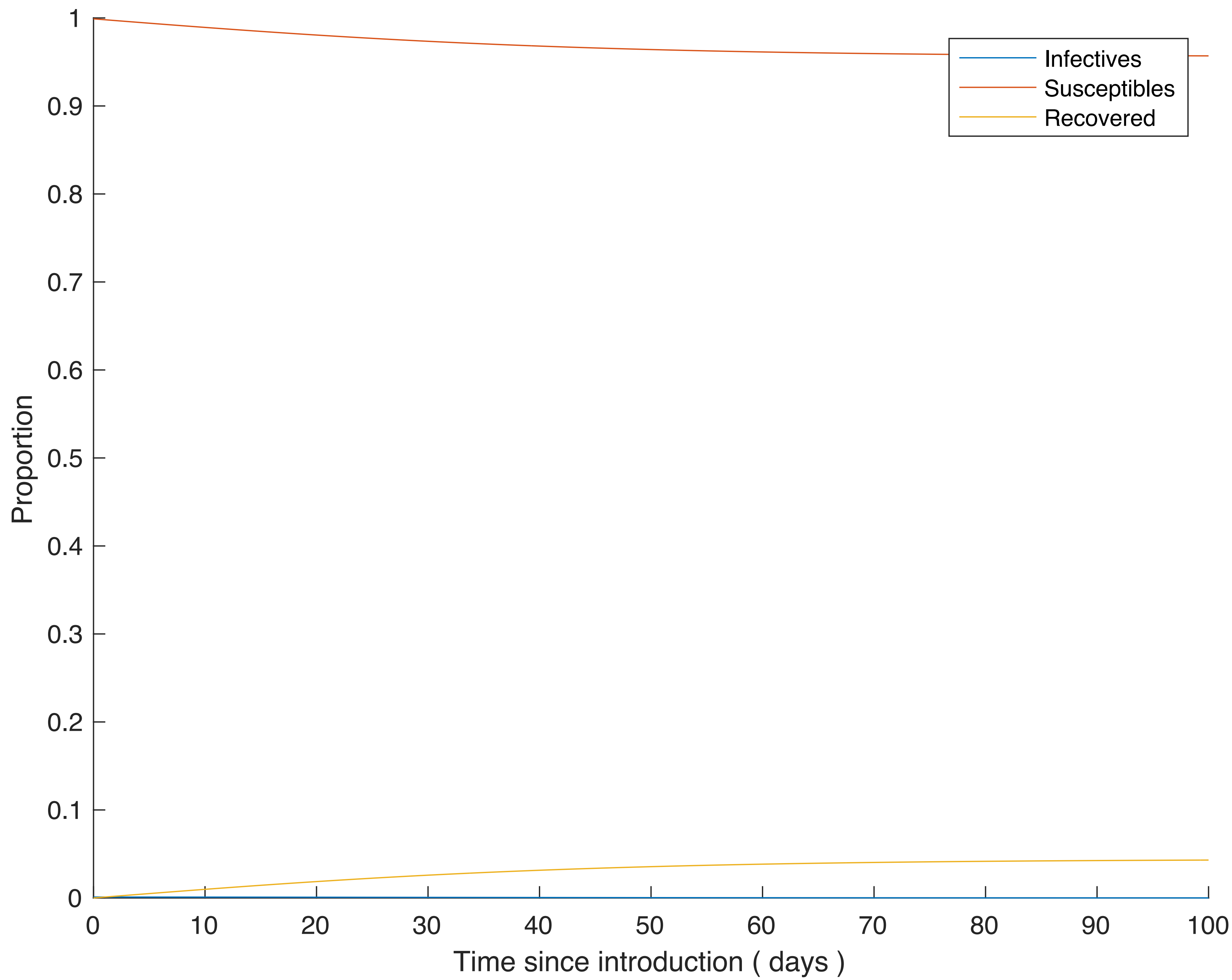


Basic Reproductive Ratio ($\mathscr{R}_0$)

- Indicates how many new hosts an infected host will infect in a population where all the hosts are susceptible.
- Epidemiological measure of parasite fitness.
- For an epidemic to occur in a susceptible population $\mathscr{R}_0 > 1$.
- Effective reproductive rate - considering a population made up of both susceptible and non-susceptible hosts.

