Report on the Design of “The Oariginal”

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“The Oariginal”

Proposed Design

“The Oariginal” is based off the extrusion of a 2d curve, but has a protrusion on both the bow and stern, that make the boat more three dimensional, which will increase the boat’s speed.

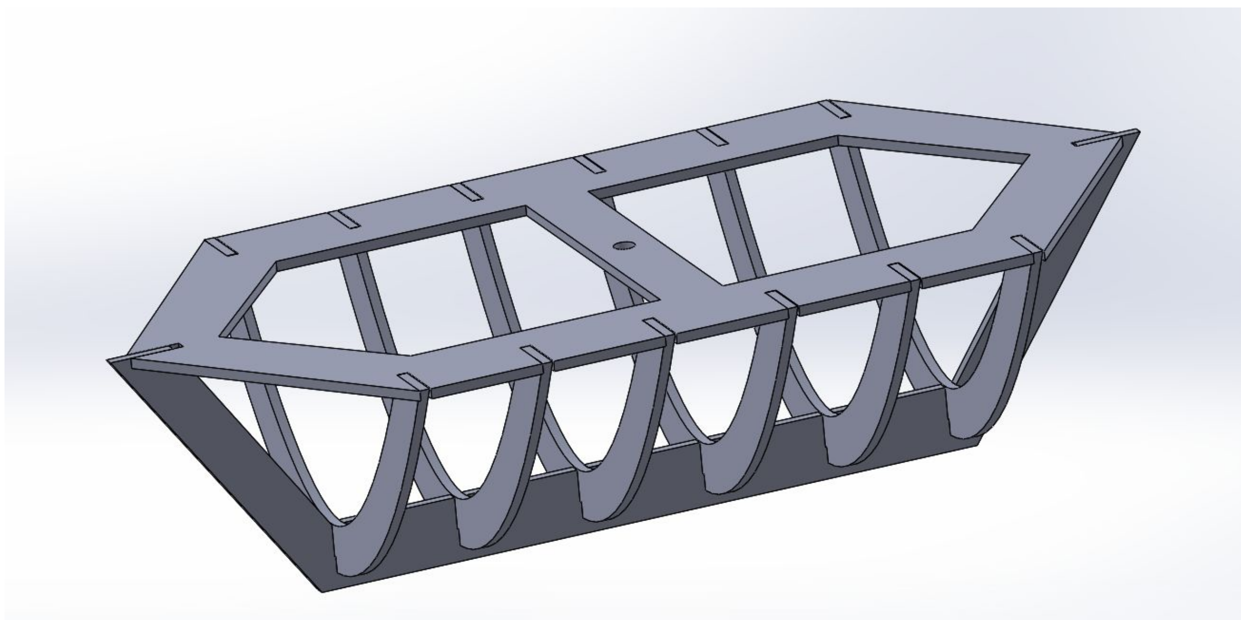


Figure 1: The final Solidworks assembly of The Oariginal

The main part of the boat’s hull is defined by the equation

$$y = \frac{1}{8}x^2$$

between $x = -8$ and $x = 8$.

This means that the boat is 16 cm across in the x direction and 8 cm tall in the y direction. The main shape of the boat is extruded by 20 cm in the z direction, however the protrusion on the bow and stern put the total z depth at 32 cm. We picked this boat shape because it makes the boat twice as wide as it is tall. That is good because it keeps the center of mass low enough that it can easily fall below the center of buoyancy once the ballast is added. We also picked this curve because it is a good compromise between having a very stable but slow flat bottomed boat, and having a fast but unstable thinner hull.

The ballast is important in keeping the center of mass of the boat below the center of buoyancy when the boat is floating flat. We used the maximum allowed amount of ballast (1000 g). The ballast is positioned in the center of the bottom of the boat, with a height of about ½ cm from the bottom of the boat. The y height of the ballast in the boat was another parameter that we adjusted as we refine the design. We originally were going to put it at around 2 cm, and refined that a little bit once we got the center of mass of just the boat from the solidworks cad model.

