

**PV is conserved** is our strongest  
statement to explain weather

But then where does PV come from?

ATM 405/561

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# Outline

- Brief review of conservation of PV concept
  - <https://www.notion.so/miamimapes/Horizontal-vorticity-and-PV-as-explanations-for-cyclones-anticyclones-2e6d2c075dba44699dc822ca5748e2e8>
- Motivated by that, examine diabatic heating profiles with hungry eyes, and explain what you find. Practice telling an illustrated narrative well.

# Questions about it

- 1. Using the concepts from the handout, and earlier homework, explain how patches or elements of **relative vorticity** advect other patches of **relative vorticity**, under the assumption that **relative vorticity** is sorta almost conserved.
- 2. Using the concepts from the handout, and earlier homework, explain how **planetary vorticity** is converted to **relative vorticity**, so that their sum, the **absolute vorticity**, is almost conserved. Consider a loop of air moving in latitude, and explain how the different Coriolis force felt by its northern and southern side act as a torque on the fluid loop.
- 3. Using the concepts from the handout, and the reading, explain how **static stability** is converted to **absolute vorticity**, so that **potential vorticity**, their **product**, which is the essence of vortices (cyclones and anticyclones) is really really almost conserved.
- 4. Using the concepts from the handout, strategize what you will look for in vertically resolved data about diabatic heating rate in the atmosphere to explain the ultimate source of PV.

# Remember absolute vorticity?

- Absolute vorticity  $\zeta_a = f + \zeta_{rel}$
- Exists from  $f$  alone, but the  $f$  part can also be *converted* to actual wind circulation  $\zeta_{rel}$  whenever *air moves toward the equator*
- It is *amplified exponentially by convergence, and relaxed exponentially toward 0 by divergence:*

- $$\frac{D\zeta_{abs}}{Dt} = -\zeta_{abs}(\vec{\nabla}_p \cdot \vec{V})$$

Horizontal convergence also spreads material surfaces apart vertically

$$(\nabla \cdot \vec{V}) = -\frac{1}{\zeta_{abs}} \frac{D\zeta_{abs}}{Dt} = \frac{1}{A} \frac{DA}{Dt} = -\frac{1}{(\Delta p)} \frac{D\Delta p}{Dt}$$

So the **RATIO PV = -g  $\zeta_a$  ( $\partial\theta/\partial p$ )** remains unchanged by horizontal divergence!

$$\boxed{\frac{D}{Dt}(PV) = 0} - g\zeta_{abs} \frac{\partial \dot{\theta}_{diab}}{\partial p}$$

# PV is conserved -- almost

Here is the key term that generates PV on the Earth

$$\frac{D}{Dt}(PV) = 0 - g\zeta_{abs} \frac{\partial \dot{\theta}_{diab}}{\partial p}$$

In this lab, you will learn (and show me you learned) about the nature of diabatic heating in the atmosphere.

# PV is conserved -- almost

Here is the key term that generates PV on the Earth

$$\frac{D}{Dt}(PV) = 0 - g\zeta_{abs} \frac{\partial \dot{\theta}_{diab}}{\partial p}$$

1. We have available T tendencies, not theta tendencies, but can still eyeball the sense of the term above since  $\dot{\theta}_{diab} = \dot{T}_{diab} \times \theta/T$
2. For large scale motions, f is most of the absolute vorticity. Therefore, you will estimate/explain:

PV is conserved -- almost

**APPROXIMATE** term that generates PV on the Earth

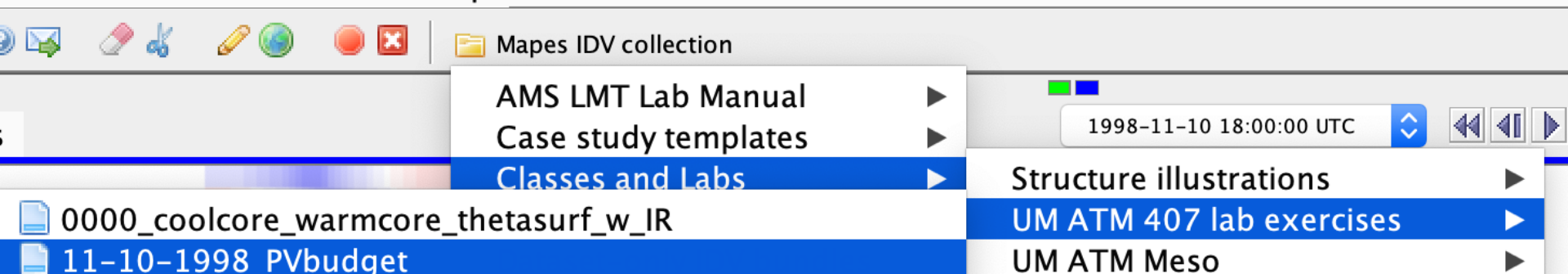
$$\frac{D}{Dt}(PV) = 0 - gf \frac{\partial \dot{T}_{diab}}{\partial p}$$

Mostly, you are looking for **WHERE HEATING RATE INCREASES WITH HEIGHT, WEIGHTED BY f**. In both hemispheres... so be careful with "cyclonic".



# Assignment part 1: global view

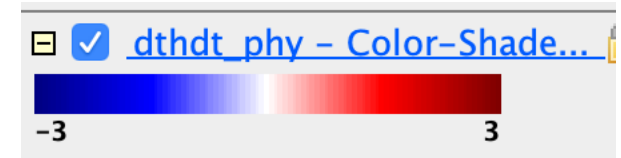
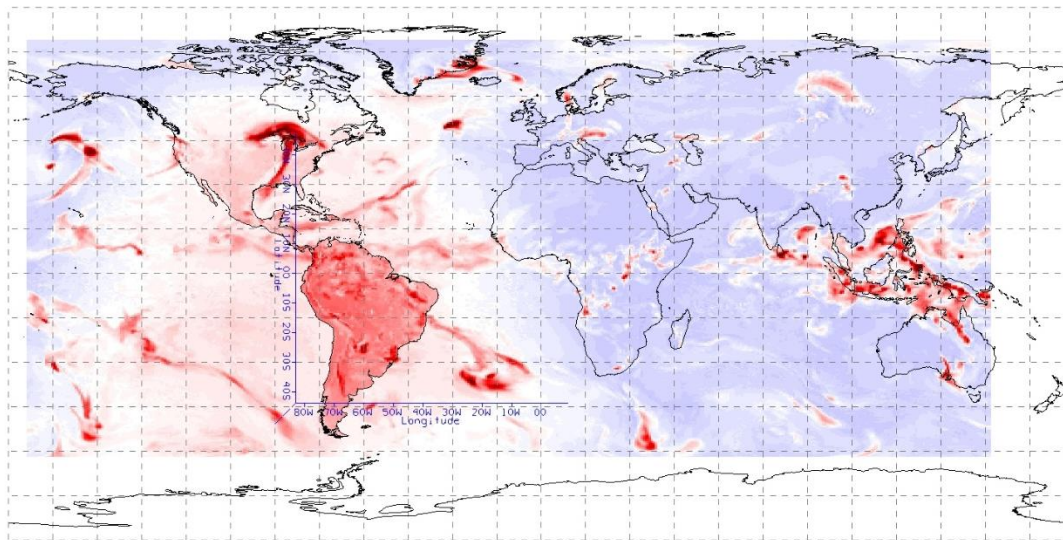
- Open the bundle 11-10-98 PV budget



- Orient yourself to its displays, in **both windows**
  - a **pole-to-pole transect** of the **zonal mean** heating rates (averaged around the whole Earth)
  - A map view with many displays.

# Assignment part 1: global view

1. What time of year is it? How can you see that fact in the **all-physics (diabatic) column-integrated heating rate** map `dthdt_phy`?

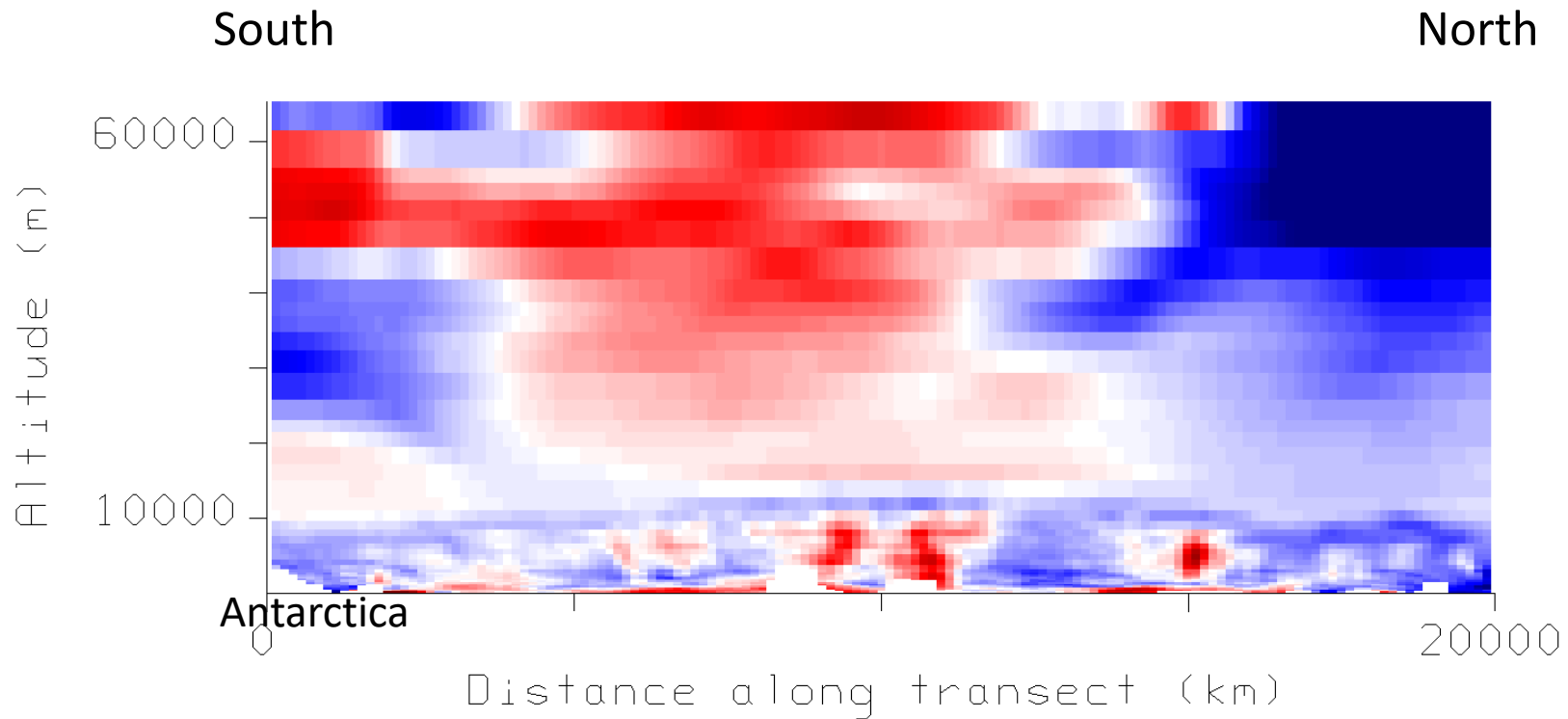


# Assignment part 1: global view

- Now turn to the Transect View window, showing average cross sections all around the Earth. Create a slide showing the transect of diabatic heating. Label it: where is the south pole, the north pole? Hint: Antarctica is mountainous. The units of all heating rates are  $\text{K/s}$ . What is the color range in  $\text{K/day}$ ?

# Assignment part 1: global view

## Diabatic heating

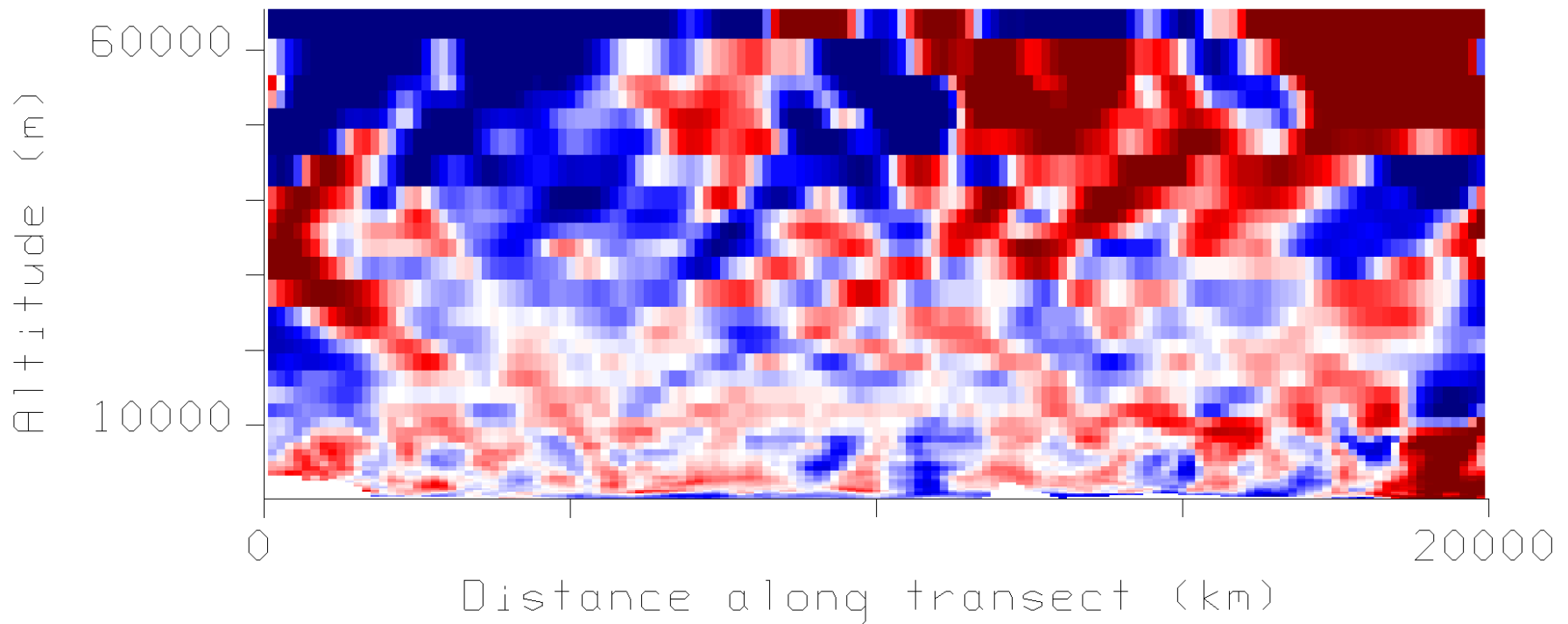


# Assignment part 1: global view

- Create slides with transect images showing each of these terms of the zonal mean heat budget.
- Use that imagery to explain the nature of all the main features in your total diabatic heating slide.
- These equations relate the terms displayed there:  
 $\partial T / \partial t = \text{dynamical} + \text{diabatic} + \text{analysis}$   
diabatic = moist + radiative + turbulence  
radiative = longwave + solar

