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Explicit Instructional Interactions: Exploring the Black Box of a Tier 2 Mathematics Intervention

Christian T. Doabler, PhD[1], Ben Clarke, PhD[2], Mike Stoolmiller, PhD[3], Derek B. Kosty, PhD[4], Hank Fien, PhD[2], Keith Smolkowski, PhD[4], and Scott K. Baker, PhD[5]

Abstract

A critical aspect of intervention research is investigating the active ingredients that underlie intensive interventions and their theories of change. This study explored the rate of instructional interactions within treatment groups to determine whether they offered explanatory power of an empirically validated Tier 2 kindergarten mathematics intervention. Direct observations were conducted in 46 interventions groups, involving approximately 228 students. Multilevel structural equation models revealed that fall mathematics achievement significantly and negatively predicted the rate of academic feedback. Specifically, intervention groups with lower student mathematics achievement at the start of the kindergarten year received higher rates of academic feedback. Analyses also suggested that latent rates of instructional interactions were not significantly correlated with gains on student mathematics outcomes. Implications are discussed in relation to specifying the underlying mechanisms to intensify mathematics interventions, documenting local adaptation of intervention implementation, and examining possible threshold effects of instructional interactions.

Keywords

systematic and explicit mathematics instruction, instructional interactions

The research agendas of prominent, federal funding agencies, such as the Institute of Education Sciences (IES) and the National Science Foundation, have led to scientific breakthroughs in the field of educational research. At the forefront of many of these advancements have been methodologically sophisticated efficacy trials (e.g., randomized controlled trials), where the primary purpose is to produce high-quality, trustworthy evidence about the impact of an intervention on targeted outcomes relative to a counterfactual or a comparison condition. Efficacy data are an integral component for building the knowledge base on effective instructional practices and educational tools. However, alone these data are insufficient to draw reliable inferences as to why an intervention works, for whom, and under what conditions.

To better understand the evidence behind an intervention's impact on student outcomes, researchers need to investigate the *active ingredients* that underlie interventions and their theories of change. Active ingredients represent the core mechanisms through which mathematics interventions operate. Investigating the active ingredients of interventions may increase researchers' capacity to explicate how or under what conditions interventions produce positive or, in some cases, negative treatment effects. The purpose of this article was to explore the theoretically specified mechanisms that comprise an empirically validated Tier 2 mathematics intervention to determine whether they had explanatory power contributing to the improvement of student mathematics outcomes within the treatment condition.

Effects of Systematic and Explicit Mathematics Interventions

Much of what the field knows about instruction for students with or at risk for mathematics difficulties (MD) has been derived from studies that examined interventions with explicit and systematic instructional foundations (Gersten et al., 2009). In many cases, the impact of these interventions on student mathematics outcomes was contrasted with

[1]The University of Texas at Austin, USA
[2]University of Oregon, Eugene, USA
[3]Michigan State University, Marquette, USA
[4]Oregon Research Institute, Eugene, USA
[5]Southern Methodist University, Dallas, TX, USA

Corresponding Author:
Christian T. Doabler, University of Texas at Austin, 1912 Speedway, SZB 228, Austin, TX 78712, USA.
Email: cdoabler@austin.utexas.edu

a business-as-usual control condition. For example, L. S. Fuchs et al. (2005) investigated the efficacy of preventive tutoring on the mathematics outcomes of 139 first-grade students with MD. Participating at-risk students (AR) were randomly assigned to a tutoring condition or a control condition that provided core mathematics only. AR students assigned to the tutoring condition received 30 min of small-group tutoring and 10 min of technology-based practice to build math facts fluency. Small-group tutoring utilized explicit mathematics instruction to teach key concepts and skills associated with whole numbers. Results significantly favored the AR students assigned to the tutoring condition over their AR control peers on standardized measures of whole number computation, problem solving, and whole number concepts and application, with effect sizes (Hedges's *g*) ranging from .14 to .70.

More recently, Bryant et al. (2011) conducted a randomized controlled study to examine the effects of a systematic and explicit Tier 2 mathematics intervention on the mathematics achievement of first-grade students with MD. Approximately 200 students were randomly assigned to a business-as-usual control condition or the treatment intervention, which focused on early numeracy concepts and skills. Statistically significant treatment effects were found on proximal measures of whole number computation and aspects of number proficiency. Bryant and colleagues reported effect sizes (Hedges's *g*) ranging from .39 to .55.

In our own work, we have studied the efficacy of the ROOTS intervention, a Tier 2 kindergarten program that was engineered to help kindergarten students with or at risk for MD develop a deep understanding of whole numbers. Central to the ROOTS intervention's capacity to meet the instructional needs of students with MD is an "instructional platform" (Simmons, 2015) that carefully integrates foundational concepts of whole number and validated principles of systematic and explicit mathematics instruction (Coyne, Kame'enui, & Carnine, 2011; Gersten et al., 2009). Recent efficacy trials funded by IES have documented the intervention's positive impact on important student mathematics outcomes (Clarke et al., 2016; Clarke et al., in press).

Our initial study of the ROOTS intervention involved 29 classrooms that were part of a larger randomized controlled trial (Clarke et al., 2016). Fourteen classrooms were in the treatment condition, and 15 were in the control condition. Teachers in both conditions were asked to nominate the five lowest performing students. A total of 140 students were determined eligible for ROOTS, with 67 students in treatment classrooms and 73 students in control classrooms. Students in the treatment classrooms received ROOTS in small-group formats (i.e., three–five students), 3 days per week for approximately 18 weeks. The intervention was delivered by school-employed paraprofessionals, who had experience working with students with learning difficulties. Control students received core mathematics instruction only. Nested time by condition analyses suggested that ROOTS students made stronger gains than their control peers across kindergarten on a standardized outcome measure of mathematics (Hedges's $g = .38$) and a set of four early numeracy curriculum-based measures (Hedges's $g = .30$).

