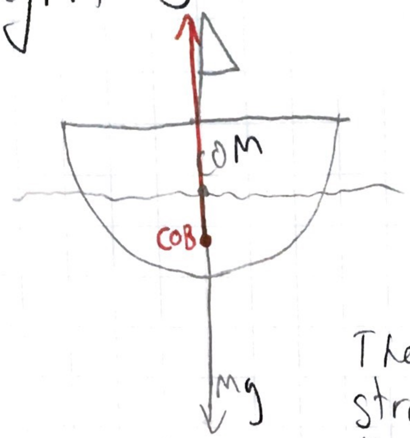
Night 5

1.



$F_{Buoyancy} - F_{gravity} = 0$

$\tau = 0$ and $\sum \vec{\tau} = 0$

The center of buoyancy needs to be in a straight line with the center of mass of the boat and the center of mass of the earth

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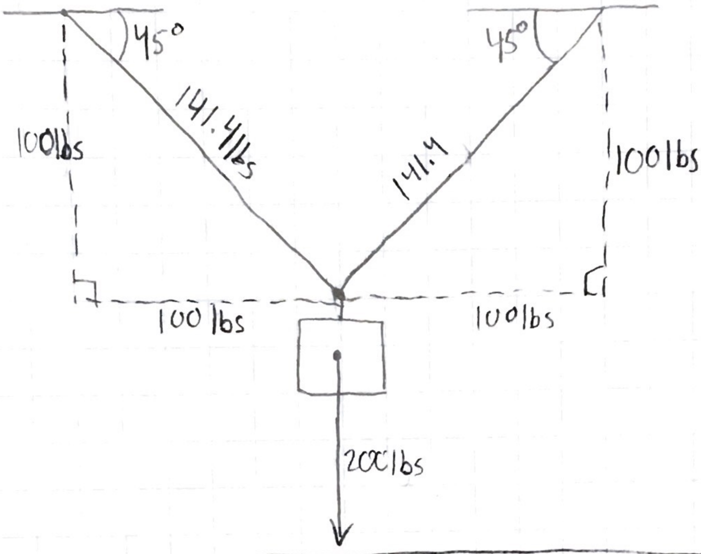
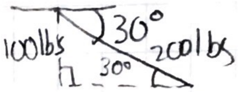
2. a)



b) As the suspension points move apart, the tension in the cables increases.



$S = \frac{O}{H}$ $\sin 30 = \frac{100}{H}$

$H = 200lbs$

---

3. a) $\int_0^{\sqrt{17}} 200\sqrt{1-\frac{x^2}{17}}\, dx = 647.66\,N$

b) The center of lift is at the center of area of the wing

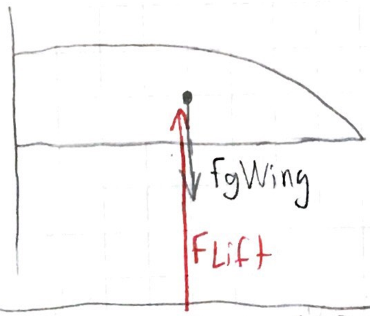
$\int_0^c 200\sqrt{1-\frac{x^2}{17}}\, dx = \frac{647.66}{2} = 323.83$

$c\sqrt{17-c^2} + 17 \times \operatorname{Arcsin}\left[\frac{c}{\sqrt{17}}\right] = 13.35$

$C = 1.665m$

$X_{COF} = \frac{1}{647.66N}\int_0^{\sqrt{17}}\int_0^{L(x)} dx\,dy$

c) Assuming the plane is moving forwards on a runway so it is experiencing some lift and some normal force so it does not fall out of the sky,



the plane is applying torque to the wing in the clockwise direction to hold up the wing

