CHAPTER THREE

Interventions in Early Mathematics: Avoiding Pollution and Dilution

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Contents

Abstract

Although specific interventions in early mathematics have been successful, few have been brought to scale successfully, especially across the challenging diversity of populations and contexts in the early childhood system in the United States. In this chapter, we analyze a theoretically based scale-up model for early mathematics that was designed to avoid the pollution and dilution that often plagues efforts to achieve broad success. We elaborate the theoretical framework by noting the junctures that are susceptible to dilution or pollution. Then we expatiate the model's guidelines to describe specifically how they were designed and implemented to mitigate pollution and dilution. Finally, we provide evidence regarding the success of these efforts.

Although specific interventions in early mathematics have been successful (Clements & Sarama, 2011), few have been brought to scale successfully, especially across the challenging diversity of populations and contexts in

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the early childhood system in the United States. In this chapter, we describe a theoretically based scale-up model for early mathematics that has avoided the pollution and dilution that often plagues efforts to achieve broad success (Clements & Sarama, 2011).

1. BACKGROUND

Taking promising interventions to scale means implementing them successfully in larger settings. Moreover, though, such scale-up goes beyond larger numbers to confront the complexity of such an enterprise. For example, there is an increase in not just the number of students and teachers, but also in both the number of categories of stakeholders and the number of different and often conflicting perspectives they hold. Therefore, we define scale-up as instantiation of an intervention in varied settings with diverse populations, addressing the needs of multiple sociopolitical stakeholders, to achieve satisfactory fidelity of implementation and, thus, intervention goals. Although this already is a substantial challenge, consider the complexity of addressing the perspectives, needs, and desires of the different categories of stakeholders, including parents, various community groups, the professional teaching community, educational leadership in early childhood and in mathematics (often these are distinct groups), and higher-level administration (Sarama & Clements, 2013). Achieving an adequate fidelity of implementation during implementation presents other challenges, such as sufficient materials, technology, professional development, and in-class support.

Also, interventions are not useful if they are not maintained after the initial thrust for the implementation. Thus, attention must be given to including transfer to local ownership and sustainability. We define *sustainability* as the length of time an innovation continues to be implemented with fidelity (cf. Baker, 2007).

Implementation, fidelity, and sustainability are threatened by both pollution and dilution. *Pollution* is the introduction of elements into the intervention environment or teaching practices that vitiate or are inimical to the intervention. *Dilution* is the gradual replacement of components of the intervention with other aspects that do not fulfill the same role as the replaced components.

The next section describes the theoretical framework we developed for our research and development work on implementation and wider scale-up, and the model we derived from it. After we define each of the 10 guidelines that constitute the TRIAD model, we describe how they were designed to mitigate the usual forces of pollution and dilution.

2. THE TRIAD MODEL

2.1 Theoretical Framework

Our theoretical framework (Sarama, Clements, Starkey, Klein, & Wakeley, 2008) is an elaboration of the *Network of Influences* framework (Sarama, Clements, & Henry, 1998). We consider successful implementation of an intervention at scale to involve multiple coordinated efforts to maintain the integrity of the vision and practices of an innovation through increasingly numerous and complex socially mediated filters.

2.1.1 Interactions

The TRIAD model goes beyond solely adopting new curricula. Instead, it scales up the support of "interactions among teachers and children around educational material" (Ball & Cohen, 1999, p. 3). This strategy creates extensive opportunities for teachers to focus on math, goals, and children's thinking and learning, which improves teachers' knowledge of subject matter, teaching, and learning, and increases child achievement (Ball & Cohen, 1999; Cohen, 1996, p. 98; Schoen, Cebulla, Finn, & Fi, 2003; Sowder, 2007). The depiction of the *Network of Influences* framework in Fig. 1 illustrates the hypothesized influences of context and implementation variables on outcomes such as teacher knowledge, child achievement, and sustainability.

2.1.2 Administrators and Other School Leaders (Fig. 1, Factors K and I)

Principal leadership is strongly correlated with levels of implementation and effectiveness (Berends, Kirby, Naftel, & McKelvey, 2001; Bodilly, 1998; Bryk, Sebring, Allensworth, Suppescu, & Easton, 2010; Fullan, 1992; Heck, Weiss, Boyd, & Howard, 2002; Kaser, Bourexis, Loucks-Horsley, & Raizen, 1999; Klingner, Ahwee, Pilonieta, & Menendez, 2003; Teddlie & Stringfield, 1993). Effective administrators provide the time for teachers to experiment, discuss, and, in general, construct their own meanings of the innovation. They communicate continuing commitment, not just in verbal form, but also in other ways, such as resource allocation (Bodilly, 1998). School mathematics and early childhood leaders can serve as essential bridges between administrators and teachers (Cobb, McClain, de Silva, & Dean, 2003). District level leaders

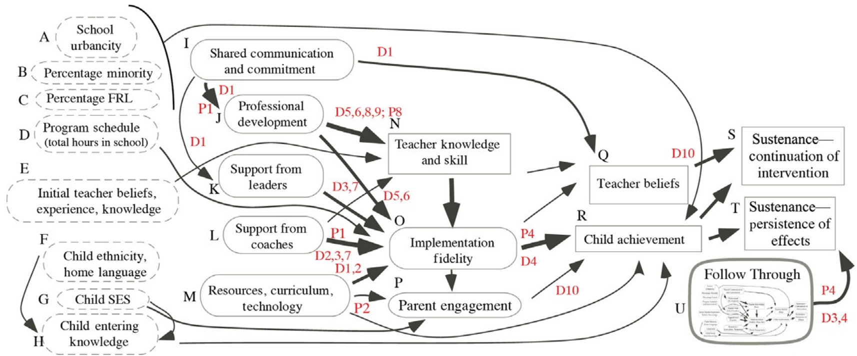


Fig. 1 Revised *Network of Influences* theoretical framework elaborated with junctures susceptible to dilution or pollution. The shaded text such as "D1" or "P2" refer to hypothesized threats (D = dilution, P = pollution) to implementation addressed by guidelines (#1, #2, etc.) of the TRIAD model. The "Follow Though" model at the bottom right is simple a microcosm of the framework. Contextual variables in *dotted* ovals include the school (A–D), teacher (E), and child (F—H) factors from the revised Network of Influences framework. For example, child socioeconomic status, or SES (G), impacts children's initial mathematics knowledge (H), which influences children's achievement (R)—an outcome variable indicated by the *solid rectangle*. Implementation variables in *solid ovals* are features that the project can encourage and support, but cannot control absolutely. For example, *heavy arrows* from professional development (J), to teacher knowledge (N), to implementation fidelity (O), to child achievement (R), indicate the strong effects in that path. Support from coaches (L) also has a strong effect on implementation fidelity, while other factors (J, K, and M) are influential, but to a moderate degree (not all small effects are depicted).

and their decisions also impact implementation and ultimately child achievement (Bodilly, 1998; Klingner et al., 2003; Snipes, Doolittle, & Herlihy, 2002; Spillane, 2000). Communication between principals and all other groups is particularly essential. Principals forgetting the study and their involvement in it and similar communication lapses can be a huge challenge, which increases with the size of the district (Foorman, Santi, & Berger, 2007).

