Exceptional Children

Vol. 76, No. 4, pp. 417-437.

Anchoring Problem-Solving and Computation Instruction in Context-Rich Learning Environments

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ABSTRACT: *Middle school students with learning disabilities in math (MLD) used two versions of Enhanced Anchored Instruction (EAI). In one condition, students learned how to compute with fractions on an as-needed basis while they worked to solve the EAI problems. In the other condition, teachers used a computer-based instructional module in place of one of the EAI problems to deliver formal fraction instruction. The results indicated that students in both instructional formats improved their fraction computational skills and that formal instruction provided an added benefit. Both instructional conditions improved students' problem-solving skills by about the same amount. The findings suggest that combining formal fraction instruction with EAI is a viable way to improve the problem-solving and computational skills of students with MLD.*

The 2007 National Assessment of Educational Progress (NAEP; Lee, Grigg, & Dion, 2007) indicated that 67% of Grade 8 students with disabilities scored below the basic level in mathematics, compared with 26% of students without disabilities. To achieve this basic level, students "should complete problems correctly with the help of structural prompts such as diagrams, charts, and graphs" and demonstrate "the appropriate use of strategies and technological tools to understand fundamental algebraic and informal geometric concepts in problem solving" (Lee et al., 2007, p. 20). Thus, scoring below basic suggests that students have few fundamental skills at the given grade level.

Many of these low-scoring students likely had a learning disability in math (MLD), because 41% of all students with disabilities have an identified learning disability (Planty et al., 2008). Educators often attribute the substandard test performances of students with MLD to a combination of weak problem-solving skills and weak computational skills. When students with MLD solve word problems, they have difficulty identifying relevant information because many have deficits in both math and reading (see Fuchs & Fuchs, 2002; Geary, 1993; Geary, Hamson, & Hoard, 2000; Muth, 1984). In particular, these students experience difficulties focusing attention on key task variables (Kauffman, 2001), self-monitoring during problem solving (Montague, Bos, & Doucette, 1991), and practicing self-management (Bricklin & Gallico, 1984; Gallico, Burns, & Grob, 1991). At the secondary level, students with MLD have trouble organizing information, make more procedural errors, and experience both long-term and short-term memory deficits (Geary, 2004).

Students with MLD also lack computational fluency. The National Longitudinal Transition Study-2 (NLTS-2; Wagner, Newman, Cameto, Levine, & Garza, 2006) found that 14% to 27% of secondary youth with MLD scored more than two standard deviations below the mean of average-achieving students across math subtests. The disability becomes even more serious during middle school and high school, when growth in computational skills slows or stops altogether. A recent study of high school students with MLD (Calhoon, Emerson, Flores, & Houchins, 2007) found that fluency scores on computation tests actually dropped in relation to grade-level expectations. Although most students with MLD had some knowledge of basic facts, their fluency scores were only 14.5% and 8.6% in fractions and decimals, respectively. Given that students with MLD gain only 1 year of achievement in math for every 2 years that they are in school (Cawley & Miller, 1989), educators must make considerable effort to upgrade these students' computational skills with whole numbers (Cawley, Parmar, Foley, Salmon, & Roy, 2001; Cawley, Parmar, Yan, & Miller, 1998), fractions (Behr, Wachsmuth, & Post, 1985), and decimals (Woodward, Baxter, & Robinson, 1999).

Instructional factors may explain the fact that large numbers of students with MLD continue to achieve at low levels. Mastropieri, Scruggs, and Shiah (1991) identified the lack of meaningful exchanges between cognitive scientists, curriculum developers, and practitioners in the disabilities literature long ago, a situation that contrasts sharply with the abundance of theoretical and experimental information about students without disabilities. For example, research in both cognitive science and math has shown that computational skills and problem-solving concepts proceed most naturally in an iterative fashion, with gains in one type of knowledge leading to improvements in the other (e.g., Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Siegler, & Alibali, 2001). Although there is debate about which form of knowledge develops first or how much of one skill set students should learn before the other, the most important finding is that they are related (Baroody & Gannon, 1984; Hiebert et al., 1996) even when students informally learn or only partially understand one set of skills and concepts (Fuson, 1990).

Over the past two decades, mathematics educators (e.g., Baroody, Feil, & Johnson, 2007; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Ginsburg, 1998; Hiebert & Carpenter, 1992; Star, 2005) have consistently emphasized the importance of developing students' computational and problem-solving skills in reciprocal ways. The National Research Council (NRC, 2001) has reinforced this viewpoint in stating that strategic competence (i.e., the ability to solve problems) and procedural fluency "are not independent; they represent different aspects of a complex whole" and "are interwoven and interdependent in the development of proficiency in mathematics" (NRC, 2001, p. 116). Most recently, the National Mathematics Advisory Panel (NMAP, 2008) recommended that students learn key math concepts (e.g., represent fractions on a number line, identify equivalent fractions) and build procedural fluency (e.g., add and subtract fractions) while learning to formulate and solve problems.

Although educators have made important progress in developing the skills of adolescents with MLD concerning equivalent fractions (e.g., Butler, Miller, Crehan, Babbitt, & Pierce, 2003), problem solving (Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra, Hoff, & Beck, 1999; Xin, Jitendra, & Deatline-Buchman, 2005), and algebra (Hutchinson, 1993; Maccini & Hughes, 2000; Witzel, Mercer, & Miller, 2003), rarely do educators teach computational and problem-solving skills together, as recommended. Among older students, interventions usually target the students' recall of math facts, procedural competency with algorithms, or their ability to solve word problems (see Miller & Hudson, 2007). Teachers of low-achieving students often put off introducing more complex and interesting problems until after the students have mastered the basics. This delay can reduce the students' motivation to learn math altogether (Jones, Wilson, & Bhojwani, 1998; Means & Knapp, 1991).

In our research using Enhanced Anchored Instruction (EAI), which we based on the pedagogical approach called *anchored instruction* (Cognition and Technology Group at Vanderbilt, 1990, 1997; Goldman, Hasselbring, & Cognition and Technology Group at Vanderbilt, 1998; Hickey, Moore, & Pellegrino, 2001), we have emphasized problem solving while paying less attention to improving students' computational skills (e.g., Bottge, Rueda, Serlin, Hung, & Kwon, 2007). EAI uses a mix of video-based problems (called *anchors*) and hands-on projects (e.g., building skateboard ramps, compost bins, or hovercrafts). Each anchored problem consists of several subproblems embedded in authentic contexts. Students must develop an understanding of the overall problem, identify the relevant pieces of information that they can use to solve the subproblems, and finally integrate this information into a solution that makes sense. Hands-on projects (e.g., skateboard ramps, hovercrafts) that accompany each video-based problem provide students with additional practice on key concepts. Thus, EAI problems directly immerse students in problem contexts, which is an important benefit for students who have difficulty in reading and math.

