Night 2: Forces & Torques

Ideas in videos I already know

- Vector has magnitude & direction
- $|A|$ is magnitude and $\hat{A}$ is unit vector representing direction
- Unit vectors (Mostly)
- adding vectors $\vec{A} = 3\hat{i} + 3\hat{j}$ $\quad \vec{B} = 3\hat{i} - 4\hat{j}$
  $\vec{A}+\vec{B} = (3+3)\hat{i} + (3-4)\hat{j} = 6\hat{i} - \hat{j}$
- scalar multiplication $\vec{A} = 2\hat{i} + 3\hat{j}$
  $3\vec{A} = 3(2\hat{i} + 3\hat{j}) = 6\hat{i} + 9\hat{j}$

Ideas in videos I had not seen before

- polar coordinates in 3D
- matlab: vector = list of numbers
- multiplying 2 vectors
  - dot product (scalar product) $\vec{A} \cdot \vec{B}$

$\vec{A} \cdot \vec{B} = |\vec{A}| \times |\vec{B}| \times \cos\varphi$

magnitude

$\hat{A} = 3\hat{i} + 4\hat{j}$ $\quad B = 7\hat{i} - 2\hat{j}$

Dot product: $\vec{A} \cdot \vec{B} = 3 \times 7 + 4 \times -2 = 13$

  - cross product (vector product) $\vec{A} \times \vec{B}$

$\hat{i} \times \hat{j} = \hat{k}$ $\quad \hat{j} \times \hat{i} = -\hat{k}$ $\quad$ FOIL it

$\hat{j} \times \hat{k} = \hat{i}$ $\quad \hat{i} \times \hat{k} = -\hat{j}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad \hat{k} \times \hat{j} = -\hat{i}$

- Force is an interaction that causes change in momentum
- 4 fundamental forces (I know what they are)
- Friction
  - static μ
  - kinetic μ
- spring forces
- gravity
- constraint forces (not called that but same concept

- the word phenomelogical
- drag / lift

$\vec{F}_D = \frac{1}{2} C_D A \rho V^2 \hat{v}$

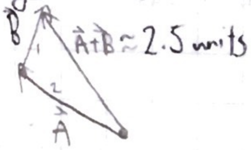
density of fluid; drag coefficient; presented area

Drag is proportionate to v^2

Vectors & Vector Operations

2. a) Dot product tells you how parallel the 2 vectors are, and how much does A point in the direction of B.
Cross product tells you how perpendicular the vectors are, and tells you about the plane in which the 2 vectors lie.

b) i)



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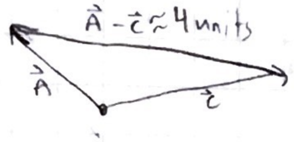


iii) $(1\times1)+(-2\times0.1) \times \cos\theta$
estimated numbers → $1-0.2 = 0.6 \times \cos\theta$

iv) $2\times1\times\sin 60 = \sqrt{3}\times3\times\sin 90 = 3\sqrt{3}$

v) it is not possible because the result of dot product is not a vector

3. $\vec{A} = 3\hat{i}+4\hat{j}$ $\quad \vec{B} = \hat{i}-\hat{j}$ $\quad \vec{C} = -5\hat{j}$

a) $|\vec{A}+\vec{B}| = (3\hat{i}+4\hat{j})+(\hat{i}+\hat{j})$
$= 4\hat{i}+5\hat{j}$

b) $\vec{A}\times\vec{C} = (3\hat{i}+4\hat{j})\times(-5\hat{j})$
$-15\hat{k}+0 = -15\hat{k}$

c) $\vec{A}\cdot\vec{B} = 3\hat{i}+4\hat{j} \cdot \hat{i}-\hat{j}$
$3\times1-4\times1 = -1$

4. a) $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

b) $\dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \hat{n}$

Force Concepts

6. a) True because the coefficient of kinetic friction is a constant that applies to moving objects. The cup and the table also stays the same friction-wise because their surface does not change. And the equation for force of friction states that in this situation, $F_{Friction} = \mu_K \times F_{Normal}$

