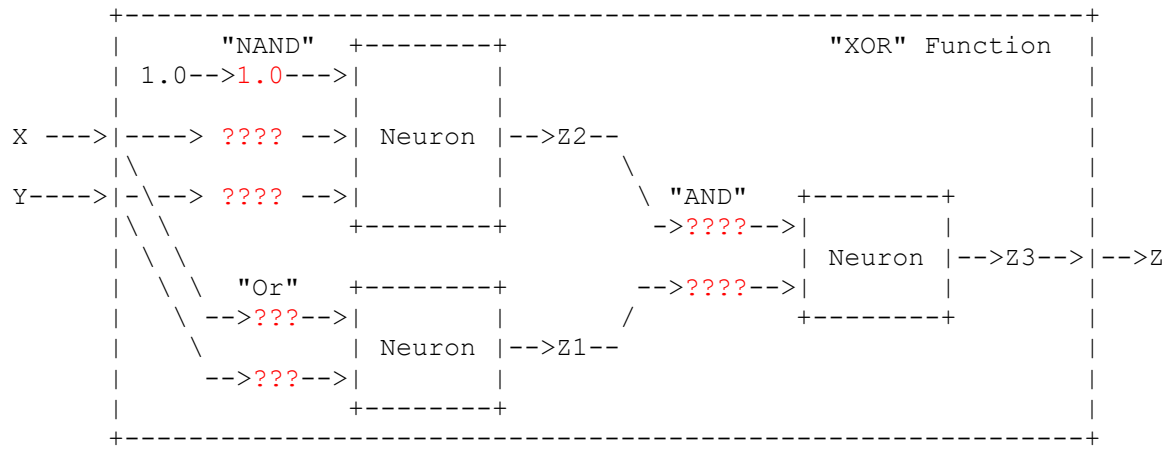
Week 10 Homework 1: Design XOR Gate

https://github.com/kathyshe/Machine-Learning/upload/main/Neural-Network/Design-XOR-Gate

Q21: Please refer to A Neural Network Primer to solve this question.

Step 1: Study the general idea on how to design XOR Gate
Step 2: Using the following rules to design your own AND Gate, OR Gate, and NAND Gate



The forward/backward process:

- Forward process
  Calculate the output Z for the given input (X,Y).
- Backward process
  Adjust weights
  - If the output Z is too low, increase the weights by 0.5, which had inputs that were "1".
  - If the output Z is too high, decrease the weights by 0.5, which had inputs that were "1".

Using step activation function

```
Z := ( W0 * C + W1 * X + W2 * Y >= T )
     where T := 1.0

if ( W0 * C + W1 * X + W2 * Y >= T )
then ouput is 1
else output = 0
The bias C for NAND is 1.0
```

Desired Function for "XOR":

OR				NAND				XOR		
X	Y	Z1		X	Y	Z2		X	Y	Z3
0	0	0		0	0	1		0	0	0
0	1	1	AND	0	1	1	=	0	1	1
1	0	1		1	0	1		1	0	1
1	1	1		1	1	0		1	1	0

Step 3: Please answer the formulas

"OR" Gate formula:

```
Z := ( W1 * X + W2 * Y >= T )
   where T := 1.0
```

Desired Function for "OR":

```
 X  Y | Z1
---------
 0  0 | 0
 0  1 | 1
 1  0 | 1
 1  1 | 1
```

Loop 1:

```
W1 = W2 = 1.0
--------------
X  Y  |  Z1
--------------
0  0  |  0    Z ok
0  1  |  1    Z ok
1  0  |  1    Z ok
1  1  |  1    Z ok
```

W1 = 1.0, W2 = 1.0 for "OR" Gate

"NAND" Gate formula:

```
Z := ( W0 * C + W1 * X + W2 * Y >= T )
        where T := 1.0
```

Desired Function for "NAND":

```
C  X  Y | Z2
------------
1  0  0 | 1
1  0  1 | 1
1  1  0 | 1
1  1  1 | 0
```