We first estimated the boat's mass by assuming we would use about 80% of the hardboard (we knew it was an overestimation), the maximum ballast of 1000 g, and the weight of the mast. We used that weight to determine the approximate buoyant force the boat would need to stay up, and estimated that we would want the boat floating about ⅓ submerged in the water. From those estimations, we estimated the volume of the boat. As we were picking the shape, we tried to keep the 2:1 width to height ratio in mind while selecting an equation. We came up with the ⅛ as a coefficient for the equation of the boat's hull once we determined the boat's approximate dimensions based off the ideal volume. Once we determined that the width of the boat should be around 16 cm, we calculated ⅛ as the coefficient that would give the boat a good width to height ratio.

Once we had modeled the boat in Mathematica and determined the boat's parameters, we built a model of the boat in Solidworks. We used that model to determine the exact amount of hardboard used, and added that mass to the weight of the mast and the weight of the ballast, to find a total mass of 1190 grams. Then, we used Mathematica to find the weighted average of the center of mass of the boat, the center of mass of the mast, and the center of mass of the ballast to find the total center of mass, which ended up being at 0 cm in the x direction, 3.4 cm in the y direction, and 10 cm in the z direction.

Finding a Stability Curve

Introduction

When a torque is applied to a boat to make it turn in the water, sometimes the boat will tilt back to its original orientation, and sometimes it will continue to tilt until it capsizes. It turns out that this behavior can be explained and predicted by physics The center of mass, depicted in figure 1 as a black dot, is the average of where the force of gravity acts on the boat. The center of buoyancy, depicted as a blue dot, is the average of where the force of buoyancy acts on the boat. You will notice that the center of buoyancy is at the center of the part of the boat that is underwater. That is because the buoyant force on the boat is equal to the weight of the water that the boat displaces.

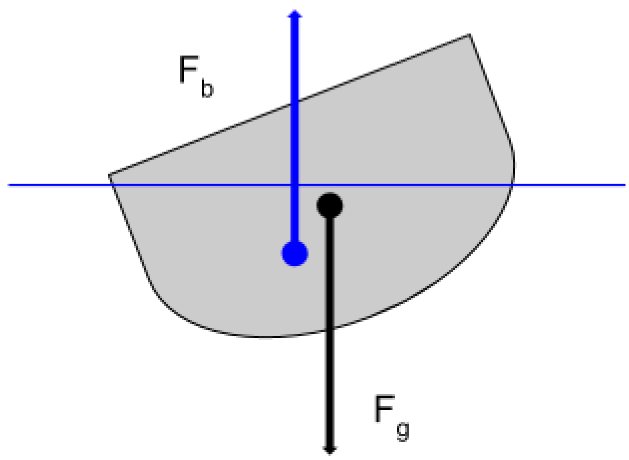


Figure 2: Free Body Diagram of a boat experiencing a righting moment

The diagram shows the boat tilted at an angle such that the center of buoyancy and center of mass are not on the same vertical line. This means that the boat must be experiencing a torque That torque is called the righting moment. The righting moment can be calculated using the center of mass and center of buoyancy of a boat.

Calculate Boat Mass and Center of Mass

The first step in determining the righting moment of the boat is to calculate the mass of the boat based off the density, and use that value, along with the mass and location of ballast, to determine the center of mass of the boat. First, since we know the density, we can use the equation

$$Mass\ of\ Boat\ = z\ distance * \int_{xmin}^{xmax} \int_{f(x)}^{ymax} Boat\ Density\ dydx$$

to find the boat's mass. In this equation, the z distance is the length of the boat in the z direction, the xmax, xmin, and ymin are the bounds of the boat, and the equation f(x) defines the curve of the boat's hull. The value of boat density that we are integrating does not include the ballast or any other mass.

