Article

Hammill Institute on Disabilities

Journal of Learning Disabilities
2016, Vol. 49(4) 354–367

DOI: 10.1177/0022219414554228
journaloflearningdisabilities.sagepub.com
SAGE

The Effects of Schema-Based Instruction on the Proportional Thinking of Students With Mathematics Difficulties With and Without Reading Difficulties

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Abstract

This study examined the effect of schema-based instruction (SBI) on the proportional problem-solving performance of students with mathematics difficulties only (MD) and students with mathematics and reading difficulties (MDRD). Specifically, we examined the responsiveness of 260 seventh grade students identified as MD or MDRD to a 6-week treatment (SBI) on measures of proportional problem solving. Results indicated that students in the SBI condition significantly outperformed students in the control condition on a measure of proportional problem solving administered at posttest (g = 0.40) and again 6 weeks later (g = 0.42). The interaction between treatment group and students' difficulty status was not significant, which indicates that SBI was equally effective for both students with MD and those with MDRD. Further analyses revealed that SBI was particularly effective at improving students' performance on items related to percents. Finally, students with MD significantly outperformed students with MDRD on all measures of proportional problem solving. These findings suggest that interventions designed to include effective instructional features (e.g., SBI) promote student understanding of mathematical ideas.

Keywords

schema-based instruction, proportional thinking, mathematics difficulties, reading difficulties

Large-scale assessment results indicate that the mathematics proficiency of secondary students with disabilities is significantly lower relative to their peers without disabilities. Findings from the recent National Assessment of Educational Progress (NAEP; National Center for Education Statistics, 2013) showed that 65% of students with disabilities performed below basic versus 21% of students without disabilities in mathematics. These results align with those of the National Longitudinal Transition Study–2 (Wagner, Newman, Cameto, Levine, & Garza, 2006), which found that secondary students with disabilities perform worse than their nondisabled peers.

One area of mathematics that is particularly difficult for many middle school students is proportional reasoning (Adjiage & Pluvinage, 2007; Fujimura, 2001; Lobato, Ellis, Charles, & Zbiek, 2010; Jitendra, Woodward, & Star, 2011; Miyakawa & Winslow, 2009; National Mathematics Advisory Panel [NMAP], 2008; Tourniaire & Pulos, 1985), which refers to understanding the multiplicative relations between quantities (ratios) as well as the "covariance of quantities and invariance of ratios" (Lamon, 2007, p. 638). Proportional reasoning is not only the capstone of elementary arithmetic but also the cornerstone of advanced mathematics (Boyer, Levine, & Huttenlocher, 2008; Lesh, Post, & Behr, 1988; NMAP, 2008). Proportional reasoning tasks are complex and encompass many different types of problems found in textbooks and in real life (e.g., basic ratios, rates, scale drawings, similar figures, linear functions, measurement conversions, percents, mark-ups, discounts; see National Council of Teachers of Mathematics, 2000; National Research Council, 2001). The centrality of proportional reasoning in middle school is emphasized in the Common Core State Standards (CCSS; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSO], 2010).

Solving even simple proportion word problems is challenging for many students, especially students with

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mathematics difficulties (MD) and those with concurrent mathematics and reading difficulties (MDRD). Even with adequate computational skills, proportional word problems are difficult because they require understanding the linguistic statements to extract relevant information to develop a representation of the problem situation. Many students do not understand "what is actually meant by a particular situation or why a solution strategy works" (Weinberg, 2002, p. 138). As such, it is important to identify useful approaches for addressing the challenges faced by these students.

Several word problem-solving interventions have been developed to help students with MD be more effective problem solvers (see Xin & Jitendra, 1999; Zhang & Xin, 2012). However, much of that research has focused on arithmetic and arithmetic story problems and relatively less on more complex problems (e.g., proportions, percents). In addition, although this research has addressed MD and development of arithmetic cognition, less attention has been directed at comorbid difficulties in both mathematics and reading. There is some evidence that variations in reading ability may explain the treatment outcomes for students with MD (e.g., Fuchs, Fuchs, & Prentice, 2004). Several researchers (Fuchs et al., 2004; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000) have documented that the functional and cognitive profiles of elementary age students with MD and those with MDRD differ and that students with MDRD evidence more pervasive difficulties in several domains (i.e., number combinations, place value, computational procedures, simple and complex arithmetic story problems) than those with MD. On more complex arithmetic problems, Fuchs et al. (2004) found that students with MDRD's "math deficits or their underlying mechanisms explained a greater proportion of variance in responsiveness to problem-solving treatment than reading deficits or their underlying mechanisms" (p. 305).

Recently, Zheng, Flynn, and Swanson (2012) summarized the literature on word problem-solving intervention studies for school-age students with MD (5–18 years). The seven group and eight single-subject design studies meeting their inclusion criteria defined MD-only students using scores below the 25th percentile cutoff on a norm-referenced mathematics test and above the 25th percentile on a norm referenced reading test; MDRD students were defined as those with scores below the 25th percentile on both norm-referenced mathematics and reading tests. Results showed large positive effect sizes (ESs) for students with MD only in both group (ES = 0.95) and single-subject design (ES = 1.45) studies compared to their counterparts in the control condition. By contrast, the mean ES for students with MDRD in the group design studies was moderate (ES = −0.45), favoring the control condition. For single-subject studies, the mean ES for MDRD students was 0.58 compared to students in the control condition. Across both group and single-subject design studies reading skills moderated treatment outcomes, with higher outcomes for students with MD only than for students with MDRD. The authors identified several instructional components (e.g., skill modeling, strategy cues, explicit practice, advance organizers, questioning, elaboration, task reduction, task difficulty control) that positively affected treatment outcomes for students with MD only and those with MDRD.

Building on this prior work, the focus of this study was on improving students' proportional reasoning skills. Specifically, we assessed the efficacy of a research-based intervention, schema-based instruction (SBI), as a function of mathematics difficulty (i.e., MD only or MDRD). In this introduction, we summarize intervention studies focusing on ratio and proportion word problems for middle school students with MD or at risk for MD (low achieving in mathematics) followed by a rationale for the study.

