Burton Examples

Kathryn Hu

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1 Time-Optimal Double Integrator

Problem formulation:

$$\min_{\mathbf{u},t_f} \int_0^{t_f} dt$$
$$[\dot{x}, \dot{y}] = [y, u],$$
$$x(t_f) = y(t_f) = 0, |u| \leq 1$$

Without fixed t_f:

$$\min_{\mathbf{u},t_f} \int_0^{1} t_f dt$$
$$[\dot{x}, \dot{y}] = [t_f \cdot y, t_f \cdot u],$$
$$x(1) = y(1) = 0, |u| \leq 1$$

2 Dubins Vehicle

Problem formulation:

$$\min_{u,t_f} \int_0^{t_f} (1 + \tfrac{1}{2}[u(t)]^2) dt$$
$$[\dot{x}, \dot{y}, \dot{\theta}] = [\cos\theta, \sin\theta, u],$$
$$|u| \leq 6, t_f \geq 0, |x(t_f)|^2 + |y(t_f)|^2 \leq (0.1)^2$$

Without fixed t_f:

$$\min_{u,t_f} \int_0^{1} t_f (1 + \tfrac{1}{2}[u(t)]^2) d\tau$$
$$[\dot{x}, \dot{y}, \dot{\theta}] = [t_f \cdot \cos\theta, t_f \cdot \sin\theta, t_f \cdot u],$$
$$|u| \leq 6, t_f \geq 0, |x(1)|^2 + |y(1)^2| \leq (0.1)^2$$

Discretized:

$$\min_{u,t_f} \sum_{k=1}^{N-1} \Delta t (1 + \tfrac{1}{2} u_i^2)$$
$$[x_{k+1}, y_{k+1}, \theta_{k+1}] = [x_k + t_f \cdot \cos\theta_k \cdot \Delta\tau, y_k + t_f \cdot \sin\theta_k \cdot \Delta\tau, \theta_k + t_f \cdot u_k \cdot \Delta\tau]$$
$$|u| \leq 6, t_f \geq 0, x_N^2 + y_N^2 \leq (0.1)^2$$