Our previous research has focused on helping students understand and use fractions. Fractions represent some of the most complex mathematical concepts that students encounter (Charalambous, & Pitta-Pantazi, 2007; Smith, 2002) and pose major obstacles to mathematical development (Behr, Harel, Post, & Lesh, 1992; Calhoon et al., 2007; Lamon, 1999; Maccini, Mulcahy, & Wilson, 2007; Post, Wachsmuth, Lesh, & Behr, 1985). For many students, misconceptions about fractions are tied to their knowledge of whole numbers (e.g., Baroody & Coslick, 1998; Mack, 1993; NRC, 2001). For example, students may view 2/3 as two whole numbers, rather than as a relationship between two quantities or they may view 1/3 as larger than 1/2 because 3 is larger than 2 (Baroody & Coslick, 1998; Behr et al., 1992). Students may also experience difficulty recognizing that the two numbers that compose a common fraction (numerator and denominator) have a multiplicative relationship rather than an additive relationship, an especially important understanding for finding equivalent, or commensurate, fractions (Steffe, 2004). Although educators can and should build on students' informal knowledge, the transition from working exclusively with whole numbers to working with fractions—especially those represented symbolically—is not a trivial one. Compounding the issue is the fact that students are likely to have less out-of-school experience on which to build with rational numbers than with whole numbers (NRC, 2001).

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In our previous work with EAI, we furnished instruction on whole-number and fraction computational skills to students on an as-needed basis. We found that although students made important gains in problem solving, this approach had minimal effects on their computational skills (Bottge, Rueda, LaRoque, Serlin, & Kwon, 2007; Bottge, Rueda, Serlin, et al., 2007). We had hoped that teachers would use the teachable moments that are frequent during EAI to provide explicit basic skills instruction. We also thought that students would show more readiness for learning the skills that they lacked if they viewed them as tools for solving the EAI problems rather than as isolated procedures. When the as-needed approach did not produce the desired results, we attempted a direct instruction approach, but most students reacted strongly against that form of instruction. In one study, high school students with severe behavior problems refused to work on solving the EAI problems altogether until we discontinued the direct instruction units and went back to teaching computational skills informally with EAI (Bottge, Rueda, & Skivington, 2006). In poststudy interviews, students told us that the direct instruction reminded them of the kind of teaching that their previous teachers had used, which they thought was ineffective and a waste of time.

Because of the difficulty that most children with and without MLD have in representational and operational transformations with fractions, our previous teaching methods with EAI were perhaps shortsighted. In a new approach, we developed a computer-based learning module and a set of concretized tools that we could use alongside our EAI problems to enhance students' fraction-computational skills. We conducted this study to measure the effects of the new version of EAI on the computational and problem-solving performances of middle school students with MLD. In the informal instruction + EAI (*informal*) classes, teachers taught fractions along with EAI on a need-to-know schedule. That method is the way that we have traditionally taught EAI. In the formal instruction + EAI (*formal*) classes, we taught students EAI with the newly developed explicit computer-based instructional modules. Specifically, we designed the study to answer the following research questions:

1. What effect does informal instruction of fractions + EAI have on the computation and problem-solving performance of students with MLD?
2. What additional contribution (if any) does formal instruction of fractions + EAI make to the computation and problem-solving performances of students with MLD?

METHOD

Participants

The participants were 54 students in Grades 6 through 8 who attended three middle schools (Craft, Francis, and Green) near large metropolitan areas in the Pacific Northwest. School enrollments ranged in size from 525 (Craft) to 1,003 (Francis) with White, non-Hispanic students making up 45% to 74% of the population; Black students, 1% to 30%; Hispanic students, 9% to 12%; Asian students, 3% to 11%; and American Indian students, 2% to 4%. The percentage of students receiving free or reduced-price lunch at Francis, Craft, and Green schools was 45%, 51%, and 68%, respectively. Between 12% and 15% of the students were receiving special education services. The Washington Assessment of Student Learning (WASL, 2009), which is administered to students at each grade level, indicated a proficiency range from 42% to 64% in reading and 24% to 36% in math across all students. Pretest scores on the Iowa Tests of Basic Skills (ITBS; Form A; University of Iowa, 2001) confirmed the low math achievement of students in this study. Grade-level equivalents based on mean standard scores of the ITBS were more than two grade levels below those of their peers on both the Computation and the Problem Solving and Data Interpretation subtests.

All participants were receiving their math instruction in self-contained special education settings because their lack of skills prevented them from learning grade-level content with their peers in general education math classes. Most of the students had an identified learning disability in math. According to the Washington State Legislature (2009), students have a learning disability if they display a large discrepancy between intellectual ability and academic achievement. We used a regressed standard score discrepancy method to develop the discrepancy tables. A few of the students did not meet the discrepancy criteria for MLD, but they had also been assigned to these classes because of their long histories of math underachievement.

Table 1 shows student information for each instructional condition. As indicated, the groups were comparable in gender, school, ethnicity, disability, subsidized lunch, and amount of special education service. Students in the informal condition were somewhat younger, but this factor did not seem to influence the effects of the study (see Results and Discussion sections). About three out of four students (76%) were receiving subsidized lunch. Students in each treatment group received more than 13 hr (780 min) of special education services a week, or about 150 min per day.

One special education teacher at each of the three schools participated in the study. All three teachers had a Bachelor's degree in Special Education and had state licensure to teach special education. The teachers had an average of 14 years of teaching experience (Francis, 4 years; Craft, 22 years; Green, 16 years). The teacher at Francis Middle School taught two classes of informal instruction and two classes of formal instruction;

TABLE 1

Description of Students in Instructional Groups

Student Characteristics	Formal + EAI (n = 29)	Informal + EAI (n = 25)	χ^2	p	t	p
Gender			0.01	.973		
Boys	21	18				
Girls	8	7				
Schools			0.95	.620		
Francis	16(2)	17(2)				
Craft	6(1)	4(1)				
Green	7(1)	4(1)				
Grade			16.16	< .001		
6		11				
7	19	10				
8	10	4				
Ethnicity			5.06	.079		
European American	13	13				
African American	4	8				
Hispanic	6	2				
Native American	5					
Asian American	1	1				
Unspecified		1				
Disability/service area				[a].449		
Learning	26	20				
ADHD	2	4				
Emotional/behavioral		1				
Speech/language	1					
Subsidized lunch	19	22	3.71	.054		
Minutes per week in special education					0.41	.681
Average	755	780				
Range	555–900	525–900				

Note. Less populated categories combined for chi-square test: Ethnicity reduced to European American, African American, and Other; Disability reduced to LD or not LD.
[a]Fisher's Exact Test.

the other two teachers both taught one class of each instructional format. All three teachers had taught EAI by using the informal method during the year before the study.

