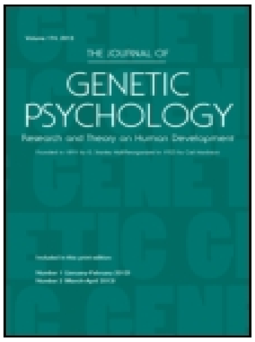

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Abstracting Sequences: Reasoning That Is a Key to Academic Achievement

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ABSTRACT. The ability to understand sequences of items may be an important cognitive ability. To test this proposition, 8 first-grade children from each of 36 classes were randomly assigned to four conditions. Some were taught sequences that represented increasing or decreasing values, or were symmetrical, or were rotations of an object through 6 or 8 positions. Control children received equal numbers of sessions on mathematics, reading, or social studies. Instruction was conducted three times weekly in 15-min sessions for seven months. In May, the children taught sequences applied their understanding to novel sequences, and scored as well or better on three standardized reading tests as the control children. They outscored all children on tests of mathematics concepts, and scored better than control children on some mathematics scales. These findings indicate that developing an understanding of sequences is a form of abstraction, probably involving fluid reasoning, that provides a foundation for academic achievement in early education.

Keywords *sequences, abstraction, reading, mathematics, achievement*

The understanding of sequences, or what follows what, may be an important aspect of cognitive development. Piaget theorized that certain types of sequences (i.e., seriation and transitivity) were milestones of the development of children's thinking. Independently of Piaget's theorizing, developers of several intelligence tests have used sequences of numbers and geometric figures of varying complexity to provide measures of the reasoning ability of children and adults. Without reference to either intelligence tests or Piaget, educators have for decades taught very simple sequences to children in the early grades of American schools, where they now form part of the Common Core of U.S. education. Theorists in education have advanced speculations, some quite cogent, that such instruction prepares children for instruction in early mathematics and facilitates their progress in understanding of mathematics concepts. Yet there have been few empirical studies of whether instruction in sequences actually produces academic progress, and the few available conflict, in part. The present research was designed to address this last point, and to describe possible relations between understanding simple sequences and the development of children's intelligence.

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INTRODUCTION

Piagetian Constructs

Understanding sequences begins to develop early in life. Inhelder and Piaget (1959/1964) applied the label *seriation* to children's first efforts to put objects in order, and recognized that seriation develops in tandem with categorization in various forms as children progressively organize their world. Seriation involves ordering items by their physical magnitude (e.g., length, width, and weight), and begins to develop around ages 4 or 5 years. The developmental trajectory of seriation begins with one-dimensional sequences in which children are able to order items based on one physical magnitude (Leiser & Gillieron, 1990; Southard & Pasnak, 1997). If asked, for example, to form an ordered line of rods that differ in length, 4-year-olds may proceed by trial and error, and if persistent may form a correctly ordered line, from shortest to longest (or vice versa). They improve on trial and error by developing and employing the method of extremum (Inhelder & Piaget, 1959/1964). This method consists of selecting the smallest object from those that are to be ordered and using it to start the line. The child then considers the objects that remain to be ordered, selects the smallest, and places it next to the one previously selected. Repeating this process, the child considers the objects that still remain to be placed in order, and selects the smallest, placing it next in line. This process may be continued indefinitely until all of the objects have been placed in line in their proper positions. While the method is very orderly, it does not involve considering the relations between objects in the seriated line: the child always selects the smallest object from those that remain to be ordered. If offered a new object that should be placed in the interior of the line, the child will have no idea how to proceed. The typical response is to place the new, medium-sized object at the end of the line, which is manifestly incorrect. It is only when a child is able to insert a new object in its proper place in the interior of an ordered line, which can only be accomplished by considering which of the objects already in the line is just shorter and which is just a bit longer, that the child is considered to be able to seriate operationally (Piaget & Inhelder, 1968/1973). In essence, they can compare neighboring objects and comprehend the relationship between them—an early form of abstraction or relational thought.

Piaget and Inhelder (1968/1973) considered transitivity to be the next important step in the understanding of serial relations. In a classic transitivity problem, a child would be allowed to compare a large red disk and a medium-sized blue disk, and note that the red disk was larger. Next, the child would be allowed to compare the blue disk with a smaller, yellow disk, and note that the blue disk was larger than the yellow. Then the child would be asked, "Which is larger, the red disk or the yellow disk?" There is no opportunity to compare the red and yellow disks directly, but by considering the size relations already known between the red and blue disk, and between the blue and yellow disk, the child can deduce that the red disk must be larger than the yellow disk. This requires relational thinking and working memory. Transitivity problems can be made more difficult by increasing the number of disks to be successively compared, but all comparisons involved changes in magnitude on one dimension.

However, there are many sequences of objects or items that do not involve physical magnitudes. These include numerical sequences, such as 1, 3, 5, 7; alphabetical sequences such as *t, s, r, q*; cause-and-effect sequences such as walk, slip, fall, rise, walk; sequences of positions as an object rotates: 45°, 90°, 135°, 180°: sequences of items that alternate, such as red, blue, red, blue, red, blue; symmetrical sequences such as car, horse, cat, truck, truck cat, horse, car; and random but repetitive sequences such as 3, 6, 2, 8, 3, 6, 2, 8, among others. Within these broad types, there are an infinite number of individual sequence rules. Understanding that items of diverse sorts may occur in predictable sequences, and comprehending the nature of such sequences may be part of the development of fluid intelligence.

Intelligence Tests

Psychologists have incorporated sequences into tests of intelligence, notably the Kaufman ABC and K-ABC II (Kaufman, Lichtenberger, Fletcher-Janzen, & Kaufman, 2005). The K-ABC and KABC-II each have a scale that requires children to understand sequences that are presented pictorially (picture arrangement and story completion). Children select several pictures that are needed to complete a story and put them in the sequence in which they belong. These measures are best interpreted as requiring children to understand a blend of cause and effect sequences with temporal sequences. These K-ABC and KABC-II scales are intended to reflect either the planning component of Luria's neuropsychological theory or (in the case of the KABC-II) the fluid reasoning component (*Gf*) of the theory developed by Cattell (1941), Horn (1991), Horn and Cattell (1966), and Carroll (1993). This is generally known as the Cattell-Horn-Carroll (CHC) theory of narrow and broad abilities. The theoretical basis of these K-ABC and KABC-II measures has been a matter of debate (Allen, Lincoln, & Kaufman, 1991), and the use of the test to measure intelligence has rested in part on the fact that it does not show as much difference between children from different cultures as other tests (Wenegrat & Bernier, 2013).

