

Evaluating Three Elementary Mathematics Programs for Presence of Eight Research-Based Instructional Design Principles

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Abstract

The present review builds on earlier research that evaluated the curricular features of core math programs to improve the performances of students with or at risk for mathematics difficulties. In this review, three elementary math programs, at Grades 2 and 4, were evaluated for the presence of eight instructional principles. Math intervention studies have empirically validated these principles for promoting math proficiency of students struggling with mathematics. Data were collected via a researcher-developed scoring rubric. Findings indicate that adherence to the instructional principles varied markedly within and across programs. In addition, the results indicated that the current textbooks contain a general lack of explicit instruction and provide too few practice opportunities to teach material to mastery. Implications for future curricular reviews and enhancing core math instruction are discussed.

Keywords

instructional design, math textbooks, learning disabilities, mathematics instruction, mathematics curriculum

Accumulating evidence indicates that many children experience an early and lasting onset of difficulties in mathematics (Aud & Hannes, 2010; Morgan, Farkas, & Wu, 2009). The National Center for Education Statistics (2009), for example, reports that just 39% of Grade 4 students scored at or above the *proficient* level on the 2009 National Assessment of Educational Progress (NAEP). At the same grade level, 19% of students with disabilities met this critical benchmark. The data further suggest that student performances worsen in the later grades. On the Grade 8 NAEP, just 34% of students met proficiency. Only 8% of students with disabilities in Grade 8 scored at or above the proficiency level.

One plausible way to address the relatively low math achievement of U.S. children is to enhance the quality of core math instruction delivered in general education classrooms. Fundamental to reaching this goal and thus boosting core math instruction is implementing effective math textbooks. Math textbooks play a prominent role in general education classrooms (Reys & Reys, 2006). On a daily basis, math textbooks largely define the type of math content taught and are likely to represent the main source of math instruction that students receive in a given school year. Textbooks should meet the instructional needs of struggling students as well as those students who are typically achieving. Textbooks also influence the ease and manner in which teachers deliver effective core instruction. They provide teachers with the instructional foundation from which modifications can be added to increase

instructional intensity for struggling learners (Chard & Jungjohann, 2006; Doabler et al., in press). The design of textbooks also helps determine the extent to which students are actively engaged in learning. In particular, textbooks can provide guided opportunities for students to interact with teachers and peers in critical math content (Carnine, 1997).

Despite the important roles of core textbooks, concerns linger over the lack of coherence in U.S. math instruction (Doabler et al., in press; Schmidt, Houang, & Cogan, 2002). An underlying worry is that the textbooks used in core math instruction fail to align with the diverse learning needs of students with or at risk for math difficulties (MD). When textbooks fall short in instructional quality and are not anchored in the principles of instruction that are empirically validated to increase student achievement, it is likely that these students will experience persistent difficulties. Thus, it is important to understand whether math textbooks provide teachers with a solid instructional base for teaching key math concepts and skills to students struggling with mathematics.

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Core Math Instruction and Its Role in Math Proficiency

Core math instruction has a unique role in helping students become proficient in mathematics. Math proficiency comprises two knowledge forms: conceptual knowledge and procedural knowledge (Kilpatrick, Swafford, & Findell, 2001; Wu, 1999). Conceptual knowledge involves understanding the relationship between representations of math concepts and abstract symbols. The latter knowledge form consists of the ability to perform math procedures fluently and effortlessly. A student with procedural knowledge can retrieve answers to basic number combinations by memory. In contrast, a student who lacks procedural fluency will rely on less sophisticated strategies, such as finger counting, to solve basic number combinations. A consistent finding of research is that students with MD often have difficulty developing conceptual knowledge and acquiring procedural fluency (Fuchs et al., 2010; Gersten, Jordan, & Flojo, 2005).

What features of core math instruction are necessary to promote math proficiency for students with MD? A common recommendation among researchers (Schmidt et al., 2002), mathematicians (Van de Walle, 2001; Wu, 2009), and expert panels (Kilpatrick et al., 2001; National Math Advisory Panel [NMAP], 2008) is for core instruction to address a select set of key topics and to teach those topics more thoroughly. A second recommendation is to provide in-depth and systematic exposure of fundamental content that is crucial for later math learning. The recently released Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010), for example, expect first-grade students to understand the relationship between addition and subtraction. Consequently, core math instruction in first grade should devote sufficient instruction toward teaching students a deep, rich understanding of the operations of addition and subtraction. Instruction should also be coherent and reflect the hierarchical nature of mathematics (Schmidt et al., 2002). Coherent instruction strategically connects new concepts and skills with previously learned material by addressing the necessary component skills (e.g., addition with no renaming) before introducing the more complex topic (e.g., addition with renaming).

Core math instruction should also reflect the principles of instruction that experimental research has empirically validated to promote math proficiency for typically achieving students (Haas, 2005; NMAP, 2008) and students with or at risk for MD (S. Baker, Gersten, & Lee, 2002; Gersten, Chard et al., 2009). These principles include (a) explicit and systematic instruction, (b) visual representations of mathematical ideas, (c) opportunities for students to verbalize their thought processes, and (d) carefully selected and sequenced teaching examples. One of the most consistent findings of this research base is that students with MD benefit more from consistent use of explicit and systematic

instruction compared with other instructional techniques (NMAP, 2008). In a recent practice guide, released by the Institute of Education Sciences (IES), a recommendation was made for classrooms to include explicit and systematic instruction when teaching struggling learners (Gersten, Beckmann, et al., 2009). IES considered the level of empirical evidence supporting this recommendation as strong for raising mean achievement levels. According to the practice guide, explicit and systematic instruction provides students with clear teacher demonstrations, scaffolded instruction, guided practice, academic feedback, and cumulative review.