$F_{grav} = 1600N$ $F_g - F_{lift} = 952.34N$

$\tau = F \times distance$

$= 952.34N \times 1.665m$

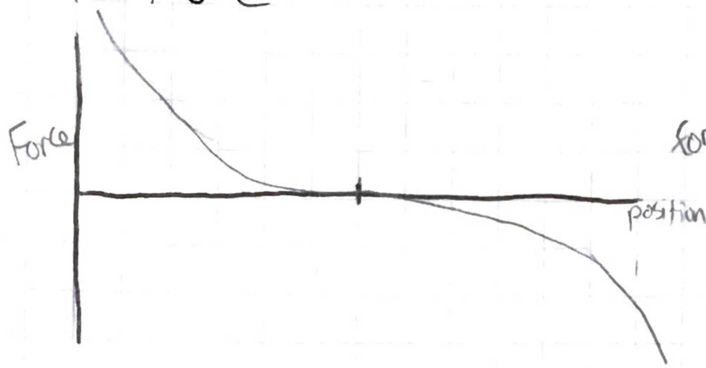
$1585.65 N \times M$

The plane is applying a clockwise torque of 1585.65 N×m to the plane wing

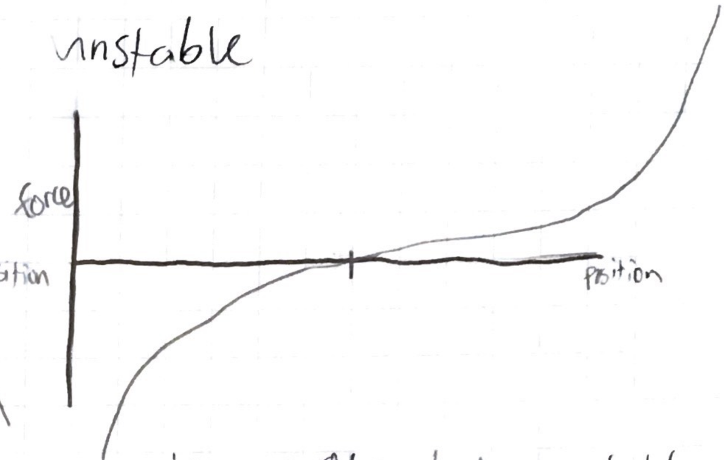
Night 5 = Night 5 + 1

4. a) stable
   b) unstable
   c) stable
   d) unstable

5. stable



unstable



When an object in stable equilibrium experiences a force/torque, it will then experience another force/torque that will bring the object back to equilibrium

$F_{push} + F_{reaction} = 0$

when an object in unstable equilibrium experiences a force or torque, it will experienca more forces and torques that pu it away from equilibrium

$F_{push} + F_{reaction} \neq 0$

$|F_{reaction} + F_{push}| > |F_{push}|$

6–8: Origin = COM of boat, $M_{boat}: 100N$, $F_{Buoyancy}: 100N$
↺ calculate net torque $\tau = r \times F$ and identify as either restoring or destabilizi

6. a) $(-2\hat{i}\ -1\hat{j}) \times (100\hat{j})$
   $-200\hat{k}$ Nxm stabilizing
   b) $(-1\hat{i} - 2\hat{j}) \times (100\hat{j})$
   $-100\hat{k}$ Nxm stabilizing
   c) $(-2\hat{j}) \times (100\hat{j})$
   0 Nxm neither
   d) $(1\hat{i} - 2\hat{j}) \times (100\hat{j})$
   $100\hat{k}$ Nxm destabilizing
   e) not super stable but not horribly unstable

.com

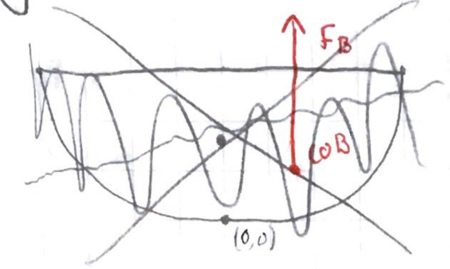
7. a) $(-1\hat{j}) \times (100\hat{j})$
   0 Nxm, neither
   b) $(-1\hat{j}) \times (100\hat{j})$
   0 Nxm, neither
   c) $(-1\hat{j}) \times (100\hat{j})$
   0 Nxm, neither
   d) $(-1\hat{j}) \times (100\hat{j})$
   0 Nxm, neither
   e) probably a cylindrical boat

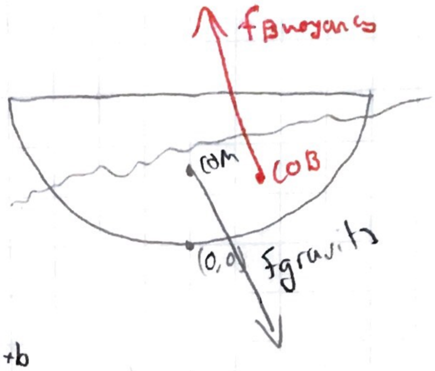
8. a) $(-2\hat{i}, -0.5\hat{j}) \times (100\hat{j})$
   $-200\hat{k}$ Nxm stabilizing
   b) $(-1.5\hat{i}, -1\hat{j}) \times (100\hat{j})$
   $-150\hat{k}$ Nxm stabilizing
   c) $(-1\hat{i}, -1.5\hat{j}) \times (100\hat{j})$
   $-100\hat{k}$ Nxm stabilizing
   d) $(-0.5\hat{i}, -2\hat{j}) \times (100\hat{j})$
   $-50\hat{k}$ Nxm stabilizing
   e) super stable

9. 6 = winner
   7 = log
   8 = raft

Night 5

2. a)





b) $V(d,\theta) = 10 \int_{-\sqrt{\frac{2}{\sqrt{3}}+2b}}^{\sqrt{\frac{2}{\sqrt{3}}+2b}} \int_{2(\frac{x}{2})^2}^{x\tan\theta+b} dy\,dx$

c) $xCOB = 10 \times \int_{-\sqrt{\frac{2}{\sqrt{3}}+2b}}^{\sqrt{\frac{2}{\sqrt{3}}+2b}} \int_{2(\frac{x}{2})^2}^{x\tan\theta+b} x \times 1000\, dx\,dy$

