Night 3

1)

Function	Derivitive	Antiderivitive
Kx^n	Knx^{n-1}	$\frac{K}{n+1}x^{n+1}+C$
$\sin x$	$\cos x$	$-\cos x+C$
$\cos x$	$-\sin x$	$\sin x+C$
b^x	$(\ln b)\times(b^x)$	$\frac{b^x}{\ln b}+C$
$\ln(x)$	$\frac{1}{x}$	$x\ln x-x+C$
e^x also $\mathrm{Exp}[x]$	e^x	e^x+C

2) $4x^2+3x^2-5x+4=f(x)$

$8x+6x-5=f'(x)$

$=14x-5=f'(x)$

3) $f(x)=(x^3-1)^{100}$ $\quad f(x)=x^{100}\quad f'(x)=100x^{99}$

$g(x)=x^3-1\quad g'(x)=3x^2$

$f'(x)=100(x^3-1)^{99}\times 3x^2$

$f'(x)=300x^2(x^3-1)^{99}$

4) $\int_0^4\sqrt{2x+1}\,dx$ $\quad u=2x+1$

$du=2dx$

$\int_1^9\sqrt{u}\;\frac{du}{2}$

$\frac{2}{3}u^{3/2}\times\frac{1}{2}\,du$

$\frac{1}{3}(2x+1)^{3/2}\Big|_0^4$

$\frac{1}{3}(9)^{3/2}-\frac{1}{3}(1)^{3/2}$

$9-\frac{1}{3}=8\frac{2}{3}$

5) $\sqrt{x}(1-x)$ $\quad f(x)=\sqrt{x}\quad g(x)=(1-x)$

$f'(x)=\frac{x^{-1/2}}{2}\quad g'(x)=-1$

$\frac{d}{dx}=\left(\frac{x^{-1/2}}{2}\right)(1-x)+(-1)(\sqrt{x})$

$\frac{x^{-1/2}(1-x)}{2}-\sqrt{x}=\frac{(1-x)}{2\sqrt{x}}-\sqrt{x}$

6) $\int_1^2 xe^{-x}\,dx$

7) a) $f'(t)=Ake^{kt}$

$\int f(t)=\frac{Ae^{kt}}{k}+c$

b) $f'(x)=m$

$\int f(x)=\frac{m}{2}x^2+bx+c$

c) $f'(x)=anx^{n-1}$

$\int f(x)=\frac{a}{n+1}x^{n+1}+bx+c$

c) $f'(x)=2g(x-h)$

$\int f(x)=$

Night 3 continued

8) $\int_0^1 x^n\,dx = \frac{1}{n+1}x^{n+1} + c\Big|_0^1$

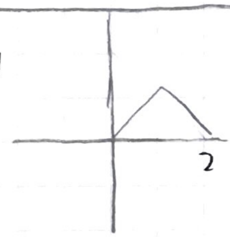
$\int_0^1 \sin x\,dx = -\cos x + c\Big|_0^1$

$\int_0^1 \cos x\,dx = \sin x + c\Big|_0^1$

$\int_0^1 e^x\,dx = x\ln x - x + c\Big|_0^1$

$\int_0^1 \ln x\,dx = e^x + c\Big|_0^1$

11) a) $\int_0^1\int_y^{2-y} 1\,dx\,dy$



$\int_0^1 x\Big|_y^{2-y}\,dy$

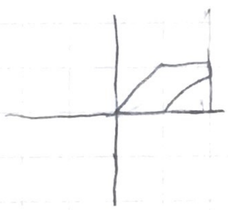
$\int_0^1 (2-y)-y\,dy$

$\int_0^1 -2y+2\,dy$

$-y^2+2y\Big|_0^1$

$-1^2+2(1)=1$

b) $\int_0^1\int_y^{\exp y} 1\,dx\,dy$



$\int_0^1 x\Big|_y^{e^y}\,dy$

$\int_0^1 e^y - y\,dy$

$e^y - \frac{1}{2}y^2\Big|_0^1$

$2.72 - \frac{1}{2} \quad -1 = 1.22$

13) a) $\int_1^2\int_0^{2x}\frac{4y}{x^3+2}\,dy\,dx$

$\frac{4}{x^3+2}\int_0^{2x} y\,dy$

$\frac{4}{x^3+2}\left(\frac{1}{2}y^2\Big|_0^{2x}\right)$

$\frac{4}{x^3+2}\times X^2$

$\int_1^2 \frac{2x^2+4}{x^3+2}\,dx$ $\qquad u = x^3+2$

$\int_3^8 \frac{\cancel{x^2}+4}{u}\times\frac{du}{3\cancel{x^2}}$ $\qquad du = 3x^2\,dx$