When calculating the mass of The Oariginal, we used Mathematica's integrate function to set a variable called "boatmass" equal to the length of the boat in the z direction, multiplied by the integral of density over the two dimensional region of the boat as shown below.

```
boatmass = zmax1 * Integrate[density, {x, y} ∈ boat];
```

Once we have found the mass of the boat, we are able to calculate the center of mass. Since we need to know the center of mass in both the x and y directions, it is subjectively easiest to use two separate equations to calculate the center of mass. One of the equations calculates the center of mass in the x direction:

$$x\ Center\ of\ Mass\ =\ (1 \div Mass\ of\ Boat) * z\ distance * \int_{xmin}^{xmax} \int_{f(x)}^{ymax} x * Boat\ Density\ dydx$$

and the other calculates the center of mass in the y direction:

$$y\ Center\ of\ Mass\ =\ (1 \div Mass\ of\ Boat) * z\ distance * \int_{xmin}^{xmax} \int_{f(x)}^{ymax} x * Boat\ Density\ dydx$$

In Mathematica, the syntax that we used to calculate the x and y center of mass of the boat was this:

```
xcomboat = (1 / boatmass) * zmax1 * Integrate[x * density, {x, y} ∈ boat];

ycomboat = N[(1 / boatmass) * zmax1 * Integrate[y * density, {x, y} ∈ boat]];
```

Once we have determined the x and y center of mass, we will use these values, along with the coordinates and mass of the ballast to calculate the total center of mass of the boat. We will do so by multiplying each mass by the x or y coordinate of it's center of mass, and summing those values Finally, we will multiply that sum by 1 divided by the total mass. By using Mathematica, and assuming that the z center of mass is located at the center of the boat in the z direction, we found that the center of mass had (x, y, z) coordinates of (0, 3.17276, 10). In this case, it would be at 10 cm.

Calculating Center of Buoyancy

Now that we have the center of mass of the boat and ballast, we just need to calculate the center of buoyancy before we can calculate the righting moment. First, we will define the region of water in terms of draft. This means that if the boat is flat at an angle of zero, we can use the draft as the upward bound of the region in the y direction. However, if the boat is floating at an a angle θ, we will need to use the equation

$$y\ =\ Tan(\theta) * x + draft$$

as the upper bound in the y direction. In this equation, $Tan(\theta)$ acts as the slope of the line, and the draft is the y coordinate of the line when y = 0. We keep the boat constant and tilt the water in this frame of reference because it makes the calculations easier. It would be difficult to redefine the center of mass every time the boat tilted. Next, we will define the submerged area of the boat as an intersection between the water and the boat. This means that the lower bound in the y direction would be $f(x)$, the function defining the hull of the boat.

Next, we will find the mass of the water displaced by the boat. According to Archimedes' principle, the mass of the water displaced by the boat will be equal to the buoyant force, which is the value we really need. Then, we will set the buoyant force equal to to the integral

$$Buoyant\ Force = \int_{-g(draft)}^{g(draft)} \int_{f(x)}^{draft} dA$$

which defines buoyant force in terms of draft. In this equation, g(x) is equal to $f^{-1}(x)$, since we solve f(x) for draft. Now that we know buoyant force from determining the mass of the displaced water, we can plug in the buoyant force and solve for draft.

In Mathematica, our code for finding draft is shown below.

```
draft = N[d /. FindRoot[displacement == totalmass, {d, -20, 20}],
```

We set the variable for draft equal to the result of setting the displacement equal to the total mass, with the range of draft restricted between -20 and 20 cm. Using this code, for a heel angle of 30 degrees, we found a draft value of 4.3 cm.

Now that we know the draft, we now know the equation for the top bound of the underwater region of the boat, giving us all that we need to solve for center of buoyancy. The top bound for the underwater region of the boat is $y = Tan(\theta) * x + draft$ and the bottom bound is f(x) which defines the hull of the boat. Now, we can integrate over the submerged area in order to find the center of buoyancy.

By using Mathematica to integrate over the submerged area, we found a center of buoyancy of (2.2, 3.5, 10) for a heel angle of 30 degrees.

Calculating Righting Moment

Now that we know both the center of mass and the center of buoyancy, it is quite easy to find the righting moment. We just need to find the difference between the two centers, and then multiply that difference by the force of buoyancy:

$$Righting\ Moment = (Center\ of\ Buoyancy - Center\ of\ Mass) * (Force\ of\ Buoyancy)$$ [1]

In Mathematica, the equation is pretty similar, but with slightly different syntax. It takes the difference between the Center of Buoyancy and Center of Mass, and crosses the resulting vector with the force of buoyancy to get the torque.