Intervention Procedures

Instruction consisted of two experimental conditions: informal (informal + EAI) and formal (formal instruction + EAI). The three participating teachers attended a 2-day workshop during the summer preceding the school year to become familiar with the lesson plans and the new materials that we had developed for each condition. A middle school math teacher conducted the workshop. He had taught with EAI in his class for 4 years, had a master's degree, and had 15 years of teaching experience. As mentioned previously, the teachers had been using EAI in the traditional way (i.e., informal) for at least 1 year. Binders for teachers contained daily lesson plans and daily checklists of tasks for each unit.

The informal and formal instructional units—including the EAI portion of the instruction—each spanned 24 days at each school. Lengths of class periods were the same within each school and very similar across schools (Francis, 50 min; Green, 54 min; Craft, 55 min). At the beginning of each class period, the teachers led students in a 10-min warm-up activity to review concepts that they had worked on the previous day and to introduce new material. For the remainder of class, each group of students worked on a laptop computer, which contained the learning tools (i.e., scaffolds) that they could use to help solve each of the subproblems presented in the video-based anchor. The teacher circulated among the groups, answering questions and posing new ones. After each group solved the media-based anchor, they worked together on solving the hands-on problem.

Informal Instruction. Informal instruction consisted of three EAI units that required using fractions (e.g., adding and subtracting). The introductory unit was an 8-min-long video anchor called *Bart's Pet Project* that opens with Bart reading a book and occasionally listening to the television. A television advertisement announces sale prices of construction lumber and landscape timbers. The next scene shows Bart counting his money. His friend Billy arrives, and together they discuss Bart's plan to buy a pet. Bart is unsure whether he has enough money to buy both a pet and a cage to put it in. In the next scene, Bart and Billy are in a pet store looking at the prices of pets and cages. As Bart is about to leave the store, he notices a brochure that shows how to build a pet cage. The last scene takes place in Bart's garage, where he and Billy are measuring several lengths of wood for the cage. Bart's challenge is to construct a cage out of 2″ × 2″ lumber and be sure that he cuts the wood in the most economical combinations so that he has enough money left over to buy a pet. The students spent 8 days on this problem.

After solving Bart's Pet Project, the students worked sequentially on two EAI activities—*Fraction of the Cost (FOC)* and *Hovercraft Challenge.* The problems in FOC are analogous to those in Bart's Pet Project, although they are more complex. The FOC scenario describes students who would like to build a skateboard ramp but are unsure whether they have enough money to purchase the wood and materials. The problem requires students to find the solution that is most economical and is within their budget. Students worked together to gather information from the video and access detailed schematic plans that accompany the video. The computer program contains an assortment of learning scaffolds, such as 3-D ramp frames that students can manipulate to see all sides. FOC took students 6 days to solve.

Both Bart's Pet Project and FOC require similar sets of skills. To figure out the pet-cage and the skateboard-ramp problems, students need to

- Read a tape measure.
- Interpret three-dimensional building plans.
- Compute with fractions and whole numbers.
- Understand bank statements and calculate balances.
- Understand and construct tables of materials.
- Compute combinations of wood lengths.
- Convert feet and inches to inches.

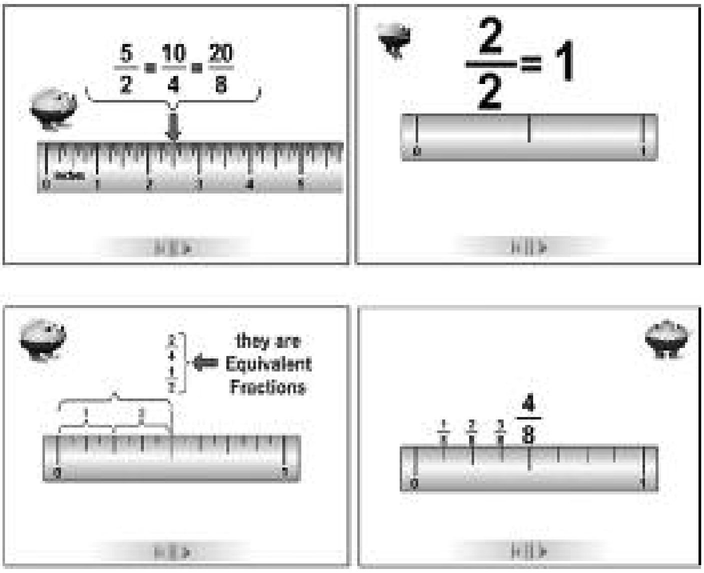
The main difference between the two problems is the level of difficulty.

The third unit is a hands-on problem called the *Hovercraft Challenge*, in which students design and construct a rollover cage out of PVC pipe for a hovercraft that they will ride when they successfully complete the project. A leaf blower powers the hovercraft, which inserts into a plywood platform; the leaf blower blows air through holes in a plastic sheet and lifts the base off the floor. Students designed their rollover cages on graph paper and then used their knowledge of ratios to determine the lengths of the pipe that they needed for the hovercraft. Students also had to stay within a predetermined budget, so they had to determine the total price of the pipe and connectors (45 degree, 90 degree, Ts). Students then cut the pieces that they needed for constructing their rollover cage and assembled it. On the final day of the activity, students attached their rollover cage to the hovercraft and rode it in the hallways of the school. The skills and concepts needed for solving the *Hovercraft Challenge* parallel those in Bart's Pet Project and FOC. Students worked on the hovercraft unit for 10 days.

Formal Instruction. Like the informal condition, the formal condition consisted of three units. Two of the units were the same as those taught in the informal condition (FOC and Hovercraft). However, in place of Bart's Pet Project, teachers taught a unit that provided explicit instruction on key fraction concepts. Research on the development of students' understanding of fractions and effective instructional strategies (e.g., Baroody & Coslick, 1998; Smith, 2002)

FIGURE 1

Screen Shots From Fractions at Work



guided the development of the unit, which helped develop students' understanding of fraction notation and equivalence and helped boost their ability to compute simple and mixed fractions.

