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Review

Learning executive function and early mathematics: Directions of causal relations

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ABSTRACT

Although there has been much recent attention to young children's development of executive function and early mathematics, few studies have integrated the two. Here we review the evidence regarding executive function and mathematic achievement in the early years. After defining the executive function processes we consider, we briefly address the question of whether executive function can be taught in schools. We then turn to the relations between executive function and achievement. We begin with a review of the larger literature on correlations between the two, both concurrent and predictive. This leads to the fewer but more directly educationally-relevant causal studies. We conclude that developing both executive function processes and mathematical proficiencies is essential for young children and suggest that high-quality mathematics education may have the dual benefit of teaching this important content area and developing executive function processes.

Contents

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1. Introduction

To learn and to solve problems, people need resources. One type of resource allows them to control, supervise, or regulate their own thinking, and behavior. Such *executive function* (EF) processes develop most rapidly in the early childhood years. Others are domain-specific resources, such as mathematics proficiencies (National Research Council, 2001). In this paper, we address the interrelations of these two as they contribute to young children's learning and development in mathematics. We begin with brief illustrations of why each is important.

Cognitive processes such as EF appear prima facie to be connected to students' achievement in school. Children need to plan ahead, focus attention, and remember past experiences. According to some, EF processes constitute "a major characteristic of productive mathematics learning" (De Corte, Mason, Depaepe, & Verschaffel, 2011, p. 155). Such EF processes support children's learning across subject matter areas, but may be particularly important to mathematics. As one example, when the initial reading of an arithmetic problem is not the correct one, children need to *inhibit* the first impulse to answer (incorrectly) and carefully examine the problem. Consider the following problem, "There were six birds in a tree. Three birds already flew away. How many birds were there from the start?" Children have to inhibit the immediate desire to subtract engendered by the phrase "flew away" and instead calculate the sum (through addition, counting on, other other strategies). Over the last 100 years, the demand for the application of such EF processes as inhibition has increased in mathematics education (Baker et al., 2010). Together, these processes allow children to complete tasks even when facing difficulties in problem solving and/or learning, fatigue, distraction, or decreased motivation (Blair & Razza, 2007; Neuenschwander, Röthlisberger, Cimeli, & Roebers, 2012). It is thus unsurprising that Kindergarten teachers say that such EF processes (albeit not by that name) are as important as academics (Bassok, Latham, & Rorem, 2016). Most teachers rate EF components such as inhibition and attention shifting, as important for math thinking and learning, and these ratings increase with teaching experience (Gilmore & Cragg, 2014). Thus, on argument is that EF development is a prerequisite for learning mathematics.

Of course, domain-specific resources, such as mathematical proficiencies, must be developed for children to progress in acquiring mathematical knowledge and problem-solving competencies (Passolunghi & Lanfranchi, 2012; Sarama & Clements, 2009). Mathematics proficiencies include five intertwined strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning (capacity for logical thought, reflection, explanation, and justification), and productive disposition (National Research Council, 2001). Early mathematical proficiency has been identified as the best predictor of later knowledge of mathematics achievement (Koponen, Salmi, Eklund, & Aro, 2013; Passolunghi, Vercelloni, & Schadee, 2007). The mathematics that children know when they enter kindergarten and first grade predicts their mathematics achievement for years to come and throughout their school career (National Mathematics Advisory Panel, 2008). What they know in math even predicts their reading achievement—better than early literacy skills (Duncan et al., 2007; Duncan & Magnuson, 2011; Koponen et al., 2013). Mathematics, including the strands of strategic competence and adaptive reasoning (Nunes, Bryant, Barros, & Sylva, 2012; Piaget, 1970), appears to be a core component of cognition (Clements & Sarama, 2011). This suggests that high-quality mathematics education experiences may simultaneously develop mathematical proficiencies and at least some EF processes.

In this paper, we examine the literature on the directionality of the relation between EF and math to better understand how the development of both proficiencies may be supported. First, we briefly examine EF, defining EF processes and addressing the question of whether EF can be taught so as to ascertain the malleability of EF (which is critical to our questions of trainability and directionality of any effects of training). The next two sections turn to relations between executive function and achievement, reviewing correlational and causal studies, respectively. We address several questions. Are EF and achievement reliably related? Is EF a prerequisite to learning mathematics and thus must be developed or taught first? Or, does thinking and learning about mathematics help develop EF? Finally, we conclude that developing both executive function processes and mathematical proficiencies is essential for young children and suggest that high-quality mathematics education may have the dual benefit of teaching this important content area and developing EF processes.

Because we were interested in a conceptual analysis of these domains, we conducted a narrative review with systematic search procedures (see the online Supplement for detailed information). This review differs from and extends previous discussions (Bull & Lee, 2014; De Corte et al., 2011) because it is (a) focused on the early years, a time the development of EF is considered rapid, critical, and foundational; and (b) focused on the *directionality* of the correlational *and* causal relations between EF and math (we also report relations involving literacy, because the comparison to math illuminates these relationships).

2. Executive function

Researchers and other educators have used the term EF to refer to the processes involved in intentionally controlling ones' impulses, attention, thinking, and behavior. Although the field lacks a common set of processes and definitions, the broad concept of EF can be viewed as a unity functionally, but has also been analyzed into several processes (Best, Miller, & Naglieri, 2011; Huizinga, Dolan, & van der Molen, 2006; Lehto, Juujarvi, Kooistra, & Pulkkinen, 2003; Miyake, Friedman, Emerson, Witzki, & Howerter, 2000; Raver, 2013; Schoemaker, Bunte, Espy, Deković, & Matthys, 2014). Three processes are frequently distinguished.

