Game Theory and Real Options:
Analysis of Land Value and Strategic Decisions
in Real Estate Development

by

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Bachelor of Science in Architecture, University of Virginia, 2003

Submitted to the Program in Real Estate Development in Conjunction with
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ABSTRACT

This thesis investigates the use of the game theory and the real options theory in real estate development at the strategic level, trying empirically to explain different economic observations among different metropolitan cities and different property types.

The real options theory provides a rich theoretical framework to analyze investment values in real estate development. It takes the market uncertainty into consideration, while the widely used neoclassical NPV valuation method takes a deterministic approach. A simplified real options valuation model is set up in this thesis to calculate the option premium value of waiting for developers. However, since it is done in a monopolistic setting, the strategic interaction aspect of real estate development will be analyzed using the game theory model. The interaction of the game theory model and the real options model will provide a comprehensive and powerful framework to study the timing strategy of developers.

Using data spanning quarterly from 1995 to 2013 among 5 property types (single-family house, apartment, industrial, office, and retail) and 44 MSAs, this thesis analyzes the relationships empirically between the volatility of underlying assets, the land cost ratio, the option premium value, and the timing of development. The aims of the study are twofold. First, the study compares different market characteristics among different MSAs and different property types from the option game theoretic perspective. Second, it analyzes the effect and the use of the game theory and the real options theory in the context of real estate development.

Thesis Supervisor: Walter Torous
Title: Senior Lecturer, Center for Real Estate

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1.0 Chapter 1: Introduction

1.1 Purpose of the thesis

This thesis investigates the use of game theory and real options in real estate development at a strategic level, backed up by empirical data to try to explain different economic observations among different regions and metropolitan cities.

The real options approach to analyzing investment under uncertainty has become part of the mainstream literature of financial economics. Essentially, the real options approach to analyze the opportunity to invest in a project is analogous to an American call option on the investment opportunity. Once that analogy is made, the vast and rigorous machinery of financial options theory can be applied to analyze such investment option. The real options approach is well summarized in Dixit and Pindyck (1994) and Trigeorgis (1996). The most well-known result of the real options literature is the invalidation of the standard net present value (NPV) rule of investing in any project with a non-negative net present value. The optimal investment rule, as described in the real options literature, is to invest when the asset value exceeds the investment cost by a potentially large option premium. While the widely used neoclassical NPV valuation method takes a deterministic approach, the real options valuation method takes into account the option value created by uncertain future outcome.

Since the 1980s, there have been lots of interest in academic research in real options valuation methods. A vast array of models and frameworks have been studied and proposed. Titman (1985), Williams (1991), and Trigeorgis (1996) provide some of the most influential conceptual frameworks in the field, especially as applied to real estate development. While the real options valuation gives a more comprehensive picture of investment value as compared to the neoclassical NPV valuation method, it is done in a monopolistic setting. That is partially the reason why the real options valuation is not applied in real-world situations as often as it should be because economic markets are rarely purely monopolistic. The action of firm A will be affected by the action of firm B.

The option value as described in real options pricing methods is not an accurate representation of project value as valuated by firms. Grenadier (1996), and Schwartz and Torous (2007) demonstrate that competitions among real estate developers erodes the option value, and illustrate that the real options valuation method alone is not comprehensive enough to reflect real-world situations. Strategic interactions are essential components to be considered in the valuation process. Therefore, the application of the game theory in the real options analysis will provide important insights at a strategic level. And this thesis empirically compares the implication of the game theory and the real options in real estate development among 4 regions (East, Midwest, South, and West), 44 metropolitan cities, and 4 property types (Apartment, Industrial, Office, and Retail) for commercial markets, and single-family houses for residential markets.

1.2 Research motivation & hypothesis

Real estate development is one of the classic applications of the real options. As in Titman (1985) and Williams (1991), the development of real estate is analogous to an American call option on a building, where the exercise price is equal to the construction cost. An option is a contract or situation that gives its holder the right but not the obligation to buy (if a call) or sell (if a put) a specified asset (e.g. common stock or project) by paying a specified cost (the exercise or strike price) on or before a specified date (the expiration or maturity date). If the option can be exercised before the maturity, it is an American option; if only at the maturity, a European option. In real estate development, if the value of a building is higher than the cost of construction, the residual value is what developers can pay for the land to make it a zero NPV project. In other words, the land value is analogous to the option value. By holding on to the piece of land and deferring development, the intrinsic value of the land would be higher than the residual value as demonstrated in Quigg (1993). Some numerical examples will be illustrated in following chapters. Taken literally, the standard, myopic real options approach implies that developers should ignore the construction behavior of their competitors. However, in real estate markets, developers are likely to face considerable competitions from competitors, and the development activities of competitors will have a fundamental impact on one's development options. By extending the options framework to account for strategic interaction, a much richer set of investment implications is obtained. While the standard real options models dictate that a developer should wait until the development option is considerably in-the-money, which means that the value of a building is much larger than the cost of construction, competitions and the fear of pre-emption will likely force developers to build much earlier. In addition, while standard real options models imply that developments will be simultaneous, game-theoretic models allow for the possibility of sequential developments. Competitive models of real estate development can also help explain boom-and-bust behavior in commercial constructions, as well as why rational developers may construct new buildings in the face of declining demand and market values as illustrated in Grenadier (1996).

Regarding the interaction of the real options and the game theory, the timing strategy of real estate developers has long been a complex issue to study, affected by many external economics forces. This thesis focuses on the relationships of the land value, the land cost ratio, and the volatility of underlying assets in different regions and metropolitan cities, trying to explain developers' timing strategy in different markets depending on those variables from an option game theoretic perspective. Intuitively, when the volatility of underlying assets is high and the land cost ratio is high, the timing option value will be high too, which implies that developers should wait to observe the market trend better before exercising the option to develop or redevelop the piece of land. A large set of data spanning from as early as 1995 to 2013 across 4 regions, 44 metropolitan cities, and 4 property types will be used to analyze the relationship and discrepancy between the observed data and results predicted by the option game theoretic approach.

1.3 Research methodology

Asset values of 4 property types (Apartment, Industrial, Office, and Retail) in 30 Metropolitan Statistical Areas (MSA) spanning from 2001 to 2013 are collected from Real Capital Analytics, Inc (RCA) for the commercial portion of the study. Construction or replacement cost data spanning from 1993 to 2013 is collected from RS Means. Asset values of single-family houses in 44 MSAs spanning from 1995 to 2013 are collected from Lincoln Institute of Land Policy for the residential portion of the study. For the commercial portion, the construction activity is measured by number of square feet completed (industrial, office, and retail) and number of units completed (apartment), with data collected from CBRE. For the residential portion, the construction activity is measured by number of permits issued for single-family houses, with data collected from U.S. Census Bureau.

Using the real options pricing model described in detail in section 2.1.1, the option premium, defined as the difference between the option value of waiting and the residual land value, is calculated for each property use in each MSA within a certain given time period. The option premium is a value indicating theoretically the magnitude of benefits for a developer to wait, rather than to develop now. This is a better indicator than the absolute option value of waiting because the option premium eliminates the distorted effect of different asset value ranges for different uses in different MSAs. For example, given everything the same, higher asset values by definition will always give higher option values because the subject in question is of higher values, which will falsely imply that office developers will always have a higher benefit of waiting compared to industrial developers because of their inherently higher absolute option values of waiting. When compared across different uses and MSAs, the option premium should be used to measure the magnitude of incentive for developers to wait.

To study the relationship between the option premium, the land cost ratio, and the timing strategy of developers, the following regression is applied to measure the degree of correlation:

$$CA = \alpha + \beta (C_p) + \gamma (LCR) + \varepsilon$$

Where

CA	= Construction Activity	
C_p	= Option Premium value	
	$= C_t - (S_t - K_t)$	
LCR	= Land Cost Ratio	
	$= \frac{S-K}{S}$	
S	= property asset value	
K	= construction / replacement cost	

In this thesis, two levels of relationships are studied: (1) across time within a MSA, and (2) across MSAs within a time period for both the commercial and residential markets. Sections 4.0 and 5.0 will further elaborate all the details.

1.4 Results & interpretation

As presented in detail in sections 4.0 and 5.0, for commercial markets, the higher the asset value volatility is, the higher the option premium value will be, and the lower the level of construction activities will result. Meanwhile, the higher the land cost ratio is, the lower the level of construction activities will result. At an aggregate level across multiple MSAs, apartment developments are the most sensitive to the option premium value, followed by industrial and office developments. Retail developments are the least sensitive to the option premium value. Compared between the time-series (across time within a MSA) study and the cross-sectional (across multiple MSAs within a time period) study, the time-series study seems to show higher sensitivity to the option premium value than the cross-sectional study does. The time-series study also shows more significant regression results than the cross-sectional study does (sections 4.1 and 5.1.1).

For residential markets, results between the time-series study and the cross-sectional study are not as consistent as those of the commercial markets study. In the time-series study, the higher the asset value volatility is, the higher the option premium value will be, and the lower the level of construction activities will result. Meanwhile, the lower the land cost ratio is, the lower the level of construction activities will result. However, the cross-sectional study shows the opposite. Compared between the time-series study and the cross-sectional study, the time-series study again seems to show higher sensitivity to the option premium value than the cross-sectional study does. Between commercial markets and residential markets, the regression results show that residential markets are more sensitive to the option premium value than commercial markets do. Residential markets also show more significant regression results than commercial markets do (sections 4.2 and 5.1.2). Overall, the correlation is a lot stronger between the option premium value and the level of construction activity than that between the land cost ratio and the level of construction activity. In the Variance Inflation Factor tests for the issue of multi-collinearity, both results of commercial markets and residential markets show that the correlation between the option premium value and the land cost ratio is not strong or problematic enough to distort their effect on the level of construction activity (section 5.3).

2.0 Chapter 2: Overview of Fundamentals

2.1 Real Options

The real options valuation, also often termed the real options analysis (ROV or ROA), applies option valuation techniques to capital budgeting decisions. A real option itself is the right, but not the obligation, to undertake certain business initiatives, such as deferring, abandoning, expanding, staging, or contracting a capital investment project. Dixit and Pindyck (1994) have explained investment decisions in detail in their book. They site that most investment decisions share three important characteristics. First, the investment is partially or completely irreversible. Second, there is uncertainty over the future rewards from the investment. Third, some flexibility about the timing of the investment usually exists. These three characteristics interact to determine the optimal decisions for any investor.

Within the neoclassical theory of investment, the net present value (NPV) theory, has not recognized the interaction between irreversibility, uncertainty, and the choice of timing. Real world investments seem less sensitive to changes in the interest rate and the tax policy, and much more sensitive to the volatility and the uncertainty over the economic environment. A growing body of literature has shown that the ability to delay an irreversible investment can profoundly affect the decision to invest. The traditional Discounted Cash Flow (DCF) method dictates that companies should not execute any negative NPV project. The new view of investment opportunities as options has shown that the traditional NPV rule can give very wrong answers unless all relevant option values are included in the NPV. Note that if choices are investing now or never, the standard NPV rule applies because there is no option to wait years.

2.1.1 Real Options Pricing Model setup

When applied in real estate development, the real options can be considered an American call option on the asset value of a building with the exercise price being the construction or replacement cost. When the construction or replacement cost is subtracted from the asset value of the building, the residual value is the maximum price that a developer should pay for the piece of land to make it a zero NPV project. As stated above, this classic NPV approach neglects the option value of waiting for future development. If the real options pricing model is applied in the decision-making process, the option value should be considered as well. In each period, developers should compare the residual value with the option value to better understand their optimal action. The option premium value (C_p) is defined as the difference between the residual value and the option value. If the option premium value is positive, it means that the option value is higher than the residual value, and the option to wait is more valuable than the decision to develop the piece of land in this period, vice versa. A fundamental and pure form of the real options pricing model will be used in this thesis to examine empirically the relationship between the land value and the timing strategy of developers. Time t=0 is defined as the initial period when the decision is about to be made. The property asset value (S_t) will either go up by a multiple (u) to ($S_{t+0.25} = uS_t$) with the probability (p) or go down by a multiple (d) to ($S_{t+0.25} = dS_t$) with the probability (1-p). The construction or replacement cost will increase at an average growth rate throughout the periods. Since it is an American call option, the option value (C_t) is defined as the maximum value between the residual value and the option of waiting till next period. Key equations are as follows:

$$S_t = e^{-rt} [\ p \times uS_t + (1-p) \times dS_t\ ]$$

$$K_t = e^{-gt} K_{t+0.25}$$

$$C_t = \max \{\ S_t - K_t\ ,\ e^{-rt} [\ p \times C^u{}_{t+0.25} + (1-p) \times C^d{}_{t+0.25}]\ \}$$

$$C_p = C_t - (S_t - K_t\ )$$

$$p = \frac{e^{rt} - d}{u-d}$$

$$u = e^{\sigma\sqrt{t}}$$

$$d = e^{-\sigma\sqrt{t}}$$

Where:

S	=	property asset value per sq.ft.
K	=	construction / replacement cost per sq.ft.
C	=	option value of waiting
C_p	=	option premium value
p	=	probability of value moving upward
u	=	multiple of value moving upward
d	=	multiple of value moving downward
σ	=	volatility of value
r	=	risk-free rate
g	=	construction cost growth rate

Volatility of value or cost is calculated using historical data with the following equations:

$$\Delta S = \ln\left(\frac{S_t}{S_{t-0.25}}\right)$$

$$SD = \sqrt{\frac{\sum(S-\bar{S})^2}{n-1}}$$

$$\sigma = SD \times \sqrt{4} \qquad \text{(annualized volatility for quarterly data)}$$

See Figure 2.0 for the binomial tree setup.