For much of the explicit instructional sequence, teachers taught students with a computer-based program called *Fractions at Work*, which contains seven chapters divided into self-contained units. We developed *Fractions at Work* for use with EAI to help students understand concepts (e.g., fraction equivalence) and enhance their computational skills (e.g., adding and subtracting fractions). Figure 1 shows screen shots of the program. The first chapter of *Fractions at Work* describes properties of fractions, including their purpose and function. The goal of the second chapter is to help students understand the concept of equivalence. The computer screens use fractions on an interactive tape measure to show how the value of fractions depends on the number of parts into which an inch is divided (i.e., denominator) and the number of these parts available (i.e., numerator). In the third chapter, the program shows students how to add simple fractions that have the same denominator. *Fractions at Work* uses multiple examples to emphasize why adding denominators is not appropriate.

Chapters 4 and 5 describe how to add simple fractions with unlike denominators. The program first teaches students to determine whether they can multiply the smaller denominator by some number to make it equal to the larger denominator. If they can, the program then shows students how they can multiply the numerator and denominator by the same number. Numerous examples help students understand that this procedure does not change the value of the fraction. When it is not possible to use the larger denominator as the common denominator, *Fractions at Work* shows students how to find least common multiples. Chapter 6 leads students through a series of steps on adding and subtracting mixed numbers, including procedures for renaming and simplifying fractions. The last chapter summarizes the content in the previous chapters and includes an assessment to determine students' understanding. Throughout the program, the reporting function of the software provides teachers with assessments of each student's skills.

In addition to working with the program *Fractions at Work*, students used concrete materials such as fraction strips to help them understand the concept of equivalence. Teachers gave narrow strips of tagboard to students and asked them to imagine that one of the strips was a long candy bar that two people wanted to share. Students represented how to share the candy bar by folding the strip in half, which they labeled 1/2. The students repeated the process with the other three strips—candy bars that four, eight, and sixteen people wanted to share—by using repeated folding to show each segment, which they then labeled. When students had labeled all four of the fraction strips, the teacher posed questions about equivalence, relative size, and the relationship between numerator and denominator, for example, What are other names for 1? and What is another name for 3/8? In the days that followed, students used their fraction strips to solve such problems as the following: If you have 3/4 of a candy bar and your friend has 1/2 of a candy bar, who has more candy? and How much more does she have? Students used their strips to show and discuss how to add and subtract fractions. Teaching the formal instruction unit took 8 days, which was the same number of days that Bart's Pet Project took.

Measures

We used two researcher-developed criterion-referenced tests and two standardized norm-referenced achievement measures to assess students' performance before and after instruction. During the intervention training, the teachers learned how to administer the tests. Teachers used scripted directions for test administrations, which a classroom observer monitored. Graduate assistants received training so that they could score the tests by using protocols. We calculated interrater reliability on 20% of pretests and posttests and computed it by dividing the number of agreements by the total number of agreements and disagreements and multiplying by 100 (Sulzer-Azaroff & Mayer, 1977).

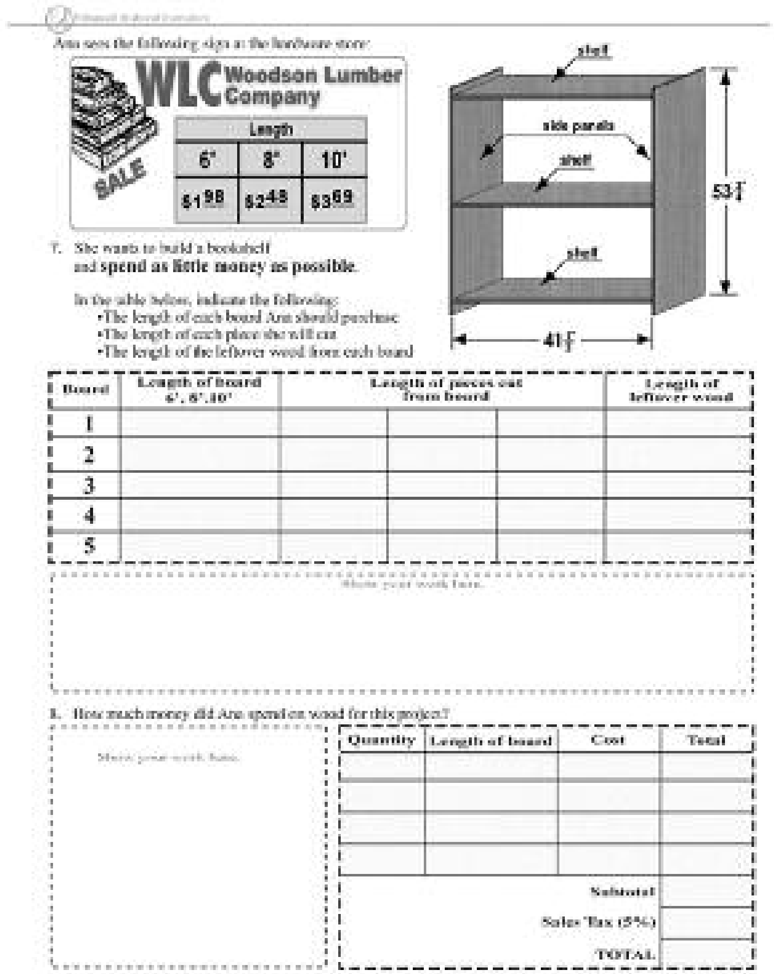
Fractions Computation Test. Students in each instructional condition took the 20-item (14 addition, 6 subtraction), 50-point criterion-referenced Fractions Computation Test before and after instruction. Items asked students to add and subtract fractions with like denominators (e.g., 1/4 + 3/4), unlike denominators in which the larger denominator could serve as the common denominator (e.g., 7/8 – 1/4), and unlike denominators where neither could serve as the common denominator (e.g., 8 2/9 + 2 1/2). Items also differed on whether the fractions appear on a ruler (e.g., 3/4 + 7/8) or do not appear on a ruler (e.g., 1/5 + 1/3). The test included simple fractions and mixed numbers as well as addition of three fractions (e.g., 1/16 + 1/8 + 1/2). The instructions asked students to reduce their answers to simplest form and to show all their work. Students could earn two or three points per item, depending on whether simplification of the answer was necessary. Students received one point for correctly identifying equivalent fractions with common denominators (on items where this was applicable) even if the final answer was incorrect. Students had as much time as they needed during a typical 50-min class period to complete the test.

The internal consistency estimate (α) of the Fractions Computation Test was .97, which is within the range of internal consistency estimates of this same test used in three previous studies: .94 (Bottge, 1999), .91 (Bottge, Heinrichs, Mehta, & Hung, 2002), and .97 (Bottge et al., 2004). The interrater reliability was 98% (range 53%–100%).