[1] Note: the force of buoyancy will always be perpendicular to and pointing towards the waterline.

```
torque = Cross[cob - com, buoyancy][[3]];
```

For an angle of 30 degrees, we ended up with a torque of 21,618 Nxm.

Results

Diagram of boat at different Heel Angles

Key:
COM = Center of Mass (black dot)
COB = Center of Buoyancy (blue dot)
Fb = force of buoyancy (blue arrow)
Fg = force of gravity (black arrow)
Orange line: hull of boat
Blue line: waterline

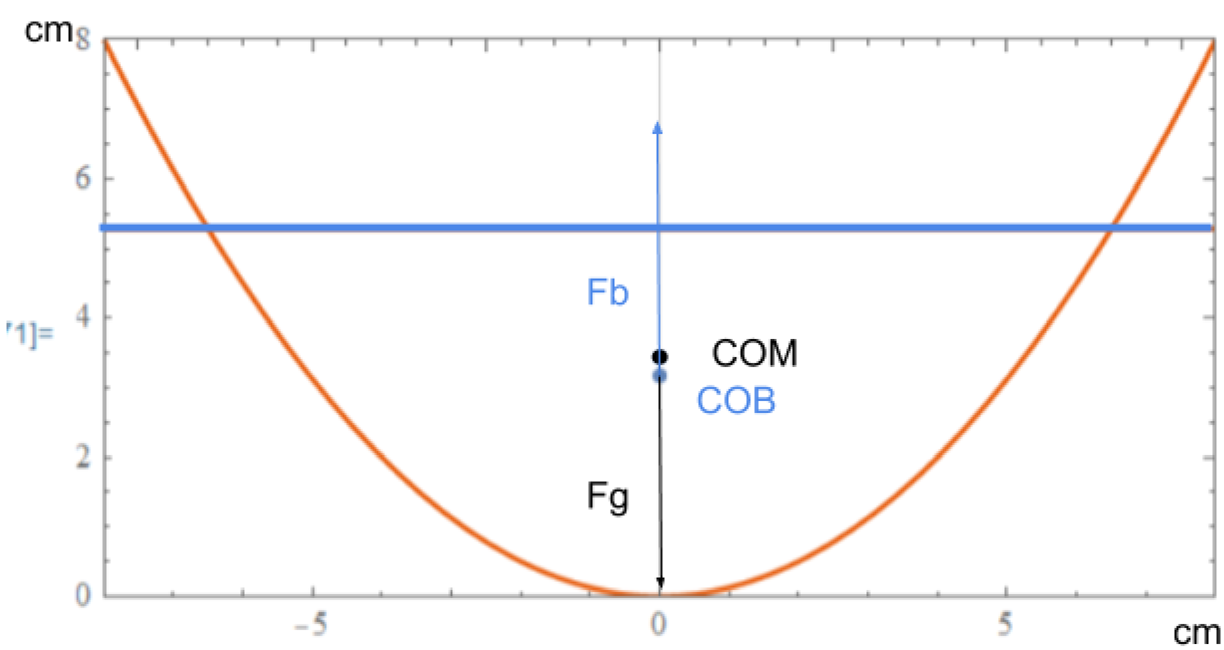


Figure 3: Plot of boat at 0 degree heel angle. The boat is in static equilibrium in this diagram, and is experiencing no torque as the forces of gravity and buoyancy on the boat are in line with each other.

