TWO LAYER ENERGY BALANCE MODEL WITH CARBON CYCLING

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Abstract. The two-layer energy balance model (EMB) is useful tool as warming is not instantaneous. Instead, the oceans act as a heat reservoir, delaying the effects of carbon emissions. We had added a simple carbon cycle to better calculate the radiative forcings caused by current emissions, and used it to test the effects of different emissions scenarios. In each, we found a linear relationship between cumulative carbon emissions and global temperature.

1. Introduction

As we produce CO_2 and other greenhouse gases, we allow less radiation to escape our atmosphere to space, and thus trap more heat within. This causes an increased radiative forcing within the earth system, and thus an increase in global temperature. We may consider an Energy Balance Model (EBM) that conserves incoming and outgoing energy from the radiation exchange between the earth and the sun: as the radiative forcing increases, the earth absorbs more energy than it radiates back out, so the surface temperatures increase.

Our oceans, however, act as heat reservoirs that absorb a significant amount of energy and delay the timescale of warming. We may consider the ocean as having two layers, surface and deep, and examine the relationship between the heat exchange between them.

We may also consider the effect of different emissions scenarios. One of the most impactful radiative forcings is that of CO_2, which can be calculated based on the current atmospheric carbon concentration. Once carbon is released into the atmosphere, it is absorbed back into the earth system on a few separate timescales, and thus continues to have an effect years after the initial emission [2]. Using a model for carbon cycling, we may calculate our associated radiative forcing and use our EBM to find surface temperatures.

Given our emissions scenario, we may find the cumulative carbon emissions over the years. Then, using our EBM, we may calculate global temperatures, and compare the relationship between them. Previous scholarship notes the linear relationship between cumulative carbon emissions and temperature under nearly any scenario, but has not found a good explanation as to why [3].

2. Methods

2.1. **2 Layer Energy Balance Model.** We may derive our change in temperature of the surface ocean: the increase from the radiative forcing R_f (Wm^{-2}); the effect of the feedback parameter λ ($Wm^{-2}K^{-1}$) on current temperature; and the decrease from the heat leaving towards the deep ocean, which is a function of the current temperatures of both layers based on the mixing efficiency between them $\gamma(T - T_O)$, where γ is in units of $Wm^{-2}K^{-1}$. This sum is then divided by the effective heat capacity of the surface, C. We may define $C = \rho c_w h$, where ρ is the density of water ($1025\ kgm^{-3}$), c_w is the specific heat ($2850\ Jkg^{-1}K^{-1}$), and h is the depth of our layer (either 70

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m or 1100 m). This yields units of $JK^{-1}m^{-2}$ For simplicity, we write this as a multiplier on $\frac{dT}{dt}$, and thus both sides are in Wm^{-2}. Thus this yields

$$C\frac{dT}{dt} = R_f + \lambda T - \gamma(T - T_O) \tag{1}$$

Since the deep ocean is only connected to the system via the heat coming from the surface layer, the only term is a positive forcing from $\gamma(T - T_o)$. This equation is similarly scaled by a C term:

$$C_O\frac{dT}{dt} = \gamma(T - T_O) \tag{2}$$

2.2. **Carbon Cycling.** Next, carbon is cycled through the atmosphere on three different timescales, with a certain percentage of the initial carbon pulse disappearing at each rate. Thus when we have a pulse of emissions, the carbon concentration changes according to

$$C(t) = C_0 + E(t)(a_0 + \sum_i a_i e^{\frac{-t}{\tau_i}}) \tag{3}$$

where E is the carbon emission, $a_0 = .217$, $a_1 = .259$, $a_2 = .338$, $a_3 = .186$, $\tau_1 = 172.9$, $\tau_2 = 18.57$ and $\tau_3 = 1.186$ [2]. Note that $\sum_i a_i \neq 1$, which is because the remaining percentage of carbon will persist in the atmosphere on a much longer timescale than we are concerned with. From the current carbon concentration, we may calculate the associated radiative forcing via

$$R_f = 5.35\ln(\frac{C}{C_0}) \tag{4}$$

We may then use this value of R_f in 1 to calculate the temperature change as a function of carbon emissions.