Research on Ratio, Proportion, and Percent Word Problem Solving

Few studies have been conducted to assess the efficacy of interventions on the proportional reasoning skills of students with MD or at risk for MD. A study by Moore and Carnine (1989) represents a first step in understanding how students at risk for MD develop proportional reasoning. The researchers contrasted two curricula (active teaching with curriculum design [ATCD] and active teaching with basals [ATB]) for teaching ratio and proportion word problems to low-achieving high school students, including students with mathematics disabilities (had individualized education programs in math) in a randomized controlled trial. Results indicated that the ATCD group outperformed the ATB group on a researcher designed posttest and retention test (2 weeks later). Although the results suggest that students could be successful on a limited range of proportion problems using a direct translation strategy, this has been criticized in the problem-solving literature because it does not emphasize the mathematical relations between objects in the problem text needed to set up the mathematical equation (see Hegarty, Mayer, & Monk, 1995; Reed, 1999; Woodward et al., 2012). Furthermore, the findings are confounded by the study design in that the medium of delivery of instruction (interactive videodisc presentation of the ATCD vs. teacher presentation of the ATB) rather than the intervention could possibly explain the between-group differences.

Xin and colleagues conducted two randomized controlled studies with students with MD (scored at least 1 *SD* or more below the mean on a standardized mathematics achievement test). The intervention incorporated instructional strategies derived from the mathematics education research and the cognitive development research in mathematics to teach students to solve ratio (e.g., multiplicative compare) and proportion word problems. Two researchers and two special education teachers provided all instruction, which included 12 sessions, each session lasting approximately 1 hr. In the first study, Xin, Jitendra, and Deatline-Buchman (2005) randomly assigned students with MD to SBI or a comparison tutoring (i.e., general strategy instruction [GSI]) condition. SBI intervention focused on explicit mathematical modeling of problem solving by priming the underlying problem structure and representing the mathematical situation using schematic diagrams that highlighted the mathematical relations between objects in the problem text. Students in the SBI group scored significantly higher than students instructed in GSI at immediate posttest and on the retention tests (ranged from 1 week to 3 months later). On a measure of novel word problems, a statistically significant effect was found favoring SBI students.

In a subsequent study, Xin et al. (2011) developed schematic diagrams to mathematically model the multiplicative concept of factor × factor = product to solve multiplication problems involving comparisons (ratio) and equal groups (proportion). On a measure of word problem solving, results revealed large, statistically significant effects favoring the treatment group at posttest and delayed posttest (1 to 2 weeks later) compared to students in the comparison group that received typical textbook problem-solving instruction. However, on the standardized problem-solving test, there was no significant difference between groups.

Jitendra and colleagues moved beyond simple ratio and proportion problems to design SBI units on ratio, proportion, and percent problems that included more tasks (e.g., meaning of ratios, equivalent ratios, rates, fractions, decimals, and percents, scale drawings, percent of change) than in previous studies (e.g., Xin et al., 2005). In addition to the previous components embedded in SBI (i.e., explicit mathematical modeling, focus on the underlying problem structure, use of schematic diagrams), their multicomponent intervention emphasized metacognitive strategy use and procedural flexibility (e.g., when to use cross-multiplication, unit rate, and equivalent fractions strategies based on numbers in the problem). Two studies were conducted in typical seventh grade mathematics classrooms, with teachers providing all instruction. In the first study (Jitendra et al., 2009), the mathematical tasks focused specifically on ratio and proportion. Blocking by achievement level (i.e., high, average, and low based on grades in mathematics from the previous school year), eight classrooms were randomly assigned to either SBI or a "business as usual" control condition. Instruction included 10 daily lessons, with each lesson about 50 min. Students were pretested and posttested on a researcher-designed measure of ratio and proportion problem solving and a state-administered test of mathematics achievement. Results indicated that students in the SBI condition outscored students in the control condition on the problem-solving immediate posttest and delayed posttest (4 months later). Despite the lack of a significant treatment group by achievement level interaction effect, the scores of low-achieving students in the SBI group were similar to those of low-achieving students in the control group. On the test of mathematics achievement, results revealed no significant difference between groups.

The second study (Jitendra & Star, 2012) focused on the topic of percent. Data were collected from four (two high-achieving and two low-achieving classrooms) of the eight classrooms in the Jitendra et al. (2009) study. Instruction included nine daily lessons, with each lesson about 50 min. On a measure of percent problem solving, high-achieving students in the SBI condition statistically outperformed high-achieving students in the control condition; no statistically significant difference was found for low-achieving students, Results for the transfer test indicated no statistically significant differences for either high- or low-achieving students. Unlike previous studies that were implemented for about 750 min on average and included a limited range of topics, low-achieving students in the Jitendra et al. (2009) and Jitendra and Star (2012) studies did not benefit from SBI possibly due to the short duration (475 min) of the intervention focusing on a wide range of tasks.

Overview of the Present Study

In sum, findings of the few intervention studies on teaching ratio and proportion problem solving are inconsistent with regard to the benefits for students with MD. None of these studies, however, assessed whether proportional problem-solving interventions are efficacious for students with MDRD. Given that students with MD and students with MDRD respond differentially to intervention (see Zheng et al., 2012), it is important to test the efficacy of the intervention for both students with MD and those with MDRD. As most instruction for students with MD and those with MDRD often occurs in general education classrooms, it is also crucial to assess differential responsiveness to proportional problem-solving intervention in this setting. Furthermore, the poor outcomes for students at risk for MD in the two SBI studies conducted in typical mathematics classrooms (Jitendra et al., 2009; Jitendra & Star, 2012) may require scheduling more time for the development of schemata (i.e., conceptual learning), metacognitive strategy knowledge, and procedural flexibility. The present study extends previous research on SBI and investigated the effectiveness of SBI on the proportional problem-solving performance of students with MD and students with MDRD. Specifically, we examined students' responsiveness to a 6-week treatment (SBI vs. control group) and considered the role of students' difficulty status (MD, MDRD) on the following problem-solving measures: immediate, retention, and transfer assessments. This study addressed the following questions:

1a. What are the immediate and retention effects of SBI for seventh grade students with MD and those with MDRD compared to students randomly assigned to a business as usual control on mathematical problem-solving?

1b. Is SBI equally effective at improving the mathematical problem-solving performance of students with MD relative to students with MDRD on the immediate and retention posttests?

2a. What is the effect of SBI on a test of transfer for seventh grade students with MD and those with MDRD?

2b. Is SBI equally effective at improving the problem-solving performance of students with MD relative to students with MDRD?

Method

Research Context, Design, and Participants

The current study focuses on a subgroup of students from a previous study (Jitendra, Star, Dupuis, & Rodriguez, 2013) that included a heterogeneous pool of seventh grade students. Jitendra et al. (2013) used a randomized controlled pretest–intervention–posttest design with a retention test. A total of 15 seventh grade math teachers from six middle schools in three districts elected to participate in the larger study. Each teacher taught one to five sections of seventh grade mathematics. Blocking by teacher at each school, each teacher's classrooms were randomly assigned to the treatment (SBI) or control condition. To assess whether the random assignment performed on the larger sample held in the current subsample, we performed a series of analyses examining differences between the treatment and control groups on each of the pretest variables. Results indicated no statistically significant differences (all $p > .17$) on any of the variables, indicating group equivalence at pretest for the current sample.