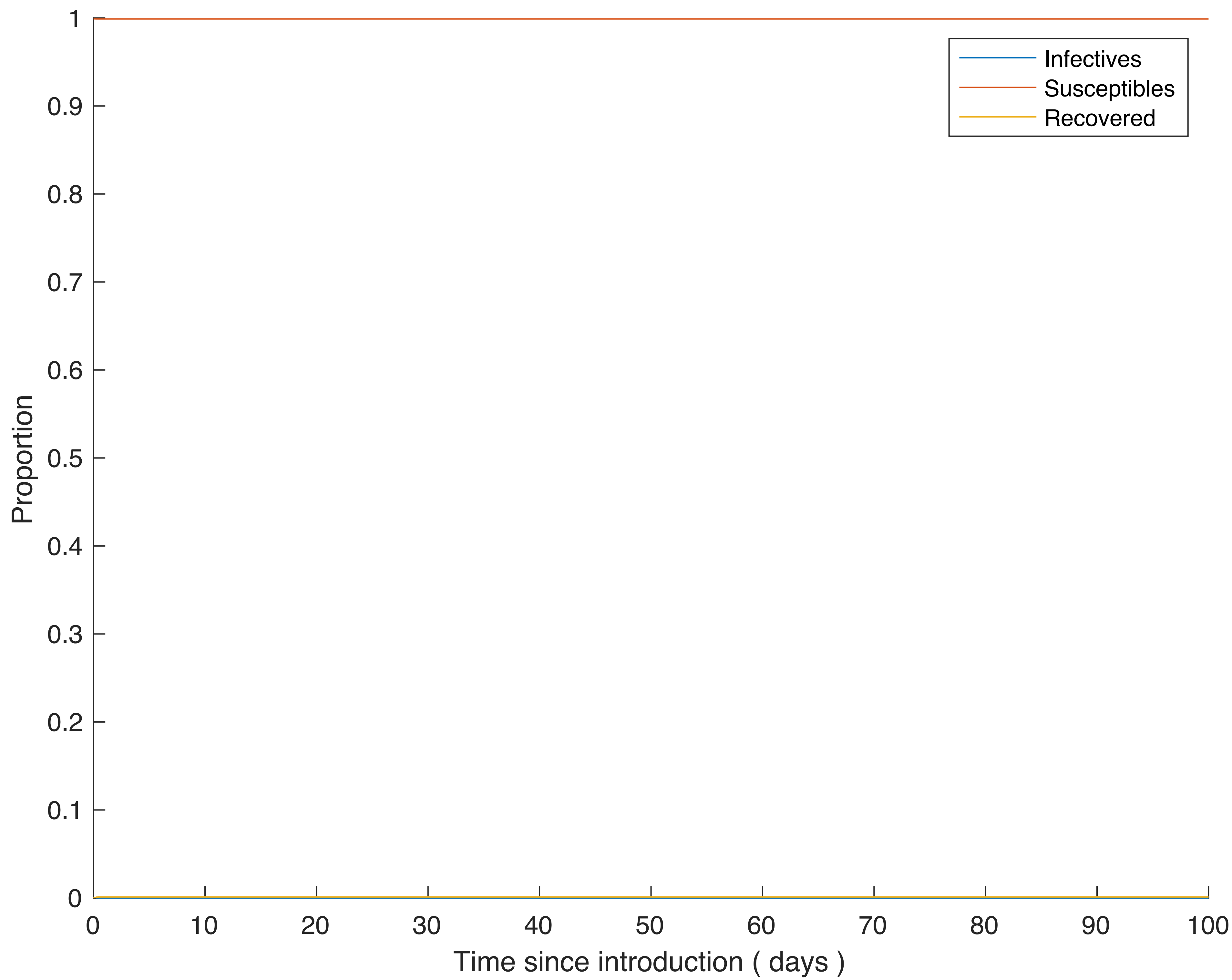
Computed as a ratio of known rates over time.

