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The Essence of Early Childhood Mathematics Education and the Professional Development Needed to Support It

Barbrina B. Ertle[1], Herbert P. Ginsburg[1], Maria I. Cordero[2], Tracy M. Curran[1], Leslie Manlapig[1], and Melissa Morgenlander[1]

[1]*Department of Human Development, Teachers College, Columbia University, New York, NY 10027, USA*
[2]*New York City's Administration for Children's Services, New York, NY 10038, USA*

Many events occurring over the last 25 years or so have influenced the way in which the field of early childhood education (ECE) views what is meant by 'mathematics' and how it should be taught to young children (usually ages 3, 4, and 5), at least in the US. This chapter begins by describing several social, political, and research influences that have produced a dramatic change in both the perception and the reality of early math education (EME) in the US. Then the paper shows how the new view of EME requires reconceptualizing the mathematics that should be taught to young children and how it should be taught. Implementing the new EME in turn requires new approaches to professional development. We describe one such approach that we employed with a group of teachers who were attempting to implement the *Big Math for Little Kids* curriculum (Ginsburg et al., 2003) at a publicly funded early care and education agency in New York City. We show how that experience affected the group's attitudes, beliefs, and behaviors and discuss the implications for supporting focused professional development.

SOCIAL, POLITICAL, AND RESEARCH INFLUENCES

In 1986, the National Association for the Education of Young Children (NAEYC) issued a position statement on *Developmentally Appropriate Practice in Early Childhood Programs Serving Children from Birth Through Age 8* (Bredekamp & Copple, 1986) that deeply shaped the thinking of ECE policy-makers, administrators and practitioners in the US. The document strongly opposed the 1980's trend of increased formal instruction for young children and instead advocated 'developmentally appropriate' practice (DAP), in which teachers do not lecture or verbally instruct, but instead serve as guides or facilitators. DAP involves providing children with a rich environment and utilizing teachable moments to extend learning. Furthermore, mathematical activities were mostly considered developmentally inappropriate, except when they were integrated with activities such as building with blocks or playing with sand or water.

The NAEYC position statement had a profound impact on ECE in the US (although more explicit early childhood mathematics instruction was employed elsewhere, e.g., Aubrey 1997). The definition of DAP was quickly incorporated into policy. For example, in October 1986, the New York City Agency for Child Development (ACD) used the NAEYC position statement to create a program assessment instrument to assess DAP in over 350 center-based child care centers in New York City.

However, as the century turned, important political and social tensions were brewing in the US that would have a critical effect on ECE. For example, the federal 'No Child Left Behind' legislation (2001) strengthened accountability requirements for the academic achievement of elementary and secondary students. And the country had also become engaged in school reform driven by standards set not only by professional organizations but also by the federal and state governments.

Of equal importance was research on cognition, learning, and child development during the last half of the 20th century. In 1999, the National Research Council commissioned the Committee on Early Childhood Pedagogy to review behavioral and social science research that had clear implications for the education of young children. The findings from this extensive report – *Eager to Learn: Educating our Preschoolers* – indicated that young children were more capable learners than the current practices reflected and that more challenging educational practices in the preschool years could have a positive impact on school learning (Bowman et al., 2001).

For example, considerable research (Ginsburg et al., 2006) has shown that young children develop a relatively powerful 'informal' mathematics before they enter school. They can competently deal with some very abstract ideas

(Ginsburg & Ertle, 2008), such as cardinality (Gelman & Gallistel, 1986) and addition (Brush, 1978; Groen & Resnick, 1977). Indeed, contrary to some interpretations, Piaget did not propose that young children were totally 'concrete' in their thinking. In fact, Piaget claimed that in many respects young children were overly abstract (Piaget & Inhelder, 1969). Taken as a whole, contemporary research suggests that young children are capable of learning more challenging and abstract mathematics than are usually assumed. It is therefore not necessarily developmentally inappropriate to engage in EME (although like anything else, the teaching of early mathematics can be done badly).

Research has shown that not only are children capable of learning challenging mathematics but doing so can be useful, and indeed essential, for subsequent education (Bowman et al., 2001). Recently, Duncan et al. (2007) showed that early mathematics skills are even more powerful predictors of later school success than reading abilities. Research has also shown that early intervention programs can be effective in reducing achievement disparities in mathematics (e.g., Dowker, 2001; Reynolds, 1995), suggesting that beginning instruction as early as possible could potentially reduce or prevent later mathematical difficulties and the need for remediation. EME can play a major role in preparing children for later success in school.