The most recent study of ROOTS utilized a randomized block design (Clarke et al., in press), with 290 students randomly assigned within classroom to one of three conditions: (a) a ROOTS intervention group with a 2:1 student–tutor ratio ($n = 58$), (b) a ROOTS intervention group with a 5:1 student–tutor ratio ($n = 145$), or (c) a no-treatment control condition ($n = 87$). Students in the ROOTS groups continued to receive district-approved core mathematics instruction. Control students, however, received district-approved core mathematics instruction only. Results showed that ROOTS students demonstrated greater mathematics achievement gains than their peers in the control condition on four of the six outcome measures. Specifically, reported effect sizes (Hedges's *g*) ranged from .28 to .74 (Clarke et al., in press).

It appears from our studies and other efficacy studies of early mathematics interventions (e.g., Bryant et al., 2008; L. S. Fuchs, Fuchs, & Compton, 2012; Sood & Jitendra, 2013) that systematically designed and explicitly delivered mathematics interventions are critical for students with or at risk for MD. In the case of ROOTS, the evidence regarding the intervention's impact on student mathematics achievement was generated in efficacy trials that focused primarily on comparisons between the treatment and control conditions (Clarke et al., 2016; Clarke et al., in press). Consequently, much is still to learn about the underlying mechanisms behind ROOTS and its theory of change.

The Theory of Change for the ROOTS Intervention

In the fields of curriculum development and intervention research, a well-specified theory of change represents the underlying mechanisms of an intervention that are hypothesized to increase student achievement when implemented as intended (Doabler, Clarke, et al., 2015). Figure 1 depicts the theory of change for the ROOTS intervention. As depicted, the intervention involves three key tenets: (a) intervention components, (b) high-quality instructional interactions, and (c) proximal and distal student outcomes. Two intervention components comprise the ROOTS program: (a) whole number concepts and skills, and (b) validated principles of systematic and explicit mathematics instructions. When systematically integrated, these two components are anticipated to facilitate high-quality *instructional interactions* between teachers and students, and among students, around critical concepts and skills of whole numbers. Such interactions are hypothesized to mediate the effects of ROOTS on proximal and distal

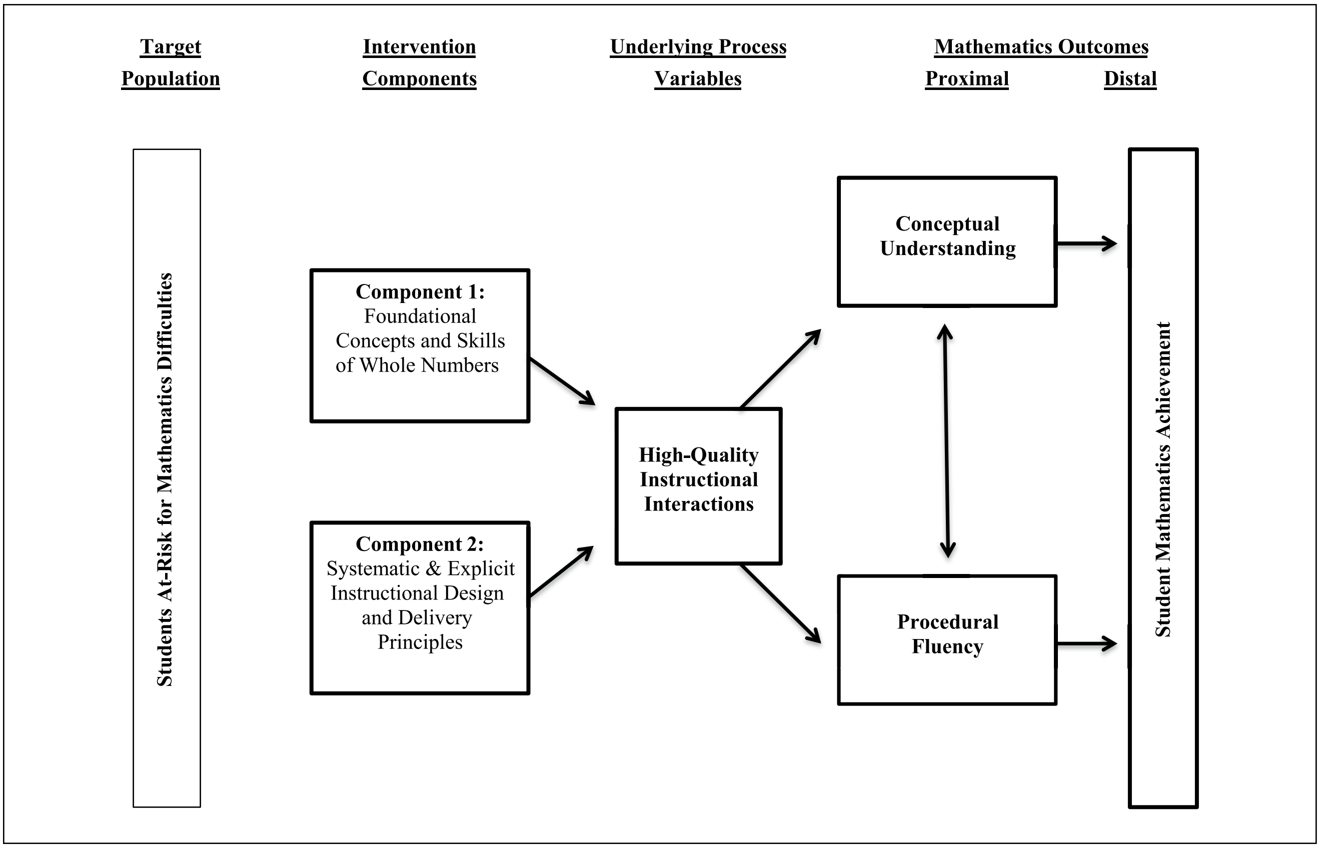

Figure 1. ROOTS theory of change.

mathematics outcomes. Proximal mathematics outcomes include students' conceptual understanding of key concepts and procedural fluency with mathematical problems. In Figure 1, student mathematics achievement is depicted as the distal outcome.

Mathematical Content of the ROOTS Intervention

ROOTS is a 50-lesson, Tier 2 kindergarten intervention program that was designed to support students in developing early mathematics proficiency (Clarke et al., 2016). Each 20-min lesson consists of four to five mathematics activities that center on three domains of whole number understanding identified in the Common Core State Standards (CCSS) Initiative (2010): (a) Counting and Cardinality (b) Operations and Algebraic Thinking, and (c) Number and Operations in Base Ten. School-based personnel deliver the lessons in small-group formats, with two to five students per group.