2.3. **Carbon Emissions Scenarios.** Information on historical global emissions was found online by courtesy of Our World in Data [1]. This dataset contains annual emissions in tons.

Alternative carbon emissions scenarios were calculated by hand to fit the bounds described by the historical dataset in order to aid comparison further on.

3. Results

3.1. **Carbon Emissions Scenarios.** In order to rigorously test the linear relationship between cumulative carbon emissions and temperature, we created a variety of scenarios that were reasonable considering the bounds of the historical emissions record, plotted in Figure 1 below:

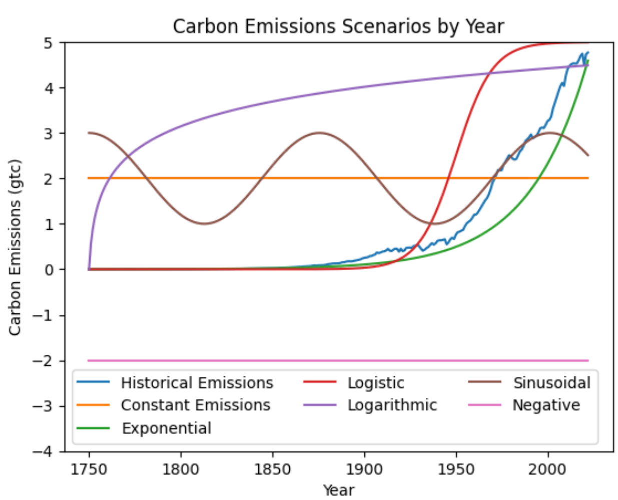
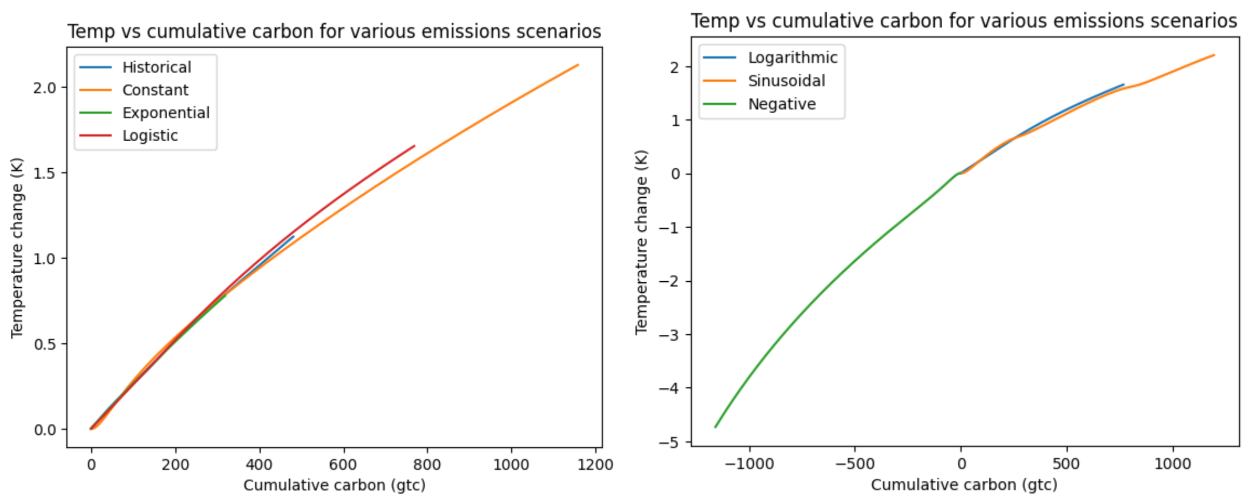


Figure 1. Potential Emissions Scenarios

Each scenario occurred over the same timeline as the historical record, solely for ease of comparison. For each emissions scenario, we discretized the function such that we had an annual metric, in gigatons of carbon (gtc), to describe emissions. We used this discrete vector in 3 to yield an annual $C(t)$ that described the current carbon concentration in parts per million (ppm). Then, using 4 and our initial carbon concentration (always taken to be 280ppm, the pre-industrial revolution estimates) we calculated the yearly R_f, which we were then able to use in our two-layer EBM.