$\frac{4}{3}\int_3^8 \frac{1}{u}\,du$

$\frac{4}{3}\left(\ln u\Big|_3^8\right)$

$\frac{4}{3}(\ln 8 - \ln 3) = 1.308$

b) $\int_0^1\int_0^{x^2} x\cos y\,dy\,dx$

$x\int_0^{x^2}\cos y\,dy$

$x\left(\sin y\Big|_0^{x^2}\right)$

$x\sin x^2$

$\int_0^1 x\sin x^2\,dy$

$u = x^2$

$du = x\,dx$

$\int_0^1 x\sin u\,\frac{du}{\cancel{x}}$

$\int_0^1 \sin u\,du$

$-\cos u\Big|_0^1$

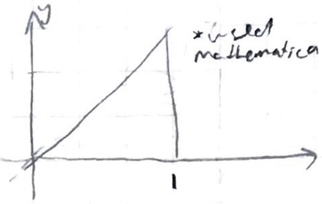
$-\cos 1 + \cos 0$

0.4597

9) $y = x^2$

$\int_0^{\sqrt{d}} (d) - (x^2)\,dx$

10) a)



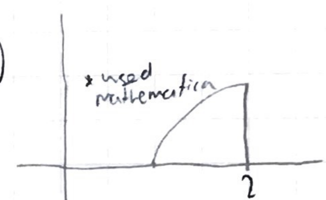
$\int_0^1\int_0^x 1\,dy\,dx$

$\int_0^1 y\Big|_0^x\,dx$

$\int_0^1 0 - x\,dx$

$\frac{1}{2}x^2\Big|_0^1 = 1/2$

b)



$\int_1^2\int_0^{\ln x} 1\,dy\,dx$

$\int_1^2 y\Big|_0^{\ln x}\,dx$

$\int_1^2 \ln x\,dx$

$(2\ln 2 - 2) - (1\ln 1 - 1)$

$= 0.386$

12) a) $\int_0^1\int_y^1 1\,dx\,dy$

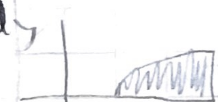
$\int_0^1 x\Big|_y^1\,dy$

$\int_0^1 1 - y\,dy$

$y - \frac{1}{2}y^2\Big|_0^1$

$1 - \frac{1}{2} = 1/2$

b) $\int_0^{\ln 2}\int_{e^y}^2 1\,dx\,dy$



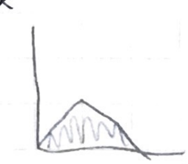
$x\Big|_{e^y}^2$

$\int_0^{\ln 2} 2 - e^y\,dy$

$2y - e^y\Big|_0^{\ln 2}$

$2\ln 2 - 2 - 1 = 2\ln 2 - 3$

a) $\int_0^1\left(\int_0^1 x\,dy\right)dx + \int_1^2\left(\int_1^2 -x\,dy\right)dx$



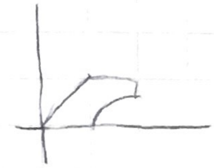
$\int_0^1 xy\Big|_0^1\,dx + \int_1^2 -xy\Big|_1^2\,dx$

$\int_0^1 x\,dx + \int_1^2 -x\,dx$

$\frac{1}{2}x^2\Big|_0^1 + -\frac{1}{2}x^2\Big|_1^2$

$\frac{1}{2} \qquad -2 + \frac{1}{2} = -1$

b) $\int_0^1\int_0^1 x\,dy\,dx + \int_1^2\int_{\ln x}^2 1\,dy\,dx$



$\int_0^1 xy\Big|_0^1\,dx + \int_1^2 y\Big|_{\ln x}^2\,dx$

$\int_0^1 x\,dx + \int_1^2 2 - \ln x\,dx$

$\frac{1}{2}x^2\Big|_0^1 + 2 - x\ln x - x\Big|_1^2$

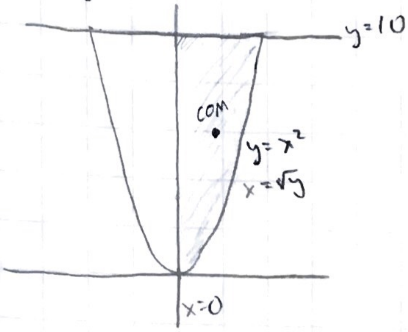
$\frac{1}{2} + (2 - 2\ln 2 - 2) - (2 - \ln 1 - 1)$

$\frac{1}{2} + -1.386 - 1 = -1.886$

Night 3 + t

13) $y = x^2$, $y=10$, $x=0$

density of Alluminum $= 2.7 g/cm^3$



$$\int_0^{10}\int_0^{\sqrt{y}} 2.7\, dx\, dy = M$$

$$2.7x\Big|_0^{\sqrt{y}}$$

$$\int_0^{10} 2.7\sqrt{y}\, dy$$

$$\int_0^{10} 2.7y^{1/2}\, dy$$

$$2.7 \times \frac{2}{3} y^{3/2}\Big|_0^{10}$$

$M = 56.9g$

$$X_{com} = \frac{1}{56.9}\int_0^{10}\int_0^{\sqrt{y}} x \times 2.7\, dx\, dy$$

$X_{com} = 1.186$ cm

$$y_{com} = \frac{1}{56.9}\int_0^{10}\int_0^{y} y \times 2.7\, dx\, dy$$

$y_{com} = 6.002$ cm

Single Variable Calculus part of assignment

Exercise 1

In[13]:= `D[x^n, x]`

Out[13]= $n\, x^{-1+n}$

In[11]:= `Integrate[x^n, x]`

Out[11]= $\frac{x^{1+n}}{1+n}$

In[18]:= `D[Sin[x], x]`

Out[18]= Cos[x]