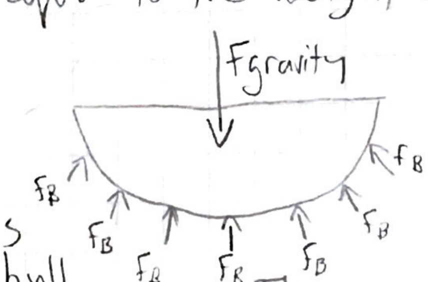
b) false. $\mu_s \times N$ gives the maximum force you could apply to the cup without it moving. I.e. the maximum static frictional force. Since you are probably not pushing as hard as the maximum force of friction, in this case, $F_{friction}$ is equal to the force you are applying to the cup. $F_{friction}$ cannot be greater than the force you are applying to the cup.

c) In this case, $F_{Normal} = F_{gravity}$ = Mass of object × g aka $9.8 m/s^2$

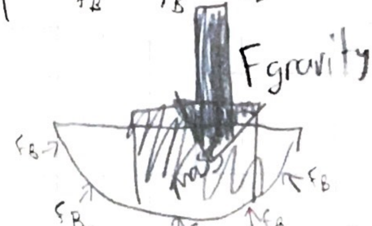
d) Since the ground is applying force to accelerate the object to a velocity of 0m/s, and F=ma, the acceleration is greater than $9.8 m/s^2$, so the normal force would be greater than mg

e) Hydrostatic pressure is the pressure that water exerts on itself due to its own weight. The top layer of water is pushing down on the next layer of water which is pushing down on the next one all the way to the bottom. Now imagine you have a watter bottle that you fill halfway with water, and then put it on its side in a pool. The waterline inside the bottle and outside the bottle line up pretty closely. This is because the amount of force that is pushing up on the water bottle is equal to the weight of the water in the bottle (plus the weight of the bottle). Since the water bottle was floating at rest, and the force of buoyancy = weight, you could measure the weight of the water and the bottle in order to get the buoyancy. This demonstrates Archemedes principle: that the buoyant force on an object is equal to the weight of the fluid displaced by that object.

Note: $F_{gravity}$ is distributed by the sturdiness of the boat's hull



$F_{gravity}$ = Sum of $F_{Buoyancy}$ while boat is at rest



Here, $F_{gravity}$ is greater than the sum of $F_{Buoyancy}$ so the boat is sinking

Calculating forces

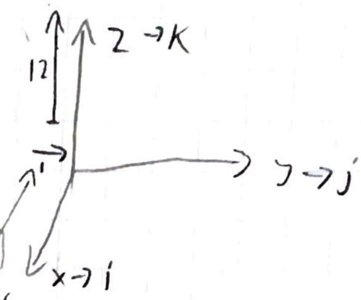
7. a) $F_{rope} = 70\text{ lbs}$ $\vec{AB} = -10\hat{i} + 1\hat{j} + 12\hat{k}$

$\sqrt{10^2 + 1^2 + 12^2} = 15.65$

$70 / 15.65 = 4.47$

$\vec{AB} \times 4.47$

$4.47\vec{AB} = -44.72\hat{i} + 4.47\hat{j} + 53.67\hat{k}$ in lbs



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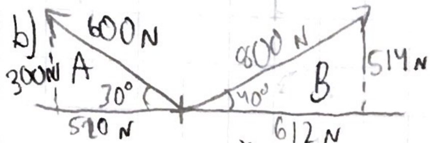


$S = \frac{O}{H}$ $\sin 30 = \frac{x}{600}$

$C = \frac{A}{H}$ $\sin 40 = \frac{x}{800}$

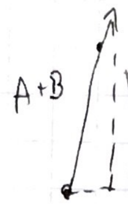
$T = \frac{O}{A}$

A = −520, 300

B = 612, 514

A + B = 92, 814

$\sqrt{92^2 + 814^2} = 819.2N$



$\boxed{\text{Magnitude} = 819.2N \quad \theta = 83.55°}$

$\sin \theta = \frac{O}{H}$

$\sin \theta = \frac{814}{819.2}$ $\theta = 83.55$

Free Body Diagrams

Table of concepts

What is in the system?

Distributed mass? Single point mass?

Cut a system out of the rest of the universe

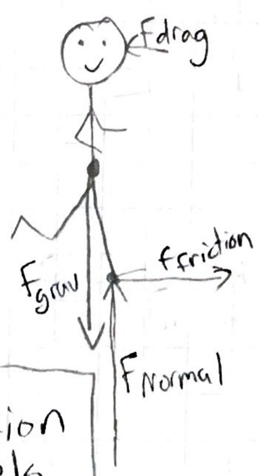
Make list of interactions maybe

Are forces balanced?