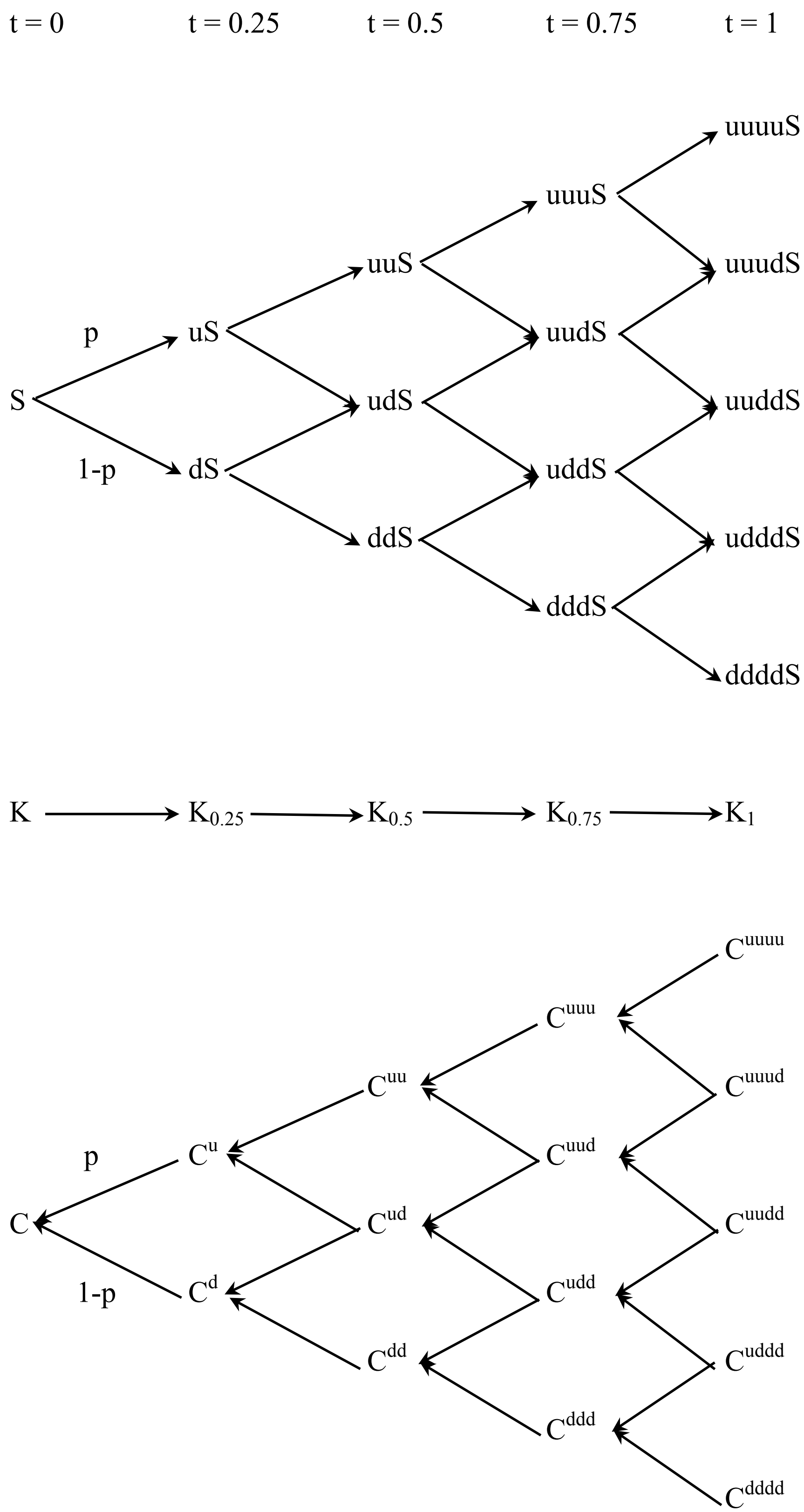

Figure 2.0: Real Options Binomial Tree Diagram

A numerical example is illustrated as follows. In a given city, the volatility of asset value of office buildings is 20% and the growth rate of construction or replacement cost is 5%. At t=0, the asset value of office building is \$100 per sq.ft. while the construction cost is \$70 per sq.ft. The Risk-free rate is assumed to be 5%. Using equations presented above, the real options binomial tree is as follows:

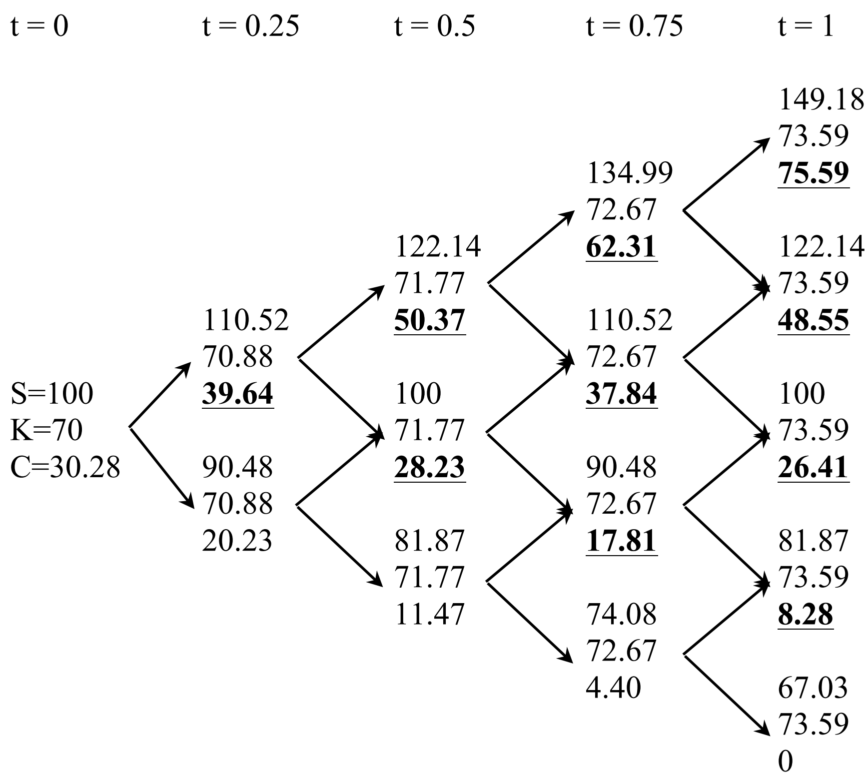

Figure 2.1: Numerical Example Binomial Tree Result

In this example, the residual land value is \$30 per sq.ft. while the option value of waiting is \$30.28 per sq.ft. The option premium value (C_p) is \$0.28 per sq.ft. This implies that office developers in this city should rationally wait to develop, rather than to exercise the option to develop at t=0. In theory, if the real options pricing model is applied to evaluate the land value, office developers in that city should be logically willing to pay up to \$30.28 per sq.ft. on average for a piece of land to account for the option value. In fact, Quigg, L. (1993) examines the empirical predictions of a real options pricing model using a large sample of market prices. She finds empirical supports for a model that incorporates the option to wait to develop land. The option model has explanatory power for predicting transactions prices over and above the residual value. Market prices reflect a premium for the option to wait to invest that has a mean value of 6% in their samples.

Option values shown in bold and underlined indicate nodes where early exercise is optimal. In other words, in those nodes, the option premium value is compressed to a point which equals the residual land value. Therefore, developers have no incentive to wait but to exercise the option to develop, as illustrated in the binomial tree below:

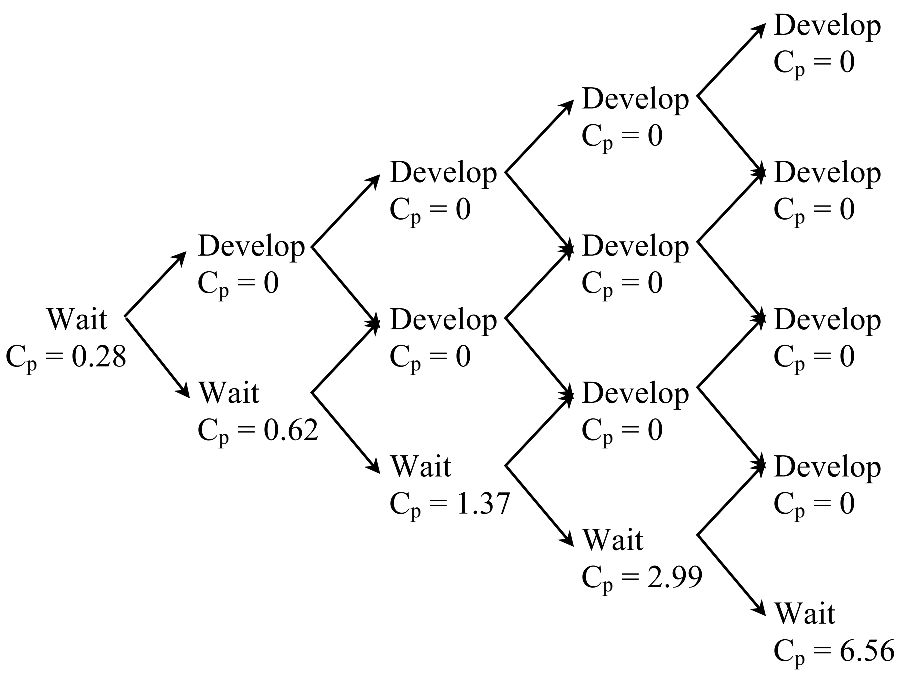

Figure 2.2: Optimal Actions Binomial Tree Result

2.1.2 Strategic implication of the real options pricing model

As illustrated with the model in Section 2.1.1, when the residual land value is less than the option value of waiting, developers should wait to develop. Based on the model, the magnitude of the option premium (C_p) is affected by the following variables: (1) the volatility of asset value, σ_s, (2) the growth rate of construction or replacement cost, g, (3) the maturity date of option, T, and (4) the risk-free rate, r.

(1) Volatility of asset value, σ_s

The option premium is an increasing function of the volatility of asset value:

$$\frac{\partial C_p}{\partial \sigma_s} > 0$$

As the volatility of asset value increases, the option premium value will increase as well. That makes intuitive sense because the option of waiting becomes more valuable if the future asset value is more uncertain. In cities with a higher volatility of asset values, more developers are expected to wait before they decide to exercise the option when asset values are more favorable to them.

(2) Growth rate of construction or replacement cost, g

The option premium is a decreasing function of the growth rate of the construction or replacement cost:

$$\frac{\partial C_p}{\partial g} < 0$$

As the growth rate of the construction or replacement cost increases, the option premium value will decrease. Intuitively, when the construction cost escalates faster, developers have a stronger incentive to start construction sooner rather than later. The option value of waiting is diminished if the growth rate of the construction cost is high. Therefore, in cities where the construction or replacement cost increases faster, more developers are expected to exercise the development option earlier.

(3) Maturity date of option, T

The option premium is an increasing function of the maturity date of option, T:

$$\frac{\partial \mathrm{C_p}}{\partial T} > 0$$

As the duration of option-exercising periods increases, the option premium value will increase. Conceptually, if external factors in the market allow developers to wait longer, the option premium will be higher, all things being equal. The Maturity date of the real options in real estate development can be considered a time by which developers lose the flexibility of choosing between developing immediately and waiting to develop. That maturity date depends on external factors in the market such as the level of competitions, land policy revisions, and a shift in market demand and internal factors of the developer such as the corporate investment time frame, the development schedule, and various business strategies. Grenadier, S. R., and Wang, N. (2006) interestingly show that developers are very impatient about choices in the short-term, but are quite patient when choosing between long-term alternatives. That paper indirectly demonstrates that when the maturity date of the option is further away from now, the option premium value will be higher and thus developers tend to keep the option of waiting more. It is noted that in this study, the maturity date of option is kept the same for comparison purposes across different cities and property types.

(4) Risk-free rate, r

The option premium is an increasing function of the risk-free rate, r:

$$\frac{\partial \mathrm{C_p}}{\partial r} > 0$$

As the risk-free rate increases, the option premium value will increase. Intuitively, when the risk-free rate increases, the opportunity cost of investment will increase too. That makes the option value of waiting higher, vice versa. Therefore, when the risk-free rate is low, more developers will tend to exercise their option to develop earlier. It is noted that this model assumes a constant risk-free rate for comparison purposes across different cities and property types.

As the volatility of asset value and the growth rate of the construction or replacement cost are different among different cities and different property types, the timing behavior of developers are influenced by different option premium values under different situations. This thesis will examine if developers behave according to results predicted by the real options pricing model presented in previous sections. Then, the game theory presented in coming sections will be applied to understand further any discrepancy found in our empirical study.

2.2 Game Theory

The game theory is the study of strategic decision-making. It is a discipline that studies situations of strategic interaction. In many real-world situations, the action of one player affects the pay-off of other players. The application of the game theory provides a strategic framework to analyze the best response of each player's action. In the game theory, a solution concept is a formal rule predicting which strategies will be adopted by players, therefore predicting the result of the game. A strategy consists of a rule specifying which actions a player should take given the actions taken by other players. The most commonly used solution concepts are equilibrium concepts. An equilibrium is a configuration of strategies where each player's strategy is his best response to the strategies of all the other players. The principle concept is illustrated in the following simplified example.