First, *attention shifting and cognitive flexibility* involves switching a "mental set" from one aspect of a situation to another as the situation requires. A simple example in mathematics is counting by different units (e.g., feet and inches, to find a total length). Cognitive flexibility is similarly involved in avoiding "functional fixedness"; for example, the tendency to see represented objects only in terms of their canonical function. An example in mathematics of the lack of cognitive flexibility is repeating the same solution strategy even after it has failed.

Second, *inhibitory control* involves suppressing unproductive responses or strategies, such as controlling a proponent response (e.g., the first solution or answer that occurs to you, as in the "six birds in a tree" example) to think about better strategies or ideas. Ignoring visually salient extraneous information in a mathematics word problem is another example.

Third, *working memory* involves a system that is responsible for the short-term holding and processing of information. The EF process is often identified with an emphasis on *updating* working memory as new information is processed; that is, maintaining, manipulating, and adding relevant information often while engaging in another cognitively demanding task. Students solving a measurement problem may have to keep the problem situation and their solution in mind while they perform a necessary computation, interpret the result of the computation in terms of the measurement units, and then apply that to the problem context to solve the problem.

Not all studies have identified the same EF factors, for example, one validated separate latent factors for working memory and attention shifting, but no coherent latent factor for inhibition

(Huizinga et al., 2006.) However, all three of these EF processes probably affect motivation, persistence, and a positive disposition toward learning (Clements & Sarama, 2015; Vitiello, Greenfield, Munis, & George, 2011).

A second critical component of this review is whether EF is malleable. If not, the main topic of this review, how can educators help children to learn both EF and mathematics competencies, is arguably moot. If EF can be improved through interventions, how? In this section, we consider interventions that target EF skills. The rapid development of EF in the early years has prompted for a call for interventions during that period although no age is too late (Zelazo & Carlson, 2012). Studies have shown enhancement of such capabilities with at least three categories of interventions, computer games, direct training of specific EF tasks, and particular curricula or programs.

The first category is the use of computer games. General reviews report that findings are limited and mixed (Otero, Barker, & Naglieri, 2014; Razza & Raymond, 2015). Here we examine recent studies on using computer games to increase EF in young children. In one study, both 4- and 6-year-old children showed increases in executive attention immediately after five days of training on specific exercises (Rueda, Rothbart, McCandliss, Saccomanno, & Posner, 2008). For example, the first exercise involved tracking a cat and moving through a maze. Anticipation exercises involved anticipating the movement of a duck across a pond by moving the cat to where the duck should emerge. Stimulus discrimination exercises consisted of remembering and selecting a multi-attribute item. In conflict resolution exercises, children had to move their joystick to pick out the larger of two arrays, which was made up of smaller digits. The researchers concluded that the executive attention network develops under strong genetic control, but it can be improved by educational interventions.

In another study of computer games, pre-K children received training of either visual-spatial working memory or inhibition for five weeks (The Spatial Reasoning Study Group, 2015; Thorell, Lindqvist, Nutley, Bohlin, & Klingberg, 2009). To train working memory, a number of visual stimuli were presented sequentially on a computer screen and children had to remember both their location and order. Three games were used to train inhibition, including go/no-go tasks in which the child was told to respond when a certain stimulus was presented, but to make no response when another stimulus was presented. Children trained on working memory improved on trained tasks and non-trained tests of spatial and verbal working memory, as well as transfer effects to attention. Training on inhibition only improved performance on the trained tasks (Thorell et al., 2009). Similar computer games lead to an increase in some, but not all, EF processes for 6-year-olds (Goldin et al., 2014). Finally, 20 or more days of training on computer activities increased working memory scores of elementary-aged children diagnosed with ADHD and also improved their response inhibition (Klingberg et al., 2005; Klingberg, Forssberg, & Westerberg, 2002). In summary, closely targeted computer games appear to increase at least some EF processes although results are not consistent and the broader and long-term benefits are unknown.

The second, relate, category is direct training of specific EF assessment tasks not involving technology. For example, providing 4-year-olds with feedback and reflection training on the conflicting rule representations involved in a card-sorting task in which children have to change the attribute for sorting (Dimensional Change Card Sort or DCCS) led to substantial improvements on near transfer tasks and some indications of transfer (the omnibus task was not significant, but paired-sample *t*-tests conducted anyway showed improvement for one of two groups in one of two studies). The training occurred after any mistake—the child was asked to name the attribute, given an example of a correct sort, and then asked to resort with assistance (Espinet, Anderson, & Zelazo, 2012). Other researchers found positive effects with interventions as simple as asking 3-year-olds to repeat such change-of-criteria categorization tasks with feedback (Dowsett & Livesey, 2000). A small group intervention that attempted to teach EF directly, adapting common assessments such as dimensional card sorting, Stroop tasks, and trail-making tasks, promoted gains in working memory and cognitive flexibility for pre-Kers and in interference control for kindergartners (Röthlisberger, Neuenschwander, Cimelia, Michel, & Roebers, 2012). In summary, direct training is possible, although generalization of the purposely focused skills is uncertain.