Because at-risk kindergarten students tend to experience difficulties understanding and working with concepts and skills of whole number, the first intervention component of the ROOTS intervention invariably prioritizes developing students' early number sense. Number sense refers to a child's awareness of and fluidity with numbers (Berch, 2005; Gersten & Chard, 1999). As identified in the CCSS Initiative (2010) and recommended by experts (Gersten et al., 2009; National Research Council [NRC], 2001), foundational aspects of number sense addressed in the ROOTS intervention include using efficient counting strategies (e.g., *min* strategy), recalling answers to basic number combinations, such as 5 + 2, gaining foundations of the base-ten system, and solving simple addition and subtraction word problems.

Principles of Systematic and Explicit Instruction

The second intervention component of the ROOTS intervention attends to the instructional design and delivery principles that form the architectural foundation of the intervention. Converging evidence suggests that students with MD significantly benefit from instruction that is systematically designed, explicitly delivered, and includes a purposive selection of scaffolded student learning opportunities and visual representations of mathematical ideas (Gersten et al., 2009; National Mathematics Advisory Panel [NMAP], 2008).

The ROOTS intervention was developed with specific attention to the *systematic* and *explicit* instructional design and delivery principles that have been validated to improve the mathematics outcomes of at-risk learners (Gersten et al., 2009). These principles include (a) engaging students' prior understandings of mathematics, (b) providing vivid demonstrations and clear explanations of mathematical concepts, (c) using visual representations of mathematical ideas to promote conceptual understanding, (d) providing opportunities for practice and review to promote mathematical proficiency, and (e) delivering timely academic feedback to address student misconceptions.

Instructional Interactions in Early Mathematics

Instructional interactions are often viewed as a critical element of academic development (NRC, 2001; Pianta & Hamre, 2009) and a defining feature of systematic and explicit interventions (Gersten et al., 2009). Systematic and explicit mathematics interventions are designed to facilitate high rates of important instructional interactions between teachers and students around critical whole number concepts. Such instructional interactions represent a dynamic interplay of the teacher offering clear examples and demonstrations of new mathematics content, students working independently and with their peers in structured learning opportunities, and the teacher providing specific, informational feedback in response to student learning (Doabler, Baker, et al., 2015). For example, a systematic and explicit intervention will support teachers in facilitating structured opportunities for students to verbalize their mathematical thinking and understanding.

The ROOTS intervention facilitates three types of systematic and explicit instructional interactions: (a) overt teacher models, (b) deliberate student practice opportunities, and (c) specific academic feedback. For example, the intervention offers tutors scripted guidelines to overtly explain and demonstrate critical concepts of whole number. The intervention also assists tutors in facilitating frequent student practice opportunities, such as allowing students to verbalize how they solved a mathematics problem and use place value blocks to represent teen numbers. In addition, ROOTS offers strategies for providing academic feedback to address student errors and affirm correct responses.

Each ROOTS lesson contains between 100 and 150 prescribed instructional interactions. Such interactions vary in terms of how they are detailed in the program. Some interactions, for example, are fully scripted, providing tutors specific directions on how to pose mathematical questions and demonstrate mathematics concepts. In other cases, the intervention offers reminders for tutors to facilitate instructional interactions. For example, after a mathematical concept or skill is introduced, the intervention scripting makes recommendations for tutors to "provide individual turns," "repeat with other pairs of children," "confirm or provide corrective feedback," or "repeat with the next number sentence."

To date, we have yet to explore the nature of the instructional interactions facilitated by ROOTS and whether these critical variables explain variance in student mathematics outcomes *within* the treatment condition. Below, we define these three types of instructional interactions and briefly review the research that supports why they are particularly beneficial for at-risk learners and relevant for investigation within the ROOTS intervention groups.

Overt Teacher Models

Teacher models are intended to make new and complex mathematics content conspicuous to students (Archer & Hughes, 2011). Research suggests that teacher models are a more powerful way of presenting critical academic content relative to discovery and problem-based teaching methods (Gersten et al., 2009; Mayer, 2004). Teacher models can entail step-by-step demonstrations of mathematical procedures and explanations of complex concepts. Think-alouds are another form of teacher modeling, where the teacher makes her mathematical thinking and problem-solving processes overt to students. In ROOTS, for example, teachers might state, "I used my place value cards to show how 16 is made up of one ten and six ones."

Deliberate Student Practice Opportunities

Research on mathematics interventions has found that deliberate practice opportunities are essential for improving important mathematics outcomes (Gersten et al., 2009). In early mathematics, effective forms of student practice entail working with visual representations of mathematical ideas (e.g., place value blocks) and engaging in mathematical discourse. Visual representations, when systematically included in instruction, allow students with MD the opportunity to express mathematical ideas and transform the representations into abstract symbols (Gersten et al., 2009; NMAP, 2008). Student verbalizations are important because they permit students to convey their mathematical thinking and understanding of critical mathematics concepts before other modes of practice are instructionally appropriate, such as solving mathematics problems through written exercises (Doabler, Baker, et al., 2015). In ROOTS, tutors can prompt mathematics verbalizations from individuals and groups of students. Individual response opportunities permit tutors to monitor a student's mathematical understanding and learning, whereas group responses allow more than one student the opportunity to verbalize their mathematical thinking.

Specific Academic Feedback

Academic feedback refers to a tutor providing specific informational feedback about a student response or action. The ROOTS intervention offers teachers the strategies for how to provide timely corrective and affirmative feedback that is specific enough to extend learning opportunities, decrease the likelihood of later misconceptions, and address specific knowledge gaps. Research has demonstrated that teachers can increase mathematics achievement by providing specific academic feedback (Gersten et al., 2009; Vaughn & Swanson, 2015).