Problem Solving Test–Revised. The authors developed a 48-point Problem Solving Test to assess the skills and concepts that students learned with EAI. The items aligned to the National Council of Teachers of Mathematics (NCTM, 2000) standards recommended for students in Grades 6 to 8 (e.g., Numbers and Operations, Measurement, Problem Solving, Communication, and Representation). We grouped sets of items into concept and skill areas and then weighted them according to difficulty and contribution to the solution of the overall problem. One group of items measured the ability of students to understand a bank statement and calculate 10% and 20% of the balance. Other items asked students to calculate how much wood they should buy to build a bookcase (see Figure 2). To answer this set of items, students had to interpret building plans where dimensions were in fractional measurements, decide how to use the boards to waste as little wood as possible, determine the lengths of unused boards, and calculate the total material costs, including sales tax. Within each item set, students could earn partial or full credit (1 to 6 points) depending on how complete their answers were.

The Problem Solving Test was similar to versions used in previous studies (i.e., constructed response items, full or partial credit, items weighted according to complexity), but it also differed in important ways. First, it was almost 25% longer than previous versions. Second, although the con-

FIGURE 2
Bookshelf Problem From the Problem Solving Test



cepts tested remained the same, the question formats and problem contexts in the new version differed from those that the students had worked on during instruction. For example, students had to interpret building plans that they had never seen before and transfer their applications to new contexts.

The internal consistency estimate (α) of this test was .90, which is consistent with problem-solving tests used in two previous studies: .80 (Bottge, Rueda, Serlin, et al., 2007) and .90 (Bottge & Stephens, 2009). The interrater reliability was 95% (range 70%–100%).

Standardized Tests. The ITBS (University of Iowa, 2001) math subtests (Computation, Problem Solving and Data Interpretation) measured math skills before and after instruction. According to the publisher, the tests reflect the spirit of *Principles and Standards for School Mathematics* (NCTM, 2000). The ITBS Computation subtest requires students to perform calculations with whole numbers, fractions, and decimals. The Problem Solving and Data Interpretation subtest includes one- and two-step word problems. Several items present tables and graphs of data that students must interpret to compare quantities and determine relationships. We administered the subtests according to the directions in the test administration booklet. Students could use calculators for the problem-solving tests only. According to the publisher of the ITBS, Kuder-Richardson Formula 20 (K-R20) reliability estimates at Level 12 are .814 for the Computation subtest and .842

for the Problem Solving and Data Interpretation subtest.

Research Design

A pretest–posttest cluster randomized experiment tested the efficacy of the two instructional interventions on students' ability to compute and solve problems. We randomly assigned classes of students at each of the three schools to informal or formal instruction conditions. Thus, students were nested within classes with the instructional treatment as the class-level variable. Students in each of the classes took the two researcher-developed tests (Fractions Computation and Problem Solving) and the two standardized subtests (ITBS Computation, ITBS Problem Solving and Data Interpretation) just before and after the instructional intervention.

Implementation Fidelity

We used data from multiple sources to evaluate the extent to which teachers followed the prescribed lesson plans and to help describe what may have accounted for the test results (O'Donnell, 2008). First, a university instructor and a doctoral student observed classrooms at weekly intervals throughout the study and recorded their observation notes directly into an electronic relational database by using a template in FileMaker Pro 7 that we designed especially for this purpose. We derived the content of the templates from the daily lesson plans that we had given to teachers during their 2-day training. It included space for observers to record routine information (e.g., date, school, condition, unit of instruction) and verify treatment fidelity (e.g., "completed warm-up activity in lesson plan," "provided students time to problem solve independently or with peers"). We trained observers to use the observation system in EAI classrooms not associated with the study.

We collected a total of 51 out of 192 (27%) whole-class-period observations and noted only slight variations in the way that the teachers implemented the two interventions. The most common departure from planned activities was completion of the warm-up activity. Teachers taught these activities as prescribed in the teacher manual 77% of the time. In some instances, teachers skipped the warm-up activities or adapted them to meet the specific needs of their students. Completion of main activities in the lesson plan, provision for independent work, attention to needs of individual students, and practice time during formal instruction occurred during 90%, 100%, 98%, and 98% of the observed lessons, respectively. On several occasions, teachers were unable to finish the entire lesson plan in one day for various reasons (e.g., shorter class time, behavioral problems, other school activities), so the lesson continued on the following day. We considered these adjustments to the lesson plans to be minor. Overall, there were no serious inconsistencies between what the lesson plans prescribed and how teachers carried them out.

Other fidelity checks included an electronic logbook into which one teacher described her daily activities and those of her students. The research staff also conducted 30-min telephone interviews with each teacher at the conclusion of the study. The scripted interview questions addressed three major areas, including (a) what the students learned, (b) how they implemented the curricula, and (c) specifics about implementation of the research plan. The observations, daily logbook entries, and interviews gave us confidence that teachers taught the EAI lessons mostly as intended.

Results

Table 2 shows the means and standard deviations on the four math achievement measures of students in the two instructional conditions. We used Hierarchical Linear and Nonlinear Modeling 6 (HLM6; Raudenbush, Bryk, Cheong, & Congdon, 2004) to conduct separate three-level hierarchical linear models to assess possible differences in intercept (i.e., pretest value) and slope (i.e., time of test) between the two treatment groups for each dependent variable (Raudenbush & Bryk, 2002). The structure of the model is as follows:

$$\text{Level 1 } \text{Score}_{jit} = \pi_{0i} + \pi_{1i}(\text{Time}) + \varepsilon_{jit}$$
$$\text{Level 2 } \pi_{0i} = \beta_{00j} + r_{ji}$$
$$\pi_{1i} = \beta_{10j}$$
$$\text{Level 3 } \beta_{00j} = \gamma_{000} + \gamma_{001}\,\text{Formal}_j + u_{0i}$$
$$\beta_{10j} = \gamma_{100} + \gamma_{101}\,\text{Formal}_j$$

TABLE 2

Means and Standard Deviations of Students by Instructional Group

Measure and Comparison	*Pretest* M	*Pretest* SD	*Posttest* M	*Posttest* SD
Fractions Computation Test				
Formal + EAI	3.14	4.22	30.24	10.27
Informal + EAI	1.67	2.62	17.72	13.09
Problem-Solving Test				
Formal + EAI	6.14	4.40	17.08	9.08
Informal + EAI	5.07	4.69	15.88	12.55
ITBS Math Computation Subtest				
Formal + EAI	190.00	20.01	194.68	22.76
Informal + EAI	183.52	18.39	189.64	15.39
ITBS Math Problem Solving & Data Interpretation Subtest				
Formal + EAI	188.46	20.07	195.57	29.34
Informal + EAI	182.48	12.07	190.56	26.91

Note. All tests: *Formal Instruction* (n = 29), *Informal Instruction* (n = 25). ITBS scores are standard scores.