In 2000, the National Council of Teachers of Mathematics (NCTM) responded to research findings like these by including pre-kindergarten (pre-k) in its newest Standards document, *Principles and Standards for School Mathematics*. In 2002, NAEYC partnered with NCTM on a joint position statement which outlined recommendations for the teaching of mathematics in the pre-k classroom (NAEYC/NCTM, 2002). Like earlier NAEYC position papers, the statement endorsed promoting learning through play in rich environments, utilizing teachable moments to extend learning, and even adult-guided learning experiences like theme-based projects. But the document also took the important, and to some, radical position of stating that these types of learning experiences are not enough. The teaching of mathematics needs also to involve an organized curriculum presenting deep mathematical ideas in a coherent, developmental sequence that utilizes and builds on children's prior learning and experiences.

Together, the NAEYC/NCTM position statement and the NCTM preschool standards, based as they were on the research literature, then set the groundwork for the design of developmentally appropriate early mathematics curricula in the US, including *Number Worlds* (Griffin, 2000), *Pre-K Mathematics* (Klein et al., 2002), *Big Math for Little Kids* (Ginsburg et al., 2003), *Building Blocks* (Sarama & Clements, 2004), and a Head Start curriculum (Sophian, 2004). Other countries have also seen the recent emergence of early mathematics curricula and intervention programs (Dowker, 2001; Kaufmann et al., 2003; Van de Rijt & Van Luit, 1998; Young-Loveridge, 2004).

In April 2002, the Good Start, Grow Smart initiative – the early education reform companion of the No Child Left Behind Act (2001) – called for federally funded programs to develop early learning goals that specified the skills and competencies that preschool children should possess before starting school. The initiative is intended to stimulate increased attention to the teaching of mathematics, among other subjects.

But content standards, goals, and curricula exert effects beyond expectations for children. They also change what we require of practitioners. The new policies – which represent a radical shift in perspective for many preschool teachers and ECE professionals – require teachers to engage in a more rigorous, intentional, and organized approach to early childhood mathematics education than was previously employed. This in turn requires extensive professional development: the quality of classroom practice is significantly related to teachers' education and training (Pianta et al., 2005). Yet the educational background of the ECE workforce in the US is weak – only 20–30% of center-based preschool teachers hold a bachelor's degree (Brandon & Martinez-Beck, 2006). Therefore, some federal and state programs now require higher degrees of preparation for their early childhood practitioners (Martinez-Beck & Zaslow, 2006).

In brief, new research on children's abilities and social pressures have resulted in a serious movement in the US to raise standards and expectations for EME, to create new programs of EME, to enhance those already in place, and to provide a teaching force that can implement the programs effectively.

As professionals attempt to implement EME on a wide scale, especially for low-income children, several challenges arise that must be successfully addressed if we are to achieve our goals. This chapter describes those challenges and the means for meeting them. We first discuss the *content* of EME. It is much deeper, wider, and more abstract than ordinarily assumed. Next we examine the several *components* of early mathematics which are now recommended. EME involves far more than letting children play with rich materials. We show how necessary educational activities include free play, the teachable moment, projects, and curriculum. The last of these is the most poorly understood, but potentially the most powerful (and still developmentally appropriate). The nature of *desirable* mathematics *teaching* for little children is also poorly understood. We show that in some ways it is similar to teaching at any grade level. Given our analyses of the mathematics to be taught, the major components of EME, and good teaching, we then examine teachers' *conceptions* about EME. We show that they tend to favor social and emotional development over academic subjects such as mathematics, and many are resistant to the EME movement. But these conceptions are linked to content knowledge and are entangled with socioeconomic status (SES). Then we consider the *quality* of current EME teaching and teachers' readiness to teach mathematics. We show that current teaching practices are often very poor and that considerable professional development is required to improve them. We discuss key features of *professional development* that have been shown to be useful. To illustrate the challenges of creating and implementing a program of professional development for poorly trained early childhood teachers in an inner-city setting, we describe and discuss an example, the *Big Math for Little Kids* professional development program conducted at a city-funded ECE agency in New York City. Finally, we discuss the *implications* of our findings and provide recommendations for helping a district or an agency set up a plan for professional development in EME.

