Project 8 Katie Stevens

1. $2^2 \equiv 4 \pmod{17}$ $\phi(17) = 16$
$2^4 \equiv 16 \equiv -1 \pmod{17}$
$2^8 \equiv 1 \pmod{17}$

2. $3^2 \equiv 9 \pmod{19}$ $\phi(19) = 18$
$3^3 \equiv 8 \pmod{19}$
$3^6 \equiv 64 \equiv 7 \pmod{19}$
$3^9 \equiv 56 \equiv -1 \pmod{19}$
$3^{18} \equiv 1 \pmod{19}$

3. $5^2 \equiv 6 \pmod{23}$ $\phi(23) = 22$
$5^{11} \equiv 22 \pmod{23}$
$5^{22} \equiv 1 \pmod{23}$

4. $a^{hk} \equiv 1 \pmod{n}$
If we have m such that $0 < m < hk$ and $a^m \not\equiv 1 \pmod{n}$
Then, $(a^h)^k \equiv 1 \pmod{n}$
This shows that a^h has order k.

5. If p is an odd prime divisor of n^4+1, then $n^4 \equiv -1 \bmod p$
and $n^8 \equiv 1 \bmod p$.
Euler's theorem shows that $8^{\phi(p)} \equiv 1 \bmod p$.
Therefore,
$p = 8k+1$ for some k.

6. Primitive Root of 13

$\phi(13) = 12 \rightarrow$ 4 primitive roots

$12 = 6 \cdot 2$

$1^{12/2} \equiv 1 \mod 12$

$2^{12/2} \equiv 2 \mod 12$

$2^{12/6} \equiv 8 \mod 12$

$3^{12/2} \equiv 1 \mod 12$

$4^{12/2} \equiv 1 \mod 12$

$5^{12/2} \equiv 1 \mod 12$

$6^{12/2} \equiv 11 \mod 12$

$6^{12/6} \equiv -4 \mod 12$

$7^{12/2} \equiv 5 \mod 12$

$7^{12/6} \equiv 6 \mod 12$

$8^{12/2} \equiv 1 \mod 12$

$9^{12/2} \equiv 1 \mod 12$

$10^{12/2} \equiv 1 \mod 12$

$11^{12/2} \equiv 9 \mod 12$

$11^{12/6} \equiv 8 \mod 12$

$\boxed{2, 6, 7, 11}$