- An infectious individual contacts β other people per unit time
- All of those people assumed to contract the disease
- The disease has a mean infectious period of 1/γ

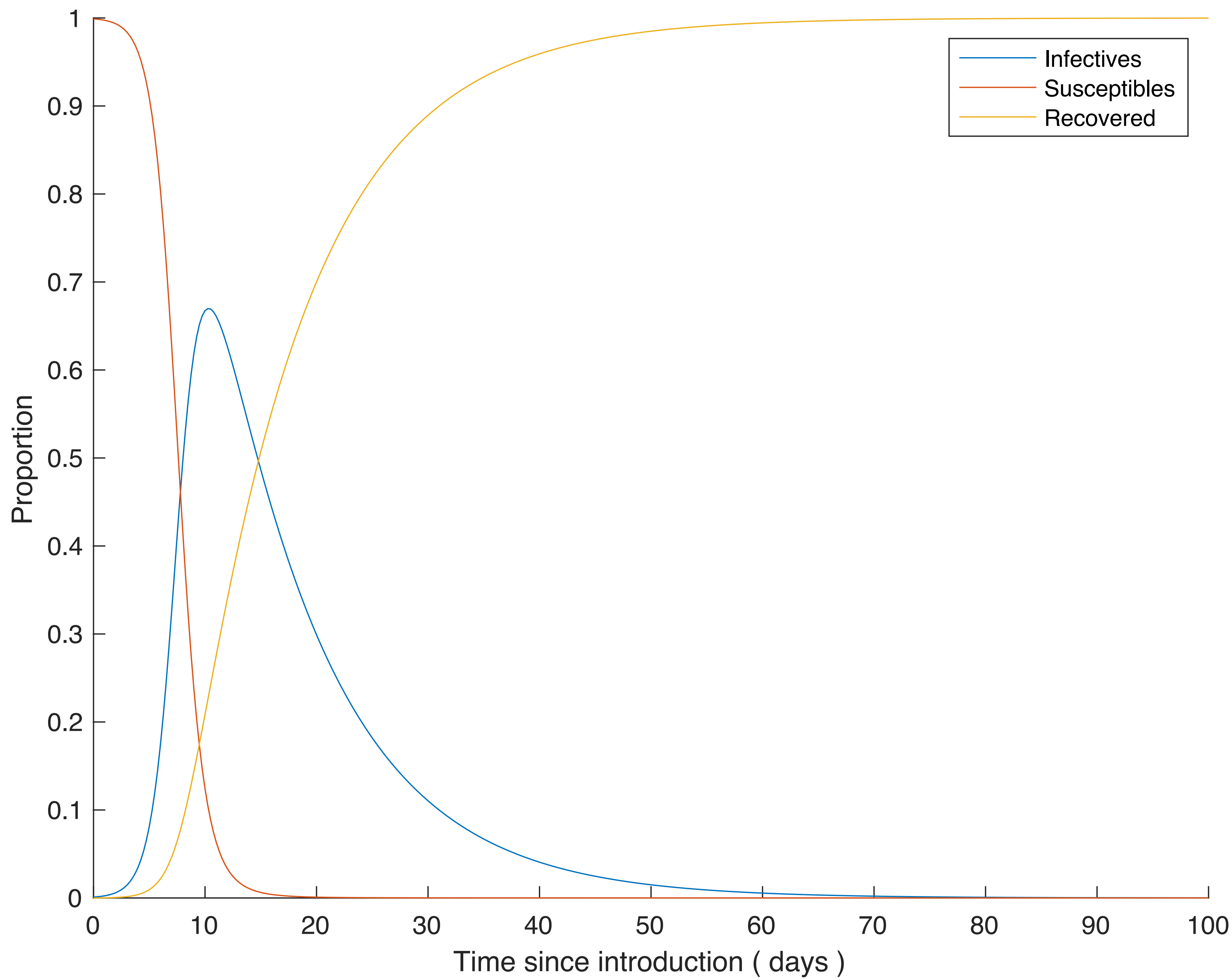


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Proportion
0
10
20
30
40
50
60
70
80
90
100
Time since introduction (days)
Infectives
Susceptibles
Recovered

$$\mathscr{R}_0 = 1$$



$$\mathscr{R}_0 = 0.1$$



$$\mathscr{R}_0 = 10$$

Calculating $\mathscr{R}_0$

- Next generation matrix

Relates the numbers of newly infected individuals in each category in consecutive generations.