Recent meta-analyses on studies that tested the effectiveness of math instruction support the use of explicit and systematic instruction for improving the learning outcomes of struggling learners (S. Baker et al., 2002; Gersten, Chard, et al., 2009; Kroesbergen & Van Luit, 2003). S. Baker et al. (2002) found that explicit instruction had the largest effects ($d = 0.65$) on math achievement for at-risk learners in the seven intervention studies reviewed. More recently, Gersten, Chard, et al. (2009) reviewed 11 studies for using explicit instruction to teach math to students with learning disabilities and reported a large effect, $g = 1.22$. In addition, Gersten and colleagues reported relatively large effects for interventions that selected and sequenced instructional examples ($g = 0.82$) and used visual representations of math concepts during instruction ($g = 0.46$).

Incorporating rich and varied practice opportunities is another key principle of core math instruction. Studies of core math instruction have found student practice opportunities, such as written exercises and math verbalizations, important for students struggling with mathematics (Gersten, Chard, et al., 2009). Math verbalizations refer to students thinking aloud in their solving of a problem. For example, a math verbalization would consist of a student explaining the decimal equivalent of a fraction. A verbalization would also be two students describing the attributes of a triangle.

Clarke et al. (2011) examined the effect of practice opportunities on student outcomes in a recent randomized control trial of a core kindergarten math curriculum, *Early Learning in Mathematics* (ELM). In the study, 65 classrooms were randomly assigned to treatment or control conditions. Results indicated that practice opportunities, including student verbalizations, provided by ELM classrooms were more effective than control classrooms in raising mean achievement levels of students with and at risk for MD. Clarke and colleagues also found that ELM classrooms with higher rates of practice opportunities were more beneficial to students with and at risk for MD than ELM classrooms with lower rates of student practice.

Status of Core Math Programs

Over the last 15 years, a body of research has exposed concerning weaknesses in U.S. math curricula. When comparing

the focus and coherence of U.S. math curricula with those of countries who are top performers on the Trends in International Mathematics and Science Study (TIMSS), Schmidt et al. (2002) found concerning disparities. For instance, the curricula of Singapore, South Korea, and Japan covered a considerably smaller set of topics at each grade level in comparison with the “laundry-list approach” found in the United States (Schmidt et al., 2002, p. 12). These countries also placed much more emphasis on building mastery with the foundations of math (e.g., whole numbers) in Grades 1 through 5 before transitioning to more complex topics (e.g., decimals). Schmidt and colleagues also found marked differences in the number of grades in which topics spanned. Whereas the top-performing TIMSS countries covered topics across an average of just 3 years, many of the topics in the United States spanned from Grades 1 to 8. With greater focus on breadth versus depth, the oft-cited comment about U.S. math curricula being a “mile wide and an inch deep” (Schmidt et al., 2002, p. 3) may be particularly apt.

Researchers in the area of special education have also identified problematic curriculum design features that might adversely affect students with MD in mastering the critical concepts of early mathematics. Carnine, Jitendra, and Silbert (1997), in an early study, compared three Grade 5 basal programs to determine if they met six factors of pedagogical effectiveness for teaching addition and subtraction of fractions. These factors centered on (a) fundamental concept identification and integration, (b) timing of introducing a big idea, (c) rate of introducing new concepts and principles, (d) clarity of teacher demonstrations, (e) effectiveness of manipulative activities, and (f) adequacy of practice and review. Findings from the investigation exposed problematic areas across the mathematics programs. According to Carnine and colleagues, these weaknesses pose “areas of serious concern for teachers with diverse learners” (p. 395). For instance, all three programs failed to identify and introduce fundamental concepts, such as understanding equivalent fractions. In addition, the programs included vague teacher demonstrations and inadequate opportunities for student practice.

Jitendra et al. (2005) investigated both instructional principles and adherence to the Process Standards proposed by the National Council of Teachers of Mathematics (NCTM, 2000; i.e., problem solving, communication, reasoning, connections, and representations) in five Grade 3 textbooks. Although problem-solving opportunities were present in most materials, Jitendra et al. found that the textbooks lacked sufficient occasions for students to verbalize their mathematical thinking. Within the context of number sense instruction, Sood and Jitendra (2007) evaluated four Grade 1 programs for their adherence to the principles of instruction, including mediated scaffolding, conspicuous instruction, teaching big ideas, judicious review. Findings from this evaluation suggest that adherence varies widely between traditional (*Harcourt Brace, Houghton Mifflin,*

Scott Foresman) and reform-based textbooks (*Everyday Mathematics*). For example, Sood and Jitendra found that traditional textbooks provided a more direct approach to instruction and academic feedback compared with the reform-based textbooks. Moreover, the reform-based programs provided fewer practice and review opportunities.