In the 42 classrooms of the 15 teachers, a total of 1,163 students participated in the study. We collected scores for these students from the fall administration of the *Minnesota Comprehensive Assessment–II* (MCA-II), a state assessment for evaluating annual yearly progress, to identify students' difficulty status. Given the variability in defining mathematics disability (students scoring one to two grade levels below their expected grade) and the questionable use of relatively high cut-scores (i.e., 35th–40th percentile ranks) for operationalizing mathematics disability, we use the term at risk of MD and operationalize it using a 25th percentile cutoff. While this cut score is often employed to designate risk for a disability, it also is used to identify students as having disabilities in the literature (see Fuchs et al., 2004; Zheng et al., 2012). Students who scored at or below the 25th percentile on both the mathematics and reading subtests of the MCA-II were designated as students with both mathematics and reading difficulties (MDRD; $n = 152$). Students who scored at or below the 25th percentile on the mathematics subtest of the MCA-II and above the 25th percentile on the reading subtest of the MCA-II were identified as students with mathematics difficulties only (MD; $n = 108$). This resulted in a sample of 260 students from 37 seventh grade classrooms and their 14 teachers in the current study. In the treatment group ($n = 149$), there were 55 students with MD only and 94 students with MDRD. In the control condition ($n = 111$), there were 53 students with MD and 58 students with MDRD.

Student demographics by treatment and difficulty status are presented in Table 1. Students' mean age at the beginning of the study was 12.86 years ($SD = 0.40$ years). The sample included 113 (43.8%) White, 81 (31.4%) Black, 36 (14.0%) Hispanic, 25 (9.7%) Asian, and 3 (1.2%) American Indian students. Of the participants, approximately 50% were eligible to receive free or reduced-price lunch, 21% were receiving special education services, and 14% were English language learners. Within each difficulty status group, there were no statistically significant differences between treatment groups on demographic variables (all $p > .06$) and screening measures (all $p > .12$).

The participating teachers (10 females and 4 males) had a mean age of 33.79 years (range = 26–55 years) and a mean of 7.1 years of experience teaching mathematics (range = 1–14 years). Of the 14 teachers, 13 were White and 1 was Black; 7 teachers had an undergraduate degree in mathematics and 9 had master's degrees. In addition, 7 teachers also held secondary education certification.

Instructional Intervention

Overview. The three districts, in which the study was implemented, used *Math Thematics* (Billstein & Williamson, 2008), *Math Course* (Larson, Boswell, Kanold, & Stiff, 2007), and *Math Connects Course* (Day, Frey, & Howard, 2009) at the middle school level. An examination of the three district-adopted mathematics textbooks (one reform-oriented and two traditional mathematics curricula) used in the control sections showed that although they covered the same topics (e.g., ratios, rates/proportions, scale drawings, percents, percent of change), the reform-oriented curriculum made extensive use of explorations of mathematical ideas in small groups, whereas the traditional curricula relied more heavily on whole-class direct instruction. Over the course of the study, teachers were expected to cover the same content on proportionality—ratios and proportions, percents—in their treatment and control sections. in both treatment and control sections, the same amount of teaching time (daily 50-min weekly sessions) was used to cover the curricular content.

In the treatment sections, teachers used the SBI curriculum and were provided with professional development to use the lessons. Professional development included 2 days of training prior to the implementation of the intervention. We describe below in greater detail the SBI curriculum and professional development.

Table 1. Student Demographic Information by Difficulty Status and Treatment.

	MD				MDRD			
	SBI		Control		SBI		Control	
Variable	*n*	%	*n*	%	*n*	%	*n*	%
Gender								
Male	25	45.5	29	55.8	49	52.7	28	48.3
Female	30	54.5	23	44.2	44	47.3	30	51.7
Ethnicity								
American Indian	0	0.0	0	0.0	2	2.2	1	1.7
Asian	5	9.1	4	7.7	10	10.8	6	10.3
Hispanic	10	18.2	4	7.7	10	11.1	12	20.7
Black	14	25.5	12	23.1	33	35.5	22	37.9
White	26	47.3	32	61.5	38	40.9	17	29.3
FRL	20	36.4	23	44.2	52	55.9	36	62.1
Special education	13	23.6	6	11.5	27	29.0	9	15.5
ELL	5	3.6	2	9.6	17	18.3	13	22.4
Age (*M*, *SD*)	12.90	(0.40)	12.91	(0.41)	12.80	(0.40)	12.86	(0.40)

Note. ELL = English language learner; FRL = eligible for free or reduced-price lunch; MD = mathematics difficulties only; MDRD = mathematics and reading difficulties; SBI = schema-based instruction.

SBI curriculum. The SBI curriculum consisted of two replacement units focused on ratios and proportions, and percents. Each unit consisted of 10 lessons, with an additional review lesson for a total of 21 lessons. Some lessons took more than one session so that all lessons could be completed in 29 sessions (6 school weeks). The SBI curriculum covered the same content taught in typical seventh grade classrooms and was aligned with the Minnesota state mathematics standards. We created detailed daily teacher guides, along with teaching materials (e.g., scripts, diagrams, problem-solving checklists) and student materials (workbook and homework book) to support teachers' implementation of activities to engage students and develop critical concepts and skills.

The SBI curriculum incorporated four instructional practices: (a) model problem solving and metacognitive strategies, (b) activate the mathematical structure of problems, (c) use diagrams to represent information in the problem text, and (d) develop procedural flexibility. With regard to the first practice, teachers explicitly modeled mathematical problem solving (problem comprehension and problem solution) using a four-step problem-solving strategy (DISC: D = Discover the problem type, I = Identify information in the problem to represent in a diagram, S = Solve the problem, C = Check the solution). In addition, metacognitive strategy use was promoted via deep-level questions for each step. For example, students identified the type of problem (ratio, proportion, or percent) by answering self-questions (e.g., Why is this a *proportion* problem? How is this problem similar to or different from the one I already solved?).