$yCOB = 10 \times \int_{-\sqrt{\frac{2}{\sqrt{3}}+2b}}^{\sqrt{\frac{2}{\sqrt{3}}+2b}} \int_{2(\frac{x}{2})^2}^{x\tan\theta+b} y \times 1000\, dx\,dy$

d) righting moment is the cross product of the vectors of the center of mass and center of buoyancy

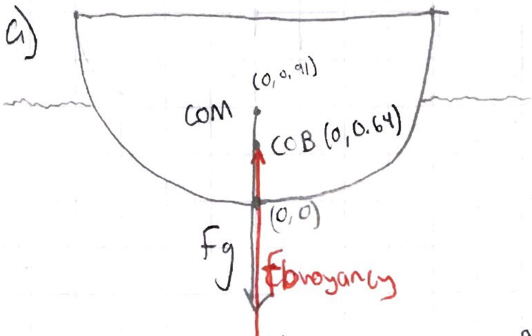
$(0\hat{i}, 0.91\hat{j}) \times (xCOB\hat{i}, yCOB\hat{j})$

COM stays the same as 0° case

Night 5 pt 3: Day 4 strikes back

0° case by hand (FBD) and with mathematica - you can use mathematica for integrals, and 30° case with FBD

1. a)



b) $M_{Boat} = 10 \times \int_{-2}^{2}\int_{2(\frac{x}{2})^2}^{2} 300\, dy\, dx$

$\int_{-2}^{2} 300y \Big|_{2(x/2)^2}^{2} dx$

$\int_{-2}^{2} 600(\frac{x}{2})^2 - 600\, dx$

$M_{Boat} = 16000$ kg

1600 kg + 5000 kg ballast

$M_{total} = 21{,}000$

c) $xCOM_{Boat} = (1/M_{Boat}) \times 10 \times \int_{-2}^{2}\int_{2(\frac{x}{2})^2}^{2} x \times 300\, dx\, dy = 0$ m

$yCOM_{Boat} = (1/M_{Boat}) \times 10 \times \int_{-2}^{2}\int_{2(\frac{x}{2})^2}^{2} y \times 300\, dy\, dx = 1.2$ m

$xCOM_{Total} = (1/M_{Total}) \times (xCOM_{Boat} \times M_{Boat} + x_{ballast} \times M_{Ballast}) = 0$ m

$yCOM_{Total} = (1/M_{Total}) \times (yCOM_{Boat} \times M_{Boat} + x_{ballast} \times M_{Ballast}) = 0.91$ m

d) $21000 \times 9.8\, m/s^2 = 205800\, N = F_{Buoyancy}$

e) $21000\, kg / 1000\, kg/m^3 = 21\, m^3$

f) $10 \times \int_{-2(\frac{draft}{2})^2}^{2(\frac{draft}{2})^2} \int_{2(\frac{x}{2})^2}^{draft} dy\, dx$

g) $21\, m^3 = 10 \int_{-\sqrt{2draft}}^{\sqrt{2draft}} \int_{2(\frac{x}{2})^2}^{draft} dy\, dx$

$\int_{2draft}^{2draft} y \Big|_{2(x/2)^2}^{draft}$

$draft - \frac{1}{2}(x)^2\, dx$

$x\, draft - \frac{1}{6}x^3 \Big|_{-(2(\frac{draft}{2}))}^{2(\frac{draft}{2})^2}$

$\cancel{\frac{1}{2}(draft)^2 \times draft - \frac{1}{6}(\frac{1}{2}draft^2)^3 - \frac{1}{2}(draft) \times draft - \frac{1}{6}(\frac{1}{2}draft^2)^3}$

mathematica says draft = 1.07m when I ~~solve~~ this integral

h) $xCOB = 10 \times \int_{-\sqrt{2draft}}^{\sqrt{2draft}} \int_{2(\frac{x}{2})^2}^{draft} x \times 1000\, dx\, dy = 0$

$yCOB = 10 \times \int_{-\sqrt{2draft}}^{\sqrt{2draft}} \int_{2(\frac{x}{2})^2}^{draft} y \times 1000\, dx\, dy = 0.644$

i) $(0\hat{i}, 0.91\hat{j}) \times (0\hat{i}, 0.64\hat{j}) = 0$

4. a) $21\ m^3$ is displaced in static equilibrium. The number does not change as a result of the boat tilting

   b) Tilt water 30° and use Regionplot3D to visualize displaced water compute volume of displaced wate

Night 5 Part 1

3

Find Mass:

```
In[ ]:= N[Integrate[200 * Sqrt[1 - ((x^2) / 17)], {x, 0, Sqrt[17]}]]
Out[ ]= 647.656
```

Trying to solve in a way that makes Mathematica freak out:

```
In[ ]:= N[Solve[Integrate[200 * Sqrt[1 - ((x^2) / 17)], {x, 0, c}] == % / 2, {c}]]
Out[ ]= $Aborted
```

Another attempt

```
Solve[Integrate[1 - 200 * Sqrt[1 - ((x^2) / 17)], {x, 0, c}] == % / 2]
```

