FOSTERING FIRST GRADERS' REASONING STRATEGIES WITH BASIC SUMS

The Value of Guided Instruction

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ABSTRACT

An intervention experiment served to evaluate the efficacy of highly guided discovery learning of relations underlying add-1 and doubles combination families and to compare the impact of such instruction with minimally guided instruction. After a pretest, 78 first graders were randomly assigned to one of three intervention conditions: highly guided add-1, highly guided doubles, or minimally guided add-1 and doubles practice-only. Each highly guided intervention served as an active control for the other. The practice-only intervention served to control for the effects of extra practice. For both the add-1 and the doubles strategies, the highly guided intervention, but not the practice-only control, was more successful (as indicated by effect size) than the active control in promoting meaningful transfer to unpracticed but related combinations. The highly guided doubles intervention, but not the highly guided add-1 intervention, produced greater transfer than the minimally guided practice-only intervention.

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THE Common Core State Standards (Council of Chief State School Officers [CCSSO], 2010) identify as central grade 1 goals: adding within 20, doing so fluently within 10, and using reasoning strategies. Fluency implies efficient (accurate and fast) production of answers. As used hereafter, the term also means appropriate and adaptive application of knowledge (e.g., selective application of a reasoning strategy/rule to new problems). Although there is general agreement that all children need to achieve fluency with basic sums and that learning reasoning strategies is an important means to this end (NCTM, 2006; NMAP, 2008; NRC, 2001), many students, particularly those struggling with mathematics, have difficulty achieving these goals (Henry & Brown, 2008; Jordan, Kaplan, Olah, & Locuniak, 2006). Thus, it is critical to find methods of instruction that promote the learning of reasoning strategies and fluency with basic sums more effectively than conventional practices. The main aim of the present study was to gauge the efficacy of software designed to help at-risk first graders achieve these goals. A secondary aim was to take a step toward identifying best practices by comparing the relative efficacy of highly guided versus minimally guided instruction.

Promoting Combination Fluency through Use of Reasoning Strategies

General Reasoning Strategies

The *meaningful learning* of a particular basic sum, or family of sums, entails three overlapping phases (Baroody, 1985; Fayol & Thevenot, 2012; Rathmell, 1978; Thornton, 1978, 1990; Verschaffel, Greer, & De Corte, 2007). Initially, children use object or verbal counting to determine the sum (Phase 1: counting strategies). Then, as a result of discovering patterns or relations, children invent reasoning strategies that they apply consciously and relatively slowly (Phase 2: deliberate reasoning strategies). Reasoning strategies can involve using known information as a premise to logically deduce a previously unknown sum. For example, using the *add-1 rule* (the sum of any whole number, n, and 1 is the number after n in the count sequence) and knowledge of number-after relations as premises, a child can deduce the sum of any unknown $n + 1$ or $1 + n$ combination for which a child knows the number-after relation (e.g., the sum of $7 + 1$ must be the number after seven, which is eight). Learning reasoning strategies aids in the meaningful memorization of combinations (Phase 3: an efficient, appropriate, and adaptive retrieval network) in two ways. First, with practice, reasoning strategies can become *automatic* (efficient and nonconscious; Jerman, 1970) and serve as a component of the retrieval system (Fayol & Thevenot, 2012). Second, the strategies provide children with an organizing framework for learning and storing both practiced and unpracticed combinations (Canobi, Reeve, & Pattison, 1998; Dowker, 2009; Sarama & Clements, 2009).

Reasoning Strategies for Children with Difficulties in Mathematics

Children at risk for academic failure frequently begin school behind their peers (Baroody, Eiland, & Thompson, 2009; NMAP, 2008) and are likely to develop mathematical skills and concepts relatively slowly throughout their academic careers and remain behind their peers (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). They are particularly susceptible to difficulties or delays in achieving fluency, even with the most basic sums (Jordan et al., 2006)—including $n + 1$ or $1 + n$ sums (Baroody, Eiland, Purpura, & Reid, 2012, 2013; Baroody, Purpura, Eiland, & Reid, 2015). Such children often do not spontaneously invent reasoning strategies—achieve Phase 2 (Swanson & Cooney, 1985; Swanson & Rhine, 1985). Fortunately, research indicates that children who struggle with learning mathematics can be taught reasoning strategies (e.g., Torbeyns, Verschaffel, & Ghesquiere, 2005; Tournaki, 2003), and there is evidence that early intervention can prevent a spiral of failure among such children (Gersten, Jordan, & Flojo, 2005).

Importance of Reasoning Strategies

Research indicates that instruction that focuses on meaningful memorization (e.g., discerning relations such as the connection between new information and existing knowledge) can more effectively promote retention of practiced basic combinations and fosters transfer to unpracticed, but related, combinations than instruction that fosters memorization by rote (Brownell, 1941; Henry & Brown, 2008; Katona, 1967; NMAP, 2008; Steinberg, 1985; Swenson, 1949; Thiele, 1938; Thornton, 1978). For example, Henry and Brown (2008) found that, whereas the use of textbooks that focused on memorizing all basic addition and subtraction facts by rote and timed tests were negatively related to learning basic combinations and the use of flash cards had no positive effect on child outcomes, teaching reasoning strategies was positively correlated with fluency gains at the end of first grade. For such reasons, there is now broad agreement that instruction on the basic combinations should focus on fostering relational learning, such as that required by learning reasoning strategies (CCSSO, 2010; NRC, 2001; Rathmell, 1978).

Basic Reasoning Strategies

The add-1 and doubles combinations are among the easiest basic sums to learn and provide a basis for more advanced mental-addition strategies such as the make-a-ten and near-doubles reasoning strategies (see reviews by Brownell, 1941; Cowan, 2003). Given the informal knowledge children bring to school, the $n + 1$ or $1 + n$ family is a logical and developmentally appropriate place to begin mental-addition instruction. At the start of school, most students are so familiar with the count sequence that they can fluently specify the number after a given number (Fuson, 1992). To achieve fluency with $n + 1$ or $1 + n$ items, they need only to connect adding 1 to their extant number-after knowledge (i.e., learn and apply the add-1 rule; Baroody, 1989, 1992; Baroody et al., 2012, 2013, 2015).

The doubles are also relatively easy to learn because they embody familiar real-world set pairs (Baroody & Coslick, 1998; Rathmell, 1978). Using a familiar everyday situation to determine the sum of a double involves analogical reasoning, the simplest and most common method of reasoning. For example, if the two rows of six eggs in a carton of a dozen eggs are analogous to $6 + 6$, then the sum of $6 + 6$ is 12 also. Moreover, the ordered sums of basic doubles are the even-numbers sequence to 18 (parallel the skip-count-by-two: "2, 4, 6 . . . 18") and are akin to the first two counts in various skip-counts (e.g., 5 + 5 can be reinforced by knowing the skip-count-by-fives: "five, ten")—common aspects of primary-level mathematics instruction (Baroody & Coslick, 1998).

Instructional Methods: Guided and Unguided Discovery of Reasoning Strategies

The NRC (2001) concluded that Phase 2 can be accelerated by teaching reasoning strategies, if done conceptually. Teaching of reasoning strategies and their rationale is often recommended by mathematics educators (Rathmell, 1978; Thornton, 1978, 1990; Thornton & Smith, 1988) and utilized in many elementary curricula, such as Everyday Mathematics (UCSMP, 2005). However, not all conceptually based instruction is equally effective (Baroody, 2003). Chi (2009) hypothesized that interactive learner activities (substantively dialoguing and considering a partner's views) are more effective in promoting learning than constructive activities (producing responses that entail ideas that go beyond provided information), which are more effective than active activities (doing something physically), which in turn are more effective than passive activities (e.g., listening or watching without using, exploring, or reflecting on the presented material).

Discovery learning may be well suited to learning basic reasoning strategies, because it can involve interactive opportunities, active learning, and constructive activities (e.g., Swenson, 1949; Thiele, 1938; Wilburn, 1949). Alfieri, Brooks, Aldrich, and Tenenbaum (2011) defined discovery learning as not providing learners with the target information or conceptual information but creating the opportunity to "find it independently . . . with only the provided materials" (p. 2). Discovery learning, though, encompasses a wide range of methods, which may not be equally effective in all cases (Baroody, 2003; Chi, 2009). *Unguided discovery* ("free play") encompasses child-determined and unstructured activities. In contrast, all forms of *partially* guided instruction involve teacher-/curriculum-chosen tasks. With *negligibly guided discovery*, the tasks are unstructured (e.g., practice with selected combinations presented in a random order and without any guidance other than feedback about correctness). *Minimally guided discovery* provides minimal structure. For instance, one of the comparison groups of the present research involved entering sums on a number list, which incidentally provides implicit structure (the sum of 6 + 1 is the number after 6 on the number list). *Moderately guided discovery* involves well-structured activities designed purposefully to provide implicit scaffolding—to direct a child's attention to mathematical regularities (e.g., number-after questions are immediately followed by a related add-1 item to create the opportunity of noticing these two aspects of knowledge are related). *Highly guided discovery* (the focus of the experimental interventions in the present study) is well structured and includes explicit questions. Although instruction does not include simply providing a strategy for determining a sum (as in direct instruction), explicit prompts or hints guide attention to a regularity or strategy (e.g., Does knowing that the number after six is seven help you to solve 6 + 1?), and feedback specifies correctness and provides some explanation of why a response is correct or incorrect. For the highly guided programs in the present study, more and more explicit hints were provided as needed.