$\mathscr{R}_0$ is the dominant eigenvalue of $K = -T\Sigma^{-1}$

- Transmission matrix (T)
- Transition matrix (Σ)

K_{ij} is the expected number / proportion of new cases with state-at-infection i generated by an individual with state-at-infection j.

- Initial growth rate of infections

For small time, we can use the approximation of $S \approx N$.

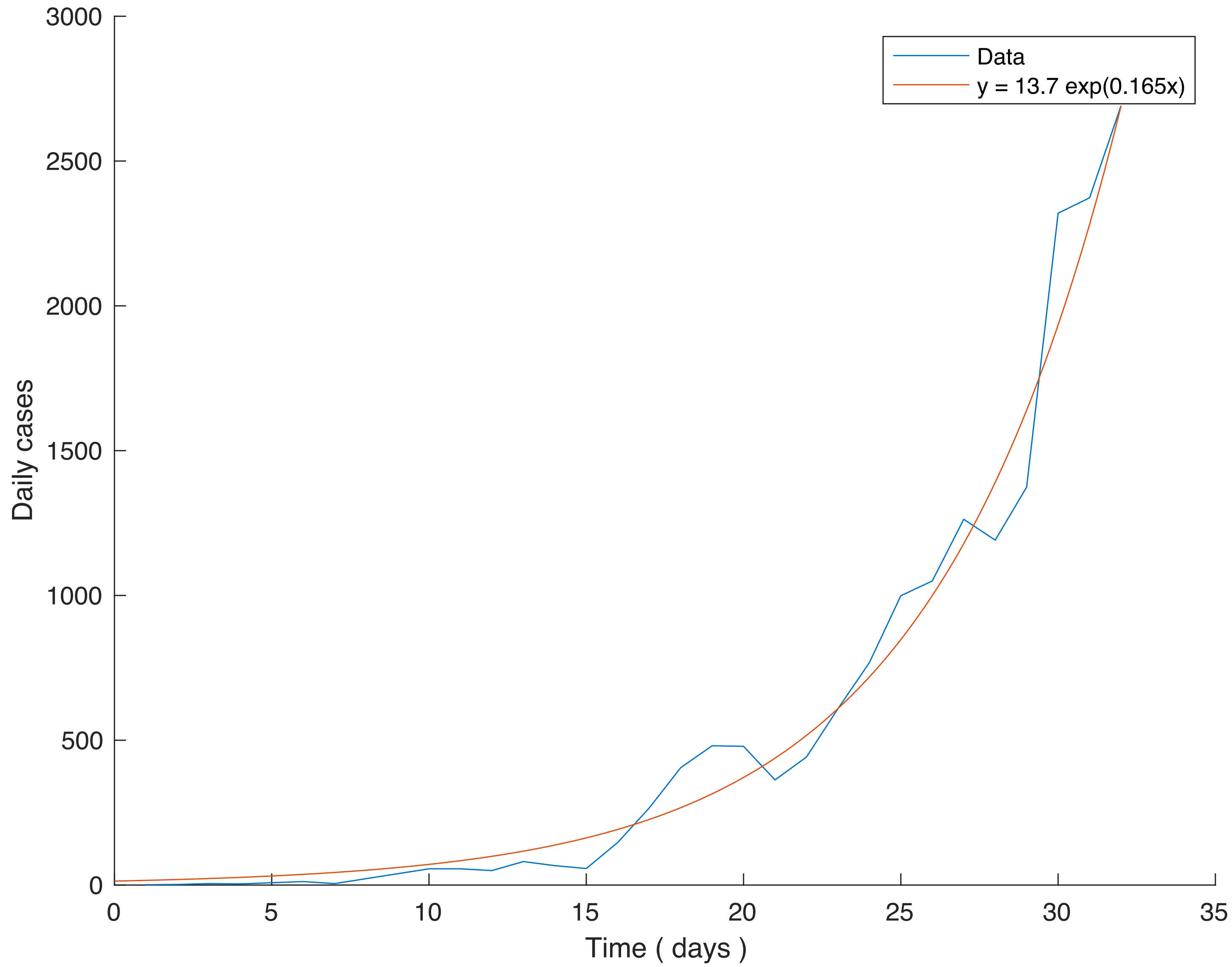
$$\frac{dI}{dt} = (\beta N - \gamma) I = (\mathscr{R}_0 - 1)\gamma I$$