The third category is the enhancement of EF capabilities by using particular curricula or early childhood programs, which also has had some small successes (Bierman, Domitrovich et al., 2008; Bierman, Nix, Greenberg, Blair, & Domitrovich, 2008; Diamond, Barnett, Thomas, & Munro, 2007; Diamond & Lee, 2011; Klingberg, 2009; Lillard & Else-Quest, 2007; Lyons & Zelazo, 2011; Otero et al., 2014; Perels, Merget-Kullmann, Wende, Schmitz, & Buchbinder, 2009; Raver et al., 2011; Riggs, Greenberg, Kusché, & Pentz, 2006; Schmitt, McClelland, Tominey, & Acock, 2015; Weiland, Ulvestad, Sachs, & Yoshikawa, 2013; Weiland & Yoshikawa, 2013). For example, specific teaching approaches have been shown to increase target EF processes, such as four different interventions involving guiding impulsive children to self-monitor their behavior by talking to themselves (Reid, Trout, & Schartz, 2005). Certain classroom interactional patterns are associated with gains in EF, including affective and cognitive classroom processes, such as a teacher being more approving, less disapproving, and using a more positive emotional tone (Fuhs, Farran, & Nesbitt, 2013). Two interventions emphasizing mindfulness appeared to raise EF functioning in young children, especially those initially low in these processes. A yoga intervention was found beneficial to 3- to 5-year-olds, especially those low in EF at the start (Razza, Bergen-Cico, & Raymond, 2015; Shapiro et al., 2014). A school-based program of mindful awareness practices (meditation followed by activities for sensory awareness, attentional regulation, and awareness of other people and the environment) similarly raised parent and teacher ratings of EF processes in second and third graders, with stronger effects for those with EF difficulties (Flook et al., 2010).

There is much yet to be understood with respect to curricular interventions for EF. For example, some early studies indicated that the *Tools of the Mind* program (Bodrova & Leong, 2007) develops EF (Barnett et al., 2008; Diamond et al., 2007). This is based on Vygotskian theory that mature, intentional dramatic play is the primary social context allowing children to practice self-regulatory behaviors (Vygotsky, 1934/1986). The use of specific pedagogical strategies that optimize such play have been claimed to improve young children's EF processes and academic achievement (Bodrova & Leong, 2005). However, four separate randomized cluster trial evaluations of the *Tools* program or the part of *Tools* targeting EF showed no effects on EF, even when implemented with good fidelity (Clements, Sarama, Layzer, Unlu, & Germeroth, 2012; Farran, Lipsey, & Wilson, 2011; Lonigan & Phillips, 2012; Morris et al., 2014). Small effects were found when *Tools* was implemented in kindergarten classrooms when EF activities were embedded into literacy, mathematics, and science learning activities (Blair & Raver, 2014). Researchers need to resolve these divergent findings; for example, they may find that *Tools* is measurably effective (on these instruments) only with intensive and extensive supports for implementation or that it is the embedding (more than the hypothesized guided play) that contributes to gains in EF processes. Even though there seems to be a correlation between them (Vieillevoye & Nader-Grosbois, 2008), there is little or no evidence that pretend play is crucial to building executive function or other processes (Lillard et al., 2013). We return to this program and these issues in a succeeding section about EF and mathematics. Similarly, another study compared two interventions, (a) training teachers to provide educational environments that were sensitive to the needs of children with working memory difficulties and (b) direct instruction, to each other and to a control condition (Elliott, Gathercole, Alloway, Holmes, & Kirkwood, 2010). Neither intervention resulted in greater gains in working memory or academic proficiencies although classroom observations suggested that children gained more if their teachers (in any condition) implemented desirable strategies (Elliott et al., 2010).

Despite these discouraging results, it may be that even though there are no significant effects for the entire population, there may be benefits for children who begin school with low executive function processes. This was the finding of a study that used whole-class games, such as a modification of "Red Light, Green Light," in which the teacher acted as a stoplight and switched the meaning of the signals, with green meaning stop, mid-game (Tominey & McClelland, 2011).

In summary, research and development has identified a handful of successful strategies for developing EF in young children, including some computer games, direct training efforts, and curricular approaches. This indicates that EF can be improved although the training may need to be narrowly focused and the expectation of results limited. That is, effects are not consistent, are often small, and often lack transfer. Further, these studies did not evaluate the long-term persistence of these effects. Finally, although gains on executive function may be important outcomes per se, our argument is that both EF processes and subject-matter proficiencies are required to support optimal learning and problem solving. Therefore, in the remainder of this article, we focus on the possible relations between them. First, are they reliably related? Second, does learning one support learning of the other?

3. Correlational relations between executive function and mathematics

The link between various manifestations of EF and academic achievement is well documented for older students (Bielaczyc, Pirolli, & Brown, 1995; Zimmerman, 2002). Until recently, less was known about how EF developed in the early years contributes to the later development of EF (National Research Council and Institute of Medicine, 2000) and academic proficiencies.

EF theoretically affects children's ability to successfully function in school settings in at least two ways. First, EF may allow children to use and further develop cognitive processes necessary for academic learning (Anghel, 2010). From this perspective, EF processes are either prerequisites or cognitive supports for learning and reasoning about mathematics. For example, competence in updating one's working memory might be considered a sine qua non for learning mathematical processes (Berg, 2008; Geary, Hoard, & Nugent, 2012; Kleemans, Segers, & Verhoeven, 2011; Monette, Bigras, & Guay, 2011; Szűcs, Devine, Soltesz, Nobes, & Gabriel, 2014). Consider the use of counting back to solve a subtraction problem. One must keep the goal (find the difference) in mind, as well as the part-whole relation, then keep track of how many 'counts' one makes (e.g., 8–2, counting back to 7 is 1 count, counting back again to 6 is 2 counts—stop and report "six"). Similarly, updating processes may support learning such strategic competence (as well as the conceptual and fluency strands) as connections between these strands, between the problem context, the mathematical structure, and the problem-solving strategies, all must be established and retained (Sarama & Clements, 2009; Siegler & Jenkins, 1989; van der Ven, 2011).