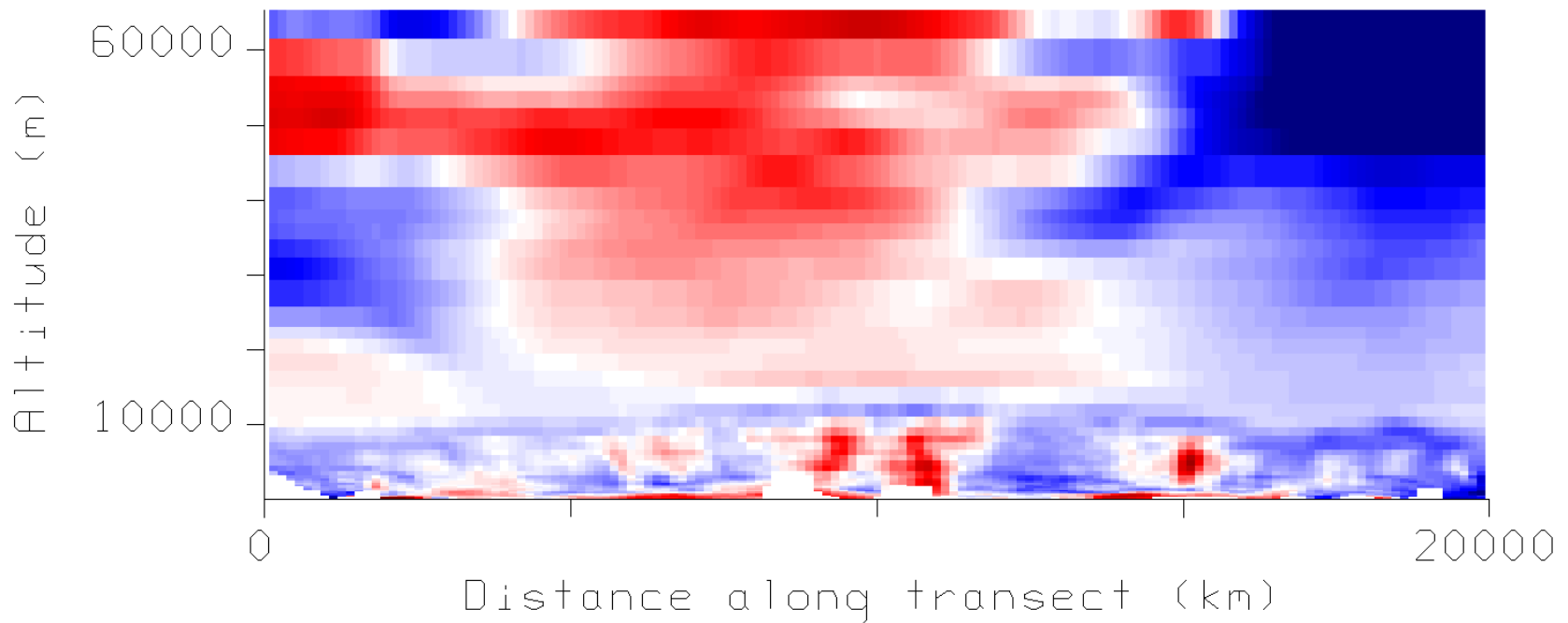
# Assignment part 1: global view

- Dynamical heating



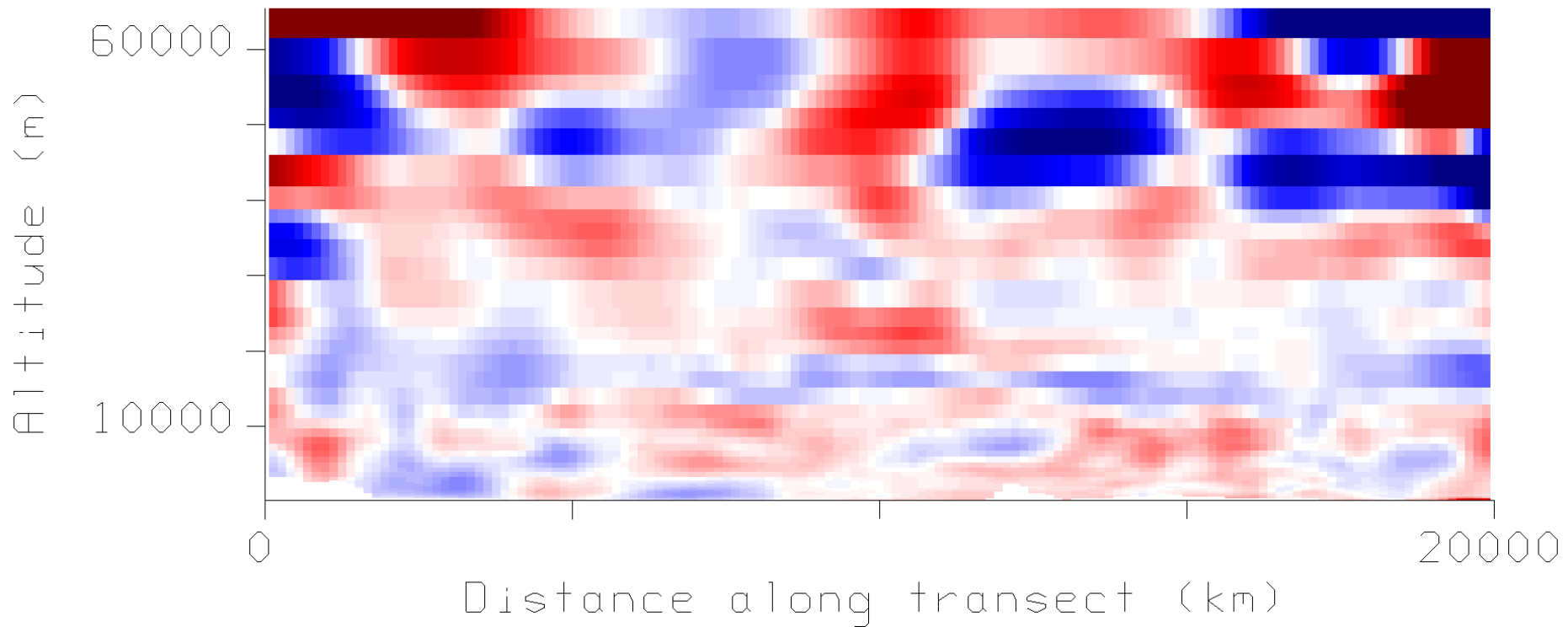
# Assignment part 1: global view

- Diabatic heating



# Assignment part 1: global view

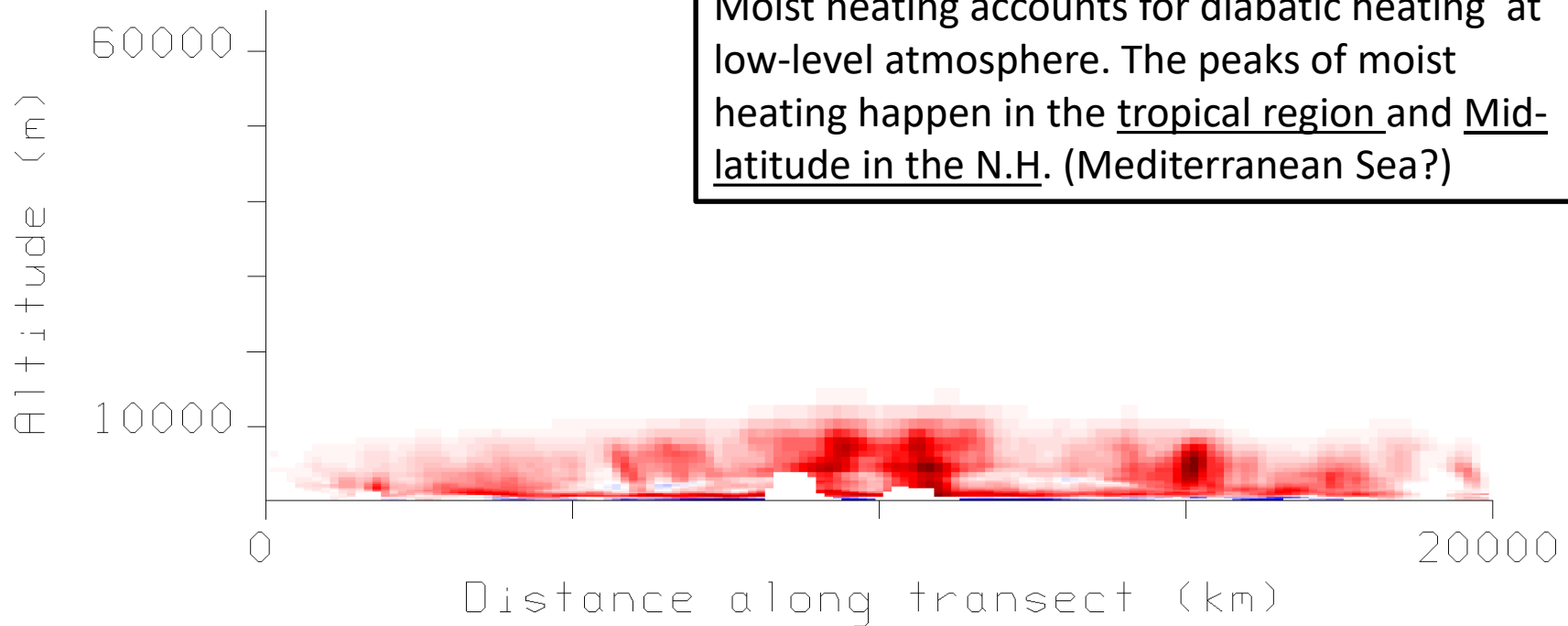
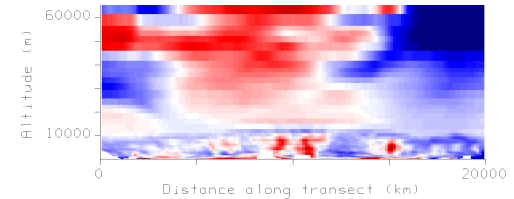
- Analysis/Model errors





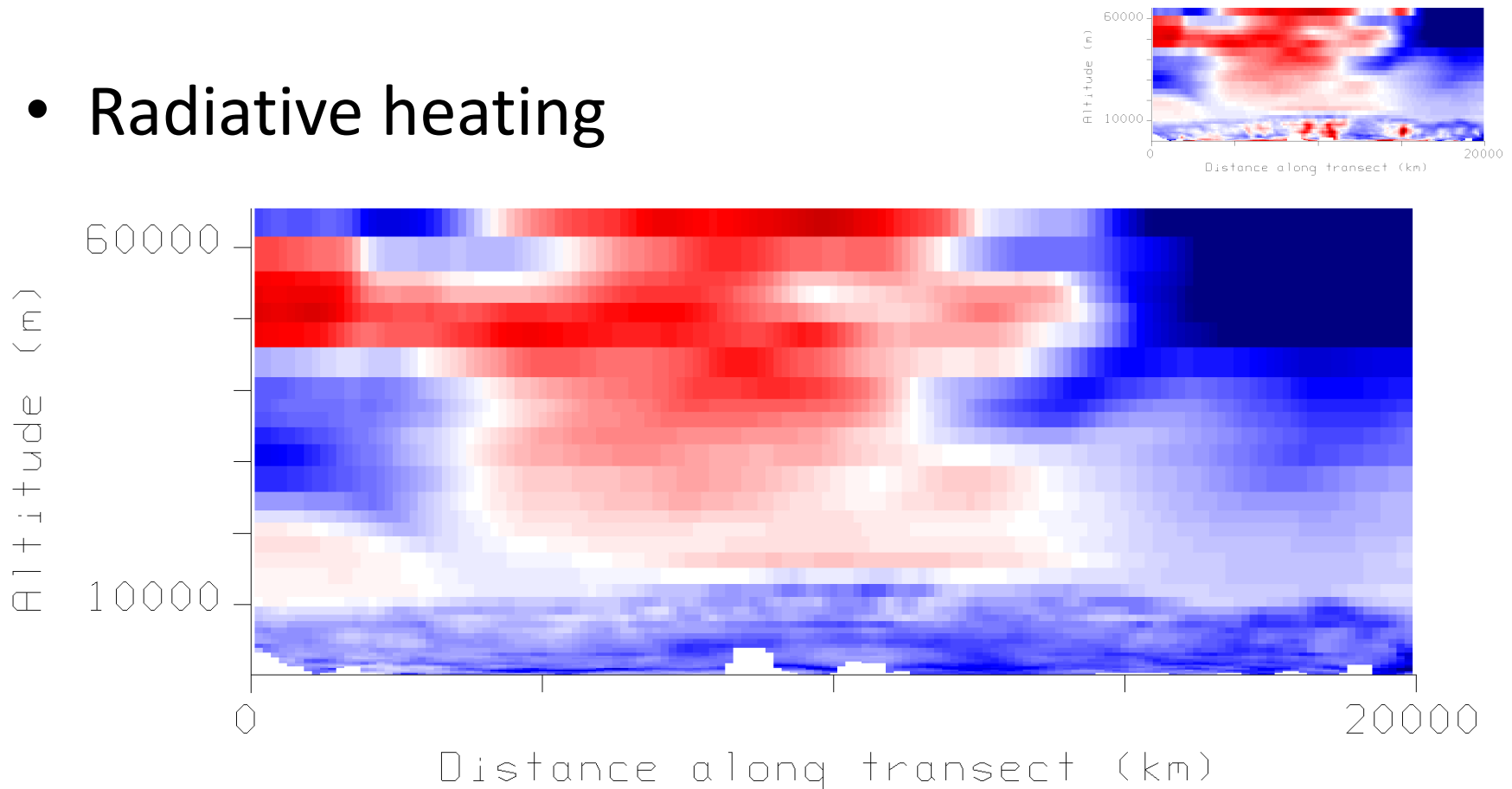
# Assignment part 1: global view

- Moist heating



# Assignment part 1: global view

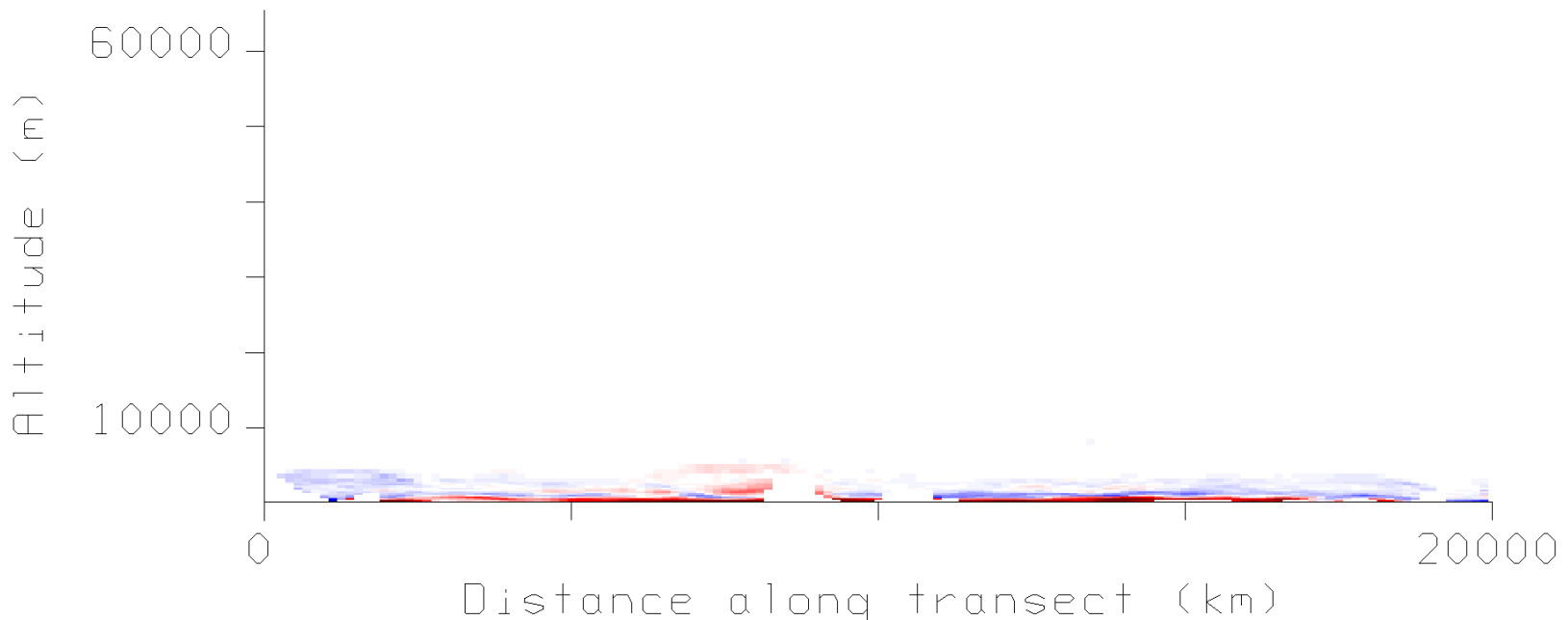
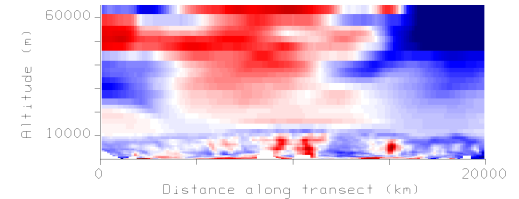
- Radiative heating



Radiative heating is the greatest part of diabatic heating happening at (10km - 60km).

