DOES THE RESPONSIVE CLASSROOM APPROACH AFFECT THE USE OF STANDARDS-BASED MATHEMATICS TEACHING PRACTICES?

Results from a Randomized Controlled Trial

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ABSTRACT

This study highlights the connections between two facets of teachers' skills—those supporting teachers' mathematical instructional interactions and those underlying social interactions within the classroom. The impact of the Responsive Classroom (RC) approach and use of RC practices on the use of standards-based mathematics teaching practices was investigated in third-grade classrooms. Eighty-eight third-grade teachers from 24 elementary schools in a large suburban district were selected from a sample of teachers participating in a larger randomized-control study. Results showed that teachers at schools assigned randomly to receive training in the RC approach showed higher use of standards-based mathematics teaching practices than teachers in control schools. These findings were supported by analyses using fidelity of implementation: greater adherence to the intervention predicted the use of more standards-based mathematics teaching practices. Findings support the use of the RC approach for creating classroom social environments that facilitate standards-based mathematical practices.

STUDENTS in the United States appear to be struggling in mathematics. Fewer than 40% of fourth graders show mathematical proficiency, according to recent results from the National Assessment of Educational Progress (Aud et al., 2010). National efforts to improve mathematics achievement require careful attention to the day-to-day interactions between teachers and children dur-

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ing mathematics instruction (Pianta, Belsky, Houts, Morrison, & NICHD-ECCRN, 2007). Such efforts are critically important given the key role teachers play in producing students' math achievement gains (Sanders & Rivers, 1996; Wright, Horn, & Sanders, 1997) and because teachers in the United States show tremendous variability in the richness of the learning environments they produce (Pianta, Belsky, Vandergrift, Houts, & Morrison, 2008). At its core, teaching is "a process of creating and fostering learning environments in which students are supported in activities that have a good chance of improving learning" (Seidel & Shavelson, 2007, p. 454). Creating and fostering learning environments in mathematics classrooms call on two interrelated teaching skills. The first is facilitation of a set of mathematics instructional interactions involving selecting useful mathematical tasks, utilizing knowledge of mathematics, and facilitating mathematical discourse in the classroom. The second requires orchestration of the social interactions between teachers and students and among students that contribute to learning. Successful facilitation of these social interactions involves awareness and enhancement of students' social and emotional skills. Efforts to improve students' learning environments in mathematics need to address both components to improve mathematics teaching practices and, ultimately, contribute to students' mathematics achievement. The present study investigates the importance of both teaching skills—those pertaining to mathematical content and those required to orchestrate social interactions within the third-grade mathematics classroom.

The Common Core State Standards Initiative (CCSSI, 2010) has established a set of national mathematics teaching objectives that address mathematical content as well as the orchestration of classroom social interactions required to learn mathematics. The CCSSI standards, which are intended to guide mathematics teaching and learning, are based on past research and the recommendations of leading experts in mathematics education, including the National Council of Teachers of Mathematics (NCTM, 2000, 2007) process standards (including problem solving, reasoning and proof, communication, connections, and representations; NCTM, 2000, 2007) and elements of the National Resource Council (Kilpatrick, Swafford, & Findell, 2001) strands of mathematical proficiency (including adaptive reasoning, conceptual understanding, procedural fluency, strategic competence, and productive disposition). A careful look at these objectives shows that both mathematical content and social interactions are instilled into the CCSSI standards. For students, succeeding in mathematics requires not only learning the prescribed content, but also developing the necessary social and self-regulatory interactions that contribute to their mathematical understanding and ability to solve problems. For example, students are expected to "construct viable arguments and critique the reasoning of others" (CCSSI, 2010, p. 6) and "use appropriate tools strategically" (CCSSI, 2010, p. 7). The successful use of these practices is dependent upon the teacher's facilitation and modeling of them. Enabling teachers in their ability to teach social and emotional skills may heighten their ability to meet these standards, which would be evidenced by the use of these standards-based math practices and stronger interactions with their students.

These guidelines have been promulgated nationally by mathematics educators with the goal of improving the quality of mathematics instruction. A series of standards-based *mathematics teaching practices* (Berry, Rimm-Kaufman, Ottmar, Walkowiak, & Merritt, 2010; Borko, Stecher, Alonzo, Moncure, & McClam, 2005; Merritt, Rimm-Kaufman, Berry, Walkowiak, & McCracken, 2010; Stein, Smith, Henningsen, & Silver, 2000) emanate from these recommendations. We operationalize standards-based mathematical teaching practices in relation to eight dimensions of instruction: structure of the lesson, multiple representations, students' use of mathematical tools, cognitive depth, mathematical discourse community, explanation and justification, problem solving, and connections and applications.

Effective mathematics teaching provides students with ample opportunities to solve challenging mathematical tasks, engage in rich mathematical discourse, and use mathematical tools to enhance understanding (NCTM, 2000, 2007). Specifically, teachers using instructional practices consistent with these teaching standards offer opportunities for student inquiry, press for depth of understanding, and teach conceptual understanding of mathematical ideas (McCaffrey et al., 2001). Further, some evidence suggests that effective use of these mathematical teaching practices helps promote students' conceptual understanding of mathematical concepts (Boaler, 1998), enhances classroom mathematical interactions, and relates to improved student learning (Fuson, Carroll, & Drueck, 2000; Stein & Lane, 1996).

Effective standards-based mathematics lessons are structured to ensure that the concepts are well linked, coherent, and lead students to a better understanding of the mathematical concept. Effective teachers provide students with appropriate amounts of time to problem solve, grapple with mathematical tasks, and think critically about the processes and procedures. Effective teachers select cognitively demanding tasks (Stein et al., 2000), which contribute to the development of mathematics fluency, deepen their conceptual understanding, and require students to connect procedures to other mathematical concepts. Discourse, or how students and teachers communicate about mathematics, is central to what students learn. Effective teachers successfully facilitate learning by asking appropriate questions and soliciting student ideas to support the students' mathematical discourse and thinking (Hufferd-Ackles, Fuson, &Sherin, 2004; Sherin, 2002; Yackel, Cobb, & Wood, 1991). Students in standards-based classrooms often work in groups, communicate and debate with others, write about their thoughts, or present their ideas to the class (NCTM, 2000). Further, students are often expected to participate in productive mathematical discussions (Stein, Engle, Smith, & Hughes, 2008), explain their mathematics strategies, and justify their reasoning (Cobb et al., 1991; Franke et al., 2009). Both students and teachers in standards-based classrooms use multiple representations, such as symbols, graphs, pictures, words, charts, diagrams, and physical manipulatives, and are able to demonstrate their understanding of mathematical concepts and relationships by explaining and translating between them (Bruner, 1966; Greeno & Hall, 1997; Moyer, 2001). The instruction in effective mathematics classrooms requires students to create connections between the mathematics being taught and other concepts. Effective standards-based instruction is also relevant to the students' lives and provides them with direct applications and an understanding of why the mathematics is important (NCTM, 2000). Taken together, the use of these mathematical teaching practices provides students with greater opportunities to engage in mathematically rich instruction, builds conceptual understanding, and makes them feel supported in an engaging learning environment (NCTM, 2000).