Gravity acts from center of mass

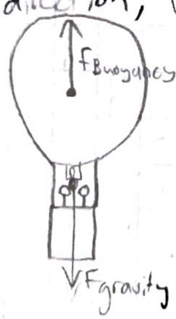
Arrows appropriate size, direction, where they attach

9. a)



Friction propels runner forward

b) i)



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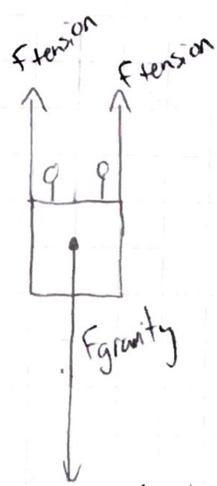

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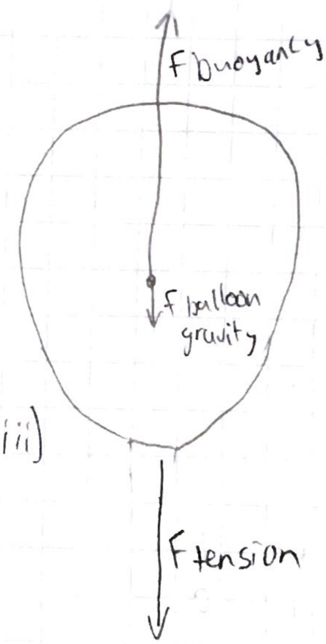
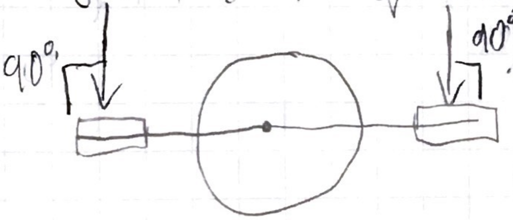


iv) The first diagram would be useful for learning about the behavior of the entire balloon-basket system. The next two diagrams would be useful in figuring out the necesary strength of the ropes

Torque Fork Pork: Ideas and calculating

10. Important ideas in the videos:
Torque is basically angular force
units are $N \times m$
$\tau = r \times F$

11. I just want to point out that pushing down with constant force as you pedal would get you nowhere. Your legs would be working against each other, which does not sound fun. However, I will answer in the spirit of the question



the greatest torque occurs when the pedal is perpendicular to the downward force applied to it.

12. I could not find a specific value for average force someone can exert on a wrench, so I am going with 100 N which is about 25 lbs which is a little more than I can bicep curl. It is also the number used in the next problem.

Torque = force × radius × ~~sin θ~~ ↙ assume force is applied straight on so this does not matter

$100 N \times m = 100 N \times r\, m$

$r = 1 m$ handle

13. Torque $\leq$ force $\times r \times \sin\theta$

$16 N \times m \leq 100 N \times 0.2 m \times \sin 60$

$16 N \times m \not\leq 17.3 N \times m$

Yes

14. $\tau = r \times F$

$1\hat{j} + 5\hat{k} \times 3\hat{i} + 2\hat{j}$

$\tau = -3\hat{k} + 15\hat{j} - 10\hat{i}$ about origin

$(\vec{r} - \vec{r}_0) \times F = \tau$

$[(1\hat{j} + 5\hat{k}) - (7\hat{i} + 3\hat{k})] \times 3\hat{i} + 2\hat{j}$

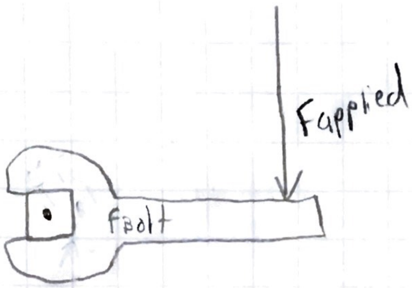
$(7\hat{i} + 1\hat{j} + 2\hat{k}) \times (3\hat{i} + 2\hat{j})$

$14\hat{k} - 3\hat{k} + 6\hat{j} - 4\hat{i}$

$11\hat{k} + 6\hat{j} - 4\hat{i} = \tau$

Wrench

15. a)



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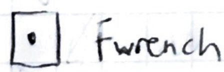

The wrench applies a torque to the nut in order to loosen it