The K-ABC and KABC-II also have a scale that presents sequences of abstract geometric figures graphically (pattern reasoning). For pattern reasoning, the children select an item (a geometric figure) that should be inserted in a sequence according to the rule that the sequence represents. The Pattern Reasoning scale is closer to seriation than the other scales; however, the scale does not limit the sequence rules to increases in physical magnitude. This scale is one of the more useful ones, having survived the pruning of scales in the revision of the K-ABC. It is a measure of fluid reasoning (*Gf*), which Kaufman et al. (2005) described as "solving novel problems by using reasoning abilities such as induction and deduction" (p. 15). In the context of sequences, inductive reasoning is essential to deriving the rule of the sequence from the individual items that comprise it, while deductive reasoning is essential to applying this rule to determine where a new item belongs in the sequence. As these two abilities develop, a child's ability to understand increasingly complex sequence correspondingly improves. Hence, understanding sequences must involve at least the *Gf* broad stratum component of the CHC theory of cognitive abilities.

Patterning Instruction

Strengthening a child's ability to understand sequences may improve this aspect of reasoning. And, inasmuch as better reasoning produces better academic achievement, it follows that improving children's to understand sequences could produces academic achievement. Exactly this enterprise has been undertaken for the last half century in the schools of the United States, Great Britain, and Australia (Clements & Sarama, 2007c; Orton, 1999; Papic, 2007). The sequences employed have been simple alternations, such as abababab, abbabbabb, aabbaabb, and typically involve colors, shapes, or small items commonly found in classrooms. This instruction, called patterning, has been endorsed by national organizations (e.g., the joint position statement of the National Association for Education of Young Children and the National Council of Teachers of Mathematics [2010]) and is considered to be an essential element of children's mathematical development. "If you ask any kindergarten teacher, he or she is likely to consider the study of patterns to be an essential part of the mathematics program" (Economopolous, 1998, p. 230).

Educational theorists have not referred to the construct of fluid intelligence in describing the potential effectiveness of improved patterning or to the role of patterns in IQ tests. They have instead described more specifically the potential role in early mathematics of understanding patterns of simple alternations. White, Alexander, and Daugherty (1998) theorized that such patterning was a progenitor of early analogical reasoning and consequently of mathematics. In their theoretical formulation, learning the relations between items in alternating patterns led to recognizing the pattern of relations between mathematical symbols. White et al. analyzed children's test performance and found that patterning was a better predictor of early mathematics than other abilities, such as recognizing numbers and comparing quantities.

Other educators have focused on the role of understanding alternating patterns in prealgebra—the interpretation and understanding of addition and subtraction sentences (equations), as opposed to the memorization of addition and subtraction facts. Clements and Sarama (2007a, 2007b, 2007c) emphasized the role of understanding simple alternations such as ababab in bringing order to otherwise unorganized collections of items. Clements and Sarama (2007a) theorized that "algebra begins with a search for patterns. Identifying patterns helps bring order, cohesion, and predictability to seemingly unorganized situations and allows one to recognize relationships and make generalizations" (p. 507). By learning that the rule has items that alternate, regardless of what the particular items may be, children develop algebraic insight (i.e., that the same relation may apply to diverse concrete elements). Thus, according to Clements and Sarama (2007b), "recognition and analysis of patterns are important components of the young child's intellectual development because they provide a foundation for the development of algebraic thinking" (p. 524). Baroody's (1993) observation that children give the same name to alternating patterns that have different elements supports this idea. Using the same name for patterns with different physical elements, such as colors and shapes, is in essence naming a variable, and hence is an early approach to algebra.

There have been surprisingly few studies of whether helping children to understand patterns actually improves their understanding of early mathematics or any other subject matter. The earliest is an unpublished dissertation, in which Herman (1973) gave 71 inner-city African American and Hispanic–Latino kindergartners 24 lessons on alternating patterns. Their results on the numbers subtest of the Metropolitan Readiness Test (Hildreth, Griffiths, & McGauvran, 1969) were compared with those of control kindergartners from another inner-city school who had somewhat lower, but roughly comparable Peabody Picture Vocabulary Test scores. She found a small, but statistically significant, difference in favor of the kindergartners taught patterning.

A second dissertation, published three decades later (Hendricks, Trueblood, & Pasnak, 2006), improved on Herman's study in three ways. First, children in Grade 1 were randomly assigned to experimental and control groups. Second, the experimental children were taught a large variety of sequences. Third, the control children had sessions matched in timing and extent to those of the experimental children. Their sessions differed only in what they were taught, which reflected subject matter recommended by their teachers as especially useful.

The experimental group was taught 480 sequences that ranged from simple linear orderings to multidimensional matrices and included letters, numbers, shapes, colors clock faces, beads, and animal stickers. The sequences involved regularly and irregularly increasing and decreasing sequences of letters and numbers, and rotations of objects through six or eight positions. The sequences, which were presented via computer, felt board, construction paper, and wooden frames, became increasingly more difficult, in terms of length, number of missing items, and number of dimensions as the instruction progressed. There were also cause and effect sequences presented via script cards. The instruction continued four days per week from the end of October through the middle of March. Testing in May revealed differences in the raw scores of the experimental and control groups on the Diagnostic Ability Battery (DAB-2; Newcomer, 1990) that favored the experimental group, but were not statistically significant. However, a multivariate analysis of covariance that equalized IQ scores yielded significant differences on the mathematics, written language, and total achievement DAB scales. It is noteworthy that the difference on the Written Language scale was the first empirical evidence that improving children's ability to understand sequences could improve basic reading and writing skills—*Grw* in the CHC—although Sarama and Clements (2004) predicted that such an outcome was likely.

Neither dissertation seems to have had any impact on teaching sequences in elementary schools. However, evidence that instruction on complex sequences can produce improvement in children's academic achievement is beginning to accumulate. In a recent study by Kidd et al. (2013), eight of the lowest scoring first-grade children in each of the 20 classrooms were randomly assigned to one of four instructional conditions designed to improve reading, mathematics, social studies, or the understanding of sequences. The children received their assigned instruction three times per week, for 15 min per day, for the majority of the school year. The children in the experimental condition were provided instruction on sequences. The elements of the sequences consisted of colors, shapes, letters, numbers, and computer-generated depictions of everyday objects. Some sequences showed a rotating object in various positions, some were alternations, others showed regular increases or decreases, others were symmetrical, while others followed some arbitrary rule. At the end of the year, children who were taught sequences outperformed those who received any other form of instruction on the Woodcock-Johnson III (W-J; McGrew & Woodcock, 2001) mathematics concepts scale 18 A. On the mathematics concepts scale 18 B, the children who were taught either sequences or mathematics scored significantly higher than those taught social studies. However, there were no significant differences on the W-J applied problems (mathematics) scale 10, or on any of three W-J reading scales (comprehension, fluency, and letter-word). This study suggests that instruction on complex sequences can substantially improve children's understanding of mathematics concepts and holds promise that future instruction may contain elements that foster cognitive development and overall academic gains.