More recently, Bryant et al. (2008) investigated the instructional quality of four widely used mathematics textbooks by examining applications of several instructional principles. Specifically, Bryant and colleagues looked at the clarity of objectives, the amount of instructional practice, and academic feedback. The authors also focused on features of vocabulary instruction, teaching of prerequisite skills, number of skills taught, inclusion of progress monitoring measures, and the recommended use of mathematical manipulatives. Comparable with the previously described studies (i.e., Carnine et al., 1997; Jitendra et al., 2005; Sood & Jitendra, 2007), Bryant and his colleagues found that most curricula did not align with the effective principles of instruction. All of the programs achieved an “acceptable” rating for the feature of providing clear instructional objectives; however, most demonstrated limited presence of the remaining features.

In summary, the findings of these textbook evaluation studies offer implications relevant to the design of core mathematics programs, particularly in highlighting what instructional features textbooks should contain. A common theme among these reviews is that textbook designers need to increase the explicitness of instruction and include more opportunities for student practice. Short of this, textbooks will fail to fulfill their role in helping prevent MD and meeting the needs of students struggling with mathematics.

Purpose of the Review

In this article, we evaluate math textbooks of three popular programs for the presence of eight principles of instruction, at Grades 2 and 4. Previous curricular reviews have suggested that elementary math textbooks lack the principles of instruction deemed important for students with MD (Bryant et al., 2008; Carnine et al., 1997; Sood & Jitendra, 2007). However, in attempt to meet the new demands on math learning (e.g., Common Core State Standards, 2010), today’s math textbooks have become arguably more comprehensive than even those published just a few years ago. Therefore, it is important to *expand this research base by using a comprehensive review process for examining more recently published textbooks to determine if they contain the principles of instruction that can meet the needs of students struggling to learn mathematics*. This information may prove useful to teachers to ensure that students with or at risk for MD are placed in programs that incorporate research-based features of instructional design and delivery. It may also have major implications for teachers who

work with dated core programs or even current programs that do not systematically attend to the design principles that are evaluated in our curricular review. Consequently, to effectively teach students with diverse instructional needs, educators will need to understand these principles of instruction to modify and adapt even the top math programs. Therefore, the following research question guided the review: To what extent do commercially available, core math textbooks attend to the principles of instruction for teaching key math concepts and skills to students with or at risk for MD?

Method

As part of a larger curricular review for a school district located in the Northwest region of the United States, core mathematics textbooks from three different publishers were examined at the Grades 2 and 4 levels. The larger review included Grade 8 math programs. All three programs were approved for the California and Oregon adoption list for mathematics. These grade levels were selected because they serve as critical junctures in preparing students for later mathematics (Wu, 2009). Programs were selected by the school district based on interest in placing them on a districtwide adoption list. At the time of this review, Programs A and B were being used in several schools in the district. These two programs (Programs A and B) are leading sellers in the U.S. elementary textbook market (Reys & Reys, 2006). The third (Program C) represents the national curriculum of a top-performing country on the TIMSS and is a relative newcomer on the U.S. curriculum market compared with the other two programs. The programs are considered distinct from one another given their structure, expectations for student learning, and instructional approach for teaching elementary school mathematics. Program A is a reform-based program, whereas Programs B and C are more traditional-based programs. Similar to Bryant et al. (2008) and Carnine et al. (1997), the present review does not disclose the names of the textbook publishers. This way, the review focused on individual textbooks rather than between-program comparisons.

Three key topics per grade level were analyzed for the review. Topics analyzed at Grade 2 were (a) place value—up to 1,000, (b) addition with renaming, and (c) telling time—increments of 5 min. At Grade 4, topics included (a) decimal equivalent for common fractions, (b) multiplication of multidigit numbers, and (c) estimating and measuring area. We considered three topics at each grade level as a brief snapshot for how a textbook may function across a given school year for teaching critical math content. Topics were selected based on their inclusion in the NCTM Focal Points (2006) and the Common Core State Standards (2010), and because they are fundamental for developing mathematics proficiencies in the elementary curriculum and

preparing students for coursework in algebra (Van de Walle, 2001; Wu, 2009). For each topic, two raters independently reviewed the core lesson and any additional lessons directly associated with the topic. Teacher manuals and student textbooks were examined for the current review.

The review team consisted of three former general and special education teachers, and a practicing school psychologist, all of whom hold doctoral degrees. It is important to note that raters were not employed by the school district, which commissioned and funded the current review. The four raters were divided into two review teams, with the lead author serving on the second review team. The first team reviewed Programs A and B at the Grades 2 and 4 levels, whereas the second team reviewed Program C at the same levels.

Rating and Evaluation Procedures

An eight-item scoring rubric was developed to systematically evaluate the textbooks (see Table 1). The rubric was based on findings of scientific research on effective math interventions for students with MD (S. Baker et al., 2002; Gersten, Chard, et al., 2009; NMAP, 2008). Identified within these respective literatures as beneficial for teaching students struggling with mathematics were features of explicit instruction and opportunities for student practice. It is worth noting that several school districts, located in the northwest region of the United States, successfully piloted the rubric during curricular adoption processes. Preliminary evidence from the pilot test suggested that reviewers could reach satisfactory agreement levels (i.e., above .85 agreement) for using the items to measure the instructional design elements of elementary math programs.

The next section describes eight research-based principles of instruction: preteaching of prerequisite skills, teaching of math vocabulary, explicit instruction, selection of instructional examples, math models to build conceptual understanding, multiple and varied practice and review opportunities, teacher-provided academic feedback, and formative feedback loops.