Out[ ]=

$$\left\{\left\{c \to -\left(17 \Big/ \left(300\sqrt{17} + 1275\sqrt{17}\,\text{ArcTan}[4] + \sqrt{1525087 + 13005000\,\text{ArcTan}[4] + 27635625\,\text{ArcTan}[4]^2}\right)^{1/3}\right) - \left(300\sqrt{17} + 1275\sqrt{17}\,\text{ArcTan}[4] + \sqrt{1525087 + 13005000\,\text{ArcTan}[4] + 27635625\,\text{ArcTan}[4]^2}\right)^{1/3}\right\},\right.$$

$$\left\{c \to \left(17\left(1 + i\sqrt{3}\right)\right) \Big/ \left(2\left(300\sqrt{17} + 1275\sqrt{17}\,\text{ArcTan}[4] + \sqrt{1525087 + 13005000\,\text{ArcTan}[4] + 27635625\,\text{ArcTan}[4]^2}\right)^{1/3}\right) + \frac{1}{2}\left(1 - i\sqrt{3}\right)\left(300\sqrt{17} + 1275\sqrt{17}\,\text{ArcTan}[4] + \sqrt{1525087 + 13005000\,\text{ArcTan}[4] + 27635625\,\text{ArcTan}[4]^2}\right)^{1/3}\right\},$$

$$\left.\left\{c \to \left(17\left(1 - i\sqrt{3}\right)\right) \Big/ \left(2\left(300\sqrt{17} + 1275\sqrt{17}\,\text{ArcTan}[4] + \sqrt{1525087 + 13005000\,\text{ArcTan}[4] + 27635625\,\text{ArcTan}[4]^2}\right)^{1/3}\right) + \frac{1}{2}\left(1 + i\sqrt{3}\right)\left(300\sqrt{17} + 1275\sqrt{17}\,\text{ArcTan}[4] + \sqrt{1525087 + 13005000\,\text{ArcTan}[4] + 27635625\,\text{ArcTan}[4]^2}\right)^{1/3}\right\}\right\}$$

```
In[ ]:= N[%]
Out[ ]= {{c → -26.0824}, {c → 13.0412 - 21.4294 i}, {c → 13.0412 + 21.4294 i}}
```

I am going to take a random guess and say that the center of lift is probably not at -26 meters.

Finding just the integral (Mathematica can do this)

In[ ]:= `200 * Integrate[Sqrt[1 - (1 / 17) * (x^2)], x]`

Out[ ]= $$\frac{100\left(x\sqrt{17-x^2}+17\,\text{ArcSin}\left[\frac{x}{\sqrt{17}}\right]\right)}{\sqrt{17}}$$

Solving for the bottom limit which is 0. The answer is 0, so we don't have to worry about it.

In[ ]:= $$\frac{100\left(0\sqrt{17-0^2}+17\,\text{ArcSin}\left[\frac{0}{\sqrt{17}}\right]\right)}{\sqrt{17}}$$

Out[ ]= 0

Trying to solve for the top limit. Mathematica does not like me doing this.

In[ ]:= $$\text{Solve}\left[\frac{100\left(c\sqrt{17-c^2}+17\,\text{ArcSin}\left[\frac{c}{\sqrt{17}}\right]\right)}{\sqrt{17}}==321.835\right]$$

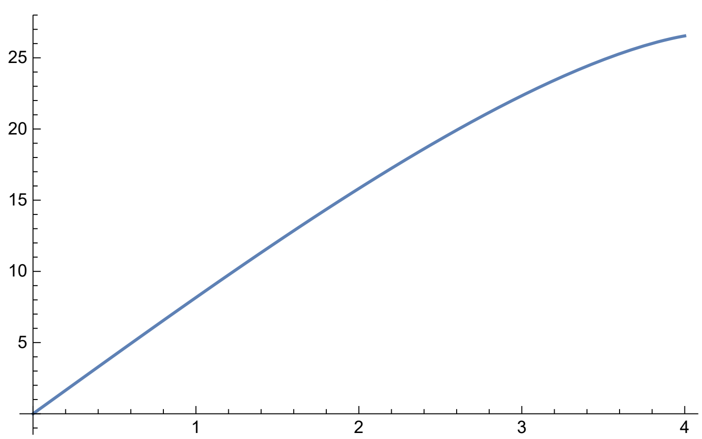
Out[ ]= `$Aborted`

This takes too long and makes my computer sound like it is about to prepare for takeoff so it does not work.

Ok, time to solve this in a really stupid way. Here is a graph of the function but slightly simplified. Now, I am looking for where the function is equal to 13.35. But for some reason Mathematica won't do it for me so...