There are a variety of possible reasons for the difference in outcomes obtained by Kidd et al. (2013) and Hendricks et al. (2006). Kidd et al. (2013) simplified and reduced the plethora of materials and presentations used by Hendricks et al. (2006), a change that might well reduce the children's ability to generalize the principles they had learned from sequences to the material on the achievement tests. The differences in the standardized tests used as dependent variables might also account for the differences in results. Kidd et al. (2013) noted that the control reading condition—direct instruction in reading for most of the school year—also produced no significant differences on the W-J reading scales. This suggests that these scales may not have been sensitive enough to reflect any gains that may have occurred. Consequently Kidd et al. (2014) conducted a similar experiment using new reading measures, and reported significant improvements in reading as well as mathematics, the outcome anticipated by Sarama and Clements (2004).

Hence, the effects of teaching children sequences more complicated than alternating patterns deserve further empirical investigation, which was undertaken in the present study. Because *patterning* is a term applied only to alternating patterns for the last half century, we prefer the term sequentiation for more complicated patterns. Our goal was to provide a thorough test of whether teaching children sequences improved children's achievement in mathematics and reading. Improved sequentiation could improve achievement if it improved certain broad-spectrum components identified in the CHC theory of intelligence. Sequentiation fits the definition of fluid intelligence (*Gf*), because it is the learning of a rule by induction and applying it to new situations by deduction. Hence, the overall importance of abstracting sequences may be that it requires flexibility applying principles to new problems in new contexts. The first and basic principle is that there may be a rule that defines a sequence; the second principle is that there are many such rules. The child's task is to determine what the sequence rule is in a particular case and then to apply it, without concern for the particular concrete items in the sequence. This is a level of abstract reasoning that helps the child solve new problems in the absence of instruction expressly on those particular problems (Carroll, 1993). To the extent that a child's competence in sequentiation is high, whether acquired naturally or via instruction, the child will be able to understand how general ideas can be combined with facts already learned (crystallized intelligence, or *Gc*) to understand new lessons taught in the classroom. When children can apply what they are taught without a great deal of instruction, it is ordinarily because their fluid intelligence and crystallized intelligence are both sufficient to allow them to succeed. The importance of sequentiation, or progress in sequentiation, may be that it strengthens *Gf*.

Although educators have not identified what they are teaching as fluid reasoning, sequentiation exemplifies the characteristics of this form of reasoning, even though the single (abababab) and double alternations (aabbaabb) used in classrooms are much simpler. The results of Hendricks et al. (2006), Kidd et al. (2013), and Kidd et al. (2014) suggest that understanding sequences also involve broad-spectrum abilities *Gq* (quantitative reasoning), and a component of *Grw*—reading ability—in the CHC theory of intelligence. In our view, to be meaningful any improvements in these abilities should be reflected in tests of sequentiation, mathematics, and reading.

Hypotheses

Hypothesis 1: Teaching sequences to children would improve understanding of sequences other than those that the children were taught.

Hypothesis 2: Teaching sequences to children would improve mathematics.

Hypothesis 3: Teaching sequences to children would improve reading.

METHOD

Participants

After obtaining parental consent, 826 first-grade students enrolled in an urban public school system in the mid-Atlantic region were administered a screening test comprised of 48 sequence problems. After attrition, 132 boys and 114 girls remained in the study. The mean age for these children was 6 years 5.97 months (*SD* = 2.57 years). Eighty-six participants were African American, 87 were Hispanic–Latino, 44 were Middle Eastern, and 29 identified as other. Most participants in this study came from immigrant families, lived in subsidized housing, or received free or reduced lunch. After the original screening test was administered, eight children who scored the lowest in each of 36 classes were chosen to participate in the study. These students were randomly assigned to receive instruction in sequencing, mathematics, reading, or social students, three times per week for 15 min from October to April.

Experimental Design

The eight children in each class who scored lowest on the screening test were formed into pairs and randomly assigned to be taught sequences or reading or mathematics or social studies. Hence, teacher and classroom effects were controlled, because each teacher had the same number of children in each of the four instructional conditions. Expectations, Hawthorne effects, and other sources of differences between groups were likewise controlled, because all children were engaged in constructive activities that could be expected to yield positive results.

The sequencing instruction required minimal skill and was intentionally designed so the instruction could be easily replicated. This instructional method was aimed at teaching children to identify each type of sequential pattern, attend to the different elements, and fill in the missing element at the beginning, middle, or end of each complex sequence. The effects of this experimental instruction were compared with those of three kinds of control instruction.

The first control group was taught mathematics, which primarily consisted of activities involving number sense and operations. It was expected that the mathematics group would outperform those in the reading and social studies control groups on mathematics measures, if the instruction was effective. Likewise, a better understanding of sequences should result in better mathematics scores according to White et al. (1998) and many educators (e.g., Clements & Sarama, 2007b; Economopolous, 1998; Hendricks et al., 2006; Kidd et al. 2013; Kidd et al., 2014; Papic, 2007). However, it was likely that the mathematics group would score lower on sequencing and reading measures if the sequencing and reading instruction affected year-end reading scores.

The second control group was taught reading, which included phonics instruction. It was expected that because these students received extra instruction in reading, they would score higher on the reading measures than the children taught mathematics and social studies. If the ability to understand sequences enhanced reading abilities, the children taught sequencing may also score higher on reading measures than those taught mathematics or social studies (Hendricks et al., 2006; Kidd et al., 2014).

The third control group was taught social studies, which consisted of cutting, pasting, drawing on worksheets, and making collages. The social studies control group was unlikely to have an advantage in sequencing, reading or mathematics measures. This control group provided a baseline for measurement of the effects of sequencing instruction and of the other forms of control instruction.

Materials

Tests

The researcher generated patterning test used by Kidd et al. (2013) was used in this research. It had 48 five-item sequences of letters, numbers, time clock faces, or rotated objects presented in rows or columns on a flip chart. Each had a missing item in the beginning, middle, or end of the sequence. Each student was presented with each sequence and asked to select the item or object that was missing from four possible alternatives that were provided. For example, a child was presented with E, G, ?, K, M, and asked to select the missing item from D, I, L, Q. Because randomization would inevitably result in some types of sequences appearing earlier in the test than others, the type of sequence, orientation and steps between sequence items were carefully counterbalanced.

The 12-item far generalization test developed by Kidd et al. (2013) was used to measure each student's ability to apply his or her knowledge and abstract level of thinking to sequences of quite different types than those used in instruction. Symmetrical and double ascending sequences were composed of playing cards and dice laid out on a table. An example of a symmetrical problem involved the use of dice that were presented in the following order: 1 and 3 pips, 4 and 5 pips, 2 and 6 pips, 6 and 2 pips, 5 and 4 pips, –?–. An example of a double ascending problem using number cards had the numbers in the following order: –?–, 6, 5, 7, 6, 8. The sequences were employed in an order counterbalanced for type, dice, and cards. Each problem contained one missing element in the first, middle, or last position in the sequence, and students were provided with four alternative choices to fill in the missing element.