Prerequisite skills. Teaching prerequisite skills is an important and necessary step for connecting previously learned information with newly acquired knowledge (Bransford, Brown, Cocking, Donovan, & Pellegrino, 2000; Kame'enui & Simmons, 1999). For instance, learning how to add and subtract multidigit numerals requires prior knowledge of the base-10 system (Van de Walle, 2001). A textbook, therefore, should preteach concepts of place value before introducing computational algorithms for multidigit addition and subtraction (Caldwell, Karp, & Bay-Williams, 2011). In the review, raters examined whether textbooks specifically identified prerequisite skills necessary for successful acquisition of the new material. Raters also considered whether opportunities to teach the prerequisite skills were

Table 1. Scoring Rubric

Principle	1	2	3	4	Score
Prerequisite skills	Does not identify prerequisite skills or provide warm-up activities.	Identifies prerequisite skills, but provides warm-up activities unrelated to target topic.	Identifies prerequisite skills, however, offers limited warm-up activities to engage students' background skills.	Clearly identifies prerequisite skills and provides sufficient warm-up activities to engage students' background skills.	
Math vocabulary	Does not identify vocabulary words.	Identifies key vocabulary but at the beginning of the chapter or unit rather than individual lessons.	Identifies and directly embeds (e.g., underlines) key vocabulary. Does not provide opportunities for students to use vocabulary.	Identifies and directly embeds (e.g., underlines) key vocabulary within lesson. Also, offers opportunities for students to use vocabulary.	
Explicit instruction	Does not provide opportunities for teacher-led instruction. Instructional activities use a discovery learning approach.	Provides limited opportunities for teacher-led instruction.	Provides opportunities for teacher-led instruction, but limited directions on how to teach the target topic.	Systematically provides opportunities for teacher-led instruction. Offers explicit directions on how to directly teach the target topic.	
Instructional examples	Does not provide teaching examples.	Provides poorly selected teaching examples. Examples are either irrelevant to student practice.	Provides clear teaching examples, however, examples are limited in number (i.e., only two examples).	Provides clear and sufficient number of teaching examples. Also, examples are at least as complex as student practice.	
Math models	Does not incorporate math models.	Incorporates math models for student use only. Does not incorporate models during teacher-led instruction.	Incorporates math models for teacher and student use. Provides limited directions on how teachers should use models during teacher-led instruction.	Appropriately incorporates models throughout the lessons (e.g., C-R-A approach). Provides clear directions on how to teach with the models.	
Practice opportunities and cumulative review	Does not provide practice and review opportunities.	Provides opportunities for math verbalizations but those offered are limited in structure (e.g., peer discourse). Includes limited opportunities for written exercises.	Provides opportunities for math verbalizations but are limited in number. Includes opportunities for written exercises, but does not allow for discrimination practice.	Provides sufficient opportunities for math verbalizations, including occasions for group and individual responses. Also, includes written exercises and discrimination practice.	
Academic feedback	Does not provide academic feedback or correction procedures.	Includes only brief <i>hints</i> for anticipating student errors and misconceptions.	Provides <i>hints</i> to anticipate student errors and misconceptions. Procedures for correcting errors and reteaching are limited.	Provides <i>hints</i> to anticipate student errors and misconceptions. Provides procedures for academic and corrective feedback. Also, offers reteaching strategies.	
Formative feedback loops	Does not provide assessments or opportunities to check for student understanding.	Provides end-of-unit assessments only.	Provides opportunities to check for student understanding and identifies an acceptable criterion for moving on with instruction.	Provides frequent opportunities to check for student understanding and identifies an acceptable criterion for moving on with instruction. Also, it has procedures to link test results.	

Note: C-R-A = concrete–representational–abstract.

provided. This could be accomplished through a warm-up activity. Textbooks earned the highest rating if opportunities were provided to engage student background knowledge and teach prerequisite skills.

Math vocabulary. Occasions to build and use math vocabulary are beneficial for both teachers and students. For teachers, it provides explicit opportunities to teach precise definitions of math vocabulary. Imprecise definitions or mathematically inappropriate language taught can lead to later misconceptions (Wu, 2009). For students, math vocabulary instruction facilitates engagement in important mathematical discourse, such as math verbalizations (Gersten, Chard, et al., 2009). Raters examined whether lessons explicitly identified key vocabulary, and provided accurate and student-accessible definitions. Raters also examined if sufficient opportunities for students to use the vocabulary

during classroom discourse were offered (Donovan & Bransford, 2005). Textbooks earned the highest rating if key vocabulary was identified and opportunities for use were provided.

Explicit instruction. There is converging evidence that explicit and systematic instruction is the most effective method for teaching students with or at risk for MD (S. Baker et al., 2002; Gersten, Chard, et al., 2009; NMAP, 2008). For our purposes, explicit and systematic instruction consisted of concise teacher directions, opportunities for model demonstrations, scaffolded instruction, guided practice, and teacher-student interactions. Raters examined whether lessons contained sufficient opportunities to directly teach key math concepts and skills. Raters also examined if the instructional opportunities were scaffolded across the lessons to promote learner independence.