Second, EF may assist children in conforming to classroom rules and in benefiting from learning in various social contexts (e.g., in large and small groups, in cooperative dyads, individually, etc.). Evidence suggests that EF promotes or inhibits children's interactions with others and thus affects learning directly and indirectly through a child's social skills and problem behaviors (Montroy, Bowles, Skibbe, & Foster, 2014).

3.1. Concurrent correlations

Consistent with these theoretical mechanisms, several studies have shown positive correlations between EF and achievement in young children (Alloway & Alloway, 2010; Alloway, Alloway, & Wootan, 2014; Bierman, Nix et al., 2008; Blair, Protzko, & Ursache, 2011; Blair & Razza, 2007; Bull, Espy, Wiebe, Sheffield, & Nelson, 2011; Bull, Johnston, & Roy, 1999; Cameron et al., 2012; Cerda, Im, & Hughes, 2014; Cortina et al., 2013; Gathercole & Pickering, 2000; Gawrilow et al., 2014; Kilday, 2010; McClelland et al., 2014; Ng, Tamis-LeMonda, Yoshikawa, & Sze, 2014; Roebers, Cimeli, Röthlisberger, & Neuenschwander, 2012; Welsh, Nix, Blair, Bierman, & Nelson, 2010), although there are exceptions (Edens & Potter, 2013). Pre-Kers' EF, including inhibitory control and attention-shifting, are related to measures of mathematics and literacy ability in kindergarten (controlling for general intelligence, Blair & Razza, 2007). Although EF was predictive of later mathematics for all ages, it was not predictive once processing speed was taken into account for 3-year-olds but did predict above and beyond processing speed for older children (4–5 years), once it stabilized (Clark et al., 2014).

In one study pre-Kers with higher behavioral EF, including attention, working memory, and inhibitory control (mainly the last) achieved at higher levels in literacy, language, and mathematics (controlling for age and gender, McClelland et al., 2007). However, the relation between behavioral regulation measured in the fall of the pre-K year and spring pre-K achievement was not significant after fall achievement was entered (McClelland et al., 2007). In a study of Kindergartners with low entering number sense, children's initial attention and EF processes predicted later math knowledge (Hassinger-Das, Jordan, Glutting, Irwin, & Dyson, 2014). Finally, impaired EF beyond impaired IQ, predicted mathematics and attention problems for preterm children (Aarnoudse-Moens, Weisglas-Kuperus, Duivenvoorden, van Goudoever, & Oosterlaan, 2013).

Measures of EF have been associated with both mathematics and reading achievement (Alloway & Alloway, 2010; Best et al., 2011; Blair et al., 2011; Blair & Razza, 2007; Neuenschwander et al., 2012; Weatherholt, Harris, Burns, & Clement, 2006), as well as science achievement (Nayfeld, Fuccillo, & Greenfield, 2013). For example, EF processes correlated with both mathematics and reading achievement and similarly across a wide age span (5 to 17 years), suggesting they are general cognitive processes that significantly support academic learning (Best et al., 2011; Neuenschwander et al., 2012). Similarly, measures of inhibitory control and attention-shifting were related to measures of both mathematics and literacy ability in kindergarten (controlling for general intelligence, Blair & Razza, 2007). Of struggling first graders in an effective reading intervention, those retained in the grade showed significantly weaker EF skills (Dombek & Connor, 2012).

However, more studies show that EF is more highly associated with mathematics than literacy or language (Blair et al., 2011; Blair, Ursache, Greenberg, Vernon-Feagans, & The Family Life Project Investigators, 2015; Fuhs, Farran, & Nesbitt, 2015; McClelland et al., 2014; Monette et al., 2011; Sasser, Bierman, & Heinrichs, 2014; von Suchodoletz & Gunzenhauser, 2013), or with mathematics but only with the early learning of literacy, rather than higher-level skills of English literature (Gathercole, Pickering, Knight, & Stegmann, 2004). For example, Chilean pre-Kers' initial EF processes were statistically significant predictors of higher emergent mathematics skills, but not of emergent literacy skills at the end of pre-K (Barata, 2010). Kindergartners' mathematics achievement is more highly associated with some measures of EF than literacy and language achievement is (Ponitz, McClelland, Matthews, & Morrison, 2009).

Given EF's apparent importance to mathematics achievement relative to other subjects, the question arises of whether different EF processes relate to mathematics to the same degree. Some research indicates that most EF processes correlate significantly with mathematics achievement (Bull & Scerif, 2001). One theoretical model (Szűcs et al., 2014) places EF processes that control the workflow of working memory activities at the core of the model of mathematical ability (see also Berg, 2008; Geary et al., 2012; Kleemans et al., 2011; Monette et al., 2011). Others suggest a greater or lesser role for particular EF processes. For example, inhibitory control appeared to play a particularly important role in mathematics (Blair & Razza, 2007; Jenks, van Lieshout, & de Moor, 2012; Ng et al., 2014, recall the "six birds in a tree" example). One study showed that performance on an Approximate Number System (ANS) comparison task was no longer a significant predictor of mathematics achievement after inhibition skills had been accounted for (Gilmore et al., 2013). This is striking, given that ANS has been postulated to be a key early foundation (and thus predictor) of later mathematics achievement (Halberda & Feigenson, 2008).

Other researchers emphasize that updating working memory predicts performance on complex and unfamiliar tasks (Davidse, De Jong, & Bus, 2015; Geary, 2011; Jenks et al., 2012; Neuenschwander et al., 2012). Similarly, updating working memory was found to be a key EF process in mathematics learning in a Netherlands study (van der Ven, Kroesbergen, Boom, & Leseman, 2012). Both updating working memory and short-term visual memory of pre-Kers predicted 3rd grade mathematics achievement in U.S. students (Bull, Espy, & Wiebe, 2008) and were lower in 9-year-olds having difficulty with arithmetic (McLean & Hitch, 1999). Similarly, short-term visual memory predicted number identification, quantity discrimination, and missing number achievement in entering kindergartners (Decker, 2011).