The mathematics and reading standardized tests employed by Kidd et al. (2014) were used to measure school achievement, as they had yielded the more positive results than those used by Kidd et al. (2013), and hence might be better suited to show the effects, if any, of instruction. Three tests assessed mathematical achievement—the KeyMath-3 test (Connoly, 2007) and the Woodcock-Johnson III mathematics concepts scales A and B (W-J 18A and W-J 18B). Three tests also measured reading achievement—the Gray Oral Reading Test-4 (GORT; Wiederholt & Bryant, 2011), the Test of Word Reading Efficiency (TOWRE; Torgerson, Wagner, & Rashotte, 1999), and the Test of Early Reading Ability-3 (TERA; Reid, Hresko, & Hammill, 2001).

Convergent validity of the KeyMath-3 ranged from .66 to .80 with the ITBS and from .67 to .75 the Kaufman Test of Educational Achievement II (Kaufman & Kaufman, 2004). Reliability of the KeyMath-3 total test score is .97, and median reliabilities for test–retest reliabilities on the KeyMath-3 subscales is .86 for younger examinees.

The W-J III scales 18 A and 18 B measure a child's understanding of mathematical concepts and number series. The W-J is a highly regarded and widely used test that has the largest standardization sample of any individually administered test. The manual (McGrew & Woodcock, 2001) gives a reliability coefficient of .84 for seven-year-olds and convergent validity coefficients of .71–.64 with the diagnostic assessment system, .68–.70 with the Wechsler Individual Achievement Test (Psychological Corporation, 1992), and .62–.66 with the Kaufman Test of Educational Achievement (Kaufman & Kaufman, 1985).

The GORT-4 is a test that effectively measures students' oral reading skills. The GORT-4 reliabilities are high, ranging from .85 to .95 on test–retest comparisons and from .91 to .97 on content sampling. Validity inferences for the GORT-4 are based on comparisons with six other standardized tests; the oral reading quotient yields a median of .63.

The TOWRE measure assesses a student's ability to pronounce printed words accurately and fluently. According to the manual, the TOWRE has exemplary reliability coefficients that range from .90 to .99. Concurrent validity with the Woodcock Reading Mastery Tests-Revised (Woodcock, 1987) was .85–.98, and the predictive validity correlated with the GORT-3 scores was .75–.80.

The TERA-3 evaluates young children's reading abilities and mastery of early developing reading skills. The TERA-3 reliabilities are also high, ranging between .83 and .95. Concurrent validity measures can be drawn from correlations with teacher judgments, the Woodcock Reading Mastery Test-Revised-Normative Update (WRMT-NU/R; Woodcock, 1998), SAT-4 (Psychological Corporation, 1996), and range from .40 to .66. These are lower than those of the GORT and TOWRE, but nonetheless respectable.

Instructional materials

The materials used by Kidd et al. (2013) were employed to teach students progressive sequences involving increasing numbers of elements, sizes, or values, rotating figures, symmetrical patterns, and random sequences that did not have an underlying functional relationship, but repeated themselves. Foam alphabet letters, plastic geometric figures, magnetic numbers, dice, and digitally depicted elements, among many others, were used to teach sequential patterns. Elements were varied within each type of sequence. Manipulatives, note cards, whiteboards, and laptops were used to present these sequences.

Children in the reading control group read poems and engaged in discussions designed to promote fluency, word recognition, and comprehension. A Whispy Reader (Quackenbush, Culver City, CA, USA), a small curved tube that children hold to their mouth and ear much like a telephone receiver, was used so they could hear themselves read without disrupting others. In addition, note cards were used to promote the use of word families when decoding unfamiliar words (e.g., the -*at* family included cards such as *bat, cat, chat, fat, hat*). Instruction focused on word families found in the poems and included examining familiar words (e.g., cat) and then using what is known about sound-symbol correspondence to decode unfamiliar words (e.g., flat).

Materials found in most first grade classrooms were used for the control mathematics instruction. These included number cards, coins, clock faces, and shape manipulatives. Maps, puzzles, mazes, activity pages, and materials for cutting and pasting were used to provide activities that fostered a deeper understanding of social studies.

Procedure

In the cooperating school system, children spent 90 min of their morning rotating to several predetermined centers where they engaged in and received different forms of instruction. Children remained at each center for approximately 15 min, until they were asked to move to the next center. The majority of these centers were student directed and engaged children in developing academic skills, such as writing, and their understandings of curricular concepts, such as science. Dependent upon the teacher's daily lesson plan, one or two centers were teacher directed wherein the teacher provided instruction. In one center, the teacher provided all of the experimental and control instruction, in sequencing, mathematics, reading, and social studies, to the eight students in each classroom who participated in the experiment. This instruction was observed daily by project managers, and records were kept of each child's progress. All forms of instruction were quite familiar to the teachers and their assistants, and were conventional for the participating school system. The only difference between the sequencing instruction and the traditional patterning instruction employed in this school system was that the patterns used in this research were more complex than simple alternations.

The research was conducted for approximately 60 min three days per week at one designated center. The students in the research rotated to this center in their classroom for 15 min. Two children received instruction on sequencing, two children received instruction on mathematics, two children received instruction on reading, and two children received instruction on social students. The order in which each pair of students came to the center was carefully counterbalanced to ensure that children received their form of instruction first, second, third, or fourth equally as often as their counterparts.

Sequencing instruction

This instruction was modeled after that of Kidd et al. (2013). Symmetrical sequences, progressions with increasing numbers of elements, sizes, or values, rotations, and random repeating sequences were introduced in that order during the sequencing instructional sessions. Sequence problems were presented to the children one at a time, and each problem contained one missing element in the beginning, middle, or end of the sequence. The children were to select one of four options they were given to complete the sequence. Performance was scaffolded through explanation and repetition until each child was able to demonstrate mastery by selecting the correct option for each sequence type on their first attempt across three consecutive sessions. When a child had mastered symmetrical sequences to that criterion, instruction on progressions of increasing numbers of elements began. When a child had mastered that type of sequence, instruction on sequences whose elements increased in size began, and so forth.

To strengthen each child's ability to identify and complete simple and complex sequences, the representation of sequences was presented on note cards and laptops. The children identified the missing element in the sequence problem displayed on the corresponding form of representation. Likewise, children were taught to create and extend sequences using manipulatives and white boards. The teacher would start a sequence and provide children with manipulatives in order for them to complete or extend it. Children were also provided with the opportunity to create a sequence for their teacher or another child to complete. White boards were also used for these same purposes, the only difference was that the sequences were drawn on a white board, as opposed to being created or extended from manipulatives. This type of instruction was part of the normal program of studies (POS) for this school system. Employing it with the sequences used in this research required no special training of the teachers or their assistants.