In[19]:= `Integrate[Sin[x], x]`

Out[19]= -Cos[x]

In[20]:= `D[Cos[x], x]`

Out[20]= -Sin[x]

In[21]:= `Integrate[Sin[x], x]`

Out[21]= -Cos[x]

In[22]:= `D[Exp[x], x]`

Out[22]= e^x

In[23]:= `Integrate[Exp[x], x]`

Out[23]= e^x

In[24]:= `D[Log[x], x]`

Out[24]= $\frac{1}{x}$

In[25]:= `Integrate[Log[x], x]`

Out[25]= -x + x Log[x]

In[26]:= `D[b^x, x]`

Out[26]= b^x Log[b]

In[27]:=
```
Integrate[b^x, x]
```
Out[27]= $\frac{b^x}{\text{Log}[b]}$

Exercise 2

In[28]:=
```
D[(4 x^2) + (3 x^2) - 5 x + 4, x]
```
Out[28]= $-5 + 14 x$

Exercise 3

In[29]:=
```
D[((x^3) - 1)^100, x]
```
Out[29]= $300 x^2 \left(-1 + x^3\right)^{99}$

Exercise 4

In[31]:=
```
Integrate[Sqrt[2 x + 1], {x, 0, 4}]
```
Out[31]= $\frac{26}{3}$

Exercise 5

In[32]:=
```
D[Sqrt[x] * (1 - x), x]
```
Out[32]= $\frac{1 - x}{2 \sqrt{x}} - \sqrt{x}$

Exercise 6

In[33]:=
```
Integrate[x * Exp[-x], {x, 1, 2}]
```
Out[33]= $\frac{-3 + 2 e}{e^2}$

Exercise 7

In[35]:=
```
D[A * Exp[k * t], t]
```
Out[35]= $A e^{k t} k$

In[36]:=
```
Integrate[A * Exp[k * t], t]
```
Out[36]= $\frac{A e^{k t}}{k}$

In[37]:=
```
D[m * x + b, x]
```
Out[37]= m

In[38]:= `Integrate[m * x + b, x]`

Out[38]= $b\,x + \frac{m\,x^2}{2}$

In[39]:= `D[a * (x^n) + b, x]`

Out[39]= $a\,n\,x^{-1+n}$

In[40]:= `Integrate[a * (x^n) + b, x]`

Out[40]= $b\,x + \frac{a\,x^{1+n}}{1+n}$

In[45]:=
```
Clear[A, ω, φ]
D[A * Sin[ω * t + φ], t]
```

Out[46]= $A\,\omega\,\mathrm{Cos}[\varphi + t\,\omega]$

In[47]:= `Integrate[A * Sin[ω * t + φ], t]`

Out[47]= $A\left(-\frac{\mathrm{Cos}[\varphi]\,\mathrm{Cos}[t\,\omega]}{\omega} + \frac{\mathrm{Sin}[\varphi]\,\mathrm{Sin}[t\,\omega]}{\omega}\right)$

In[48]:= `D[g * (x - h)^2 + k, x]`

Out[48]= $2\,g\,(-h + x)$

In[49]:= `Integrate[g * (x - h)^2 + k, x]`

Out[49]= $g\,h^2\,x + k\,x - g\,h\,x^2 + \frac{g\,x^3}{3}$

Exercise 8

In[50]:= `Integrate[x^n, {x, 0, 1}]`

Out[50]= $\mathrm{ConditionalExpression}\left[\frac{1}{1+n}, \mathrm{Re}[n] > -1\right]$