For the second practice (activating the mathematical structure of problems), students were encouraged to read, retell, and examine information in the problem to identify the problem type (ratio, proportion, or percent) as well as think about how problems within and across types are similar (e.g., both ratio and proportion problems involve a multiplicative comparison of quantities) or different (e.g., ratio problems are confined to a single situation; proportion problems, which describe a statement of equality between two ratios/rates, allow one to think about the ways that the two situations are the same).

The third practice focused on connecting the problem to a certain schematic diagram and using the appropriate diagram to represent the problem such that the problem representation showed the mathematical relations between quantities in the problem text. Specifically, instruction emphasized identifying information critical to solving the problem to represent using the schematic diagram. With regard to the fourth practice, instruction focused on developing students' procedural flexibility, including explicit teaching of multiple solution methods and being cognizant of specific methods that are more efficient than others. See the appendix for a sample script from Lesson 16 to illustrate the nature and content of SBI. The scripts were used as a resource for teaching; teachers were not expected to read them while teaching.

Professional development. The project investigators were responsible for training teachers. The purpose of the 2-day-long training sessions was to introduce the project and each unit of the SBI curriculum and to emphasize the importance of adhering to the "business as usual" curriculum in the control sections. The training sessions gave teachers an opportunity to review the materials aligned with the curricular units and to learn about implementing the intervention. The sessions focused on first engaging teachers in a discussion of how their students would approach ratio, proportion, and percent problems as well as analyzing expected student solutions, explanations, and difficulties. Next, teachers learned about key features of the SBI intervention (e.g., recognizing problem types, generating estimates before solving the problem, knowing multiple strategies and selecting the most efficient based on the numbers in the problem) to support student learning. Throughout the training, teachers viewed multiple short video segments that illustrated implementation of the SBI intervention by a teacher from the previous year. The video segments were used to address the importance of implementing SBI intervention faithfully without the need to read the teacher guide verbatim. Both sessions focused on not only how to use the four practices but also eliciting student discussions to develop mathematical reasoning skills.

Treatment Fidelity

To assess fidelity of treatment and identify diffusion of treatment, if any, in the control sections, Jitendra et al. (2013) developed two measures: one for the treatment condition (5 items measuring general teaching behaviors and 15 items corresponding to critical elements of SBI) and one for the control condition (same 5 items measuring general teaching behaviors and 4 items corresponding to the critical elements of SBI). All items on both measures were scored dichotomously, indicating the presence or absence of the respective teaching behavior. Three lessons for each teacher were videotaped during the 6 weeks of the study. Interrater reliability computed via a second rater observing the videotape and rating one randomly selected videotape of a teacher per condition averaged 99% for SBI and 98% for the control condition.

The results for items representing general teaching behaviors (e.g., sets purpose, provides positive feedback) indicated that, on average, SBI teachers (M = 0.97) engaged in these behaviors slightly more often than control teachers (M = 0.91). Regarding the items corresponding to the critical elements of SBI, the data suggest that instruction in control sections (M = 0.07) did not overlap with instruction in treatment sections; teachers in treatment sections (M = 0.87) consistently implemented SBI at a moderately high level over time to allow us to attribute group differences to the implementation of SBI.

Measures

Mathematical problem-solving test. All students were assessed on mathematical problem solving using a researcher-developed assessment that was developed using released items from the Trends in International Mathematics and Science Study (TIMSS), NAEP, and past state mathematics tests (see Jitendra et al., 2013). The 21 multiple-choice items (11 ratio and proportion, 10 percent) and two short-response items (ratio and percent of change) aligned with the seventh grade curriculum covered (ratio, proportion, and percent) during the 6 weeks the intervention occurred. The same test was used at pretest, posttest, and delayed posttest (6 weeks following the intervention). The multiple-choice items were scored as correct or incorrect. The two short-response items were scored using a rubric, which emphasized correct reasoning; based on the difficulty of the items, responses for the ratio and percent of change short-answer problems were scored on a 4-point and a 6-point scale, respectively. The possible total score on this test was 31 points. For the short-response items, interscorer agreement on 20% of explanations was 98% and 94%, respectively, in the larger study. We estimated reliability by fitting the parallel, tau-equivalent, and congeneric measurement models to the pretest, posttest, and delayed posttest separately. Results indicated that the congeneric model fit the data best with all root mean square error of approximation values less than .04 and all goodness-of-fit index values greater than .95. Reliability estimates (coefficient omega) from the congeneric model (Dunn, Baguley, & Brunsden, 2014) were .58 at pretest, .68 at posttest, and .66 at delayed posttest for the present sample.

Transfer test. The items for the transfer test were also selected from items released from NAEP, TIMSS, and past state mathematics tests. The 18-item test included novel and challenging items (e.g., probability), which were not directly aligned with the content covered during the intervention. The transfer test assessed the extent to which the intervention promoted transfer to novel problems having the same mathematical structure as learned problems (ratios) but of a different type or having a modified problem structure (e.g., probability). The same test was used as a pretest and posttest. Coefficient omega was .60 at pretest and .63 at posttest for the present sample.

Data Collection

All assessments were group administered in students' classrooms by teachers with the research assistants in attendance to ensure adherence to test administration procedures. Reliability of test administration using an observation protocol form (e.g., teacher distributes calculators, reads all instructions aloud, circulates and monitors students, instructs students who finish early to read quietly at their desks) exceeded .95 (number of items completed divided by total number of items) for all tests at pre- and posttreatment. The mathematical problem solving (MPS) and transfer tests were given within 2 weeks prior to and within 2 weeks following the intervention. In addition, the MPS was administered 6 weeks after the intervention.

Data Analysis

To address the research questions, we fit a series of mixed-effects two-way ANCOVA models to the MPS posttest (total score, ratio and proportion subscore, percent subscore), MPS delayed posttest (total score, ratio and proportion subscore, percent subscore), and the transfer posttest. In each model, treatment (SBI or control), difficulty status (MDRD or MD), and the interaction between treatment and difficulty status were treated as fixed effects; teachers were treated as a random effect, and the respective pretest was treated as a covariate. Teacher effects were included in the models to account for within-teacher dependencies. For significant teacher effects, variance components were estimated using maximum likelihood estimation. ESs were calculated following Hedge's g (What Works Clearinghouse [WWC], 2014).

Results

Table 2 shows the unadjusted and adjusted means and standard deviations for the MPS and transfer tests by treatment and difficulty status; Table 3 shows the ESs for treatment by difficulty status. Figures 1 and 2 show percentage correct improvement from pre- to posttreatment on the MPS posttest and delayed posttest.