# Assignment part 1: global view

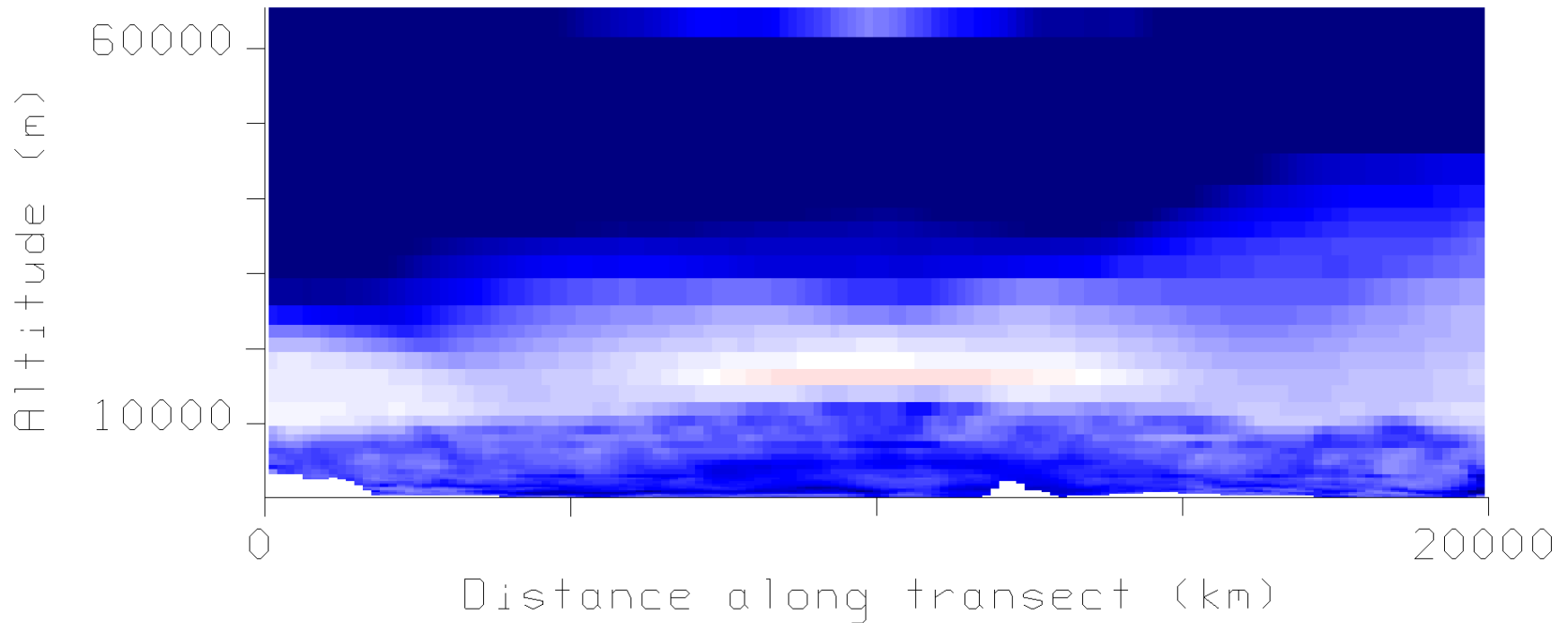
- Turbulence heating



Turbulence heating is the smallest part of diabatic heating happening at low-level atmosphere.

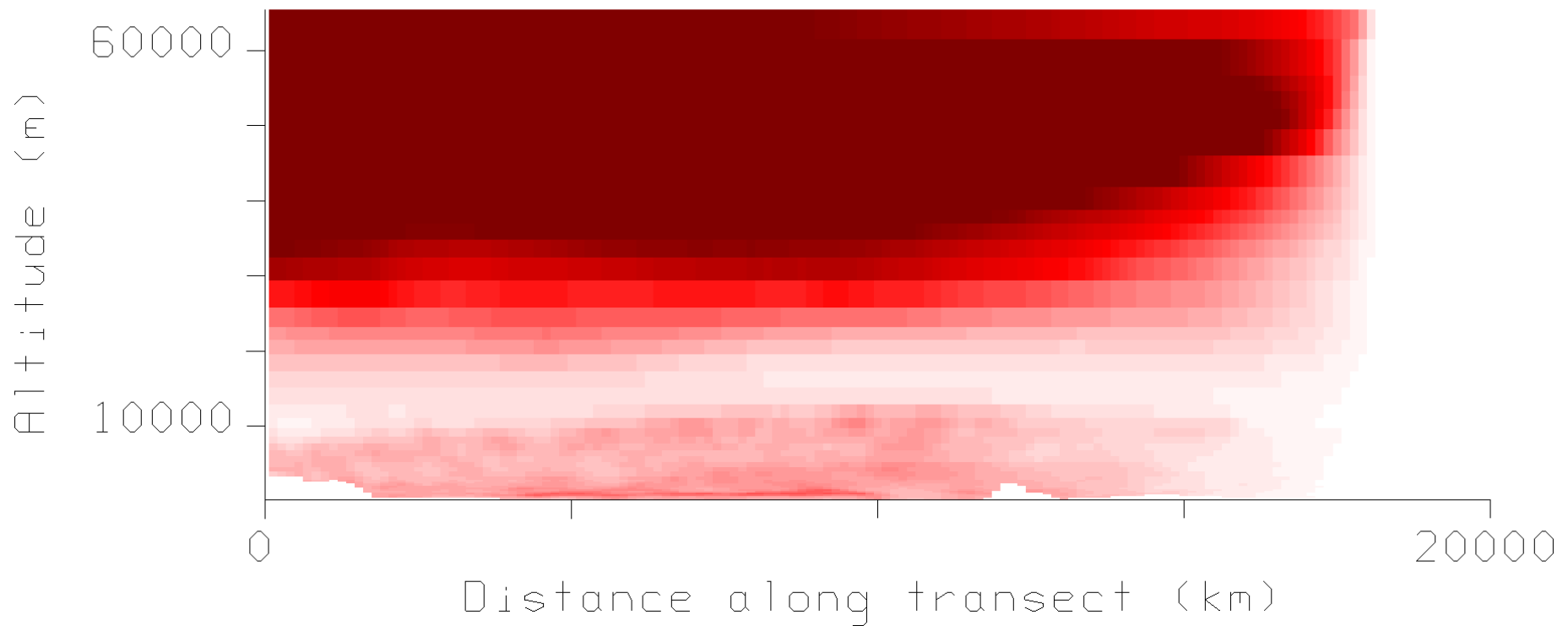
# Assignment part 1: global view

- Longwave heating



# Assignment part 1: global view

- Shortwave heating

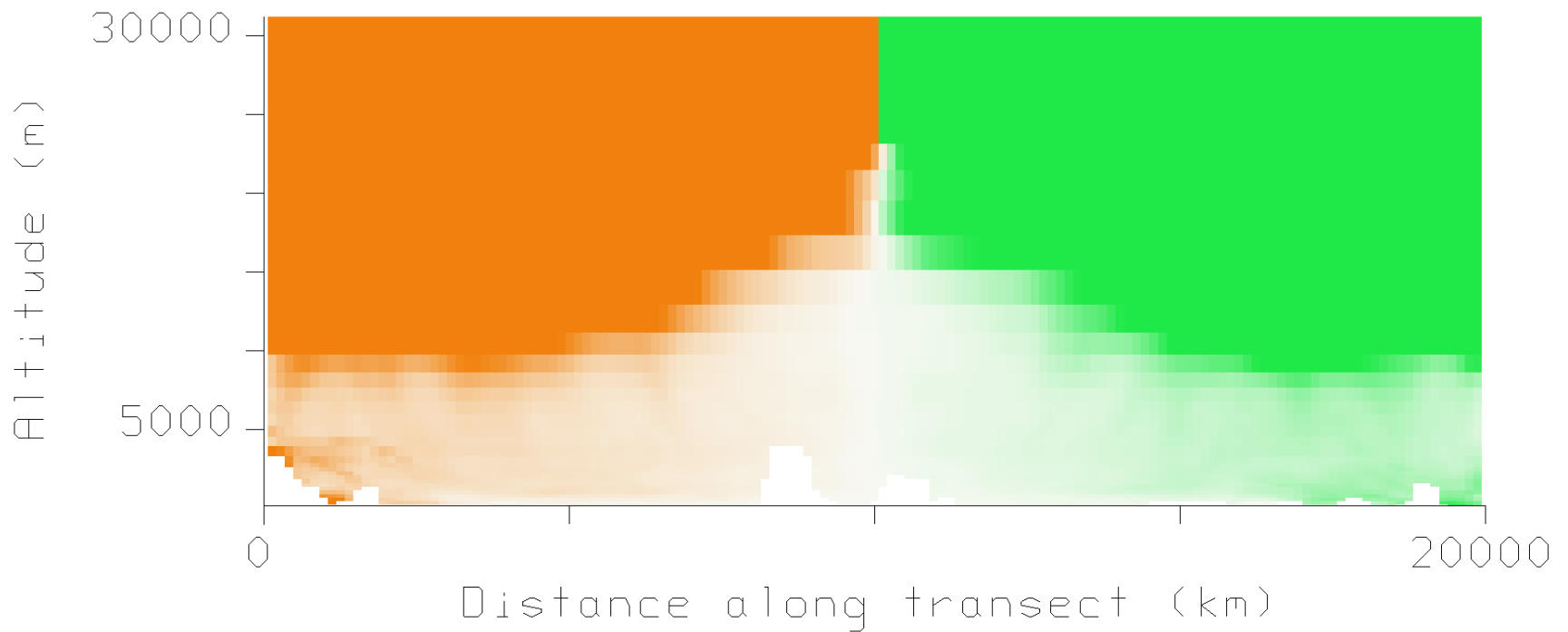


# Assignment part 1: global view

- From your total diabatic heating, indicate areas where PV tendency is positive and negative. Also label these areas as cyclonic or anticyclonic tendencies.
- Does the *PV transect itself* resemble areas where your *PV tendency* is strong?
- Is PV mostly contained in the vorticity factor of its product, or the static stability factor? Show and label images to explain your answer.

# Assignment part 1: global view

- PV



## Assignment part 2: Local view

- Now activate the *cross section displays in the map view window*.
- These are like the cross sections you have examined before: you can drag them around to storms or other features
- Drag them to north-south positions that slice through tropical and higher latitude weather features that interest you (as seen on the other displays).



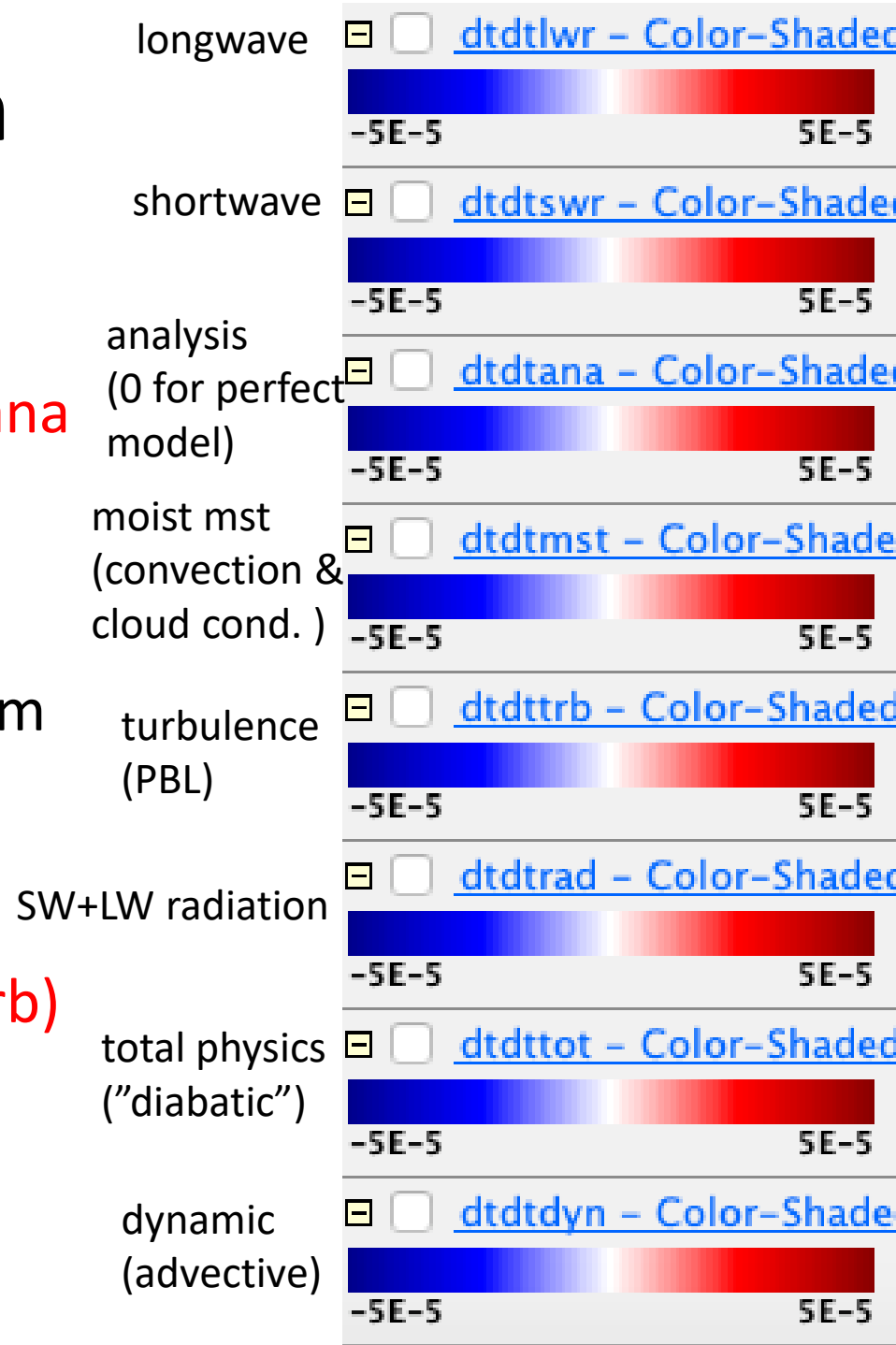
# Legend explanation for cross sections

$$\partial T / \partial t = \text{dtdt\_tot (physics)} + \text{dtdt\_dyn (advection)} + \text{dtdy\_ana}$$