In[51]:= `Integrate[Sin[x], {x, 0, 1}]`

Out[51]= $1 - \mathrm{Cos}[1]$

In[52]:= `Integrate[Cos[x], {x, 0, 1}]`

Out[52]= $\mathrm{Sin}[1]$

In[53]:= `Integrate[Exp[x], {x, 0, 1}]`

Out[53]= $-1 + e$

In[54]:= `Integrate[Log[x], {x, 0, 1}]`

Out[54]= -1

Exercise 9

In[55]:=
```
Integrate[d - (x^2), x]
```

Out[55]= $d\,x - \frac{x^3}{3}$

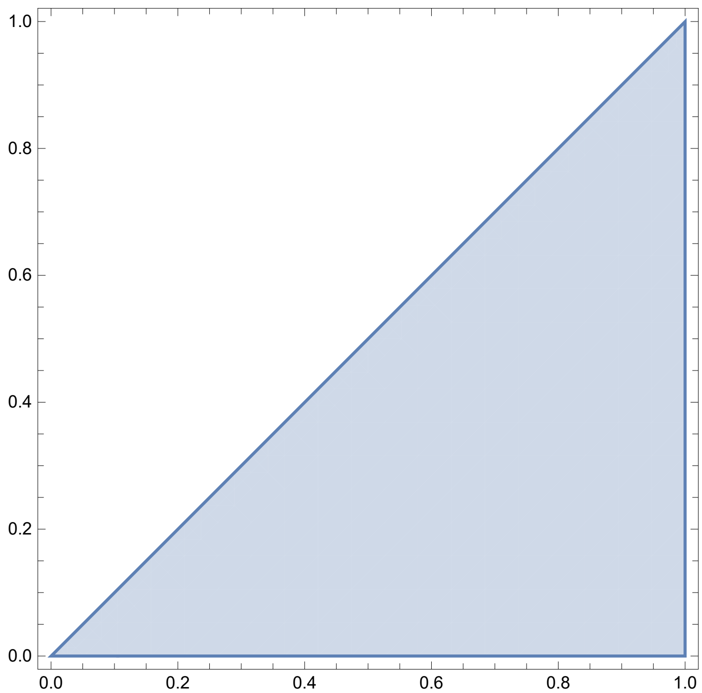
Exercise 10

In[57]:=
```
Integrate[Integrate[1, {y, 0, x} ], {x, 0, 1}]
```

Out[57]= $\frac{1}{2}$

In[67]:=
```
RegionPlot[ 0 < x < 1 && 0 < y < x, {x, 0, 1}, {y, 0, 1}]
```

Out[67]=

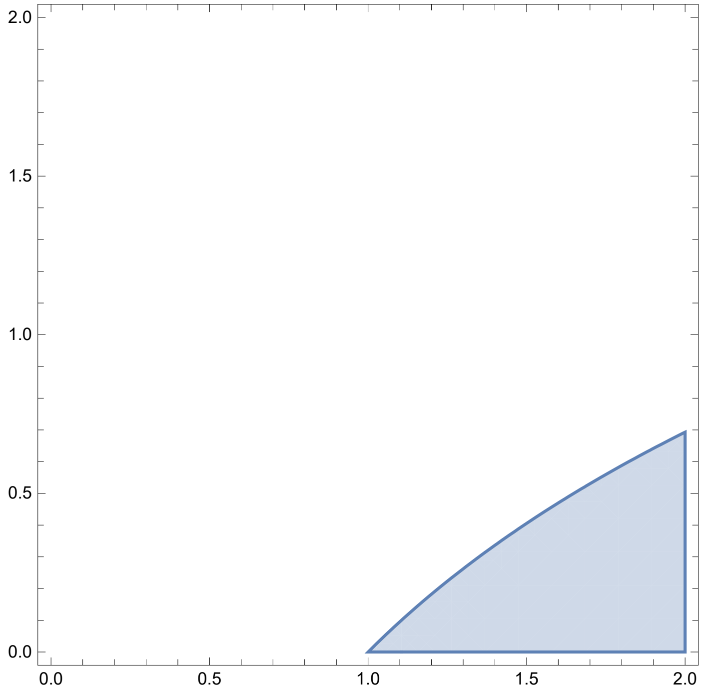


In[63]:=
```
Integrate[Integrate[1, {y, 0, Log[x]} ], {x, 1, 2}]
```

Out[63]= $-1 + \mathrm{Log}[4]$

In[68]:=