Despite the clarity of the vision for describing effective standards-based mathematics teaching practices, the guidelines offer little direction on how to create classroom social conditions conducive to mathematics instruction (NCTM, 2007). For instance, the first standard in the Common Core Standards for Mathematical Practices (CCSSI, 2010, p. 6) states that students should "make sense of problems and persevere in solving them." To promote this standard, teachers need to teach self-regulatory skills so students show self-directed behavior as they solve problems. In addition, teachers need to provide affective support to contribute to students' sustained motivation. The third standard requires students to "construct viable arguments and critique the reasoning of others" (CCSSI, 2010, p. 6). This standard requires teachers to instruct students in the mathematical content, but also to teach students how to listen to each other, take turns in conversation, ask productive and relevant questions, and be willing to take academic risks. The fifth standard states that students need to learn to "use tools appropriately" (CCSSM, 2010), involving solving problems by manipulating concrete materials (e.g., ruler, protractor, or pattern blocks). Teachers implementing this standard need to manage behavior in their classrooms effectively so that children learn how to use the materials and behave autonomously as they take out and put away their tools. Further, teachers need to have taught self-regulatory skills so that students direct their attention to the salient features of the task.

Taken together, mathematics teachers are expected to provide strong "classroom interactions among teachers and students around content directed toward facilitating students' achievement of learning goals" (Hiebert & Grouws, 2007, p. 377). However, standards-based practices present sizable challenges for teachers (Remillard, 2000; Sherin & Drake, 2009). Standards-based mathematics guidelines not only require that teachers have the ability to teach mathematics content effectively, but also require a classroom climate that cultivates learning and provides opportunities for this type of mathematics instruction (Grossman, Schoenfeld, & Lee, 2005). Strategies to facilitate social interactions among students and manage classrooms effectively are notably absent from most mathematics curricula and state or national standards (Shuell, 1986), requiring attention to interventions outside of mathematics curricula to improve teachers' ability to create classroom conditions conducive to mathematics learning. As a result, we turn our attention to social interventions designed to provide teachers with strategies that enhance classroom interactions and promote such social and noncognitive skills and consider their contribution to the use of standards-based mathematics instructional practices.

The Responsive Classroom Approach

The present study examines the impact of a social and emotional learning intervention, the Responsive Classroom (RC) approach, on the implementation of standards-based mathematics teaching practices. The RC approach is designed to help teachers create a supportive and safe environment for learning. Findings from a quasi-experimental study of the RC approach suggest that it contributes to children's social competence as well as academic achievement (Rimm-Kaufman, Fan, Chiu, & You, 2007). Specifically, teachers' use of RC practices appears to foster prosocial skills and assertion in the classroom (Rimm-Kaufman et al., 2007), and both behaviors correspond to skills required when teachers use standards-based mathematics practices.

Further, many of the goals of the RC approach are consistent with the cognitive, social, and organizational skills needed to implement standards-based mathematics teaching practices. The RC approach offers a set of basic principles that prioritize social and academic learning, focus on the process of learning (not only the product), point to the importance of social interactions to support cognitive growth, describe a set of critical social skills for children to learn, emphasize the importance of teachers knowing the individual characteristics and the families of the children in their classroom, and encourage support among the adults within the school community (Northeast Foundation for Children [NEFC], 2011). Ten RC practices emanate from these principles, as described in RC manuals (NEFC, 2007a). For example, "morning meetings" are 20-minute class meetings involving greetings, a socially engaging activity, and news sharing, all designed to engage children in the school day, create a sense of community, produce opportunities for students to listen to one another, and teach prosocial and self-regulatory skills. *Interactive modeling* is a technique where teachers teach children to notice and internalize behaviors expected of them through stating goals, collecting ideas, modeling appropriate behaviors, asking students what they noticed, and having the class practice the behavior together. *Guided discovery* is used to introduce students to new class materials in a way that fosters responsible but also creative use of those materials.

The RC approach is widely used, with over 90,000 teachers trained in the approach. One notable component of the RC approach is its rich training infrastructure. The NEFC offers introductory trainings (RC1), intermediate trainings (RC2), and sessions for teachers who want to become trainers to sustain the RC approach in their school or district. Subsequent to each week-long training session, RC coaches meet with newly trained teachers in their school two to three times during the year, observe specific RC components, and make recommendations. The approach is described in a thorough and structured manner in manuals (NEFC, 2007a, 2007b). Further, the training is very intensive, consistent with the fact that teacher change is challenging to produce and difficult to sustain over time (Fullan, 2001). Despite its prevalence, the present study is based upon the first randomized controlled trial examining the impact of the RC approach on mathematics teaching practices.

Social and Emotional Learning Interventions and Instructional Practice

Several studies have examined the relation between social, emotional, and behavioral learning interventions and the quality of teacher-student interactions. For instance, Brown, Jones, LaRusso, and Aber (2010) examined the efficacy of the 4Rs program, a school-based intervention that embeds social and emotional learning into literacy development, on the quality of teacher-student interactions in third grade. The 4Rs program provides teachers with literacy instructional practices, while emphasizing the building of supportive relationships between students and teachers, the establishment of positive social norms, and clear classroom rules and routines. Findings showed positive intervention effects of 4Rs; teachers who received the 4R intervention provided higher-quality emotional support and instructional support than did control teachers. These intervention effects were present, even after controlling for classroom- and school-level factors, such as teacher burnout and teacher emotional ability, that may predict teacher-student interaction quality. Raver and colleagues (2008) reported findings from a randomized controlled trial that classroom behavioral intervention in preschool classrooms improved the emotional and organizational climate of the classroom. Results suggest that training in the intervention improved the emotional support, classroom organization, and quality of instruction in the classroom. Another study examining the contribution of the RC approach on classroom processes showed that random assignment to RC versus a control group did not relate to quality of teacher-student interactions; however, teachers' use of RC practices mediated the relation between assignment to the RC approach and improved classroom practices (Abry, Rimm-Kaufman, Larsen, & Brewer, in press). Results from such studies show that use of specific interventions produces change in teacher-student instructional and social interactions; however, little is known about how these interventions can alter interactions within the mathematics classroom.