(**ana** is *analysis*; a "missing" tendency needed to make the observed  $\frac{\partial T}{\partial t}$ ; reflecting the sum of all model errors)

diabatic **tot** = moist (mst) + radiative (rad) + turbulence (trb)

**rad** = lwr + swr

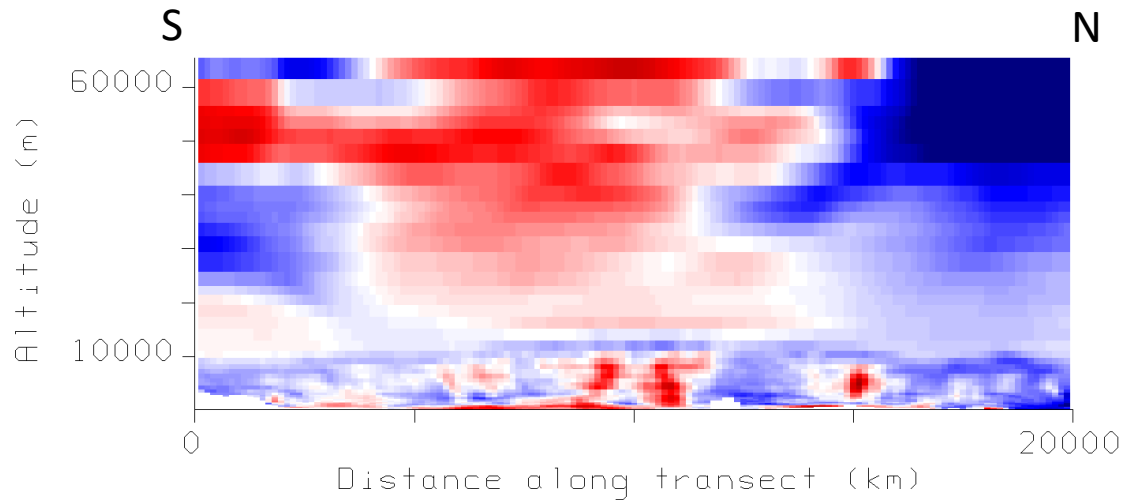


# Assignment part 2: Local view

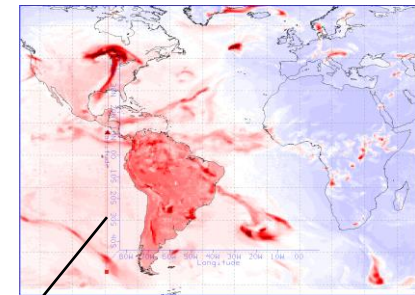
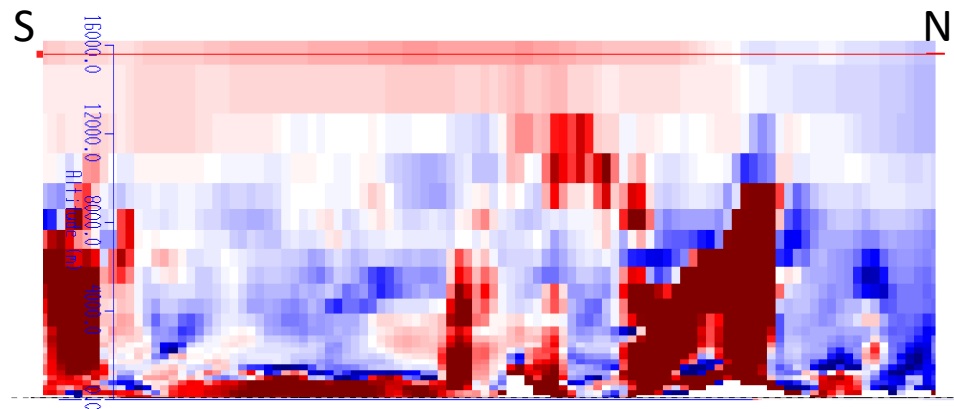
- Make comparison figures between the global average transects and your local cross-sections, and address these questions:
  - Does the moist heating (condensation related processes) correspond to the high cloudiness you infer from the OLR display?
  - Does its vertical gradient imply large PV sources in any places relevant to storm-scale PV budgets?

# Diabatic heating

- global average transects

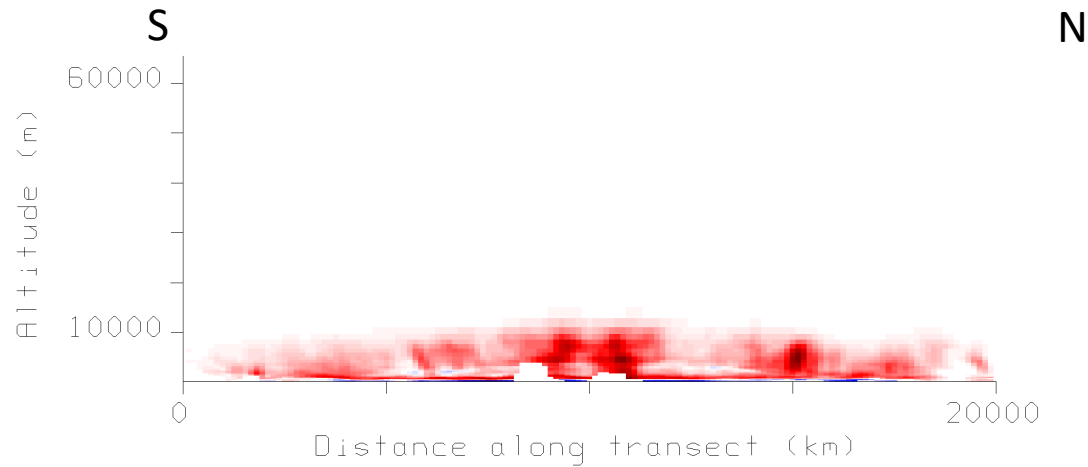


- local cross-sections

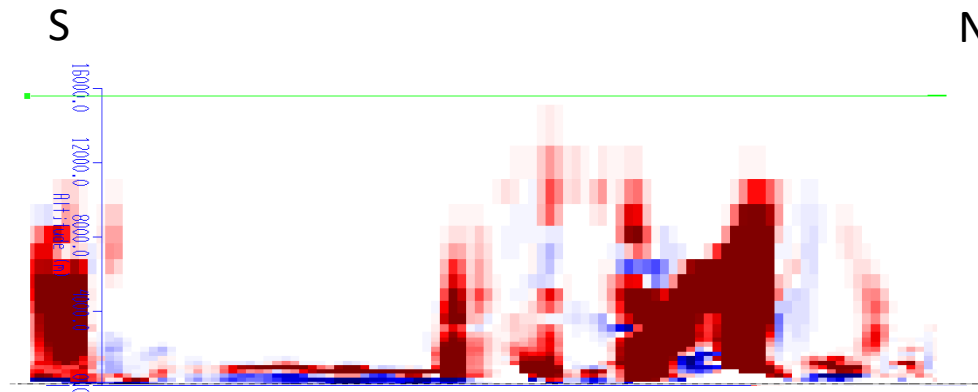


# Moist heating

- global average transects



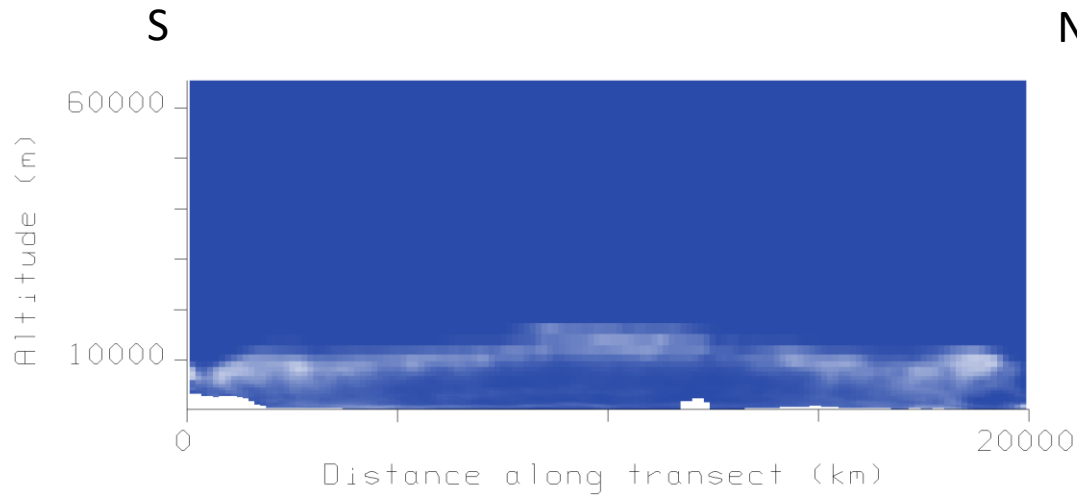
- local cross-sections



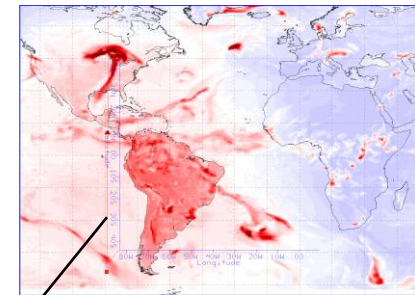
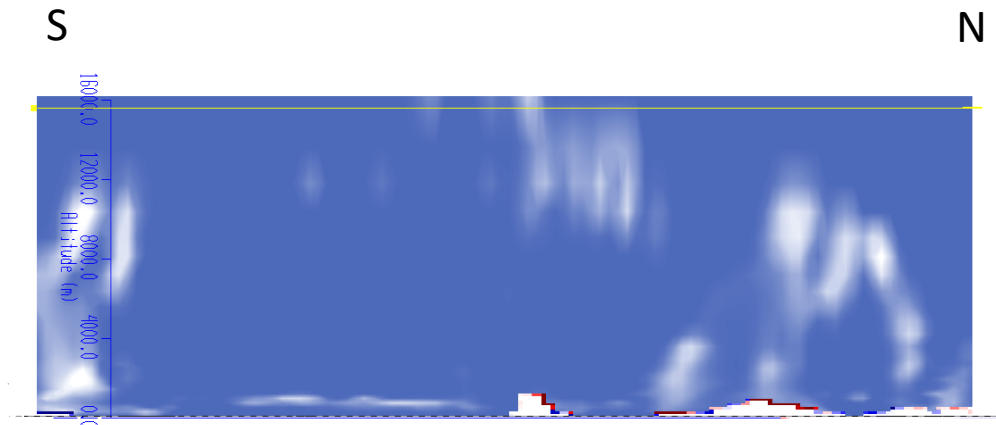
# Cloud fraction

High moist heating area is associated with high cloudiness.

- global average transects



- local cross-sections

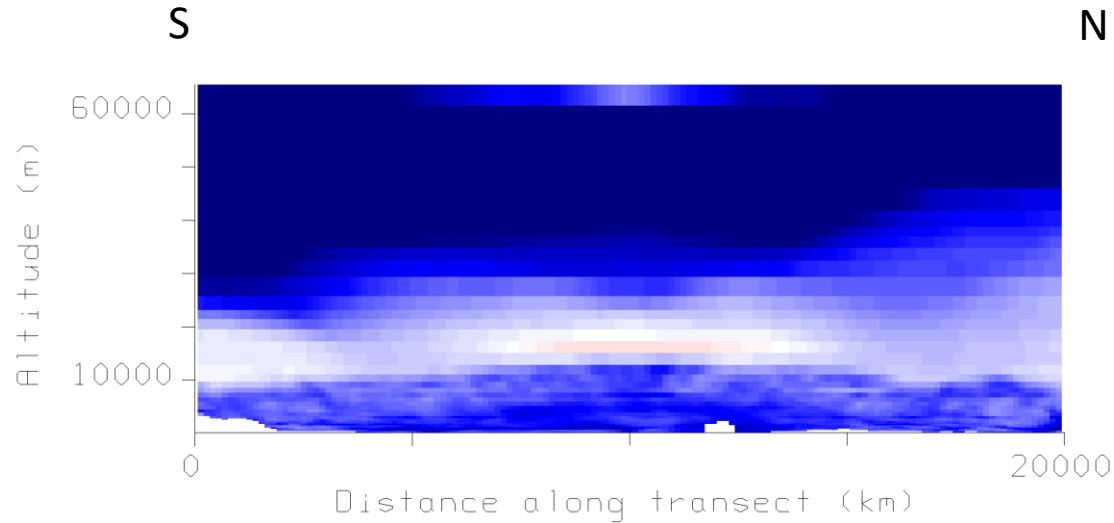


# Assignment part 2: Local view

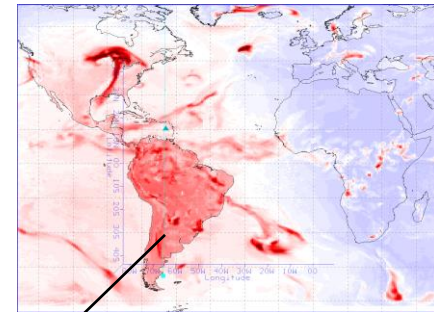
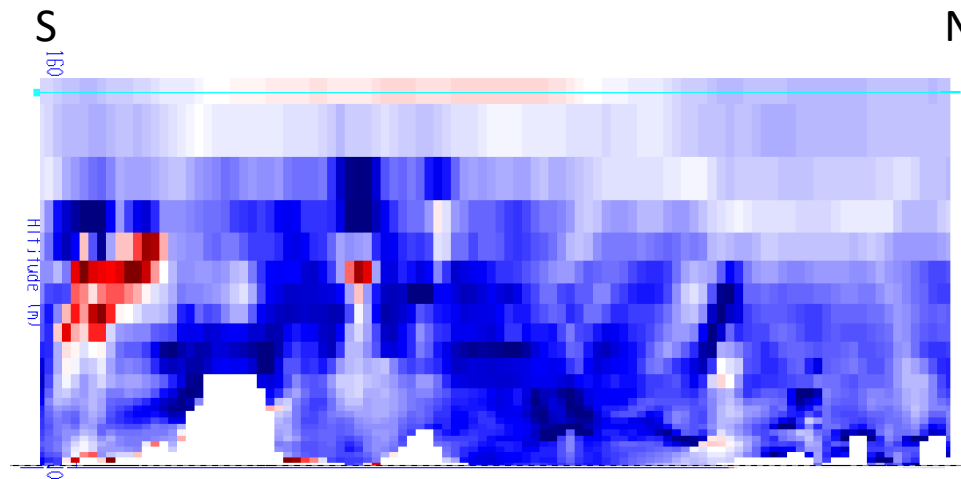
- Make comparison figures between the global average transects and your local cross-sections
  - LW radiation can be understood as water vapor cooling, cloud top cooling, and cloud base warming. Use cloudiness and longwave cross-section images in tandem to show an example of a place where cloud effects are dominant
  - What do these strong vertical heating gradients imply as a PV source?