Mathematics instruction

The mathematics instruction was shaped by the POS employed in the cooperating school system. Several concepts and skills, such as number recognition, shape identification, counting, addition and subtraction, and fractions, were introduced in daily mathematics activities. Each day, a different lesson was presented with a corresponding activity. Each mathematics session began with an informal assessment to determine the student's understanding of the concept or skill addressed in the planned activity. The teacher provided a quick review of the skill to ensure the student had the fundamentals needed to complete the chosen task. This information enabled the teacher to establish an appropriate starting point and modify instruction accordingly. The teacher provided the student with a simplified version of the initial task or a more challenging extension, in order to individualize their instruction to best meet the needs of the child. At the conclusion of the session, the teacher ended with one final question that addressed the overall focus of the activity.

Reading instruction

The reading instruction was also shaped by the POS employed in this school system. It addressed several aspects of literacy, including vocabulary development, decoding skills, comprehension skills, and fluency. More specifically, each session focused on one poem that had a particular phonetic ending (i.e., *-ay*, *-ed*, *-est*, *-ump*). On the first day, the session began with an interactive discussion designed to promote oral language and conversational skills. Following the discussion, children used the Whispy Readers to read the focus poem from the previous week. As the children read the poem, the teacher monitored each child's ability to read the poem, while prompting them with questions to foster a deeper understanding of the text. Afterward, the teacher engaged the children in a discussion by asking several basic questions about the poem, such as "Why do you think the truck never got stuck?"

Then, the teacher introduced a new poem that would be referenced throughout the week. The purpose of the following activities was threefold: to improve students' comprehension, fluency, and vocabulary. On the first day, the teacher began by reading the new poem aloud to the children, and discussing different elements within the poem to facilitate comprehension. On the second day, the teacher and children engaged in shared reading. The teacher emphasized fluency, and elaborated on unfamiliar vocabulary to enhance the students' understanding of the poem. On the third day, the children independently read the poem. The teacher generated a discussion to strengthen all three skills, comprehension, fluency, and vocabulary development.

To build upon each poetry reading, the teacher integrated a phonics activity that emphasized rhyming words. At the end of each session, the teacher presented rhyming words flashcards with the same rime or word family (e.g., *–uck*) as the focus poem for that week. The teacher helped students make a connection between the word sequences and patterns found on the note cards to those found in the poem. To conclude each session, the teacher and students summarized the poem, reviewed the activities, and reflected on what was learned.

TABLE 1
Descriptive Statistics for Sequences and Far Generalization Tests

	Sequences test		*Far generalization test*	
Group	M	SD	M	SD
Sequences	27.21	6.71	7.14	3.02
Reading	15.33[a]	4.19	3.85[a]	2.48
Mathematics	13.33[a]	3.31	3.68[a]	2.37
Social studies	13.61a	5.23	4.55a	2.76

[a]$p < .001$ in comparisons with sequences group.

Social studies instruction

The social studies instruction was less structured than the sequencing, mathematics, and reading groups, and reflected activities integrated into the social studies curriculum of the participating schools. The social studies lessons varied each day, and activities aligned with the content were presented. Social studies content included civics, geography, famous people, and important events. The instruction involved frequent interactions between the teacher and students in which additional guidance and assistance was provided to foster meaningful engagement in the lessons. The teacher assisted students with drawing, coloring activity sheets, puzzles, creating collages, filling in maps, completing puzzles, and using various cut-and-paste materials to create pictures related to the social studies concepts emphasized.

Testing

All instruction on sequences and control instruction was completed in early May. School psychologists who were blind to the instruction the children had received subsequently administered the sequencing, reading, and mathematics tests in counterbalanced order.

RESULTS

For all comparisons, a p value of .05 was used to evaluate statistical significance; in accordance with the American Psychological Association's (2010) latest recommendations, exact p values are given for significant outcomes. The results for the sequences test and the far generalization sequences test are shown in Table 1. The children taught sequences scored approximately twice as well on the test made of sequences that were novel, but similar in principle, to those which they had been taught, as the children in the other instructional conditions. The overall F for a one-way analysis of variance (ANOVA) was significant, $F(3, 242) = 108.247$, $p < .001$, $\eta^2 = .57$. This is a large effect (Cohen, 1992), who recommended as a rule of thumb that the same values be applied to r and η^2 effect sizes—small (.10), medium (.20), and large (.50). Here, the instructional condition accounts for more than half of the variance. The Bonferroni comparisons of the children taught sequences with each of the other conditions were all significant ($p < .001$). There were no differences between the children in the other three conditions.

The outcomes for the far generalization test were similar. The mean for the sequences condition is not quite twice that of the means for the other conditions. The overall difference is significant, $F(3, 237) = 25.07$, $p < .001$, $\eta^2 = .24$. This is a smaller (medium) effect, accounting for less than a quarter of the variance, but the Bonferroni comparisons between the outcomes for the sequences instruction and the other forms of instruction yielded $p < .001$ in all cases. There were no differences between the other three conditions.

Composite Analysis of Mathematics Scales

A multivariate analysis of variance (MANOVA) for mathematics showed a significant multivariate main effect for mathematics, Wilks's $\lambda = .327$, $F(27, 663.599) = 11.456$, $p < .001$, $\eta_p{}^2 = .311$. Power to detect the effect was 1.000. This approach showed that differences on all mathematics scales except data problems, computation, and numeration were significant (see Table 2).

Analyses for Individual Mathematics Scales

Inasmuch as different scales have produced different results in previous research, ANOVA was applied to each scale in turn and followed by Bonferroni protected post hoc tests whether or not comparisons were a priori or a posteriori. Tests were bidirectional even though a good case could be made for unidirectional tests in most instances. Hence, this analysis is quite conservative.

Table 3 shows the results for the mathematics scales. On the W-J 18A mathematics concepts scale, the overall F was significant, $F(3, 242) = 89.055$, $p <.001$, $\eta^2 = .525$, a large effect accounting for more than half the variance on this scale. Bonferroni comparisons showed that the instruction on sequences produced scores significantly higher than those of any other condition, $p < .001$ in all cases. The scores for the mathematics instruction were significantly higher than those for the reading or social studies instruction, $p < .001$ in both cases. Perhaps surprisingly, the scores from the reading condition were significantly better than those from the social studies condition, $p < .003$.

For the W-J 18B mathematics concepts scale, the overall F was again significant, $F(3, 242) = 45.015$, $p < .001$, $\eta^2 = .358$ (a medium effect). On this scale, the children in the sequences condition again outscored all others ($p < .001$ in all comparison)s. The children in the mathematics condition outscored those in the reading ($p < .003$) and social studies ($p < .001$) conditions. Those in the reading and social studies conditions did not differ significantly.