Prior Evidence of Instructional Effectiveness

Evidence regarding the efficacy of different types of discovery learning is mixed. On one hand, Kirschner, Sweller, and Clark (2006) concluded that minimally guided discovery methods were ineffective and inefficient compared to instructional methods involving guided or direct instruction. On the other hand, prior intervention studies have indicated that negligibly guided discovery can help typically developing kindergartners to discover the relatively obvious (salient) regularities underlying certain basic combination families (e.g., the add-0 and add-1 families) and apply these rules adaptively with an untimed mental-addition task (Baroody, 1989, 1992). Furthermore, a more recent intervention study (using a timed mental-addition task) indicated minimally and moderately guided discovery were equally successful in helping at-risk children discover the relatively salient add-1 combination family (Baroody et al., 2013).

Baroody and colleagues (2013) surmised that transforming the moderately guided program into a highly guided intervention might result in a significant difference over minimally guided instruction. Consistent with this hypothesis, Hmelo-Silver, Duncan, and Chinn (2007) argued that Kirschner et al.'s (2006) category of discovery learning was extremely broad/undifferentiated and that highly scaffolded problem-based or inquiry-based instruction is effective. In addition, Alfieri et al.'s (2011) meta-analysis of 164 studies revealed that explicit instruction was generally more effective than unguided discovery, but that enhanced (guided) discovery was more effective than other forms of instruction. However, they noted that those differences were moderated by the content of instruction—and the differences in outcomes based on instructional methods were less clear for mathematics and for young children overall.

Relatively little research has been conducted to evaluate the impact of discovery learning for doubles relations. Baroody et al. (2013) found that after regular classroom instruction or supplemental minimally guided practice, children with a risk factor for learning difficulties still had mastered less than two-thirds of doubles at the end of first grade. Baroody et al. (2013) further found that (as gauged by effect size) moderately guided doubles intervention had limited meaningful transfer when compared to an active control group that did not practice any doubles relations (i.e., produced improvement on the unpracticed small double $4 + 4$ but not the unpracticed large doubles $7 + 7$ and $8 + 8$). With a revised program, Baroody, Eiland, Bajwa, and Baroody (2009) also found significant, but modest, effects of a moderately guided doubles intervention for both practiced and unpracticed doubles when compared to an active control group. However, for similar reasons to the add-1 intervention, there is need to evaluate the efficacy of a *highly* guided doubles intervention (as opposed to the moderately guided in the previous study) and to contrast it with a minimally guided practice group to account for the effects of practice. Such an evaluation is a key step in identifying best practices for promoting fluency with basic reasoning strategies.

Present Study

The primary aim of the present research was to evaluate whether the revised add-1 and doubles programs were efficacious in promoting the deliberate and fluent use of add-1 and doubles strategies (Phase 2 and Phase 3, respectively) with first graders—most of whom were at risk of later mathematical difficulties as defined by low mathematics performance and/or low family income. This was achieved by comparing an experimental (highly guided instruction) group on a targeted strategy to two comparison groups: (*a*) an active control, which received an experimental intervention on a different reasoning strategy and only regular classroom instruction and practice on the targeted strategy, and (*b*) a practice-only control group, which received the same amount of practice on the targeted items as did the experimental groups. The latter control served to discount the alternative hypothesis that the experimental effect was merely due to extra practice. It also effectively served to compare the effects of the highly guided discovery learning to minimally guided discovery learning. The present research addressed three main hypotheses:

> **Hypothesis 1**: Highly guided add-1 instruction would be more efficacious in promoting progress toward fluency or even fluency itself with practiced and, more importantly (because transfer is indicative that general reasoning strategy has been learned), related but unpracticed add-1 items than the doubles intervention (active control).
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> **Hypothesis 2**: Highly guided doubles instruction would be more efficacious in promoting progress toward fluency or even fluency itself with practiced and, more importantly, unpracticed doubles items than add-1 training (active control).
>
> **Hypothesis 3**: Highly guided discovery of the add-1 rule or doubles relations would be more helpful in promoting progress toward fluency or even fluency itself with practiced and unpracticed add-1 and doubles items than minimally guided instruction (unstructured practice involving entering a sum on a number line and with feedback about correctness only).

Method

Participants

Participants were recruited from five elementary schools from two midsized midwestern cities. Parental consent forms were returned for 164 first graders, of which 78 children were identified as eligible to participate in the study. Eligibility was defined as not fluent on more than 50% of the $n + 1$ / $1 + n$ items ($M = 13\%$) or the doubles ($M = 15\%$) on the mental-addition pretest. A total of seven students did not complete the study because they either moved (five students) or refused to participate (two students). The seven students who dropped out of the study scored lower on the TEMA-3 (Test of Early Mathematics Ability—Third Edition) than the students who completed the study ($F(1, 76) = 5.03$, $p = .028$). However, attrition was equally distributed across groups and was primarily due to factors outside of the study (e.g., family mobility). Descriptive information on participants who completed the study can be found in Table 1. Participants ranged in age from 6.0 to 8.1 years old (mean = 6.6); 57.7% were female. The majority of children were African American (56.3%; 26.7% Caucasian; 8.5% Hispanic; 8.5% mixed, unknown, or other race). In addition, 84.6% of participants were eligible for free or reduced-price lunch, and approximately 52.1% of participants had a score in the bottom 25th percentile on the TEMA-3.

A total of 13 classes in five schools participated in this study. Two classes in School 1 ($n = 21$) and one class in School 2 ($n = 7$) used Math Expressions (Fuson, 2006). A second class in School 2 ($n = 11$) used Saxon Math (Larson, 2008). Nine classes in Schools 3 to 5 ($n = 32$) used Everyday Mathematics (UCSMP, 2005). Details on each of the curricula can be found in Table 2.

Measures

General mathematics achievement test. The TEMA-3 (Ginsburg & Baroody, 2003) is a manually and individually administered, nationally standardized test of math achievement for 3- to 8-year-olds. The test measures informal and formal concepts and skills in the domains of: numbering, number comparison, numeral literacy, combination fluency, and calculation ($\alpha = .94$).

Mental-addition test. The mental-addition testing was done in the context of computer games developed for the project and included four primary item types: (*a*) practiced add-1 items used in the add-1 and practice-only interventions (1 + 3, 3 + 1, 1 + 4, 4 + 1, 1 + 7, 7 + 1, 1 + 8, 8 + 1), (*b*) unpracticed (transfer) add-1 items (1 + 5, 5 + 1, 1 + 6, 6 + 1, 1 + 9, 9 + 1), (*c*) practiced doubles items used in the doubles and practice-only interventions (2 + 2, 3 + 3, 5 + 5, 6 + 6, 7 + 7), and (*d*) unpracticed doubles items (4 + 4, 8 + 8, 9 + 9, 10 + 10, 12 + 12). In addition, filler items (2 + 8, 3 + 4, 3 + 5, 4 + 3, 5 + 3, 7 + 2), which served as non-examples of the mathematical relations and practiced in all conditions, were tested but not included in the analyses. No group practiced the transfer items—the unpracticed

Table 1. Characteristics of the Participants by Condition

	Highly Guided Add-1 ($n = 25$)	Highly Guided Doubles ($n = 24$)	Minimally Guided Practice ($n = 22$)
Age range	6.1–7.3	6.1–8.1	6.0–7.1
Mean age	6.6	6.6	6.6
Number of boys : girls	14 : 11	6 : 18	10 : 12
TEMA-3 range	71–109	75–114	70–112
Mean TEMA-3	88	92	91
Free/reduced lunch eligible	21	21	18
Black	12	15	13
Hispanic	1	2	3
Multiracial	2	2	2
English as a Second Language	2	1	2
Attrition	2 moved	1 moved; 1 refused	2 moved; 1 refused

Table 2. Summary of the Mental-Addition Intervention and Practice Provided in Each of the Three Curricula Used in the Schools