2.1.3 Communication

Communication, collaboration, and agreement among all groups are essential. Our own previous research revealed multiple missed opportunities for facilitation of innovations due to the divergent beliefs of social groups, even about ostensibly observable "facts," such as "there is adequate technology available" (Sarama et al., 1998).

2.1.4 Teachers and Professional Development (Fig. 1, Factors E, N, and Q)

Research suggests that the most critical feature of a high-quality educational environment is a knowledgeable and responsive adult and that high-quality professional development is essential to innovation (Fig. 1J and N; Darling-Hammond, 1997; Ferguson, 1991; National Research Council, 2007; Sarama & DiBiase, 2004; Schoen et al., 2003; Sowder, 2007). Scaling up such professional development has special challenges and opportunities given the early childhood setting, as noted previously. Our model specifies only a weak effect of *initial* teacher expertise (Fig. 1, dotted oval E) because of the low level of mathematics content and pedagogical content knowledge of most pre-K teachers, regardless of background (Copley, 2004; Sarama, 2002; Sarama & DiBiase, 2004), which is consistent with previous research (e.g., Bryk et al., 2010). Changes in beliefs follow changes in practice (boxes N, Q, Showers, Joyce, & Bennett, 1987); moreover, we believe that changes in beliefs help *sustain* teacher practices (Fig. 1, boxes Q and S).

Research-based solutions to these challenges can be found (Corcoran, 2007; Klingner et al., 2003; NAEYC, 2002; National Research Council, 2007; Peisner-Feinberg et al., 1999; Sarama, 2002; US Department of Education, 1999). Use of theory, demonstrations, practice, and feedback, especially from coaches, quadruples the positive effects of information-only training (e.g., strong effects from J and L to N and O in Fig. 1, Foorman et al., 2007; Pellegrino, 2007; Showers et al., 1987). Effective professional development eschews "one-shot" interventions, begins with a specific curriculum (Fig. 1M), embodies the kind of flexible, interactive teaching styles that work well with children, and weaves together math content, pedagogy, and knowledge of child development and family relationships (Baroody & Coslick, 1998; Schoen et al., 2003; Sowder, 2007). The professional development in TRIAD provides a promising path for developing teachers' understanding of learning, teaching, curriculum, and assessment by focusing on research-based models of children's thinking and learning. Research indicates the efficacy of such cognitive models (Bredekamp, 2004; Carpenter & Franke, 2004; Hiebert, 1999; Klingner et al., 2003), especially compared with other approaches such as process–product models (Lawless & Pellegrino, 2007). Research-based *learning trajectories* are TRIAD's core (Clements, Sarama, & DiBiase, 2003). We defined learning trajectories as "descriptions of children's thinking and learning…and a related, conjectured route through a set of instructional tasks" (Clements & Sarama, 2004, p. 83). Thus, learning trajectories have three components: a goal (that is, an aspect of a mathematical domain children should learn), a developmental progression, or learning path through which children move through levels of thinking, and instruction that helps them move along that path (Fig. 2 provides an example). Learning trajectories are at the heart of both TRIAD's math curriculum and its professional development. Learning trajectories help teachers

The *goal* connects learning trajectories to the important *big ideas* in mathematics. The relevant big idea is that numbers can be used to tell how many, describe order, and measure; they involve numerous relations and can be represented in various ways. The specific goals include understanding that subitizing can be used to tell how many (perceptual subitizing), that a quantity can consist of parts and can be broken apart (decomposed) into the parts, and that those parts can be combined (composed) to form the whole (conceptual subitizing). Specific objectives are for children to achieve those understandings and fluency in perceptual and conceptual subitizing.

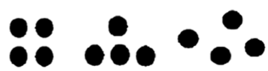
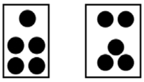
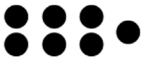
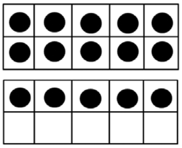
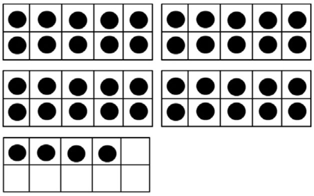
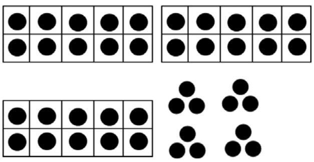
Age (years)	Developmental progression	Sample instructional tasks
0–1	**Preexplicit number.** Within the first year, the child is sensitive to small numbers, but does not have explicit, intentional knowledge of number.	*Noticing collections*. Provide a rich sensory environment, use words such as *more,* and use actions of adding objects.
1–2	**Small-collection namer**. Names groups of 1 to 2, sometimes 3. Shown a pair of shoes, says, "Two shoes."	*Board games–small numbers*. Play board games with a special die (number cube) or spinner that shows only 1, 2, and 3 dots (then add 0 to it).
3	**Maker of small collections**. Nonverbally makes a small collection (no more than 4, usually 1–3) with the same number as another collection via mental model (i.e., not necessarily by matching). Might also be verbal. When shown a collection of 3, makes another collection of 3.	*Get the number*. Ask children to get the right number of crackers or some other item for a small number of children.
4	**Perceptual subitizer to 4.** Instantly recognizes collections up to 4 briefly shown, and verbally names the number of items. When shown objects briefly, says, "Four."	*Snapshots*. Play "Snapshots" with collections of one to four objects, arranged in line or other simple arrangement, asking children to respond verbally with the number name. Start with the smaller quantities and easier arrangements, moving to those of moderate difficulty only when children are fully competent and confident.  easy medium difficult
5	**Perceptual subitizer to 5.** Instantly recognizes briefly shown collections up to 5 items and verbally names the number of items. Recognizes and uses spatial and numeric structures beyond the situations in which they were already experienced. Shown 5 objects briefly, says, "Five."	*Snapshots*: Play **"**Snapshots**"** with dot cards, starting with easy arrangements, moving to more difficult arrangements when children are able to do so.  easy medium difficult
5	**Conceptual subitizer to 5**. Verbally labels all arrangements to about 5, when shown only briefly. Asked "Why?" says, "Because I saw 3 and 2, and so I said 5."	*Snapshots*. Use different arrangements of the various modifications of "Snapshots" to develop conceptual subitizing and ideas about addition and subtraction. The goal is to encourage children to see the addends and

Fig. 2 A learning trajectory for recognition of number and subitizing.