# Longwave heating

- global average transects



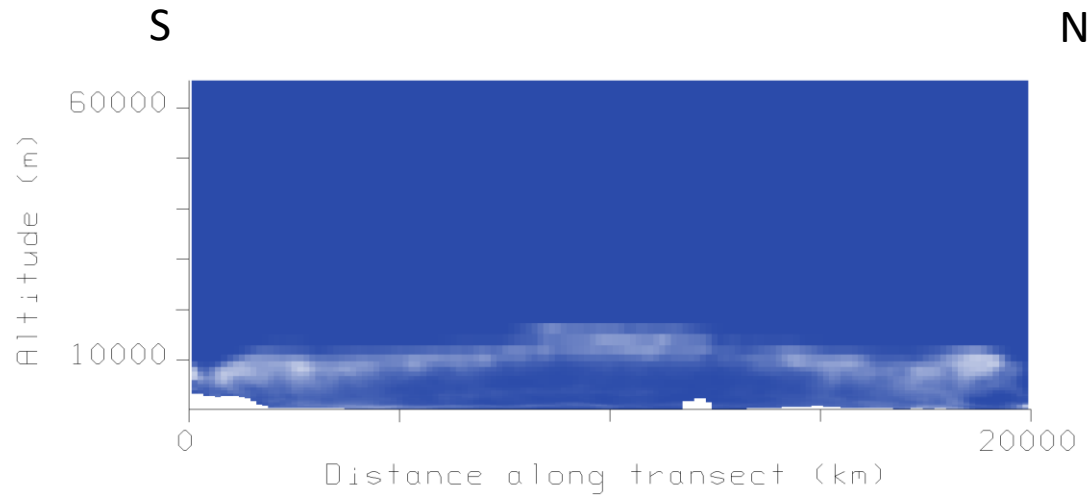
- local cross-sections



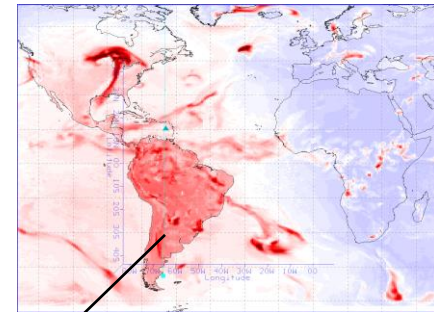
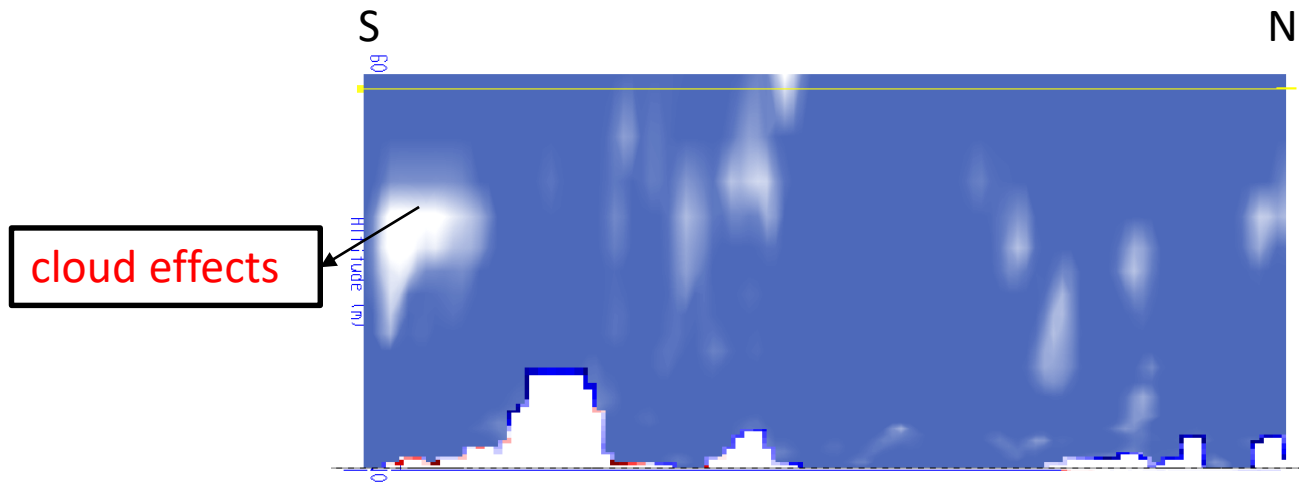
Lon:  $-65^{\circ}$

# Cloud fraction

- global average transects



- local cross-sections





# “The Primitive Equations”

(meaning elemental, fundamental)

$$\frac{D}{Dt} \vec{V}_h = -f \hat{k} \times \vec{V}_h - \vec{\nabla}_p \Phi + \vec{F}_r \quad \text{F=MA in the HORIZONTAL}$$

$$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p} \quad \text{HYDROSTATIC}$$

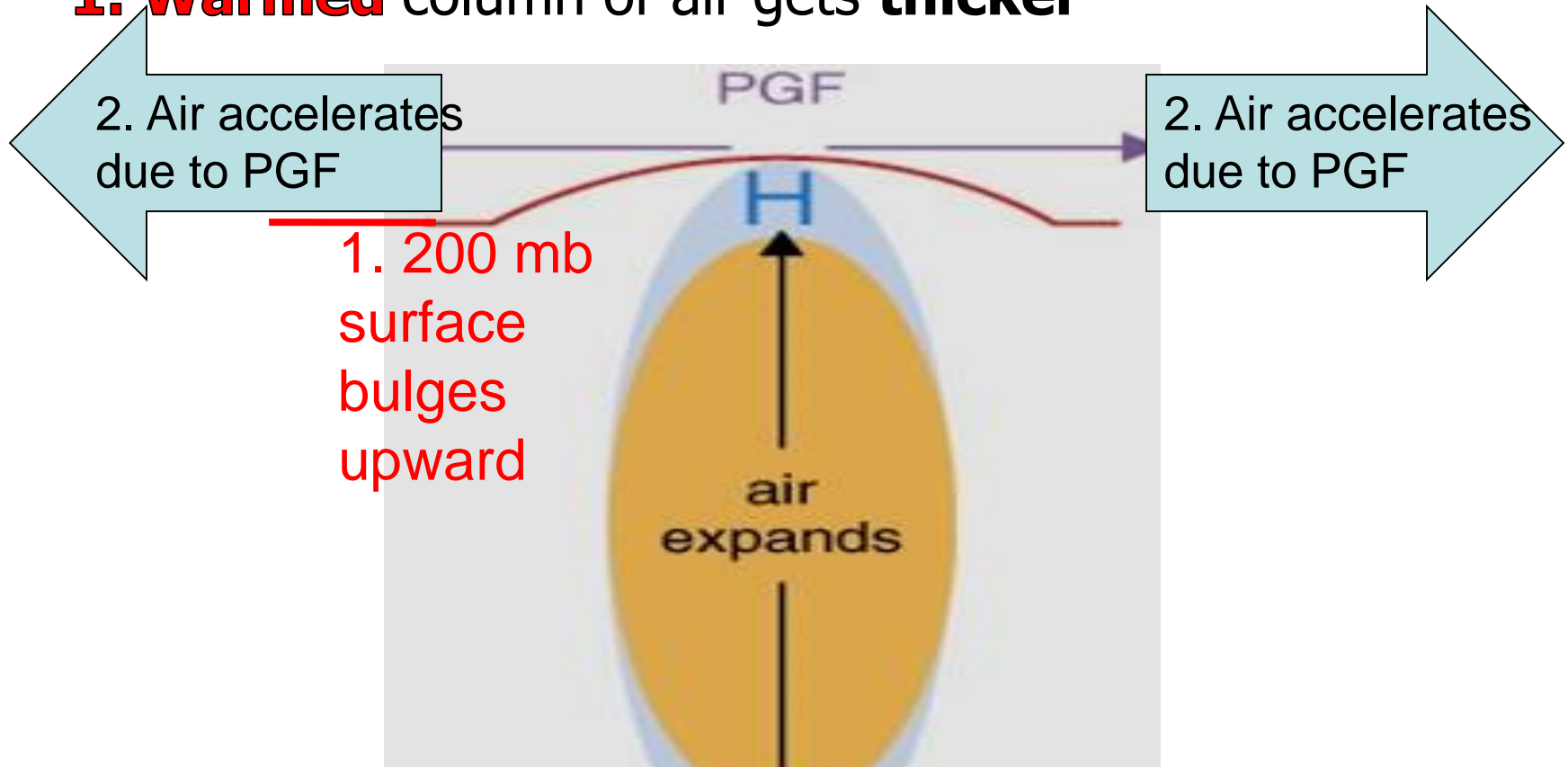
(w/ ideal gas law to eliminate  $\rho$ )

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \quad \text{MASS CONSERVATION}$$

$$\frac{\partial T}{\partial t} = -\vec{V} \cdot \vec{\nabla}_p T - \omega S_p + \frac{J}{C_p} \quad \text{FIRST LAW OF THERMO}$$

# How heated air rises and a warm core vortex develops: the Primitive Equation view. 7 logical steps

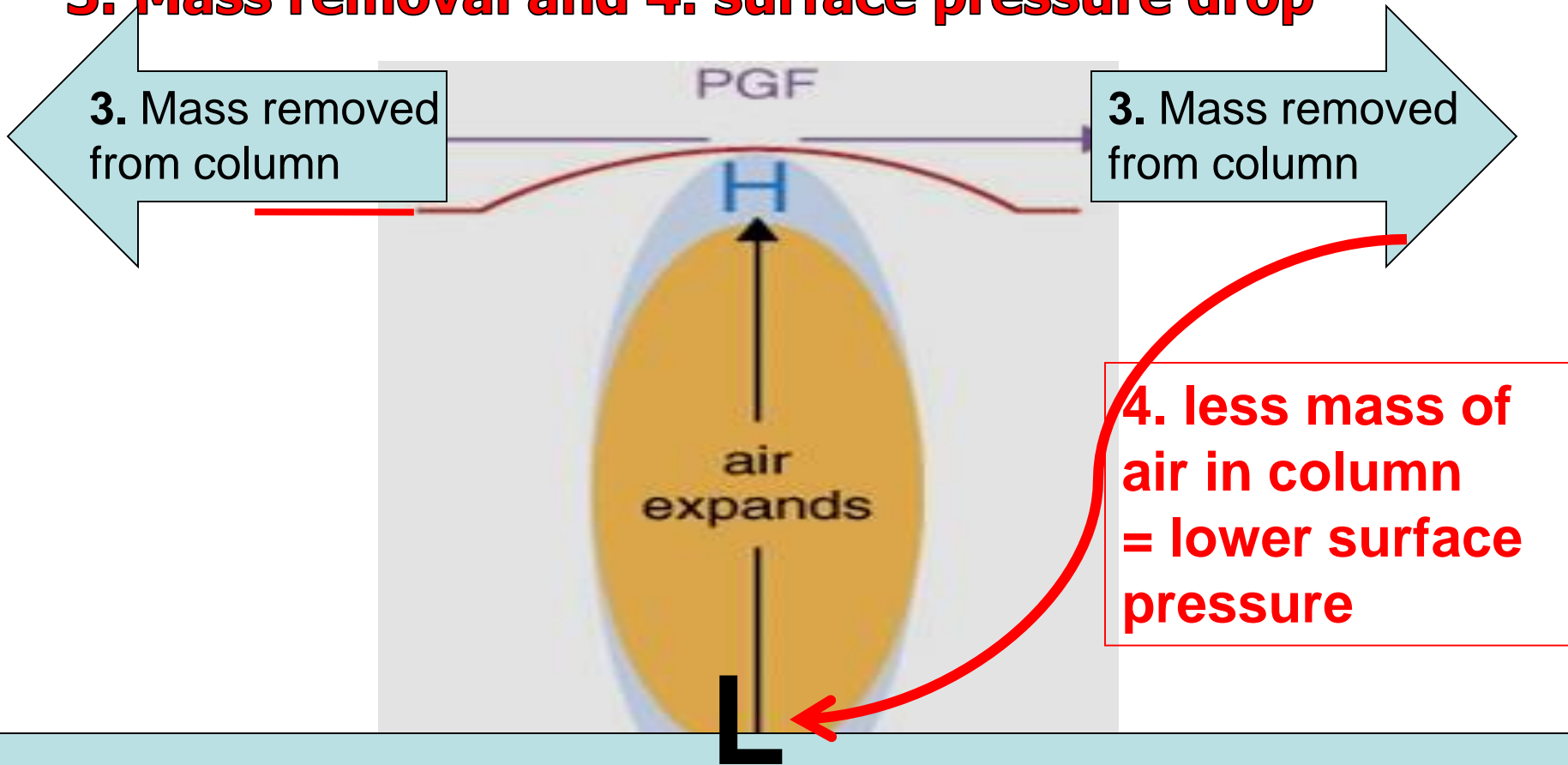
**1. Warmed** column of air gets **thicker**



0. Heating (maybe latent heating by condensation in a patch of convection over warm water someplace)

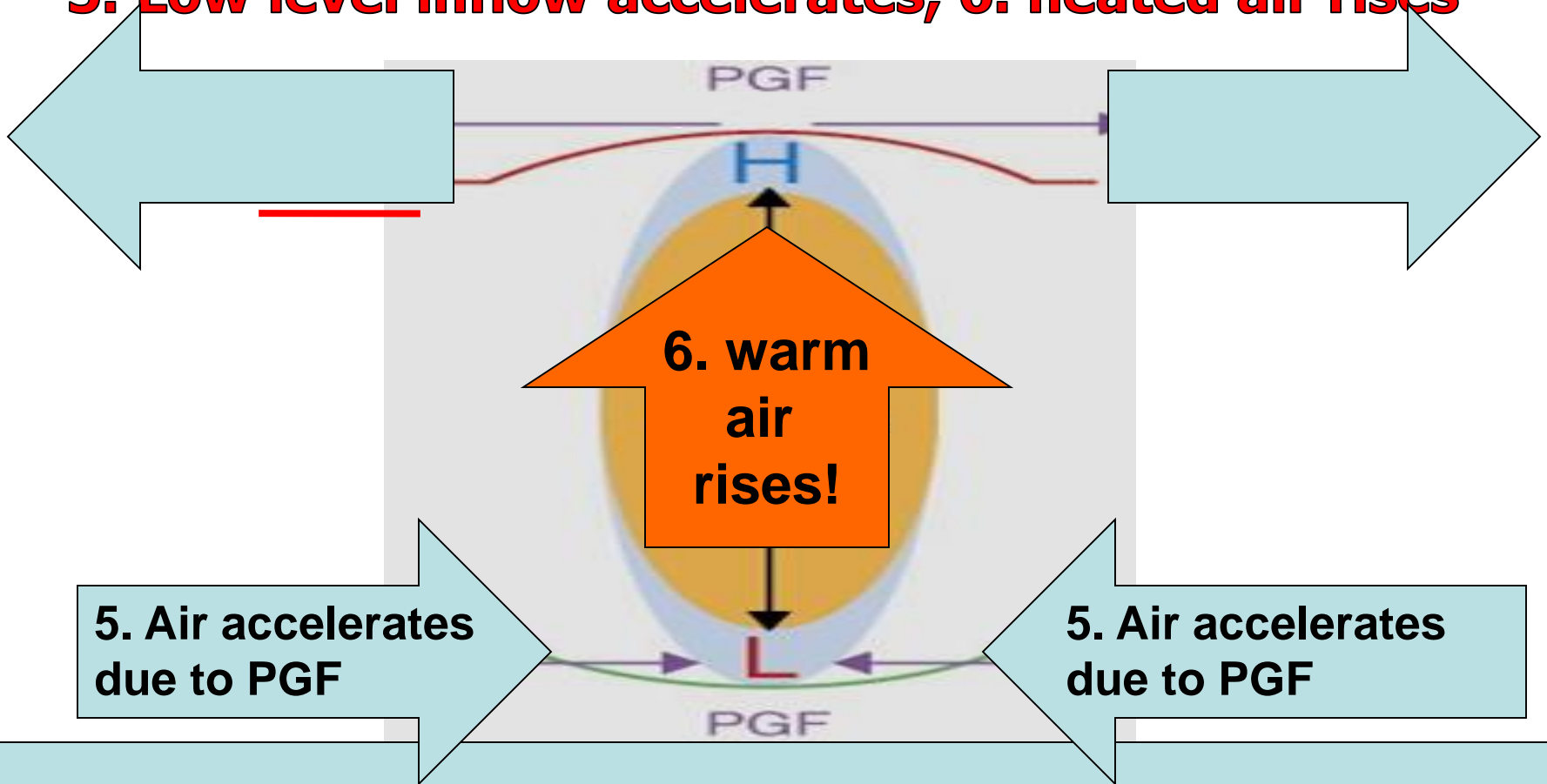
# How heated air rises and a warm core vortex develops: the Primitive Equation view. 7 logical steps