In an efficient market, assuming it is a symmetric game, both rational player 1 and player 2 have the option to invest now or to wait to invest in next period. The pay-off matrix of player 1 and player 2 is shown in Table 2.0, with pay-offs on the left referring to those of player 2 while pay-offs on the right referring to those of player 1.

		Player 1	
		Invest	Wait
Player 2	Invest	3 , 3	4 , 2
	Wait	2 , 4	1 , 1

Table 2.0: Game Payoff Matrix Example

The game pay-off matrix describes four possible outcomes. If both players invest, each of them gets a pay-off equal to 3. If player 1 invests while player 2 waits, they will get pay-offs of 4 and 2 respectively. If both players wait, they will each get a pay-off of 1. In this symmetric game, when player 1 invests, the best response of player 2 is to invest as well to maximize its pay-off. When player 1 waits, the best response of player 2 is to invest because the pay-off of investing now is bigger than that of waiting. Applying this logic to all possible scenarios, there is an equilibrium, as indicated with a bold box, in this game, which is that both players should invest now. In this simplified symmetric game, the application of game theoretical principles shows that the best response for both players is to invest now because that is the equilibrium of this game. A few other examples in later sections will show that there may be multiple equilibria in other games and discuss its implications.

2.2.1 Payoff Matrix Setup for Real Estate Development

In real estate development, the real-world situation is usually too complex for one model to analyze the optimal development strategy accurately. However, the interaction of the real option valuation and the game theory can provide a strategic framework to help better understand the interactive investment opportunity and to offer important strategic implications. The following model will be used to analyze the empirical data presented in this thesis.

In this simplified setting, two developers have the option to develop now or to wait to develop in next period. If a developer develops, its pay-off is defined as the property asset value minus the construction or replacement cost. If a developer waits, its pay-off is defined as the option value of waiting. See Table 2.1 for the pay-off matrix of these two developers.

		Developer 1	
		Develop	Wait
Developer 2	Develop	$\omega_{D2}\, S_t - K_t e^{-rt}$, $\omega_{D1}\, S_t - K_t e^{-rt}$	$\omega_{L2}\, S_t - K_t e^{-rt}$, C_{t1}^{fw}
	Wait	C_{t2}^{fw} , $\omega_{L1}\, S_t - K_t e^{-rt}$	C_{t2} , C_{t1}

Table 2.1: Game Theory Model Setup

Where:	S_t	= property asset value per sq.ft. at time = t
	K_t	= construction / replacement cost per sq.ft. at time = t
	ω_{D1}	= proportion of value for developer 1 when both developers develop
	ω_{D2}	= proportion of value for developer 2 when both developers develop
	ω_{L1}	= proportion of value for developer 1 when developer 1 develops first (Leader)
	ω_{L2}	= proportion of value for developer 2 when developer 2 develops first (Leader)
	ω_{F1}	= proportion of value for developer 1 when developer 1 waits (Follower)
	ω_{F2}	= proportion of value for developer 2 when developer 2 waits (Follower)
	C_{t1}	= option value of waiting for developer 1 when both developers wait (no Leader)
	C_{t2}	= option value of waiting for developer 2 when both developers wait (no Leader)
	C_{t1}^{fw}	= option value of waiting for developer 1 when developer 1 waits (Follower) $= \max(\omega_{F1} S_{t+1} - K_{t+1}e^{-r(t+1)}, e^{-r(t+1)} [q\, C^{u}_{(t+1)1}{}^{fw} + (1\text{-}q)\, C^{d}_{(t+1)1}{}^{fw}])$
	C_{t2}^{fw}	= option value of waiting for developer 2 when developer 2 waits (Follower) $= \max(\omega_{F2} S_{t+1} - K_{t+1}e^{-r(t+1)}, e^{-r(t+1)} [q\, C^{u}_{(t+1)2}{}^{fw} + (1\text{-}q)\, C^{d}_{(t+1)2}{}^{fw}])$

The pay-off matrix is designed to capture a few important elements in the strategic planning process of real estate development. At a conceptual level, the simplified model reflects factors including the nature of developers, the nature of market, and the nature of timing.

Firstly, for example, if two office developers are players of the game, when they decide to develop at the same time, their pay-offs will depend on their strength to capture the market demand, which will be reflected by ω_{D1} and ω_{D2} (where $\omega_{D1} < \omega_{D2} < 1$ if developer 2 is a stronger player). When two complementary developers, such as office and retail developers, are players of the game, the pie will actually grow bigger when they develop at the same time, reflected by ω_{D1} and ω_{D2} (where $\omega_{D1} \leq \geq \omega_{D2} > 1$). Therefore, the nature of developers would affect their pay-offs, which affects their optimal strategies accordingly.

Secondly, market conditions affect how developers react to the market demand. For example, if the demand for residential space in certain submarkets is finite and known, developing alone will always yield a higher pay-off than competing with other developers to split the market, as indicated by ω_{D1} and ω_{L1} (where $\omega_{D1} < \omega_{L1}$). In a different scenario, if the demand for retail space will increase through collaboration, retail developers will both get a higher pay-off when they develop together, as indicated by ω_{D1} and ω_{L1} (where $\omega_{D1} > \omega_{L1}$). Therefore, the nature of market has strategic implications for actions of developers from a game theoretic perspective.

Thirdly, like in other types of investments, the timing is essential to successful development projects. It is instrumental in determining the pay-offs in interactive games. For example, if the demand for more office space is obvious and the volatility of office building value is low, developers have less incentive to wait, i.e. they would rather be a Leader than a Follower to capture the First Mover Advantage (FMA) as reflected by ω_{L1} and ω_{F1} (where $\omega_{L1} > \omega_{F1}$). However, if the demand is uncertain and the new market requires synergy to enhance new demand, developers have more incentive to wait, i.e. they would rather be a Follower than a Leader to capture the Second Mover Advantage (SMA) as reflected by ω_{L1} and ω_{F1} (where $\omega_{L1} < \omega_{F1}$). Therefore, the nature of timing affects interactive decisions of developers. As illustrated above, these three important factors are indicated by variables ω_{Di}, ω_{Li}, and ω_{Fi} in the model. In the next section, their strategic implications will be discussed based on different multi-equilibria scenarios and a numerical example will be used at the end of the section to tie the real options valuation model and the game theory model together to make the framework clearer.

2.2.2 Strategic implication of the Real Estate Development Game Theory Payoff Matrix

From a game theoretic perspective, the action of Developer 1 and that of Developer 2 are mutually affected by each other. If both developers are rational players, they should all choose the equilibrium action to maximize their pay-offs. Once in equilibrium, no one can improve its pay-off by switching its decision. In the context of real estate development, the real option is irreversible. That is, once a developer decides to "develop", it cannot switch back to the "wait" option. Therefore, developers are sometimes stuck in the sub-optimal situation. Based on the simplified model presented in the previous section, there are four equilibrium or multi-equilibria scenarios in a symmetric game with different variations in an asymmetric game. See Tables 2.2a to 2.2d for the four scenarios.

		Developer 1	
		Develop	Wait
Developer 2	Develop	C , C'	A , D'
	Wait	D , A'	B , B'

Table 2.2a: Scenario 1: Develop-Develop Equilibrium

		Developer 1	
		Develop	Wait
Developer 2	Develop	B , B'	D , C'
	Wait	C , D'	A , A'

Table 2.2b: Scenario 2: Develop-Develop / Wait-Wait Equilibrium

Developer 1

		Develop	Wait
Developer 2	Develop	B , B’	$\boxed{\text{C , A’}}$
	Wait	$\boxed{\text{A , C’}}$	D , D’

Table 2.2c: Scenario 3: Develop-Wait / Wait-Develop Equilibrium

Developer 1

		Develop	Wait
Developer 2	Develop	D , D’	B , C’
	Wait	C , B’	$\boxed{\text{A , A’}}$

Table 2.2d: Scenario 4: Wait-Wait Equilibrium

Where: Value of A > B > C > D and

Value of A’ > B’ > C’ > D’ and

Whether A > A’ or B > B’ or C > C’ or D > D’ depends

on relative value of ω_{D1} & ω_{D2}, ω_{L1} & ω_{L2}, and ω_{F1} & ω_{F2}.

As explained in section 2.2, the equilibrium state in this model can be determined by comparing the pay-offs of each player given the action taken by another player. For example, in scenario 1, if Developer 1 decides to “develop”, the best response of Developer 2 should be to “develop” as well because C > D. If Developer 1 decides to “wait”, the best response of Developer 2 should still be to “develop” because A > B. Meanwhile, if Developer 2 decides to “develop”, the best response of Developer 1 should be to “develop” as well because C’ > D’. If Developer 2 decides to “wait”, the best response of Developer 1 should still be to “develop” because A’ > B’. In other words, there is a dominant strategy in scenario 1, which is to “develop”. In the model presented in Section 2.2.1, there would be only four possible equilibrium scenarios: Develop-Develop Equilibrium, Develop-Develop / Wait-Wait Equilibrium, Develop-Wait / Wait-

Develop Equilibrium, and Wait-Wait Equilibrium. Within each equilibrium, there are different variations of pay-off matrices, which depend on the relative value of ω_{Di}, ω_{Li}, and ω_{Fi}. To simplify the analysis process, only the specific cases represented above will be discussed. However, the same logic and analysis approach can be applied to all matrix variations.

Scenario 1: Develop-Develop Equilibrium

In this scenario, several characteristics of the nature of market and the nature of timing can be observed from the pay-off matrix. Developers can get the highest pay-off when they decide to develop first while the other party decides to wait. It is clear that there are First Mover Advantages (FMA) in this market. However, as indicated in the matrix above, Develop-Wait is not an equilibrium state. The other developer will not choose to wait (getting a D), but rationally decide to compete instead (getting a C). It is also interesting to observe that the Develop-Develop Equilibrium does not yield the highest possible pay-offs to both parties. Both developers can get a higher pay-off by choosing to wait together in this period. It is a classic Prisoner's Dilemma situation. Similarly in real estate development, collaborations between developers can actually yield a higher return for both players. However, this is not a stable equilibrium state because either party always has a strong incentive to cheat (to develop first in order to gain FMA). Once one developer cheats, the other will rationally choose to develop as well, and thus revert the game back to an equilibrium state. At a conceptual level, it explains why developers choose to develop to compete even though the option value of waiting is actually higher, especially if there is a clear First Mover Advantage in this market.

Scenario 2: Develop-Develop / Wait-Wait Equilibrium

The market implication in this multi-equilibria scenario is somewhat similar to scenario 1. The key difference is that there is no FMA in this market as indicated by a lower pay-off if one developer develops first ($\omega_{Li} < \omega_{Fi}$). This market implies a much higher volatility of asset values, which makes the waiting option much more valuable. However, collaboration between developers to develop together seems to create a good synergy for high pay-offs too ($\omega_{Di} > \omega_{Li}$ and $\omega_{Di} > \omega_{Fi}$). In multi-equilibria games, determining which action to take usually involves tactics of signaling and commitment. In this particular case, both developers get a higher pay-off if they decide to wait together in this period. This usually happens if the market is highly volatile and there is uncertainty about current and future demand.

Scenario 3: Develop-Wait / Wait-Develop Equilibrium

This market is characterized by a strong Second Mover Advantage (SMA). The developer who decides to wait gets a higher pay-off than the developer who develops first ($\omega_{Li} << \omega_{Fi}$). The pay-off matrix implies that this market has a low volatility of asset values because the option values for both developers to wait are low. There is a clear demand for development but exogenous factors make the one who makes the first move much riskier than the one who follows. If the game is in a particular site, it will be similar to the game of chicken, in which neither party wants to develop first, but waiting together is even worse because of the loss of profit opportunity. This situation usually happens to developers who complement each other. For example, residential developers want to have enough retail activities to drive a higher residential demand before they develop while retail developers want to have enough residents to drive a higher shopping demand before they develop.

Scenario 4: Wait-Wait Equilibrium

In this market, a combination of very weak demand and high volatility of asset value contributes to very high option values of waiting. Waiting is the dominant strategy for both developers in this market, and developing to compete yields a very low pay-off (very low ω_{Di}). This market usually occurs during recession. Since $\omega_{Di} < \omega_{Li}$, when the option value of waiting falls below certain trigger values, developers will want to develop first to capture FMA. Gradually, it will lead to a development cascade when all developers decide to develop, partially creating the cyclic nature of real estate development.

A numerical example is illustrated as followed. Using the same assumptions as presented in section 2.1.1, in a given city, the volatility of the asset value of office buildings is 20% and the growth rate of the construction or replacement cost is 5%. At t=0, the asset value of office building is \$100 per sq.ft. while the construction cost is \$70 per sq.ft. The risk-free rate is assumed to be 5%. In this city, assuming office developer 1 is a stronger developer and is better able to capture the market demand if it competes with office developer 2, then ω_{D1} will be bigger than ω_{D2}. In this example, ω_{D1} is 0.9 and ω_{D2} is 0.85. In this market, there is also a clear First Mover Advantage (FMA) because of the obvious, but finite, office demand. That implies that whoever develops first will benefit from the market condition. Therefore, ω_{Li} will be bigger than ω_{Di}, and in turn, ω_{Di} will be bigger than ω_{Fi}. Since developer 1 is a stronger player than developer 2, it can be assumed that ω_{L1} is 1.15, ω_{L2} is 1.1, while ω_{F1} is 0.8, and ω_{F2} is 0.75. Using the real options valuation model presented in 2.1.1 to calculate C_{t1}, C_{t2}, C_{t1}^{fw} and C_{t2}^{fw}, where C_{ti} is the option value of waiting for developer i when both developers wait and C_{ti}^{fw} is the option value of waiting for developer i when developer i waits while the other developer develops (= max ($\omega_{F1}\ S_{t+1} - K_{t+1}e^{-r(t+1)}$, $e^{-r(t+1)}$ [q $C^{u}_{(t+1)1}{}^{fw}$ + (1-q) $C^{d}_{(t+1)1}{}^{fw}$]), the game theory payoff matrix is shown as follows:

Developer 1

		Develop	Wait
Developer 2	Develop	15.00 , 20.00	40.00 , 12.50
	Wait	8.47 , 45.00	30.28 , 30.28

Table 2.3: Numerical Example Game Payoff Matrix Result

As the payoff matrix shows, the equilibrium between developer 1 and developer 2 is that they will develop together. This market is as depicted in Scenario 1: Develop-Develop Equilibrium. It is a classic Prisoner's Dilemma situation. In this case, the game theory explains why developers still choose to develop even though their option value of waiting is actually higher.