Teacher change is challenging to produce, and teachers show variability in their uptake of social and emotional interventions (Century, Rudnick, & Freeman, 2010; Greenberg, 2010). Simply receiving training in the intervention may not be sufficient to produce change in classrooms. Thus it becomes important to measure fidelity of implementation, operationalized here as teachers' frequency of use of RC practices and adherence to the RC approach. Existing work suggests that implementation problems in social emotional learning (SEL) interventions can dilute intervention effects on children's social and academic outcomes (Durlak, Weissberg, Dymnicki, Taylor, & Schellinger, 2011). Similarly, examining the role of fidelity of implementation may be an important consideration in predicting the use of standards-based mathematics teaching practices. We measure the use of RC practices in both intervention and control groups because of work suggesting that actual differences in use of RC practices between intervention and control conditions may be smaller than differences expected theoretically (Hulleman & Cordray, 2009). In the intervention group, fidelity of implementation assesses variability in use of RC practices, whereas in the control group, fidelity of implementation measures the degree to which teachers in the control group may be using practices so closely resembling RC practices that they are likely to have comparable effects. Measures of fidelity of implementation reflect naturally occurring variability within the sample of teachers; thus, findings help explain patterns of association but do not permit causal inferences.

In considering the impact of the RC approach, it is important to consider teacher attributes other than fidelity that could contribute to teacher practice. In addition, classroom environment and teacher knowledge and beliefs stand out as factors that prove promising as predictors of mathematics teaching practices. These constructs do not operate in isolation; therefore, it is important to consider how these constructs operate collectively in order to understand how teachers can promote learning.

Other Teacher Attributes and School Factors

Mathematics instruction is remarkably complex, and teachers' approach to mathematics instructional interactions and practice reflects a complex set of teacher attributes, including teachers' knowledge, beliefs, and attitudes (Van der Sandt, 2007). Mathematical knowledge for teaching (MKT) refers to the subject matter knowledge and pedagogical content knowledge that is necessary for teaching mathematics (Ball, Thames, & Phelps, 2008). Higher MKT has been associated with the quality of mathematics instruction, the use of mathematical teaching practices (Charalambous, 2010; Hill, Ball, Blunk, Goffney, & Rowan, 2008; Hill, Schilling, & Ball, 2004), and student achievement (Hill, Rowan, & Ball, 2005). However, mathematical knowledge alone is unlikely to predict mathematics teaching practices. Teaching is an intensely psychological process, and teachers vary in their perception of their abilities as mathematics teachers.

Teachers' psychological processes, such as their self-efficacy for teaching mathematics, influence their ability to use standards-based mathematical teaching practices (Ross & Bruce, 2007). Mathematics teaching self-efficacy comprises two subconstructs: *personal teaching efficacy* (PMTE) and *outcome expectancy* (MTOE) (Ashton & Webb, 1982; Bandura, 1977; Enochs, Smith, & Huinker, 2000; Gibson & Dembo, 1984). PMTE refers to teachers' belief in their ability to be effective teachers and bring about student learning (Gibson & Dembo, 1984), while MTOE is defined as the belief that effective teaching can bring about student learning, despite external factors such as home environment or family background (Ashton, 1985; Gibson & Dembo, 1984). Research on self-efficacy and mathematics indicates that teachers with higher mathematics self-efficacy are more likely to use standards-based mathematics teaching practices (Brown, 2005; Haney, Czerniak & Lumpe, 1996; Riggs & Enochs, 1990; Spidek, Givvin, Salmon, & MacGyvers, 2001). Thus, teachers reporting higher self-efficacy in teaching mathematics also report that they support more student risk taking, use more inquiry-based learning, use more student-centered teaching strategies, attend to student prior knowledge, support equity, and encourage collaboration between students and teachers. In addition to teacher beliefs and knowledge, there are additional teacher attributes and school contextual factors that are likely to influence the ways that teachers approach mathematics instruction in the classroom (Cobb, McClain, Lamberg, & Dean, 2003), as described below.

Decisions about resource allocation and the policies implemented in school have implications for how teachers interact with children as a function of the resources available to them. For example, today's focus on accountability, increasing student achievement scores and classroom achievement levels, teacher experience, or meeting national annual yearly progress (AYP) requirements can create a stressful environment that is likely to make teachers feel more pressure toward improving achievement, yet less efficacious about their success in the classroom (Valli & Buese, 2007). These stresses may contribute to the types of mathematics teaching practices teachers provide. Further, identification for Title 1 funds has implications for the types of services that are present at the school to improve instruction and the student population that the school serves. As a result, it is important to account for these contextual factors when examining predictors of standards-based mathematics teaching practices.

The Present Study

The present study examines the extent to which the RC approach strengthens classroom social interactions and facilitates teachers' use of standards-based mathematics teaching practices. We hypothesized that teachers trained in RC practices and teachers using those practices with fidelity would use more standards-based mathematics teaching practices in the classroom than teachers using "business as usual" approaches to social and emotional learning and/or classroom management. In conducting these analyses, we considered other teacher and school factors that have been associated with mathematics instruction. Specifically, we focused attention on MKT, PTME, and MTOE in relation to standards-based mathematics teaching practices, even after controlling for the presence of the RC intervention. In addition, we controlled for teacher characteristics (i.e., years of teaching experience, average classroom mathematics achievement) and school attributes (i.e., AYP status, Title 1 funding) that are often linked to instructional practice.

Method

Participants

The present study is part of a larger 3-year, longitudinal cluster, randomized controlled study, the Responsive Classroom Efficacy Study (RCES), examining the impact of the RC approach on classroom instruction and student achievement in the third, fourth, and fifth grades. Twenty-four schools in a mid-Atlantic school district were enrolled into RCES because of their willingness to receive training in the RC approach and participate in a research study. Schools were matched and randomized into intervention ($n = 13$) and control ($n = 11$) schools. After randomization, groups were compared across demographic and school characteristics. Free and reduced-price lunch (FRL) and racial composition were fairly comparable across intervention (27.63% FRL, 59% racial minority) and control schools (24.53% FRL, 53% racial minority). Nine of the 24 schools (3 control, 6 RC) received Title 1 funding, and 6 of the 24 schools (1 control, 5 RC) did not meet AYP in 2008. Data for the present study were collected from third-grade teachers and classrooms in these 24 schools.