In a related vein, several studies have identified lack of both of these two processes – inhibition and working memory – as specific deficits for children of lower mathematical ability, resulting in their having difficulty switching and evaluating new strategies for dealing with a particular task (Bull & Scerif, 2001; Lan, Legare, Ponitz, Li, & Morrison, 2011, found similar results). Working with primary school students, one inhibition and three working memory tasks predicted success in mathematics (Toll, van der Ven, Kroesbergen, & Van Luit, 2010). The working memory tasks also predicted math learning disabilities, even over and above the predictive value of earlier mathematical abilities (Toll et al., 2010). Thus, an argument can be made that the two components of EF, updating of working memory and inhibitory control, play particularly important roles in the learning of mathematics for young children.

EF processes may also be differentially associated with distinct mathematical topics. Perhaps unsurprisingly, given the task demands, complex EF tasks correlated more with solving word problems than with calculation (Best et al., 2011). Complex EF processes appear to play a role in acquisition of novel procedures and the development of automatic access to arithmetic facts (LeFevre et al., 2013; Rosenberg-Lee, Barth, & Menon, 2011). Updating predicted both patterning and number skills, with analyses suggesting that better number and arithmetic skills are unlikely to support learning of patterning without increases in updating processes (Lee et al., 2012). Primary-school children with poor updating abilities may have difficulty making connections that support learning and using more advanced strategy (van der Ven, 2011). Patterning was related to cognitive flexibility in first graders (Bock et al., 2015). As children following a developmental progression from counting to arithmetic operations, the initial strong association between short-term memory and counting weaken and planning becomes more strongly associated (Kilday, 2010). Finally, different aspects of working memory may be related to different mathematical areas. In one study, the visual—spatial sketchpad predicted magnitude judgments and number writing but updating (central executive) tasks explained variance in addition (Simmons, Willis, & Adams, 2012). Both working memory in general and its individual components (verbal memory and visual-spatial memory) were related to arithmetic calculation, even controlling for processing speed and short-term memory in 3–6th graders (Berg, 2008). Another study of 3rd and 5th graders who have arithmetic difficulties suggested that it is active, not passive, working memory that matters (Passolunghi & Cornoldi, 2008). That is arithmetic relates to the manipulation of stored information. The importance of active working memory was also found in a study of 1st graders (Passolunghi, Cargnelutti, & Pastore, 2014). Both calculation estimation and calculation with renaming appeared to require high working memory resources that varied. Horizontally presented addition problems required more verbal working memory resources, whereas vertically presented addition problems required visuospatial working memory, especially for estimation tasks. These resources may be particularly important when renaming is involved (Caviola, Mammarella, Cornoldi, & Lucangeli, 2012).

Finally, others agree that factors such as working memory and processing speed are important, but also remind the field that domain-specific proficiencies, such as numerical competence are critical contributors to subsequent math achievement (Passolunghi & Lanfranchi, 2012). Even fine motor/spatial skills predicted mathematics and literacy achievement above and beyond measures of EF (Cameron et al., 2012). Similarly, a combination of EF and spatial skills of pre-Kers predicted 70% of the variance of later mathematics performance, with the spatial skills uniquely predicting 27% of the variance in mathematics competence (Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014). Thus, both quantitative and spatial competence (The Spatial Reasoning Study Group, 2015) may form, along with EF, important foundations for later mathematics learning. A caveat to such comparisons, and a caution to interpreting the results of all these studies, is that measurement of EF processes are not as well-developed or tested psychometrically as measures of mathematics proficiencies, which may affect and perhaps attenuate correlations.

3.2. *Predictive relations: EF to math*

Although significant correlations may suggest causal connection in which EF processes support learning of academic content (Best et al., 2011; Fuhs et al., 2015; George & Greenfield, 2005), some argue that longitudinal correlations make a stronger case. Remembering that despite their suggestive longitudinal nature, these studies *are* correlational, not causal, we examine such research for further clues as to relations between EF competencies and later mathematics achievement. For example, some claim that EF predicts (and thus may contribute to) later academic achievement rather than academic learning predicting later EF learning (Best et al., 2011; Clark, Pritchard, & Woodward, 2010; LeFevre et al., 2013). For example, EF processes of entering pre-Kers predicted end-of-the-year literacy skills and (somewhat) mediated the effect of the pre-K program (Bierman, Nix et al., 2008). In another study, EF at age 3 predicted mathematics performance in kindergarten (controlling for earlier informal numeracy, socioeconomic status, language and processing speed, Clark et al., 2010). Measures of attention were more predictive of later achievement than behavioral measures through the elementary grades (Grimm, Steele, Mashburn, Burchinal, & Pianta, 2010). In another study, pre-K EF was not predictive, but growth in EF skills did predict 1st grade math (and reading) achievement (Davidse et al., 2015). Strikingly, pre-K attention span-persistence skills predicted mathematics achievement at age 21 (controlling for achievement at age 7 and a variety of other variables, McClelland, Acock, Piccinin, Rhea, & Stallings, 2013). However, another study found that measures of attention in early childhood were not predictive of 5th grade achievement or socio-emotional development (Sabol & Pianta, 2012). It may be that only comorbidity between attention and other social behaviors predicts achievement. Early deficits in working memory were negatively predictive and thus may be more detrimental to development (Sabol & Pianta, 2012).