```
RegionPlot[ 1 < x < 2 && 0 < y < Log[x], {x, 0, 2}, {y, 0, 2}]
```

Out[68]=



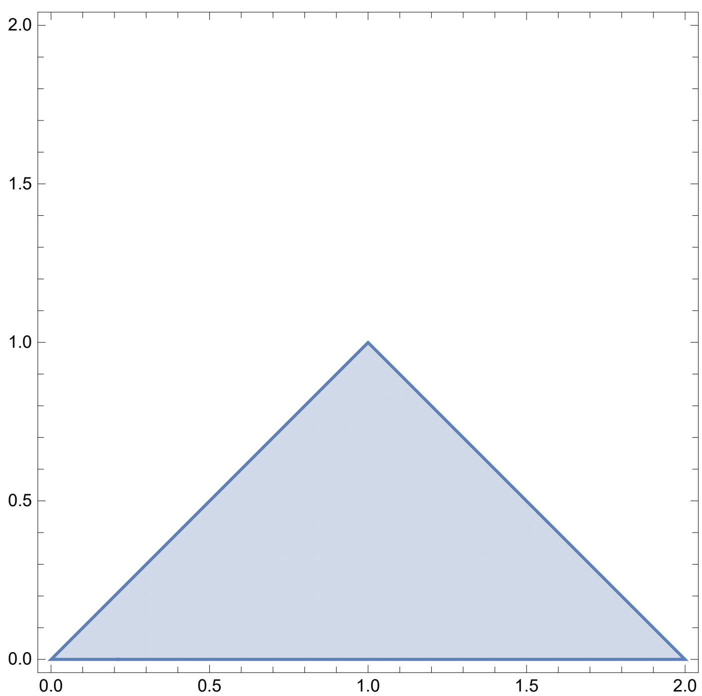
Exercise 11

In[59]:=
```
Integrate[Integrate[1, {x, y, 2 - y} ], {y, 0, 1}]
```

Out[59]= 1

In[77]:=
```
RegionPlot[ 0 < y < 1 && y < x < (2 - y), {x, 0, 2}, {y, 0, 2}]
```

Out[77]=

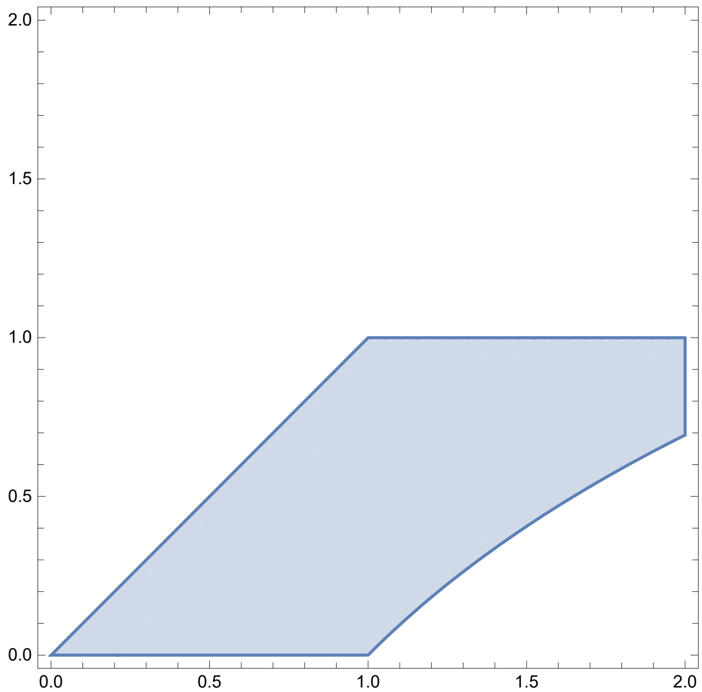


In[72]:=
```
Integrate[Integrate[1, {x, y, Exp[y]} ], {y, 0, 1}]
```

Out[72]= $-\frac{3}{2} + e$

In[76]:=
```
RegionPlot[ 0 < y < 1 && y < x < Exp[y], {x, 0, 2}, {y, 0, 2}]
```

Out[76]=



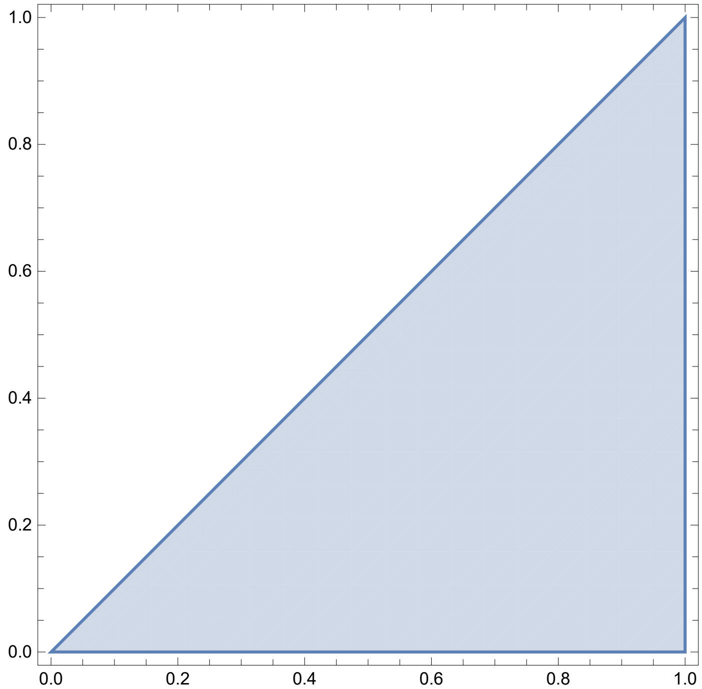
Exercise 12

In[78]:=
```
Integrate[Integrate[1, {x, y, 1} ], {y, 0, 1}]
```

Out[78]= $\frac{1}{2}$

In[79]:=
```