All third-grade teachers who taught mathematics during the 2008–2009 school year at the 24 schools were invited to participate in this study ($n = 100$). Out of the 100 teachers, 94 consented to participate, representing a 94% response rate. An additional six third-grade teachers were excluded from the study because they were immersion teachers and taught mathematics in a foreign language (Spanish and/or French). Teacher participants in the present study were 88 third-grade teachers from 24 schools (83 female, 5 male; 83% White, 6.8% African American, 1.1% Hispanic, and 9.1% of another ethnicity). Roughly half ($n = 43$; 49%) of the teachers taught in schools receiving the RC intervention. With regard to highest degree obtained, 30.7% had a bachelor's degree only, and 69.3% held a master's or other graduate degree. Teachers in the present sample had fewer years of teaching experience ($M = 8.87$ years, $SD = 8.10$, range = 1–35 years) compared to comparable national samples ($M =$ 14.2 years) (Livingston, 2007). T-tests indicated that highest degree obtained and years teaching experience did not differ between the control and intervention groups ($p > .05$).

Procedures

Prior to data collection (summer 2008), all third-grade teachers assigned to the RC condition completed a one-week RC1 training institute constituting 35 hours of instruction and professional development. The training was provided in groups of 20 by trained consultants from the Northeast Foundation for Children. During this training, participants learned strategies for implementing key practices of the RC approach, including morning meeting, rule creation, interactive modeling, positive teacher language, and logical consequences. In addition, during the 2008–2009 school year, teachers received one-day workshops, three consultations and classroom visits by their RC coach, as well as email and phone communication with their coaches. During the coaching visits, teachers were observed and given feedback about their use of RC practices. In addition, the coach conducted a lesson in their class, held debriefing sessions and mini-workshops, and led meetings with teachers and administrators. The 45 control teachers received no training on RC practices and continued instruction using business-as-usual approaches. The schools reported using a combination of textbooks (e.g., Everyday Mathematics, Silver Burdett, Scott Foresman) and the district testing frameworks and pacing guides as tools to direct instruction.

Data were collected from three sources: district data, online teacher-report questionnaires, and classroom observations conducted and coded by research assistants. In spring 2009, teachers responded to an online questionnaire, providing information about their demographic characteristics, teaching experience, and training, as well as their mathematics self-efficacy beliefs and use of RC teaching practices. They also completed an online assessment of their MKT in spring 2009. The questionnaire and MKT assessment required roughly 1 hour of teachers' time, and teachers received $100 for participating in the study.

During the 2008–2009 school year, research assistants observed and videotaped all third-grade teachers for three complete mathematics lessons and two morning observations (all approximately 60 minutes). Three 3-month windows of observation were established, and teachers were observed once for math and morning instruction during each observation window: fall (September to November), winter (December to February), and spring (March to May). The multiple observations in a large sample of teachers allow us to consider the dynamic, interactive aspects of mathematics teaching practices, rather than examining instruction specific to a classroom context or curriculum. Upon the completion of videotaping, tapes were sent to the laboratory for off-site observational coding by coders blind to the assignment of the teachers.

Measures

Mathematics Scan. Mathematics teaching practices were assessed using the Mathematics Scan (M-Scan; Berry et al., 2010). The M-Scan is an observational measure designed to assess the use of standards-based mathematics teaching practices in classroom instruction. This measure was developed based on past research in mathematics education, the NCTM standards, observational work by Borko et al. (2005), and the Classroom Assessment Scoring System (CLASS) (Pianta, LaParo, & Hamre, 2008). The M-Scan measure includes eight dimensions of standards-based mathematics teaching practices, each coded on a Likert scale of 1–7 representing low (1, 2), medium (3, 4, 5), and high (6, 7) use of practices. The eight dimensions are structure of the lesson, multiple representations, students' use of mathematical tools, cognitive depth, mathematical discourse community, explanation and justification, problem solving, and connections and applications. A multidimensional coding guide provides keyword descriptors and anchors for each dimension, characterizing low, medium, and high use of standards-based mathematics teaching practices (see App. A, table A1, for a sample dimension from the coding guide).

The development process (initiated in 2007) and validity work conducted on the M-Scan are described elsewhere. To summarize briefly, the M-Scan instrument was validated using several methods (Walkowiak, Berry, Meyer, McCracken, & Rimm-

Kaufman, 2010), indicating the usefulness of the measure for assessing standards-based mathematics teaching practices. First, analyses by content experts in mathematics and mathematics education confirmed that the M-Scan dimensions and indicators represent important components of mathematics instruction. Next, an analysis of the response processes of coders indicated that 87.7% of the coders' rationales for assigning scores were aligned with the coding descriptors of the M-Scan. Third, patterns of convergence and divergence were found with existing observational measures of mathematics instruction, including the construct of inquiry-based instruction as measured by the Reformed Teaching Observation Protocol (RTOP) (Sawada et al., 2002), instructional support as measured by the CLASS (Pianta, LaParo, & Hamre, 2008), and Inside the Classroom (Horizon Research, 2000; Weiss, Pasley, Smith, Banilower, & Heck, 2003).

Each mathematics observation (n = 264) was coded for the full mathematics lesson by trained and reliable coders. To achieve reliability, coders attended an in-depth training where they learned about the development of the M-Scan measure, the coding protocol, and important elements of standards-based mathematics instruction (NCTM, 2000, 2007) and the state mathematics standards. In addition, coders watched several video clips that exemplified different levels of use for each of the M-Scan dimensions and coded eight 1-hour classroom observations. Coders were required to meet 80% exact match before their codes were used as data.

Coders were blind to the intervention status of the teacher, and no coder watched more than two lessons taught by the same teacher. Coders watched the first 30 minutes of the tape, taking anecdotal notes throughout the 30-minute segment, to record what occurred during the lesson. These notes were used to provide examples and references when completing the M-Scan coding. After 30 minutes, coders stopped to reflect on what they saw and made "soft codes" to rate the first part of the lesson. Next, coders watched the remainder of the tape, following the same procedures as the first 30-minute segment. Upon completion of the whole lesson, coders assigned codes from 1 to 7 for each dimension on the M-Scan measures.

Exploratory and confirmatory factor analyses (EFA and CFA) were conducted to determine the structural validity of the M-Scan. Both EFA and CFA indicated that all eight dimensions of the M-Scan loaded onto one latent construct (mathematics teaching practices). Based on these findings, a composite mean score of overall use of mathematics teaching practices was created for each teacher. First, a mean score for each dimension was calculated by averaging the scores for each dimension over the 3 sampled days. Next, these composite means for each of the eight dimensions were averaged, resulting in an average M-Scan value for each teacher. Potential values of average M-Scan scores could range from 1 (low implementation) to 7 (high implementation). Measures of internal consistency (alphas) ranged from 0.90 to 0.93.