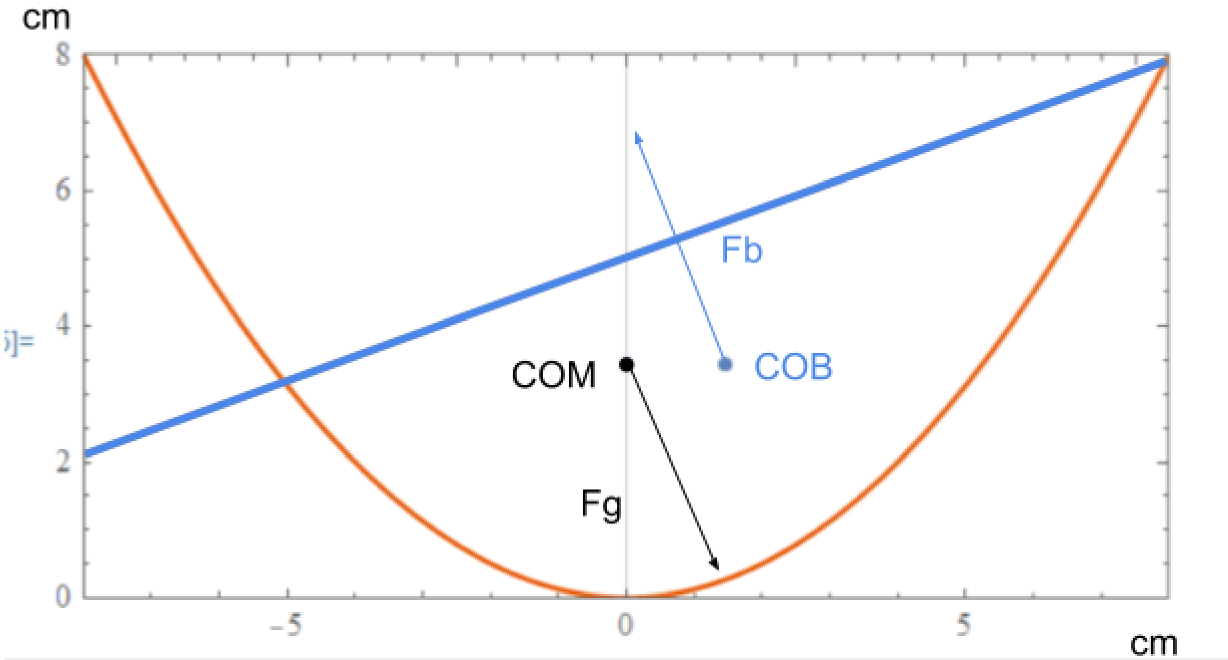


Figure 4: Plot of boat at 30 degree heel angle. The frame of reference is the boat, so the waterline appears to tilt around the boat, but in reality the boat is what is really tilting. The boat in this diagram is experiencing a righting moment, and is not in equilibrium because the forces of gravity and buoyancy are not in line with each other.