Instructional examples. To optimize student understanding, textbooks should provide clear and appropriate teaching examples (Chard & Jungjohann, 2006; Doabler et al., 2012). These examples should be systematically selected and ordered throughout the lessons. In a meta-analysis of instructional interventions for students with learning disabilities, Gersten, Chard, et al. (2009) calculated a mean effect size of .82 for intervention studies that included carefully sequenced instructional examples. For the current review, raters considered whether instruction began with simpler teaching examples rather than more complex ones. Raters also examined if lessons provided a sufficient range of positive teaching examples, with limited use of negative examples. Finally, raters considered whether teaching examples were as complex as the ones used during student independent practice (Chard & Jungjohann, 2006).

Math models. An effective method for building student conceptual understanding is a concrete–representational–abstract (C-R-A) process of instruction (Hudson & Miller, 2006). Within a C-R-A context, instruction systematically includes opportunities for students to use visual representations of mathematics such as number lines, tally marks, strip diagrams, and counting blocks. Math models help students struggling with mathematics to understand the relationship between math representations and abstract symbols. Gersten, Chard, et al. (2009) reported a mean effect size of .46 for studies that used visual representations while problem solving. Raters examined whether lessons judiciously incorporated visual models of math. They also considered whether lessons scaffolded the models as learner independence was purported to develop.

Practice opportunities and cumulative review. Purposefully designed review and practice facilitates student automaticity of math skills and maintenance of previously learned material (Kilpatrick et al., 2001). Review and practice opportunities should provide teachers with accurate snapshots of student performance and understanding. Raters examined whether practice opportunities for math verbalizations were present and sufficient enough to facilitate student learning. For example, raters searched for cues or prompts that directed teachers to have students read problems, answer questions, and justify answers. Raters also considered whether written practice activities covered new and previously learned material, and incorporated opportunities for students to discriminate when and when not to apply newly learned skills. Discrimination practice is considered important for generalizing math skills in other contexts (Stein, Silbert, & Carnine, 2006).

Academic feedback. Addressing student errors is an important aspect of math instruction (Stein et al., 2006). Although mistakes are expected to occur in learning, unattended errors are likely to lead to later misconceptions in math. As a result, teachers require procedures for correcting errors, eliciting correct responses, and modeling correct responses (Hudson &

Miller, 2006). Raters examined lessons to determine whether there were opportunities to address student misconceptions during the lesson. Raters also examined if error correction procedures were provided.

Formative feedback loops. Scientific studies suggest that providing teachers with recommendations for how to adjust instruction based on student performance data has a significant impact on the outcomes of students with MD. In a recent meta-analysis of math intervention research for students with math disabilities, Gersten, Chard, et al. (2009) found a small but statistically significant effect size (.23) for providing performance feedback information to teachers. When interventions provided teachers with more in-depth information, such as the type of content to teach and when to progress to new material, the synthesis revealed stronger effects. Raters examined whether textbooks offered procedures to make data-driven decisions. For example, raters considered whether specific guidelines were provided to assist teachers in deciding whether it was appropriate to progress to the next lesson or reteach a target topic based on student performance data.

Item Scoring

For scoring the principles of instruction, raters applied a 4-point rating scale. A 4-point rating was used over a dichotomous “yes/no checklist” and a 3-point rating scale to account for potential variability of textbook quality and prevent raters from consistently selecting a middlemost rating. A rating of 1, the lowest score, indicated that the item was completely missing from the textbook. For example, a textbook received the lowest rating if lessons contained no teaching examples. In contrast, a rating of 4, the highest score, suggested that full criteria were met for the principle. For instance, a textbook earned the highest rating if teaching examples were sufficient and carefully sequenced to provide in-depth coverage of a key topic.

The 2- and 3-point ratings were incorporated to more finely account for variation of the principles with textbooks. A rating of 2 indicated that the item appeared in less than 50% of the instructional material. For example, a textbook earned a score of 2 if teaching examples were present in the introduction of the lesson but missing during the problem-solving sections. A rating of 3, the minimum acceptable score, indicated an inconsistent application or presence of a principle. For example, a textbook earned a 3 if there was a sufficient amount of teaching examples, but the examples provided were sometimes less complex than those problems included in the student independent practice sections.

Summary scores were derived for each of the eight principles (e.g., teaches prerequisite skills) by averaging the scores from two raters. At each grade level, each instructional principle had three reported means, one for each topic (i.e., place value, addition with renaming, telling time in second grade). Therefore, for a given program, at each grade level, there were

Table 2. Mean Scores for Principles of Instruction at Grade 2

Principle	Program A			Program B			Program C		
	PV	AD	TT	PV	AD	TT	PV	AD	TT
Prerequisite skills	2.0	2.0	1.5	1.0	4.0	1.0	3.0	3.0	1.0
Math vocabulary ^a	2.0	1.5	2.0	1.0	3.0	2.0	3.0	3.0	3.0
Explicit instruction ^a	2.0	2.5	2.0	3.0	3.0	2.5	3.0	3.0	3.0
Instructional examples ^b	2.0	2.5	2.5	3.0	3.0	3.0	3.0	3.0	3.0
Math models ^b	3.0	3.0	3.0	3.0	3.0	3.0	4.0	4.0	4.0
Practice opportunities and cumulative review	2.0	2.0	2.0	3.0	3.0	2.5	3.0	3.0	2.0
Academic feedback	1.0	2.0	1.0	1.0	2.0	2.0	3.0	2.0	1.0
Formative feedback loops	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0

Note: PV = place value; AD = addition with renaming; TT = telling time.

^aInstructional principle met by at least one textbook across all three topics.