RegionPlot[ 0 < y < 1 && y < x < 1, {x, 0, 1}, {y, 0, 1}]
```

Out[79]=

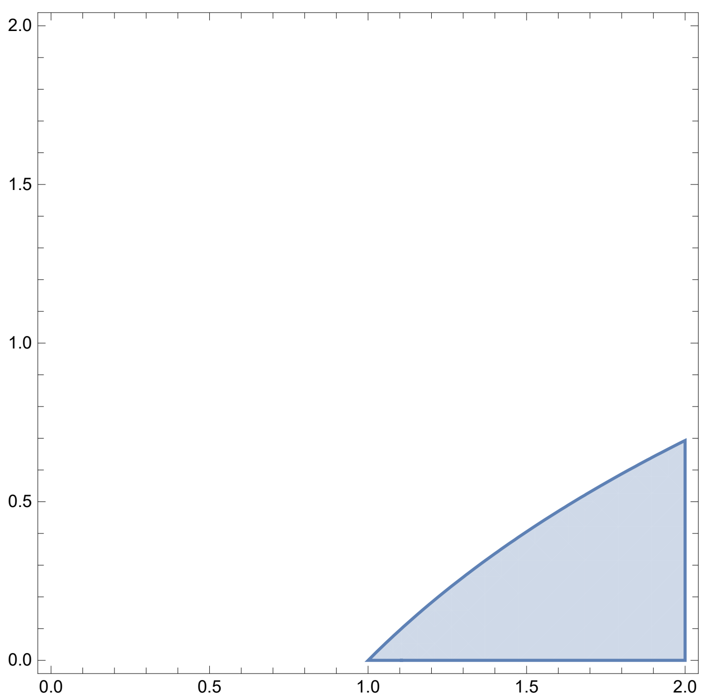


In[80]:=
```
Integrate[Integrate[1, {x, Exp[y], 2} ], {y, 0, Log[2]}]
```

Out[80]= $-1 + \text{Log}[4]$

In[82]:=
```
RegionPlot[ 0 < y < Log[2] && Exp[y] < x < 2, {x, 0, 2}, {y, 0, 2}]
```

Out[82]=

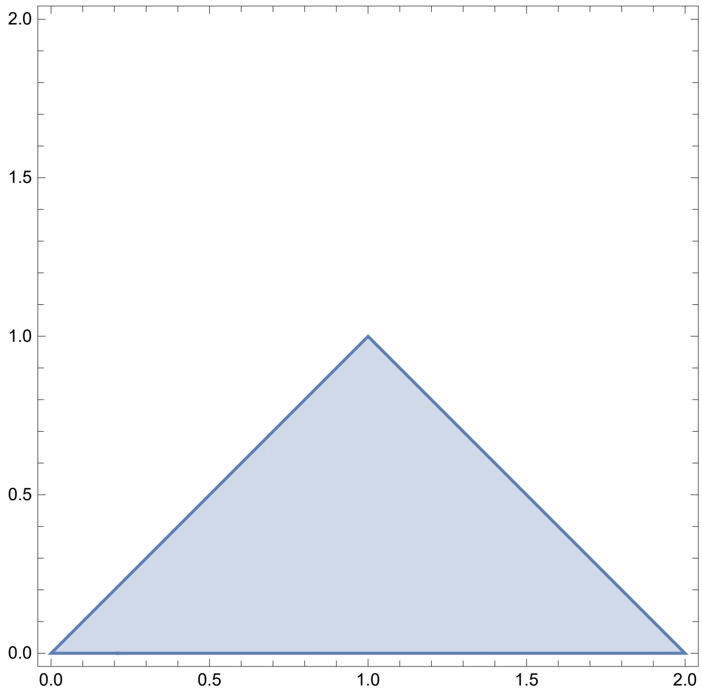


In[83]:=
```
Integrate[Integrate[x, {y, 0, 1} ], {x, 0, 1}] +
 Integrate[Integrate[-x, {y, 1, 2} ], {x, 1, 2}]
```

Out[83]= -1

In[84]:=
```
RegionPlot[ 0 < y < 1 && y < x < (2 - y), {x, 0, 2}, {y, 0, 2}]
```

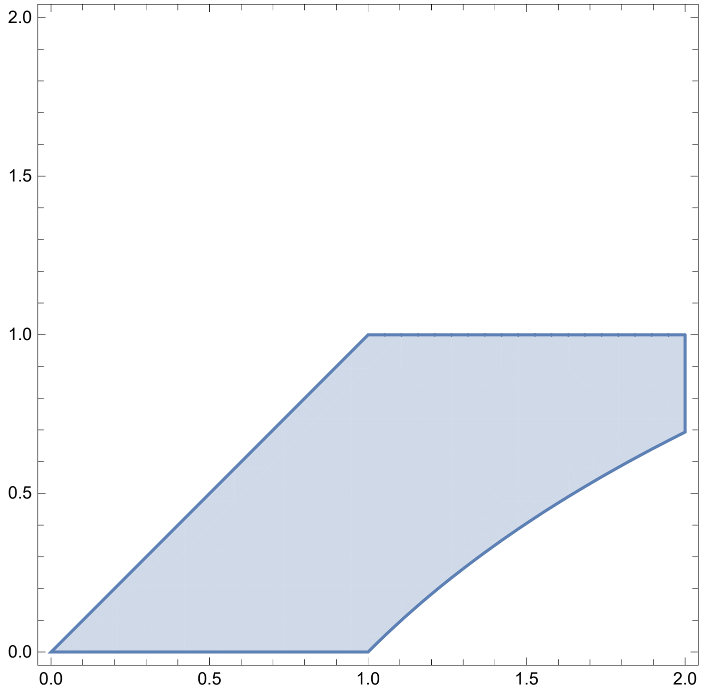
Out[84]=



In[86]:=
```
Integrate[Integrate[x, {y, 0, 1} ], {x, 0, 1}] +
 Integrate[Integrate[1, {y, Log[x], 2} ], {x, 1, 2}]
```