It denotes i as student in class j, t as the tth score for student i (pretest, posttest), and time as either 0 for pretest score or 1 for posttest score. Formal $_j$ is 0 if class j received informal instruction and 1 if class j received formal instruction because the type of instruction is a class-level variable (Level 3). Therefore, γ_{000} represents the mean initial status (pretest) for the informal condition, γ_{001} represents the average incremental initial status for the formal instruction, γ_{100} represents mean growth rate from pretest to posttest for the informal instruction, and γ_{101} represents the average additional growth rate for formal instruction. The model contains only one random effect on intercept in the classroom level because the analysis makes the typical assumption of homogeneity of variance, specifically that the variability within classes is constant over time. The model reports effect size (ES) on change scores of the informal instructional group (γ_{100}) and for the additional increase attributable to formal instruction (γ_{101}). We computed it by dividing the growth rates by the standard deviation of the difference scores for both informal and formal instruction

$$\left(ES = \frac{\gamma}{\sqrt{S^2_{pre} + S^2_{post} - 2\rho_{pre,post} S_{pre} S_{post}}} \right)$$

Table 3 shows that pretest scores between students in the formal and informal classes did not differ on the Fractions Computation Test. Estimated posttest scores were 17.51 for the informal group and 28.68 for the formal group. The informal group scored significantly higher (about 16 points) on the posttest than on the pretest (ES = 1.14). Students in the formal group scored an average additional 11 points over students in the informal group (ES = 0.81).

To obtain a more complete picture of the nature of students' computation, we further analyzed results at the item/procedural level. Table 4 shows the means and standard deviations of students in each instructional condition for items according to procedural type. We conducted a series of analyses of variance (ANOVAs) with repeated measures on each item type to determine how students in each instructional condition fared from pretest to posttest. Table 5 shows the comparisons of improvement scores by instructional condition across test times. Students in the formal condition made greater gains than students in the informal group on all but one skill (i.e., computing with fractions that have the same denominators).

Table 6 indicates that pretest scores on the Problem-Solving Test did not show a difference between the formal and informal conditions. The estimated average posttest score of students in the informal group was about 18 points, which was

TABLE 3

HLM 3-Level Model of Treatment Effects on the Fractions Computation Test

Fixed Effects		*Estimate*	SE	t	df	p
Initial status	*(Informal + EAI)* γ_{000}	1.80	1.87	0.96	6	0.373
Initial difference	*(Formal + EAI)* γ_{001}	1.49	2.59	0.58	6	0.585
Growth rate	*(Informal + EAI)* γ_{100}	15.71	2.25	6.97	96	< 0.001
Incremental growth rate	*(Formal + EAI)* γ_{101}	11.18	3.24	3.57	96	0.001

Random Effects		*Variance*	χ^2	df	p
Initial status		61.02			
Level 2	*Student*	8.44	53.38	42	.112
	Class	2.14	10.93	6	.090

about 11 points higher than their pretest score and statistically significant. The average posttest score of students in the formal group was about the same as that of students in the informal group. The informal group showed a significant improvement from pretest to posttest (ES = 1.16), but no additional increase occurred for students in the formal group (ES = 0.03). Figure 3 shows a graphic representation of results for both researcher-developed tests.

On the ITBS subtests, we found no pretest differences between students in the formal and informal groups, no statistically significant gains on posttest scores, and no additional benefit from being in the formal group on the ITBS subtests. On the Computation subtest (Table 7), estimates of pretest and posttest scores were 183.70 and 188.35 for the informal group and 189.84 and 190.12 for the formal group, respectively. There were also no differences on the Data Interpretation and Problem-Solving subtest (Table 8). Estimated pretest and posttest scores increased from 181.47 to 189.43 for students in the informal group and were about the same on the pretest (186.92) and posttest (186.97) for students in the formal group.

Because some students in the informal group were younger than students in the formal group, we conducted a post hoc analysis to determine whether we could find an association between achievement differences and grade level. We found no differences between sixth-grade students and students in seventh and eighth grades on pretest, posttest, and difference scores on the Fractions Computation Test and the two ITBS subtests. Thus, the evidence does not suggest that the statistically significant improvement on the Fractions Computation Test by students in the formal group was associated with grade level. A somewhat different finding occurred for the Problem Solving Test. Although sixth-grade students did not differ from the other students on the pretest, $t(22) = 0.96$, $p = .37$, they made less improvement, $t(22) = 2.88$, $p < .01$, and they scored lower on the posttest, $t(22) = 2.80$, $p = .01$, than the older students. As previously indicated, the HLM analysis found no difference between the formal and informal groups on the Problem Solving Test. However, if an equal number of older students had been in the informal group, their problem-solving scores might have surpassed those of students in the formal group. This phenomenon would make sense because they learned one additional EAI problem.

DISCUSSION

We designed this study to test a new version of EAI for achieving the dual purpose of advancing students' computational skills with fractions and developing their problem-solving skills. In previous studies with EAI (e.g., Bottge, Rueda, Serlin, et al., 2007), students with MLD approximated the performance of general education students on

TABLE 4

Means and Standard Deviations on Fractions Computation Test by Item Type, Instructional Group, and Time of Test

	Formal + EAI		Informal + EAI	
Skill Comparison and Time of Test	M	SD	M	SD
Addition (35)				
Pretest	2.00		<1.00	
Posttest	23.21		13.52	
Difference	21.21	7.16	12.60	9.27
Subtraction (15)				
Pretest	1.09		<1.00	
Posttest	7.03		4.20	
Difference	5.94	3.74	3.32	4.12
Simplify				
Need to simplify (30)				
Pretest	2.21		1.37	
Posttest	17.55		10.48	
Difference	15.34	5.39	9.11	7.52
No need to simplify (20)				
Pretest	<1.00		<1.00	
Posttest	12.70		7.24	
Difference	11.76	4.58	6.92	5.60
Denominator				
Same (10)				
Pretest	2.48		1.67	
Posttest	7.48		6.16	
Difference	5.00	3.09	4.49	3.61
Larger (35)				
Pretest	<1.00		<1.00	
Posttest	20.24		10.68	
Difference	19.69	7.40	10.68	10.29
New (5)				
Pretest	<1.00		<1.00	
Posttest	2.52		<1.00	
Difference	2.41	1.68	0.88	1.64
Ruler				
Related to (35)				
Pretest	2.24		1.37	
Posttest	20.83		12.48	
Difference	18.59	6.62	11.11	9.40
Unrelated to (15)				
Pretest	<1.00		<1.00	
Posttest	9.41		5.24	
Difference	8.55	3.41	4.92	3.85

Note. Numbers in parentheses ( ) indicate total number of points possible.

the problem-solving measure, but their computational skills remained disappointingly low. The new version of EAI added explicit instructional modules for teaching the meaning and purpose of fractions, the parts of fractions, and the procedures for adding and subtracting fractions.