Curriculum	General	$n + 1$ / $1 + n$ Items	Doubles
Everyday Mathematics (UCSMP, 2005)	Spends weeks on memorizing the "easy facts" (e.g., Unit 2 Lesson 3, Unit 4 Lessons 11 & 12, Unit 5 Lessons 10 & 11; Unit 6, Lesson 1): $n + 0$, $n + 1$, and doubles and items that sum to 10. The $0 + 0$ to $9 + 9$ items are practiced often (e.g., Unit 3 Lesson 14 and Unit 4 Lesson 11 use dominoes); the notion of a turnaround fact (the same numbers are being added so they have the same sum) is introduced to lessen the number of addition combinations to be learned (e.g., Unit 5 Lesson 11). Worksheets with n + 0 in row 1, n + 1 in row 2, and doubles along a diagonal—each shaded in a different color—assigned repeatedly.	$n + 1$ is related to the sum and is always the number after the starting number. Unit 1 Lesson 1 introduces 1 more by practicing number-after and adding 1 separately or implicitly (e.g., using a number line to add 1). In Unit 3 Lesson 8, a teacher explains: "Counting up by 1s is like adding 1 to each number to get the next number."	Doubles are highlighted as the numbers along the diagonal and are always even. Skip-counting, such as counting by twos (2, 4, 6 . . .) and fives (5, 10 . . .), is practiced (e.g., Unit 3 Lesson 9).
Math Expressions (Fuson, 2006)	Addition is introduced in terms of "decomposition" (e.g., the "break-aparts of the number 4" = 1 + 3, 2 + 2, and 3 + 1). Attention is drawn to commuted items ("switch facts"). Counting-on is encouraged initially. Mountain Math is used to introduce missing-addend addition. Visualization of finger representations and groups of 5 is encouraged (e.g., for 5 + 3: 5 fingers plus another 3).	One whole-class activity explores + 1 and –1 patterns: "Exploring 1 more and 1 less" (Activity 3 in Lesson 7 Unit 1). Children hold a number card with a numeral 1 to 5, lined up in order. To underscore that each number is 1 more than its predecessor, a child steps forward at the appropriate time. After "Here's 1," "Here's 1 and 1 more" is said, the second child steps forward. Practice is again provided in the "Consolidation" lesson (Lesson 17).	Doubles are taught in terms of the even numbers and "equal sharing" in Lesson 16 Unit 1. Defined as "two groups [with] equal shares" (p. 94); pictured as paired drawings (an number of items split into two equal groups). The flash doubles (e.g., to 5 fingers + 5 fingers = 10 fingers and later to 9 + 9 = 18), doubling a single or multiple squares on graph paper, relating doubles pattern to counting by 2s and dice patterns, decomposing a double such as 8 into two 4s is covered in Lessons 1 (Investigate Doubles) and 2 (Problem Solve with Doubles) of Unit 7.
Saxon Math (Larson, 2008)	Facts are introduced and practiced as a family. Teachers instructed to ask: "Do you see a pattern?" or "Who would like to share something you learned today?" Examples of satisfactory (and unsatisfactory) explanations are not included in the teacher's guide. Structured practice gives way to unstructured practice. Practice via worksheets is used.	Initially, the add-1 facts are introduced and practiced as a family (1 + 0 to 9 + 0 and 1 + 9 to 0 + 9). The structured practice and teacher prompts serve as minimally guided or implicit discovery of the add-1 rule.	Doubles introduced in Lesson 76 ("Showing Doubles Plus 1 Facts") and defined as "the numbers being added are the same." However, no effort is made to relate the doubles to other aspects of knowledge.

Note.—Information cited in this table appears in the Teacher's Manual but not student worksheets.

add-1 and unpracticed doubles. Each testing session consisted of a test set of 10 items, computer or manual reward game, second test set of 10 items, and final reward game. The mental-addition items were presented in a random order, except that two items with the same addends or sum were not presented one after another, commuted items were not presented in the same session, and types of items were evenly distributed across sets.

Data were gathered on strategy, response time, and response (accuracy). Testers identified whether a child used a counting, a reasoning, or an undetermined strategy. A counting strategy entailed representing one or both addends before the sum count or representing *both* addends during the sum count (Fuson, 1992). The criterion for counting was evidence of counting objects (e.g., fingers), verbally citing any of the counting sequence, or subvocally counting accompanied by successive movement of a finger or the eyes. A reasoning strategy was scored if there was evidence a child used deductive reasoning (e.g., for $5 + 1$ saying, “Six is the number after five, so five plus one equals six”). Response time was the interval between when a trial was presented (initiating a computer stopwatch) and when the child responded (when the tester depressed the key to stop the stopwatch). The tester entered the child’s answer via a number pad. An accurate response was defined as the correct sum that was not a false positive due to a response bias. A response bias was defined as inflexibly responding with the same strategy on trials in a testing session where such a strategy was inappropriate. Accounting for response biases was important to prevent overinterpretation of the effects of the interventions—as use of family-specific response biases may be more likely on instruction of specific relational strategies. For a detailed discussion of the adverse impact of response biases in evaluating effects of interventions and for detailed procedures in determining response biases, see Baroody, Purpura, Reid, and Eiland (2011). All testing sessions were videotaped, and a sampling of the sessions was rescored by another member of the project staff. Agreement in scoring was excellent (>99%).

For each combination family of interest, the strategy, response time, and accuracy data were used to generate two measures for gauging the impact of the experimental interventions. The *rate of fluent retrieval*, used previously by Baroody and colleagues (2012, 2013, 2014), served to gauge achievement of Phase 3. Fluency was defined as an accurate, undetermined, or reasoned response in less than 3 seconds with no evidence of counting. This measure has ecological validity because the ultimate goal of instruction is fluency. However, an intervention program may be partially successful if participants learn a reasoning strategy and use it appropriately and adaptively, if not efficiently (i.e., achieve Phase 2, if not Phase 3). In order to provide a more complete picture of the impact of the intervention programs, a new measure, namely a *fluency index* (F-index), was developed to gauge overall progress toward retrieval fluency by taking into account use of Phase 1 to Phase 3 strategies (see Table 3 for a description of the scoring procedure).

Interventions

The interventions each consisted of five stages. Stages I and II, common to all participants, served to ensure children had the necessary computer skills (e.g., how to use the virtual manipulative and enter responses) and mental-arithmetic pre-

Table 3. Scoring Details for the F-index

Score	Description
5 points	Behavior consistent with the achievement of Phase 3—automatic retrieval: efficient (<3 seconds), no evidence of overt counting or overt reasoning.
4 points	Behavior consistent with the transition from Phase 2 to Phase 3—overt but efficient reasoning (<3 seconds).
3 points	Behavior consistent with the achievement of Phase 2—overt (deliberate) reasoning in 3 or more seconds but less than 6 seconds.
2 points	Behavior consistent with advanced Phase 1 achievement or the transition from Phase 1 to Phase 2—solution method could not be determined, and the response was 3–6 seconds or highly deliberate reasoning strategy in 6 or more seconds but less than 15 seconds.
1 point	Behavior consistent with the achievement of Phase 1—method of solving could not be determined, and the response was in 6 or more seconds, but less than 15 seconds or the child overtly used a counting strategy.
0 points	Behavior indicating no effective mental-addition strategy—incorrect answer, false positive due to a response bias, or no response after 15 seconds.

requisite skills (Phase 1 counting strategies) to use and benefit from the computerized Stages III to V interventions. Stages III to V differed by condition in terms of the type of instruction or combination family.

Highly guided interventions. For the two highly guided interventions, Stage III was designed to help children devise a particular reasoning strategy (achieve Phase 2). Stage IV served to consolidate Phase 2 and begin the process internalizing the strategy it (i.e., facilitate the transition to Phase 3). The aim of Stage V was to promote the achievement of Phase 3 (fluency with the strategy). See Table 4 for further details. The add-1 intervention focused on constructing the add-1 rule and received no instruction on doubles relations or practice with the doubles. The opposite was true for the doubles relations.