Taken together, these three types of instructional interactions (i.e., teacher models, student practice opportunities, and academic feedback) are of particular interest because they provide an ideal platform for exploring the "black box" of an empirically validated Tier 2 mathematics intervention. Examination of instructional interactions can also help ascertain how to intensify early mathematics interventions for students with MD and, in turn, provide the instructional experiences that lie at the heart of multitiered systems of supports (MTSS) in mathematics and Response to Intervention (RtI) service delivery models (D. Fuchs & Fuchs, 2015; L. S. Fuchs & Vaughn, 2012). For example, the benefit of certain behaviors, like practice opportunities, may have maximum effect on intensifying an intervention when delivered below a particular rate of occurrence. However, practice opportunities may have limited value in intensifying an intervention after a certain threshold is reached. Moreover, greater importance may lie with the frequency of explicit teacher models. We note that the role of these behaviors may interact with other factors such as the initial skill level of the group. For example, higher rates of practice opportunities for lower performing groups may be critical whereas they may have less relative value for higher performing groups.

Purpose of the Study

A growing research literature on early mathematics interventions has begun to crystallize the importance of systematic and explicit instruction for students with or at risk for MD (Gersten et al., 2009). However, to our knowledge, no intervention studies have examined whether rates of instructional interactions offer explanatory power for detecting differences on outcomes within the treatment condition. This study sought to address this blank spot in the field of mathematics intervention research by examining whether instructional interactions within the ROOTS mathematics intervention contributed to the improvement of student mathematics achievement within the treatment condition. Two research questions were addressed in the study.

> **Research Question 1:** Does the rate of instructional interactions within the ROOTS intervention predict gains in student mathematics outcomes, net of mathematics achievement at the start of the year?
>
> **Research Question 2:** What is the relationship between student mathematics achievement at the start of kindergarten and the subsequent rate of instructional interactions during the ROOTS intervention?

Method

Participants

Forty-six ROOTS intervention groups from 14 kindergarten classrooms in Oregon and 32 kindergarten classrooms in Texas participated in the study. Each group represented one kindergarten classroom. Of the 46 classrooms, 33 were located in public schools, eight in private schools, and five in charter public schools. All charter and private school classrooms were located in Texas. Public school classrooms were located in schools eligible for Title 1 funding and had populations where an average of 76% qualified for free or reduced-price lunch programs.

Each ROOTS group was taught by a school-employed paraprofessional. Of the 46 tutors, 11 held bachelor's degrees and four had associate's degrees. Twenty-three of the tutors held only high school diplomas. Eight of the 46 tutors were certified in elementary education. The majority of the tutors were females (98%) and identified their race as White. No attrition among tutors occurred during the study.

In the efficacy trial, the five lowest performing students in each of the 46 treatment classrooms were nominated by the teacher to receive the ROOTS intervention in addition to their core (Tier 1) mathematics instruction. The nomination process entailed a three-step process. First, to be considered eligible for the intervention, a student had to have a pretest score below the 40th percentile on the *Test of Early Mathematics Ability–Third Edition* (TEMA-3). From those students who qualified for intervention, teachers were provided with student scores from a battery of curriculum-based measures that assessed students' number proficiencies (see "Measures" section). Teachers then selected up to five students who demonstrated low performances on the number sense measures.

Of the 228 nominated ROOTS students, 77% were White, 8% Black, and 15% were missing race data. In addition, 30% were English learners, 50% were male, and 58% identified their ethnicity as Hispanic. The average age of the ROOTS students at pretest was 5.5 years (SD = 0.3). Nearly 60% of the ROOTS students scored below the 10th percentile on a standardized mathematics measure in the fall of their kindergarten year.

Research Design

This study analyzed data collected during an IES-funded efficacy trial of the ROOTS kindergarten mathematics intervention (Baker, Chard, Clarke, Smolkowski, & Fien, 2008). The study took place in Oregon and Texas, respectively, during the 2009–2010 and 2010–2011 school years. Blocking on schools, the efficacy trial randomly assigned 94 full-day kindergarten classrooms to either treatment (n = 46) or control (n = 48) conditions. Each of the 46 treatment classrooms provided one ROOTS intervention group. Because the purpose of this article was to explore the explanatory power of instructional interactions facilitated within the ROOTS condition, our analyses focused exclusively on observation and student performance data collected in the 46 ROOTS groups.

Procedures

Implementation of ROOTS. School-based tutors delivered the ROOTS intervention in small-group formats (5:1 student–tutor ratio), 3 days per week (20-min sessions) for approximately 18 weeks. All lessons were scripted to enhance implementation and retain the preciseness of the mathematical content. Each ROOTS lesson contained between four and five activities. The opening activity entailed the "Nifty Fifty" activity and its purpose was to promote students' number identification skills and use of efficient counting strategies with a (1–50) number chart. Each Nifty Fifty activity corresponded to the number of lessons completed in the intervention program. For example, in Lesson 18, tutors used the Nifty Fifty activity to help children count and identify numbers up to 18. The opening activities also supported students' conceptual understanding of and procedural fluency with rational counting (i.e., one-to-one correspondence) and identifying whether one group of objects is greater than, less than, or equal to another group of objects. The second activity typically introduced a new mathematical concept or skill that was central to the lesson's overall objective. For this activity, tutors used concrete objects (e.g., counting blocks or number lines) to explicitly demonstrate and explain the targeted concept or skill. The third and fourth activities involved either continuous practice of the second activity or a review of previously learned material. The final activity was a brief worksheet activity that tutors used to review the lesson's content. Each worksheet included a "note home" (in both English and Spanish) to provide students with additional practice opportunities outside of school. Finally, to document individual student performance across ROOTS, tutors administered the intervention's curriculum-embedded assessments every five lessons.

ROOTS training. All tutors received three curriculum workshops on intervention implementation and small-group management techniques. Each 4-hr workshop offered tutors practice opportunities in using explicit teacher modeling, providing academic feedback, and facilitating multiple practice opportunities to struggling learners. The workshops also informed tutors that extended and more frequent instructional interactions than those prescribed in the ROOTS program were permissible as long as the interactions (a) centered on the targeted mathematics content and (b) did not interfere with other aspects of implementation fidelity (e.g., completing the lesson within 20 min). That is, when deemed necessary, tutors were allowed to go above and beyond the ROOTS program. For example, if students were struggling to grasp a particular mathematical concept, tutors were encouraged to redemonstrate the concept or provide additional practice opportunities to promote higher rates of student success. To further bolster implementation, in-class coaching was provided during across the 18-week intervention. At the conclusion of each workshop and prior to heading out into the field, tutors had to complete a real-time checkout. This checkout had tutors implement a lesson activity and receive feedback on their delivery of instruction from research staff. Criteria were not specified for the training checkouts.