		the sum. 
5	**Conceptual subitizer to 10**. Verbally labels most briefly shown arrangements to 6, then up to 10, using groups. "In my mind, I made two groups of 3 and one more, so 7."	*Snapshots*. Play "Snapshots" with larger quantities to develop ideas about addition and subtraction. 
6	**Conceptual subitizer to 20**. Verbally labels structured arrangements up to 20, shown only briefly, using groups. "I saw three 5s, so 5, 10, 15."	*Ten frame addition snapshots*: Briefly show 2 ten frames to help children visualize addition combinations. 
7	**Conceptual subitizer with place value and skip counting**. Verbally labels structured arrangements, shown only briefly, using groups, skip counting, and place value. "I saw groups of 10s and 2s, so 10, 20, 30, 40, 42, 44,. . . 44!"	*Ten frame addition snapshots*. Briefly show several ten frames. 
8	**Conceptual subitizer with place value and multiplication**. Verbally labels structured arrangements shown only briefly, using groups, multiplication, and place value. "I saw groups of 10s and 3s, so I thought, 3 tens is 30 and 4 threes is 12, so 42 in all."	*Snapshots with dots*. Play "Snapshots" with structured groups that support the use of increasingly sophisticated mental strategies and operations. 

(Adapted from Clements & Sarama, 2014, pp. 17–20; copyright 2014, *Learning and teaching early math: The learning trajectories approach* (2nd ed.) by Clements & Sarama. Adapted with permission of Taylor and Francis Group, LLC, a division of Informa plc.)

Fig. 2—Cont'd

focus on the "conceptual storyline" of the curriculum, a critical element that is often missed (Heck et al., 2002; Weiss, 2002). They facilitate teachers' learning about mathematics, how children think about and learn this math, and how such learning is supported by the curriculum and its teaching strategies. They address domain-specific components of learning and teaching that have the strongest impact on cognitive outcomes (Lawless & Pellegrino, 2007). By illuminating potential developmental progressions, they bring coherence and consistency to goals, curricula, and assessments (for a comprehensive description and review of research on these learning trajectories, see Sarama and Clements (2009a), and for the instructional component).

2.1.5 Children and Their Families (Fig. 1, Factors F, G, and P)

Interventions are more effective if they involve parents, especially by providing activities to do with their children (Bryk et al., 2010; Galindo & Sonnenschein, 2015; Halpern, 2004; Ramey & Ramey, 1998). As with pre-K teachers, parents have a limited view of the breadth of math appropriate for young children (Sarama, 2002). Low-income parents, compared to middle-income parents, believe that math education is the responsibility of the preschool and that children cannot learn aspects of math that research indicates they can learn (Starkey et al., 1999).

2.1.6 Resources, Curriculum, and Technology

The print curriculum, research-based learning trajectories that link developmental progressions to connected instruction, manipulatives, supporting manuals, the *Building Blocks* software activities and management system, and the professional development website, all components of the mathematics intervention, provided a central focus around which the reform revolved. Scale-up is more likely to be successful if resources are targeted to implementing a single curriculum well (Snipes et al., 2002).

2.2 The TRIAD Model's 10 Guidelines

Building on our theoretical framework and review of research, we created a scale-up model for scale-up called TRIAD (technology-enhanced, research-based, instruction, assessment, and professional development, Sarama et al., 2008). The TRIAD model scales up the support of "interactions among teachers and children around educational material" (Ball & Cohen, 1999, p. 3). This strategy creates extensive opportunities for teachers to focus on mathematics, goals, and students' thinking and learning, which improves teachers' knowledge of subject matter, teaching, and learning, and increases student achievement (Ball & Cohen, 1999; Cohen, 1996, p. 98; Schoen et al., 2003; Sowder, 2007). This section summarizes 10 guidelines we followed in creating the TRIAD intervention (Sarama et al., 2008) and the following section describes how each was expected to minimize pollution and dilution.

1. *Involve, and promote communication among, key groups around a shared vision of the innovation* (Hall & Hord, 2001). Emphasize connections between the project's goals, national and state standards, and greater societal need. Promote clarity of these goals and of all participants' responsibilities. School and project staff must share goals and a vision of the intervention (Bryk et al., 2010; Cobb et al., 2003).
2. *Promote equity* through equitable recruitment and selection of participants, allocation of resources, and use of curriculum and instructional strategies that have demonstrated success with underrepresented populations (Kaser et al., 1999).
3. *Plan for the long term*. Recognizing that scale-up is not just an increase in number, but also of complexity, provide continuous, adaptive support over an extended time. Communicate clearly that change is not an event, but a process (Hall & Hord, 2001).
4. *Focus on instructional change that promotes depth of children's thinking, placing learning trajectories at the core* of the teacher/child/curriculum triad to ensure that curriculum, materials, instructional strategies, and assessments are aligned with national and state standards and a vision of high-quality math education, each other, (c) "best practice" as determined by research, especially formative assessment (Ball & Cohen, 1999; Bodilly, 1998; Bryk et al., 2010; Fullan, 2000; Kaser et al., 1999; National Mathematics Advisory Panel, 2008; Raudenbush, 2008; Sowder, 2007).
5. *Build expectations and camaraderie to support a consensus around adaptation.* Promote "buy-in" in multiple ways, such as dealing with all participants as equal partners and distributing resources to support the project. Establish and maintain cohort groups. Facilitate teachers visiting successful implementation sites. Build local leadership by involving principals and encouraging teachers to become teacher leaders (Berends et al., 2001; Borman, Hewes, Overman, & Brown, 2003; Elmore, 1996b; Fullan, 2000; Glennan, Bodilly, Galegher, & Kerr, 2004; Hall & Hord, 2001.)
6. *Provide professional development that is ongoing, intentional, reflective, focused on mathematics content knowledge and children's thinking, grounded in particular curriculum materials, situated in the classroom and the school.* Encourage

sharing, risk taking, and learning from and with peers. Aim at preparing to teach a specific curriculum and develop teachers' knowledge and beliefs that the curriculum is appropriate and its goals are valued and attainable. Situate work in the classroom, formatively evaluating teachers' fidelity of implementation and providing feedback and support from coaches in real time (Bodilly, 1998; Borman et al., 2003; Bryk et al., 2010; Cohen, 1996; Elmore, 1996b; Garet, Porter, Desimone, Birman, & Yoon, 2001; Guskey, 2000; Hall & Hord, 2001; Kaser et al., 1999; Klingner et al., 2003; Pellegrino, 2007; Schoen et al., 2003; Showers et al., 1987; Sowder, 2007). This also provides a common language for teachers in working with each other and other groups (Bryk et al., 2010).