For the KeyMath algebra scale (prealgebra at this grade level), the overall F was significant, $F(3, 241) = 8.509$, $p < .001$, $\eta^2 = .10$. The overall effect was small, but the children taught sequences outscored the children taught mathematics ($p = .005$), reading ($p < .001$), or social studies ($p < .001$). No other differences approached significance.

On the other KeyMath scales, overall Fs were significant for the foundations of problem solving scale, $F(3, 238) = 3.814$, $p = .011$, $\eta^2 = .05$; the addition–subtraction scale,

TABLE 2
Multivariate Analysis of Variance Tests of Between-Subjects Effects for Mathematics

Dependent variable	*Type III SS*	df	*MS*	F	p	$\eta_p{}^2$
Woodcock-Johnson III						
Subtest 18 A	1695.008[a]	3	565.00	85.722	.000	.523
Subtest 18 B	1049.096[b]	3	349.70	43.477	.000	.357
KeyMath-3						
Applied problems	84.774[c]	3	28.258	2.655	.049	.033
Data problems	108.310[d]	3	36.103	2.094	.102	.026
Computation	58.420[e]	3	19.473	1.595	.191	.020
Foundations	78.441[f]	3	26.147	3.638	.014	.044
Numeration	100.193[g]	3	33.398	2.317	.076	.029
Algebra	280.582[h]	3	93.527	8.659	.000	.100
Measurement	154.916[i]	3	51.639	2.976	.032	.037
Error						
Woodcock-Johnson III						
Subtest 18 A	1548.909	235	6.591			
Subtest 18 B	1890.168	235	8.043			
KeyMath-3						
Applied problems	2501.401	235	10.644			
Data problems	4052.393	235	17.244			
Computation	2868.266	235	12.205			
Foundations	1688.831	235	7.187			
Numeration	3386.669	235	14.411			
Algebra	2538.405	235	10.802			
Measurement	4077.017	235	17.349			

[a]$R^2 = .523$ (Adjusted $R^2 = .516$).
[b]$R^2 = .357$ (Adjusted $R^2 = .349$).
[c]$R^2 = .033$ (Adjusted $R^2 = .020$).
[d]$R^2 = .026$ (Adjusted $R^2 = .014$).
[e]$R^2 = .020$ (Adjusted $R^2 = .007$).
[f]$R^2 = .044$ (Adjusted $R^2 = .032$).
[g]$R^2 = .029$ (Adjusted $R^2 = .016$).
[h]$R^2 = .100$ (Adjusted $R^2 = .088$).
[g]$R^2 = .037$ (Adjusted $R^2 = .024$).

$F(3, 239) = 4.706$, $p = .003$, $\eta^2 = .06$; the measurement scale, $F(3, 241) = 2.975$, $p = .032$, $\eta^2 = .04$; and the applied problem solving scale, $F(3, 239) = 2.731$, $p = .032$, $\eta^2 = .03$. On the foundations of problem solving scale, the children taught sequences scored significantly higher than the children in the reading group ($p < .03$) but no other differences were significant. On the other three scales, the only significant Bonferroni differences were between the children taught sequences and those taught social studies: $p = .002$ for addition–subtraction, $p = .03$ for measurement, and $p = .035$ for applied problem solving. No other differences on these scales were significant, and, although the scores of the children taught sequences were highest in an absolute sense, there were no significant differences on the numeration, computation, or data problems scales.

TABLE 3
Descriptive Statistics and Grade Equivalencies for Woodcock-Johnson and KeyMath Scales

Woodcock-Johnson III mathematics concepts

	Subtest 18 A		*Subtest 18 B*	
Group	M	SD	M	SD
Sequences	16.35	2.63	13.95	3.01
Reading[ab]	10.97	2.32	9.38[ab]	3.11
Mathematics[a]	13.84	2.27	11.20[a]	2.74
Social studies[abc]	9.34	2.10	8.47[ab]	2.37

KeyMath Scales

	Algebra			*Foundations*			*Applied problems*		
Group	M	SD	GE	M	SD	GE	M	SD	GE
Sequences	9.21	3.79	2.4	7.84	2.59	2.2	8.36	3.44	1.7
Reading[a]	6.97	2.92	1.8	6.43[a]	2.63	1.6	7.35	3.27	1.5
Mathematics[a]	7.12	3.13	1.8	7.47	3.31	2.0	7.77	3.57	1.6
Social studies[a]	6.37	3.39	1.5	6.58	3.02	1.7	6.69[a]	2.92	1.4

	Addition/subtraction			*Measurement*			*Computation*		
Group	M	SD	GE	M	SD	GE	M	SD	GE
Sequences	9.90	2.80	2.2	11.14	4.29	2.1	7.51	3.21	2.0
Reading	8.71	3.03	1.9	9.56	4.28	1.9	6.38	3.51	1.8
Mathematics	8.62	3.30	1.9	9.64	4.36	1.9	6.39	3.65	1.8
Social studies[a]	7.88	3.07	1.7	8.97[a]	3.91	1.7	6.29	3.80	1.8

	Data problems			*Numeration*		
Group	M	SD	GE	M	SD	GE
Sequences	9.27	4.37	1.9	12.57	3.93	1.9
Reading	8.27	3.93	1.6	11.06	4.00	1.6
Mathematics	7.82	4.13	1.4	11.64	4.12	1.7
Social studies	7.39	4.19	1.3	10.92	3.55	1.6

Note. The manual does not provide grade equivalencies for the Woodcock-Johnson (W-J) scales. GE = grade equivalent.

[a]p <. 05 for a posteriori independent comparisons with sequences.

[a]p <. 05 for a posteriori independent comparisons with mathematics.

[a]p <. 05 for a posteriori independent comparisons with reading.

Composite Analysis for Reading Scales

A MANOVA for the reading scales showed a significant multivariate main effect for reading, Wilks's $\lambda = .793$, $F(12, 561.191) = 4.294$, $p < .001$, $\eta_p{}^2 = .075$. Power to detect the effect was .999. Differences for the TERA and TOWRE word scales were significant (see Table 4).

Analyses for Individual Reading Scales

The outcomes for the individual reading scales are also shown in Table 5. On the TOWRE word scale, the overall F was significant, $F(3, 240) = 9.426$, $p < .001$, $\eta^2 = .11$. This is another small

TABLE 4
Multivariate Analysis of Variance Tests of Between-Subjects Effects for Reading

	Type III SS	df	*MS*	F	p	$\eta_p{}^2$
Dependent variable						
GORT	135.213[a]	3	45.071	0.990	.398	.014
TERA	1050.693[b]	3	350.231	12.737	.000	.151
TOWRE Word	5685.086[c]	3	1895.029	9.040	.000	.112
TOWRE Phonemic	448.241[d]	3	149.414	0.791	.500	.011
Error						
GORT	9787.015	215	45.521			
TERA	5911.864	215	27.497			
TOWRE Word	45069.334	215	209.625			
TOWRE Phonemic	40589.549	215	188.789			

Note. GORT = Gray Oral Reading Test-4; TERA = Test of Early Reading Ability-3; TOWRE = Test of Word Reading Efficiency.