^bInstructional principle met by two or more textbooks across all three topics.

24 summary scores. Across the three programs, at each grade level, a total of 72 summary scores were calculated for the review. A mean score of 3 or greater was considered an acceptable score.

Interrater Reliability

Prior to the review, the three raters met with the lead author (the fourth rater) for 3 hr to discuss the review process and the scoring rubric. To establish initial interrater reliability and to familiarize raters with the review process, all four raters independently analyzed a pilot lesson that addressed addition with renaming from a second-grade textbook not included in the study. Reliability was calculated as the number of agreements divided by the number of agreements and disagreements multiplied by 100. This resulted in a mean interrater agreement of 90% for the pilot lesson. For the study, average interrater reliability across all programs (A, B, and C) was 83%. As an additional check for interrater agreement, the lead author, who served on the second review team, independently reviewed all lessons from Grades 2 and 4 from Programs A and B (approximately 66% of all lessons included in the study). The interrater reliability between the lead author and the first review team was 85%. Raters met periodically throughout the study either in person, through email, or over the phone to compare ratings and discuss the review. It is important to note that these rater discussions did not influence the interrater calculations.

Results

Raters reviewed three elementary math programs at the Grades 2 and 4 levels for presence of eight principles of instruction. Three topics were examined per grade level. Each rater independently applied an eight-item rubric with a 4-point rating scale. A rating of 1 represented the lowest

score; a rating of 4 represented the highest score. Meeting the minimum criteria for a selected topic required a mean score of 3 or greater. Tables 2 and 3 present summary scores for Grades 2 and 4, respectively. At Grade 2, 32 out of 72 (44%) summary scores met the minimum criteria. At Grade 4, 24 out of 72 (33%) summary scores met the minimum requirements for acceptability. Findings for the presence of each principle are presented below.

Prerequisite Skills

The first principle of instruction requires that the prerequisite skills be addressed before the introduction of the targeted topic. This review must entail the skills and concepts necessary for success with the lesson content. Moreover, the review must be presented through a warm-up activity. At the second-grade level, no textbook we reviewed met the minimum criteria for teaching the prerequisite skills of place value and telling time. Program B for Grade 2 demonstrated an acceptable score for addition with renaming. Among the Grade 4 textbooks, only one (Program B) met the minimum requirement for this instructional principle across all three topics. Program C met the minimum requirement for teaching the prerequisites of multiplication and fractions. For the remaining Grade 4 textbook, prerequisite skills were inconsistently addressed across the topics.

Math Vocabulary

The second principle of instruction focuses on teaching key math vocabulary. This principle requires vocabulary to be defined in precise mathematical language and opportunities to practice using the vocabulary to be offered throughout the lessons. One of the Grade 2 textbooks taught key math vocabulary across the targeted topics (Program C), whereas the remaining two textbooks were mainly inconsistent in

Table 3. Mean Scores for Principles of Instruction at Grade 4

Principle	Program A			Program B			Program C		
	MP	FR	AR	MP	FR	AR	MP	FR	AR
Prerequisite skills ^a	2.0	1.0	1.0	4.0	3.0	3.0	3.0	3.0	2.0
Math vocabulary ^a	3.0	3.0	3.0	3.0	2.5	2.5	3.0	3.0	1.5
Explicit instruction	2.0	2.0	1.5	3.0	2.5	2.5	3.0	3.0	2.0
Instructional examples	2.0	2.0	2.0	2.0	2.0	2.0	2.5	2.5	2.5
Math models ^a	2.0	2.0	2.0	3.0	2.0	2.0	3.0	3.0	3.0
Practice opportunities and cumulative review	2.5	2.5	2.5	3.0	2.5	2.5	2.5	2.5	2.5
Academic feedback ^a	2.0	2.0	2.0	3.0	3.0	3.0	3.0	3.0	1.0
Formative feedback loops	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0

Note: MP = multiplication; FR = fractions; AR = measuring area.

^aInstructional principle met by at least one textbook across all three topics.

identifying vocabulary. With one exception (Program B, Grade 2), raters noted that direct methods for teaching vocabulary did not exist for the other Grade 2 textbooks. At Grade 4, one textbook (Program A) met the minimum criteria for teaching vocabulary across the three topics. Program C adequately addressed math vocabulary in the lessons related to fractions and measuring area, whereas Program B met criteria in multiplication.

Explicit Instruction

The third principle, and arguably one of the most important for students with MD, has to do with explicit and systematic instruction. This principle requires sufficient teacher-directed opportunities throughout each lesson, such as think-alouds and frequent teacher–student interactions. Lessons must be clear enough so that a teacher could model and demonstrate the key components of the target topic. Of all the textbooks, only one (Program C, Grade 2) demonstrated an explicit and systematic instructional approach across all three topics. Two textbooks (Program B, Grade 2; Program C, Grade 4) met criteria in two of the topics. The remaining textbooks contained either few or no teacher-directed opportunities. For example, Program A’s Grades 2 and 4 textbooks used a discovery learning approach. Raters noted that lessons within these textbooks typically began with whole-class discussion or child-directed activities and then transitioned to student independent work.