7. *Ensure school leaders are a central force supporting the innovation* and provide *teachers continuous feedback that children are learning what they are taught and that these learnings* are valued. Leaders, especially principals, must show that the innovation is a high priority, through statements, resources, and continued commitment to permanency of the effort. An innovation champion leads the effort within each organization (Bodilly, 1998; Bryk et al., 2010; Glennan et al., 2004; Hall & Hord, 2001; Rogers, 2003, p. 434; Sarama et al., 1998).
8. *Give latitude for adaptation to teachers and schools, but maintain integrity.* Emphasize the similarities of the curriculum with sound practice and what teachers already are doing. Discourage uncoordinated innovations (Fullan, 2000; Huberman, 1992; Sarama et al., 1998; Snipes et al., 2002).
9. *Provide incentives for all participants, including intrinsic and extrinsic motivators linked to project work*, such as external expectations—from standards to validation from administrators. Show how the innovation is advantageous to and compatible with teachers' experiences and needs (Berends et al., 2001; Borman et al., 2003; Cohen, 1996; Darling-Hammond, 1996; Elmore, 1996a; Mohrman & Lawler III, 1996; Rogers, 2003).
10. *Maintain frequent, repeated communication, assessment (checking up), and follow-through efforts* emphasizing the purpose, expectations, and visions of the project, and involve key groups in continual improvement through cycles of data collection and problem solving (Fullan, 1992; Hall & Hord, 2001; Huberman, 1992; Kaser et al., 1999; Snipes et al., 2002). Throughout, connections with parents and community groups are especially important, to meet immediate and long-range (sustainability) goals.

2.3 How the TRIAD Implementation Was Designed to Militate Against Pollution and Dilution

This section addresses the TRIAD model regarding pollution and dilution. That is, for each of these 10 guidelines constituting the model, we describe the sources of possible pollution or dilution and then describe how the guideline was expected to minimize the threat to high-quality implementation.

1. Involve, and promote communication among, key groups around a shared vision of the innovation. This guideline jumpstarts and later institutionalizes the intervention; for example, by establishing a common understanding of the goals and how they are consistent with state or district standards and by planning for the ongoing socialization and training of new teachers (Elmore, 1996b; Fullan, 2000; Huberman, 1992; Kaser et al., 1999; Klingner et al., 2003; Sarama et al., 1998). To begin, the TRIAD model includes initial meetings with administration as well as teachers, including a presentation with a brochure and a question and answer session. Planning for this meeting includes preliminary discussions with key staff to ensure that the relationships between the district's goals and other instructional guidelines and those of the intervention can be explicated and defended. After the meeting, presenters emphasize that to move forward, all levels of administration must agree to participate. (Ideally, at least 80% of teachers, as well as parents, would agree to participate. However, for the purposes of generalizability, we did not ask teachers to sign consent forms in the largest study described here. This way we could generalize our findings to districts that used that model but had reluctant or hostile teachers.)

   Once agreements are obtained and the project started, ongoing planning with all key groups that minimizes dilution (see D1's in Fig. 1) includes specifics around the intervention, including clear, written specifications concerning the time and supports needed to implement (e.g., all teachers and paraprofessionals are scheduled to attend all trainings; sufficient resources are available in every classroom, from blocks to computers; admin). Furthermore, dilution may occur due to rapid turnover of preschool teachers, introducing an increasing proportion of teachers who are not involved in the initial implementation. Also, pollution (see P1's in Fig. 1) can occur if new (and also old) teachers introduce inconsistent elements into their practice. Therefore, plans are made for training and socializing new teachers, including a mentor system and including district supervisors and coaches in the project's training sessions so they would be well versed in introducing the intervention to any new teachers and to monitoring and supporting all teachers past the end of the project.

2. Promote equity. This ensures that every teacher has the support, both physical and personnel, necessary to teach and continue to teach the intervention properly. Without this, both pollution and dilution are likely (see D2's and P2 in Fig. 1), especially given the challenges present-day teachers face in implementing mathematics interventions. That is, without comprehensive curricular resources, training, and coaching, fidelity is unlikely to be achieved and, maintained; unfortunately, inequitable distribution of resources frequently plagues preschools, especially those serving children from lower SES communities. We worked with administrators to ensure all resources were provided to each teacher and coaches filled in for any missing materials throughout the implementation period.
3. Plan for the long term. The TRIAD professional development model includes regular professional development meetings, bi-weekly coaching visits, and peer coaching. Year 1 professional development is a "gentle introduction," in that child data are not collected and coaches have time to slowly get teachers comfortable with all aspects of the intervention. "One-shot" interventions may have a temporary effect, but frequently are diluted or even abandoned soon thereafter (Bodilly, 1998; Cuban & Usdan, 2003). TRIAD uses a dynamic, multilevel, feedback, and self-corrective strategy (Bryk et al., 2010; Coburn, 2003; Guskey, 2000). That is, all work with teachers is designed so that data about teachers' understandings and engagement is collected in real time and used to alter plans for the following sessions. Interviews with principals occur throughout the project, to ensure they not only remember the project (Foorman et al., 2007) but provide feedback on what is progressing successfully and what is not in their building (see guideline #1). In addition, administrators are provided with tools to offer additional support to teachers in their building and fulfill district requirements (to monitor instructional time and quality of instruction).
4. Focus on instructional change that promotes depth of children's thinking, placing learning trajectories at the core of the teacher/child/curriculum. This is the core of the TRIAD approach so ensuring this guideline is critical. Implemented well, this focus helps avoid pollution, mainly, by ensuring fidelity by helping teachers not only use the

effective instructional strategy of formative assessment (Penuel & Shepard, 2016) but also by helping teacher understand not only how to teach a given activity, but why—in other words, connecting their growing knowledge of how children think about and learning mathematics to the characteristics of a given activity that promote this learning. With an integrated understanding of the mathematics, children's developmental progressions, and instruction, teachers are less likely to introduce lethal mutations, changes to the core math content, or pedagogy that weakens the activity. This focus also prevents dilution, however, because teachers who observe and understand the depth of children's thinking and learning given these activities are more likely to continue to use the program with fidelity over years (Clements, Sarama, Wolfe, & Spitler, 2015).

5. Build expectations and camaraderie to support a consensus around adaptation. Dilution is less likely when teachers are part of a learning–teaching community. In TRIAD, teachers are intermingled (across schools) in small group work, but they also work in school clusters for other small groups. In this way, teachers are exposed to ideas from peers at other sites but had adequate time to address school specific concerns and address implementation as a team.
6. Provide professional development that is ongoing, intentional, reflective, focused on mathematics content knowledge and children's thinking, grounded in particular curriculum materials, situated in the classroom and the school. As guideline #4 is a main focus, this guideline is TRIAD's main leverage point—sine qua non of a high-quality implementation. That is, without well-trained and committed teachers, dilution (to the degree of no implementation) and/or pollution (low fidelity) is highly likely. TRAID's professional-development model includes both repeated (e.g., >10) full-day sessions of training in regular meetings and frequent coaching. Training includes all three components of each learning trajectory, the goal, the developmental progression, and the instructional activities and strategies (Fig. 2). All three are critical. To understand the goal, teachers study the mathematical content. Without knowledge of the mathematical content, pollution is almost assured. For example, teachers may believe that any series of colors in fabric is "a pattern" (Fox, 2005) or that "two triangles put together always make a square" (Thomas, 1982). In TRIAD, teachers learn core mathematics concepts and procedures for each topic. For example, they study the system of verbal counting based on cycling through 10 digits

and the concept of place value (i.e., content like that presented in National Research Council, 2009; Wu, 2011).