Fidelity of implementation to RC. The fidelity of implementation (FOI) latent variable score comprised three indicators: an observed measure of fidelity to RC practices (C-POM) and two self-report measures of FOI (perceived adherence to RC practices and a frequency measure of the use of RC practices).

The Classroom Practices Observational Measure (C-POM) (Abry, Brewer, Nathanson, Sawyer, & Rimm-Kaufman, 2010) assessed 16 classroom practices that are supported by RC, including routines, rules and procedures, and the organization of physical classroom space, on a three-point rating scale (1 = not at all characteristic, 2 = moderately characteristic, 3 = very characteristic). Each item was written using language (void of RC) to ensure that the measure tapped teachers' use of specific classroom practices in both the control and intervention conditions. Research assistants coded C-POM live while observing five 60-minute observations per teacher during the year. The full 16-item measure was administered during two morning observations, while an abbreviated 10-item version (which excluded items pertaining to morning RC practices such as morning meeting) was administered during the three mathematics observations. A final C-POM composite was created by averaging the scores of all items over all time points (Cronbach's alpha = .85). To establish interreliability, the research assistants exceeded 80% exact agreement for all coders on a minimum of eight 60-minute segments and were evaluated on an ongoing basis through monthly calibration meetings.

Teachers were also asked to complete the online Classroom Practices Teacher Survey (CPTS; Nathanson, Sawyer, & Rimm-Kaufman, 2007) during the last 8 weeks of school. This self-report measure of FOI asked teachers to reflect on their perceived adherence to RC practices (46 items, Cronbach's alpha = 0.90) and report on their frequency of use of RC practices (11 items, Cronbach's alpha = 0.84). Composite scores for each of the two constructs were created by averaging the items.

To create the FOI variable used in these analyses, we conducted a confirmatory factor analysis (CFA) using a Bayesian estimator and then created a final factor score. The factor loadings were .83, .73, and .78 for the observed fidelity measure, self-report adherence fidelity measure, and self-report frequency fidelity measure, respectively. The posterior predictive *p*-value was .18, indicating good model fit (Muthén & Asparouhov, 2011).

Mathematics Teacher Efficacy Beliefs Inventory. To measure teachers' self-efficacy about teaching mathematics, teachers completed the abbreviated version of the 21-item Mathematics Teacher Efficacy Beliefs Inventory (MTEBI; Enochs et al., 2000). Results of a CFA showed that the inventory measured two constructs of mathematics teaching efficacy: PMTE and MTOE. PMTE (13 items) reflects teachers' beliefs about their ability to teach effectively (e.g., "Even if I try very hard, I will not teach mathematics as well as I will most subjects"). MTOE (8 items) tapped teachers' belief that effective teaching could have a positive effect on student learning (e.g., "When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort"). For this study, the response scale ranged from "strongly disagree" (score of 1) to "strongly agree" (score of 4). MTEBI composite scores ranging from 1 to 4 were created for PTME and MTOE by averaging the teachers' responses across the 13 and 8 items. The internal consistency reliabilities, using Cronbach's alpha, for PMTE and MTOE constructs were 0.88 and 0.78, respectively, consistent with past studies using the MTEBI (Enochs et al., 2000; Utley, Moseley, & Bryant, 2005).

Mathematical Knowledge for Teaching Assessment (MKTA). The MKTA (Hill et al., 2004) consisted of an online 13-item assessment related to number and operations (K–6). The MKT is a multiple-choice assessment requiring teachers to evaluate unusual solution methods, represent mathematical content, and identify adequate mathematical explanations. The MKT scores are reported using item response theory (IRT) scales for the number and operations items, ranging from −2.0 to 2.0. Each teacher's score represents the number of standard deviation units he or she scored from the larger national sample mean. Reliability values were consistent with other samples (Hill et al., 2004), with Cronbach's alphas ranging from 0.71 to 0.84.

Average classroom achievement. To control for the average baseline achievement level of each classroom, the children's second-grade achievement scores were aggregated to create a measure of average classroom achievement. Second-grade mathematics achievement was measured using the Stanford Achievement Test, 10th edition (Stanford-10; Harcourt Educational Measurement, 2002). The assessment was administered to all children in participating third-grade classrooms ($n = 2{,}028$) in April of their second grade. A standardized score was generated for each child by the manufacturer (range 1–999). We aggregated the students' scores by teacher ID to create an average classroom achievement variable.

Teacher demographic measure. Teachers provided information about their age, ethnicity, years of teaching experience, and educational training. These variables were used to describe the sample of teachers. Teaching experience was also entered as a teacher-level covariate in both models.

Approach to Analysis

First, descriptive statistics and correlations were calculated to determine means and variability for each variable and relations between each construct. Next, the unconditional model (Model 1) tested whether a two-level hierarchical linear modeling (HLM) model or ordinary least-squares (OLS) regression were more suitable. The intraclass correlation coefficient (ICC = 0.13), coupled with the nesting of teachers in schools, led us to use two-level HLM (Raudenbusch, Bryk, Cheong, Congdon, & du Toit, 2004) to analyze our outcomes. Model 2 tested the impact of the RC approach on the use of standards-based practices and included teacher-level variables (MKT, PMTE, MTOE, teacher experience, and average classroom achievement) and school-level variables (school assignment [RC vs. control], AYP status, and Title 1 status). Model 3 included the FOI variable. All level 1 variables were grand-mean centered, while all three level 2 (school) predictors were uncentered.

Model 3 in HLM form has two linear regression equations, one at the teacher level and one at the school level. The teacher level has the following form:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(\mathrm{MKT}_{1ij}) + \beta_{2j}(\mathrm{PMTE}_{2ij}) + \beta_{3j}(\mathrm{MTOE}_{3ij}) + \beta_{4j}(\text{teacher experience}_{4ij}) + \beta_{5j}(\text{average classroom achievement}_{5ij}) + \beta_{6j}(\mathrm{FOI}_{6ij}) + e_{ij}.$$

The school level formula has the following form:

$$Y_{ij} = \gamma_{00} + \gamma_{01}(\mathrm{AYP}_j) + \gamma_{02}(\text{Title } 1_j) + \gamma_{02}(\mathrm{RC}_j) + \delta_{ij},$$

where i is teachers 1 through n, and j is schools 1 through n.