Measure the initial growth rate 'r'.

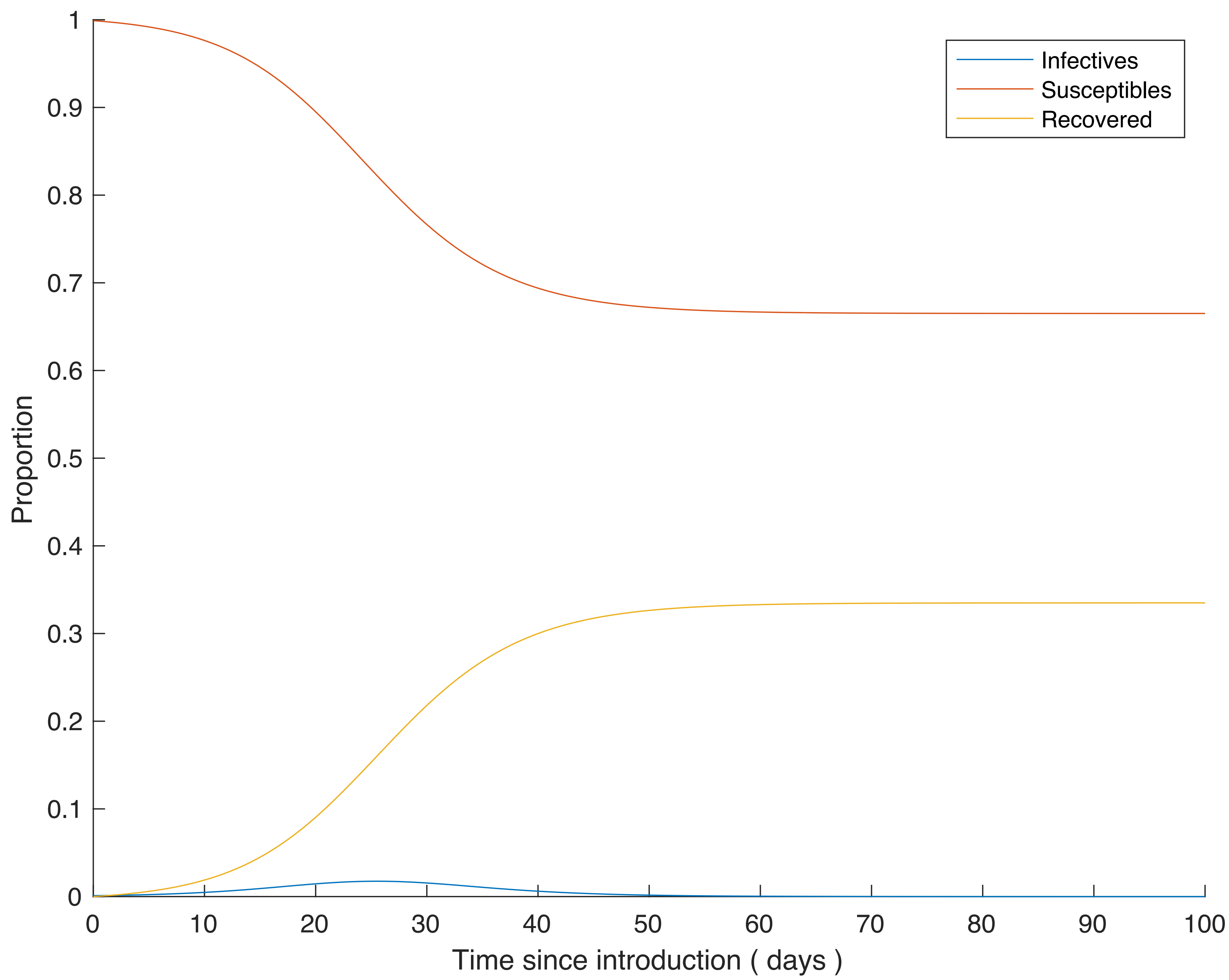
$$\mathscr{R}_0 \approx 1 + \frac{r}{\gamma}$$