Since our two-layer EBM is already set up to take in a yearly measurement for R_f, we could get predictions for both surface and deep ocean temperatures for each emissions scenario. We chose to compare surface temperatures to cumulative carbon instead of deep ocean temperatures as the surface temperatures more closely match atmospheric temperatures. The cumulative carbon emission was found for each year by summing up all years of carbon emissions prior.

3.2. **Annual temperature vs cumulative carbon emissions.** Using the models described above, we inputted the particular emissions scenario, calculated the associated radiative forcing across all years, and used that to inform our two-layer EBM. This yielded many different warming patterns, depending on the emissions pattern. For each scenario, we calculated the cumulative emissions, and matched that up with the associated temperature for that year, then graphed them.



(A) Realistic Emissions Scenarios (B) Unrealistic Emissions Scenarios

FIGURE 2. Temperature vs cumulative carbon emissions under different emissions scenarios

For a majority of our scenarios, we got a good linear relationship between global temperature (using the surface temperature calculated in 1) and cumulative carbon emissions. In most cases, the lines are so close together it's hard to see where one situation leaves off and another begins. Nearly all of our responses are linear, with similar slope: we get about one degree of warming for every 500 cumulative gigatons of carbon emitted into the atmosphere. The only exception is in Figure 2b, where the negative emissions scenario does yield some nonlinearity as we get higher amounts of carbon taken out from the atmosphere.

4. Discussion and conclusions

The more realistic carbon scenarios in Figure 2a had a more linear relationship. These scenarios are considered realistic as they either match up well with historical data or describe a common climate change SSP (logistic, which could describe a leveling off of emissions). The most unrealistic, negative emissions, would eventually yield a negative carbon concentration, which could have caused the nonlinearities. As of the model's current state, there is no mechanism with which to cap the amount of carbon in the atmosphere nor to prevent that amount from becoming negative, both of which are unrealistic given the physical world. Thus we can say that for all realistic carbon emissions scenarios, we get a linear fit with similar slope.

To see how this works in our particular model, we may consider one pulse of CO_2. This pulse causes a particular temperature response that is completely independent of the emissions scenario we are in. Thus we may consider an arbitrary emissions scenario to be a weighted sum of pulses, and thus the temperature response to be a sum with the same weights. Thus we may generalize to say that for any (realistic) emissions scenario, we will always have a linear relationship between cumulative carbon and temperature [3].

Many other, more complicated earth system models also support this relationship, though the cause of it is hotly debated due to all the nonlinear processes that we had to go through before getting from carbon emission to temperature. Some scientists have suggested that this linear relationship due to the fact that the same physical processes drive both heat and carbon removal by the ocean, however other analysis does not support this explanation. Instead, this relationship appears to be a 'chance result of the many interacting physical and biogeochemical processes in the earth system, and not something which is amenable to simple physical explanation' [3].

Due to the simplicity of our model, it is far more likely that our linear relationship is a result of assuming the ocean heat uptake to be constant, as well as neglecting the more complex processes that govern carbon uptake by the land and other sources. However, the fact that this relationship is present in our very simple model, and in so many more complication ones, suggests that the linear slope we have derived is a good approximation of real life.

References

[1] Annual total (fossil fuels and land-use change) emissions of carbon dioxide. `https://ourworldindata.org/co2-emissions`, 2024.

[2] F. J. et al. Carbon dioxide and climate impulse response functions for the computation of greenhouse gas metrics: a multi-model analysis. *Atmospheric Chemistry and Physics*, 13(5):2793–2825, 2013.

[3] N. P. Gillett. Warming proportional to cumulative carbon emissions not explained by heat and carbon sharing mixing processes. *Nature Communication*, 14, 2023.