A key instructional use of learning trajectories is in formative assessment, an empirically supported pedagogical strategy (National Mathematics Advisory Panel, 2008; Penuel & Shepard, 2016). However, dilution is probable if teachers cannot accurately assess children's developmental level or select appropriate tasks and instructional strategies based on children's levels—the second and third components of a learning trajectory. To learn about the developmental progression, teachers analyze multiple video segments illustrating each level and discuss the mental "actions on objects" that constitute the defining cognitive components of each level; order tasks corresponding to those levels; and practice diagnosis in teams, with a couple of teachers exemplifying behaviors of children at different levels, and one teacher identifying the level of each. Furthermore, teachers need training in understanding, administering, and especially using data from new assessment strategies (Foorman et al., 2007). TRIAD training focuses mainly on the curriculum-embedded assessment of Building Blocks' "Small Group Record Sheets," but also provides teachers with tasks appropriate for clinical interviews.

Formative assessment requires more than identifying children's levels of thinking. Teachers must select and modify instructional activities and strategies that are appropriate and effective for each level. To learn about instructional tasks and strategies, teachers practice the curriculum's activities, but also analyze them to establish and justify their connection to a particular level of the developmental progression. They extend the team practice sessions to use their diagnoses to select and to modify activities to match instructional tasks to developmental levels of individuals.

Across all forms of professional development, the focus is on children's thinking and learning. Conversations about children learning serve as way to address implementation issues. Although early mathematics is often an uncomfortable topic for early childhood educators, the newness of the learning trajectories for all participants helps establish a sense of shared learning and community. Each session in the last third of professional development includes scheduled time to discuss "learning stories" (Perry, Perry, Dockett, & Harley, 2007). Teachers show their record keeping on small group record sheets, and sometimes videos, and discuss their use of learning trajectories in teaching children, including challenges, questions, and successes. These discussions promote peer learning and collaboration and also motivate peers to solve implementation difficulties.

Finally, we do not train the teachers and immediately begin evaluating the project. Instead, an entire year is spent in professional development and implementation without evaluations as a "gentle introduction" without the pressure of assessments.

7. Ensure school leaders are a central force supporting the innovation and provide teachers continuous feedback that children are learning what they are taught and that these learnings are valued. School leaders deal with myriad issues and problems and a single intervention can easily lose their attention (Foorman et al., 2007), vitiating the support teachers need from their leaders. In TRIAD, communication is initially established (see guideline #1) and is not maintained to ensure that school leaders provide support, encouragement, and feedback to teachers. Principals receive, and are offered training on, a "walk-through" fidelity instrument, which when used, heightened teachers' perception of the importance the leader assigned to the intervention. Leaders also include supervisors of early childhood education and mathematics education. Coaches from both these fields are invited to all trainings. All leaders are taught about the structure and research of the intervention, to avoid dilution and pollution that can occur when leaders tell teachers they are "professionals who can choose to include any activity from any source."
8. Give latitude for adaptation to teachers and schools, but maintain integrity. Adaptation to local contexts can be important, as can adaptation to children's interests and backgrounds. However, it is easy for teachers to mistakenly dilute or pollute instructional activities in these attempts. TRIAD training addresses this issue by explicitly comparing productive adaptations to lethal mutations (Brown & Campione, 1996). For example, teachers learn to change surface features (e.g., to fit a class "theme") without altering the core mathematical or pedagogical components of the instructional activities (Winter & Szulanski, 2001). They learn to modify an activity to fit children's interests and needs without (permanently) reducing the cognitive demand of instructional tasks after their initial introduction (Stein, Brover, & Hennigsen, 1996). The best productive adaptations are then shared among the sites and added to future training sessions.

From a school perspective, our fidelity form, including the "walk-through" version for principals, includes pacing guidelines. However, these forms are not based on compliance alone. That is, the emphasis is on integrity and pacing, but with latitude to make changes. For example, higher, not lower, scores are given to teachers who modify the schedule following children's development and learning. Similarly, higher scores are given for modifying activities, but only for productive adaptations.

9. Provide incentives for all participants, including intrinsic and extrinsic motivators linked to project work. TRIAD implementations connect the intervention to the standards and school reports teachers use. Early successes, even anecdotal, with teachers and children are shared with principals so they remain committed and pass on the complements to their teachers. Discussing the small group record sheets and sharing the learning stories not only motivate those implementing, but also subtly pressure nonimplementers to "up their game." Training sessions are designed to be happy and productive, providing opportunities to work with peers, to discuss education over good food, and generally to be treated as professionals. For example, they are asked to share their ideas for productive adaptations and trainers are serious about using feedback from them to trainings.

10. Maintain frequent, repeated communication, assessment (checking up), and follow-through efforts. Of course, assuming that simple introduction and training will lead to complete and continued implementation is invalid. Throughout the TRIAD implementation, project staff maintained connections with not only teachers and leaders, as described previously, but also with parents and local media.

3. RESEARCH EVALUATIONS: DID THE TRIAD DESIGN MITIGATE DILUTION AND POLLUTION?

Our colleagues and we have conducted several studies to evaluate instantiations of the TRIAD model and its components (Clements & Sarama, 2008a, 2008b; Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Clements, Sarama, Wolfe, & Spitler, 2013; Sarama & Clements, 2009b; Sarama et al., 2008). Most of these studies used cluster randomized trials designs. Although financial and logistical constraints did not allow data collection that would directly address every source of dilution and pollution identified in Fig. 1 and in the previous section, successful outcomes do provide evidence that these challenges were addressed successfully.

3.1 Initial Instantiation and Evaluations of the TRIAD Model

The first evaluations were limited, designed to complete procedures and assessments and serve as a proof of concept that the model could be effective. In the first, we randomly assigned 25 preschool classrooms serving children at-risk for later school failure to either TRIAD or control groups (Sarama et al., 2008), including about half Head Start and half public preschool programs. This was successful so we implemented it more completely with 36 teachers, which we describe here (Clements & Sarama, 2008a).