[a]$R^2 = .014$ (Adjusted $R^2 = .000$).

[b]$R^2 = .151$ (Adjusted $R^2 = .139$).

[c]$R^2 = .112$ (Adjusted $R^2 = .100$).

[d]$R^2 = .011$ (Adjusted $R^2 = -.003$).

effect. Bonferroni comparisons showed that the children taught sequences outscored those taught mathematics or social studies ($p = .006$ and $p < .001$, respectively). The children taught reading also outscored those taught social studies ($p = .002$).

TABLE 5
Descriptive Statistics and GE for Reading

	TOWRE word			*TOWRE phonemics*		
Group	M	SD	GE	M	SD	GE
Sequences	46.94	14.63	2.6	21.83	14.29	3.2
Reading	43.40	15.86	2.4	21.59	15.40	3.2
Mathematics[b]	37.69	14.93	2.2	19.66	11.48	2.6
Social studies[ad]	33.25	15.99	2.0	18.44	12.46	2.6
	GORT			*TERA meaning*		
Group	M	SD	GE	M	SD	GE
Sequences	12.34	6.98	1.7	17.31	5.90	1.4
Reading	10.65	6.67	1.6	16.77	5.39	1.4
Mathematics	11.31	6.39	1.6	13.69[ac]	5.20	1.2
Social studies	10.64	6.65	1.6	11.95[ab]	4.68	1.0

Note. An average grade equivalent (GE) for a child at the eighth month of Grade 1 is 1.8. GORT = Gray Oral Reading Test-4; TERA = Test of Early Reading Ability-3; TOWRE = Test of Word Reading Efficiency.

[a]$p < .001$ for comparisons with sequences.

[b]$p < .01$ for comparisons with sequences.

[c]$p < .001$ for comparisons with reading.

[d]$p < .01$ for comparisons with reading.

On the TERA, $F(3, 234) = 13.533$, $p < .001$, $\eta^2 = .15$. The instruction on sequences produced significantly higher scores than the mathematics or social studies instruction ($p < .001$ in both cases). The reading instruction also produced scores significantly higher than those resulting from mathematics instruction ($p < .010$) or social studies instruction ($p < .001$). The mathematics and social studies conditions did not differ significantly.

In sum, on the reading scales, the scores of the children taught sequences were significantly higher than those of the children taught mathematics or social studies in four of eight comparisons, but were never significantly higher than those of the children taught reading. The children receiving the control reading instruction produced scores significantly higher on the reading scales than those of the children taught mathematics or social studies in three of eight comparisons.

DISCUSSION

Performance on Sequences

It is clear that the first-grade students who received instruction on sequences were able to make significant progress in learning complex sequences. The students applied their knowledge and understanding of complex sequences similar in principle, although different in details, to those with which they had been taught. More impressively, they generalized their understanding to sequences much different from those they had been taught. These students demonstrated an advanced understanding of sequences, which they were able to apply to novel sequences presented in different media. However, there remained much room for improvement. Although the children taught sequences attained scores nearly twice as high as the children in the control groups on both the sequencing test and the far generalization test, they solved little better than half of the problems correctly. The control children solved only about a quarter of the problems correctly—little better than chance. Hence, it is an open question whether this sample, which was comprised of the lowest scoring students in the class, was the most appropriate sample to demonstrate the effects of instruction on sequences.

The improvements made on identifying, creating, and extending sequences resulted from the student's ability to recognize the sequence rules previously learned despite differences in the concrete items that constituted the sequences, thus demonstrating algebraic insight (Clements & Sarama, 2007b). By applying the rule to sequences with different elements and determining the appropriate choice to fill in the missing element, students demonstrated the use of deductive and inductive reasoning skills. Based on children's performance on the far generalization test, both forms of reasoning appear to have improved, indicating a functional difference in *Gf*, as children comprehended the rules governing sequences were quite unlike those they had initially been taught. That required the deductive reasoning that analysts of IQ tests have posited as essential to the solution of sequence problems (Kaufman et al., 2005; Roid, 2003). Solutions to these problems required an inductive formulation of the sequence rule, followed by the application of the rule to the dice or playing cards that constituted the elements of the sequence, in the absence of feedback. Although the students' performance was far from perfect, it seems clear that the children taught sequences developed stronger reasoning skills, at least in the context of sequences, while the control children's performance on these novel sequences was at a chance level. The possibility that solving sequences involves deductive and inductive aspects of *Gf* is a viable explanation of these results, although it was not directly tested.

Improvement in Mathematics

The biggest gain in mathematics was in mathematics concepts. This is congruent with the claims of educators that instructing preschool and kindergarten children on simple sequences with alternating elements may improve aspects of their mathematical reasoning and prealgebraic skills (Clements & Sarama, 2007a, 2007b, 2007c; Orton, 1999; Papic, 2007; Threlfall, 2004). While evidence that sequencing produces improved understanding of mathematical concepts is sparse, the outcomes of the present research replicate the findings of Hendricks et al. (2006), Kidd et al. (2013), and Kidd et al. (2014) further supporting the notion that understanding more complicated sequences improves children's understanding of mathematics concepts. The consistency of the findings in these three studies, each of which employed different measures of quantitative ability, suggests that a broad-spectrum ability, *Gq* in the CHC theory, has probably been improved.

The instruction on sequences also produced advantages in other aspects of first-grade mathematics in this study, although the significant differences Kidd et al. (2014) reported on the Numeration, Data problems, and Computation scales were not replicated here. This could be a sample difference, but is most likely due to the conservative analysis used here. Kidd et al. (2014) reported least significant difference comparisons, and conducted one-tailed tests in most instances. However, there were significant differences on several mathematics scales here. These, along with the advantage on the prealgebra measure, and the results of Hendricks et al. (2006) on the DAB-2 contrasts with the result of Kidd et al. (2013), who found no effect on the only mathematics achievement measure (W-J 10) they employed. It appears that different scales differ in their sensitivity to the advantages of improved understanding of sequences.

Even though it did not produce scores on the W-J mathematics concepts measures equivalent to those produced by instruction on sequences, the control mathematics instruction was nonetheless effective. This is demonstrated by the significant differences the mathematics instruction produced on the W-J mathematics concepts scales, compared to the reading or social studies instruction. However, differences between the children receiving mathematics instruction and those receiving reading or social studies instruction were not statistically significant on the KeyMath scales. This may reflect the challenge students face with understanding mathematics problems if they are not helped to develop the abstract thinking and reasoning required.