In[ ]:= $$\text{Plot}\left[c\sqrt{17-c^2}+17\,\text{ArcSin}\left[\frac{c}{\sqrt{17}}\right],\ \{c,\ 0,\ 4\}\right]$$

Out[ ]=

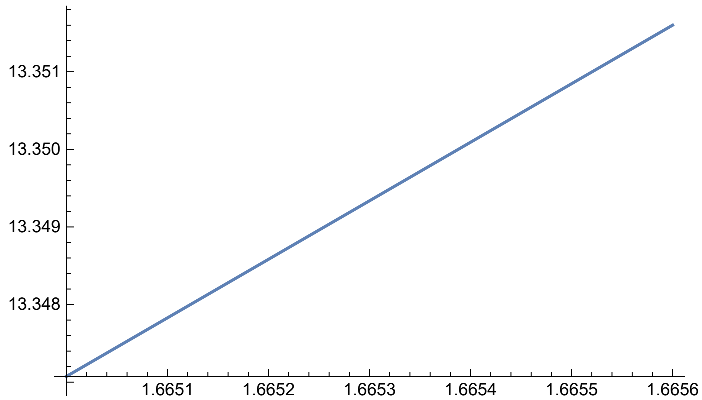


Here is the same graph zoomed in so that I can get close enough to the answer. I told you it would be really stupid.

In[ ]:=
```
Plot[c Sqrt[17 - c^2] + 17 ArcSin[c/Sqrt[17]], {c, 1.665, 1.6656}]
```

$$\text{Plot}\left[c\sqrt{17-c^2}+17\,\text{ArcSin}\left[\frac{c}{\sqrt{17}}\right],\ \{c,\ 1.665,\ 1.6656\}\right]$$

Out[ ]=



c=1.665

Night 5 Part 2

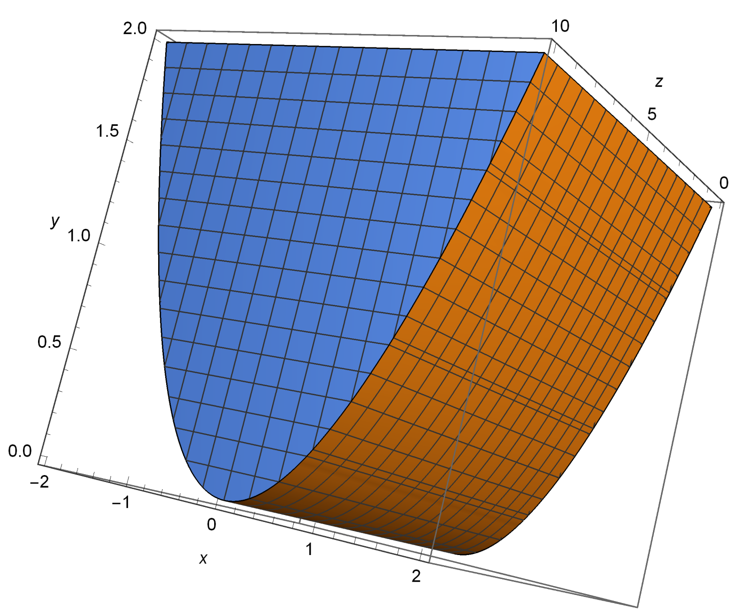
Defining Variables

In[101]:=
```
Clear[f, density, xmin1, xmax1, ymin1, ymax1]
f = 2 * ((x / 2) ^2);
density = 300;
ballastmass = 5000;
xmin1 = -2;
xmax1 = 2;
ymin1 = 0;
ymax1 = 2;
zmin1 = 0;
zmax1 = 10;
xballast = 0;
yballast = 0;
```

3D Plot of Boat

In[130]:=
```
RegionPlot3D[zmin1 ≤ z ≤ zmax1 && f ≤ y ≤ ymax1 && xmin1 ≤ x ≤ xmax1,
 {x, xmin1, xmax1}, {y, ymin1, ymax1}, {z, zmin1, zmax1},
 PlotPoints → 100,  Axes → True, AxesLabel → {x, y, z}]
```

Out[ ]=



Defining Boat region

```
boat = ImplicitRegion[f < y < ymax1 && xmin1 < x < xmax1 && ymin1 < y < ymax1, {x, y}]
```

Out[∘]= ImplicitRegion$\left[\frac{x^2}{2} < y < 2 \,\&\&\, -2 < x < 2 \,\&\&\, 0 < y < 2,\ \{x, y\}\right]$

Defining water region

In[47]:=
```
water = ImplicitRegion[ymin1 < y < d && xmin1 < x < xmax1 && ymin1 < y < ymax1, {x, y}]
```

Out[∘]= ImplicitRegion$[0 < y < d \,\&\&\, -2 < x < 2 \,\&\&\, 0 < y < 2,\ \{x, y\}]$

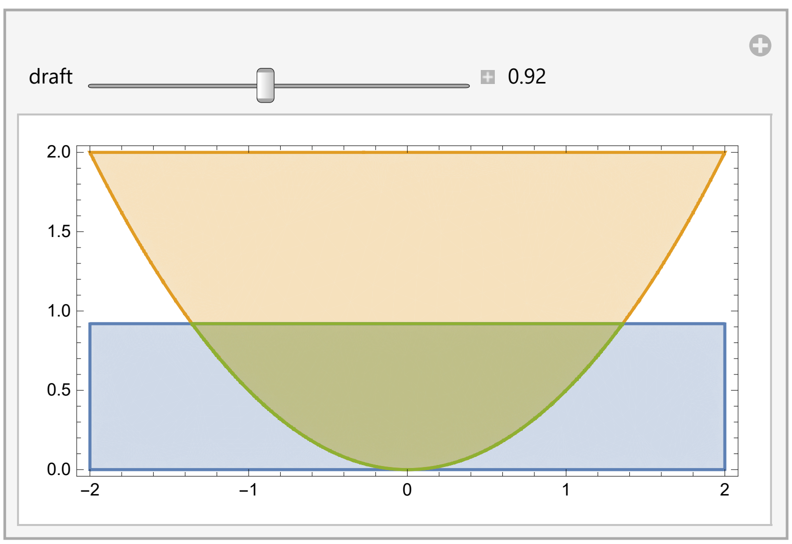
Defining Submerged region of boat as intersection of boat and water