In summary, there is a pattern in which EF predicts later mathematics achievement. A meta-analysis documented that predictive, as well as concurrent, unconditional correlations between EF skills and achievement are consistent across subject matter (averaging .31 for math), children's age, type of measurement (naturalistic or laboratory), and subcomponent of EF (Jacob & Parkinson, 2015). Another final case is the Chicago School Readiness Project, which trained teachers in strategies such as implementing clear rules and routines and redirecting negative behavior. This intervention improved low-income children's EF skills and provided partial support for the notion that these improvements mediated gains in academic readiness (Raver et al., 2011).

3.3. Predictive relations: math to EF

Although this pattern of EF predicting later mathematics seems convincing, there are limitations in both the design of some of these studies and results from other studies that call into question the unidimensional interpretation of the relations between EF and mathematical proficiencies. For example, in a longitudinal study, EF measures including working memory, inhibition, and planning and monitoring measures were predictive of mathematics (and reading) achievement into the primary grades (Bull et al., 2008). Both working memory and spatial working memory predicted mathematics achievement at each time point. However, this study did not test early mathematics skills (or instructional activities) and later EF processes, so only one possible causal connection could possibly be suggested by the data (Clark et al., 2010; LeFevre et al., 2013).

In a similar vein, the discussion of the previously-discussed correlational study (McClelland et al., 2007) focuses exclusively on the role of behavioral EF in supporting achievement in mathematics, even suggesting that strengthening these skills "prior to kindergarten may be an effective way to ensure that children also have a foundation of early academic skills" (McClelland et al., 2007). Although reported in the correlational matrix, the text does not mention that fall academic achievement predicts spring EF just as well as the reverse. Other correlational studies (Blair & Razza, 2007) could similarly be interpreted differently, without presupposing a one-directional influence. For example, higher levels of attention in kindergarten were found to be associated with more productive engagement in the classroom throughout elementary school, beyond any child and familial factors (Pagani, Fitzpatrick, & Parent, 2012). However, early math skills also predicted classroom engagement (Pagani et al., 2012).

Supporting the hypothesis of a bidirectional relation, a longitudinal study showed that EF (updating working memory) and mathematics achievement predict each other (van der Ven et al., 2012). Interestingly, this may not be true of literacy proficiencies (Fuhs, Nesbitt, Farran, & Dong, 2014; Ponitz et al., 2009; Weiland, Barata, & Yoshikawa, 2014; Welsh et al., 2010). Researchers investigated associations between growth in EF (working memory and attention control) and domain-specific (emergent literacy and numeracy) proficiencies across the pre-K year and their relative contributions to kindergarten reading and math achievement (Welsh et al., 2010). They found that working memory and attention control predicted growth in emergent literacy and numeracy skills during the pre-K year and that growth in these domain-general cognitive skills made unique contributions to the prediction of pre-K math and reading achievement (controlling for growth in domain-specific proficiencies). However, they also found that pre-K numeracy, but not emergent literacy (which may be more crystalized types of knowledge—mainly fluency and some concepts), made unique, reciprocal contributions to the growth of pre-K EF processes. Again, early numeracy knowledge predicted later EF (controlling for initial EF). In a related study, EF skills at the beginning of pre-K significantly predicted receptive vocabulary skills at the end of pre-K, not the reverse (Weiland et al., 2014). Finally, an analysis of longitudinal data found that the strongest association was between early and later mathematics achievement (Watts et al., 2015). However, mediators only account for 39% of this association, and most of that was prediction of later fractional knowledge. Contrary to the expectation that EF would play a significant mediational role, none of the EF measures predicted later math competence. However, early mathematics achievement predicted all measures of EF (Watts et al., 2015). In the correlational study discussed previously, not only teacher interaction but also teacher instruction in mathematics and literacy were associated with EF gains (Fuhs et al., 2013).

In summary, the results of correlational, longitudinal analyses are mixed, with some suggesting only that EF contributes to academic achievement (often not even testing the opposite direction of influence), but many other suggesting that early mathematics proficiencies or experiences can predict later-developing EF processes. Again, the suggestive longitudinal nature of these results cannot indicate causal relations, and as such this research corpus can only suggest co-mutual relations between the two domains. The causal relation can only be tested with other methods, such as the randomized control experiment or regression discontinuity designs. The next section addresses such studies, first those that taught EF to support learning mathematics, and then the converse: teaching math to learn EF, as well as mathematics.

4. Learning EF and mathematics: evaluating causal relations

4.1. Teaching EF to affect children's learning of mathematics

Despite the suggestive evidence of correlational analyses, researchers find surprisingly little evidence for causal relations between EF and achievement. Jacob and Parkinson (2015) even suggest that levels of EF may be a proxy for other unobserved characteristics of the child, such as socio-economic status or a parent's level of education Their meta-analysis reported that predictive studies that accounted for both child background characteristics and IQ showed more than a 2/3 decrease in the associations between EF and achievement, as well as a loss of statistical significance (Jacob & Parkinson, 2015). Similarly, they conclude that experimental studies of programs teaching EF, such as *Tools of the Mind*, show no reliable evidence of positive impacts on EF nor resultant effects on achievement. As an example, consider an experiment previously cited that found that the *Tools* curriculum improved classroom quality (Barnett et al., 2008). Lower scores on a measure of problem behaviors were interpreted as indicating improvement on children's EF; however, there were minimum effects on achievement measures (the largest effect sizes were on language), which were not statistically significant when adjusted for hierarchical structure (classroom grouping of students). This includes no significant effects on mathematics, even though the curriculum includes activities designed to promote mathematics skills as well as EF skills (Barnett et al., 2008). Thus, there is no evidence that any cognitive EF skills were increased or that they facilitated learning. (Note that the results of the same study, Barnett et al., 2008; were interpreted differently by the authors of the research and by Jacob and Parkinson.)