The results of the HLM analyses indicated that students in the informal group, who learned fractions on an as-needed basis, made statistically significant improvement on the Fractions Computation Test, which has not happened in previous studies with EAI. Although this improvement was

TABLE 5

Improvement Scores on Fractions Computation Test for Formal Versus Informal Instruction

Measure and Comparison	*Contrast Estimate*	SE	*Observed* t	p *Value*
Addition				
Formal versus informal	8.57	2.11	4.06	$p < .001$*
Subtraction				
Formal versus informal	2.61	1.08	2.57	$p = .013$*
Simplify				
Need to simplify				
Formal versus informal	6.34	1.66	3.81	$p < .001$*
No need to simplify				
Formal versus informal	4.84	1.31	3.69	$p = .001$*
Denominator				
Same				
Formal versus informal	0.64	0.89	0.72	$p = .473$
Larger				
Formal versus informal	9.01	2.27	3.97	$p < .001$*
New				
Formal versus informal	1.53	0.43	3.60	$p = .001$*
Ruler				
Related to				
Formal versus informal	7.55	2.07	3.65	$p = .001$*
Unrelated to				
Formal versus informal	3.63	0.93	3.90	$p < .001$*

*Significant, $\alpha = .05$ familywise, Holm (1979).

encouraging, it may not be practically important because the mean posttest score was below what we would consider a satisfactory performance level. A more promising finding was that the combination of explicit instruction of fractions and EAI provided an additional boost for students in fraction computation on overall test score and subskill comparisons.

On the Problem-Solving Test, students in both instructional conditions made statistically significant gains on the posttests, and no performance difference between groups occurred. Substituting *Fractions at Work* for Bart's Pet Project did not seem to depress the problem-solving performance of students in the formal group. However, post hoc analysis indicated that differences in the groups were attributable to grade level, so the added problem-solving instruction could have generated higher problem-solving scores for informal instruction if the two groups had been similar in age.

As for the standardized measures, the Computation Subtest showed an upward but nonsignificant trend, as did the Problem-Solving and Data Interpretation Subtest. Because the ITBS Computation Test measures a wide range of skills (e.g., whole-number operations, decimals) in addition to fraction computation, it is not surprising that the gains that students made on the criterion-referenced Fractions Computation Test did not show up on the standardized measure. Although the researcher-developed and standardized tests purported to measure the same construct (i.e., problem solving), differences in the format and content of the ITBS (e.g., standard word problems) may have limited students' ability to show what they learned from EAI. As Schoenfeld (2006) observed, results from traditional standardized measures can lead to false negatives because the lack of test coverage and the way that problems are represented limit their use for capturing what students really understand. That statement may be especially true for low-achieving students.

We view these results with guarded optimism because, unlike in our previous research with EAI, students showed elevated achievement in problem solving and computation. Accomplishing both

TABLE 6

HLM 3-Level Model of Treatment Effects on the Problem-Solving Test

Fixed Effects		Estimate	SE	t	df	p
Initial status	*(Informal + EAI)* γ_{000}	6.51	2.89	2.25	6	0.064
Initial difference	*(Formal + EAI)* γ_{001}	–0.27	4.06	–0.07	6	0.949
Growth rate	*(Informal + EAI)* γ_{100}	11.29	1.87	6.02	96	< 0.001
Incremental growth rate	*(Formal + EAI)* γ_{101}	–0.29	2.60	–0.11	96	0.911
Random Effects		*Variance*		χ^2	df	p
Initial status		42.20				
Level 2	*Student*	2.06		46.06	42	0.307
Level 3	*Class*	25.50		62.01	6	< 0.001

objectives has been difficult because the adolescents in our studies have reacted strongly against learning fractions, especially when educators used explicit instructional methods. We did not detect these attitudes in the present study. All three teachers reported that the combination of *Fractions at Work* and concretized instruction (e.g., fraction strips) helped students understand what fractions represent conceptually and how to work with them procedurally. Some students could find equivalent fractions in their heads. In fact three of them improved so much that they went back to their regular math class and remained there the rest of the school year.

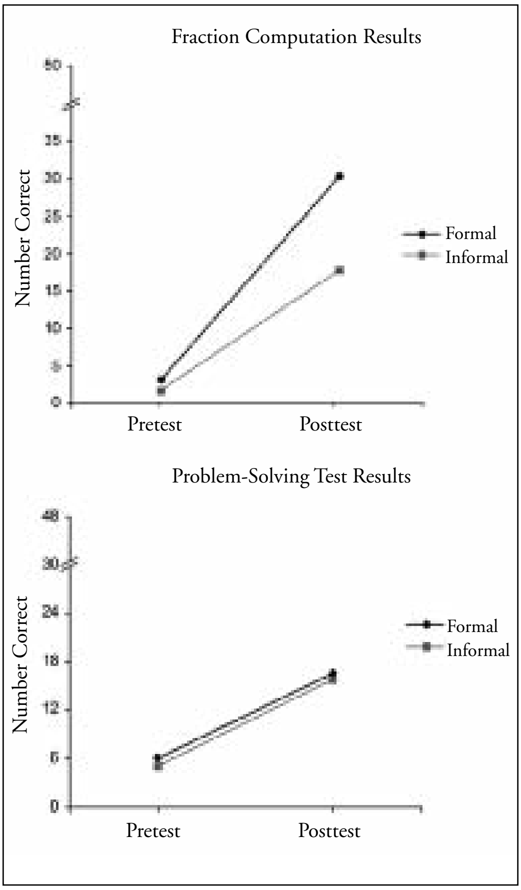
We think that we can attribute these results, in part, to elements of the Key Model for learning mathematics (Bottge, 2001), which guided the design of the problem-solving and explicit teaching modules. Important principles of the model include situating authentic-like problems in engaging contexts, tapping and building on students' previous knowledge, and affording students opportunities to apply their knowledge in a variety of contexts. The design properties of the media-based and hands-on EAI curriculum align closely with those described in the Institute of Education Sciences (IES) *Practice Guide for Organizing Instruction to Improve Student Learning* (Pashler et. al., 2007).

However, the findings of this study and recent reviews (Maccini et al., 2007; Woodward, 2004) suggest that our design models may also need modification. For example, if we follow the advice of researchers in cognitive science and mathematics education, the revised model would consider the reciprocal benefits of teaching computation and problem solving in complementary ways. Both the NRC (2001) and NMAP (2008) acknowledged the reinforcing properties and importance of developing students' conceptual understanding and procedural fluency (e.g., computation) along with strategic competence and adaptive reasoning (e.g., problem solving).