In[48]:=
```
submerged = RegionIntersection[boat, water]
```

Out[∘]= ImplicitRegion$\left[-2 < x < 2 \,\&\&\, 0 < y < 2 \,\&\&\, 0 < y < d \,\&\&\, \frac{x^2}{2} < y < 2,\ \{x, y\}\right]$

Manipulate plot where you can adjust draft

In[51]:=
```
Manipulate[RegionPlot[{water /. {d → dd}, boat, submerged /. {d → dd}},
  AspectRatio → Automatic], {{dd, 1, "draft"}, 0, 2, Appearance → "Labeled"}]
```



··· RegionPlot: RegionPlot called with 1 argument; 3 arguments are expected.

Figuring out total mass of boat by integration and adding ballastmass

∈ sign is escape, e, l, escape

```
In[81]:= boatmass = 10 * Integrate[density, {x, y} ∈ boat]
Out[•]= 16 000

In[82]:= totalmass = boatmass + ballastmass
Out[•]= 21 000
```

Finding COMs

```
In[127]:= xcomboat = (1 / boatmass) * 10 * Integrate[x * density, {x, y} ∈ boat]
Out[•]= 0
```

Just boat

```
In[85]:= ycomboat = N[(1 / boatmass) * 10 * Integrate[y * density, {x, y} ∈ boat]]
Out[•]= 1.2
```

COM of combined boat and ballast: 2 ways, second one takes into account ballast x or y coordinate

```
In[131]:= xcom = N[(1 / totalmass) * 10 * Integrate[x * density, {x, y} ∈ boat]]
          xcom = 1 / (total) * (xcomboat * boatmass + xballast * ballastmass)
Out[•]= 0.
Out[•]= 0

In[133]:= ycom = N[(1 / totalmass) * 10 * Integrate[y * density, {x, y} ∈ boat]]
          ycom = 1 / (totalmass) * (ycomboat * boatmass + yballast * ballastmass)
Out[•]= 0.914286
Out[•]= 0.914286
```

Finding Buoyant force with d as a variable

```
In[87]:= displacement = 10 * Integrate[1000, {x, y} ∈ submerged, Assumptions → {ymin1 < d < ymax1}]
```

Out[•]= $\frac{40\,000}{3}\sqrt{2}\ d^{3/2}$

Figuring out waterline based on displacement

```
In[89]:= waterline = NSolve[displacement == mass, d, Reals]
Out[•]= {{d → 1.07443}}
```

With flat waterline, there is only one answer that satisfies the condition

```
In[91]:= draft = N[d /. waterline[[1]]]
Out[•]= 1.07443
```

Finding Center of buoyancy

In[97]:=
```
cob = 10 * Integrate[1000 * {x, y},
    {x, y} ∈ submerged, Assumptions → {ymin1 < d < ymax1}] / displacement
```

Out[]= $\{0, \frac{3\,d}{5}\}$

In[99]:=
```
cob /. {d → draft}
```

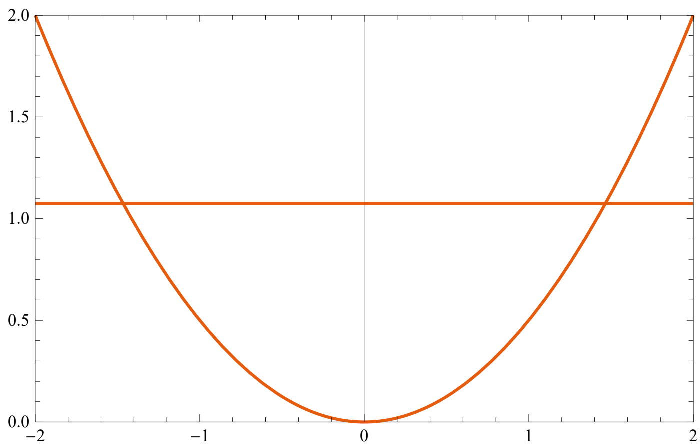
Out[]= {0, 0.644656}

In[100]:=
```
ycob = cob[[2]]
```

Out[]= $\frac{3\,d}{5}$

Plotting boat with waterline

In[126]:=
```
Show[Plot[f, {x, xmin1, xmax1}, PlotRange → {{xmin1, xmax1}, {ymin1, ymax1}},
  PlotTheme → "Scientific"], Plot[draft, {x, xmin1, xmax1},
  PlotRange → {{xmin1, xmax1}, {ymin1, ymax1}}, PlotTheme → "Scientific"] ]
```