MPS Posttest

On the total posttest score (ratio and proportion, percent), results indicated a statistically significant effect for treatment, $F(1, 242) = 11.31$, $p = .001$, favoring the SBI students ($g = 0.40$). The effect for difficulty status was also statistically significant, $F(1, 242) = 6.21$, $p = .013$, favoring the MD students ($g = 0.30$). However, the interaction between treatment and difficulty status was not statistically significant, $F(1, 242) = 0.43$, $p = .513$. The random effect for teachers was statistically significant, $F(13, 242) = 3.02$, $p < .001$. The estimated variance component for teachers was 1.81, with teachers accounting for 11% of the variance in posttest scores. The pretest was statistically significant, $F(1, 242) = 63.05$, $p < .001$.

On the ratio and proportion posttest subscore, results indicated no statistically significant effect for treatment, $F(1, 239) = 3.81$, $p = .052$. The effect for difficulty status, $F(1, 239) = 7.02$, $p = .009$, was statistically significant, favoring the MD students ($g = 0.33$). The treatment by difficulty status interaction, $F(1, 239) = 0.15$, $p = .699$, was not statistically significant. The random teacher effect, $F(13, 239) = 1.92$, $p = .029$, and the pretest, $F(1, 239) = 59.46$, $p < .001$, were statistically significant. The estimated variance component for teachers was 0.37, with teachers accounting for 6% of the variance in ratio and proportion posttest scores.

On the percent posttest subscore, results indicated a statistically significant effect for treatment, $F(1, 238) = 10.00$, $p = .002$, favoring SBI students ($g = 0.42$). Similarly, the effect for difficulty status, $F(1, 238) = 3.97$, $p = .048$, was statistically significant favoring MD students ($g = 0.42$). However, the interaction between treatment and difficulty status, $F(1, 238) = 0.68$, $p = .409$, was not statistically significant. The random teacher effect was statistically significant, $F(13, 238) = 2.41$, $p = .005$; the variance component associated with teacher was 0.47, indicating that teachers account for 8% of the variance in posttest percent scores. The pretest was statistically significant, $F(1, 238) = 9.32$, $p = .003$.

MPS Delayed Posttest

On the total delayed posttest score, results indicated a statistically significant effect for treatment, $F(1, 242) = 11.20$, $p = .001$, favoring the SBI students ($g = 0.42$). The effect for difficulty status was statistically significant, $F(1, 242) = 6.19$, $p = .009$, favoring MD students ($g = 0.31$). However, the interaction between treatment and difficulty status was not statistically significant, $F(1, 242) = 1.00$, $p = .319$. The random effect for teachers was statistically significant, $F(13, 242) = 2.23$, $p = .009$. The estimated variance component for teachers was 1.04, with teachers accounting for 6% of the variance in delayed posttest scores. The pretest was statistically significant, $F(1, 242) = 45.96$, $p < .001$.

On the ratio and proportion delayed subscore, results indicated a statistically significant effect for treatment, $F(1, 236) = 5.03$, $p = .026$, favoring SBI students ($g = 0.29$). The effect for difficulty status, $F(1, 236) = 11.04$, $p = .001$, was statistically significant favoring MD students ($g = 0.43$). The treatment by difficulty status interaction, $F(1, 236) = 4.72$, $p = .031$, and the random teacher effect, $F(13, 236) = 2.00$, $p = .021$, were statistically significant. The estimated variance component for teachers was 0.32, with teachers accounting for 5% of the variance in ratio and proportion delayed posttest scores. The pretest results was statistically significant, $F(1, 236) = 22.64$, $p < .001$.

On the percent delayed posttest subscore, results indicated a statistically significant effect for treatment, $F(1, 235) = 6.56$, $p = .011$, favoring SBI students ($g = 0.39$). The effect for difficulty status, $F(1, 235) = 2.20$, $p = .140$, the interaction between treatment and difficulty status, $F(1, 235) = 0.13$, $p = .724$, and the random teacher effect, $F(13, 235) = 0.85$, $p = .606$, were not statistically significant. The pretest was statistically significant, $F(1, 235) = 13.70$, $p < .001$.

Transfer Test

Results for the transfer test did not indicate a statistically significant effect for treatment, $F(1, 237) = 2.16$, $p = .143$.

Table 2. Means and Standard Deviations by Difficulty Status and Treatment.

	MD				MDRD				Total			
	SBI (*n* = 55)		Control (*n* = 53)		SBI (*n* = 94)		Control (*n* = 58)		SBI (*n* = 149)		Control (*n* = 111)	
Measure	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Screening measures[a]												
MCA-II mathematics	638.29	4.31	639.06	4.13	633.12	8.53	634.45	7.86	635.03	7.66	636.65	6.74
MCA-II reading	654.47	4.73	654.64	5.06	639.23	6.22	640.86	6.24	644.86	9.32	647.44	8.95
MPS total[b]												
Pretest	10.42	4.27	10.68	4.54	8.50	3.19	9.16	3.61	9.21	3.73	9.88	4.13
Posttest	15.56	5.21	13.53	4.70	12.31	4.36	11.81	3.87	13.51	4.93	12.63	4.35
Adjusted posttest	14.90	—	12.69	—	13.22	—	11.68	—	14.06	—	12.18	—
Delayed posttest	14.45	4.89	13.43	4.56	12.19	4.55	10.83	3.67	13.03	4.79	12.07	4.30
Adjusted delayed posttest	14.25	—	12.86	—	13.40	—	10.97	—	13.82	—	11.91	—
MPS ratio and proportion[b]												
Pretest	6.69	2.73	6.87	3.41	5.51	2.43	5.90	2.49	5.98	2.56	6.42	2.94
Posttest	9.39	2.95	8.55	3.21	7.41	2.85	7.38	2.43	8.14	3.03	7.94	2.88
Adjusted posttest	8.34	—	8.00	—	7.78	—	7.20	—	8.31	—	7.60	—
Delayed posttest	9.02	2.99	8.90	2.86	7.74	2.81	6.69	2.51	8.32	2.79	7.74	2.89
Adjusted delayed posttest	8.65	—	8.55	—	8.20	—	6.64	—	8.42	—	7.60	—
MPS percent[b]												
Pretest	4.00	2.12	3.81	2.16	2.99	1.77	3.26	1.81	3.38	1.95	3.55	1.98
Posttest	6.46	3.16	4.98	2.45	4.89	2.25	4.43	2.25	5.47	2.72	4.69	2.35
Adjusted posttest	6.24	—	4.89	—	5.32	—	4.49	—	5.78	—	4.69	—
Delayed posttest	5.70	2.49	4.79	2.29	4.45	2.65	4.14	2.15	4.97	2.61	4.45	2.23
Adjusted delayed posttest	5.71	—	4.72	—	5.11	—	4.35	—	5.41	—	4.53	—
Transfer total[c]												
Pretest	8.65	2.44	8.83	2.49	7.55	3.02	6.59	2.28	8.01	2.80	7.73	2.53
Posttest	10.43	2.64	9.64	2.60	8.34	2.83	7.88	2.38	9.16	2.84	8.72	2.63
Adjusted posttest	9.82	—	9.43	—	8.49	—	8.47	—	9.16	—	8.95	—