Consistent with the recommendation of the National Mathematics Advisory Panel (NMAP, 2008), Stage III was structured, and Stage IV was partially structured. Structuring included sequentially arranging problems to highlight a relation and where that relation was applicable. For example, for the highly guided add-1 intervention, answering a number-after-n question (e.g., "What number comes after 3 when we count?") was immediately followed by answering a related $n + 1$ item (e.g., "$3 + 1 = ?$"). The $1 + n$ item was posed next to prompt recognition of additive commutativity and the applicability of the add-1 rule to $1 + n$ items. An $n + 0$ or $0 + n$ item and an $n + m$ item served as non-examples of the add-1 rule to discourage overgeneralizing the rule (i.e., to instill its appropriate use). Children actively determined sums and reconsidered answers in the face of feedback regarding correctness only. Children were sometimes asked if the answer to certain problems (e.g., number after 3) helped to solve other problems (e.g., $3 + 1$). However, the connection between the two was never explicitly stated as would be done in direct instruction (e.g., they were never explicitly told that $n + 1$ or $1 + n$ is the number after n). Similarly, in the highly guided doubles intervention, the doubles combinations were connected to prior knowledge about even numbers (fairly sharing a collection between two characters) and skip-counting (e.g., counting by 2s, 3s, 4s). Example screenshots for each stage of the add-1 and doubles interventions can be found in Appendix A. Accompanying the screen shots are de-

Table 4. Five Stages of Computer-Based Mental-Arithmetic Intervention

Stage Name	Description
Stage I: Concrete Intervention (7 sessions; ~3.5 weeks)	Aim: Support Phase 1 learning (use of counting strategies) and ensure recognition and understanding of the formal symbolism for addition and subtraction (e.g., 7 + 1 or 8–2) by connecting it to meaningful situations and their own informal solutions. Plan: During each of the seven sessions, children completed three sets of tasks. These tasks included learning how to navigate the program (e.g., use the mouse), introducing virtual manipulatives (e.g., record a score using a 10 frames and dots), solving word problems (relating expressions or equations to a concrete model), and relating part-whole terminology to equations and composition and decomposition.
Stage II: Estimation Intervention (8 sessions; ~4 weeks)	Aim: Serve as a developmental bridge between using Phase 1 strategies promoted by Stage I intervention (i.e., informal counting-based strategies with objects such as fingers or a 10 frame) and using the more advanced Phases 2 and 3 strategies (i.e., using mental-arithmetic strategies involving reasoning or retrieval). Plan: Numerical estimation (approximating the size of a single collection) was introduced first in sets 8 and 9. The stage begins with children visually estimating the number of carrots or frogs. Arithmetic estimation (approximating the size of sums and differences) was introduced in sets 10–12.
Stage III: Strategy Intervention (8 sessions; ~4 weeks)	Aim: Promote Phase 2 arithmetic knowledge by helping children discover the relations that underlie a reasoning strategy. Plan: In this stage, items were presented concretely (number lists) and symbolically (4 + 1 = ?). For the guided conditions, the items were juxtaposed with related problems. A child was encouraged to determine an answer in the manner of his/her choice (e.g., using a number list or fingers). No time limit was set for children to respond, and re-dos for incorrect responses were done concretely.
Stage IV: Strategy Practice (8 sessions; ~4 weeks)	Aim: Promote Phase 3 arithmetic knowledge. To promote this knowledge, items were presented symbolically, and children were encouraged to make an initial response mentally and quickly (“make a smart guess as quickly as you can”). Plan: Related items were juxtaposed or immediately followed one another only some of the time. Concrete solutions (number lists) were only to be used as a backup for determining the exact answer (correcting an incorrect response) on second attempts or re-dos. In addition, children were encouraged, through hints in the feedback, to use the relationship they are being trained in to solve problems in a timely fashion (e.g., “After 3 is 4 and 3 and 1 more is 4.”).
Stage V: Strategy Fluency (8 sessions; ~4 weeks)	Aim: Reinforce Phase 3 knowledge. In this stage, the child was encouraged to make a good guess (“smart guess”) as accurately and quickly as possible. Plan: As fluency is the goal of this stage, *mental* arithmetic was emphasized. The games were intended to underscore the importance of a *fast* and *smart* guess and the disadvantages of counting. For an initial incorrect response, the child was given a second chance to revise his or her answer mentally (a second-chance guess), but was not provided with hints or manipulatives.

Note.—Stages I and II = common preparatory intervention; Stages III to V = condition-specific experimental intervention.

scriptions of the process by which the strategies were highlighted in the highly guided interventions.

Practice-only intervention. The intervention for the minimally guided practice-only group consisted of engaging in similar games/activities. As with the highly guided interventions, Stage III involved no time limits; Stage IV had a generous time limit of 6 seconds; and the criterion for success in Stage V included a response time limit of 4 seconds. Although this comparison group practiced both add-1 and doubles items, the ordering of items was random, there was no relational training, and children only received feedback on correctness of their response.

Procedure

General overview. All project testing and intervention was conducted at project computer stations in a hallway outside a child's classroom or in a room dedicated to the project. Pullouts occurred in nonliteracy time blocks, including mathematics instruction and playtime per the request of the districts. All intervention instructions and feedback were both presented on the screen and read to the child by a trainer.

Of the 164 recruited children, three were excluded because of a severe developmental delay and nonavailability due to multiple out-of-classroom services, and another refused to participate in initial screening. The remaining 160 participants had sufficient understanding of English to complete the interventions. The preparatory (Stage I and II) instruction was 7.5 weeks long (15 half-hour sessions). During the period of preparatory intervention, participants completed the TEMA-3. After the preparatory intervention, participants completed the computer-based mental-addition pretest. Pretesting identified 78 eligible participants (fluent on half or fewer of the add-1 and doubles items), who were then randomly assigned *within* school to one of three intervention conditions: (*a*) highly guided add-1 ($n = 27$), (*b*) highly guided doubles ($n = 26$), or (*c*) minimally guided practice of both add-1 and doubles combinations ($n = 25$). After assignment, seven participants dropped out of the study either because their family moved or because the child refused to participate. Attrition was comparable across groups (see Table 1).

The experimental (Stage III to V) intervention took place during twice weekly 30-minute sessions over 12 weeks. Intervention sessions for all conditions took the same amount of time, and items common to a highly guided condition and the practice-only condition were practiced equally. Instruction was implemented on computers to maximize treatment fidelity by ensuring that all students in each condition received identical instructions and item ordering. Furthermore, the computerized programs also ensured that, for the highly guided programs, students were systematically exposed to each step in the highly guided learning process. All participants were retested on the mental-arithmetic items 2 weeks after the end of the primary intervention.

Analytic procedure. As the two primary (highly guided add-1 and highly guided doubles) groups targeted different relational families, each served as an active control group for the other and a means of testing the efficacy of the highly guided interventions. The minimally guided practice group also served as an instructional comparison group in both sets of analyses to determine if highly guided instruction resulted in better outcomes than minimally guided practice with the items. ANCOVAs, using pretest mental-arithmetic fluency index, pretest TEMA-3 standard score, and age as the covariates, were conducted to compare posttest performance of each group on targeted practiced and unpracticed combinations. As children were randomly assigned to condition within school, condition was not expected to be confounded with school. However, all analyses were also conducted with school and class/teacher as random-effect covariates. School was not found to be significant for any of the analyses and was not included in the analyses below. Teacher was found to be significant for one analysis, and it has been included in the model for which it was significant (noted below). Unconditional intraclass correlation coefficients (ICCs) were calculated for each outcome variable at both the teacher and school levels. All ICCs were small (<.10). The Benjamini-Hochberg correction was applied to correct for Type I error due to multiple comparisons.

Given the importance of reporting effect size (Lipsey et al., 2012; Wilkinson and the APA Task Force on Statistical Inference, 1999), efficacy was evaluated by using Hedges's g. Statistical significance (p values) and effect size are presented in the results section, and interpretations in the discussion section are largely based on effect sizes. Effect sizes were calculated for all specific contrasts of interest using posttest means that were adjusted to account for pretest and all covariates. Per the Institute of Education Sciences (2014) What Works Clearinghouse (WWC) criteria, a Hedges's g that exceeds .25 was considered indicative of substantively important practice.

Results

Preliminary Analyses

No significant pretest differences were found among the groups on practiced add-1 items ($F(2, 68) = .29, p = .750$), unpracticed add-1 items ($F(2, 68) = .75, p = .474$), practiced doubles ($F(2, 68) = .03, p = .968$), unpracticed doubles ($F(2, 68) = 1.96$, $p = .147$), or on the TEMA-3 ($F(2, 68) = .73, p = .486$). Means and standard deviations of the fluency index score for practiced and unpracticed (transfer) combinations by combination type and condition are detailed in Table 5. A summary of all effect sizes can be found in Table 6.

Table 5. Pretest and Adjusted Posttest Mean Fluency Index and Standard Deviations for Group Performance on Each Set of Arithmetic Combinations

	Highly Guided Add-1 (n = 25)				Highly Guided Doubles (n = 24)				Minimally Guided Practice (n = 22)			
	Pretest		Adjusted Posttest		Pretest		Adjusted Posttest		Pretest		Adjusted Posttest	
Combination Family	*M*	*SD*	*M*[a]	*SD*	*M*	*SD*	*M*[b]	*SD*	*M*	*SD*	*M*[c]	*SD*
Practiced $n + 1/1 + n$	1.02	1.07	3.57/3.66	1.40	.81	.90	2.88/2.90	1.77	.94	.83	4.00/3.95	1.29
Unpracticed $n + 1/1 + n$	1.17	1.08	3.20/3.29	1.52	.91	.93	2.67/2.77	1.71	1.26	1.03	3.12/3.12	1.71
Practiced doubles	1.25	1.16	2.23/2.28	1.29	1.28	1.26	3.76/3.83	1.46	1.34	1.14	3.82/3.72	1.28
Unpracticed doubles	.35	.55	1.23/1.31	.92	.49	.74	1.68/1.91	1.26	.76	.85	1.11/1.03	1.15

Note.—Posttest means are adjusted for age, TEMA-3, and pretest. Some adjusted posttest means are different for different comparisons because adjustments are based only on participants involved in specific comparisons.