Results

Means and standard deviations for all variables are presented in table 1. On average, use of standards-based mathematics teaching practices was moderate ($M = 3.29$, $SD = 0.89$). Correlations between pairs of teacher-level predictors revealed three statistically significant correlations: Use of standards-based mathematics teaching practices was related positively to scores on MKT ($r = 0.27$), PMTE ($r = 0.27$), and FOI

Table 1. Descriptive Statistics and Correlations for All Variables

Variable	1	2	3	4	5	6	7	8	9	10
1. Mathematics teaching practices	–									
2. Mathematical knowledge for teaching	.27*	–								
3. Personal mathematics teaching efficacy	.27*	.13	–							
4. Math teaching outcome expectancy	.10	.17	.12	–						
5. Average classroom math achievement	.11	.03	.21	.13	–					
6. Years teaching experience	.01	−.14	.15	.11	.10	–				
7. Fidelity of implementation of RC practices	.45**	.00	.22	−.30	−.07	−.12	–			
8. Title 1 (0 = no, 1 = yes)	.24*	−.08	.04	−.32**	−.44**	.06	.26**	–		
9. School met AYP (0 = yes, 1 = no)	.00	.00	−.17	−.06	−.28**	.10	.22**	.36**	–	
10. Intervention (1) vs. control (0)	.15	−.10	−.12	−.19	−.16	−.13	.59**	.23*	.37**	–
Mean	3.29	−.13	3.40	2.68	580.20	8.87	−.04	.40	.22	.49
Standard deviation	.89	.73	.37	.34	28.21	8.10	.99	.49	.41	.50
Minimum	1.29	−1.64	2.69	1.50	532.20	1.00	−1.94	.00	.00	.00
Maximum	5.58	1.64	4.00	3.38	652.50	35.00	2.08	1.00	1.00	1.00
N	88.00	81.00	83.00	83.00	87.00	61.00	79.00	24.00[a]	24.00[a]	24.00[a]

[a] Refers to school sample size.
* $p < .05$.
** $p < .01$.

($r = 0.45$). MTOE was not correlated to use of standards-based mathematics teaching practices or PMTE ($p > .05$).

Table 2 displays the results of the three two-level HLM models. The unconditional model (Model 1) predicting mathematics teaching practices had an ICC of 0.13, indicating that 13% of the total variance was at the school level and that most (87%) of the variability in teacher practice was at the teacher level. Model 2 accounted for 26% of the teacher-level variance and 76% of the school-level variance in mathematics teaching practices. Model 3 accounted for 33% of the teacher-level variance and 85% of the school-level variance in mathematics teaching practices.

Model 2: Impact of the RC Approach

Results for Model 2 reflect a significant effect at the school level (level 2) ($B = 0.39$, $p < .05$), suggesting that the RC approach positively impacted the use of standards-based mathematics teaching practice in schools. On average, teachers in the intervention group used more standards-based mathematics teaching practices (roughly 4/5 of a standard deviation) than comparison schools. Second, MKT, PMTE, and MTOE were all positively related to the use of standards-based mathematics teaching practices (all p-values $< .05$). In addition, teachers in classrooms with higher-achieving students and teachers in Title 1 schools used more standards-based mathematics teaching practices ($B = 0.75$, p-values $< .05$).

Table 2. Results from Two-Level HLM Model Examining the Impact of the RC Approach and the Contribution of MKT, Teacher Self-Efficacy, FOI, and School Contextual Factors on Mathematics Teaching Practices

	Mathematics Teaching Practices (M-Scan)								
	Model 1 Unconditional Model			Model 2 RC Effects			Model 3 Fidelity of RC		
	B	*SE*	*β*	*B*	*SE*	*β*	*B*	*SE*	*β*
Intercept	3.26	.12	10.51	3.26	.12	10.51	3.26	.12	10.51
Level 1 variables: Teacher level:									
MKT				.35**	.17	.31**	.34**	.14	.30**
Personal mathematics teaching efficacy				.35**	.21	.16**	.13	.21	.08
Math teaching outcome expectancy				.46**	.27	.19**	.23	.24	.12
Years teaching experience				.00	.01	.04	.01	.02	.1
Average classroom math achievement				.01**	.00	.25**	.00	.00	−.05
Fidelity of implementation of RC practices							.37**	.10	.44**
Level 2: School level:									
AYP status (0 = no, 1 = yes)				−.29	.21	−.61	−.40**	.20	−.83**
Title 1 funding (0 = no, 1 = yes)				.75**	.19	1.62**	.35**	.15	.80**
RC intervention (1) control (0)				.39**	.18	.83**	.02	.20	.06
Variance components:									
ICC			.13						
Teacher-level variance explained (%)						.26			.33
School-level variance explained (%)						.76			.85

Note.—$N = 88$ at teacher level and $N = 24$ at school level. Average cluster size = 3.67.
* $p < .05$.
** $p < .01$.

AYP and teacher experience did not predict the use of standards-based mathematics teaching practices (p-values $> .05$).

Model 3: Fidelity of Implementation of the RC Approach

Model 3 showed a main effect of fidelity to RC practices on the use of standards-based mathematics teaching practices, where teachers who implemented more RC-like practices in their teaching were observed using more standards-based mathematical teaching practices ($B = 0.37$, $p < .05$) in their mathematics instruction. MKT was positively related to mathematics teaching practice: Teachers who demonstrated an advantage in MKT also were observed using more standards-based mathematics teaching practices in their instruction. After accounting for FOI, neither measure of mathematics teacher self-efficacy nor average classroom achievement predicted effective mathematics teaching practice (p-values $> .05$). Further, working in a school that did not meet AYP status related to lower use of standards-based mathematical teaching practices; however, teachers who worked in schools receiving Title 1 funding were more likely to use more standards-based mathematical teaching practices.

Discussion

Several findings emerge in relation to examining the extent to which the RC approach facilitates teachers' use of standards-based mathematics teaching practices.

First, we find that teachers randomly assigned to the intervention group, and thus trained in the RC approach, use more standards-based mathematics teaching practices than teachers in the control group. In the context of this finding, we also identify the salience of teachers' MKT, PMTE, and MTOE in predicting more standards-based mathematical teaching practices. Second, results show that teachers showing higher fidelity of implementation of the RC approach (in both intervention and control groups) use more standards-based mathematics teaching practices. When we consider individual differences in teachers' FOI, MKT appears to be linked to use of standards-based mathematics practices, but teacher beliefs (PMTE and MTOE) do not appear to be associated. Third, several classroom- and school-level contextual characteristics relate to higher use of standards-based mathematical teaching practices, findings that can be interpreted in relation to district characteristics. For instance, in both analyses, availability of Title 1 funding related to increased use of standards-based mathematics teaching practice. In examining the impact of random assignment to the RC versus control conditions, classrooms with higher average classroom mathematics achievement link to higher use of standards-based mathematics teaching practices.