Out[86]= $\frac{7}{2} - \text{Log}[4]$

In[85]:=
```
RegionPlot[ 0 < y < 1 && y < x < Exp[y], {x, 0, 2}, {y, 0, 2}]
```

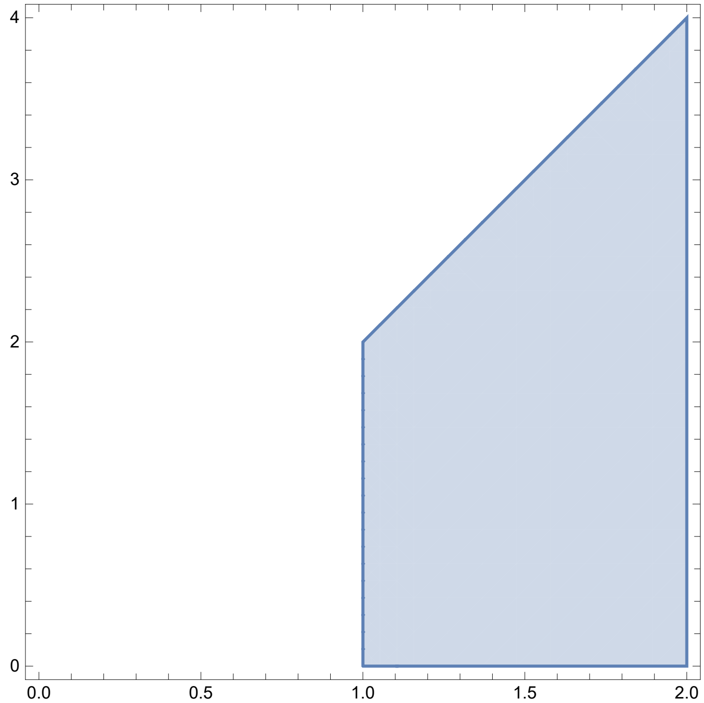
Out[85]=


Exercise 13

In[87]:=
```
Integrate[Integrate[(4 y) / ((x^3) + 3), {y, 0, 2 x} ], {x, 1, 2}]
```

Out[87]= $\frac{8}{3}\,\text{Log}\left[\frac{11}{4}\right]$

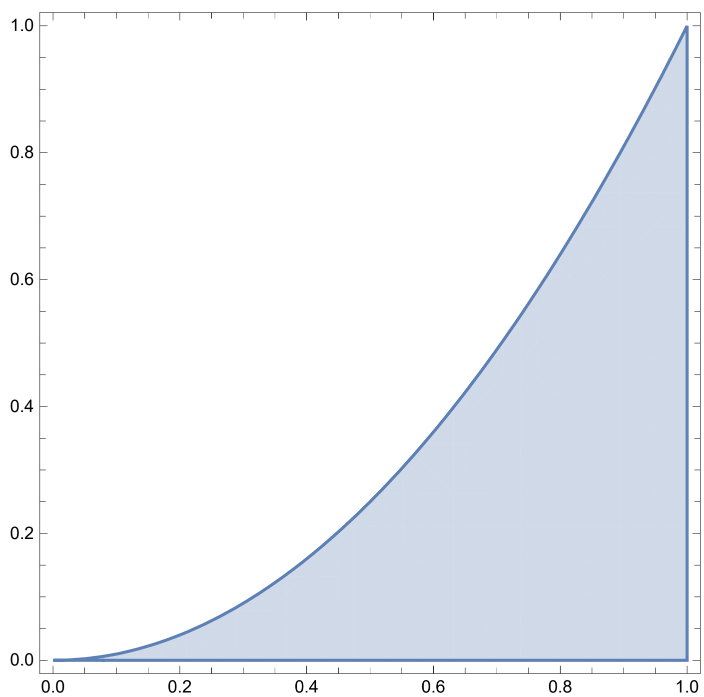
In[92]:=
```
RegionPlot[ 1 < x < 2 && 0 < y < 2 * x, {x, 0, 2}, {y, 0, 4}]
```

Out[92]= 

In[89]:=
```
Integrate[Integrate[x * Cos[y], {y, 0, x^2} ], {x, 0, 1}]
```

Out[89]= $\mathrm{Sin}\left[\frac{1}{2}\right]^2$

In[91]:=
```
RegionPlot[ 0 < x < 1 && 0 < y < x^2, {x, 0, 1}, {y, 0, 1}]
```

Out[91]= 

Exercise 14

In[94]:=
```
Integrate[Integrate[2.7, {x, 0, Sqrt[y]} ], {y, 0, 10}]
```
Out[94]= 56.921

In[97]:=
```
Integrate[Integrate[x * 2.7, {x, 0, Sqrt[y]} ], {y, 0, 10}] / 56.9
```
Out[97]= 1.18629

In[98]:=
```
Integrate[Integrate[y * 2.7, {x, 0, Sqrt[y]} ], {y, 0, 10}] / 56.9
```
Out[98]= 6.00221