[a] The first mean is for Highly Guided Add-1 vs. Highly Guided Doubles, and the second is for Highly Guided Add-1 vs. Minimally Guided Practice.

[b] The first mean is for Highly Guided Add-1 vs. Highly Guided Doubles, and the second mean is for Highly Guided Doubles vs. Minimally Guided Practice.

[c] The first mean is for the Highly Guided Doubles vs. Minimally Guided Practice, and the second mean is for the Highly Guided Add-1 vs. Minimally Guided Practice.

Table 6. Summary of Statistically and Meaningfully Significant Effects for Intervention Conditions

	Add-1		Doubles	
	Practiced	Unpracticed	Practiced	Unpracticed
Research question 1—efficacy of highly guided add-1:				
Highly guided add-1 vs. highly guided doubles (active control)	**.43**	**.33**		
Minimally guided practice vs. highly guided doubles (active control)	**.69**$^{+}$	.18		
Research question 2—efficacy of highly guided doubles:				
Highly guided doubles vs. highly guided add-1 (active control)			**1.09*****	**.40**$^{+}$
Minimally guided practiced vs. highly guided add-1 (active control)			**1.03*****	−.26
Research question 3—highly guided vs. minimally guided practice:				
Highly guided add-1 vs. minimally guided practice	−.21	.11		
Highly guided doubles vs. minimally guided practice			.00	**.65****

Note.—Boldface effect sizes (Hedges's *g*) indicate the differences met WWC effect-size standards for substantively important effects.

$^{+}$ $p < .10$.

* $p < .05$.

** $p < .01$.

*** $p < .001$.

Primary Analyses

The results of the primary analysis are summarized in Table 6 and detailed below. Analyses were conducted using both the F-index and fluency rate (correctly responded in <3 seconds without evidence of a response bias; a score of 4 or 5 on the F-index). As the results of the fluency rate analyses largely mirrored those of the F-index and because the F-index takes into account incremental increases in a child's knowledge, only the F-index results are presented.

> Question 1: Was highly guided discovery efficacious in nurturing fluency with practiced *and*, more importantly, related but unpracticed (transfer) add-1 items?

For practiced add-1 combinations, planned contrasts revealed that the highly guided add-1 group did not statistically significantly outperform the highly guided doubles (active-control) group (which did not practice the combinations) at the delayed posttest ($F(1, 44) = 2.61$, $p = .113$, Hedges's $g = .43$). The minimally guided practice group did outperform the highly guided doubles (active-control) group ($F(1, 41) = 7.43$, $p = .009$, Hedges's $g = .69$); however, this difference was no longer statistically significant after applying the Benjamini-Hochberg correction. Importantly, the effect size difference for both comparisons (i.e., Hedges's $g \geq .25$) is indicative of substantively important effects according to the WWC guidelines (IES, 2014).

For unpracticed add-1 items, no statistically significant difference was found between the highly guided add-1 group and the highly guided doubles (active-control) group ($F(1, 44) = 1.61$, $p = .212$, Hedges's $g = .33$) or between the minimally guided practice group and the highly guided doubles (active-control) group when including teacher as a random-effect covariate ($F(1, 40) = .39$, $p = .539$, Hedges's $g = .18$). Note that the effect size for the first, but not the second, comparison exceeded the criterion for substantively important effects (i.e., Hedges's $g \geq .25$) set by the WWC guidelines (IES, 2014).

> Question 2: Was highly guided discovery learning efficacious in promoting fluency with practiced and, more importantly, related but unpracticed (transfer) doubles?

For practiced doubles items, both the highly guided doubles group and the minimally guided practice group outperformed the highly guided add-1 (active control) group ($F(1, 44) = 25.78$, $p < .001$, Hedges's $g = 1.09$; $F(1, 42) = 27.90$, $p < .001$, Hedges's $g = 1.03$, respectively).

For unpracticed doubles items, the highly guided doubles group did not outperform the highly guided add-1 (active control) group ($F(1, 44) = 3.39$, $p = .072$, Hedges's $g = .40$), but this difference approached statistical significance. The minimally guided practice group did not significantly outperform the highly guided add-1 (active control) group ($F(1, 42) = 1.26$, $p = .269$, Hedges's $g = -.26$). However, both effect sizes exceeded the criterion for substantively important effects (IES, 2014). Interestingly, the minimally guided practice group performed worse on the unpracticed items than did the highly guided add-1 (active control) group, likely because the minimally guided practice group was somewhat higher at pretest than the other groups, but did not outperform the highly guided add-1 (active control) group at posttest.

> Question 3: Was highly guided discovery of the add-1 rule or doubles relations more helpful in promoting fluency with practiced and, more importantly, related but unpracticed items than minimally guided discovery learning?

No significant differences were found between the highly guided add-1 group and minimally guided practice group for either practiced add-1 items ($F(1, 42) = .72$, $p = .401$, Hedges's $g = -.21$) or unpracticed add-1 combinations ($F(1, 42) = .17$, $p = .683$, Hedges's $g = .11$). In contrast, although the highly guided doubles group did not significantly outperform the minimally guided practice group for the practiced doubles ($F(1, 41) = .00$, $p = .990$, Hedges's $g = .00$), it did for unpracticed doubles ($F(1, 41) = 8.67$, $p = .005$, Hedges's $g = .65$).

Discussion

The effect size findings in the present study largely support the hypotheses that highly guided discovery learning is efficacious in promoting knowledge of practiced and unpracticed (transfer) doubles knowledge. The findings for promoting knowledge of trained and untrained add-1 knowledge were less clear, but still promising. Importantly, the benefit of highly guided over minimally guided intervention was not as straightforward as hypothesized. Discussed in turn are the hypotheses, key findings, limitations, implications, and future directions.

> **Hypotheses 1 and 2**: Highly guided discovery would be more efficacious in promoting fluency with practiced and, more importantly, related but unpracticed combinations.

Add-1 Programs

Although the effects of the add-1 programs were not statistically significant (potentially due to power issues related to the small sample size), there were still meaningful and substantively important effect size differences between the groups (IES, 2014; Lipsey et al. 2012). Both the highly guided and minimally guided add-1 interventions produced meaningful improvement in practiced add-1 items compared to the highly guided doubles (active-control) group that did not practice add-1 items (Hedges's $g = .43$ and .69, respectively), but only the highly guided intervention produced meaningful transfer to unpracticed items (Hedges's $g = .33$ and .18, respectively). The results with the unpracticed add-1 combinations indicate that the highly guided add-1 intervention, but not the minimally guided practice intervention, was potentially efficacious in promoting transfer of a general add-1 rule. These group differences were found even with an extremely strong counterfactual. The curriculum for all except seven children (see Table 2 for curricula foci) actively taught add-1 in at least a cursory manner, and all groups made dramatic improvement in add-1 knowledge across the study (see Table 5). The noted effect size differences were found *on top* of high-quality curricula.

It could be argued that children can calculate add-1 sums quickly and that efficient counting strategies, rather than application of the add-1 rule, account for the transfer. Although it is not possible to distinguish between the efficient use of the add-1 rule and abstract counting-on strategy (e.g., for $5 + 1$, counting "Five, six" or simply "Six"), research indicates that the add-1 rule develops prior to, and provides a basis for inventing, the abstract counting-on (Baroody, 1995; Bråten, 1996). Specifically, within days of discovering the add-1 rule, children begin counting-on two and then three (e.g., "$5 + 3$, six [is one more], seven [is two more], eight [is three more]"). Discovery of the add-1 rule appears to serve as a scaffold for realizing that they can start the sum count with the cardinal value of an addend instead of "one" in the context of adding and integrating this insight with the keeping-track process necessary to execute abstract counting-on. That is, for $5 + 3$, a child might reason that if the sum of $5 + 1$ is the number after 5 is 6, then the sum of $5 + 3$ must be three numbers after five: "6 is 1 more, 7 is 2 more, 8 is 3 more"). The second count or keeping-track process ("is 1 more, is 2 more, is 3 more"), which serves to determine when to stop the sum count, may be explicitly verbalized or executed implicitly (subvocally or mentally). The exceptionally fast reaction times for add-1 items registered by older children and adults indicate that they are using an even more efficient process than fact recall or rapid counting, namely a rule-based strategy (Barrouillet & Thevenot, 2013; Campbell & Beech, 2014; Fayol & Thevenot, 2012).