Note. MCA = *Minnesota Comprehensive Assessment*; MD = mathematics difficulties only; MDRD = mathematics and reading difficulties; MPS = mathematical problem solving; SBI = schema-based instruction.
[a]MCA-II scores are standard scores. [b]MPS scores are raw scores; the maximum possible points for the total score, ratio and proportion subscore, and percent subscore is 31, 15, and 16, respectively. [c]Transfer scores are raw scores; the maximum possible points is 18.

Table 3. Effect Sizes for Treatment by Difficulty Status.

Measure	MD	MDRD
MPS total		
Adjusted posttest	0.44	0.32
Adjusted delayed posttest	0.29	0.57
MPS ratio and proportion		
Adjusted posttest	0.11	0.21
Adjusted delayed posttest	0.03	0.57
MPS percent		
Adjusted posttest	0.47	0.37
Adjusted delayed posttest	0.41	0.31
Transfer total		
Adjusted posttest	0.15	0.01

Note. MD = mathematics difficulties only; MDRD = mathematics and reading difficulties; MPS = mathematical problem solving.

The effect for difficulty status, $F(1, 237) = 11.02$, $p = .001$, was statistically significant, favoring students with MD ($g = 0.43$). The interaction between treatment and difficulty status, $F(1, 237) = 1.20$, $p = .275$, and the effect of teachers, $F(13, 237) = 0.68$, $p = .782$, were not statistically significant. The pretest was statistically significant, $F(1, 237) = 48.93$, $p < .001$.

Discussion

The current study is a retrospective randomized controlled design as students had already completed participation in the Jitendra et al. (2013) study. In this study, we were interested in determining whether the positive effects (immediate, retention, and transfer) of SBI intervention in the previous efficacy study would uphold for students with MD

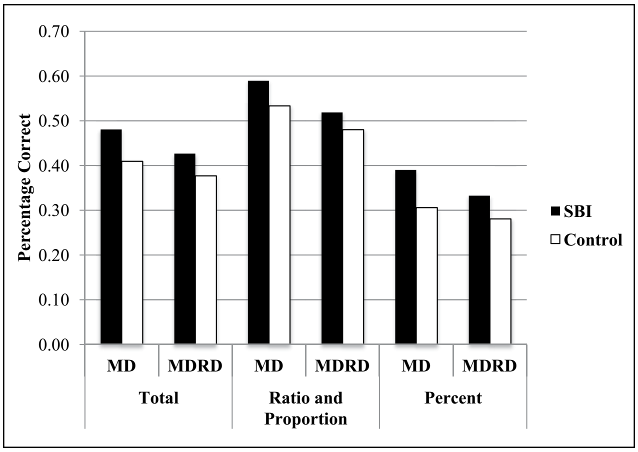


Figure 1. Adjusted mathematical problem solving posttest percentage correct scores by difficulty status and treatment.
Note. MD = mathematics difficulties only; MDRD = mathematics and reading difficulties; SBI = schema-based instruction.

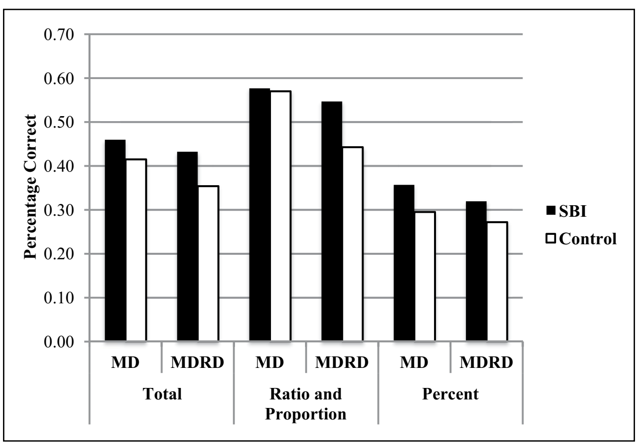


Figure 2. Adjusted mathematical problem solving delayed posttest percentage correct scores by difficulty status and treatment.
Note. MD = mathematics difficulties only; MDRD = mathematics and reading difficulties; SBI = schema-based instruction.

and students with MDRD. We also wanted to know whether the treatment effects would be strong for not only the total scores (ratio, proportion, and percent problem solving) examined in the efficacy study, but also for the subscores for ratio and proportion, and percent. We used the incoming MCA-II mathematics and reading scores to identify students with MD and MDRD. Analyses comparing the performance of these students indicated that students with MDRD showed more deficits than students with MD on the mathematics (d = 0.84) and reading (d = 2.66) subtests of the MCA-II. Similar to the efficacy study, no other main effect differentiated the treatment groups and there was no interaction between treatment group and difficulty status; thus, using this database to evaluate the efficacy of SBI for seventh grade students with MD and students with MDRD seems appropriate.

Similar to findings reported in Jitendra et al. (2013), we found that outcomes closely aligned with the intervention content were sensitive to group differences. Specifically, intervention students made significant gains on the MPS posttest (g = 0.40) and delayed posttest (g = 0.42) with regard to total scores (ratio, proportion, and percent problem solving). Although the effects sizes are smaller than those in Jitendra et al. (2013; g = 1.24 and 1.27 for posttest and delayed posttest), the effects in the current study are nonetheless considered substantively important (see WWC, 2014). That is, the ESs of 0.40 on the MPS posttest and 0.42 on the delayed posttest mean that, on average, a student in the SBI group is expected to score about 16 percentile points higher than a student in the control group. With regard to ratio and proportion subscores, the intervention showed a positive effect on the delayed posttest (g = 0.29; translates to an average student in the SBI group scoring 11 percentile points higher than an average student in the control group); there was no significant finding on the posttest. For the percent subscores, there was a significant effect of SBI compared to the control condition on both the MPS posttest (g = 0.42) and delayed posttest (g = 0.39).