Second, the new model should acknowledge the importance of developing curricula specifically for secondary school students who have grown to dislike math and put very little effort into learning it. One of five attributes that the NRC (2001) associated with math proficiency was *productive disposition*, which conveys to students the sensible, useful, and worthwhile purposes for learning math. To stimulate this feeling in adolescents with MLD, general educators and special educators alike need to consider using instructional methods and materials that align more closely with what adolescents regard as purposeful. Older students need to see explicit connections between what teachers expect them to learn and how it will help them in the future. This redesign will most likely include the expanded use of technology and more complex math content, which in turn will require more staff development (Loveless, 2004; Maccini & Gagnon, 2006; Stodden, Galloway, & Stodden, 2003).

FIGURE 3

Results of Problem-Solving and Fractions Computation Tests



Finally, the new model will need to address the logistics of conducting research with newer instructional methods like EAI. With the current emphasis on testing, administrators often give educational research low priority, which logically impedes classroom-based research (Chval, Reys, Reys, Tarr, & Chavez, 2006). In previous studies, math teachers and special education teachers have agreed to implement EAI because they understood that one EAI problem provides them with opportunities to teach several skills and concepts at once. For example, Fraction of the Cost aligns with such NCTM (2000) middle school standards as Number and Operations (i.e., work flexibly with fractions to solve problems), Geometry (i.e., investigate relationships by drawing, measuring, and visualizing geometric shapes), Measurement (i.e., understand relationships among units and convert from one unit to another), Problem Solving (i.e., solve problems that arise in mathematics and other contexts), and Communication (i.e., formulate explanations of problems). The challenge will be to create new understanding among school officials that the term *basic* now includes much more than just computational fluency.

Limitations

The results of this study hold promise for developing more effective math curricula for adolescents with MLD, but several limitations exist. First, teachers taught both intervention conditions in an effort to reduce variability in background and teaching skills across teachers. Teachers might have inadvertently used methods meant for one condition in the other condition, although fidelity checks did not detect this problem.

Second, although we did not intend the Problem-Solving Test to be a mastery test, the students' low performance surprised us. The growth was statistically significant, but posttest scores of students in both groups were less than half the total number of points possible. In a companion study conducted during the same year (Bottge & Stephens, 2009), we found that this test was very difficult even for general education middle school students. The mean pretest and posttest scores of the general education students were about 8 points above those of students in this study. Gain scores were about the same for both groups of students (10 points).

Finally, we did not design our study to identify many of the possible mediating variables (Hiebert & Grouws, 2007) that may have led to our findings, such as time on task and specifics related to the nature of teacher-student interactions. Thus, we are unable to identify with precision the specific features of instructional practices that were more or less responsible for our findings.

Implications for Practice

Research findings and policy reports (e.g., *America's Perfect Storm: Three Forces Changing Our Nation's Future*, Educational Testing Service, 2007; *Tough Choices or Tough Times: The Report of the New Commission on the Skills of the American Workforce*, National Center on Education and the

TABLE 7

HLM 3-Level Model of Treatment Effects on the ITBS Computation Subtest

Fixed Effects		*Estimate*	SE	t	df	p
Initial status	*(Informal + EAI)* γ_{000}	183.70	3.82	48.09	6	< 0.001
Initial difference	*(Formal + EAI)* γ_{001}	6.14	5.24	1.17	6	0.287
Growth rate	*(Informal + EAI)* γ_{100}	4.65	4.06	1.15	96	0.255
Incremental growth rate	*(Formal + EAI)* γ_{101}	1.77	5.57	0.32	96	0.751
Random Effects		*Variance*		χ^2	df	p
Initial status		189.51				
Level 2	*Student*	145.13		106.97	41	< 0.001
Level 3	*Class*	0.15		6.78	6	0.341

Economy, 2007; *Building Academic Skills in Context: Testing the Value of Enhanced Math Learning in CTE*, National Research Center for Career and Technical Education, 2006) have advocated greater emphasis on problem solving because new jobs will require workers to cooperate with their coworkers to solve problems by using technological tools. The development of EAI over the past several years represents our response to the spirit of these recommendations for helping students with MLD accomplish tasks that align better with workplace expectations. Early in the development of EAI, it included only a video-based problem and one related hands-on problem. We gradually added cognitive supports that students could use to reduce the cognitive load generated by the complexity of the problems.

When instruction is too constrained and repetitive, it can dampen students' enthusiasm to learn.

However, good problem solvers must also be able to compute accurately, which is a skill that we found that we could address in combination with EAI. In effect, we have combined theoretical

TABLE 8

HLM 3-Level Model of Treatment Effects on the ITBS Problem-Solving and Data Interpretation Subtest

Fixed Effects		*Estimate*	SE	t	df	p
Initial status	*(Informal + EAI)* γ_{000}	181.47	4.64	39.14	6	< 0.001
Initial difference	*(Formal + EAI)* γ_{001}	5.45	6.37	0.85	6	0.426
Growth rate	*(Informal + EAI)* γ_{100}	7.96	5.62	1.42	94	0.160
Incremental growth rate	*(Formal + EAI)* γ_{101}	–2.46	7.71	-0.32	94	0.751
Random Effects		*Variance*		χ^2	df	p
Initial Status		363.21				
Level 2	*Student*	86.46		59.75	41	0.029
Level 3	*Class*	7.00		9.78	6	0.133

and instructional orientations that serve several purposes. As Woodward (2004) points out, it is important in special education to consider instructional methods that maintain a balance between direct and constructivist philosophies. The move to constructivist orientations often inhibits the learning of students who need more guided instruction. However, when instruction is too constrained and repetitive, it can dampen students' enthusiasm to learn—especially the enthusiasm of older students (Boaler, 2002). Perhaps the most important implication of this study is that both types of instruction can coexist; and when they do, they can produce encouraging results. The challenge for special educators is to strike a balance of instructional methods that develop a wide range of math skills that low-achieving students can apply in future transitional settings (e.g., postsecondary education, work).

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The research reported in this article was supported by a grant from the U.S. Department of Education, Institute of Education Sciences Cognition and Student Learning (CASL) Program, Award No. R305H040032. Any opinions, findings, or conclusions are those of the authors and do not necessarily reflect the view of the supporting agency.

We thank Ann Black and Jill Hallows at the Special Education Technology Center, Central Washington University, and the dedicated teachers who participated in this research.

Manuscript received August 2008; accepted August 2009.