THE CONTENT OF EARLY MATHEMATICS

In many of today's early childhood classrooms and day care centers, children's exposure to mathematics content is fairly narrow, often limited to the learning of small numbers and simple shapes (Balfanz, 1999). However, EME should not be limited to such topics. The leading professional organizations now recommend that EME involve 'big ideas' of mathematics in the areas of number, geometry, measurement, and algebra – particularly pattern (NAEYC/NCTM, 2002; NCTM, 2000). These areas encompass some very deep mathematics, even at the level appropriate for young children (Ginsburg & Ertle, 2008). Number, for example, includes such concepts as the counting words ('one, two, three,…'), the ordinal positions ('first, second, third,…'), the idea of cardinal value (how many are there?), and the various operations on number like addition and subtraction. Even if we examine only elements of number commonly presented in today's early childhood classrooms, the depth of the mathematics can be made evident. Consider the counting words and enumeration (determining how many).

Before we can determine 'how many' are in a group of objects, we first need to know the counting words (in English, 'one, two, three,…'). In most languages, the first ten number words or so are completely arbitrary, but beyond that they assume a pattern, as most languages use a base-ten system to organize numbers. Consequently, despite the need to memorize the first ten numbers, learning to count is not merely an act of memorization. It involves learning the building blocks of an elegant pattern, derived from the base ten system, that once understood, permits counting to very high numbers with little additional memorization. For example, the counting words are organized into groups of tens and ones, and the names for the tens are derived from the names of the units. Once the names for the tens (ten, twenty, thirty,…) are known, the unit words (one, two, three,…) can easily be appended. The result is a simple and elegant pattern, which is even simpler and more elegant in East Asian languages (Miller & Parades, 1996). However, when children first tackle the act of counting, the very elements that make the counting system so simple to adults are not yet known. Children must grapple with memorizing the first set of numbers and with trying to understand why a fixed order is required before its necessity and utility become apparent in the elegance of the system.

Once the counting words are known, they can then be used to enumerate, or 'count' a group of objects. Enumeration also entails a number of underlying complexities that many adults take for granted. One is the idea that a group of things to be counted can contain any variety of things – real or imagined; they do not have to be limited to similar objects, like the set of three red circles that children are commonly presented with. But possibly the most difficult idea for a child to grapple with when counting a set of objects is that the numbers do not specifically name the items being counted; instead they describe the cardinal value of the items. So when counting a set of three objects – a ball, a blue car, and a wood block – you may point to the ball and say 'one,' the car and say 'two', the block and say 'three,' but you could just as easily begin counting with the block. A child must come to understand that pointing to the objects and saying 'one, two, three' is different in a crucial respect from pointing to them and saying 'round,' 'blue,' and 'wood.' Names designate objects; counting words refer to quantity.

And that is just the beginning of the depth of the ideas found within the domain of number. The other areas of mathematics contain equally deep ideas and complexities. Early mathematics is both basic and deep. And as we have seen, young children can deal with ideas like these, even without explicit instruction (Ginsburg et al., 2006).

THE DIFFERENT COMPONENTS OF EARLY MATHEMATICS

If early mathematics is deep, how should it be taught? A new NAEYC guide clarifies guidelines for DAP (Copple & Bredekamp, 2006): 'The idea that there is very little or no structure in a DAP classroom is a misconception. Again, in reality the opposite is true. To be developmentally appropriate, a program must be thoughtfully structured to build on and advance children's competence' (p. 60). The issue then arises as to the form this structure should take.

Drawing in part on the Joint Position Statement (NAEYC/NCTM, 2002), we propose five levels of classroom structure necessary for promoting mathematics learning: a rich environment, play, the teachable moment, projects, and curriculum.

The first is the *environment*. The preschool classroom (or 'day care center' – we use the terms synonymously) should contain a rich variety of materials that can afford mathematics and other learning. It should be divided into areas such as blocks, water table, and science table which encourage children's play, exploration, and discovery. There has never been much controversy about the need for a rich environment. Widespread agreement on this requirement has resulted in the extensive use of the *Early Childhood Environment Rating Scale (ECERS)* (Harms et al., 1998), which mainly provides a rating of the quality of the preschool classroom environment. But a rich environment by itself is not enough. The crucial factor is not just what the environment makes possible, but what children *do* in a rich environment.