Current conditions in elementary mathematics classrooms in the United States demand that researchers examine the complex factors that predict the use of standards-based mathematical teaching practices. The present study casts light on the connections between two facets of teachers' skills—those supporting teachers' mathematical instructional interactions and those underlying social interactions within the classroom. Inferences drawn are strengthened by the use of a randomized controlled trial design, permitting causal inference of the RC approach on the use of standards-based mathematics teaching practices, as well as the multimethod approach to data collection utilizing teacher report, direct assessment, and classroom observational data. Findings suggest the complementarity between the RC approach and teachers' use of standards-based mathematics teaching practices.

The RC Approach and Mathematics Teaching Practices

Results from this randomized controlled trial of the RC approach demonstrate the impact of the RC approach on the use of standards-based mathematics teaching practices. More specifically, teachers in schools who received the RC intervention were observed using more standards-based mathematics teaching practices in their teaching than teachers in the control group (corresponding to a 0.39 *SD* difference). Findings fit with past research suggesting that school-based social and emotional interventions can have positive effects on teacher practice (Brown et al., 2010; Raver et al., 2008). Although the RC approach is a capacity-building intervention, not designed to improve mathematics instruction specifically, it appears to affect instructional practice in the subject domain of mathematics. The fidelity-of-implementation findings suggest that teachers who implement more RC practices in their classroom also use more standards-based mathematics teaching practices; for every 1 *SD* increase in FOI, teachers were observed using 0.37 *SD* more standards-based mathematics teaching practices in their instruction. The FOI measures assessed teachers' use of RC practices, using both teacher report and observational methods. For instance, measures tapped teacher practices such as engaging children in classroom rule making, offering children opportunities to make individual choices in their coursework, and using reminders and redirection to decrease misbehavior (as opposed to punishments such as loss of recess that are unrelated to the misbehavior). Use of such practices may set the stage for standards-based mathematics teaching practices. Alternatively, the qualities of teachers that contribute to their implementation of RC practices may also lead to a propensity toward standards-based practices. The fidelity-of-implementation findings do not permit causal inference; however, they do help give a clearer picture into actual practices within the classroom that link RC practices and standards-based mathematics teaching practices.

One possible explanation for these results could be that the use of RC practices help support the teaching of self-regulatory skills. Many children do not come to school with the appropriate social and self-regulatory skills necessary for academic success (Rimm-Kaufman, 2004). RC practices, such as rule creation and interactive modeling, provide students with concrete expectations and examples of how to regulate their behavior and interact with each other and the curriculum. As one example, interactive modeling involves teaching children to notice and internalize expected behaviors through a series of steps in relation to a desired behavior, including direct statement of the behavior, gathering student ideas about the behavior, teacher or student demonstration of the behavior, gathering student ideas about what they noticed, student demonstration of the behavior, and, finally, student practice of the behavior (NEFC, 2007a). During mathematics class, teachers may use interactive modeling to teach students how to engage with one another pertaining to a mathematics game. Students may acquire skills such as waiting for their turn, helping each other if a peer makes a mathematical mistake, and using the materials properly. Thus the interactive modeling practice provides explicit instruction in self-control and turn taking, which are necessary for successful student-centered mathematics instruction, as promoted by the national standards (CCSSI, 2010; NCTM, 2000). Thus, explicitly teaching students these learning-related self-regulatory skills could reduce several challenges that teachers face when using standards-based mathematics teaching practices in their instruction.

A second plausible explanation could be that the use of RC practices helps build important student social skills (e.g., listening, speaking, communicating) and relationships between students, which are necessary for creating a mathematical discourse community. Many of the NCTM and Common Core standards assume that students are able to successfully interact and communicate with others about mathematical ideas. However, without a supportive and caring classroom environment, students may not feel comfortable sharing their ideas. Emphasizing student caring and teacher-student relationships, as they are enhanced by RC, creates an environment that promotes the use of mathematical discourse. One RC practice, positive teacher language, may be especially important for establishing a mathematically caring community where students are comfortable sharing their ideas. Through promoting a sense of community and mathematical caring relations (Hackenberg, 2010) in the classroom, students may show enhanced engagement, participation, and participation in mathematics (Borman & Overman, 2004; Hughes & Kwok, 2007). As a result, students may be more likely to take risks, communicate their ideas with others, and actively explain and justify their thinking. When students have stronger social skills and are better able to communicate with others, there are more opportunities for peer interaction and classroom discourse to occur (Yackel et al., 1991).

Perhaps teachers in more social classrooms are more likely to facilitate the lesson, allowing students to guide the discussions and engage in more collaborative discourse. In contrast, teachers who have students without adequate social and noncognitive skills may teach in a more traditional style, limiting classroom discourse and relying on more basic procedures and instructional techniques.

A third explanation could be that the RC approach provides teachers with the strategies, tools, and resources that support classroom organization and behavior management. In turn, teachers may experience fewer barriers in their efforts to enact many of the mathematics teaching practices that they are asked to administer in the classroom (i.e., cooperative learning, problem solving, and use of mathematical tools). While we cannot point to one-to-one correspondence between RC practices and actual mathematical instructional interactions, it is plausible that training in the RC approach expands teachers' teaching skills and competencies and/or helps teachers create a classroom environment more conducive to standards-based practices. Teachers trained in RC are taught to establish an organized learning environment that is responsive to students' needs, sets clear guidelines for how students should behave, and has high expectations for all students. When classrooms are better managed, teachers may be more capable of providing supportive learning environments that emphasize collaborative learning, discourse, and the use of instructional materials (such as manipulatives). CCSSM and the NCTM standards suggest that students should use mathematical tools and representations, and translate back and forth between the representation and the mathematical idea. However, without previously teaching students proper use of these tools, teachers may struggle to effectively incorporate the use of mathematical tools and representations at a high level. For example, teachers often report spending too much time passing out materials, dealing with behavioral issues, or preventing students from playing with the materials inappropriately, leaving little time to successfully complete the task and emphasize the translation of the tools to the mathematical content (McLeod, Fisher, & Hoover, 2003). RC practices such as guided discovery and interactive modeling may provide teachers with the necessary organizational skills to better prepare students to use these tools appropriately, leaving more time to focus on mathematics content. In addition, providing teachers with clear classroom-management strategies gives teachers more time to focus on more engaging mathematics instruction. In turn, when students are more engaged and better able to work cooperatively, teachers may be more likely to provide high-quality learning opportunities to their students. When the students are socially prepared to engage in high-quality tasks and mathematical discourse, teachers can shift their focus from directing instruction and managing behavior to facilitating high-quality standards-based learning (Stein, Grover, & Henningsen, 1996).