As illustrated above, the intrinsic values of ω_{Di}, ω_{Li}, and ω_{Fi} shape different market conditions. Together with the option value of waiting, which is largely influenced by the volatility of asset value in certain given markets, the game matrix creates a rich framework to analyze the timing strategy of real estate development at a strategic level. However, real-world situations are usually more complex than this simplified model. A few previous works by Grenadier (1996), Trigeorgis (1996) and Schwartz and Torous (2007) study some aspects of this option-game theoretic approach. It is particularly insightful to see how competitions erode option values and create a force to switch Wait-Wait Equilibrium as indicated in scenario 3 to Develop-Develop Equilibrium. In the next section, a more comprehensive and in-depth review will be presented to show how previous works have contributed to the development and understanding of the real options valuation and the game theory, particularly in the context of real estate development.

3.0 Chapter 3: Literature review

3.1 Real Options in Real Estate Development

Since the 1980s, there has been much interest in academic research about real options valuation methods and game theory application in the business world. A vast array of models and frameworks have been studied and proposed. Option theory was first applied to real estate by Titman (1985). "Urban Land Prices under Uncertainty" by Titman (1985) provides a valuation model for pricing vacant lots in urban areas. An implication of this relationship between uncertainty and vacant land values is that increased uncertainty leads to a decrease in building activity in the current period. This model also provides insights into the role of real estate speculators who purchase vacant lots, and rather than develop them immediately, choose to keep them vacant for a period of time. The framework developed in this paper can also be extended to analyze other issues relating to real estate pricing under uncertainty. For example, it can be used to determine optimal time to demolish smaller building for redevelopment, and be used to analyze the effect of uncertainty on the optimal durability of buildings. This paper applies option valuation methods developed by Black and Scholes (1973) and Merton (1973). "Real Estate Development as an Option" by Williams (1991) solves the option pricing problem analytically and numerically for the optimal date and density of development, the optimal date of abandonment, and the resulting market values of the developed and undeveloped properties. "Mixed-Uses and the Redevelopment Option." by Childs, P. D., Riddiough, T., and Triantis, A. J. (1995) considers how the potential for mixing uses and redevelopment impact property value. Operating flexibility of this type is found to significantly increase property value when the correlation between payouts from different property types is low or when redevelopment costs are low. The ability to mix uses and redevelop over time is also shown to affect the timing of initial land development.

3.2 Empirical Testing of Real Options Models in Real Estate Development

Until Quigg (1993), there had not been much empirical studies of real option models in real estate development. "Empirical Testing of Real Option-Pricing Models" by Quigg, L. (1993) examines the empirical predictions of a real option-pricing model using a large sample of market prices. She finds empirical support for a model that incorporates the option to wait to develop land. The option model has explanatory power for predicting transactions prices over and above the intrinsic value. Market prices reflect a premium for the option to wait to invest that has a mean value of 6% in their sample. She also estimates the implied standard deviations for individual commercial property prices ranging from 18 to 28% per years. "Uncertainty and the Rate of Commercial Real Estate Development." by Holland, S., Ott, S., and Riddiough, T. (1995) empirically examines the relationship between uncertainty and investment using commercial real estate data. To sort out long- versus short-run effects of asset volatility on investment decisions, they extend the standard real options model to determine the probability of investment over a particular time horizon. In doing so, they find that an increase in asset volatility can either increase or decrease the probability of development, although the anticipated negative short-run relationship is confirmed when the land is "ripe" for development (i.e., near the development hurdle value). The role of uncertainty in determining the rate of real investment is then tested using aggregate data. By developing two measures of property value volatility, they empirically confirm the expected strong relationship between changes in uncertainty and the rate of development activity. "Effects of Uncertainty on the Investment Decision: An Examination of the Option-Based Investment Model Using Japanese Real Estate." by Yoshida, J. (1999) examines the validity of the option-based investment model as opposed to the neoclassical investment model in the decision-making of commercial real estate development, using aggregate real estate data from Japan, focusing on the effect of uncertainty. It concludes that various kinds of real options must be incorporated in investment and economic models. "Empirical Testing of Real Option-Pricing Models Using Land Price Index in Japan." by Yamazaki, R. (2000) examines the way uncertainty plays a role in built land prices. This paper provides basic real option pricing models of land prices on the demand side in central Tokyo. The model in this research analyzes micro land prices covering individual lot data provided by the Land Price Index. Since land prices are determined by both macro economic environment and micro lot-specific attributes, this paper utilizes both time-series economic data and cross-sectional (micro) data including uncertainty terms. In addition to the total uncertainty in asset prices, this research also gives some ideas of cross-sectional uncertainty in land price variations by utilizing cross-sectional amenity variables. These cross-sectional and time-series variables including these two uncertainty variables are pooled and the Ordinary Least Squares method is conducted. The results from the option-based models favor the application of the real option theory in land prices. The total uncertainty with respect to built asset return has a substantial effect on increasing land prices, which implies that an increase in uncertainty leads to an increase in land prices.

3.3 Game Theory in Real Estate Development

With the real options theory studied extensively as applied to real estate development and the business world in general, scholars started to question its incomplete application and limitation. Strategic interaction seems to have huge implication of the explanatory power of the real options model. That is how game theory is used to extend the real options model. "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets" by Grenadier, S. R. (1996) develops an equilibrium framework for strategic option exercise games. He focuses on a particular example: the timing of real estate development. An analysis of the equilibrium exercise policies of developers provides insights into the forces that shape market behavior. The model isolates the factors that make some markets prone to bursts of concentrated development. The model also provides an explanation for why some markets may experience building booms in the face of declining demand and property values. While such behavior is often regarded as irrational overbuilding, the model provides a rational foundation for such exercise patterns. "Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms." by Grenadier, S. R. (2000) provides a very general and tractable solution approach for deriving the equilibrium investment strategies of firms in a continuous-time Cournot-Nash framework. The impact of competition on exercise strategies is dramatic. For example, while standard real options models emphasize that a valuable "option to wait" leads firms to invest only at large positive NPV, the impact of competition drastically erodes the value of the option to wait and leads to investment at very near the zero NPV threshold. The Nash equilibrium exercise strategies are shown to display the useful property that they are equivalent to those derived in an "artificial" perfectly competitive industry under a modified demand curve. This transformation permits a simplified solution approach for the inclusion of various realistic features into the model, such as time-to-build. "An Equilibrium Analysis of Real Estate Leases" by Grenadier, S. R. (2003) provides a unified equilibrium approach to valuing a wide variety of commercial real estate lease contracts. Using a game-theoretic variant of real options analysis, the underlying real estate asset market is modeled as a continuous-time Nash equilibrium in which developers make construction decisions under demand uncertainty. Then, using the economic notion that leasing simply represents the purchase of the use of the asset over a specified time frame, it uses a contingent-claims approach to value many of the most common real estate leasing arrangements. "Investment Under Uncertainty and Time-Inconsistent Preferences" by Grenadier, S. R., and Wang, N. (2006) extends the real options framework to model the investment timing decisions of entrepreneurs with such time-inconsistent preferences; developers are very impatient about choices in the short-term, but are quite patient when choosing between long-term alternatives. Two opposing forces determine investment timing: while evolving uncertainty induces entrepreneurs to defer investment in order to take advantage of the option to wait, their time-inconsistent preferences motivate them to invest earlier in order to avoid the time-inconsistent behavior they will display in the future. They find that the precise trade-off between these two forces depends on such factors as whether entrepreneurs are sophisticated or naïve in their expectations regarding their future time-inconsistent behavior, as well as whether the payoff from investment occurs all at once or over time. They extend the model to consider equilibrium investment behavior for an industry comprised of time-inconsistent entrepreneurs. Such an equilibrium involves the dual problem of entrepreneurs playing dynamic games against competitors as well as against their own future selves. "Real Options and Games: Competition, Alliances and Other Applications of Valuation and Strategy." by Smit, H., and Trigeorgis, L. (2006) illustrates the use of real options valuation and game theory principles to analyze prototypical investment opportunities involving important competitive / strategic decisions under uncertainty. It uses examples from innovation cases, alliances and acquisitions to discuss strategic and competitive aspects, relevant in a range of industries like consumer electronics and telecom. It particularly focuses on whether it is optimal to compete independently or coordinate / collaborate via strategic alliances. “Commercial Office Space: Tests of a Real Options Model with Competitive Interactions.” by Schwartz, E. S., and Torous, W. N. (2007) tests a real options model with competitive interactions using an extensive commercial real estate data base. The competitive nature of the local real estate market as proxied by the market’s Herfindahl ratio is found to have a significant effect on building starts: larger values of the Herfindahl ratio, consistent with less competition, are associated with fewer building starts. In particular, a one standard deviation increase in this ratio leads to a 25.9% decreases in the number of new building starts. Other variables suggested by the real options model, such as the volatility of local lease rates, are also found to be important. “Irreversible investment, real options, and competition: Evidence from real estate development.” by Bulan, L., Mayer, C., and Somerville, C. (2008) examines the extent to which uncertainty delays investment, and the effect of competition on this relationship, using a sample of 1214 condominium developments in Vancouver, Canada built from 1979 to1998. They find that increases in both idiosyncratic and systematic risk lead developers to delay new real estate investments. Empirically, a one-standard deviation increase in the return volatility reduces the probability of investment by 13 percent, equivalent to a 9 percent decline in real prices. Increases in the number of potential competitors located near a project negate the negative relationship between idiosyncratic risk and development. These results support models in which competition erodes option values and provide clear evidence for the real options framework over alternatives such as the neoclassical NPV valuation method.

3.4 Land Value Studies

"Insights on the Effect of Land Use Choice: The Perpetual Option on the Best of Two Underlying Assets" by Geltner, D., Riddiough, T. and Stojanovic, S (1996) considers the effect of land use choice on speculative land value and on development timing as reflected in the optimal "hurdle ratio" which triggers immediate development. They found that land use choice may add over 40 percent to land value under typical economic circumstances. The conditions for optimal development of the land become markedly more difficult to achieve when the two land uses have similar values. In fact, development will never occur when the two land use choices have equal value. "Swings in Commercial and Residential Land Prices in the United States" by Mulhall, M., Nichols, J., and Oliner, S. (2012) uses a large dataset of land sales dating back to the mid-1990s to construct land price indexes for 23 MSAs in the United States and for the aggregate of those MSAs. The price indexes show a dramatic increase in both commercial and residential land prices over several years prior to their peak in 2006-07 and a steep descent since then. These fluctuations have exceeded those in well-known indexes of home prices and commercial real estate prices. Because those indexes price a bundle of land and structures, this comparison implies that land prices have been more volatile than structures prices over this period. This result is a key element of the land leverage hypothesis, which holds that home prices and commercial property prices will be more volatile, all else equal, in areas where land represents a larger share of real estate value.

4.0 Chapter 4: Methodology & Data Collection

4.1 Commercial Real Estate Asset and Land Value Data

Asset values of 4 property types (Apartment, Industrial, Office, and Retail) in 30 Metropolitan Statistical Areas (MSA) spanning from 2001 to 2013 are collected from Real Capital Analytics, Inc (RCA) for the commercial portion of the study. The construction or replacement cost data spanning from 1993 to 2013 is collected from RS Means. The residual land value is defined as the difference between the asset value and the construction or replacement cost. For the commercial portion, the construction activity is measured by number of square feet completed (industrial, office, and retail) and number of units completed (apartment), with data collected from CBRE.

The volatility (σ) of value or cost is calculated using historical data with the following equations:

$$\Delta S = \ln\left(\frac{S_t}{S_{t-0.25}}\right)$$

$$SD = \sqrt{\frac{\Sigma(S-\bar{S})^2}{n-1}}$$

$$\sigma = SD \times \sqrt{4} \quad \text{(annualized volatility for quarterly data)}$$

4.1.1 Use of Real Capital Analytics (RCA) data

The RCA asset value data is transaction-based, not appraisal-based, and it is based on independent reports of properties and portfolios \$2.5 million and greater. Quarterly non-smoothed data from 2001 to 2013 is used in the analysis of this paper.