In all probability, counting strategies less sophisticated and efficient than abstract counting-on would have been scored as slow, a counting strategy, or both—that is, as nonfluent. Even concrete counting-on would require more physical modeling than abstract counting-on. For 5 + 3, for example, entails representing an addend ahead of time by, perhaps, putting up three fingers, then stating the cardinal value of the other addend "five," and count on from there on the fingers previously put up: "six" [pointing to the first finger], "seven" [pointing to the second finger], "eight" [pointing to the second finger]. Note that concrete counting-on does not require a second count to keep track of how far to count beyond the cardinal value of the first addend because children simply stop counting-on when they run out of previously raised fingers. As children would have to represent an addend ahead of time (e.g., for 5 + 1, put five fingers or one finger up), this would take time and would probably be seen by a trained observer. Counting-all strategies, whether concrete or abstract, would be time consuming and noticeable. For 5 + 1, for example, a concrete counting-all strategy entails representing both the 5 and 1 and then counting both representations to determine the sum. Using an abstract counting-all strategy to determine the sum of an add-1 item (e.g., for 5 + 1, counting: "1, 2, 3, 4, 5; [and 1 more] is 6" or "1, 2 [is 1 more], 3 [is 2 more], 4 [is 3 more], 5 [is 4 more], 6 [is 5 more]) would take about 6 seconds to execute (see Baroody, Tiilikainen & Tai, [2006], for a detailed discussion of the differences among concrete and abstract counting-all and counting-on strategies).

Doubles Programs

The results clearly indicate that the highly guided doubles intervention was efficacious. This intervention promoted notably greater and meaningful improvements in doubles knowledge with practiced and, more importantly, unpracticed doubles items than did the highly guided add-1 (active control) group. The results with the unpracticed doubles combinations indicate transfer due to the highly guided doubles group achieving greater levels of doubles knowledge with the sums of unpracticed doubles. In comparison, whereas the minimally guided practice group clearly gained greater doubles knowledge with practiced doubles combinations than did the highly guided add-1 (active control) group, the minimally guided practice group did not exhibit transfer on unpracticed doubles items. Essentially, both the highly guided doubles and minimally guided practice groups improved on practiced items compared to the highly guided add-1 (active control group), but only the highly guided doubles intervention resulted in transfer to unpracticed problems. These results were found above and beyond standard classroom instruction, which—with the exception for one classroom (seven children)—practiced doubles items as part of the regular curriculum.

> **Hypothesis 3**: Highly guided discovery learning would be more helpful in promoting fluency with practiced and unpracticed items than minimally guided discovery learning.

Surprisingly, the highly guided add-1 intervention used in the present study was not clearly more effective than the minimally guided add-1 intervention at promoting fluency with practiced and unpracticed add-1 items. In contrast, the results of the doubles intervention indicate that, although the highly guided doubles intervention was not more efficacious than minimally guided practice with the *practiced* doubles, the highly guided instruction was significantly more successful in fostering knowledge with the *unpracticed (transfer)* doubles. The difference between the doubles and add-1 results may be due to three potential explanations.

First, relatively salient regularities such as the connection between well-known number-after relations and adding 1 may require little guidance. In contrast, the doubles strategy may be more difficult to learn, even with guided instruction, because the doubles connection to the even numbers, everyday analogies, and skip-counting (e.g., recognizing that the 4 + 4 can be determined by skip-counting "four, eight") may not be salient, in part, because primary-school-age children are unfamiliar or nonfluent with such prerequisite knowledge. As Kirschner et al. (2006) noted, although guided instruction is generally more effective than unguided instruction, this advantage recedes "when learners have sufficiently high prior knowledge to provide 'internal' guidance" (p. 75).

Second, in addition to the fact that both the highly guided add-1 and minimally guided practice-only programs involved supplemental practice with adding with 1, both programs also required a participant to enter the sums of such items on a number list and used a number list to provide visual feedback. These visual analogies implicitly but directly embody the add-1 rule. Specifically, adding 1 to a number (moving 1 cell more to the right on a number list) has a sum that is the number after in the counting sequence (results in landing on the cell of the next larger number on the number list). Thus, even the semi-random drill of the practice intervention could be enough structure to enable children to discern the regularity. In contrast, there is no direct relation between the visual representation of a number list and doubles relations. As such, a representation was probably not helpful in itself, the practice condition might better be considered negligibly guided discovery (involving targeted practice only), which appears to be insufficient to induce the nonsalient doubles relations.

Third, the regular classroom instruction for add-1 combinations may be more in-depth than that for the doubles. This in-depth classroom instruction plus the extra practice with add-1 combinations by the minimally guided practice group may not have elevated their level of relational learning (as indicated by transfer to unpracticed add-1 item) enough to produce a meaningful difference over the highly guided doubles (active control) group. However, it may have elevated the minimally guided group's transfer enough to reduce the difference with the highly guided group below a meaningful level.

Regardless of the explanation, the relative efficacy of highly guided and minimally guided discovery learning may not be as straightforward as Alfieri et al. (2011) imply. The results in this study may help to explain Alfieri et al.'s finding that outcomes based on the type of discovery learning were moderated by domain and age—that the magnitude of effect differences between explicit, guided, and unstructured discovery were smaller for certain academic domains (notably mathematics) and for different ages (notably for younger children). The domain of early childhood mathematics offers many relatively salient relations that children can likely discover without much guidance, whereas other domains and higher level mathematics involve relations that are less salient and require more guidance to notice. For example, in our prior research on other relational strategies, highly guided discovery learning was more efficacious for the less salient subtraction-as-addition and use-a-10 strategies (Baroody, Purpura, Eiland, & Reid, 2014; Baroody, Purpura, Eiland, Reid, & Paliwal, 2016). These differences in the efficacy of instructional relations indicate that a more fine-grained approach may be needed in structuring instructional plans, as different methods may be needed to maximize children's learning of the different combination families.

Conclusion

The Role of Highly Guided Practice

The overall results indicate that the frequency of practice may not be the most important factor in the meaningful memorization of basic combinations, but that discovery of mathematical relations may play an important role in enabling children to flexibly solve new (unpracticed), but related, combinations. The results, particularly for doubles, are also consistent with the view that guided-discovery learning has unique beneficial effects on achieving transfer to novel problems.

The transfer produced by both highly guided interventions is consistent with the hypothesis that practice may not only be a method for strengthening knowledge of individual number facts by rote memorization, but an opportunity to enrich memory of a combination by actively creating new connections with it. Each time a known fact or relation is recalled, new information can be connected to old information—changing the existing form of the old information—thereby increasing an individual's ability to recall both forms of knowledge (Nader & Hardt, 2009). For example, recalling the total number of fingers and thumbs on two hands while solving $5 + 5 = ?$ may help children to construct or strengthen knowledge of the doubles tactic—relating doubles combinations to familiar real-world pairs of a set. Recognizing the connection between known pairs and doubles may be what allows the generalization of other real-world pairs to their respective doubles combination (e.g., two rows of six eggs [$6 + 6$] equals 12). Such a representation allows children to use their (automatic) knowledge of the doubles strategy to efficiently deduce the sum of any $n + n$ item for which they know an analogous, everyday pair. These processes of active memory embellishment may have been supported or prompted by the structure of the guided intervention.

Limitations of the Present Research

First, as the same project personnel conducted the intervention and testing, one possible limitation of the present study was "experimenter bias." Arguments against such bias are the uneven or disappointing results (e.g., the highly guided add-1 group did not outperform the minimally guided practice group). Second, the majority of children in this study were in classrooms that utilized curricula that, to some degree, practiced add-1 and doubles items. However, the effects of these interventions were found above and beyond typical instructions—indicating that the interventions, particularly the highly guided doubles intervention, are highly efficacious. A third limitation of the present research was that the full range of discovery learning (e.g., minimally vs. moderately vs. highly guided discovery) was not directly compared with each other or with direct/explicit intervention of reasoning strategies. Thus, the findings from this study cannot necessarily be used as a means to say that highly guided instruction is the most effective form of instruction.

Implications of Findings and Future Directions

Learning reasoning strategies is generally viewed as a key vehicle for helping children achieve the meaningful memorization of the basic sums (NMAP, 2008). However, best practices for helping primary-grade children achieve these ends remain unclear.

The relative merits of discovery learning. Currently, the most prevalent curricular approach to teaching reasoning strategies for unknown sums is essentially fully guided (direct, explicit) instruction in which the teacher describes the strategy and its rationale, then provides targeted practice of the strategy with supervision and feedback, and finally furnishes "mixed" practice that could involve other strategies (Math Expressions [Fuson, 2006]; Go Math 2012 [Houghton Mifflin Harcourt, 2012]; Saxon Math [Larson, 2008]; Math Connects [Macmillan/McGraw Hill, 2012]; Everyday Mathematics [UCSMP, 2005]). The conventional approach involving fully guided instruction has, to date, not been compared to partially guided instruction (structured or unstructured discovery learning). The current study provides a foundation for such comparison by demonstrating the efficacy of the highly guided (structured discovery) add-1 and doubles programs and the minimally guided add-1 program.