Implementation fidelity. Implementation fidelity of ROOTS was assessed 3 times in each group across the 18-week intervention. The fidelity of implementation measure was specifically designed to target mathematics activities within each ROOTS lesson. Using a 3-point rating scale, which ranged from 0 (*did not implement*) to 0.5 (*partial implementation*) to 1.0 (*full implementation*), observers coded whether tutors taught key design components described within each lesson activity. For example, tutors received a full implementation rating in the opening activity if they used the Nifty Fifty Chart and Quick Numeral cards. Fidelity scores were computed as the mean across all lesson activities. Clarke et al. (in press) reported moderate to high levels of implementation fidelity (M = 0.95, SD = 0.07).

Measures

Student measures. Students were assessed at pretest and posttest on a general outcome measure of students' procedural and conceptual knowledge of whole numbers, and a set of early mathematics curriculum-based measures that focused on discrete skills of number sense. Trained staff administered all student measures.

The TEMA-3 (Pro-Ed, 2007) is a standardized, norm-referenced, individually administered measure of beginning mathematical ability (i.e., ages 3 to 8 years 11 months) with a population mean and standard deviation of 100 and 15, respectively. The TEMA-3 reports test–retest reliability data of .82 and .93 using alternative forms. For concurrent validity with other mathematics outcome measures, the TEMA-3 reports coefficients ranging from .54 to .91. Standard scores were used in subsequent analyses.

Early Numeracy Curriculum-Based Measurement (EN-CBM; Clarke & Shinn, 2004) consists of four, 1-min

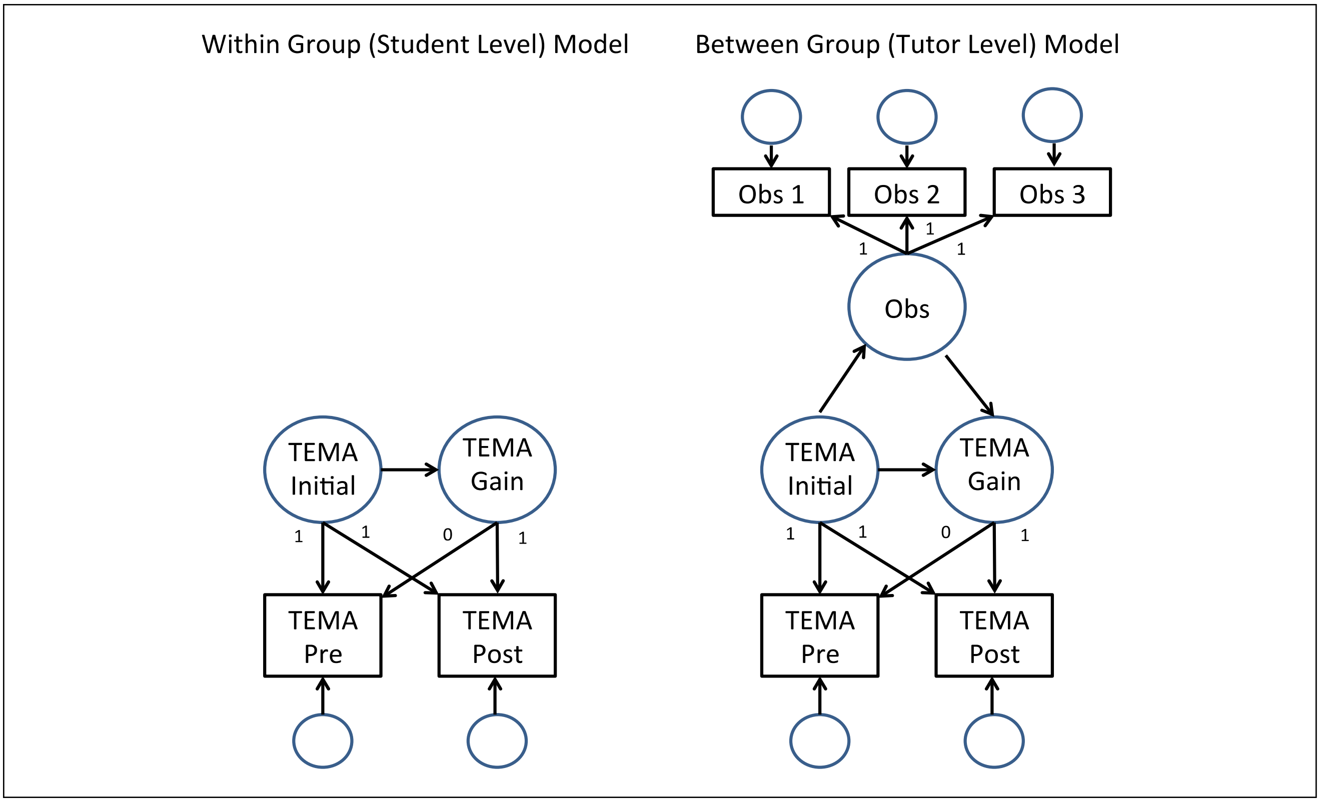

Figure 2. Path diagram of MLSEM.
Note. MLSEM = multilevel structural equation model; TEMA = *Test of Early Mathematics Ability*.

fluency-based measures: Oral Counting, Number Identification, Quantity Discrimination, and Missing Number. Clarke et al. (2016) report average test–retest reliability of EN-CBM as .89, and concurrent validity coefficients between EN-CBM total scores and the TEMA-3 scores at pretest (r = .87) and posttest (r = .81). A total EN-CBM score was computed as the sum across all subtests and used in subsequent analyses.

Observations of instructional interactions. Trained observers administered the Classroom Observations of Student-Teacher Interactions–Mathematics instrument (COSTI-M; Doabler, Baker, et al., 2015) to document the frequency of instructional interactions. The COSTI-M represents a modified version of an observation system designed by Smolkowski and Gunn (2012). Observational data were collected during the 20-min ROOTS intervention sessions on the real-time occurrences of: (a) teacher models, (b) group responses, (c) individual responses, and (d) teacher-provided academic feedback. Incident rates of the instructional interactions were calculated by dividing the frequency of instructional interactions in an observed lesson by the duration of the observation in minutes.