Loop 1:

```
W0 = 0.0
W1 = W2 = 1.0
--------------
C  X  Y  |  Z2
--------------
1  0  0  |  0    Z too low, increase W0 by 0.5, W0=0.5
1  0  1  |  1    Z ok
1  1  0  |  1    Z ok
1  1  1  |  1    Z too high, decrease all by 0.5, W0=0.0, W1=0.5, W2=0.5
```

Loop 2:
W0 = 0.0
W1 = W2 = 0.5

C	X	Y	Z2	
1	0	0	0	Z too low, increase W0 by 0.5, W0=0.5
1	0	1	0	Z too low, increase W0, W2 by 0.5, W0=1.0, W2=1.0
1	1	0	0	Z too low, increase W0, W1 by 0.5, W0=1.5, W1=1.0
1	1	1	1	Z too high, decrease all by 0.5, W0=1.0, W1=0.5, W2=0.5

Loop 3:
W0 = 1.0
W1 = W2 = 0.5

C	X	Y	Z2	
1	0	0	1	Z ok
1	0	1	1	Z ok
1	1	0	1	Z ok
1	1	1	1	Z too high, decrease all by 0.5, W0=0.5, W1=0.0, W2=0.0

Loop 4:
W0 = 0.5
W1 = W2 = 0.0

C	X	Y	Z2	
1	0	0	0	Z too low, increase W0 by 0.5, W0=1.0
1	0	1	0	Z too low, increase W0, W2 by 0.5, W0=1.5, W2=0.5
1	1	0	0	Z too low, increase W0, W1 by 0.5, W0=2.0, W1=0.5
1	1	1	0	Z ok

Loop 5:
W0 = 2.0
W1 = W2 = 0.5

C	X	Y	Z2	
1	0	0	1	Z ok
1	0	1	1	Z ok
1	1	0	1	Z ok
1	1	1	1	Z too high, decrease all by 0.5, W0=1.5, W1=0.0, W2=0.0

Loop 6:
W0 = 1.5
W1 = W2 = 0.0

C	X	Y	Z2

```
1  0  0  |  1    Z ok
1  0  1  |  1    Z ok
1  1  0  |  1    Z ok
1  1  1  |  1    Z too high, decrease all by 0.5, W1=1.0, W2=-0.5, W3=-0.5
```

Loop 7:

W0 = 1.0
W1 = W2 = -0.5

```
--------------
C  X  Y  |  Z2
--------------
1  0  0  |  1    Z ok
1  0  1  |  0    Z too low, increase W0, W2 by 0.5, W0=1.5, W2=0.0
1  1  0  |  0    Z too low, increase W0, W1 by 0.5, W0=2.0, W1=0.0
1  1  1  |  0    Z ok
```

Loop 8:

W0 = 2.0
W1 = W2 = 0.0

```
--------------
C  X  Y  |  Z2
--------------
1  0  0  |  1    Z ok
1  0  1  |  1    Z ok
1  1  0  |  1    Z ok
1  1  1  |  1    Z too high, decrease all by 0.5, W1=1.5, W2=-0.5, W3=-0.5
```

Loop 9:

W0 = 1.5
W1 = W2 = -0.5

```
--------------
C  X  Y  |  Z2
--------------
1  0  0  |  1    Z ok
1  0  1  |  1    Z ok
1  1  0  |  1    Z ok
1  1  1  |  0    Z ok!
```

W0 = 1.5, W1 = -0.5, W2 = -0.5 for "NAND" Gate

"AND" Gate formula:

```
Z := ( W1 * X + W2 * Y >= T )
   where T := 1.0
```

Desired Function for "AND":

X	Y	Z3
0	0	0
0	1	0
1	0	0
1	1	1

Loop 1:

W1 = W2 = 1.0

X	Y	Z3	
0	0	0	Z ok
0	1	1	Z too high, decrease W2 by 0.5, W2=0.5
1	0	1	Z too high, decrease W1 by 0.5, W1=0.5
1	1	1	Z ok

Loop 2:

W1 = W2 = 0.5

X	Y	Z3	
0	0	0	Z ok
0	1	1	Z ok
1	0	1	Z ok
1	1	1	Z ok!