Undoubtedly, both highly guided programs can be improved further by, for example, supplementing the existing active and constructive activities with interactive activities, such as dialoguing with a virtual character, considering its contribution, and revising errors based on scaffolding or feedback (Chi, 2009). Eliciting explanations is an effective way to foster strategy learning and generalization (Chi, de Leeuw, Chiu, & LaVancher, 1994; Crowley & Siegler, 1999). See Hmelo-Silver et al. (2007) for additional forms of scaffolding. The efficacy of such changes needs to be evaluated to determine the most efficacious way of utilizing highly guided instruction. Once the most efficacious version of highly guided instruction is identified, it should be compared to explicit or fully guided instruction. Findings from that work will have direct implications for curriculum development and general classroom practices.

The value of computer-based interventions. It could be argued that, despite the gains made in the highly guided interventions, the computer-based interventions were time-intensive and, thus, may have limited practical value. However, the positive effects on retention and transfer were achieved with relatively little practice (i.e., with only approximately 27 trials for each practiced combination and only using a subset of items). Moreover, the investment of student time (about 8 hours in actual instruction) required by the current programs, which further refinement of the programs may reduce, needs to be weighed against the costs of the active controls' missed educational opportunities. Current textbooks and teacher professional development typically do not (sufficiently) focus on the connection between adding 1 and children's existing number-after relations or the relations between the doubles and the even numbers or everyday analogies of the doubles. For example, the Math Expressions text (Fuson, 2006) used by four of the participating classes in this study has a single lesson on adding 1 and does not connect such addition to number-after relations. The Everyday Mathematics program (UCSMP, 2005) used by nine of the participating classes in this study recommends teaching the number-after rule, but the single lesson on the topic involves practicing number-after relations and adding 1 separately. (See Table 2 for a more detailed analysis of these two curricula). The primary tool for helping children memorize the doubles in Saxon Math curriculum (Larson, 2008), used by one participating class, is the rhyming but otherwise meaningless "Doubles Rap" (e.g., $1 + 1 = 2$, Oooooh! $2 + 2 = 4$, More! $3 + 3 = 6$, Kicks!). Without (adequate) meaningful instruction, it is unclear what the costs of additional instruction, repetitious drill, and repeated review would be that would enable the active control participants to catch up to their peers in the experimental conditions.

It is an empirical question whether the interventions embodied in the programs (e.g., encouraging students to record their add-1 sums on a number list, directing their attention to the connection between adding 1 and the number-after relations by practicing one after the other or the sums of doubles to the even numbers, encouraging analogical reasoning to determine the sums of doubles) may be effectively taught via textbook-based teacher instruction with equal or better efficacy and efficiency than stand-alone computer-based intervention. There are several reasons, though, to believe that the latter would ultimately be more cost effective. Stand-alone programs would require less teacher professional development, time, or effort to use. For example, teachers would only periodically need to check on students' progress through the programs. Moreover, the highly guided programs might better ensure students systematically receive each particular step of sophisticated, sequenced instruction they need.

Summary

In conclusion, the effect sizes found in the present research indicate that both highly guided or minimally guided discovery is a more useful tool for fostering the learning of a general add-1 reasoning strategy than regular classroom instruction. However, only highly guided instruction is a more useful tool for helping at-risk first graders discover the doubles relations and apply this knowledge to unpracticed combinations. Unlike the doubles strategies, minimally guided instruction may have been sufficient to promote learning of the add-1 rule because the relation involved is highly salient, entering answers on a number list directly models the rule, and classroom instruction may have reinforced this strategy. The evidence of doubles transfer supports the positions outlined by both the NMAP (2008) and the number sense view (Baroody, 1985; Baroody, Bajwa, & Eiland, 2009; Gersten & Chard, 1999; Jordan, 2007) that (*a*) instruction designed to help students see arithmetic regularities is useful, and (*b*) reasoning strategies can be an efficient, appropriate, and adaptive basis for determining the solutions to basic combinations. Ultimately, these findings provide key foundations for future research centered on refining and evaluation the structure of key curricula and general classroom practices.

Appendix

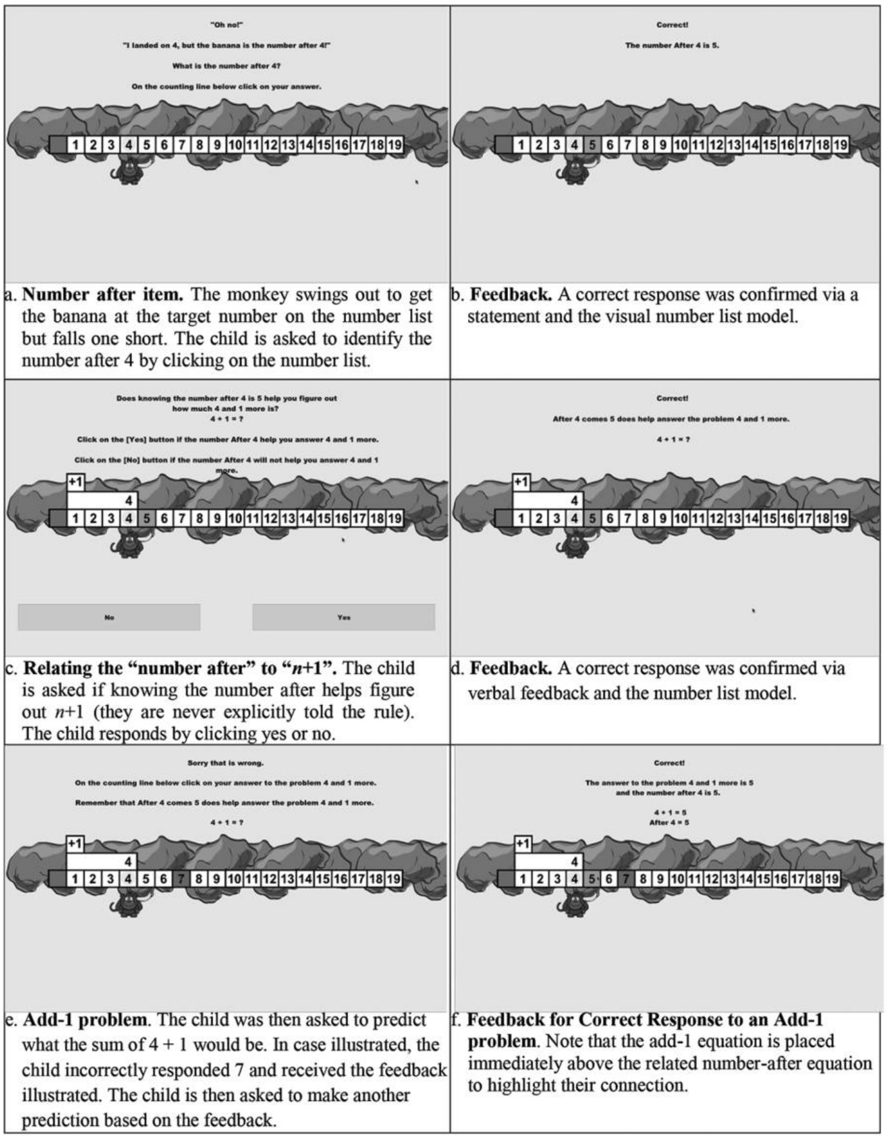

Figure A1. Stage III: Connecting number after and $n + 1$ (monkey game). A color version of this figure is available online.