The Role of Teacher Attributes

To assume that training in the RC approach alone facilitates the use of standards-based mathematics teaching practices would be overly simplistic. This finding also builds upon existing research in mathematics education in that teacher attributes such as MKT, PMTE, and MTOE play an important role in predicting mathematics teaching practices aligned with standards-based instruction (Hill et al., 2004; Spidek et al., 2001; Sztajn, 2003). One way that teacher beliefs and knowledge may strengthen teacher practice is through the planning and selection of high-quality mathematical tasks for students. Teachers are responsible for engaging students by selecting cognitively demanding mathematical tasks that deepen understanding and contribute to the development of mathematical proficiency. In addition to selecting the task, teachers must also design and organize the lesson to be conceptually coherent. A teacher with higher MKT and efficacy may be more likely to assign well-chosen tasks to students, facilitate open-ended mathematical discussion, and use more modalities to represent ideas (Charalambous, 2010). However, without adequate knowledge or efficacy, a teacher may be more likely to assign problems that encourage only one strategy to solve each problem, or give worksheets with exercises for which they are practicing an already learned procedure.

It is worth noting that the findings may reflect a bidirectional relationship between teacher practice and efficacy, as described by Haney, Lumpe, and Czerniak (2002). Not only may teachers who feel more efficacious about their teaching implement more standards-based mathematics teaching practices in their instruction, but also teachers who use more standards-based mathematics teaching practices may feel more efficacious about their ability to teach students mathematics, despite their backgrounds. Further research investigating the direction(s) of these complex pathways and the extent to which mathematics teachers' beliefs and behaviors reflect actual mathematics achievement growth in elementary school classrooms will lend insight to the development of future interventions.

Other School Factors

Other school factors, such as Title 1 funding, were related to improved mathematics teaching practices. It is possible that Title 1 schools allocated resources in such a way as to improve mathematics teaching practices (e.g., providing mathematics specialists, coaches, resources, and professional development). Not achieving AYP goals may create pressure and nervousness within the school that may lead to reliance on more traditional mathematical practices to raise student achievement (Hamilton, Stetcher, & Berends, 2005). Both findings suggest the importance of considering school conditions that create a context for student learning.

Limitations and Directions for Further Inquiry

Several limitations warrant mention. First, we sampled use of standards-based mathematics teaching practices on 3 days during the school year. The decision to sample 3 days was based on resource limitations and trade-offs associated with such intensive data-collection efforts, as well as guidance based upon other observational work in elementary classrooms (Pianta, LaParo, & Hamre, 2008; Stuhlman, Hamre, Downer, & Pianta, 2009). However, future mixed-methods work may be necessary to assess the implications of the sampling decision. Second, due to design limitations, we cannot make causal inferences about the relations between mathematical knowledge for teaching, personal mathematics teaching efficacy, mathematics teacher outcome expectancy, and mathematics teaching practices. Third, although this study finds a positive relationship between fidelity of implementation of RC and mathematics teaching practices, it does not allow us to determine which components of RC (e.g., morning meeting, rules and logical consequences, academic choice) are responsible for the increased use of standards-based mathematical classroom practices.

Future research is required to examine mediation, moderation, and the extent to which these social and instructional constructs predict student achievement, both directly and indirectly. Focusing on the mechanisms that explain the relations between school and teacher attributes and classroom practice will provide insights to direct future practice and policy. In addition, future work should investigate the role of noncognitive skills, which set the stage for successful teaching and learning. Given the disparities between current levels of achievement and national mathematics benchmarks, teachers do not only need to be knowledgeable and efficacious about teaching mathematics content and pedagogy, but must also have the necessary supports to deal with the more social issues that affect everyday mathematics teaching. Identifying and describing the social and mathematical challenges that teachers face in relation to using standards-based mathematics instruction represents an important step for improving instruction. This study supports the complementary nature of both the social and instructional skills required to teach mathematics effectively. While more traditional approaches to professional development emphasize improving teachers' knowledge of mathematics content and pedagogy, focusing on social interventions may also cultivate growth in teacher practice and support widespread efforts to improve high-quality mathematics instruction for all students. In addition to ensuring that pre-service and in-service teachers develop the adequate pedagogical content knowledge necessary for teaching, training and professional development programs may also choose to provide ample opportunities for teachers to improve their day-to-day classroom practices and interactions with children.

Appendix A

Table A1. Sample Coding Guide

Mathematical Discourse Community: The extent to which the classroom social norms foster a sense of community in which students feel free to express their mathematical ideas honestly and openly. The extent to which the teacher and students "talk mathematics," and students are expected to communicate their mathematical thinking clearly to their peers and teacher, both orally and in writing, using the language of mathematics. NOTE: There is a "high bar" on this dimension because there is an expectation for students to have an active role in promoting discourse; this should not be only the teacher's role. This is in contrast to Explanation/Justification. The rating does take into account whether discourse focuses on mathematics content but not the cognitive depth of that content.

	Low (1, 2)	Medium (3, 4, 5)	High (6, 7)
Teacher's use of discourse	The majority of math discussion in the classroom is directed from the teacher to the students.	Some of the math discussion in the classroom includes student participation, but some is teacher-initiated.	Throughout the math discussion in the classroom, students consistently participate.
	Teachers are wholly responsible for the content of math discussions.	Teachers are partially responsible for the content of math discussions.	Students play a substantive role in directing the content of math discussions.
Sense of mathematics community through student talk	When students talk, they rarely share mathematical thinking[a] and language. Student-to-student talk rarely or never occurs.	When students talk, they sometimes share mathematical thinking[a] and language. Student-to-student talk sometimes occurs.	When students talk, they often share mathematical thinking[a] and language. Student-to-student talk frequently occurs.
Questions	All of the teacher's questions have known/correct answers.	Most of the teacher's questions have known/correct answers, but some are focused on mathematical thinking.[a]	Some of the teacher's questions have known/correct answers, but many are focused on mathematical thinking.[a]

Source.—From Berry et al. (2010).
[a] Mathematical thinking = processes, strategies, and/or solutions.

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