W1 = 0.5, W2 = 0.5 for "AND" Gate

"XOR" Gate formula (plugging W0, W1, and W2s from previous gates):

```
Z1 := X  "Or"   Y
Z2 := X  "NAND" Y
Z := Z3 := Z1 "AND" Z2
Z := ( X "Or" Y ) "AND" ( X "NAND" Y )
Z := ( 1.0 * X + 1.0 * Y >= 1.0 ) "AND"
     ( 1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0 )
Z := ( 0.5 * ( 1.0 * X + 1.0 * Y >= 1.0 ) +
       0.5 * ( 1.5 + -0.5 * X + -0.5 * Y >= 1.0 ) >= 1.0 )
```

Desired Function for "XOR":

X	Y	Z3
0	0	0
0	1	1
1	0	1
1	1	0

Step 4: Please prove that your designed XOR gate works

When X=0, Y=0:

```
Z3 = (0.5 * ( 1.0 * 0 + 1.0 * 0 >= 1.0 ) +
      0.5 * ( 1.5 + -0.5 * 0 + -0.5 * 0 >= 1.0 ) >= 1.0 )
   = (0.5 * ( 0.0 >= 1.0 ) +
      0.5 * ( 1.5 >= 1.0 ) >= 1.0 )
   = (0.5 * ( false ) +
      0.5 * ( true ) >= 1.0 )
   = 0.5 * 0 + 0.5 * 1 >= 1.0
   = 0.5 >= 1.0
   = false
   = 0  ➔ same as desired
```

When X=0, Y=1:

```
Z3 = (0.5 * ( 1.0 * 0 + 1.0 * 1 >= 1.0 ) +
      0.5 * ( 1.5 + -0.5 * 0 + -0.5 * 1 >= 1.0 ) >= 1.0 )
   = (0.5 * ( 1.0 >= 1.0 ) +
      0.5 * ( 1.0 >= 1.0 ) >= 1.0 )
   = (0.5 * ( true ) +
      0.5 * ( true ) >= 1.0 )
   = 0.5 * 1 + 0.5 * 1 >= 1.0
   = 1.0 >= 1.0
   = true
   = 1  ➔ same as desired
```

When X=1, Y=0:

```
Z3 = (0.5 * ( 1.0 * 1 + 1.0 * 0 >= 1.0 ) +
      0.5 * ( 1.5 + -0.5 * 1 + -0.5 * 0 >= 1.0 ) >= 1.0 )
   = (0.5 * ( 1.0 >= 1.0 ) +
      0.5 * ( 1.0 >= 1.0 ) >= 1.0 )
   = (0.5 * ( true ) +
      0.5 * ( true ) >= 1.0 )
   = 0.5 * 1 + 0.5 * 1 >= 1.0
   = 1.0 >= 1.0
   = true
   = 1  ➔ same as desired
```

When X=1, Y=1:

```
Z3 = (0.5 * ( 1.0 * 1 + 1.0 * 1 >= 1.0 ) +
      0.5 * ( 1.5 + -0.5 * 1 + -0.5 * 1 >= 1.0 ) >= 1.0 )
   = (0.5 * ( 2.0 >= 1.0 ) +
      0.5 * ( 0.5 >= 1.0 ) >= 1.0 )
   = (0.5 * ( true ) +
      0.5 * ( false ) >= 1.0 )
   = 0.5 * 1 + 0.5 * 0 >= 1.0
   = 0.5 >= 1.0
   = false
   = 0 ➔ same as desired
```

All Z3 same as desired in "XOR" Gate!