Instructional Examples

The fourth principle requires textbooks to contain effective instructional examples that are teacher-directed. The examples must be clear, sufficient, and appropriately sequenced to facilitate student understanding. Of all the textbooks, two met this instructional principle across all three topics (Program B, Grade 2; Program C, Grade 2). The instructional examples included in these textbooks provided in-depth coverage of

the topics reviewed. Instructional examples within the remaining Grade 2 textbooks were found insufficient to promote student learning. Similar patterns were noted for the Grade 4 textbooks. For example, several textbooks were missing fundamental teaching examples within the introductory lessons of each topic. Others used complex examples at the introduction of each topic and thus posed potential confusion for students with MD.

Math Models

The fifth principle of instruction has to do with the incorporation of math models during instruction. The principle requires models to be judiciously selected, appropriately sequenced, and sufficiently applied to build student conceptual understanding. All three of the Grade 2 textbooks met this principle across the target topics by providing a C-R-A sequence. Of the Grade 4 textbooks, one (Program C) met the minimum criteria in all three topics. With the exception of Program B (Grade 4) in teaching multiplication, math models were inconsistently used in the remaining textbooks. For example, Program A’s Grade 4 textbook incorporated math models for student use but failed to include them for teacher demonstration.

Practice Opportunities and Cumulative Review

The sixth principle requires lessons to provide frequent opportunities for student practice and cumulative review. Practice opportunities must be in the form of student verbalizations and written exercises. Although raters noted that practice opportunities existed across the textbooks, none of the six textbooks provided sufficient practice to promote learning mastery across all of the target topics. At Grade 2, Programs B and C did provide adequate practice for teaching place value and addition with renaming. At Grade 4, Program B demonstrated an acceptable score in multiplication. Raters

also noted that review activities lacked discrimination practice across all textbooks.

Academic Feedback

The seventh principle has to do with teacher-provided academic feedback. This principle requires lessons to offer correction procedures for addressing student errors and potential misconceptions. Only one textbook (Program B, Grade 4) met the minimum criteria for this principle across all three topics. Raters noted that the textbook provided opportunities or “Error alerts” to anticipate student errors. For Program C at Grades 2 and 4, one and two of the topics, respectively, provided academic feedback opportunities. The remaining textbooks were missing correction procedures for teachers.

Formative Feedback Loops

The eighth principle requires textbooks to provide procedures for linking assessment results with instructional decision making. Formative feedback loops must offer specific procedures for teachers on how to proceed or not proceed based on student performance data. Whereas all six textbooks offered end-of-unit assessments, none of the textbooks included procedures for linking assessment data with instructional decision making.

Discussion

Mathematics textbooks are cornerstones of core math instruction. When well designed, their instructional framework helps teachers deliver effective instruction so that all students, including students with MD, gain cognitive access to fundamental content. In contrast, poorly designed textbooks offer little instructional support to teachers and are likely to be insufficient to teach students with MD.

The purpose of this curricular review was to examine three popular elementary math programs, at Grades 2 and 4, for presence of eight instructional principles. Our contention was that a high-quality program, or one to consider for core math instruction, should demonstrate acceptable evidence of the principles across three important topics. Results from this review indicate that presence of the principles was scant. Our findings also highlight the challenges teachers face in using current math programs to address the needs of students struggling to learn mathematics proficiently. In many ways, our findings mirror those of earlier curricular reviews (e.g., Bryant et al., 2008), which suggest that commercially available texts are falling short in serving as a foundation of effective core math instruction.

Adherence to the Principles of Instruction

When examining for adherence to the principles of instruction, several findings are noteworthy. Most notably, textbooks

were missing consistent opportunities for explicit and systematic instruction. A potential result of missing this principle in a core math textbook is providing math instruction that is confusing for students, especially those with MD.

We also found that most programs provided limited opportunities for students to engage in discrimination practice (Stein et al., 2006), that is, discriminating between new and previously learned problem types. For example, students should practice when and when not to “rename” in multidigit addition problems. Our review also revealed few procedures or guidance for teachers on when and how to provide academic feedback to students (e.g., when to provide corrective feedback or reteach complex problems). One program did provide prompts to anticipate common student misconceptions and/or learning problems. However, given their obscure location and lack of explicitness about how to avoid the misconception, raters agreed that it would be difficult for teachers to take advantage of these prompts during instruction. Errors are bound to occur in learning new and complex mathematical content. Therefore, curricula should overtly provide teachers with procedures for providing academic feedback. At the very least, these procedures will help remind teachers to correct student errors and misconceptions, and when to provide extra practice opportunities before introducing new material (Stein et al., 2006).

Differences of ratings were also noted within math programs. The Grade 2 textbooks demonstrated slightly stronger adherence to the principles of instruction than their Grade 4 counterparts. One possible explanation for this within-program variation is that publishing companies use different curriculum teams across grade levels to design and develop materials. Another possibility is the content differences and associated task difficulties between the grades. Regardless of the reason, be it differing curriculum designers or lack of communication between the teams, these findings are disconcerting given that two of the programs included in the present review constitute approximately one fourth of U.S. elementary textbook sales (Reys & Reys, 2006). Our findings suggest that a considerable amount of students are receiving markedly different experiences in math instruction as they progress through the grade levels, with possibly less coherent instruction in the later grades. Inconsistent instruction across grade levels will likely pose difficulties for most learners, especially those at risk for math failure.

Finally, the review revealed that none of the textbooks offered procedures for linking assessment results with instructional decision making. These results are discouraging because teachers can save precious instructional time when they use data on a regular basis. Moreover, without these critical information links, teachers may unknowingly begin to teach more sophisticated topics before students have mastered less sophisticated ones or may spend too much time on a topic that students have already mastered.