Improvement in Reading

The large and statistically significant superiority on the GORT that Kidd et al. (2014) found for the children taught sequentiation was not replicated here. Otherwise, the results for reading are essentially similar. The significant difference on the TOWRE and TERA found here and by Kidd et al. (2014), as well as that on the DAB-2 by Hendricks et al. (2006), indicates that understanding sequences involves some aspects of *Grw* broad-spectrum ability. Sarama and Clements (2004) suggested the possibility that the simple sequences of alternating elements taught to young children could have an indirect effect on reading, and spoke to the need for empirical evidence.

This suggestion is reasonable, because developmental psychologists and educators have long known that children's mastery of the serial ordering involved in seriation—a simpler, very basic kind of sequencing—predicts reading ability (see review by Waller, 1977). Improving a child's understanding of sequences could in turn improve the child's reading, because the order of letters in words and the order of words in a sentence determine the meaning of the word or sentence. It remains unclear why some of the well-respected standardized tests showed an effect (TERA and TOWRE) and others did not—the GORT here and the W-J scales used by Kidd et al. (2013). It is noteworthy that the control reading instruction, while clearly effective, as shown by the TERA and TOWRE, also had no effect on GORT scores.

The TOWRE's WORD scale is a combination vocabulary and speed measure, while the TOWRE PHONEMICS scale is a combination of phonics and speed. Scores reflect how many words or phonemes a child can read in 1 min. Both may reflect in part the *Gs* (processing speed) and *CDS* (correct decision speed) of the Cattell-Horn formulation, and/or *Gs* (broad cognitive speediness) and *Gt* (decision/reaction time/speed) in Carroll's three-stratum theory. This seems unlikely, given the lack of any significant differences on the phonemics scale. The difference in outcomes on the word and phonemics scales may instead reflect the importance of the ordering of letters in words. The direction of the effects on both scales is the same, but words are longer than phonemes, and better processing of the sequence of letters may have been more important for improving children's vocabularies by the end of first grade, reflecting an improvement in this aspect of *Grw*. A more complete answer to the question of why different reading measures yielded different results in Hendricks et al. (2006), Kidd et al. (2013), Kidd et al. (2014), and the present experiment awaits better explanations from test developers of just what these reading tests measure.

Importance of Instruction in Sequences

Absolute differences in the scores on reading measures produced by instruction on sequences and reading were not statistically significant. Only the differences in mathematics concepts and prealgebra produced by instruction on sequences and mathematics were significant. Hence, the reading instruction and mathematics instruction that teachers already know how to do is as efficacious or nearly as efficacious in producing progress in reading and mathematics as instruction in sequences. The advantage of the latter lies in its impact on both mathematics and reading. In effect, this effort to improve one aspect of cognitive development has two applications in the classroom. This broad impact indicates that there was a general improvement in overall ability from the instruction in sequentiation. This general improvement is likely to be in fluid intelligence, as better *Gf* would be useful in applying classroom instruction in both mathematics and reading. Further, the progress was most substantial on the scales most conceptual in nature (W-J 18 A and W-J 18 B), indicating that *Gf* played a bigger role than various aspects of *Gc*. Hence, the overall importance of sequentiation may be that it is an expression of *Gf* and perhaps part of the basis for developing *Gf*. The results obtained here are most readily explained by positing improvement in such a basic ability.

The advantage to the children in receiving instruction in sequences is substantial in practice, although small on most scales in a statistical sense. Any curricular innovation that produces an advantage of one or two months in grade equivalents is likely to be very welcome to educators. The advantage of the children taught sequences ranged from two to eight months, (see Tables 3 and 5). Moreover, instruction on sequences is very similar to the traditional instruction on patterning, requiring no additional training or resources.

Instructional Context

A critical feature of the current research is that the experimental instruction constituted a small part of a school day devoted to instruction in which reading and mathematics were incorporated across the curriculum and integrated into various lessons and activities throughout the day. It is difficult to envision that instruction in sequences alone will produce better reading or mathematics achievement; concurrent instruction in reading or mathematics is needed for mastery of sequences to have an effect in either sphere. The major difference the instruction in sequences can produce is in the development of conceptual abilities, and the difference in mathematics concepts it produced was not small, statistically or otherwise. It is important to consider children's conceptual development and their ability to understand the classroom instruction. That may well have been an issue for the children in this research, who were in approximately the bottom third of their class at the outset of the school year. Whether it was an application of *Gf, Gq*, or *Grw*, or some other aspect of reasoning that the instruction on sequences enhanced, it seems clear that a better understanding of sequences yielded progress in reading and mathematics. The control children, lacking this understanding, might too often have had to deal with daily classroom teaching that was in some part beyond their grasp.

Limitations

An important limitation of the application of this research is that instruction must match children's cognitive level in order for them to benefit from the instruction. Any cognitive intervention can fail if it requires reasoning too advanced for the children at which it is directed, or too far below their current level of functioning. Pasnak, Hansbarger, Dodson, Hart, and Blaha (1996) found that the same cognitive instruction, delivered simultaneously in two school systems, failed in one and succeeded in the other. Herman's (1973) cognitive instruction was much more effective for one of the ethnicities in her sample than for the other. These findings reinforce the importance of matching particular interventions to children's cognitive levels.

Another, more obvious limitation is that the intervention might have been more successful if the children were taught different sequences than those taught here, or if the sequences were presented in a different manner. For example, Kidd et al. (2013) found that sequences of letters were learned much more easily if presented in horizontal orientations, while number sequences were learned more easily in vertical orientations. The effects were large, and were more pronounced for more difficult sequences. There are a myriad of possible sequences and presentations, and it is inevitable that some will be more productive than others. The present research shows that instruction in sequences can be effective, but not how effective be it can be.

Another characteristic of the current research that may be important is that brief sessions were conducted for most of the school year. The patterning instruction usually offered to young children typically involves longer sessions for 3–6 weeks. Whether instruction on sequences would be effective in such a time frame remains to be determined. In any event effects on reading and mathematics from instruction on sequences are not likely to be immediate. There must be time for children to apply their improved cognitive ability to the regular classroom mathematics and reading instruction. Pasnak (1987) found that there was no effect on end-of-the-year scores from a spring intervention; however, that same intervention during the fall had a substantial effect on year-end scores (Pasnak, McCutcheon, Holt, & Campbell, 1991).

We conclude that better mastery of sequences can be developed through instruction and that it can have effects on reading and mathematics that have substantial practical importance. The conditions under which these effects can be achieved are not presently well defined, and the mechanisms by which improved mastery of sequences affects reading and mathematics are not known, although the CHC theory appears to offer at least some suggestions. There is much to be done, by applied developmental psychologists and by educators.

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