To determine the effects of SBI for students with MD relative to students with MDRD on the immediate and retention posttests, we examined the interaction of treatment group by difficulty status results (see Table 3). With the exception of ratio and proportion scores on the delayed posttest (only students with MDRD in the SBI condition significantly outperformed the MDRD students in the control condition), the findings with regard to the interaction effect for all other scores on the MPS measure were not significant, suggesting that SBI was equally effective for both MD and MDRD students. Given the positive effect for SBI, we calculated the ESs for SBI students with MD and MDRD contrasting with students with MD and MDRD in the control condition. Although SBI students with MD and MDRD made substantive gains ($g > 0.25$) on the total score on both posttest and delayed posttest compared to their counterparts in the control condition, these students' responsiveness to treatment was differentiated by their performance on the subscores. SBI students with MDRD showed positive effects on both ratio and proportion, and percent scores on the posttest and delayed posttest. By contrast, SBI students with MD demonstrated positive effects on percent scores only on the posttest and delayed posttest. Overall, the ESs on the total score at posttest were similar for both students with MD (0.44) and those with MDRD (0.32) and contrast with the ESs (0.95 and −0.45 for students with MD and MDRD, respectively) reported in the Zheng et al. (2012) synthesis of group design studies. Although the relatively small ESs for our sample of students with MD is expected given that the content covered was complex, it is important to unpack the results for students with MDRD. Based on the MD/MDRD literature in mathematics, students with MDRD consistently perform worse than students with MD only.

What could account for SBI's positive effect for students with MDRD in this study? Effective instructional features (explicit and consistent procedures for solving word problems) and appropriate scaffolds (e.g., schematic diagrams, checklists) might explain the findings. Despite the linguistic demands associated with word problem solving, effectively scaffolding the representation and solution processes by providing diagrams that are appropriate for the task and teaching students to represent the problem situation as well as using problem-solving checklists to support them as they solved problems may have reduced the cognitive load placed on students (Berends & van Lieshout, 2009; Lee, Ng, & Ng, 2009). Evidence of the benefits of diagrams and checklists was demonstrated through SBI students' ratings. Students were asked to rate four items each related to diagrams (e.g., The diagrams helped me to organize information and understand how to solve problems) and the DISC four-step procedure (e.g., I found the DISC four-step procedure helpful in checking my understanding of how to solve word problems) on a 1 to 4 scale (4 = *strongly agree*, 1 = *strongly disagree*). The mean ratings of both students with MD (M = 3.31 for diagrams and 3.29 for DISC) and MDRD (M = 3.10 for diagrams and 3.36 for DISC) were moderate to moderately high.

We also examined the effects of SBI on the transfer measure. This assessment revealed no significant difference between the intervention and control group. This finding replicated the result from Jitendra et al. (2013).

Factors Related to Effectiveness of SBI

The stronger response of students with MD and MDRD in the SBI condition compared to their counterparts in the control condition, especially on percent problems that are considered difficult for many middle school students (Lembke & Reys, 1994; Parker & Leinhardt, 1995), may be explained in a number of ways. First, the SBI approach's focus on the development of both conceptual understanding and *deep procedural knowledge* (see Baroody, Feil, & Johnson, 2007; Star, 2005, 2007) may have contributed to the positive outcomes for students with MD and MDRD. In SBI, development of conceptual understanding focused on having students recognize the underlying problem structure (i.e., ratio, proportion, percent) and teaching them how to represent the mathematical relation in the problem text using a few types of schematic diagrams prior to solving the problem; procedural knowledge focused on procedural fluency, which is characterized as "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (Kilpatrick, Swafford, & Findell, 2001, p. 121). Students' flexibility in strategy use, which is crucial for solving a wide range of problems, was supported in SBI by teaching multiple solution strategies (cross-multiplication with ratios, unit rates, equivalent fractions), including when and how to use them based on the numbers in the problem.

Second, even though the focus in the current study was on complex tasks that require students to reason at high levels, the benefits of SBI for both students with MD and MDRD may be explained by adequate time (6 weeks) allocated to not only studying challenging mathematics but also learning and applying the instructional strategies in the multicomponent intervention. Furthermore, the positive findings on the more difficult percent tasks suggest that there was value added to students' problem-solving performance as a result of participating in SBI intervention over time. These findings are in contrast to those of previous studies (Jitendra et al., 2009; Jitendra & Star, 2012). The brief SBI intervention (about 2 weeks) in those studies as well as the lack of close proximity of teaching the closely related topics (ratio, proportion, and percent) were not effective for students at risk for MD. It is possible that students in previous studies may not have mastered the essential components of SBI such as recognizing the structure of the different problem types or flexibly using multiple strategies. The brief SBI intervention, compared to the longer intervention in the current study, may not have been optimally aligned with the needs of students at risk for MD who may have attention and working memory deficits. Jitendra and Star (2012), for example, explain their less than positive findings for students at risk for MD by noting that learning multiple strategies in the short time may have overly taxed their cognitive resources, and it may have been challenging for these students to split their attention and working memory resources between "learning the new schema (percent) and integrating new information with prior knowledge of other salient schemata [ratio, proportion] that were not fully acquired" (p. 157).

Furthermore, this study provides some evidence that ambitious mathematics practices need to occur over a longer period for students with MD and those with MDRD than their general education peers within the context of the current Common Core framework of high standards. Although the scope of the current study does not include an examination of which features of SBI are responsible for influencing the outcomes, explicitly connecting the mathematics practices (e.g., make sense of problems, model with mathematics, look for and make use of structure) to the mathematics content recommended in the CCSS (NGA & CCSO, 2010) may be one explanation for the study's positive findings for both students with MD and those with MDRD.

Implications and Directions for Practice and Future Research

A limitation that should be considered in interpreting these findings and planning for future research in mathematics intervention for teaching complex content to students with MD and MDRD in general education classrooms is the issue of low reliability estimates for the measures (particularly on the pretests). Compared to the adequate reliability estimates for the larger sample in Jitendra et al. (2013), the low reliability for the current study sample may be due to the homogeneity of the sample of students with MD and MDRD. Although the reliability coefficients were sufficient for making group comparisons, the low reliabilities may underestimate the treatment effects (see Thorndike, 1977).