Some programs, such as the Head Start REDI project and Chicago School Readiness Program had a small impact on some of the measured EF skills, but the designs did not permit conclusions regarding the influence of those skills on achievement. Findings of some of the training studies previously discussed suggest a positive effect. For example, the computer games that led to an increase in some EF processes also led to higher ratings of achievement by teachers (Goldin et al., 2014), although no subject-matter proficiencies were measured directly. Similarly, english language learners in the 8-week self-regulation intervention in Head Start classrooms using movement and music games (Schmitt et al., 2015) also increased in math achievement, but no other children appeared to benefit.

Stronger effects were found for an intervention that provided first graders with computerized attention training or computerized academic training. Both positively affected attention according to a teacher rating scale. However, this impact was greater for the *academic* training. Further, only the academic training affected achievement (Rabiner, Murray, Skinner, & Malone, 2010). That is, teaching subject matter resulted in not only learning reading and math fluencies, but improved one EF process more than directly training that process—a point to which we return.

4.2. Other approaches to teaching EF and mathematics

Other causal studies suggest alternative approaches. For example, teaching children to use better EF processes *within a subject-matter context* increases learning (Iseman & Naglieri, 2011; Naglieri & Gottling, 1995). Students with learning disabilities and mild mental impairments benefit from verbalizing and reflecting on their strategies on arithmetic computation worksheets (Naglieri & Johnson, 2000). Effects are stronger for those with low planning skills (Naglieri & Gottling, 1997). Mental rotation training improved math performance in 6–8-year-olds (Cheng & Mix, 2012) although it is uncertain whether visual-spatial working memory or other spatial skills were responsible. Finally, students benefit from instruction on EF strategies to read mathematics word problems with comprehension (Capraro, Capraro, & Rupley, 2011; De Corte et al., 2011; who show similar results using a wider definition of self-regulation and involving older students; Fuchs, Fuchs, & Prentice, 2004).

4.3. The converse: teaching math to develop math and EF

These studies, along with predictive research, suggest a bidirectional relation between learning of EF and academic proficiencies. That is, high-quality mathematics (and literacy) instructional activities may also develop EF processes. Even when effects on EF are not planned, effects have been found. For example, the combination of the *Building Blocks* (*BB*) mathematics curriculum (Clements & Sarama, 2013) and the *OWL* literacy curriculum resulted in unplanned but positive, albeit small, statistically significant impacts on EF (Weiland & Yoshikawa, 2013). This “spill- over” phenomenon supports that hypothesis that cognitively demanding curricula improves other cognitive developmental domains such as EF, even without targeting them specifically.

Two evaluations of the *Tools of the Mind* program produced consistent results in finding little effect on EF of the *Tools* program as mentioned previously, but also produced evidence regarding the potential of mathematics curricula alone. In a large-scale evaluation, (Farran et al., 2011) the *Tools* program had little effects on EF. However, the more focus the classroom and teacher had on mathematics, the greater the children’s gains in both mathematics and EF (Farran et al., 2011). The second large-scale evaluation compared three treatments. The *Tools + Building Blocks* group was hypothesized to perform better than a Building Blocks math curriculum group on EF, and on mathematics as well, given the facilitative effect of the (presumed) gains in EF (Clements et al., 2012) and both these groups were hypothesized to outperform the control group in mathematics. Results were surprising. The BB group had higher mathematics scores than either of the other groups at the end of pre-K, but they did not reach statistical significance (unfortunately, problems coordinating with the school districts prevented the use of pretests in the analyses, severely limited the power of the analyses). However, the *BB* group did outperform the control group at the end of Kindergarten. Most surprising was that the *BB* group outperformed the control groups on one of the EF measures, the Head–Toes–Knees–Shoulders (HTKS) task and outperformed the *BB + Tools* group on another (backwards digit span). Effects on the HTKS are especially noteworthy as it measures all three major components of EF, and it predicts later mathematics achievement (McClelland et al., 2014). The latter group may have found implementing these two curricula too challenging, even though a *Tools* author synthesized them.

4.4. Hypothesized mechanisms: math supporting EF learning

Our hypothesis is that optimal learning of mathematics and EF is affected by the interrelation of resources in EF processes and mathematical proficiencies (Blair, 2002; Williford, Vick Whittaker, Vitiello, & Downer, 2013). This is consistent with the bidirectional relations reported in research reviewed previously. Here, we hypothesize specific mechanisms through which mathematical activity may support children’s learning of EF processes. We present them as converses of the previously presented mechanisms (EF → math; now math → EF).

First, mathematical thinking and learning may allow children to use and further develop EF processes. From this perspective, learning new EF processes is not prerequisite to mathematical activity, but rather mathematical proficiencies and EF process are co-mutually supportive. According to our theory of hierarchic interactionalism (Sarama & Clements, 2009), children possess important, but often inchoate, premathematical and general cognitive abilities (such as EF processes) and predispositions at birth or soon thereafter that support and constrain, but do not absolutely direct, subsequent development. These initial bootstrap processes provide children the resources for the continued reciprocal construction of mathematical proficiencies and EF processes.