Data Models

Using data given on the government website - https://coronavirus.data.gov.uk/details/cases - we can model the initial growth rate of infections for coronavirus back in January 2020.



$\mathscr{R}_0 \approx 1 + \dfrac{0.165}{0.768} \approx 1.21$ and from this we can estimate $\beta \approx 0.933$



Infectives
Susceptibles
Recovered
Proportion
Time since introduction (days)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
10
20
30
40
50
60
70
80
90
100

Model Calculations

- Epidemic peak

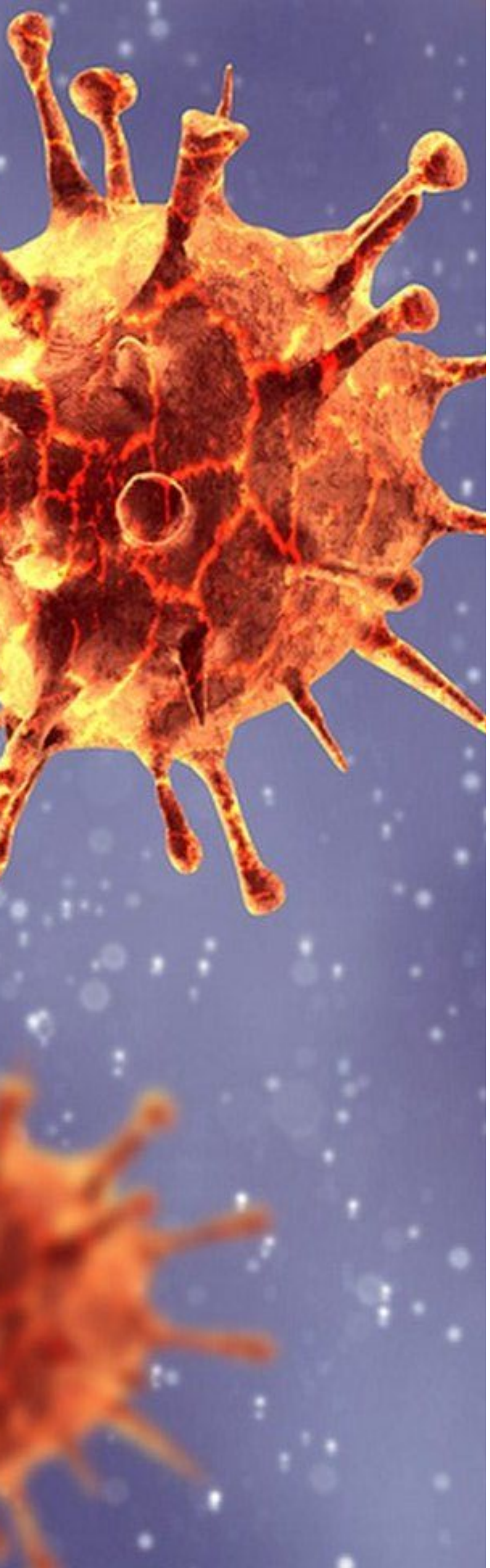
$$\frac{dI}{dt} = 0 \implies S = \frac{\gamma}{\beta}$$

$$\frac{\dot{S}}{S} = -\beta I \implies \int_0^t I\,dt = \frac{1}{\beta}\,ln\left(\frac{S_0}{S}\right)$$

$$\dot{S} + \dot{I} = -\gamma I \implies S + I = -\gamma \int_0^t I\,dt + S_0 + I_0$$

$$\therefore I = S_0 + I_0 - S + \frac{\gamma}{\beta}\,In\left(\frac{S}{S_0}\right)$$