3.1.1 Implementation

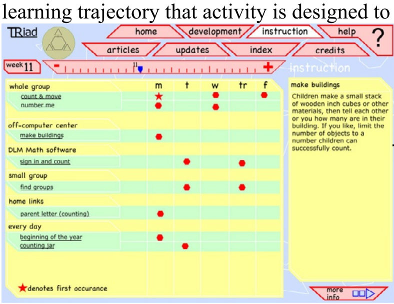
We attempted to implement most components of the TRIAD model (some, such as planning for the long term, guideline #3, and full realization of #10, were not possible in this limited scale-up). For example, we began by meeting with administrators and teachers to encourage them to have a shared understanding of the goals of the intervention (guideline #1). We provided incentives in the form of rationales for the intervention and the provision of curriculum materials and especially professional development (guideline #9).

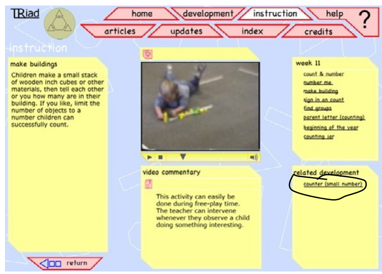
To promote equity (guideline #2), the TRIAD intervention used the Building Blocks curriculum, which had been empirically validated in classrooms serving underrepresented populations (Clements & Sarama, 2007, 2008a). Every classroom received all components and, thanks to the support of administration, (guidelines #1 and #7) adequate auxiliary materials such as computers. All teachers were provided with the same professional development (see guideline #6).

We describe this professional development in detail because, like the measures, it shared most characteristics with similar work in all future studies of TRIAD (guidelines #4 and #6). The first year was a pilot/training year, because our previous experience and others' research suggested that teachers often need at least a year of experience before completely and effectively implementing a curriculum (Berends et al., 2001; Clements & Sarama, 2014; Cobb et al., 2003). Thus, teachers participated in 8 days of professional development during the school day in the first year (a "no stress, gentle introduction" to the curriculum with no data collection by researchers). Introductory discussions emphasized the "developmental appropriateness" of the intervention's mathematics education and its importance to the teachers and children (guideline #5), especially in promoting equity (guideline #2). This work focused on the learning trajectories for each mathematical topic, usually as woven into the *Building Blocks* curriculum (guideline #4). Training addressed each of the three components of the learning trajectories. To understand the *goals*, teachers learned core mathematics concepts and procedures for each topic. For example, they studied the system of verbal counting based on cycling through 10 digits and the concept of place value (based on content like that in National Research Council, 2009).

To understand the developmental progressions of levels of thinking, teachers studied multiple video segments illustrating each level and discussed the mental "actions on objects" that constitute the defining cognitive components of each level. To understand the instructional tasks, teachers studied the tasks, and they viewed, analyzed, and discussed video of the enactments of these tasks in classrooms. A central tool to study and to connect all three components was the internet-based software application, *Building Blocks Learning Trajectories* (BBLT, see Fig. 3). Throughout the 2 years, teachers study the BBLT and engaged in discussions of these videos and correlated text from the *Building Blocks* curriculum. *BBLT* provided scalable access to the learning trajectories via descriptions, videos, and commentaries. Two sequential aspects of the learning trajectories—the developmental progressions of children's thinking and connected instruction—are linked to the others (see Fig. 2). As an example, using these approaches, teachers studied the learning trajectory for counting. Initial presentations including viewing the levels of thinking that constitute the developmental progression, using the BBLT, and discussing the attributes, both mathematical and psychological, of each level. As an illustration, these included the cardinal principle (the last counting word "tells how many"), which is often underemphasized in curricula and teaching; and, at higher levels, competencies in counting on, and other counting strategies. Teachers used the "Test Yourself" feature of the BBLT to evaluate their abilities to diagnose children's level of counting (by identifying the level evinced by children in randomly selected videos). They also used the BBLT's links to view research-based instructional strategies to promote children's progress to the next level. Teachers worked in small groups to plan how activities from their curriculum might promote, or be modified to promote, learning for the relevant levels. The coaches joined the teachers in the participated in professional development, as well as several days of training on coaching, most of which focused on the unique aspects

BBLT provides scalable access to learning trajectories—*developmental progressions* of children's thinking and *instruction*—linked to each other. For example, teachers might choose the instruction (curriculum) view and see the screen on the left. Clicking a specific activity provides a description. Clicking more info slides the screen over to reveal descriptions, several videos of the activity "in action," notes on the video, and the level of thinking in the learning trajectory that activity is designed to develop, as shown below on the right.





Clicking on the related developmental level (child's level of thinking), ringed above, switches to the development view *of that topic and that level of thinking.* Alternatively, the user may have been studying developmental sequences. After choosing development, teachers see a list of the mathematical topics and the developmental sequences. If they had chosen "Counting," then the "Counter (Small Numbers)" level, and then "More Info," they would see the same screen as above. The level of thinking is illustrated by both video of clinical interview assessments *and* video of classroom activities in which children show that level thinking (the icons above the video allow the selection of alternative video), an approach that has received empirical support (Klingner et al., 2003).

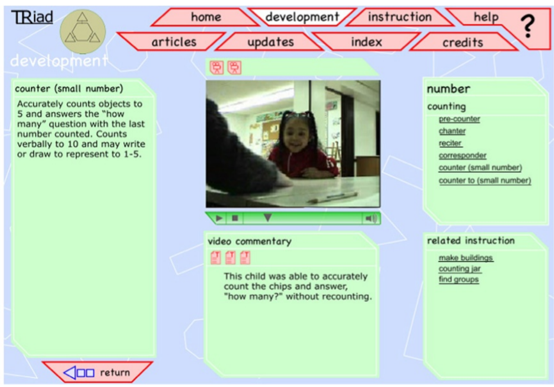


Fig. 3 Building Blocks Learning Trajectory (BBLT) web application.

of coaching early mathematics education. Coaches worked with teachers during the year to avoid dilution of the intervention and to provide teachers with continual feedback and support.

Trainers and coaches also promoted productive adaptations and avoidance of lethal mutations (guideline #8). As an example, for board games, productive adaptations could include changing the die (from 1–6 dots to 1–3 dots for children who are at a lower level, or 5–10 dots or even two cubes for children at higher levels). Such adaptations could simply be changing the theme of the board and tokens to match a class theme (e.g., a trip through a farm and farm animals). In contrast, lethal mutations include changing the game to Candy Land or other games that use a spinner with colors where children just advance to the next instance of that color, eliminating number from the activity. In subsequent sessions, teachers shared and kept lists of their own productive adaptations and discussed how they avoided lethal mutations.

In year 2, teachers and coaches participated in an additional 5 days of professional development (guidelines #6 and #10). They continued to study the learning trajectories, including discussions of how they conducted various curricular activities the previous year. As part of this work, teachers brought case studies of particular situations that occurred in their classrooms to the group to facilitate these discussions; thus, this work included elements of lesson study.