Despite the above limitation, the findings of our study have important implications for research and practice. First, results indicate that our SBI approach (e.g., focus on the mathematical structure of problems using schematic diagrams, explicit instruction on problem-solving and metacognitive strategies, procedural flexibility) appears to be a more effective intervention than typical mathematics instruction for students with MD and MDRD in inclusive classrooms to enhance their mathematical reasoning involving complex problems (ratio, proportion, and percent). Second, the study findings reveal that achieving this effect requires adequate time to study challenging mathematics in depth. Yet, our intervention appears to be relatively brief based on the recommendation in the best evidence syntheses of a 12-week criterion to ensure external validity (Slavin, 2008). However, the duration of our study is consistent with the time allocated to the topics of ratio, proportion, and percent in most middle schools. Although we believe that the positive effect of SBI for students with MD and MDRD in our study was related to the duration of the study (6 weeks), we did not experimentally test this effect of different durations of treatment. Therefore, future intervention research should consider designing studies that account for differences in treatment duration. Given the limited research on teaching complex mathematics content to students with MD and MDRD, these findings also reinforce the importance of conducting further studies in different settings and instructional contexts to "provide a more accurate, albeit more complex, picture of the overall results of an intervention" (Coyne et al., 2013).

Conclusion

In this study, we found that students with MD and MDRD in inclusive classrooms can make important gains in mathematical problem solving when instruction is used appropriately to develop both conceptual and procedural knowledge. Considering the duration and content of the SBI intervention, our findings suggest that interventions designed to include effective instructional features promote understanding of mathematical ideas and procedural flexibility to positively influence student learning.

Appendix

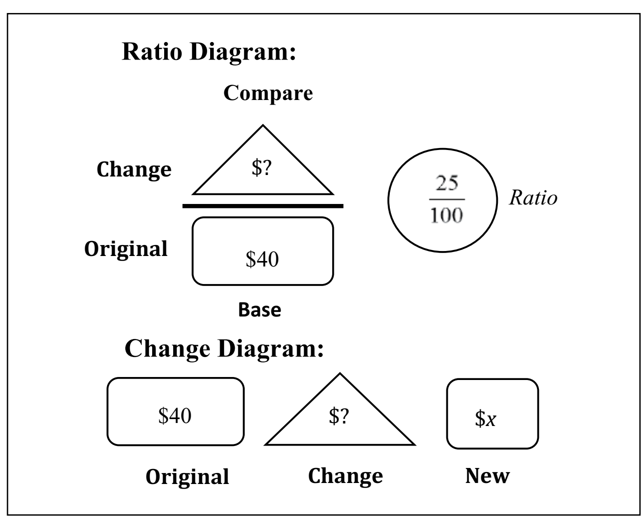
Lesson 16—Solving Mark-up and Discount Problems: Percent of Change/Problem 1 (Abbreviated)

Problem: EB Games purchases Wii memory cards for $40 each (the wholesale price). They then sell the memory cards for 25% over (i.e., markup) the price for them. What is the markup price (the retail price) of each Wii memory card at EB Games?

Teacher: Let's use these 4 steps (DISC) to solve this problem. [Point to Step 1 on the checklist.] To discover the problem type, I will read, retell, and examine information in the problem to recognize the problem type. [Read and retell the problem.] What type of a problem is this? How do you know?

Students: It is a markup problem, which is a percent of change problem. The markup rate tells about the percent of change (extra) from the original price to new price.

Teacher: Now I will ask myself if this problem is different from/similar to another problem I have already solved. This problem is similar to the tip problem we solved earlier, because we were given the tip rate (ratio value) and the original amount (taxi fare) and asked to find the total fare, including tip. This problem is different from the tip problem, because we are given the markup rate instead of the tip rate and the original price before the markup rather than the original taxi fare. [Point to Step 2 on the checklist.] Now we are ready to identify information in the problem to represent in a diagram(s).



Teacher: Great! Now we are ready to solve the problem. [Point to Step 3 on the checklist.] Let's first come up with an estimate for the answer. What is the estimate of the total retail cost of a Wii memory card that costs $40, including a mark up of 25%.

Students: I know that the total cost must be more than $40, because you need to add the 25% markup on $40. And the markup is less than 100%; a 100% markup would double the price to $80! So my estimate of the total cost would be more than $40 and less than $80.

Teacher: Excellent! I will translate the information in the diagram into a math equation. From the diagrams, we don't know the change amount and the new amount. So which one should we figure out first? Why?

Students: You can't figure out the new amount until you find the change amount. So, we first need to solve for the change amount.

Teacher: That is right. From the ratio diagram, we can set up the equation:

$$\frac{\$?}{\$40} = \frac{25}{100}$$

Now I need to figure out what strategy to use to solve for the change amount. There are several different ways to solve this problem. You can use cross-multiplication using ratios, the equivalent fractions strategy, or the unit rate strategy. What strategy works best with these numbers?

Students: The unit rate strategy. When you use the unit rate strategy, you are reasoning up or down, as shown below, to analyze the relation between the two quantities:

$$\uparrow \frac{\$?}{\$40} = \frac{25}{100} \uparrow$$

To solve for "?," first we start with the existing ratio ($\frac{25}{100}$). Because in the first ratio, the "?" is in the numerator, it might be easier if we work from the bottom to the top in the existing ratio of $\frac{25}{100}$. In other words, we ask ourselves "100 divided by what number equals 25? The answer is 4 (since 100 ÷ 25 = 4). Since you divided 100 by 4 to get 25, you divide 40 by 4 in the second ratio to get "?" So, 40 ÷ 4 = 10 (i.e., the change amount).

Teacher: Let's write 10 for "?" in both the ratio and change diagrams for the change amount. Remember, markup is always added to the original amount, so write a "+" before $10 in the change diagram. You are ready to solve for the new amount (i.e., $40 +$10). What is the complete answer to this percent of change problem?

Students: The total markup or retail price for a Wii memory card is $50.

Teacher: Good. What do you do next? [Point to Step 3 on the checklist.]

Students: Check if the answer makes sense.

Teacher: We estimated our answer to be more than $40, but less than $80. Our answer was $50, so our estimate gave us a good ballpark approximation for the actual answer. Does $50 seem right? Explain.

Students: We know that if $40 is the original price of the Wii memory card before markup, then the total mark up cost, which is $50, seems right. The change or markup was added to the original price of the Wii memory card ($40).

Teacher: You can also check the ratio 10:40 to see if the value is equal to the ratio 25:100. When you use a calculator and divide the numerator by the denominator, you get 0.25, which tells me that the two ratios are equivalent.

Acknowledgments

We gratefully acknowledge the efforts of all teachers and students who participated. The authors thank the project coordinator, Susan C. Slater, and research assistants, Mary Lindell, Fumio Someki, Beth Zukowski, Xueqin Qian, and Michael Berner for their contribution to this study.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R305K060002 to the University of Minnesota. The opinions expressed are those of the authors and do not represent views of the U.S. Department of Education.

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