By design, each ROOTS group was scheduled for observation 3 times across the 18-week study, with roughly 5 weeks separating each observation occasion. Because of logistical difficulties in forming the groups, some ROOTS groups were observed 2 times. Due to scheduling errors, three groups were observed 4 times. Combined, 125 direct observations were conducted and on 29 of these occasions two observers collected data simultaneously to assess observer agreement. Intraclass correlation coefficients (ICCs) representing observer agreement of the instructional interactions ranged from .78 to .96, which suggests substantial to nearly perfect observer agreement (Smolkowski & Gunn, 2012).

Statistical Analysis

A series of multilevel structural equation models (MLSEMs), with students and repeated observations nested within tutors, tested both research questions separately for each instructional interaction. We estimated both of the key associations by (a) regressing latent instructional interactions on fall student mathematics achievement and (b) regressing fall to spring gains in student mathematics on fall student mathematics and latent instructional interaction. The basic MLSEM is shown as a path diagram in Figure 2.

As depicted in Figure 2, a directional path is shown from fall achievement to the latent rates because fall achievement precedes the group observations in time. In

Table 1. Descriptive Statistics for Student- and Group-Level Variables.

Variables	*M* (*SD*)	Skewness	Kurtosis	Minimum	Maximum	*n*
Student level						
TEMA-3 pretest	77.25 (16.7)	0.61	−0.31	55.00	122.00	205
TEMA-3 posttest	94.62 (13.3)	−0.10	0.47	54.99	141.00	203
TEMA-3 gains	17.77 (14.1)	0.06	1.01	−38.00	54.00	186
EN-CBM pretest	40.24 (41.7)	1.39	1.36	0.00	196.00	205
EN-CBM posttest	140.54 (49.7)	−0.02	−0.84	26.00	254.00	203
EN-CBM gains	102.90 (42.7)	0.29	−0.22	−18.00	213.00	186
Group level						
TEMA-3 pretest	76.95 (13.2)	0.42	−0.92	55.40	105.25	46
TEMA-3 posttest	94.82 (10.5)	0.08	−0.30	76.25	119.67	46
TEMA-3 gains	17.83 (9.8)	1.07	1.49	1.50	47.50	46
EN-CBM pretest	39.30 (35.0)	1.19	0.45	2.50	134.00	46
EN-CBM posttest	140.50 (38.6)	−0.26	−0.99	66.67	214.33	46
EN-CBM gains	102.29 (30.8)	0.48	−0.43	53.00	178.75	46
Academic feedback	1.18 (0.5)	0.38	−0.63	0.16	2.36	46
Individual responses	1.67 (0.7)	0.97	0.34	0.67	3.58	46
Group responses	1.36 (0.6)	0.90	0.63	0.51	3.18	46
Teacher models	0.86 (0.3)	0.96	0.31	0.30	1.90	46

Note. Instructional interactions were averaged across the three observations occasions. TEMA-3 = *Test of Early Mathematics Ability–Third Edition*; EN-CBM = *Early Numeracy Curriculum-Based Measurement*.

turn, a directional path is shown between the latent rates and fall to spring achievement gains for the same reason. We used a latent variable approach for the rates to partial out time specific variation and random measurement error, correct for the anticipated modest reliability of the rates and thus get a more accurate estimate of effect size for the rates (Stoolmiller, Eddy, & Reid, 2000). Each occasion of observation served as an observed indicator of the underlying latent rate of interaction for a tutor across the intervention period. MLSEM was completed using Mplus (Muthén & Muthén, 2012) with the standard assumption of missing at random (MAR) for missing data and full information maximum likelihood estimation. All *p* values are two-tailed. Prior to and after model fitting, we performed extensive graphical inspection of univariate and bivariate distributions to identify potential violations of underlying model assumptions.

Results

Table 1 provides means, standard deviations, sample sizes, and skewness and kurtosis statistics for each student outcome, and a unit weighted composite rate score based on all the individual rates from each occasion of observation. Correlations between the composite rates of instructional interactions ranged from .25 to .58. The fall student outcomes both had mild floor effects but the fall to spring gain scores were more normally distributed. ICCs for the student outcomes were .53 and .62 for fall TEMA-3 and CBM, respectively; .52 and .45 for spring TEMA-3 and CBM, respectively; and .27 and .31 for TEMA-3 and CBM gains, respectively. The rate scores from each occasion of observation did not suffer from floor effects (many zeros) but were somewhat positively skewed and so were log transformed to better approximate normality.

Table 2 shows that across the eight models, overall model fit ranged from good to fair with RMSEA values ranging from .034 to .072, comparative fit index (CFI) values ranging from .78 to .93, and Tucker–Lewis Index (TLI) values ranging from .77 to .92. ICCs for the three repeated observations of tutor rates were .10, .33, .25, and .33 for models, individual responses, group responses, and feedback, respectively, and except for models, were significant at $p < .05$. The ICCs can be used to estimate the reliability of a composite based on the sum or average of the three observed rates using standard formulas and the reliabilities ranged from .25 to .60. These reliabilities are low enough to cause substantial bias in the estimation of the effects of the rates on subsequent achievement gains, which justifies the use of the latent variable approach for the rates.

Tables 3 and 4 summarize the multilevel models. As shown in Table 3, fall mathematics achievement (TEMA-3 and EN-CBM) was negatively and significantly correlated with only the latent rate of academic feedback. Latent rates were not significantly correlated with gains. In Table 4, for regressions, fall mathematics achievement significantly and negatively predicted the latent rate of academic feedback but all other regression effects involving the latent rate measures were nonsignificant, including the regression of fall to spring gains on latent rates.

Table 2. Fit indices for MLSEMs.

Rate (log transformed)	Outcome	χ^2	*df*	*p*	RMSEA	CFI	TLI
Teacher models	TEMA-3	27.10	15	.028	0.059	0.783	0.768
	EN-CBM	28.25	15	.020	0.062	0.793	0.779
Individual responses	TEMA-3	22.87	15	.087	0.048	0.870	0.862
	EN-CBM	24.74	15	.054	0.053	0.857	0.847
Group responses	TEMA-3	19.02	15	.213	0.034	0.926	0.921
	EN-CBM	19.58	15	.189	0.037	0.925	0.920
Academic feedback	TEMA-3	24.13	15	.063	0.052	0.877	0.869
	EN-CBM	32.55	15	.005	0.072	0.798	0.785

Note. MLSEM = multilevel structural equation model; RMSEA = root mean square error approximation; CFI = comparative fit index; TLI = Tucker–Lewis Index; TEMA-3 = *Test of Early Mathematics Ability–Third Edition*; EN-CBM = *Early Numeracy Curriculum-Based Measurement*.