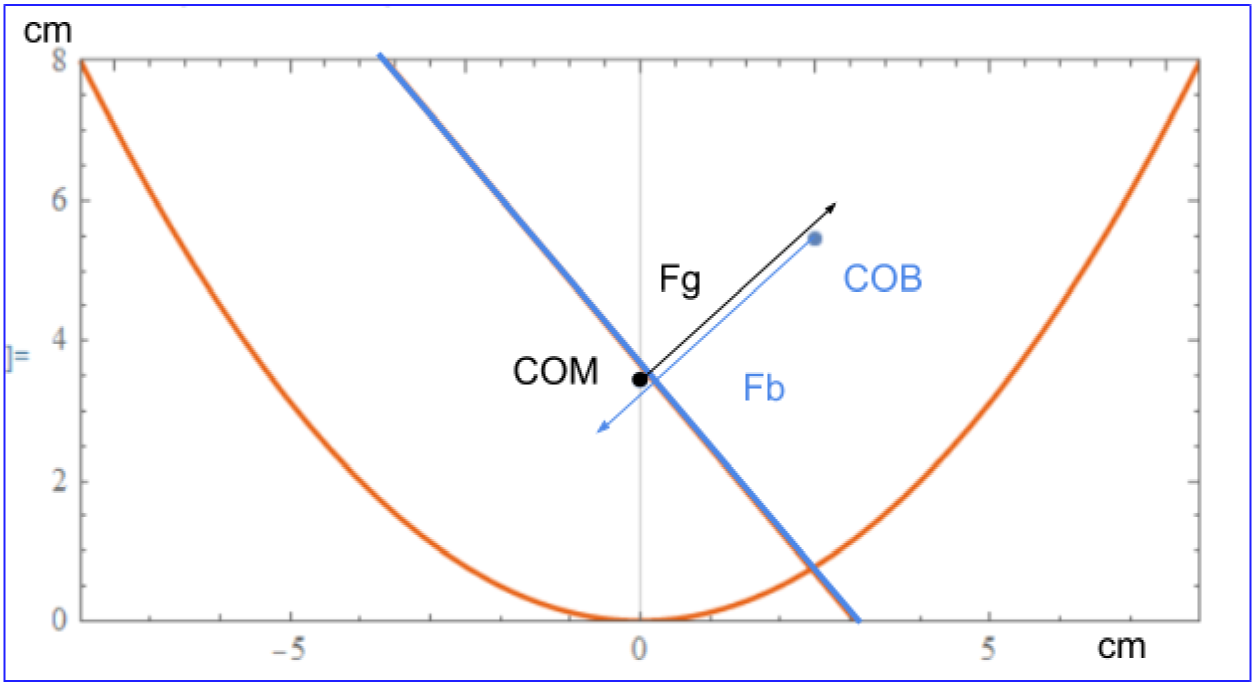


Figure 5: Plot of boat at 130 degree heel angle. In this diagram, the submerged region of the boat is represented by the area to the top right of the waterline. The forces of gravity and buoyancy are very close to each other at this angle, and the boat is experiencing a very small torque. 130 degrees is in fact very close to the angle of vanishing stability of this boat, where the torque is 0.

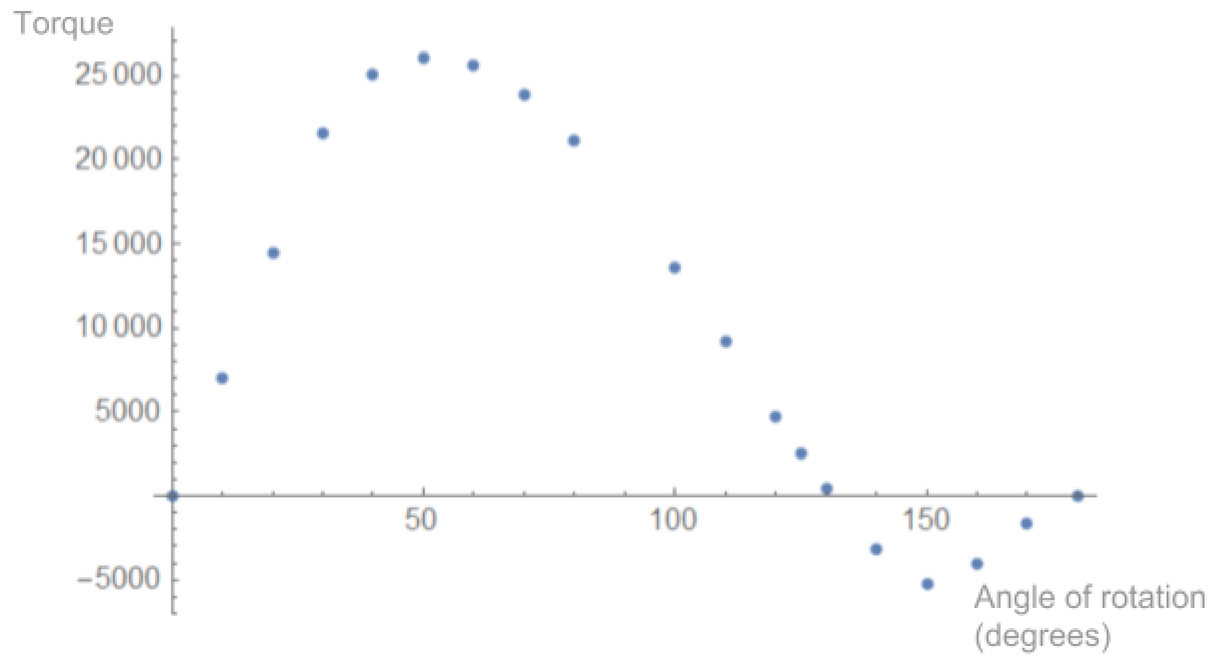


Figure 6: This graph of torque vs heel angle shows the boat's angle of vanishing stability as the point where the line crosses the x axis. For our boat, the AVS is around 130 degrees.