4.1.2 Comparison between 30 Metropolitan Statistical Areas (MSAs)

Midwest	South	East	West
Chicago	Atlanta	Baltimore	Los Angeles
Cincinnati	Charlotte	Boston	Portland
Cleveland	Dallas	Miami	San Diego
Columbus	Denver	New York	San Francisco
Detroit	Houston	Philadelphia	San Jose
Indianapolis	Memphis	Washington, D.C.	Seattle
Kansas City	Phoenix		
Minneapolis	Tampa		
Pittsburgh			
St. Louis			

Table 4.0: 30 Metropolitan Statistical Areas for Commercial Markets Analysis

Regarding the commercial markets, in general, RCA data shows that major U.S. cities such as Los Angeles, Washington DC, New York, and Boston are less volatile and less speculative than other second-tier or third-tier cities such as Pittsburgh, Detroit, Memphis, and St Louis. Overall, the volatility of the West region is the lowest and the Midwest region is the highest. RCA data demonstrates higher volatility than expected, which is probably due to idiosyncratic variations of the transaction-based data. However, for the purpose of this thesis, which studies the timing strategy of developers, transaction-based data with higher volatility across all MSAs and property types is still consistent enough to be used for comparison purposes.

Regarding the land cost ratio, it is defined as the ratio of the residual land value to the asset value. RCA data demonstrates that the asset value volatility is negatively correlated with the land cost ratio. In other words, cities such as Washington DC, New York, San Francisco, and Boston with low volatility have high land cost ratios. Cities such as Pittsburgh, Detroit, and Columbus with high volatility have low land cost ratios. In section 5.0, the relationship between these variables and the timing strategy will be discussed empirically.

4.1.3 Comparison between 4 property types

Among the 4 property types (Apartment, Industrial, Office, and Retail), RCA data shows that the asset value volatility of office buildings is the lowest, followed by industrial buildings and retails. Apartment buildings have the highest volatility. For the office market, the East and West regions have the lowest volatility, while the Midwest region has the highest volatility. For the apartment and retail markets, the South and West regions have the lowest volatility, while the Midwest region again has the highest volatility. For the industrial market, the South and West regions again have the lowest volatility, while the East region has the highest volatility.

Regarding the land cost ratio, for the office and apartment markets, the East region has the highest land cost ratio, followed by the West and South regions. The Midwest region has the lowest land cost ratio. For the retail market, the West region has the highest land cost ratio, followed by the East and South regions. The Midwest region again has the lowest land cost ratio. For the industrial market, the East region has the lowest land cost ratio. At the property type aggregate level, the data again shows that the asset value volatility is negatively correlated with the land cost ratio. Office buildings demonstrate the lowest volatility and high land cost ratios. Apartment buildings have the highest volatility and low land cost ratios. It is also true at the regional level, the East region has the lowest volatility, but the highest land cost ratio, for the office market. The regression analyses yield the following results:

$$\sigma = \alpha + \beta\ (LCR)$$

Where: σ = Asset Value Volatility

LCR = Land Cost Ratio ($\frac{S-K}{S}$)

	$\alpha =$ *(P-value)*	$\beta =$ *(P-value)*
Overall	1.13 *(4.59×10^{-49})*	-0.0377 *(6.37×10^{-1})*
Apartment	1.26 *(1.79×10^{-14})*	-0.119 *(5.16×10^{-1})*
Industrial	1.14 *(6.49×10^{-10})*	0.186 *(2.84×10^{-1})*
Office	1.23 *(5.59×10^{-13})*	-0.623 *(5.17×10^{-3})*
Retail	1.38 *(1.88×10^{-9})*	-0.426 *(1.65×10^{-1})*

Table 4.1: Volatility and Land Cost Ratio Regression Results for Commercial Markets

With the results shown above, overall, except for the industrial market, the volatility is statistically negatively correlated with the land cost ratio:

$$\frac{\partial \sigma}{\partial LCR} < 0$$

However, they are only weakly correlated, as indicated by the high P-values. The office market seems to be the only exception. Nonetheless, for any given MSA and property type, except for the industrial market, if the land value constitutes a small portion of the asset value, that asset type in that given market tends to be more volatile and relatively riskier to develop. The relationship between the volatility and the land cost ratio across 30 MSAs and 4 property types is summarized below:

		Asset Value Volatility	Land Cost Ratio
EAST	Overall	2	2
	Apartment	2	1
	Industrial	1	2
	Office	3	1
	Retail	2	2
MIDWEST	Overall	1	4
	Apartment	1	4
	Industrial	2	4
	Office	1	4
	Retail	1	4
SOUTH	Overall	3	3
	Apartment	3	3
	Industrial	3	3
	Office	2	3
	Retail	3	3
WEST	Overall	4	1
	Apartment	4	2
	Industrial	4	1
	Office	4	2
	Retail	4	1

Table 4.2: Volatility and Land Cost Ratio Summary for Commercial Markets

Where: 1 = highest

4 = lowest

4.2 Residential Real Estate Asset and Land Value Data

Asset values of single-family houses in 44 MSAs spanning quarterly from 1995 to 2013 are collected from the Lincoln Institute of Land Policy for the residential portion of the study. For the residential portion, the construction activity is measured by number of permits issued for single-family houses, with data collected from U.S. Census Bureau. Again, the volatility is calculated using the same method as presented in section 4.1. The land cost ratio is defined as the ratio of the residual land value to the house value.

4.2.1 Use of the Lincoln Institute of Land Policy (LILP) data

The data provided here contains estimates of the average value of housing, land, and structures, and price indexes for land and housing, for the average single-family detached owner-occupied housing unit in each of 44 large metropolitan areas in the United States.

The land price and quantity data are derived from data on housing values and structures costs, and from price indexes for housing and construction costs. For each of the included 44 metropolitan areas, house values and construction costs are derived using micro data from the Metropolitan American Housing Survey in a benchmark year. House values are reported directly in that survey, and construction costs are based on the age and square footage of the house. House prices are extrapolated forwards and backwards from the benchmark year using metro-area CMHPI and Case-Shiller-Weiss (when available) house price indexes. Construction costs are extrapolated forwards and backwards from the benchmark year indexes using construction cost indexes published by the R.S. Means Corporation.

4.2.2 Comparison between 44 metropolitan US cities

Midwest	**South**	**East**	**West**
Buffalo	Atlanta	Baltimore	Los Angeles
Chicago	Birmingham	Boston	Oakland
Cincinnati	Charlotte	Hartford	Portland
Cleveland	Dallas	Miami	Sacramento
Columbus	Denver	New York	San Diego
Detroit	Fort Worth	Norfolk	San Francisco
Indianapolis	Houston	Philadelphia	San Jose
Kansas City	Memphis	Providence	Seattle
Milwaukee	New Orleans	Washington, D.C.	
Minneapolis	Oklahoma City		
Pittsburgh	Phoenix		
Rochester	Salt Lake City		
St. Louis	San Antonio		
	Tampa		

Table 4.3: 44 Metropolitan Statistical Areas for Residential Markets Analysis

Regarding the single-family house market, LILP data shows opposite relationships compared to the commercial markets. Major U.S. cities such as Los Angeles, San Francisco, New York, and Boston are more volatile and more speculative than other second-tier or third-tier cities such as Pittsburgh, Detroit, Memphis, and St Louis. Overall, the volatility of the West region is the highest and the Midwest region is the lowest. LILP data demonstrates lower volatility than expected, which is probably due to the inherit smoothing effect of the appraisal-based data. However, for the purpose of this thesis, which studies the timing strategy of developers, appraisal-based data with lower volatility across all MSAs is still consistent enough to be used for comparison purposes.

Regarding the land cost ratio, the LILP data demonstrates that the volatility of house values is positively correlated with the land cost ratio. In other words, cities such as Washington DC, New York, San Francisco, and Boston with high volatility have high land cost ratios. Cities such as Pittsburgh, Detroit, and Columbus with low volatility have low land cost ratios. The regression analysis yields the following result, with P-values shown in parentheses:

$$\sigma = \underset{(2.11 \times 10^{-11})}{0.0265} + \underset{(1.06 \times 10^{-6})}{0.0402}\ LCR$$

Where: σ = Asset Value Volatility

LCR = Land Cost Ratio ($\frac{S-K}{S}$)

With the result shown above, it is confirmed that the volatility is statistically positively correlated with land cost ratio:

$$\frac{\partial \sigma}{\partial LCR} > 0$$

In other words, for single-family house markets in any given MSA, if the land value constitutes a large portion of the house value, the residential market in that given market tends to be more volatile and relatively riskier to develop. Compared to the correlation between the volatility and the land cost ratio in the commercial markets as shown in section 4.1.3, the regression result shows a much stronger correlation for the residential market, as indicated by the much lower P-values. The relationship between the volatility and the land cost ratio across 44 MSAs in the residential market is summarized below:

	Asset Value Volatility	Land Cost Ratio
EAST	2	2
MIDWEST	4	4
SOUTH	3	3
WEST	1	1

Table 4.4: Volatility and Land Cost Ratio Summary for Residential Markets

Where: 1 = highest
4 = lowest

4.3 Data Summary

As presented in sections 4.1 and 4.2, statistically, commercial markets and residential markets seem to demonstrate opposite relationship between the asset value volatility and the land cost ratio.

Volatility Table		High			Low
Commercial	Regions Overall	Midwest	East	South	West
	Apartment	Midwest	East	South	West
	Industrial	East	Midwest	South	West
	Office	Midwest	South	East	West
	Retail	Midwest	East	South	West
	Uses Overall	Apartment	Retail	Industrial	Office
	EAST	Industrial	Apartment	Retail	Office
	MIDWEST	Apartment	Retail	Office	Industrial
	SOUTH	Retail	Apartment	Office	Industrial
	WEST	Apartment	Retail	Office	Industrial

Residential	Regions Overall	West	East	South	Midwest

Table 4.5: Volatility Analysis Summary for Commercial and Residential Markets

Land Cost Ratio Table		High			Low
Commercial	Regions Overall	West	East	South	Midwest
	Apartment	East	West	South	Midwest
	Industrial	West	East	South	Midwest
	Office	East	West	South	Midwest
	Retail	West	East	South	Midwest
	Uses Overall	Retail	Office	Apartment	Industrial
	EAST	Office	Retail	Apartment	Industrial
	MIDWEST	Retail	Office	Apartment	Industrial
	SOUTH	Retail	Office	Apartment	Industrial
	WEST	Retail	Office	Apartment	Industrial
Residential	Regions Overall	West	East	South	Midwest

Table 4.6: Land Cost Ratio Analysis Summary for Commercial and Residential Markets

For clarity purposes, relative magnitudes, rather than exact numbers, are used to illustrate their relationships. In general, for commercial markets, the asset value volatility and the land cost ratio are negatively correlated statistically, while for residential markets, they are positively correlated. In section 5.0, implications of this observation on the timing strategy of developers in both commercial and residential markets will be studied and discussed.

4.4 Strengths & weaknesses of methodology

One obvious difference between the RCA data and the LILP data is that the RCA data is transaction-based while the LILP data is appraisal-based. The RCA data appears to be more volatile because it records average asset value per sq.ft. actually traded in the market regardless of the quality of the properties. Therefore, asset values can vary greatly between quarters, inherently raising the volatility level. However, transaction-based data does truly reflect actual market values of the property type in the MSA under study. For the purposes of this thesis, which studies the relationship of the volatility and the land cost ratio on the timing strategy of developers, it is reasonable that as long as the same methodology is used to compare different MSAs and different property types, the data is consistent and representative for this study. Raw recorded data, rather than any smoothed data index, is used in order to preserve the accuracy of transactional values. RCA data also comprises a very comprehensive set of data which covers 4 different uses in 30 MSAs spanning more than 10 years. It can draw powerful implications as applied to different property types in different regions.

A similar logic can be applied to the LILP data. The data appears to be less volatile because it records average appraised house values quarterly. The house values tend not to move too much between quarters, so it is not reflecting the true volatility in the market. However, if the focus of the study is on comparing the effect of volatility and land cost ratio on construction activities between different MSAs, the data set is still very comprehensive and consistent, which covers 44 MSAs spanning close to 20 years. It is also very useful to draw insightful implications as applied to one MSA across time.

With the rationales stated above kept in mind, in the next section, the game theory and the real options theory will be used to study the effect of the volatility and the land cost ratio on the timing strategy of developers empirically. At a strategic level, results will be discussed to evaluate the use of game theory and real options in real estate development. Two levels of relationship will be studied: (1) across time within a MSA, and (2) across MSAs within a time period for both the commercial and residential markets.

5.0 Chapter 5: Data analysis & Interpretation

5.1 Empirical Testing of the Real Options Pricing Model

As described in section 2.1.1, the real options pricing model is used to calculate the option premium value (C_p) for each property type in each MSA within a certain given time period. Theoretically, the option premium is a value indicating the magnitude of benefit for a developer to wait, rather than to develop now. In other words, the higher the option premium value is, the higher the benefit for a developer to wait is, the fewer the construction activity should be observed. For comparison purposes, the construction activity of each MSA will be population-adjusted, which will be measured as number of square feet completed per 1000 people or number of units completed per 1000 people, with population information collected from the U.S. Census Bureau.

To study the relationship between the option premium, the land cost ratio, and the timing strategy of developers, the following regression is applied to measure the degree of correlation:

$$CA = \alpha + \beta (C_p) + \gamma (LCR) + \varepsilon$$

Where CA = Construction Activity

C_p = Option Premium value

$= C_t - (S_t - K_t)$

LCR = Land Cost Ratio

$= \frac{S-K}{S}$

S = property asset value

K = construction / replacement cost

In this thesis, two levels of relationships are studied:

(1) Across time within a MSA

The volatility and land cost ratios of 4 property types (apartment, industrial, office, and retail) spanning quarterly from 2001 to 2013 will be used to analyze the commercial markets of Boston. The volatility and land cost ratios of single-family houses spanning quarterly from 1995 to 2013 will be used to analyze the residential market of Boston. Boston is chosen because of its completeness of available data. This time-series study will focus on how strong the correlation is between the option premium value, the land cost ratio, and the construction activity within one MSA through time. The strength of the correlation will help analyze the usefulness of using real options analysis in the timing aspect of real estate development.