Table 3. MLSEM Estimated Correlations.

Variables	Estimate	*SE*	z	*p*
Teacher models				
Log rate with initial TEMA-3	0.26	0.34	0.77	.43
TEMA-3 gains with log rate	0.06	0.38	0.16	.86
TEMA-3 gains with initial TEMA-3	−0.62	0.12	−5.01	.00
Log rate with initial EN-CBM	0.53	0.35	1.52	.12
EN-CBM gains with log rate	−0.37	0.38	−0.99	.31
EN-CBM gains with initial EN-CBM	−0.34	0.17	−1.97	.04
Individual responses				
Log rate with initial TEMA-3	0.10	0.21	0.50	.61
TEMA-3 gains with log rate	−0.10	0.24	−0.40	.68
TEMA-3 gains with initial TEMA-3	−0.62	0.12	−5.01	.00
Log rate with initial EN-CBM	0.08	0.20	0.42	.67
EN-CBM gains with log rate	−0.18	0.23	−0.76	.44
EN-CBM gains with initial EN-CBM	−0.34	0.17	−1.96	.04
Group responses				
Log rate with initial TEMA-3	−0.40	0.21	−1.89	.05
TEMA-3 gains with log rate	0.26	0.26	1.01	.31
TEMA-3 gains with initial TEMA-3	−0.63	0.12	−5.04	.00
Log rate with initial EN-CBM	−0.29	0.21	−1.38	.16
EN-CBM gains with log rate	−0.23	0.25	−0.91	.36
EN-CBM gains with initial EN-CBM	−0.34	0.17	−1.97	.04
Academic feedback				
Log rate with initial TEMA-3	−0.71	0.15	−4.65	.00
TEMA-3 gains with log rate	0.42	0.23	1.83	.06
TEMA-3 gains with initial TEMA-3	−0.63	0.12	−5.06	.00
Log rate with initial EN-CBM	−0.61	0.15	−3.85	.00
EN-CBM gains with log rate	−0.06	0.23	−0.28	.77
EN-CBM gains with initial EN-CBM	−0.35	0.17	−2.02	.04

Note. TEMA-3 = *Test of Early Mathematics Ability–Third Edition*; EN-CBM = *Early Numeracy Curriculum-Based Measurement*.

To assess the robustness of our results, we reestimated our MLSEMs, including a binary indicator variable, to distinguish public from other types of schools. The only significant effects obtained were on the initial math skill level (public schools substantially lower on both CBM and TEMA) and the rest of the results were quite similar to those shown in Tables 3 and 4.

Discussion

There is general consensus in the field that instructional interactions around critical mathematics content constitute the sinew of students' development of early mathematics proficiency (Gersten et al., 2009; NMAP, 2008; NRC, 2001; Pianta & Hamre, 2009). However, little is known about the

Table 4. MLSEM Estimated Regressions.

Variables	Estimate	*SE*	*z*	*p*
Teacher models				
Log rate on initial TEMA-3	0.02	0.03	0.80	.42
TEMA-3 gains on log rate	1.56	2.50	0.62	.53
TEMA-3 gains on initial TEMA-3	−0.42	0.12	−3.40	.00
Log rate on initial EN-CBM	0.02	0.01	1.73	.08
EN-CBM gains on log rate	−5.31	11.34	−0.46	.64
EN-CBM gains on initial EN-CBM	−0.14	0.27	−0.53	.59
Individual responses				
Log rate on initial TEMA-3	0.02	0.04	0.501	.61
TEMA-3 gains on log rate	−0.09	0.58	−0.169	.86
TEMA-3 gains on initial TEMA-3	−0.38	0.10	−3.798	.00
Log rate on initial EN-CBM	0.00	0.01	0.429	.66
EN-CBM gains on log rate	−1.48	2.28	−0.648	.51
EN-CBM gains on initial EN-CBM	−0.24	0.13	−1.752	.08
Group responses				
Log rate on initial TEMA-3	−0.07	0.04	−1.772	.07
TEMA-3 gains on log rate	0.06	0.84	0.072	.94
TEMA-3 gains on initial TEMA-3	−0.38	0.11	−3.194	.00
Log rate on initial EN-CBM	−0.01	0.01	−1.325	.18
EN-CBM gains on log rate	−4.10	3.19	−1.285	.19
EN-CBM gains on initial EN-CBM	−0.33	0.15	−2.191	.02
Academic feedback				
Log rate on initial TEMA-3	−0.07	0.01	−3.746	.00
TEMA-3 gains on log rate	−0.32	2.04	−0.156	.87
TEMA-3 gains on initial TEMA-3	−0.40	0.19	−2.143	.03
Log rate on initial EN-CBM	−0.02	0.00	−3.268	.00
EN-CBM gains on log rate	−8.67	6.56	−1.323	.18
EN-CBM gains on initial EN-CBM	−0.35	0.17	−2.026	.04

Note. MLSEM = multilevel structural equation model; TEMA-3 = *Test of Early Mathematics Ability–Third Edition*; EN-CBM = *Early Numeracy Curriculum-Based Measurement*.

explanatory power of instructional interactions within the treatment conditions of intervention studies. This study explored whether instructional interactions facilitated by a scripted intervention program might provide insight into whether or not differences in rates of instructional interactions explained variance in student mathematics outcomes within the ROOTS condition.

Two research questions were addressed in the study. The first research question explored whether rates of teacher models, student practice opportunities, and academic feedback within the treatment condition predicted meaningful improvements in the TEMA-3 and EN-CBM. We hypothesized that differences in the rate of systematic and explicit instructional interactions would predict gains in important mathematics outcomes. Results suggested that rates of teacher models, feedback, group responses, and individual responses within the intervention condition were not associated with gains in the TEMA-3 and EN-CBM.