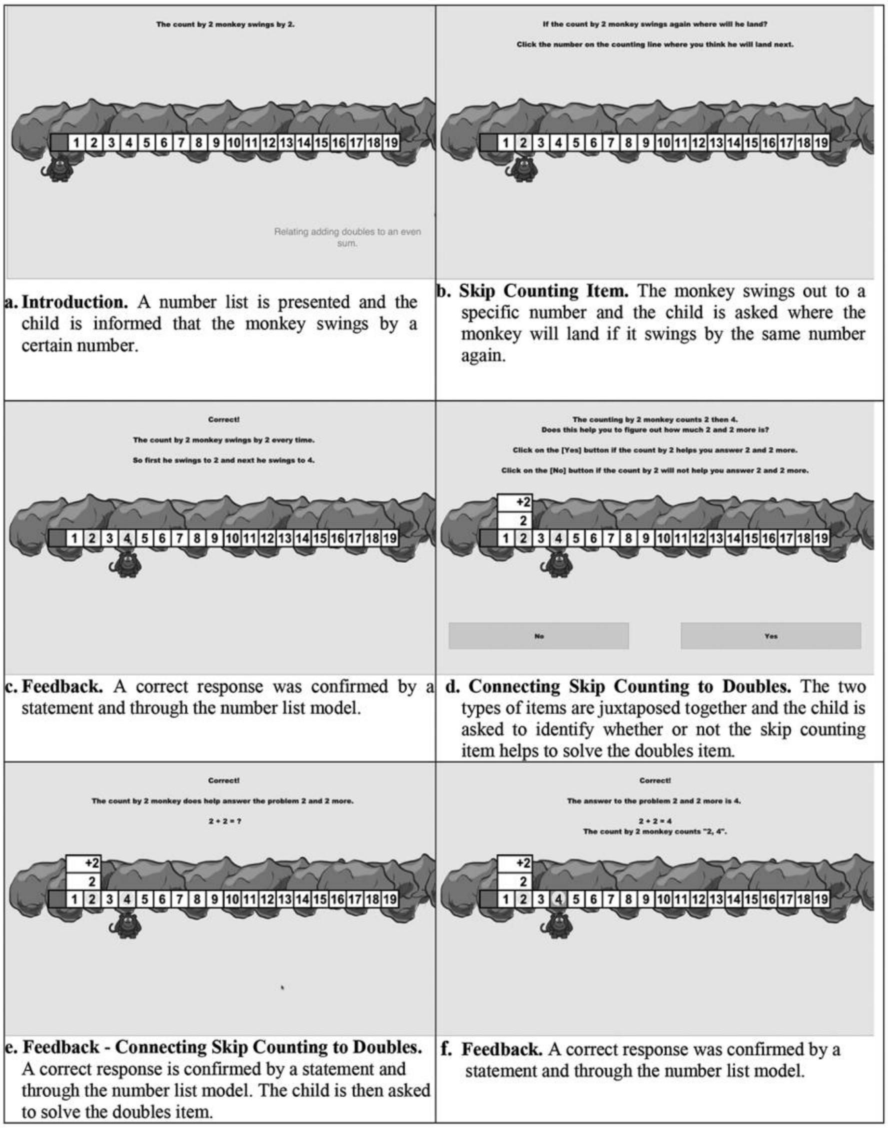

Figure A2. Stage III: Connecting skip-counting to doubles (monkey game). A color version of this figure is available online.

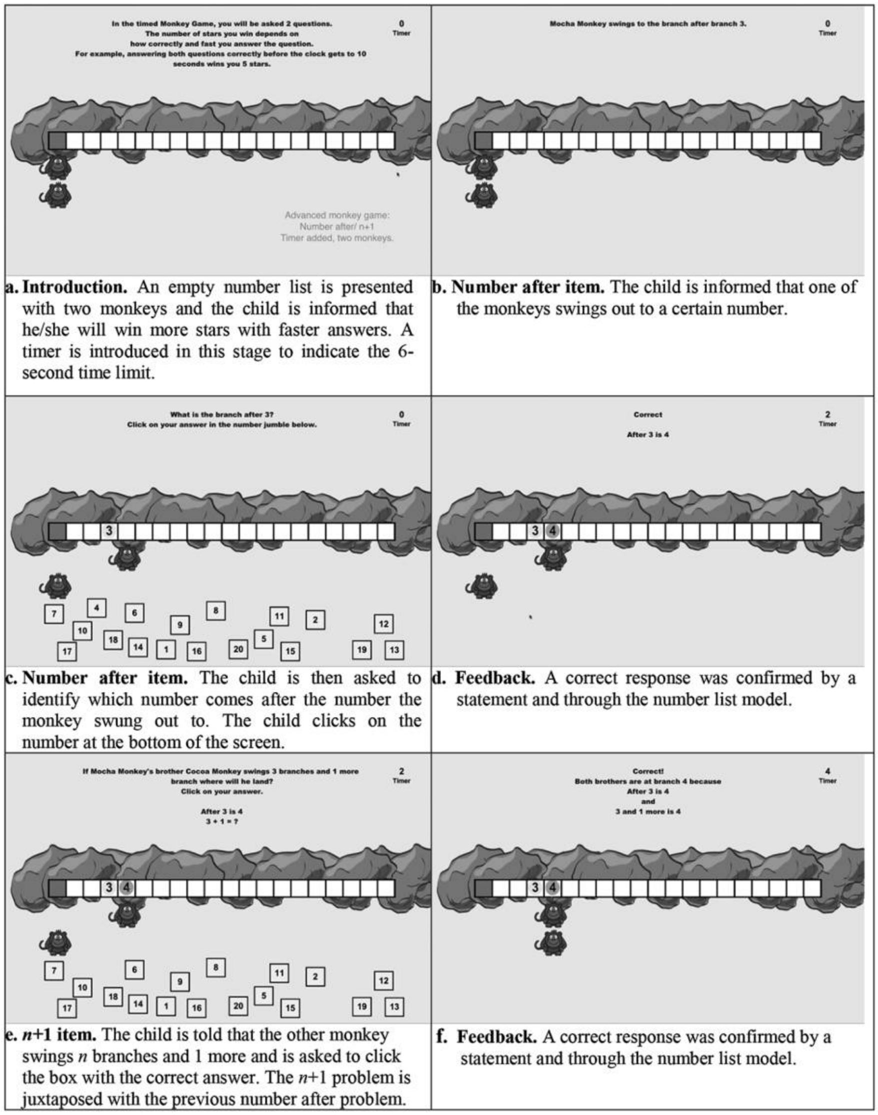

Figure A3. Stage IV: Implicitly connecting number after and $n + 1$ (monkey game). A color version of this figure is available online.

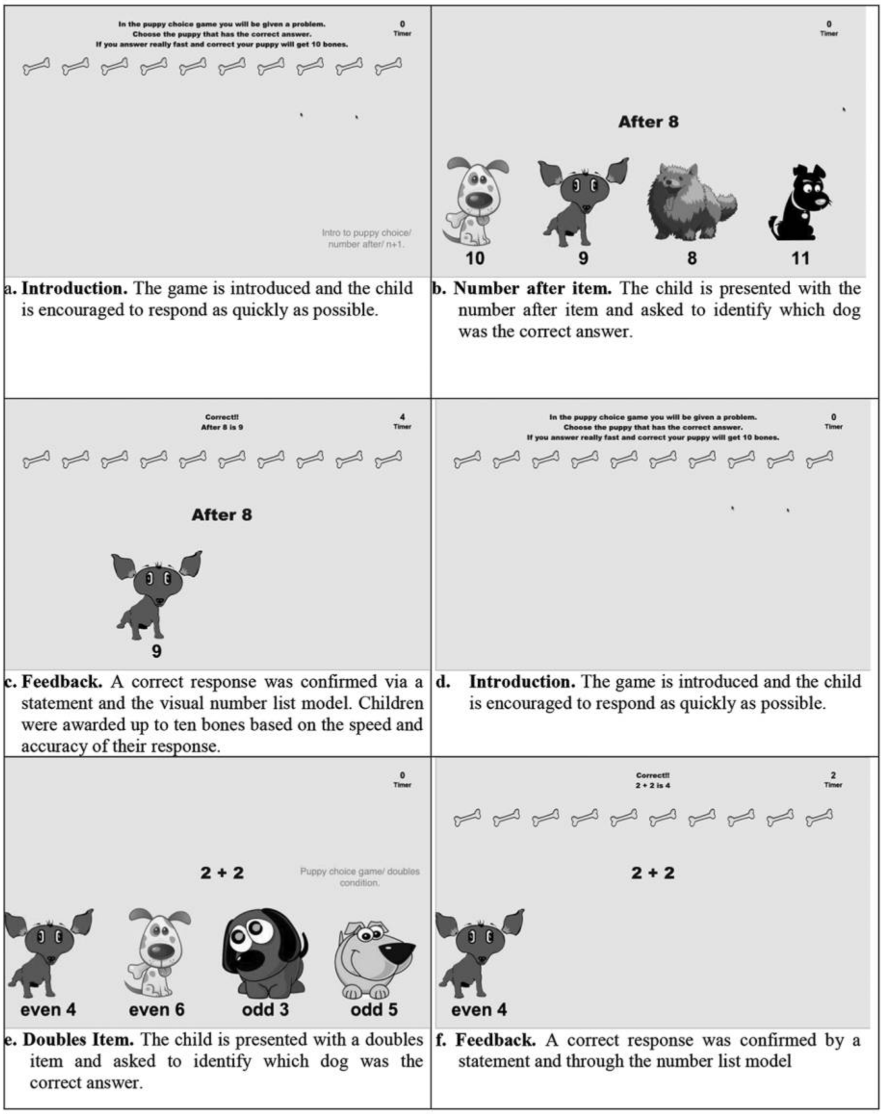

Figure A4. Stage V: Practice using the number after and $n + 1$ (panels *a* to *c*) and the doubles relations (panels *d* to *f*) in puppy choice game. A color version of this figure is available online.

Note

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