Out[]=


In[]:=
```
10 * (1 / totalmass) Integrate[Integrate[1000 * y, {y, 2 * (x / 2) ^2, draft}],
  {x, - (2 * draft) ^ (1 / 2), (2 * draft) ^ (1 / 2) }]
```

Out[]= 0.644656

Night 5 Part 3 Start

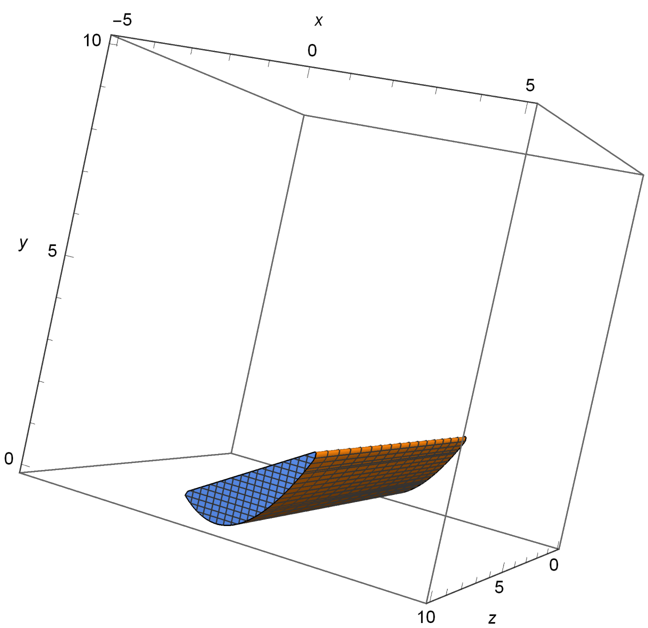
Defining Variables

In[249]:=
```
Clear[f, density, xmin1, xmax1, ymin1, ymax1, zmin1, zmax1, xballast, yballast,
  ballastmass, θ, boat, water, submerged, boatmass, totalmass, xcomboat, ycomboat,
  xcom, ycom, displacement, waterline, b, d, draft, cob, xcob, ycob, x, y]
f  =  2 * ((x / 2) ^2);
density  = 300;
ballastmass  =  5000;
xmin1  = -2;
xmax1 = 2;
ymin1 = 0;
ymax1 = 2;
zmin1 = 0;
zmax1 = 10;
xballast = 0;
yballast = 0;
θ  = 30 Degree;
```

3D Plot of Boat’s submerged region (ESTIMATE. NOT ACTUALLY)

In[262]:=
```
RegionPlot3D[zmin1 ≤ z ≤ zmax1 && f ≤ y ≤ Tan[θ] * x + 1 && xmin1 ≤ x ≤ xmax1,  {x,  -5,  5},
 {y,  0,  10}, {z, zmin1, zmax1}, PlotPoints → 100,  Axes → True,  AxesLabel → {x, y, z}]
```

Out[262]=



Defining Boat region

In[263]:=
```
boat = ImplicitRegion[f < y < ymax1 && xmin1 < x < xmax1 && ymin1 < y < ymax1, {x, y}]
```

Out[263]= ImplicitRegion$\left[\frac{x^2}{2} < y < 2 \,\&\&\, -2 < x < 2 \,\&\&\, 0 < y < 2,\ \{x, y\}\right]$

Defining water region

In[264]:=
```
water = ImplicitRegion[ymin1 < y < Tan[θ] + b && xmin1 < x < xmax1 && ymin1 < y < ymax1,  {x, y}]
```

Out[264]= ImplicitRegion$\left[0 < y < \frac{1}{\sqrt{3}} + b \,\&\&\, -2 < x < 2 \,\&\&\, 0 < y < 2,\ \{x, y\}\right]$

Defining Submerged region of boat as intersection of boat and water

In[265]:=
```
submerged = RegionIntersection[boat, water]
```

Out[265]= ImplicitRegion$\left[-2 < x < 2 \,\&\&\, 0 < y < 2 \,\&\&\, 0 < y < \frac{1}{\sqrt{3}} + b \,\&\&\, \frac{x^2}{2} < y < 2,\ \{x, y\}\right]$

Figuring out total mass of boat by integration and adding ballastmass

∈ sign is escape, e, l, escape

In[266]:=
```
boatmass = 10 * Integrate[density, {x, y} ∈ boat]
```

Out[266]= 16 000

In[267]:=
```
totalmass = boatmass + ballastmass
```

Out[267]= 21 000

Finding COMs

In[268]:=
```
xcomboat = (1 / boatmass) * 10 * Integrate[x * density, {x, y} ∈ boat]
```

Out[268]= 0

Just boat

In[269]:=
```
ycomboat = N[(1 / boatmass) * 10 * Integrate[y * density, {x, y} ∈ boat]]
```

Out[269]= 1.2

COM of combined boat and ballast: 2 ways, second one takes into account ballast x or y coordinate

In[270]:=
```
xcom = N[(1 / totalmass) * 10 * Integrate[x * density, {x, y} ∈ boat]]
xcom = 1 / (total) * (xcomboat * boatmass + xballast * ballastmass)
```

Out[270]= 0.