## 3. Mass removal and 4. surface pressure drop

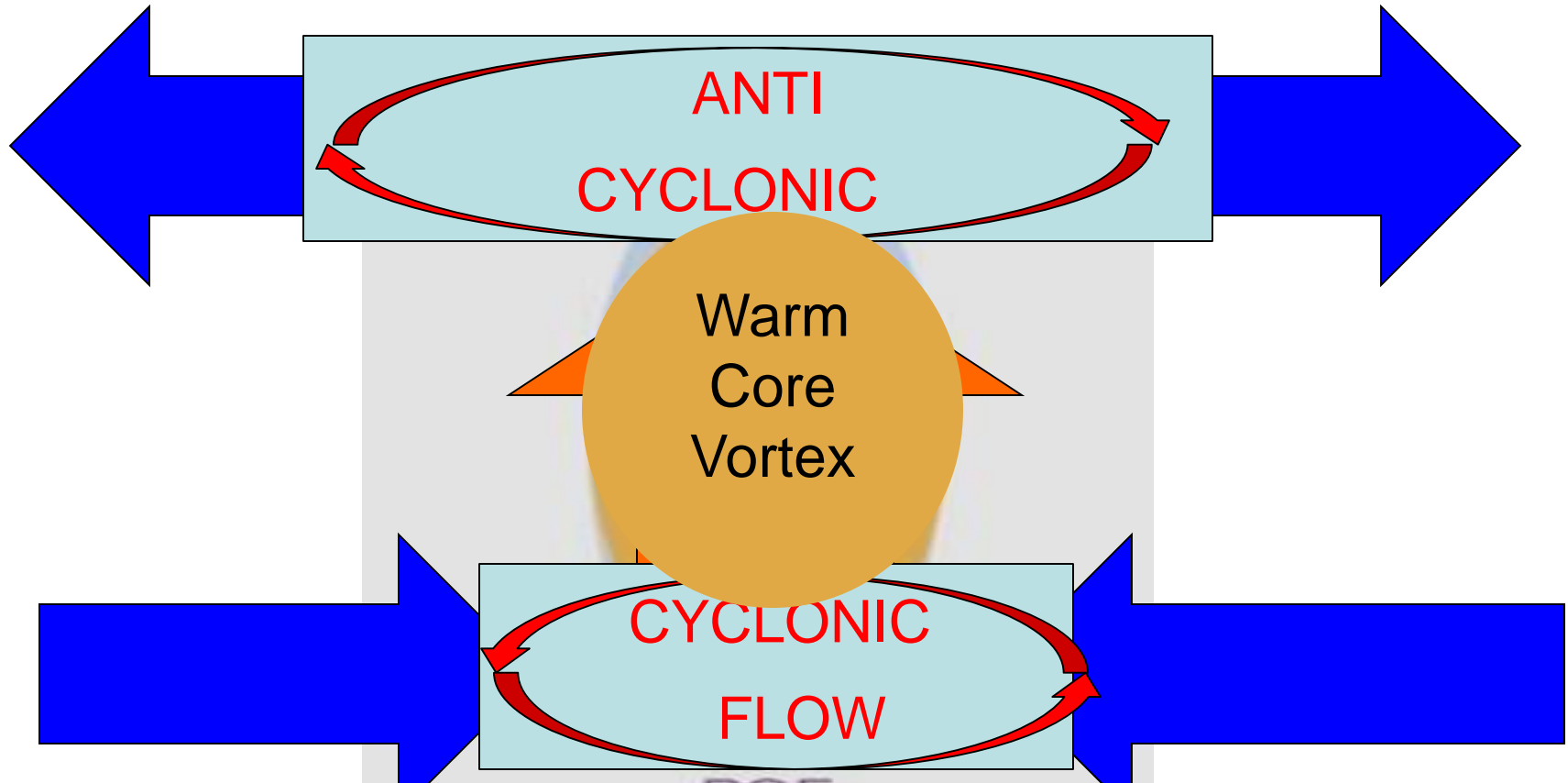


# How heated air rises and a warm core vortex develops: the Primitive Equation view. 7 logical steps

**5. Low level inflow accelerates, 6. heated air rises**



# How heated air rises and a warm core vortex develops: the Primitive Equation view. 7 logical steps



**7. Coriolis turns flow to right**

HW: use The Primitive Equations to compute how a local heating  $J$  drives flow in an initially motionless atmosphere

$$\frac{D_h T}{Dt} \boxed{\phantom{0}} = J/C_p$$

**1.  $J$  causes  $T$  to increase**  
 net change of  $T$  =  
 amount of heat added/ $C_p$

$$\frac{\partial \Phi}{\partial p} = - \frac{RT}{p}$$

**2. Warmer  $T$  causes increased thickness** of the heated column

$$\frac{D}{Dt} \vec{V}_h = \boxed{\phantom{0}} - \vec{\nabla}_p \Phi$$

**3. High  $\Phi$  over hot column pushes wind outward**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

**4. Surface pressure drops**  
 (remember,  $\omega = Dp/Dt$ ; Holton eq. 3.44)

HW: use The Primitive Equations to compute how a local heating  $J$  drives flow in an initially motionless atmosphere

$$\frac{D}{Dt} \vec{V}_h = \boxed{\phantom{0}} - \vec{\nabla}_p \Phi$$

**5. Low  $\Phi$  under hot column pulls wind inward**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

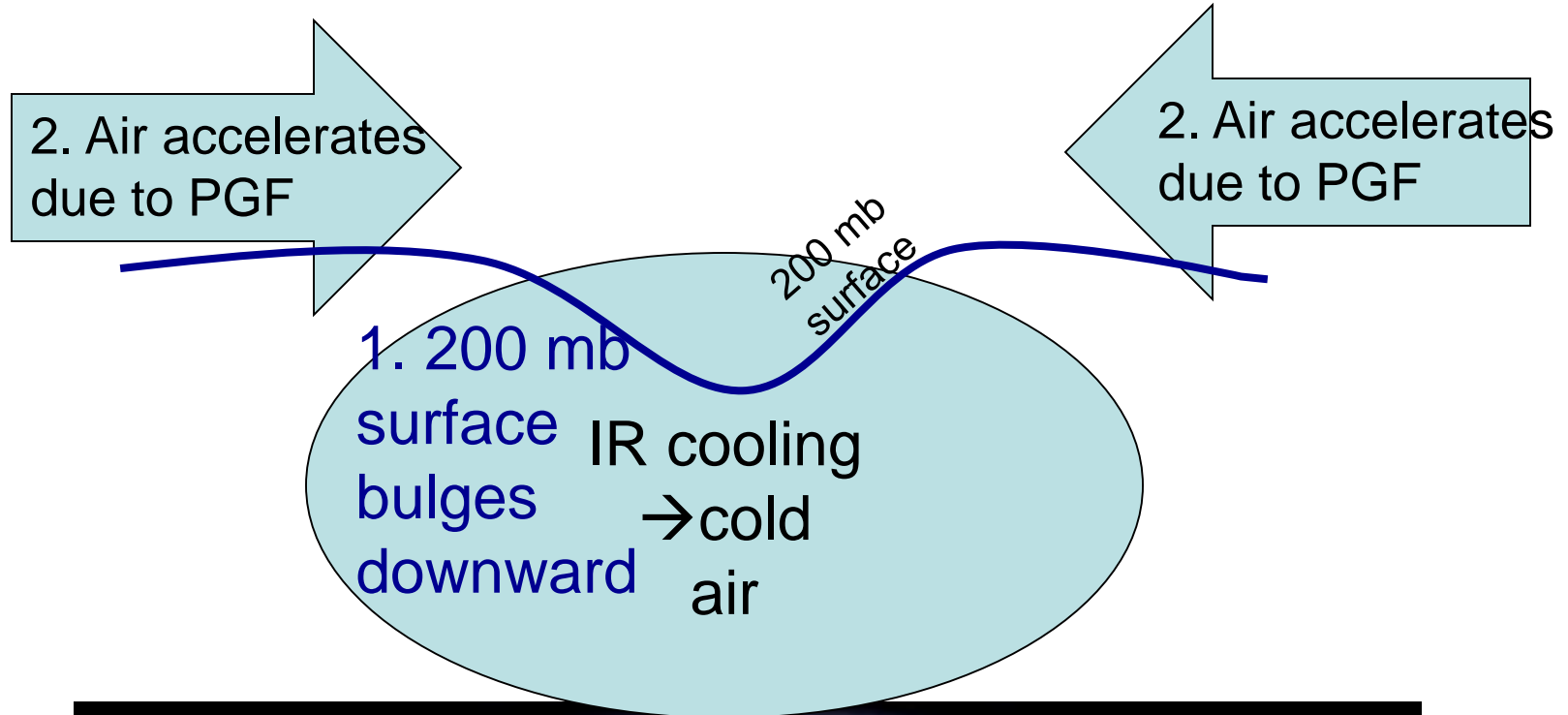
**6. Hot air rises (finally!)**  
 $\omega \sim -\rho g w$

$$\frac{D}{Dt} \vec{V}_h = -f \hat{k} \times \vec{V}_h \boxed{\phantom{0}}$$

**7. Coriolis force turns inflowing and outflowing air to make round-and-round flow**

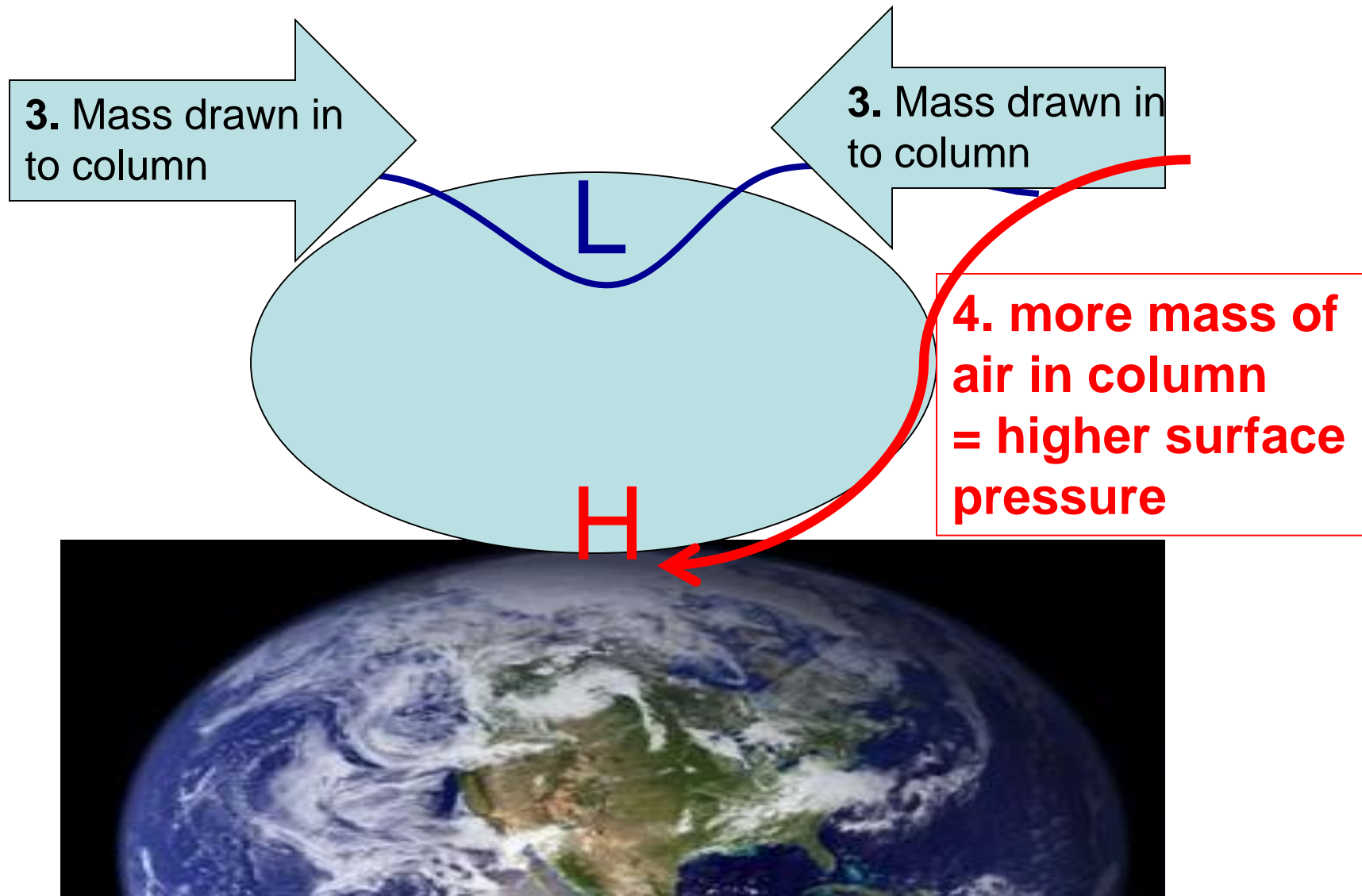
# How cooled air sinks and a cool core vortex develops: the Primitive Equation view. 7 logical steps