It is essential to point out that our findings should not be construed as devaluing the critical importance of instructional interactions when designing and delivering Tier 2 mathematics interventions. The evidence is clear that these interactions are validated and necessary mechanisms of Tier 2 mathematics interventions. However, the evidence is less clear about their role for contributing to student mathematics outcomes within intervention programs. A number of interesting questions arise from these findings that warrant future consideration.

Our ability to demonstrate predictive utility of the instructional interactions may have been affected by possible threshold effects. Once instructional interactions reach a certain level or rate of delivery within soundly designed interventions such as ROOTS, they may fail to make a substantively important impact on mathematics growth. Take the rate of individual response opportunities provided in the ROOTS groups as a possible example of this threshold effect. Results suggested that individual students received close to two opportunities per minute to verbalize or demonstrate their mathematical thinking. The benefit of mathematics verbalizations may be maximized below a rate of two opportunities per minute and offer diminishing returns above that threshold. Thus, future research to address threshold effects is warranted. Additional research is also needed to examine other key aspects of instructional interactions such as timing or spacing (Rohrer & Pashler, 2010). For example, it may be that greater spacing of individual responses opportunities across the 20-min ROOTS lessons has a more pronounced effect on student mathematics outcomes than when the total number of opportunities exceeds a specified threshold of mathematics practice.

The quality of instructional interactions may have also influenced the predictive utility of the instructional interactions. Prior observational research suggests that student mathematics achievement is related to the quality of instructional interactions provided in mathematics instruction (Doabler, Baker, et al., 2015; Pianta & Hamre, 2009). However, a serious limitation or challenge may lie with the COSTI-M observation instrument and its theoretical principles. The COSTI-M is limited to capturing the frequency of observable instructional interactions during real-time instruction and thus was unable to document whether the interactions that occurred during the ROOTS intervention were of high or low quality. Other observation tactics, such as technology-based systems may have the capacity to enable deeper investigation of instructional interactions (Connor, 2013). For example, researchers could use tablets to simultaneously code the frequency, quality, duration, and cognitive demand of instructional interactions.

Our second research question explored the relationship between the rate of instructional interactions and student mathematics achievement at the start of kindergarten. Our findings revealed that rates of teacher models, and group and individual responses were not associated with students' pretest TEMA-3 and EN-CBM scores. Academic feedback, however, was found to have a negative and significant relationship with fall mathematics achievement. Findings indicated that groups with lower student mathematics achievement at the start of kindergarten received higher rates of academic feedback. In the context of small-group instruction, academic feedback, both corrective and affirmative, is an optimal platform for teachers to differentiate learning opportunities for students with MD. When teachers provide timely, specific academic feedback they can address misconceptions of mathematics tasks and build upon students' current mathematics thinking and understanding (Gersten et al., 2009).

Notwithstanding the importance of academic feedback for students with MD, questions remain about why tutors of the lower performing groups facilitated more academic feedback. One explanation is that, while kept blind to students' TEMA-3 and EN-CBM pretest scores, these tutors made "local adaptations" (Forman et al., 2013) to ROOTS to best meet the instructional needs of their students. Tutors of the lower performing groups may have used ROOTS' curriculum-embedded assessments to gauge student performance and subsequently provide additional academic feedback than otherwise prescribed within the intervention. However, we are unable to determine whether student performance on the curriculum-embedded assessments was the driving factor behind the instructional adjustments in the lower performing groups. Future studies are therefore needed to understand how and why tutors might locally adapt ROOTS, and whether such adaptations are beneficial to students.

Because a fine line exists between program fidelity and program adaptation, deeper investigations are also warranted to distinguish the "essential" features of Tier 2 interventions that are adaptable from ones that must be delivered as designed (Forman et al., 2013). Essential features of the ROOTS intervention that tutors can adjust to provide better local fit include the provision of instructional interactions. For example, ROOTS tutors are encouraged to offer additional academic feedback to provide greater clarity and specificity of complex mathematics concepts and procedures. However, as emphasized in the ROOTS workshops, such local adaptation was permissible if it remained in the boundaries of implementation fidelity (e.g., completing each lesson within 20 min). Conversely, the mathematics content targeted within ROOTS is not flexible or adaptable as tutors are required to teach the prescribed topics.

Limitations and Implications for Research

A number of limitations must be considered when interpreting our results. One limitation is the study's sample size. Data from only 46 intervention groups were analyzed in the study. A larger sample size will likely be required to detect small yet meaningful group differences that may exist within a highly specified treatment condition. Relatedly, missing observation data is another possible limitation. Across the intervention, 16 observation occasions were missed.

Another possible limitation is that ROOTS groups were scheduled for only three observations. Although scheduling three observations is common among observational research studies (e.g., Pianta & Hamre, 2009; Smolkowski & Gunn, 2012), conducting four or more observations per instructional group may provide an unequivocal perspective of instructional interactions and the ways in which they can be manipulated to increase intervention intensity (D. Fuchs & Fuchs, 2015; Yoder & Woynaroski, 2015). Our limit of three observation occasions was primarily driven by resource constraints in the larger efficacy trial.

Finally, although our work has focused extensively on observed instructional interactions, teachers' mathematics knowledge might offer an additional way to examine variations in treatment outcomes (Ball, Hill, & Bass, 2005). It may be that teachers with in-depth understanding of mathematics may offer a potentially more intensive experience for struggling learners than from teachers who are less skilled in mathematics even though the teachers are utilizing the same instructional materials. For example, a teacher with extensive knowledge of mathematics may be able to represent whole number concepts in different ways than otherwise prescribed in mathematics interventions. Teachers who can draw on their mathematical knowledge are also likely to be more effective at linking an intervention with students' prior understandings of mathematics. Future research is warranted to investigate the role teachers' content knowledge may play in the variance of student outcomes within Tier 2 interventions.

Conclusion

There is an urgent need not only to develop effective mathematics interventions but also to systematically study the underlying mechanisms of those interventions that accelerate student achievement. Although using direct observations to document important instructional interactions can be challenging (Connor, 2013), this measurement tactic has strong potential to further unpack the black box of mathematics interventions and evaluate the extent to which students at risk for MD receive intensive mathematics instruction. Doing so will help to ensure that all learners maximize their potential in understanding mathematics.

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