Expected Performance

The boat will almost certainly float, since the weight of the water displaced by the boat when fully submerged is about twice the mass of the boat. This means that it will float about halfway underwater. Our analysis shows an angle of vanishing stability of close to 130 degrees, which is where the graph of torque vs heel angle crosses the x axis. I predict that our boat will be about average in speed. It has the advantage of being a lot smaller and compact than other boats, but it is based off a 2d extrusion, so many of the 3d boats in the class will have the advantage of being more hydrodynamic per their size. I predict our boat will be about 0.9 m/s.

Final Boat Calculations

Clearing Variables

In[879]:=
```
Clear["Global`*"]
```

Defining Variables

In[880]:=
```
f = (1/8) * x^2;
density = (28/1000);
ballastmass = 1000.;
xmin1 = -8.;
xmax1 = 8.;
ymin1 = 0.;
ymax1 = 8.;
zmin1 = 0.;
zmax1 = 26.;
xballast = 0.;
yballast = (1/2);
xmast = 0.;
ymast = (51/2);
mastmass = 97.;
θ = 30.;
```

Calculating Mass and Center of Mass

In[895]:=
```
MassFunc[f_, xmin1_, xmax1_, ymin1_, ymax1_, zmin1_, zmax1_, ballastmass_,
  xballast_, yballast_, density_, xmast_, ymast_, mastmass_] := Module[{},
  boat = ImplicitRegion[f < y < ymax1 && xmin1 < x < xmax1 && ymin1 < y < ymax1, {x, y}];

  boatmass = zmax1 * Integrate[density, {x, y} ∈ boat];

  totalmass = boatmass + ballastmass;

  xcomboat = (1 / boatmass) * zmax1 * Integrate[x * density, {x, y} ∈ boat];

  ycomboat = (583 / 100);

  xcom = 1 / (totalmass) * (xcomboat * boatmass + xballast * ballastmass);

  ycom = 1 / (totalmass) * (ycomboat * boatmass + yballast * ballastmass + ymast * mastmass);

  com = {xcom, ycom, 0};

  {boat, com, totalmass}
 ]
```

In[896]:=
```
MassFunc[f, xmin1, xmax1, ymin1, ymax1, zmin1, zmax1,
 ballastmass, xballast, yballast, density, xmast, ymast, mastmass]
```

Out[896]= $\left\{\text{ImplicitRegion}\left[\frac{x^2}{8} < y < 8. \,\&\&\, -8. < x < 8. \,\&\&\, 0. < y < 8., \{x, y\}\right], \{0., 3.14057, 0\}, 1062.12\right\}$

Function/Module

In[904]:=
```
TorqueFunc[f_, density_, ballastmass_, xmin1_, xmax1_, ymin1_, ymax1_,
  zmin1_, zmax1_, xballast_, yballast_, θ_, boat_, com_] := Module[{},
  water = ImplicitRegion[If[θ < 90, y < Tan[θ Degree] * x + d, y > Tan[θ Degree] * x + d],
    {{x, -10, 10}, {y, -10, 10}}];

  submerged = RegionIntersection[boat, water];

  displacement = 1 * zmax1 * RegionMeasure[submerged];

  draft =
   N[d /. FindRoot[displacement == totalmass, {d, -20, 20}, WorkingPrecision → 10][[1]]];

  cob = Append[RegionCentroid[submerged /. {d → draft}], 0];

  buoyancy = totalmass * 98 / 10 * {-Sin[ θ Degree], Cos[θ Degree], 0};

  torque = Cross[cob - com, buoyancy][[3]];

  torque
 ]
```