(2) Across MSAs within a time period

The volatility and land cost ratios of 4 property types (apartment, industrial, office, and retail) in 2008 will be used to analyze the commercial markets of 30 MSAs listed in section 4.1.2. The volatility and land cost ratios of single-family houses in 2008 will be used to analyze the residential market of 44 MSAs listed in section 4.2.2. 2008 is chosen because of its completeness of available data. The uncertain nature of the pre-crisis and the post-crisis in 2008 may offer significant insights of developers' timing strategy as well. This study will focus on how strong the correlation is between the option premium value, the land cost ratio, and the construction activity across different MSAs within a particular time period.

5.1.1 Regression Results - Commercial Real Estate

(1) Across time within a MSA

$$CA = \alpha + \beta (C_p) + \gamma (LCR) + \varepsilon$$

Boston Commercial Markets	α = *(P-value)*	β = *(P-value)*	γ = *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Apartment	2.217 *(8.34×10^{-2})*	-0.0069 *(4.95×10^{-1})*	-0.1133 *(9.13×10^{-1})*	-	-
Apartment w/ 2-yr lag	3.955 *(1.92×10^{-4})*	-0.0274 *(1.03×10^{-2})*	-0.6277 *(2.20×10^{-1})*	-	-
Industrial	3259.4 *(1.76×10^{-7})*	-54.637 *(5.59×10^{-5})*	-868.44 *(6.35×10^{-3})*	-	-
Industrial w/ 2-yr lag	2413.0 *(1.17×10^{-5})*	-40.105 *(1.43×10^{-3})*	-473.02 *(9.35×10^{-2})*	-	-
Office	3725.5 *(3.03×10^{-3})*	-51.348 *(8.26×10^{-3})*	-2044.0 *(6.22×10^{-2})*	-	-
Office w/ 2-yr lag	965.91 *(2.83×10^{-1})*	-13.615 *(3.57×10^{-1})*	118.58 *(8.79×10^{-1})*	-	+
Retail	839.78 *(1.33×10^{-2})*	-9.8907 *(1.18×10^{-1})*	-268.33 *(2.90×10^{-1})*	-	-
Retail w/ 2-yr lag	878.86 *(2.63×10^{-2})*	-11.525 *(1.30×10^{-1})*	-279.48 *(3.38×10^{-1})*	-	-

Table 5.0: Time-Series Analysis Regression Results for Commercial Markets

Regression results in general agree with results predicted by the real options valuation theory. However, a few regression results show that some variables are only weakly correlated, as indicated by high P-values. Comparing between the option premium value and the land cost ratio, the land cost ratio seems to demonstrate a less significant statistical result than the option premium value does. That implies that the correlation between the land cost ratio and the level of construction activity is weaker than that between the option premium value and the level of construction activity. Among the 4 property types, office and industrial markets show more significant statistical results than retail and apartment markets. That can imply that the effect of the real options theory has a stronger correlation with the timing strategy of office and industrial developers. Nonetheless, the results overall shows that the level of construction activity tends to be negatively correlated with the option premium value and the land cost ratio, which imply that if the option premium value is high, there is a bigger benefit for developers to wait, which yields to low construction activity. Similarly, if the land cost constitutes a large portion of the asset value, the construction activity in markets with that characteristic will be lower too. In the real options valuation method stated in section 2.1.2 and the statistical observation shown in section 4.1.3, the following relationships are presented:

$$\frac{\partial \mathrm{C_p}}{\partial \sigma_{\mathrm{s}}} > 0 \qquad \& \qquad \frac{\partial \sigma}{\partial LCR} < 0$$

Those relationships are not consistent with the regression results shown here. In other words, if volatility of a specific property type in a specific market is high, the option premium value will be high too, which naturally results in lower construction activity, as it is shown in the regression results. Since the volatility is statistically negatively correlated with the land cost ratio, the option premium value will be negatively correlated with the land cost ratio as well, which implies that high land cost ratios should result in higher construction activities. However, as it is shown in the regression results above, the land cost ratio and the construction activity are negatively correlated. The reason may be that either the land cost ratio and the volatility are relatively weakly correlated as shown in section 4.1.3 or other exogenous market forces may affect the relationship between the land cost ratio and level of construction, which is outside the scope of the model. In section 5.3, the potential issue of multi-collinearity of the model will be discussed and tested.

It is also noticed that the strength of the effect of the option premium value and the land cost ratios is indicated by the magnitude of the coefficients in the regression table. Since construction activity of apartment buildings is measured in number of units completed, the coefficients may appear to be a lot smaller than the rest of the property types. If average size of apartments is assumed to be 1000 sq.ft., the coefficients will be adjusted as follows:

Boston	α = *(P-value)*	β = *(P-value)*	γ = *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Apartment	2217.4 *(8.34×10^{-2})*	-6.9481 *(4.95×10^{-1})*	-113.32 *(9.13×10^{-1})*	-	-
Apartment w/ 2-yr lag	3954.6 *(1.92×10^{-4})*	-27.351 *(1.03×10^{-2})*	-627.68 *(2.20×10^{-1})*	-	-

Table 5.1: Time-Series Analysis Area-Adjusted Regression Results

Comparing the magnitude of the coefficients, the strength of the effect of option premium value and land cost ratios on the level of construction activity across different property types in Boston is summarized below:

Boston	High			Low
Option Premium	Industrial	Office	Apartment	Retail
Land Cost Ratio	Office	Industrial	Apartment	Retail

Table 5.2: Time-Series Analysis Summary

The result above can imply that if office developers and retail developers have the same option premium value in a specific project, office developers tend to wait, which gives lower construction activity in the market, while retail developers tend not to wait, which does not lower construction activity as much. In other words, the option premium value or the real options valuation matters more to office developers than to retail developers, at least in Boston. The same logic can be applied to land cost ratio and it can draw similar implications. In section 5.2, this observation will be re-visited again from a game theory perspective.

(2) Across MSAs within a time period

$$CA \quad = \alpha + \beta\,(C_p) + \gamma\,(LCR) + \varepsilon$$

MSAs Commercial Markets in 2008	$\alpha =$ *(P-value)*	$\beta =$ *(P-value)*	$\gamma =$ *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Apartment	2.6168 *(1.04×10^{-2})*	-0.0142 *(2.47×10^{-1})*	-0.0329 *(9.58×10^{-1})*	-	-
Apartment w/ 2-yr lag	0.5733 *(2.20×10^{-2})*	-0.0021 *(4.95×10^{-1})*	-0.0748 *(6.31×10^{-1})*	-	-
Industrial	1838.3 *(8.73×10^{-3})*	-12.061 *(3.09×10^{-1})*	-726.80 *(5.53×10^{-2})*	-	-
Industrial w/ 2-yr lag	334.90 *(4.18×10^{-2})*	-2.7732 *(3.31×10^{-1})*	-109.85 *(2.20×10^{-1})*	-	-
Office	590.94 *(2.55×10^{-1})*	-3.8779 *(6.18×10^{-1})*	574.50 *(3.45×10^{-1})*	-	+
Office w/ 2-yr lag	522.97 *(7.78×10^{-2})*	-5.5206 *(2.12×10^{-1})*	-105.10 *(7.56×10^{-1})*	-	-
Retail	613.74 *(3.40×10^{-2})*	-3.8820 *(3.03×10^{-1})*	-31.349 *(9.20×10^{-1})*	-	-
Retail w/ 2-yr lag	78.126 *(7.50×10^{-2})*	-0.2758 *(6.31×10^{-1})*	10.707 *(8.23×10^{-1})*	-	+

Table 5.3: Cross-Sectional Analysis Regression Results for Commercial Markets

Applying a similar process to analyze the regression results as in 5.1.1 (1), we notice that the regression results across multiple MSAs within a time period generally agree with results predicted by the real options valuation method as well. However, the regression results are not as significant as those of the time-series analysis discussed in section 5.1.1 (1). P-values are a lot higher in this across-MSA analysis. The reason may be that different MSAs have drastically different market characteristics. That makes the option premium values play a different role in different timing strategies of developers, which leads to much weaker correlation between the option premium value and the land cost ratio with the level of construction activity across different MSAs in this regression analysis. In section 5.2.1 (2), this observation will be revisited again from a game theory perspective. Coefficients for apartment buildings in the following regression table are area-adjusted, assuming the average size of apartments is 1000 sq.ft.:

MSAs	α = *(P-value)*	β = *(P-value)*	γ = *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Apartment	2616.8 *(1.04×10^{-2})*	-14.179 *(2.47×10^{-1})*	-32.852 *(9.58×10^{-1})*	-	-
Apartment w/ 2-yr lag	573.31 *(2.20×10^{-2})*	-2.0588 *(4.95×10^{-1})*	-74.786 *(6.31×10^{-1})*	-	-

Table 5.4: Cross-Sectional Analysis Area-Adjusted Regression Results

Comparing the magnitude of the coefficients, the strength of the effect of the option premium value and the land cost ratio on the level of construction activity across different property types in multiple MSAs is summarized below:

MSAs	High			Low
Option Premium	Apartment	Industrial	Office	Retail
Land Cost Ratio	Industrial	Office	Apartment	Retail

Table 5.5: Cross-Sectional Analysis Summary

At an aggregate level across different MSAs, the results seem to imply that in the U.S. on average, the option premium value matters more to apartment developers than to office developers, at least shown statistically, while the reverse may happen in certain specific MSAs such as Boston as shown in section 5.1.1 (1). Comparing the results across multiple MSAs within a time period with the results across time within a MSA, it is noticed that both the option premium value and land cost ratio have a stronger effect on the level of construction activities in the across-time analysis, based on the magnitude of coefficients. The reason will be discussed further in detail in section 5.2 using the game theory model developed in section 2.2.1.

5.1.2 Regression Results - Residential Real Estate

(1) Across time within a MSA

$$CA = \alpha + \beta (C_p) + \gamma (LCR) + \varepsilon$$

Boston Residential Markets	$\alpha =$ *(P-value)*	$\beta =$ *(P-value)*	$\gamma =$ *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Single-family house	584.56 *(1.06×10^{-1})*	-0.1846 *(6.96×10^{-7})*	2432.6 *(1.43×10^{-5})*	-	+

Table 5.6: Time-Series Analysis Regression Results for Residential Markets

Assuming the average size of single-family houses is 1,500 sq.ft., the coefficients are adjusted as follows:

Boston	$\alpha =$ *(P-value)*	$\beta =$ *(P-value)*	$\gamma =$ *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Single-family house	876,845.8 *(1.06×10^{-1})*	-276.91 *(6.96×10^{-7})*	3648,892.3 *(1.43×10^{-5})*	-	+

Table 5.7: Time-Series Analysis Area-Adjusted Regression Results

In the real options valuation method stated in section 2.1.2 and the statistical observation shown in section 4.2.2, the following relationships are presented for the residential markets:

$$\frac{\partial C_p}{\partial \sigma_s} > 0 \qquad \& \qquad \frac{\partial \sigma}{\partial LCR} > 0$$

Those relationships are not consistent with the regression results shown here. In other words, if volatility of the house value in a specific market is high, the option premium value will be high too, which naturally results in lower construction activity, as it is shown in the regression results. However, since the volatility is statistically positively correlated with the land cost ratio, the option premium value will be positively correlated with land cost ratio as well, which implies that high land cost ratio should result in lower construction activity. However, as it is shown in the regression results above, land cost ratio and construction activity are positively correlated. Therefore, it implies that either land cost ratio and volatility may be relatively weakly correlated or other exogenous market forces may affect the relationship between land cost ratio and level of construction, which is outside the scope of the model. In section 5.3, the potential issue of multi-collinearity of the model will be discussed and tested.

Compared with the commercial markets, the residential market also seems to be more sensitive to changes in the option premium value and the land cost ratio, as it is indicated by the larger magnitude of coefficients shown in the regression table above.

(2) Across MSAs within a time period

$$CA = \alpha + \beta (C_p) + \gamma (LCR) + \varepsilon$$

MSAs Residential Markets in 2008	$\alpha =$ *(P-value)*	$\beta =$ *(P-value)*	$\gamma =$ *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Single-family house	2.5655 *(6.18×10^{-1})*	0.00193 *(9.56×10^{-2})*	-11.196 *(8.37×10^{-3})*	+	-

Table 5.8: Cross-Sectional Analysis Regression Results for Residential Markets

Again, assuming the average size of single-family houses is 1,500 sq.ft., the coefficients are adjusted as follows:

MSAs	$\alpha =$ *(P-value)*	$\beta =$ *(P-value)*	$\gamma =$ *(P-value)*	$\frac{\partial CA}{\partial C_p}$	$\frac{\partial CA}{\partial LCR}$
Single-family house	3848.3 *(6.18×10^{-1})*	2.8949 *(9.56×10^{-2})*	-16,794.5 *(8.37×10^{-3})*	+	-

Table 5.9: Cross-Sectional Analysis Area-Adjusted Regression Results

What is peculiar about this regression result is that it seems to be opposite to that predicted by the real options valuation method. In this case, the option premium value is positively correlated with the level of construction activity. That means when the option premium value is high, the level of construction activity will be high too. The reason will be discussed further in detail in section 5.2 using the game theory model developed in section 2.2.1.