**1. Cooled** column of air gets **thinner**

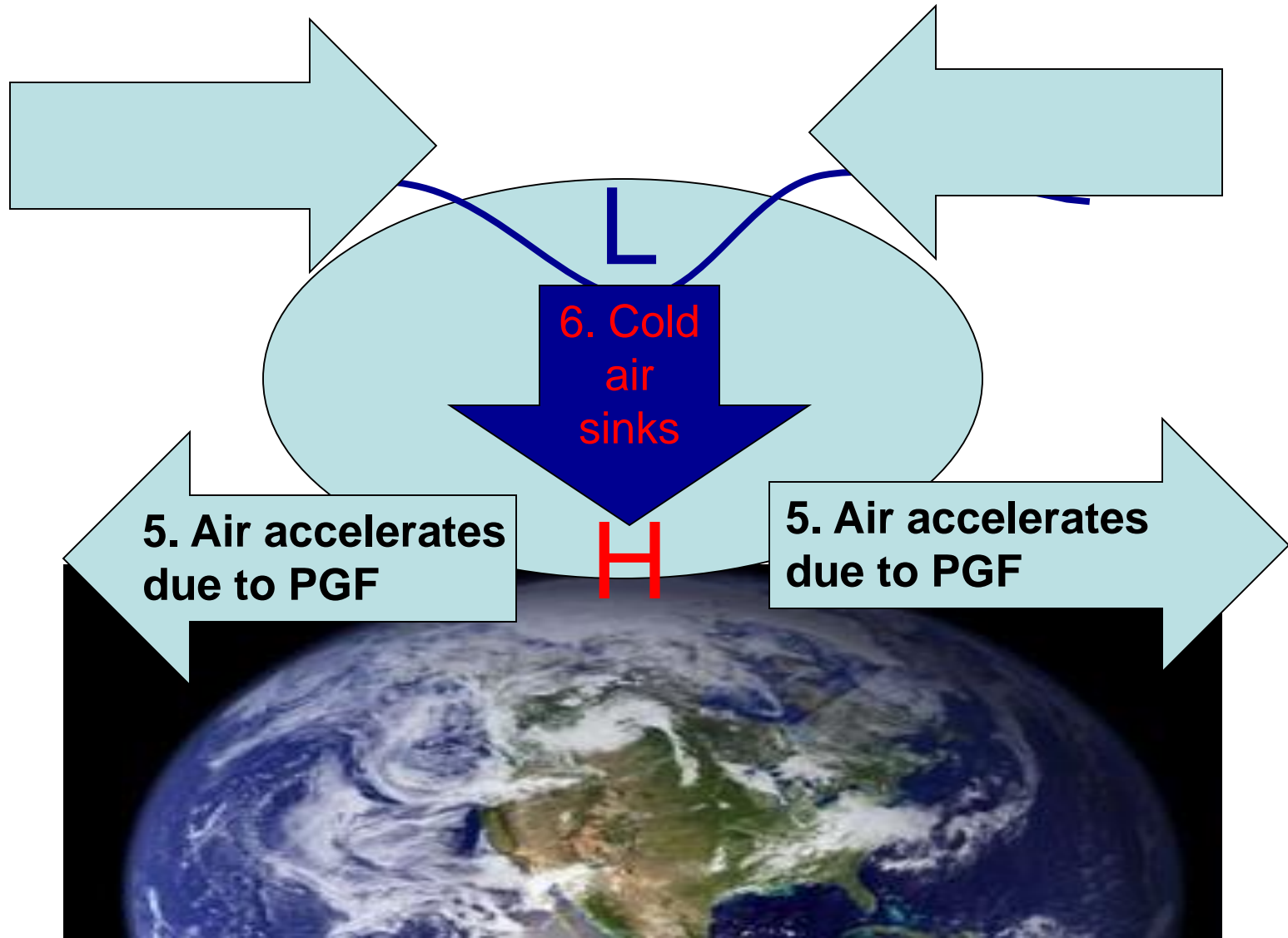




# How cooled air sinks and a cool core vortex develops: the Primitive Equation view. 7 logical steps

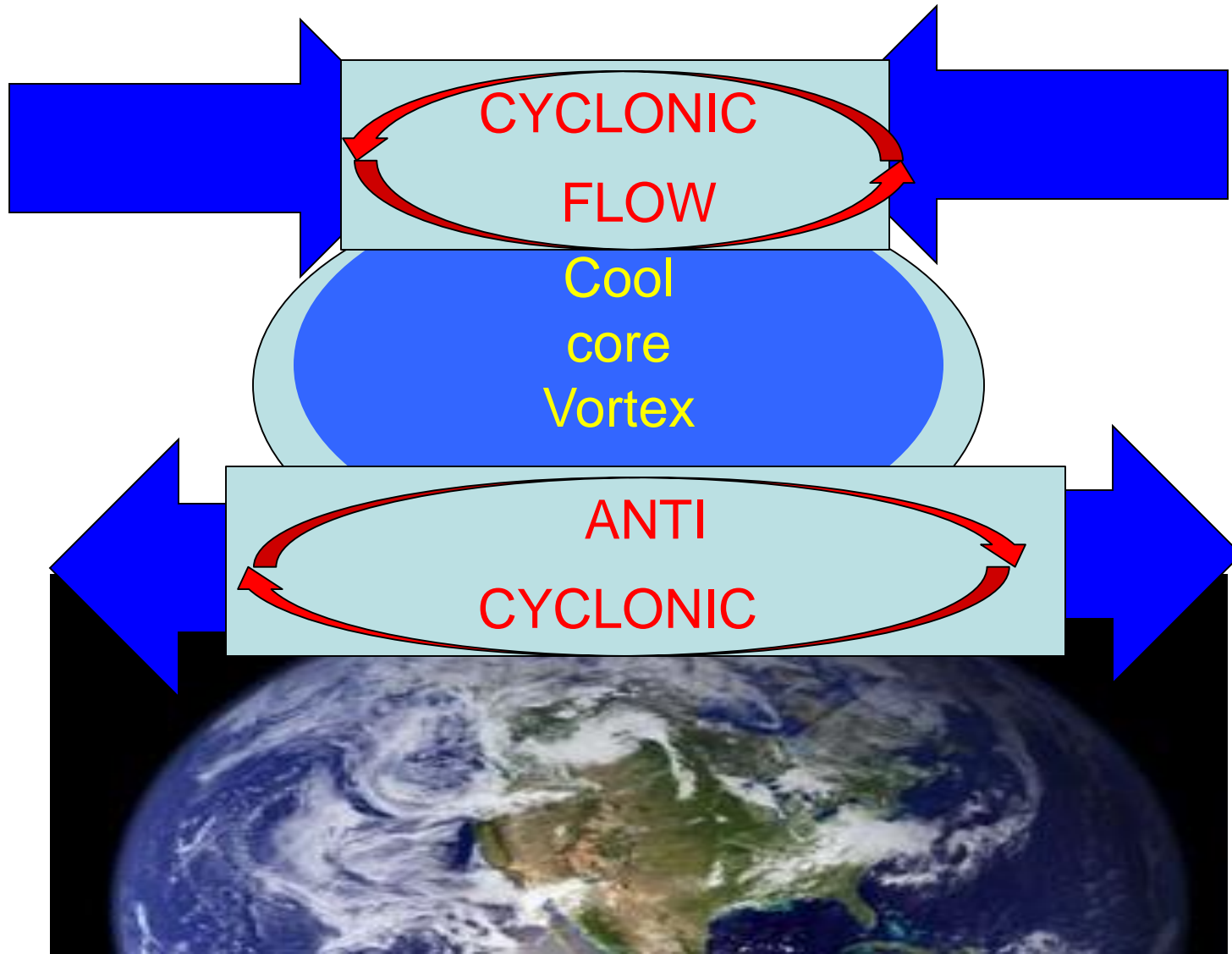


# How cooled air sinks and a cool core vortex develops: the Primitive Equation view. 7 logical steps

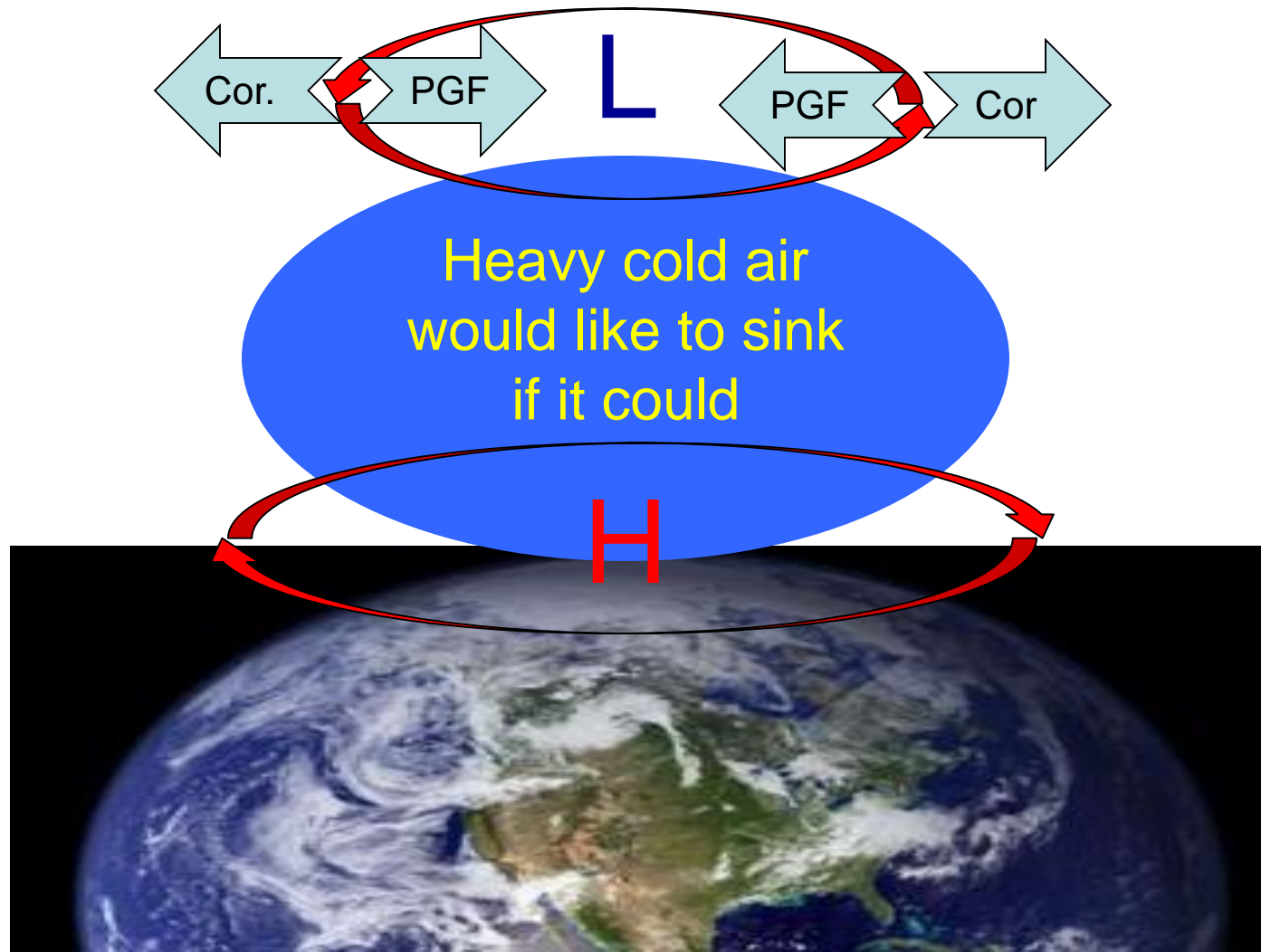


# How cooled air sinks and a cool core vortex develops:

## 7. Coriolis force turns the winds



**The geostrophically balanced polar vortex:  
The Coriolis force on the westerly jet stream  
prevents cold pool of Arctic air from sinking down  
and covering the whole Northern Hemisphere**