3.1.2 Findings

TRIAD children made significantly and substantially greater gains in mathematics achievement than the control children (effect size = 1.07). There was no evidence that contextual variables influenced these positive effects, such as the Head Start or public programs. Similarly, there was no evidence that effects differed by children's ethnicity or gender. This provides evidence that TRIAD's guidelines worked as a whole, and those addressing equity were successful in particular.

The two observational measures provide additional evidence that greater scores in achievement by the TRIAD group resulted from the implementation of the guidelines. Findings on the fidelity instrument indicate that the teachers implemented the curriculum with "good" fidelity (that is, on a 5-point Likert scales from "strongly disagree" to from "strongly agree" for each item evaluating the quality of teaching, the average was "agree"). This provides support for guidelines especially #4, #5, #6, and #8 as teachers as a group implemented the intervention with high-quality fidelity. In addition, the higher the teacher's fidelity score, the higher the average child gain in achievement in her classroom (although small variance resulted in lack of statistical significance). Furthermore, the TRIAD intervention resulted in consistently greater scores on the quality and quantity of the mathematics environments and teaching in the TRIAD, compared to control, classes. TRIAD classrooms had higher average scores on general classroom behaviors such as teaching more mathematics, showing knowledge of, enjoyment in, and enthusiasm for mathematics, and using effective management and instructional strategies. In summary, the intervention increased the quantity and quality of the mathematics environment and teaching in preschool classrooms, providing empirical support for the guidelines, especially #4, #5, #6, and #9.

3.2 Full-Scale Implementation and Evaluation of TRIAD

Our largest implementation and evaluation to date involved 1375 preschoolers in 106 classrooms serving low-resource communities in two states. Schools in two large urban cities in two states were randomly assigned to three treatment groups: TRIAD, TRIAD with Follow Through (TRIAD-FT), and control (business as usual—in both cities, this involved standards, curriculum, and expectations in preschool mathematics). The TRIAD-FT differed from the TRIAD implementation only after the preschool year, so they are combined for analyses reported here.

3.2.1 Implementation

This implementation was also the most valid and therefore generalizable because the administration of both school systems agreed to the implementation across their districts. Therefore, we could both randomly *select* and randomly assign schools (with small exceptions—schools that had worked with us in previous studies were not included in the selection pool). Further, teachers were not volunteers—a common limitation in many such studies, especially as math is often not the favorite subject of preschool teachers (Copley, 2004; Sarama & DiBiase, 2004).

We attempted a complete implementation of the TRIAD-based intervention in two school districts. We again began with extensive explanations of the intervention, including the rationale for it and the particulars of it, to broad audiences of administrators, teachers, and parents (guideline #1). In this effort, we include planning for the long term, (guideline #3); for example, by planning for training both teachers who were randomly assigned to the control group and those who are new to the district. We also more fully realized guideline #10. For example, we scheduled meetings with district and school leaders more frequently and, when possible, school boards to update them on our results and progress. As another example, parents received letters explaining what their children were doing that week and how they could support their children's learning at home. Beyond this, they were asked to give the project staff advice on the usefulness of these letters and on the way the intervention was or was not helping their children. Such efforts are important to avoid dilution of implementation over years. Leaders did respond well, buying supplemental materials at one site, and actively working on integrating the new curriculum with other curricula at the other. They released and encouraged early childhood coaches to attend the trainings. Several early childhood center leaders asked for additional support after the trainings proper.

The professional development followed the same model as previously described; here we note only the differences. We added a distinction between "mentors" (coaches who were trained by and worked closely with project staff) and in-school teacher coaches within each building. Assigning schools instead of classrooms allowed us to build consensus, support, and camaraderie within each building. Both changes were hypothesized to better implement guideline #5. As much as possible, we included each districts' early childhood and mathematics specialists in these meetings and our professional development to plan for long-term implementation. We encouraged them to rely on teachers in our sessions as teacher leaders to support wider implementation after the conclusion of the data collection (guidelines #3, #7, and #10).

3.2.2 Findings

Teachers implemented the intervention with adequate fidelity (Clements et al., 2011). Again, the modal category for mentor-rated Likert items was "agree." That is, most teachers implemented all aspects of the *Building Blocks* curriculum. Along with previous studies, this evidence indicates that interventions in preschool mathematics education, such as this one, can be successfully implemented on a large scale. Similarly, the TRIAD intervention enabled teachers to develop richer classroom environments for mathematics than those of the control classrooms as measured by the COEMET (the Classroom Observation of Early Mathematics Environment and Teaching measures the quality of the mathematics environment and activities with a half-day observation and can be used with different curricula, see Clements et al., 2011). The TRIAD classes scored significantly higher than the control classes on four components of this measure: the classroom culture subscore, the specific math activities (SMAs) subscore, the number of SMAs, and the number of computers on and working for students to use. The classroom culture subscore assesses teachers' general approach to mathematics education, indicated by "environment and interaction" variables such as responsiveness to children, use of "teachable moments" as well as "personal attributes of the teacher" variables, including appearing knowledgeable and confident about mathematics as well as showing enjoyment in, curiosity about, and enthusiasm for, teaching mathematics. These suggest that the TRIAD intervention successfully altered teachers' beliefs and dispositions, beyond specific curricular practices (guideline #6). Such practices were also positively affected, given that teachers used the computer component of the TRIAD curriculum and engaged their children in a greater number of explicit, targeted, and mathematics activities. The higher scores for the SMA subscale suggest that the SMAs that teachers in TRIAD classrooms conducted were of a higher quality than those in the control classrooms. We will return to the mediational role of these practices after examining the effects on mathematics achievement. Again, these data support the efficacy of implementing guidelines #4, #5, #6, and #8 to avoid dilution and pollution and instead implement with high fidelity, achieving high-quality mathematics teaching and environments.

Thus, children in the Building Blocks group learned more mathematics than the children in the control group (effect size, $g = 0.72$). This is significant, because, unlike most previous research, the counterfactual condition was not the "practice-as-usual" control condition involving no published mathematics curriculum and little district-wide emphasis on mathematics. Both districts had placed new emphasis on pre-K mathematics and adopted new literacy curricula that included specific mathematics components. In addition, the TRIAD intervention itself, especially the first year's "gentle introduction," generated considerable spillover of early mathematics pedagogical practices into control classrooms.

The use of curriculum demonstrated as effective with underrepresented populations, with adequate support for teachers, is supported (guideline #2). Consistent with other research, analyses support the use of curricula that have been empirically supported. Analyses also indicate that the computer software component of the *Building Blocks* curriculum makes a unique contribution to children's learning.