Testing Function

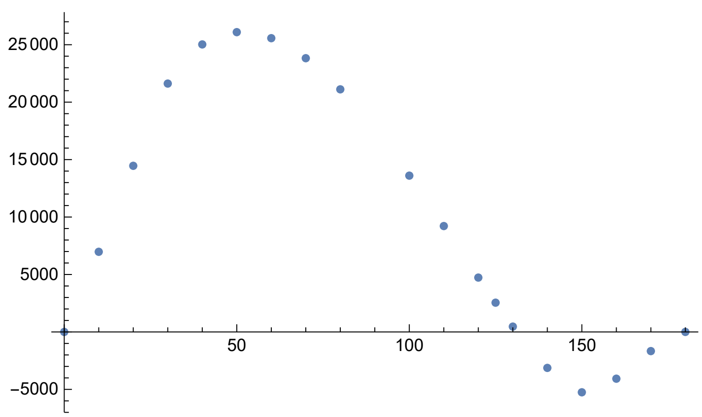
In[905]:=
```
TorqueFunc[f, density, ballastmass, xmin1, xmax1,
 ymin1, ymax1, zmin1, zmax1, xballast, yballast, θ, boat, com]
```

Out[905]= 21 618.

Plotting Results

In[899]:=
```
ListPlot[Table[{θ, TorqueFunc[f, density, ballastmass, xmin1,
    xmax1, ymin1, ymax1, zmin1, zmax1, xballast, yballast, θ, boat, com]},
  {θ, {0, 10, 20, 30, 40, 50, 60, 70, 80, 100, 110, 120, 125, 130, 140, 150, 160, 170, 180}}]]
```

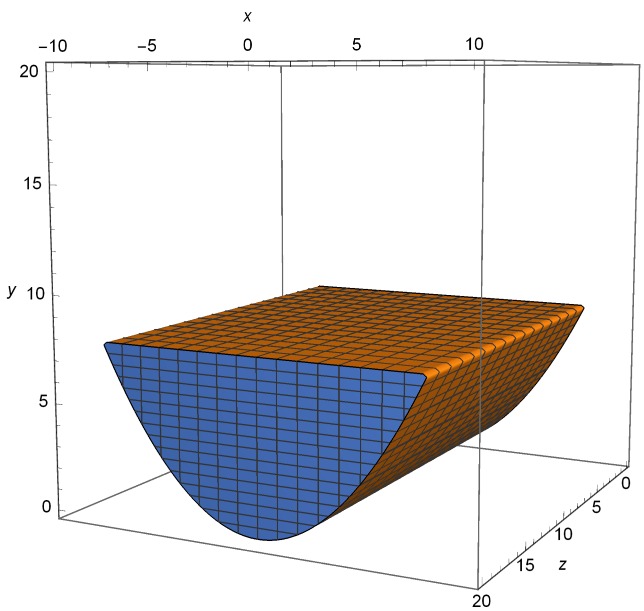
Out[899]=



Visualizing Boat Shape

In[696]:=
```
RegionPlot3D[f < y < ymax1 && xmin1 < x < xmax1 && ymin1 < y < ymax1, {x, -10, 10},
 {y, 0, 20}, {z, 0, 20}, PlotPoints → 100, Axes → True, AxesLabel → {x, y, z}]
```

Out[696]=



In[697]:=
```
plotcob = {{cob[[1]], cob[[2]]}}
plotcom = {{com[[1]], com[[2]]}}
Show[Plot[f, {x, xmin1, xmax1}, PlotRange → {{xmin1, xmax1}, {ymin1, ymax1}},
  PlotTheme → "Scientific", AspectRatio → Automatic],
 ListPlot[plotcob], ListPlot[plotcom],
 Plot[Tan[30 Degree] * x + draft, {x, xmin1, xmax1},
  PlotRange → {{xmin1, xmax1}, {ymin1, ymax1}}, PlotTheme → "Scientific"] ,
 RegionPlot[submerged /. d → draft, {x, -8, 8}, {y, 0, 8}]]
```

Out[697]= $\{\{9.40017 \times 10^{-17}, 6.32509\}\}$

Out[698]= {{0., 3.17276}}

Out[699]=