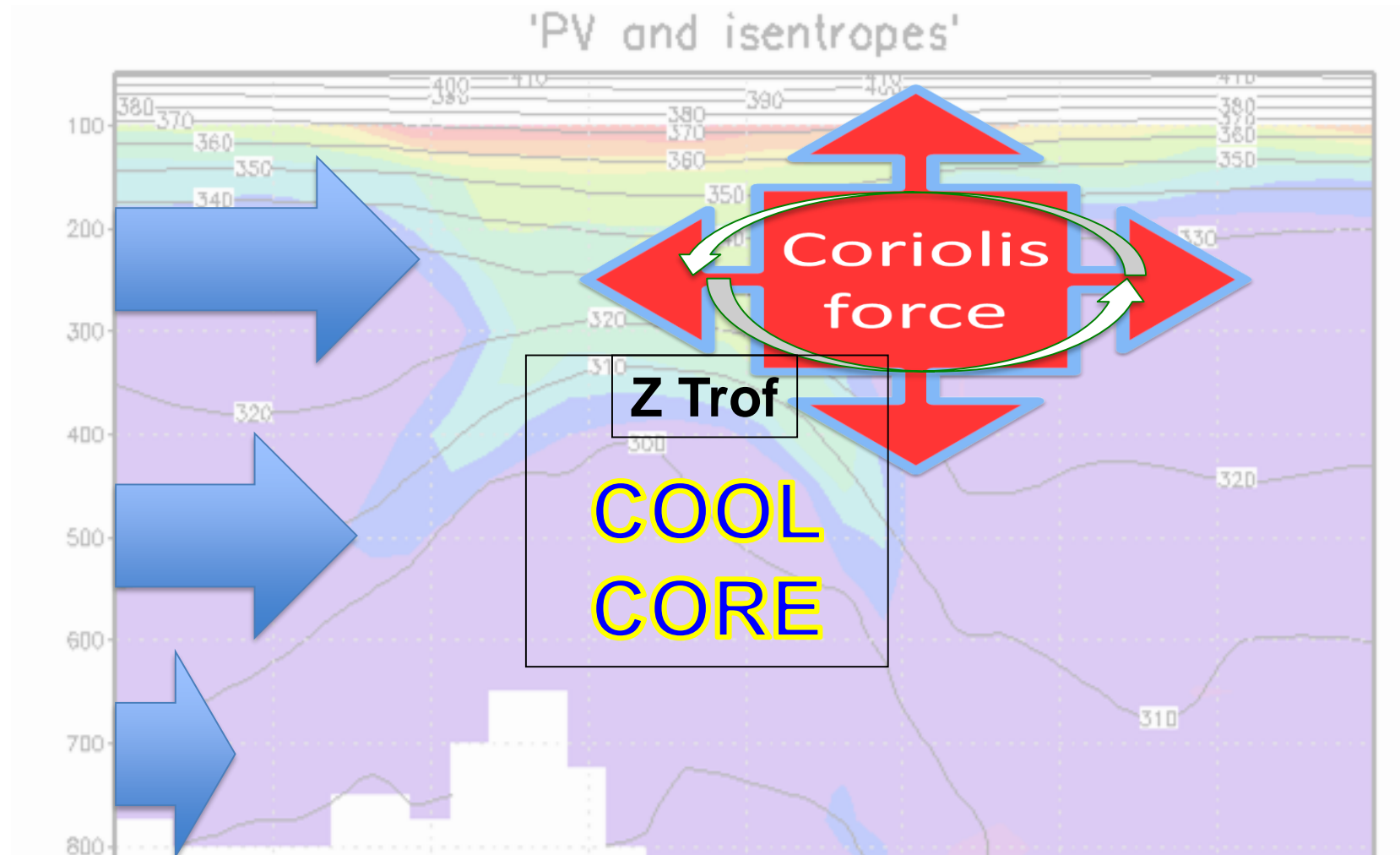


# Polar and stratospheric "Reservoirs" of $\zeta_a$ or PV

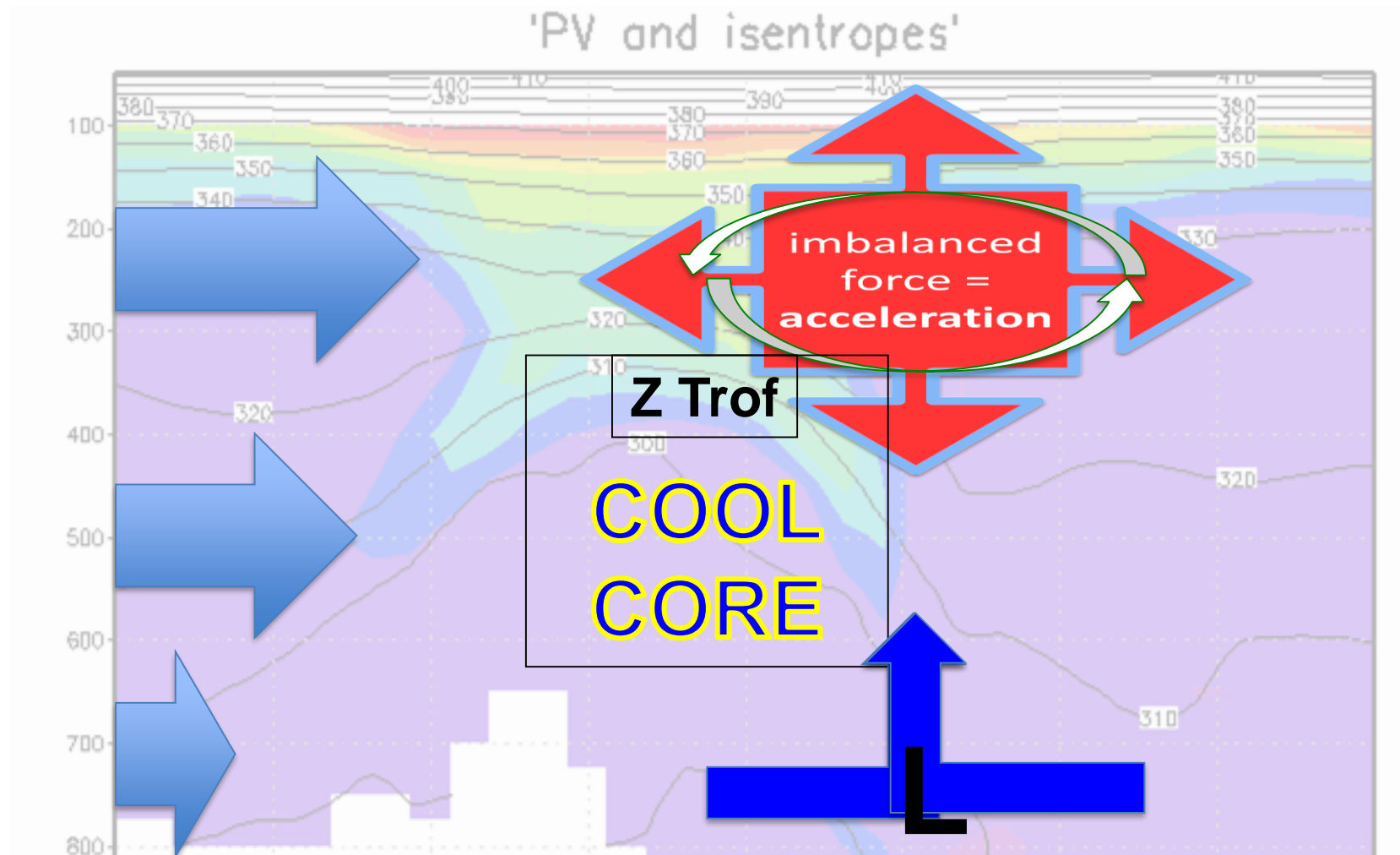
- Potential vorticity: **PV** =  $-g \zeta_a (\partial\theta/\partial p)$ 
  - The polar latitudes, where  $f$  is large, are a "reservoir" of high PV even when there is no wind!
  - The stratosphere where  $(\partial\theta/\partial p)$  is large is a "reservoir" of PV even when there is no wind!
  - When **tentacles** or **pieces** of the **polar & stratospheric PIZZA or OCTOPUS of PV** stretch or break off into the midlatitudes, they become our upper-tropospheric synoptic cyclones.



# Sheared advection breaks thermal wind balance



# Sheared advection breaks thermal wind balance



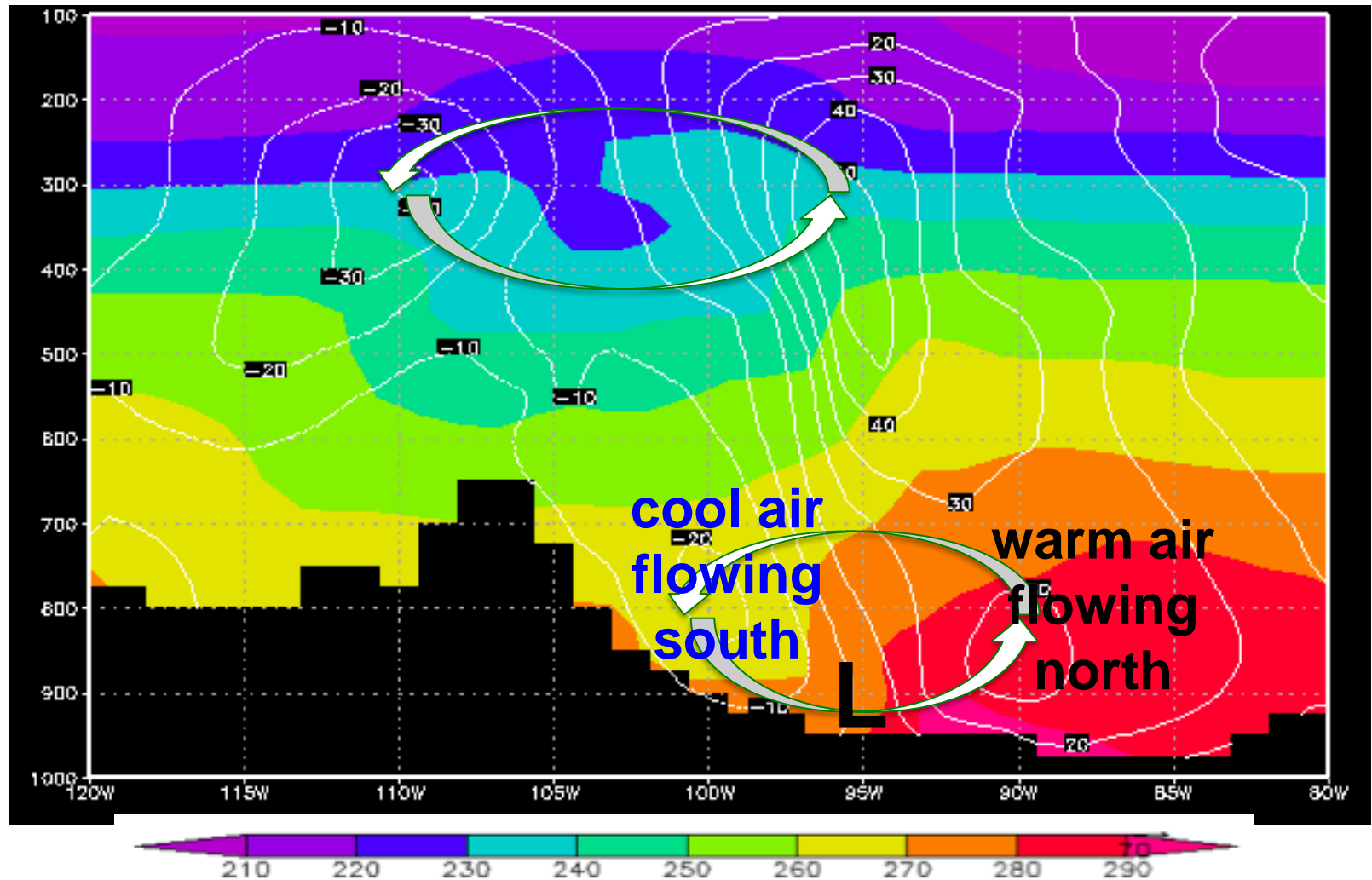


# Sheared advection

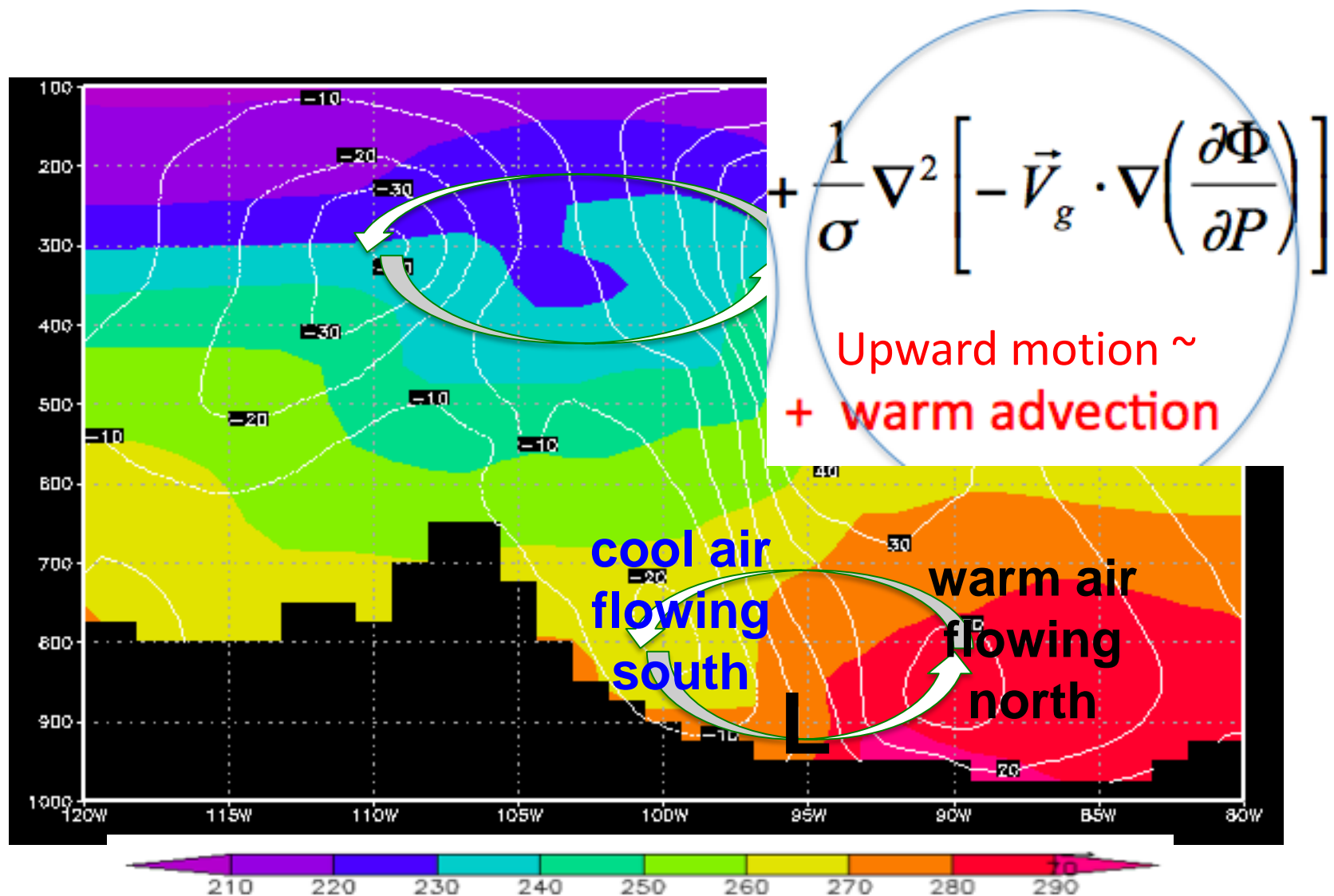
breaks thermal wind balance



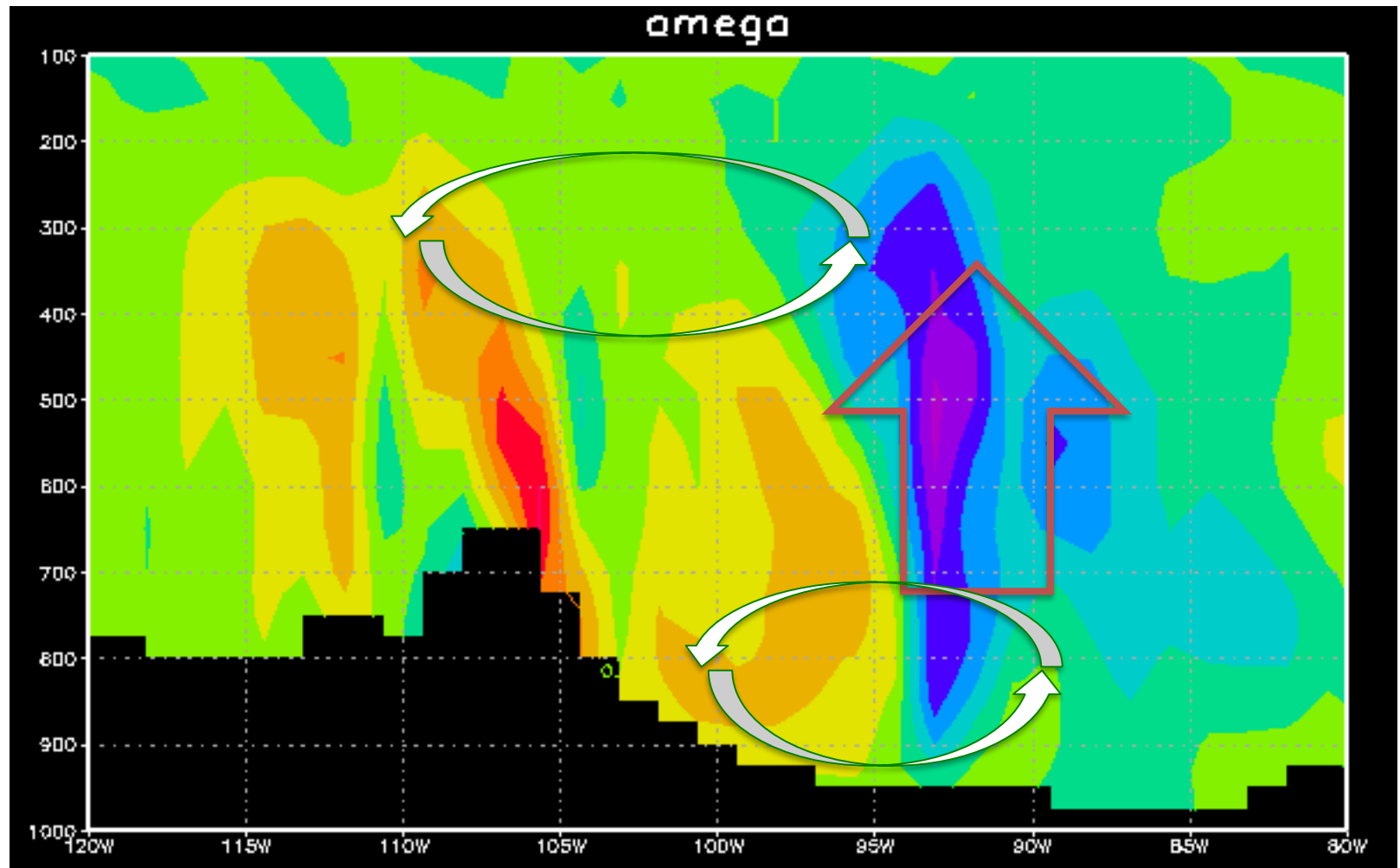
# But there is some T advection too



# But there is some T advection too



# East-west section: omega



# Unsheared advection of $T$ , $u$ , $v$ , vort, PV: no breaking of balance

'PV and isentropes'