2. Multiple Integrals

The main functions used in this section are the derivative **D**, the integral **Integrate,** and the function **RegionPlot** to visualize regions. To learn more about these functions you can either open up the Mathematica documentation in a new window, or you can get the basic syntax using a question mark before the function name and using shift+return.

```
? D
```

D[*f*, *x*] gives the partial derivative $\partial f / \partial x$.
D[*f*, {*x*, *n*}] gives the multiple derivative $\partial^n f / \partial x^n$.
D[*f*, *x*, *y*, …] gives the partial derivative $\cdots (\partial / \partial y)(\partial / \partial x) f$.
D[*f*, {*x*, *n*}, {*y*, *m*}, …] gives the multiple partial derivative $\cdots (\partial^m / \partial y^m)(\partial^n / \partial x^n) f$.
D[*f*, {{x_1, x_2, …}}] for a scalar *f* gives the vector derivative $(\partial f / \partial x_1, \partial f / \partial x_2, \ldots)$.
D[*f*, {*array*}] gives an array derivative. >>

```
? Integrate
```

Integrate[*f*, *x*] gives the indefinite integral $\int f \, dx$.
Integrate[*f*, {*x*, x_{min}, x_{max}}] gives the definite integral $\int_{x_{min}}^{x_{max}} f \, dx$.
Integrate[*f*, {*x*, x_{min}, x_{max}}, {*y*, y_{min}, y_{max}}, …] gives the multiple integral $\int_{x_{min}}^{x_{max}} dx \int_{y_{min}}^{y_{max}} dy \ldots f$.
Integrate[*f*, {*x*, *y*, …} ∈ *reg*] integrates over the geometric region *reg*. >>

```
? RegionPlot
```

RegionPlot[*pred*, {*x*, x_{min}, x_{max}}, {*y*, y_{min}, y_{max}}] makes a plot showing the region in which *pred* is True. >>

Here are some basic examples to get you started:

The derivative of x^n

```
D[x^n, x]
```

$n\, x^{-1+n}$

The anti-derivative or indefinite integral of x^n

```
Integrate[x^n, x]
```

$$\frac{x^{1+n}}{1+n}$$

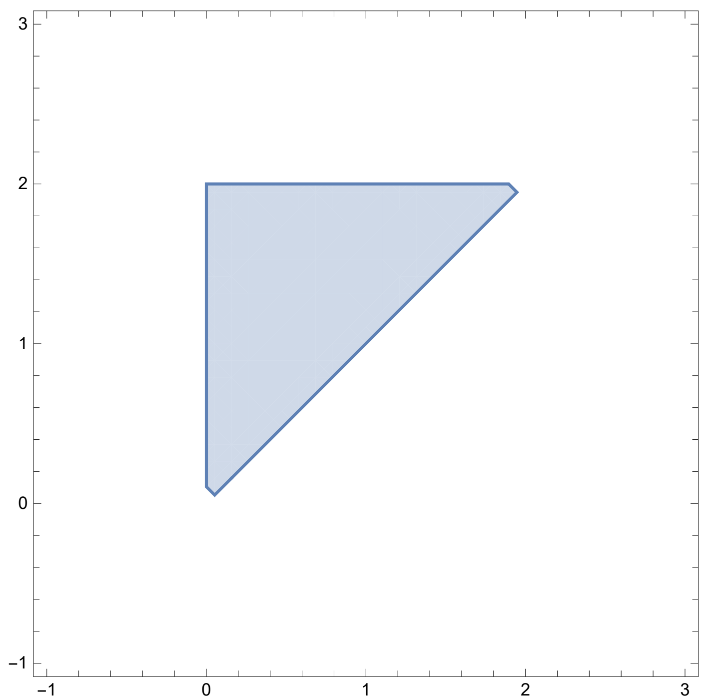
The definite integral of $x^n$from $x = 1$ to $x = 2$.

```
Integrate[x^n, {x, 1, 2}]
```

$$\frac{-1+2^{1+n}}{1+n}$$

The region in the plane bounded by $y = x,\ x = 0,\ y = 2$.

```
RegionPlot[y > x && y < 2 && x > 0, {x, -1, 3}, {y, -1, 3}]
```



The area of this region. Notice that the order of integration here really matters.

```
area = Integrate[1, {x, 0, 2}, {y, x, 2}]
```

2

The center of mass of this region, also known as the centroid for a geometrical shape.

```
xcom = Integrate[x, {x, 0, 2}, {y, x, 2}] / Integrate[1, {x, 0, 2}, {y, x, 2}]
ycom = Integrate[y, {x, 0, 2}, {y, x, 2}] / Integrate[1, {x, 0, 2}, {y, x, 2}]
```

$\frac{2}{3}$

$\frac{4}{3}$

Now we plot the plate and the center of mass of the plate and check that it makes sense.

```
Show[RegionPlot[y > x && y < 2 && x > 0, {x, -1, 3}, {y, -1, 3}], ListPlot[{{xcom, ycom}}]]
```