Mathematical activities may provide unique affordances for the development and strengthening of EF processes. For example, the solution of arithmetical problems may require children to expand their application of working memory, but also it must provide scaffolding for such extension. As one example, the contexts of story problems related to real-world experiences with which children are familiar may provide such scaffolding through the incorporation of the “narrative mode” (Bruner, 1986) of thinking, which provides sequential and interpretable situations that guide children’s translation of the situation into the logical and systematic structures of mathematics. Such activity is posited to increase not only mathematical proficiencies (all strands: concepts, fluency, strategic competence, adaptive reasoning, and productive disposition) but also EF processes, especially updating working memory and inhibition.

Further, we hypothesize that mathematical activity provides logical and engagement, which provide cognitive constraints and guidelines (such as goal sketches and constraints of logical reasoning, Gelman & Williams, 1998; Sarama & Clements, 2009; Siegler, 1996). That is, mathematics qua “logical-mathematical thinking” (Piaget, 1970, 2001) is arguably a core component of cognition and as such may evoke, enable, and exercise EF processes in young children (Clements & Sarama, 2011).

A minor but justifiable argument complements the other mechanism through which EF was hypothesized to affect academic learning (i.e., as previously described, EF assists children in conforming to rules and benefiting from learning in social contests). We hypothesize that high-quality mathematical tasks, activities, and conversations may be natural, playful, and motivating to children (Clements & Sarama, 2014; Fisher, Dobbs-Oates, Doctoroff, & Arnold, 2012; Seo & Ginsburg, 2004). And thus, serve as scaffolding by helping children build the resources and relations needed to maintain engagement in these and other educational acts (Vygotsky, 1934/1986, 1935/1978).

4.5. The nature of experiences that may support both math and EF learning

What instructional activities in mathematics may develop EF and why? Some may require children to suppress prepotent responses, manipulate abstract information, and remain cognitively flexible, as when children find all pairs of positive whole numbers that sum to six. The greater relation of EF and math, as compared to literacy, activities (Fuhs et al., 2014), may reflect that the former make greater demands on working memory and attention control, such as the ability to hold relevant information in mind, to operate on it while shifting attention appropriately among problem elements, and to inhibit automatic responding to only one aspect of a given problem. Some have postulated that 100 years of rising population mean IQ in the United States is due to the increasing cognitive demands of mathematical curricula (Blair, Gamson, Thorne, & Baker, 2005).

Importantly, EF may be developed in *learning* the mathematics in the context of challenging activities, not in "exercising" the mathematics once learned. A neuroimaging study found both commonalities and differences in the children's and adults' judgments of relative magnitude between two numerals (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005). They used the well-known "distance effect"—the more distance between two numbers, the faster and more accurate are people's judgments (e.g., "which is larger"). Children showed more activation in the right frontal regions that control attention, working memory, whereas adults show more engagement of intraparietal and posterior parietal regions, which may indicate increasingly strong and flexible mappings from numerals to the numerical quantities they represent, as well as more specialization in these regions. As another example, when children are learning arithmetic combinations, they use the frontal areas of their brains (EF and working memory) and only later use medial temporal areas (declarative memory) and the parietal areas (magnitude processing and arithmetic fact retrieval) and occipito-temporal areas (processing symbolic form) (Butterworth, Varma, & Laurillard, 2011). Thus, EF may be developed in the early stages of learning. However, it may be that EF processes are recruited (albeit not necessarily improved) in early learning—this remains to be studied.

In summary, these studies also suggest that high-quality mathematics education may have the dual benefit of teaching an important content area and developing (at least "exercising" and possibly expanding) some EF processes. A more intentional development of mathematics curricula based on recent research on EF may do both more effectively. Further, research has also identified environments (V. Fisher, Godwin, & Seltman, 2014) and teaching practices that can help children pay attention and grow in their ability to do so, as well as to develop general EF processes (see Clements & Sarama, 2014; for numerous activities so designed). Carefully guiding children to attend to specific mathematical features, such as the number in a collection or the corners of a polygon, is likely to improve their learning. The predisposition to spontaneously recognize number, for example, is a skill but also a habit of mind, including the ability to direct attention to number (Lehtinen & Hannula, 2006). These habits of mind generate further development of specific mathematical knowledge and the ability to direct attention to mathematics in situations in which it is relevant; that is, to generalize and transfer knowledge to new situations, as well as to develop EF processes.

5. Conclusions

Learning requires both affective and cognitive resources. This is true for children, especially young children, negotiating and learning in schools. EF – the ability to control and supervise one's own emotions and thinking – may be one of the most important of these resources. EF processes appear to play a role in learning. This is especially important in that mathematics curricula have increasingly required higher-order skills such as EF processes provide (Baker et al., 2010). EF develops most rapidly in the early childhood years, so educators need to use research to provide environments, curricula, and experiences that develop EF, especially for those at-risk due to low entering levels of these processes. Several approaches have showed promise, but few have been consistently successful at a practically significant level. Further, the causal evidence that interventions to develop EF will increase achievement is weak. However, the evidence of bidirectionality remains suggestive and no approaches should be abandoned. More research and development work is needed, but some combination of the two may be most beneficial to children.

A potentially promising approach to developing EF was alternately proposed in this paper. It is the hypothesis that high-quality mathematics education may have the dual benefit of teaching an important content area and developing at least some EF processes. Given that early mathematics content predicts later mathematics (as well as predicting later reading achievement better than early literacy skills do, Duncan et al., 2007) and that early EF does not (once early mathematics is factored in), but that early mathematics predicts later EF (Watts et al., 2015), this approach stands as potentially promising in both efficacy and efficiency. Given the precious, few hours children – especially those most in need – have in early childhood settings, a strategy that develops multiple critical competencies is particularly valuable. A more intentional development of mathematics curricula based on recent research on EF may contribute even more to both.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ecresq.2015.12.009.

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