At the epidemic peak we obtain $I = S_0 + I_0 + \frac{\gamma}{\beta}\left[ln\left(\frac{\gamma}{\beta S_0}\right) - 1\right]$



- Final epidemic size

The total number / proportion of the population that got infected during the course of the epidemic.

The epidemic size $f = 1 - \dfrac{S_\infty}{N} \; (= R_\infty)$

$$ln\left(\frac{N}{S_\infty}\right) = \mathscr{R}_0\left(1 - \frac{S_\infty}{N}\right) \implies R_\infty = 1 - e^{-\mathscr{R}_0 R_\infty}$$

Epidemic Model with Births and Deaths

- birth rate (b)
- death rate (δ)

Assumption:

$$b = \delta \implies N = constant = 1$$

$$\mathscr{R}_0 = \frac{\beta}{\gamma + \delta}$$

$$\frac{dS}{dt} = bN - \beta SI - \delta S$$

$$\frac{dI}{dt} = \beta SI - \gamma I - \delta I$$

$$\frac{dR}{dt} = \gamma I - \delta R$$

$$\frac{dS}{dt} = 0 \implies I = \frac{\delta}{\beta}\left(\frac{N}{S} - 1\right)$$

$$\frac{dI}{dT} = 0 \implies I = 0\,, S = \frac{\delta + \gamma}{\beta}$$

$$\frac{dR}{dt} = 0 \implies \gamma I - \delta R = 0 \implies R = \frac{\gamma}{\delta} I$$

$$S^* = \frac{N}{\mathscr{R}_0}$$

$$I^* = \frac{\delta}{\beta}(\mathscr{R}_0 - 1)$$

$$R^* = \frac{\gamma}{\beta}(\mathscr{R}_0 - 1)$$

Epidemic Model with Juveniles and Adults

- birth rate (b)
- death rate (δ)
- ageing factor (α)

$$\frac{dJ}{dt} = bA - \beta(I_J + I_A)J - \alpha J - \delta_J J$$

$$\frac{dA}{dt} = \alpha J - \beta(I_J + I_A)A - \delta_A A$$

$$\frac{dI_J}{dt} = \beta(I_J + I_A)J - (\alpha + \delta_J + \gamma_J)I_J$$

$$\frac{dI_A}{dt} = \beta(I_J + I_A)A - (\delta_A + \gamma_A)I_A + \alpha I_J$$

$$\frac{dR_J}{dt} = \gamma_J I_J - (\alpha - \delta_J)R_J$$

$$\frac{dR_A}{dt} = \gamma_A I_A + \alpha R_J - \delta_A R_A$$

$$\frac{dI_J}{dt} = \beta(I_J + I_A)J - (\alpha + \delta_J + \gamma_J)I_J$$

$$\frac{dI_A}{dt} = \beta(I_J + I_A)A - (\delta_A + \gamma_A)I_A + \alpha I_J$$

$$T = \beta \begin{pmatrix} N_J & N_J \\ N_A & N_A \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} -(\alpha + \delta_J + \gamma_J) & 0 \\ \alpha & -(\delta_A + \gamma_A) \end{pmatrix}$$

$$K = -T\Sigma^{-1}$$

$$\mathcal{R}_0 = \frac{\beta N_A(\alpha + \delta_J + \gamma_J) + \beta N_J(\alpha + \delta_A + \gamma_A)}{(\alpha + \delta_J + \gamma_J)(\delta_A + \gamma_A)}$$

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