Out[271]= 0

```
In[272]:= ycom = N[(1/totalmass) * 10 * Integrate[y * density, {x, y} ∈ boat]]
          ycom = 1/(totalmass) * (ycomboat * boatmass + yballast * ballastmass)
Out[272]= 0.914286
Out[273]= 0.914286
```

Finding Buoyant force with b as a variable

```
In[274]:= displacement = 10 * Integrate[1000, {x, y} ∈ submerged, Assumptions → {ymin1 < b < ymax1}]
```

Out[274]= $$10\left(\begin{cases} \frac{4000}{9}\sqrt{\frac{2}{3}}\left(\sqrt{3}+3\,b\right)^{3/2} & b \le \frac{1}{3}\left(6-\sqrt{3}\right) \\ \frac{16\,000}{3} & \text{True} \end{cases}\right)$$

Figuring out waterline based on displacement

```
In[275]:= waterline = NSolve[displacement == mass, b, Reals]
Out[275]= {{b → 0.497077}}
```

With flat waterline, there is only one answer that satisfies the condition

```
In[276]:= draft = N[b /. waterline[[1]]]
Out[276]= 0.497077
```

Finding Center of buoyancy

```
In[277]:= cob = 10 * Integrate[1000 * {x, y},
             {x, y} ∈ submerged, Assumptions → {ymin1 < b < ymax1}] / displacement
```

Out[277]= $$\left\{0,\ \begin{cases} 500\left(-\frac{2}{45}\sqrt{\frac{2}{3}}\left(\sqrt{3}+3\,b\right)^{5/2}+2\sqrt{\frac{2}{3}}\sqrt{\sqrt{3}+3\,b}\left(\frac{1}{3}+\frac{2\,b}{\sqrt{3}}+b^2\right)\right) & b \le \frac{1}{3}\left(6-\sqrt{3}\right) \\ 6400 & \text{True} \end{cases} \Bigg/ \begin{cases} \frac{4000}{9}\sqrt{\frac{2}{3}}\left(\sqrt{3}+3\,b\right)^{3/2} & b \le \frac{1}{3}\left(6-\sqrt{3}\right) \\ \frac{16\,000}{3} & \text{True} \end{cases}\right\}$$

```
In[278]:= cob /. {b → draft}
Out[278]= {0, 0.644656}
```

In[279]:=

```
ycob = cob[[2]]
```

Out[279]=

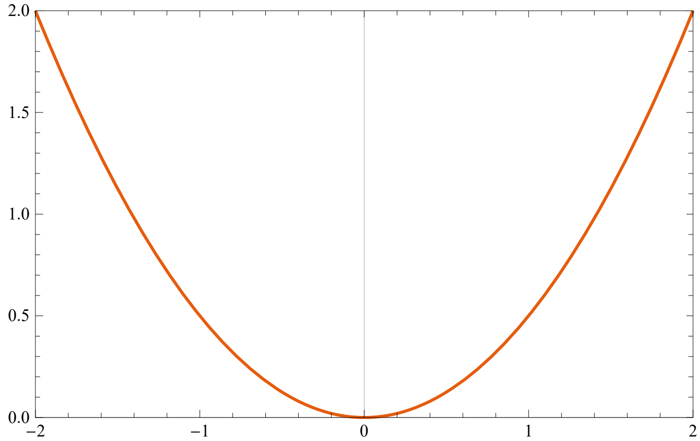
$$\begin{cases} 500\left(-\frac{2}{45}\sqrt{\frac{2}{3}}\left(\sqrt{3}+3\,b\right)^{5/2}+2\sqrt{\frac{2}{3}}\sqrt{\sqrt{3}+3\,b}\left(\frac{1}{3}+\frac{2\,b}{\sqrt{3}}+b^2\right)\right) & b\le\frac{1}{3}\left(6-\sqrt{3}\right) \\ 6400 & \text{True}\end{cases} \Big/ \begin{cases}\frac{4000}{9}\sqrt{\frac{2}{3}}\left(\sqrt{3}+3\,b\right)^{3/2} & b\le\frac{1}{3}\left(6-\sqrt{3}\right) \\ \frac{16\,000}{3} & \text{True}\end{cases}$$

Plotting boat with waterline

In[280]:=

```
Show[Plot[f, {x, xmin1, xmax1}, PlotRange → {{xmin1, xmax1}, {ymin1, ymax1}},
  PlotTheme → "Scientific"], Plot[Tan[θ] + b,  {x, xmin1, xmax1},
  PlotRange → {{xmin1, xmax1}, {ymin1, ymax1}}, PlotTheme → "Scientific"] ]
```

Out[280]=



In[∘]:=

```
10 * (1 / totalmass) Integrate[Integrate[1000 * y, {y, 2 * (x / 2) ^2, draft}],
  {x, - (2 * draft) ^ (1 / 2), (2 * draft) ^ (1 / 2)}]
```

Out[∘]= 0.644656