5.1.3 Summary of the results

For commercial markets, the higher the asset value volatility is, the higher the option premium value will be, and the lower the level of construction activities will result. Meanwhile, the higher the land cost ratio is, the lower the level of construction activities will result. At an aggregate level across multiple MSAs, apartments are the most sensitive to the option premium value, followed by industrial and office. Retail is the least sensitive to the option premium value. Compared between the time-series (across time within a MSA) study and the cross-sectional (across multiple MSAs within a time period) study, the time-series study seems to show higher sensitivity to the option premium value than the cross-sectional study does. The time-series study also shows more significant regression results than the cross-sectional study does.

For residential markets, results between the time-series study and the cross-sectional study are not as consistent as those of the commercial markets study. In the time-series study, the higher the asset value volatility is, the higher the option premium value will be, and the lower the level of construction activities will result. Meanwhile, the lower the land cost ratio is, the lower the level of construction activities will result. However, the cross-sectional study shows the opposite. Compared between the time-series study and the cross-sectional study, the time-series study again seems to show higher sensitivity to the option premium value than the cross-sectional study does. Between commercial markets and residential markets, the regression results show that residential markets are more sensitive to the option premium value than commercial markets do. Residential markets also show more significant regression results than commercial markets do.

In the next section, game theory payoff matrix as presented in section 2.2.1 will be used to discuss some of the observation, discrepancy, and implication shown in the regression results above. The study will focus on how to use the game theory model to explain and complete the observations not fully predicted by the real options model, serving as a more comprehensive framework to study the timing strategy of developers and its relationship with volatility and land cost ratio.

5.2 Application of Game Theory Payoff Matrix

As seen in section 2.1.1 and section 5.1, the real options valuation method provides insightful ways to explain developers' investment behavior and economic market situations. However, the real options model takes a monopolistic approach and neglects the interactive effect of other players and characteristics of specific markets. To enhance the model and complete the picture, game theory is used to analyze the discrepancy between the predicted results of the real options model and empirical observations.

In section 2.2.2, several equilibrium scenarios are discussed and a numerical example is used to illustrate the option game theoretic approach. Since the game theory model presented is conceptual in nature, the option game theoretic approach should only serve as a strategic framework for analysis purposes. Exact values of ω_{Di}, ω_{Li}, and ω_{Fi} will be very difficult to be determined. However, their relative values provide powerful implication to understand characteristics of the market and optimal strategy of developers. To simplify the analysis process without diluting the implication of the model, it is assumed that the given market is a symmetric game, which means that developer 1 and developer 2 have the same strength and under the same circumstances, as illustrated with the matrix below:

		Developer 1	
		Develop	Wait
Developer 2	Develop	ω_D $S_t - K_t e^{-rt}$, ω_D $S_t - K_t e^{-rt}$	ω_L $S_t - K_t e^{-rt}$, C_t^{fw}
	Wait	C_t^{fw} , ω_L $S_t - K_t e^{-rt}$	C_t , C_t

Table 5.10: Symmetric Game Theory Model Setup

Where:
- S_t = property asset value per sq.ft. at time = t
- K_t = construction / replacement cost per sq.ft. at time = t
- ω_D = proportion of value when both developers develop

ω_L = proportion of value for the Leader

ω_F = proportion of value for the Follower

C_t = option value of waiting when both developers wait (no Leader)

C_t^{fw} = option value of waiting for the Follower

$= \max(\omega_F S_{t+1} - K_{t+1}e^{-r(t+1)}, e^{-r(t+1)}[q C^u_{(t+1)}{}^{fw} + (1-q) C^d_{(t+1)}{}^{fw}])$

Although the model is set up for two developers, the framework of the model can be extended to be applied to the whole market. In theory, the aggregate results of multiple games should reflect the market condition too. In the next section, this game theory model will be used to analyze the regression results presented in section 5.1 and conceptually explain any observed situation not predicted by the real options model.

5.2.1 Strategic Implication in Commercial Real Estate

(1) Across time within a MSA

As presented in section 5.1.1 (1), in the particular case of Boston, development of 4 property types (apartment, industrial, office, and retail) in general agrees with the real options theory. However, as shown in the regression results, industrial and office development seem to be more sensitive to the option premium value and show more significant regression results than apartment and retail development do. In other words, even though option premium values are all negatively correlated with level of construction activity, decrease in the option premium value will lead to greater increase in industrial and office development than in apartment and retail development.

Using the same methodology as illustrated in the numerical example in section 2.2.2, office and retail developments in Boston will be analyzed from an option game theoretic perspective. With empirical data presented in section 4.1, the payoff matrix of office and retail developments are as follows:

Office	Develop	Wait
Develop	108 , 108	156 , 103
Wait	103 , 156	145 , 145

Table 5.11: Office Game Payoff Matrix

Retail	Develop	Wait
Develop	80 , 80	123 , 81
Wait	81 , 123	117 , 117

Table 5.12: Retail Game Payoff Matrix

For comparison purposes, ω_{Di}, ω_{Li}, and ω_{Fi} are assumed to be the same for both office and retail markets. As shown above, office market is similar to Scenario 1: Develop-Develop Equilibrium, while retail market is similar to Scenario 3: Develop-Wait Equilibrium. The intuition behind the model is that even with the same values of ω_{Di}, ω_{Li}, and ω_{Fi}, the intrinsic characteristics of different markets are influenced by asset value, construction or replacement cost, and volatility. As seen in the payoff matrix above, office developers tend to compete and develop together, even though the option value of waiting is higher, which leads to Develop-Develop Equilibrium. Meanwhile, retail developers tend to think more strategically about First Mover Advantage and Second Mover Advantage, which leads to Develop-Wait Equilibrium. The game theory model does make intuitive sense because office market does tend to compete for market demand between developers, while retail market focuses more on the success of building up a critical mass to create positive synergy, which makes the strategic timing of development more important. The game theory model also explains why the regression results show that office development in Boston is more sensitive to the option premium value than retail development does because more construction activity will result in the Develop-Develop Equilibrium of office market than the Develop-Wait Equilibrium of retail market. Thus, the model confirms the implication that level of construction activities will change more in the office market than in the retail market per unit change in the option premium value, as it is indicated by the magnitude of coefficients in the regression results.

(2) Across MSAs within a time period

As presented in section 5.1.1 (2), the regression results across multiple MSAs within a time period generally agree with results predicted by the real options valuation method. However, comparing the results across multiple MSAs within a time period with the results across time within a MSA, it is noticed that option premium values have a stronger effect on the level of construction activities in the across-time analysis, based on the magnitude of coefficients. The time-series study also shows more significant regression results than the cross-sectional study does. In other words, the results imply that even though option premium values are all negatively correlated with the level of construction activity, decrease in the option premium value will lead to greater increase in construction activities in some MSAs than in some other MSAs.

Using the same methodology as illustrated in section 5.2.1 (1) above, office markets in New York and Dallas will be analyzed from an option game theoretic perspective. With empirical data presented in section 4.1, the payoff matrix of New York office markets and Dallas office markets are as follows:

New York	Develop	Wait
Develop	552 , 552	697 , 445
Wait	445 , 697	590 , 590

Table 5.13: New York Office Game Payoff Matrix

Dallas	Develop	Wait
Develop	24 , 24	49 , 36
Wait	36 , 49	$\boxed{57 , 57}$

Table 5.14: Dallas Office Game Payoff Matrix

For comparison purposes, ω_{Di}, ω_{Li}, and ω_{Fi} are assumed to be the same for both New York and Dallas office markets. As shown above, New York office market is similar to Scenario 1: Develop-Develop Equilibrium, while Dallas office market is similar to Scenario 4: Wait-Wait Equilibrium. As seen in the payoff matrix above, New York office developers tend to compete and develop together, even though the option value of waiting is higher, which leads to a Develop-Develop Equilibrium. Meanwhile, Dallas office developers tend to value the option of waiting more and wait to develop until the market conditions become more favorable to them, which leads to a Wait-Wait Equilibrium. The game theory model does make intuitive sense because New York office market tends to have more competition, which will compress the option value of waiting in fear of pre-emption. That is also consistent with the low volatility and high land cost ratio characteristics of the market. Meanwhile Dallas office market tends to have relatively fewer competition, higher volatility, and lower land cost ratio, which is logical for developers to wait to observe the market more comprehensively before taking actions.

The game theory model thus explains why the regression results across time within a MSA are more sensitive to the option premium value than results across multiple MSAs within a time period because different MSAs have different market characteristics, and thus different equilibria in different game scenarios. It also explains why the regression results across MSAs show much higher P-values and less significant results. Therefore, the effect of option premium values on the level of construction activities is different in different MSAs, and their correlation is weaker in the cross-sectional analysis. For example, more construction activity will result in the Develop-Develop Equilibrium of New York office market than the Wait-Wait Equilibrium of Dallas office market per unit change in the option premium value.

5.2.2 Strategic Implication in Residential Real Estate

With similar methodology as presented in section 5.2.1 and empirical data as shown in section 4.2, residential markets in different MSAs are predominantly markets with Develop-Develop Equilibrium. 6 MSAs with very different volatility, land cost ratio, and market characteristics are selected for the analysis. Their game theory payoff matrices are as follows:

San Fran.	Develop	Wait
Develop	470 , 470	594 , 378
Wait	378 , 594	502 , 502

Table 5.15: San Francisco Single-Family House Game Payoff Matrix

Boston	Develop	Wait
Develop	196 , 196	262 , 149
Wait	149 , 262	215 , 215

Table 5.16: Boston Single-Family House Game Payoff Matrix

Wash. DC	Develop	Wait
Develop	166 , 166	232 , 118
Wait	118 , 232	185 , 185

Table 5.17: Washington DC Single-Family House Game Payoff Matrix

Phoenix	Develop	Wait
Develop	65 , 65	103 , 38
Wait	38 , 103	76 , 76

Table 5.18: Phoenix Single-Family House Game Payoff Matrix

New Orleans	Develop	Wait
Develop	40 , 40	64 , 24
Wait	24 , 64	47 , 47

Table 5.19: New Orleans Single-Family House Game Payoff Matrix

Atlanta	Develop	Wait
Develop	6 , 6	34 , 0
Wait	0 , 34	15 , 15

Table 5.20: Atlanta Single-Family House Game Payoff Matrix

The market characteristics of the 6 selected MSAs is summarized as follows:

MSAs	High					Low
House Value	San Francisco	Wash. DC	Boston	Phoenix	Atlanta	New Orleans
Volatility	Phoenix	San Francisco	Wash. DC	Boston	Atlanta	New Orleans
Land Cost Ratio	San Francisco	Boston	Wash. DC	Phoenix	New Orleans	Atlanta
Option Premium	Wash. DC	Atlanta	San Francisco	Boston	Phoenix	New Orleans
Construction Activity	Atlanta	Wash. DC	Phoenix	New Orleans	Boston	San Francisco

Table 5.21: Market Characteristics Summary for 6 MSAs

Per the regression results presented in section 5.1.2 (2), the option premium value is not negatively correlated with level of construction activity as it should be as predicted by the real options theory. From the game theory perspective shown above, it is noticed that 6 MSAs with very different market characteristics all tend to have Develop-Develop equilibrium. The summary table shown above does not demonstrate strong expected correlations between different parameters as implied in the real options model. In residential markets across different MSAs, the game theoretic approach seems to be more aligned with the regression results empirically, which show that option premium values do not have strong effect on the timing strategy of developers. Residential developers all tend to compete with each other and develop immediately whether they have a high option value of waiting or not. That partially explains the peculiar regression results presented in section 5.1.2 (2).

5.2.3 Summary of the results

For commercial markets, the game theory model explains and confirms some of the observations from the regression results shown in section 5.1. By analyzing different game theory payoff equilibria of different markets, it is noticed that level of construction activities will change more in office market than in retail market per unit change in the option premium value because office markets have a Develop-Develop equilibrium while retail markets have a Develop-Wait equilibrium. The game theory model also explains why the regression results across time within a MSA are more sensitive to the option premium value than results across multiple MSAs within a time period because different MSAs have different market characteristics, and thus different equilibria in different game scenarios. Therefore, the effect of option premium values on level of construction activities is different in different MSAs, and their correlation is weaker in the cross-sectional analysis. For example, more construction activity will result in the Develop-Develop Equilibrium of New York office market than the Wait-Wait Equilibrium of Dallas office market per unit change in the option premium value.

For residential markets, the game theory model shows that option premium values do not have a strong effect on the timing strategy of developers. Residential developers all tend to compete with each other and develop immediately whether they have a high option value of waiting or not, as illustrated by the 6 MSAs with very different market characteristics which all tend to have a Develop-Develop equilibrium. That partially explains why the option premium value is not negatively correlated with level of construction activity as it should be as predicted by the real options theory.

Overall, the game theory model offers powerful insights to explain things that cannot be explained by the real options theory model. In the next section, benefits and limitation of applying the game theory and the real options theory to real estate development will be discussed.

5.3 Evaluation of the use of Game Theory and Real Options in Real Estate Development

(1) Real Options Theory

In the regression analyses presented in section 5.1, the results show various strength of correlation between the level of construction activity and the option premium value and the land cost ratio.

$$CA = \alpha + \beta (C_p) + \gamma (LCR) + \varepsilon$$

Where CA = Construction Activity

C_p = Option Premium value

LCR = Land Cost Ratio

Since the option premium value and the asset value volatility are positively correlated and the volatility and the land cost ratio are correlated to a certain degree as shown in sections 4.1.3 and 4.2.2, a potential issue of multi-collinearity may distort the regression results to study the effect of the option premium value and the land cost ratio on the level of construction activity.

$$\sigma = \alpha + \beta (LCR)$$

Where: σ = Asset Value Volatility

LCR = Land Cost Ratio ($\frac{S-K}{S}$)

The multi-collinearity refers to a situation in which two or more independent variables in a multivariate regression model are approximately linearly related. To determine the degree of multi-collinearity, the Variance Inflation Factor (VIF) method can be used as illustrated below:

$$\text{VIF} = \frac{1}{1 - R^2}$$

Where: VIF = Variance Inflation Factor

R^2 = Coefficient of Determination

A common rule of thumb is that if the VIF is higher than 5, the issue of multi-collinearity becomes problematic. As it turns out, the VIF of the commercial markets is 1.0019 and that of the residential markets is 1.8266. Therefore, it is concluded that the multi-collinearity is not an issue in the model.