Limitations

Limiting factors of this study were the number of grade levels, topics, and programs reviewed, the exclusion of supplemental materials, and the lack of raters' blindness to the names of the programs included in the review. Although multiple topics were analyzed within three programs at two different grade levels, our review falls short of a comprehensive program evaluation. While it was not feasible to expand this review, future evaluations of elementary programs may benefit from analyzing a host of topics in kindergarten through Grade 5 materials. This review also included only teacher manuals and student textbooks and thus excluded supplemental materials associated with each program. Finally, raters did not perform blind reviews of the programs. In other words, reviewers were privy to the names of each of the curricula as they conducted the review. Thus, one legitimate question is whether reviewers were predisposed to favorably rate particular programs based on prior knowledge of the program. Our response to this final limitation is twofold. First, this curricular review was part of a larger review, which was commissioned and funded by a school district. Toward that end, the school district selected the programs targeted for the review. Second, a review of the literature on curricular reviews revealed that previous studies did not perform blind evaluations (i.e., Bryant et al., 2008; Carnine et al., 1997; Jitendra et al., 2005; Sood & Jitendra, 2007). Readers should interpret our findings with these limitations in mind.

Implications for Research and Practice

Our ultimate hope is that the findings of this review lead to improved core math textbooks. The only way we realistically expect for this to occur is to provide data-based information to consumers so that they can advocate for such changes in core math programs. Specifically, we encourage consumers and publishing companies to consider the theoretical frameworks of instructional design (e.g., Coyne, Kame'enui, & Carnine, 2011; Kame'enui & Simmons, 1999), the results of this review and the previous literature on math textbook analyses (e.g., Bryant et al., 2008; Sood & Jitendra, 2007), and the growing body of evidence on math instruction (e.g., Gersten Chard, et al., 2009; NMAP, 2008). We feel confident that by including the most effective principles of instruction within classroom materials, teachers will be better prepared to address a diverse range of learning needs.

Given the practical significance for the eight instructional principles, it is important that they be contained in core math instruction. In their absence, educators will need to enhance core (Tier 1) math instruction. Recently S. K. Baker, Fien, and Baker (2010) presented a model for intensifying reading instruction by using enhanced versions of Tier I core reading

materials. In many ways, the features identified in their model generalize to enhancing core math instruction for students with or at risk for MD. According to the model, an integral step in enhancing core instruction is making it more explicit and more systematic for at-risk learners. To do this, Baker et al. suggest clarifying (a) teacher to student interactions, (b) types of student responses, (c) checks for student understanding, and (d) student practice opportunities.

For example, the model proposed by S. K. Baker and colleagues (2010) recommends that teachers begin instruction with a brief explanation of the lesson's purpose and targeted content. This way, learning expectations and objectives are clear and unambiguous for students. When teaching new math concepts or skills, teachers explicitly model and demonstrate what they want students to learn. Moreover, they are expected to provide guided practice opportunities to better ensure student success.

Applied in the context of early mathematics instruction, a teacher might state, "Today we are going to learn how to count by 10s using the hundreds chart. Listen as I count from 10 to 100. I'll use the hundreds chart to help me count. When I'm finished, we will count together." Following the demonstration and guided practice, teachers are expected to provide specific and frequent feedback as students engage in response opportunities. These opportunities can occur in multiple formats but particularly important are practice opportunities that involve verbal interactions between teachers and students. Using the same math example, this would entail having individuals and groups of students count by 10s.

Given the importance of selecting and adopting an effective core math program, we strongly suggest that textbook adoption committees use a systematic, research-based approach for analyzing potential materials. We believe the approach used in this review serves as a practical and potentially replicable model for the curriculum adoption process. Retrospectively, we recognize that future curricular reviews may need to expand the evaluation of textbooks. For instance, we would suggest including a search for evidence of impact on student achievement for each program. Committees can accomplish this by examining independent reviews conducted by the What Works Clearinghouse (2009) and the Best Evidence Encyclopedia (2009). Reviews from these organizations are highly respected within the field and are available at no cost to consumers. Although conducting this type of literature search can be time intensive, it may bear further evidence in support of a particular program.

We also recommend that local and state educational agencies provide teachers with high-quality professional development and ongoing support (e.g., expert coaching) for implementing effective core math instruction. These support systems would help teachers implement core math programs with fidelity. Perhaps above all, they would provide teachers the skills necessary to modify and adapt core math textbooks to facilitate cognitive access for all students, including those

at risk for failure in mathematics. More specifically, schools should train teachers and interventionists on how to make core math instruction more explicit and systematic. Because the field of math instruction lacks an empirically based framework for enhancing core math instruction, we encourage schools to refer to the model suggested by S. K. Baker et al. (2010) to guide their training of educators. We believe this model serves as a practical guide to increase the intensity of core math instruction for at-risk learners. For example, schools can train teachers on how to increase the quantity of teacher–student interactions and provide a variety of group and individual response opportunities.

In addition, even the most well-designed math program requires a teacher with a deep understanding of the underlying principles of mathematics. Therefore, ongoing professional development should also target teachers' content and pedagogical knowledge for teaching mathematics. We contend that strong curricular programs in the hands of high-quality teachers with deep content knowledge of mathematics will significantly increase math achievement across the United States.

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