Evidence suggested that the *Building Blocks* curriculum was equally effective in classes serving low- or mixed-income families or with schools that have a higher or lower percentage of children with limited English proficiency. Also, the intervention was equally effective for girls and boys, and for children with or without IEPs. There was evidence that the intervention was differentially effective for only a single comparison: African-American children learned less than other children in the same control classrooms and African-American children learned more than other children in the same Building Blocks classrooms. It may be that this TRIAD intervention is particularly effective in ameliorating the negative effects of low expectations for learning of mathematics (National Mathematics Advisory Panel, 2008).

Thus, there is some support for the implementation avoiding dilution and pollutions from inequitable support and thereby promoting equity (guideline #2, although future research may wish to alter the intervention to close or narrow other gaps as it did for African-American children here).

3.3 Fighting Dilution Over Time: TRIAD and Sustainability

Avoiding dilution during the intervention is necessary, but it may be more important to investigate the dilution that most interventions suffer once the "push" for the intervention has passed—especially for those like TRIAD, which is introduced from outside the school district. A full concept of successful scale-up requires not only consequential implementation and diffusion but also endurance over long periods of time and a transfer of responsibility from any external organization to the internal resources of a school district (Coburn, 2003; Dearing & Meyer, 2006). Does implementation become diluted or polluted over time? Or is it *sustainable*—continued professional practice over time, with a focus on the maintenance of core beliefs and values, and the use of these core beliefs to guide adaptations (Century, Rudnick, & Freeman, 2012; Scheirer, 2005; Scheirer & Dearing, 2011; Timperley, 2011). (Persistence of effects, continuation of the effects of an intervention on individual children's trajectories of learning, is a distinct issue, discussed in other publications, Clements et al., 2017; Clements et al., 2013; Sarama, Clements, Wolfe, & Spitler, 2012; Sarama, Lange, Clements, & Wolfe, 2012.)

We evaluated the fidelity of implementation and the sustainability of TRIAD years after any support from the funded project ended in two studies. The first measured teachers' implementation 2 years later (Clements et al., 2015). Although a logical expectation would be that, after the cessation of external support and professional development provided by the intervention, teachers would show a pattern of decreasing levels of fidelity, these teachers demonstrated increasing levels of fidelity, continuing to demonstrate high levels of sustained fidelity in their implementation of the curriculum and underlying pedagogy. Different profiles of variables predicted separate aspects of sustainability, but by far the most consistent factor was teachers' recognition of children's learning. Teachers who noticed children's substantial development increased their efforts to implement all the components of the intervention with greater fidelity (Clements et al., 2015).

In our latest study, teachers again demonstrated sustained or increasing levels of fidelity as long as 6 years after the end of the intervention without support from the project (Sarama, Clements, Wolfe, & Spitler, 2016). Notable is these teachers' encouragement and support for discussions of mathematics and their use of formative assessment.

We believe there is considerable evidence that dilution especially was mitigated by implementing guidelines #1, #7, and #10: Communication among key groups around a shared vision of the innovation, ensuring school leaders are a central force, and maintaining communication, assessment, and follow-through efforts. Most project activities would not be achieved consistently throughout the districts without such communication and continued collaboration. We found it necessary to repeatedly provide higher-level administrators with updates and reminders of the projects' goals and activities. Similarly, every change in administration had to be monitored and new people introduced to the project and its successes quickly. The early introduction of the project was facilitated by prior awareness of the researchers' work and a strong commitment to raising mathematics achievement.

At the classroom level, both dilution and pollution were minimized because of a strong and consistent implementation of guidelines #6 and #7, addressing professional development and support from leaders, especially coaches. Effects of TRIAD depend on the development of teachers' skills and knowledge. The classroom culture so heavily dependent on the teacher is a consistent mediator of those positive effects. Note that we provided approximately 75 h of out-of-class teacher training as well as hours of mentoring in the classroom, which is substantially more than offered to most teachers, only 6% of whom participate in mathematics professional development for more than 24 h over a year (Birman et al., 2007). A total of 50–70 h of professional development is consistent with previous research documenting what is necessary to achieve measurable effectiveness (Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). Similarly, findings indicate that learning trajectories played a critical role in both teachers' and children's learning, supporting guideline #4: Focus on instructional change that promotes depth of children's thinking, placing learning trajectories at the core.

Finally, teachers repeatedly commented that the project staff respected and collaborated with them as professionals, important for generating a self-sustaining learning community (avoiding dilution via camaraderie support adaptation, guideline #5). After some initial trepidation, teachers came to view peer coaching within schools as a positive characteristic of the implementation. In-class support from mentors and technology support staff (guideline #6) were also appreciated and viewed as important by the teachers after they were sure they were there to help and not "police" implementation. Mentors needed to have the knowledge and skill to demonstrate the implementation of all components of the curriculum, doing activities right in teachers' classroom on many occasions and inviting discussion centered on students' learning.

4. FINAL WORDS

The goal of the TRIAD project was to increase knowledge of scaling up by conducting research that investigates the effectiveness of a research-based mathematics education intervention implemented in varied pre-K settings with diverse student populations. Perhaps most unique is TRIAD's consistent emphasis on teaching for understanding following developmental guidelines, or learning trajectories, as well as its use of technology at multiple levels. TRIAD had a moderate to strong positive effect on children's mathematics achievement and did not harm literacy skills while *increasing* most language competencies measured. There was no evidence the approach is differentially effective for participants in different states or types of programs, or for children of different SES, ethnic group, or gender. The moderate the large effect sizes we obtained is notable considering that other comprehensive reform programs, including multitiered teacher support, sustained professional development, and in-class coaching, achieve effect sizes such as 0.24 only with great effort (Balfanz, Mac Iver, & Byrnes, 2006).

Most people who discuss TRIAD focus on these findings regarding children's learning, and of course without this evidence, little else matters. However, to evaluate the ultimate effects of the project, these longitudinal results are arguably more important—if the intervention is diluted and polluted over time, is it worth the cost and effort? On the other hand, if it *grows, the number of children positively affected is potentially an order of magnitude greater.*

Teachers seemed to have internalized the program (Timperley, Wilson, Barrar, & Fung, 2007). Through the professional development and support, and then, becoming empowered by their own knowledge of the trajectories and the practices to support learners through the trajectories, they became progressively more faithful to the intended program, instead of drifting from it as time elapsed and support disappeared, dilution found in other studies (Datnow, 2005; Hargreaves, 2002).

The TRIAD model is not simply about a new curriculum or training teachers to use it. Success required complex changes, including a change in instructional structures, pedagogical strategies, and classroom communication and culture (Grubb, 2008). Given the importance of early competence in mathematics (e.g., Duncan et al., 2007; Paris, Morrison, & Miller, 2006), the TRIAD implementation described here has implications for practice and policy, as well as research. TRIAD's guidelines should be considered when planning to increase the quality and quantity of not only early mathematics but also to mitigate dilution and pollution threatening any intervention.

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