While the regression results demonstrate certain market characteristics and general consistency with the real options theory, the correlations between the application of the real options theory and the timing strategy of developers in different markets vary drastically. This conclusion is particularly obvious in the commercial cross-sectional analysis that gives very high P-values, which implies that applications of the real options theory differ widely in different MSAs and different property types. Therefore, the real options theory alone per se is shown not to be a widely used valuation method in the timing strategy of real estate development yet, and it alone has limited explanatory power on observed market conditions. Nonetheless, the benefits and limitations of the real options model will be discussed further in sections 5.3.1 and 5.3.2.

(2) Game Theory

The game theory takes interactive effect of other developers in the market into consideration. At a strategic level, it offers insightful explanations to observed market conditions that cannot be explained by the real options theory. It helps to elaborate the different characteristics of different property types in different MSAs through analyzing different equilibria of the game payoff matrices. Empirically, the matrix analyses also show that the game theory model is consistent with the regression results and reflects general market conditions. However, at an implementational level, the game theory model is very hard to be applied in real-world situations. In this thesis, the game matrices are set up as symmetric games, where both players are equal in all aspects. In the actual markets, it is almost impossible to know the exact payoffs of your opponents. That may change your optimal action completely because of an inaccurate payoff matrix. The game matrix analysis also assumes that both players are rational. If either player is irrational, the game theory approach will have limited influence on the timing strategy of developers. Nonetheless, the advisory power and limitations of the game theory model will be discussed further in sections 5.3.1 and 5.3.2.

5.3.1 Benefits and Advisory Power of the method

As demonstrated throughout the thesis, the interaction of the real options theory and the game theory provides a more comprehensive framework to analyze investment timing options in real estate development than the standard Net Present Value method.

(1) Real Options Model

The real options model forces developers to think more strategically about the benefits of waiting through the calculation of option values of waiting. It allows developers to analyze the maximum land cost they should offer through taking the volatility of the market into consideration when the option value is higher than the residual land value. It also implies that negative NPV does not necessarily mean bad investment options. The option premium value may be high enough to justify the profitability of the development option. Therefore, if the option premium value is calculated accurately based on the endogenous factors of the developer, the model allows it to offer a higher land price to outbid its opponents as demonstrated in Quigg (1993). The real options model can change the decision of developers drastically. It will have far-reaching implications on the development strategies of developers.

(2) Game Theory Model

While the real options model focuses on the development strategies internally, the game theory model analyzes them externally. Interaction with other developers is an integral part of the real estate game. The game payoff matrix offers powerful, and yet sometimes counter-intuitive, implications to advise developers with the optimal responses. It allows developers to analyze real estate development strategies from an equilibrium perspective, rather than to only focus on maximizing their own payoffs per se. The actual application of the game theory model in a business strategic setting will go beyond the quantitative aspect of the theory as it involves other strategic aspects such as signaling, sequential moves, repeated games, threats, and promises. However, the game payoff matrix provides an insightful tool to analyze development options based on different market characteristics as described in section 2.2.2.

(3) Regression Analysis from Option Game Theoretic Perspective

As illustrated in sections 4.0 and 5.0, important information reflecting the U.S. markets is extrapolated from the model using an option game theoretic approach. Those market characteristics are instrumental to help developers make the optimal decision. For example, an office developer in Boston, knowing that the Boston office market tends to have a Develop-Develop equilibrium as shown in section 5.2.1, should expect that other developers will compete to develop. In this case, its strategy of waiting may lead to losses because of pre-emption even though it may have a high option value of waiting. Therefore, the option game theoretic approach provides powerful market insights in the decision-making process.

5.3.2 Limitations of the method

Although the option game theoretic approach works well at a strategic level, there are a few hurdles that make it hard to be widely used in real estate development. Some of them have been mentioned briefly in previous sections.

(1) Real Options Model

The model used in this thesis is based on the fundamental binomial tree method. Asset values and construction or replacement costs fluctuate in a discrete fashion at hypothetical time intervals. The construction period is assumed to be simultaneous. A constant risk-free rate is assumed. There are other nuances in real estate development that are not captured in the simplified model such as permitting period duration, uniqueness of each property, transactional costs and other miscellaneous fees in the valuation process, and other exogenous market factors which may affect the option value calculation. At the aggregate level, the simplified model as presented in this thesis works well for comparing purposes across different MSAs and different property types. However, using it at the property level to calculate the exact option premium value for a specific project will be inadequate. To quantify and capture all variables in a real estate development project, which is by nature very complex, will make the real options model too complicated to be practical. That will involve lots of assumptions, which lead to high degree of idiosyncratic risk. Moreover, the real options theory takes a monopolistic setting, which does not take into account the interactive effect of other players. However, real estate markets are rarely monopolistic. Therefore, option values will be eroded by competitions as demonstrated in Grenadier (1996) and Schwartz and Torous (2007). This type of exogenous factors is not captured in real options models. That is why although the real options theory is very powerful and insightful in theory, it is not widely used in real estate development in practice yet.

(2) Game Theory Model

The simplified game theory model is based on the game payoff matrix analysis. Payoffs are calculated using the real options model setup. Therefore, the payoffs reflect any limitation embedded in the real options model as well. In practice, it will be almost impossible to accurately calculate the payoffs of the opponent and the corresponding values of ω_{Di}, ω_{Li}, and ω_{Fi}. Again, at the aggregate level, the simplified model as presented in this thesis works well for comparing purposes across different MSAs and different property types to provide strategic insights to complete the real options model. However, it will be very hard to apply it at the property level. Moreover, the game theory model as presented in this thesis does not take into account any exogenous factors such as non-project specific motives of opponents, non-payoff-maximizing business strategies of opponents, faux information signaled by opponents. It also assumes that all players are rational and their goal is to maximize payoffs. If any of the assumptions is not true, the equilibrium concluded may not be a true representation of optimal actions for each developer. That is why the game theory model is not usually applied quantitatively in real-world situations, in the context of real estate development. Nonetheless, if the model is used at the conceptual and strategic level, the accuracy of the payoffs does not need to extremely high, and the model can still provide powerful insights to reflect the market characteristics and to help developers analyze their optimal response in the market they are developing.

6.0 Chapter 6: Conclusions

6.1 Conclusion

This thesis investigates the use of the game theory and the real options theory in real estate development at the strategic level, trying empirically to explain different economic observations among different metropolitan cities and different property types.

The real options theory provides a rich theoretical framework to analyze investment values in real estate development. It takes the market uncertainty into consideration, while the widely used neoclassical NPV valuation method takes a deterministic approach. A simplified real options valuation model is set up in this thesis to calculate the option premium value of waiting for developers. However, since it is done in a monopolistic setting, the strategic interaction aspect of real estate development will be analyzed using the game theory model. The interaction of the game theory model and the real options model will provide a comprehensive and powerful framework to study the timing strategy of developers.

Using data spanning quarterly from 1995 to 2013 among 5 property types (single-family house, apartment, industrial, office, and retail) and 44 MSAs, this thesis analyzes the relationships empirically between the volatility of underlying assets, the land cost ratio, the option premium value, and the timing of development. The aims of the study are twofold. First, the study compares different market characteristics among different MSAs and different property types from the option game theoretic perspective. Second, it analyzes the effect and the use of the game theory and the real options theory in the context of real estate development. The key results and observations are summarized below:

(1) Volatility and Land Cost Ratio

In general, commercial markets and residential markets show opposite results. The data shows that major U.S. cities such as Los Angeles, Washington DC, New York, and Boston are less volatile in the commercial markets, but more volatile in the residential markets. Second-tier or third-tier cities such as Pittsburgh, Detroit, Memphis, and St Louis demonstrate the opposite. Regarding property types, overall, apartment buildings have the highest volatility, followed by retail and industrial buildings. Office buildings have the lowest volatility.

For commercial markets, the volatility is negatively correlated with the land cost ratio, but the regression result is not too significant. Therefore, they are only weakly correlated. Regarding property types, retail buildings have the highest land cost ratio, followed by office and apartment buildings. Industrial buildings have the lowest land cost ratio. For residential markets, the volatility is positively correlated with the land cost ratio, and the regression result is very significant. That means that major U.S. cities with high volatility in the residential markets also have high land cost ratios, and vice versa.

(2) Option Premium Value, Land Cost Ratio and Construction Activity

For commercial markets, the higher the asset value volatility is, the higher the option premium value will be, and the lower the level of construction activities will result. Meanwhile, the higher the land cost ratio is, the lower the level of construction activities will result. At an aggregate level across multiple MSAs, apartments are the most sensitive to option premium value, followed by industrial and office. Retail is the least sensitive to option premium value. Compared between the time-series (across time within a MSA) study and the cross-sectional (across multiple MSAs within a time period) study, the time-series study seems to show higher sensitivity to option premium value than the cross-sectional study does. The time-series study also shows more significant regression results than the cross-sectional study does.

For residential markets, results between the time-series study and the cross-sectional study are not as consistent as those of the commercial markets study. In the time-series study, the higher the asset value volatility is, the higher the option premium value will be, and the lower the level of construction activities will result. Meanwhile, the lower the land cost ratio is, the lower the level of construction activities will result. However, the cross-sectional study shows the opposite. Compared between the time-series study and the cross-sectional study, the time-series study again seems to show higher sensitivity to option premium value than the cross-sectional study does. Between commercial markets and residential markets, the regression results show that residential markets are more sensitive to option premium value than commercial markets do. Residential markets also show more significant regression results than commercial markets do. Overall, the correlation is a lot stronger between the option premium value and the level of construction activity than that between land cost ratio and the level of construction activity. In the Variance Inflation Factor tests for the issue of multi-collinearity, both results of commercial markets and residential markets show that the correlation between the option premium value and the land cost ratio is not strong and problematic enough to distort their effect on the level of construction activity.

(3) Game Theory and Real Options Theory

Overall, the game theory model offers powerful insights to explain things that cannot be explained by the real options theory model.

For commercial markets, the level of construction activities will change more in office market than in retail market per unit change in option premium value because office markets have a Develop-Develop equilibrium while retail markets have a Develop-Wait equilibrium. The game theory model also explains why the regression results across time within a MSA are more sensitive to option premium value than results across multiple MSAs within a time period because different MSAs have different market characteristics, and thus different equilibria in different game scenarios. Therefore, the effect of option premium values on level of construction activities is different in different MSAs, and their correlation is weaker in the cross-sectional analysis. For example, more construction activity will result in the Develop-Develop Equilibrium of New York office market than the Wait-Wait Equilibrium of Dallas office market per unit change in option premium value.

For residential markets, the game theory model shows that option premium values do not have a strong effect on the timing strategy of developers. Residential developers all tend to compete with each other and develop immediately whether they have a high option value of waiting or not, as illustrated by the 6 MSAs with very different market characteristics which all tend to have a Develop-Develop equilibrium. That partially explains why option premium value is not negatively correlated with level of construction activity as it should be as predicted by the real options theory.

As demonstrated throughout the thesis, the interaction of the real options theory and the game theory provides a more comprehensive framework to analyze investment timing options in real estate development than the standard Net Present Value method. The real options model forces developers to think more strategically about the benefits of waiting through the calculation of option values of waiting. It can change the decision of developers drastically and has far-reaching implications on the development strategies of developers. While the real options model focuses on the development strategies internally, the game theory model analyzes them externally. The game payoff matrix offers powerful, and yet sometimes counter-intuitive, implications to advise developers with the optimal responses. Although the option game theoretic approach works well at a strategic level, there are a few hurdles that make it hard to be widely used in real estate development. To quantify and capture all variables in a real estate development project, which is by nature very complex, will make the real options model too complicated to be practical. It is also impossible to accurately calculate the payoffs of the opponent and the corresponding values of ω_{Di}, ω_{Li}, and ω_{Fi} in the game theory model. This partially explains why the game theory and the real options theory are not as widely used in real estate development as they are in other industries such as the pharmaceutical industry. Nonetheless, at a conceptual and strategic level, the option game theoretic approach is still a very valuable and robust tool to analyze the market characteristics, to calculate the intrinsic land value, and to strategize the optimal action in response to the interactive effect between players in real estate development.

6.2 Topics for further study

There are a few questions that are worth further investigating, which are touched on briefly in this thesis but do not go in detail yet; (1) Can there be a unified model that capture the essences of the game theory and the real options theory which can be used like what the Black-Scholes formula is used in the financial industry to become a standard formula to calculate the intrinsic land value? (2) If such model exists, how would that change the real estate game? Would it make lands tradable like stocks and would there be a derivative market for real estate? (3) Is there a way to measure whether a market is in the equilibrium state as predicted by the game theory model, and whether developers in that market act accordingly? So, will the game theory become a much more quantifiable business strategy? (4) When the same methodology is used to analyze other global markets, will that give similar results to the U.S. markets, and if not, what makes the results different?

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