The second important component of EME is *play*. We know that children do indeed learn a good deal of everyday mathematics on their own (Seo & Ginsburg, 2004). Play provides valuable opportunities to explore and to undertake activities than can be surprisingly sophisticated from a mathematical point of view (Ginsburg, 2006), especially in block play (Hirsch, 1996). Play is essential for children's intellectual development generally and for mathematics learning in particular. But play is not enough. Children need experiences beyond the play to help them learn even more – especially how to communicate about their experiences and how to see them in explicit mathematical terms. Consequently, some degree of adult guidance is necessary.

The third component of EME is the *teachable moment*, a form of adult guidance that enjoys widespread acceptance in the preschool world. The teachable moment involves the teacher's careful observation of children's play in order to identify the spontaneously emerging situation that can be exploited to promote learning. The Creative Curriculum program (Dodge et al., 2002), which is extremely popular in the US, relies on use of the teachable moment.

No doubt, the teachable moment, accurately perceived and suitably addressed, can provide a superb learning experience for the child. But there are good reasons to believe that in practice the teachable moment is not an effective educational method or policy. First, most preschool teachers spend little time in the careful observation necessary to perceive such moments. Teachers tend to manage behavior during free play (Kontos, 1999) or to spend very little time with children during free play (Seo & Ginsburg, 2004); in practice, teachers do not even attempt to exploit teachable moments. Second, it is very hard to know what to do when such moments arise, especially in the unlikely event that a teacher is able to recognize the deep mathematical issues with which many children may engage. The teachable moment demands considerable and rapid creativity – a quality in short supply in all professions. It therefore seems grossly unfair to expect teachers to rely on the exploitation of teachable moments as the primary component of mathematics education. Third, extensive reliance on the teachable moment is an impractical educational policy. How can a teacher manage all the teachable moments (even if she were able to notice them) that might arise among the 15 or 20 or 25 children in a classroom during the course of a year? The task seems virtually impossible and would seem to be a less than ideal way of responding to young children's intellectual needs. In brief, exploiting the teachable moment is wonderful if it can be done, but usually teachers do not try to notice teachable moments, could not respond effectively to such moments even if noticed, and could not manage all those that could be noticed.

A fourth component involves *projects* (Katz & Chard, 1989). Projects are intended to engage children in extensive teacher initiated and guided explorations of complex topics related to the everyday world, like figuring out how to create a map of the classroom. A project of this type can involve consideration of measurement, space, perspective, representation, and a whole host of mathematical and other ideas (e.g., scientific) which have very practical application and appeal. They can help children learn that making sense of real-life problems can be stimulating and enjoyable.

Although projects can be enormously effective, they are not sufficient unto themselves:

> Teachers should ensure that the mathematics experiences woven throughout the curriculum follow logical sequences, allow depth and focus, and help children move forward in knowledge and skills. The curriculum should not become… a grab bag of any mathematics-related experiences that seem to relate to a theme or project. Rather, concepts should be developed in a coherent, planful manner (NAEYC/NCTM, 2002, p. 10).

Some kind of 'intentional curriculum' (Klein & Knitzer, 2006) is therefore necessary. Direct, intentional literacy instruction is particularly important in supporting achievement gains for low-SES children (Hamre & Pianta, 2005); the same is likely to be true for EME.

A mathematics *curriculum*, the fifth component, involves a sequence of planned activities designed to help children progress through the learning of key mathematical ideas throughout the year. The *Big Math for Little Kids* curriculum (Ginsburg et al., 2003), which provides the illustrative material for this paper, engages children first in learning key concepts of number, then shape, pattern, measurement, operations on number, and then space. Activities are offered for each day of the school year. Within each of the larger topics, the activities are arranged in order of difficulty, as indicated by research on the developmental trajectories of children's mathematics learning. Thus, in the case of number concepts, children first begin to learn number words, then encounter concepts of cardinal number, representation, and next ordinal number, in that rough order. Later activities revisit earlier concepts, for example, as when shape activities involve analysis of shapes into numbers of sides, and practice in several activities (like counting) is provided throughout the year. Research on the curriculum's effectiveness is currently underway.

In brief, EME involves a hierarchy of components. Adults provide the first two but do not intervene directly in them: a rich environment and the opportunity to play. But adults do take a more direct role in the next three components, the teachable moment, the project approach, and especially the curriculum. Consequently, the success of these components depends on the quality of teaching.

WHAT IS GOOD EARLY MATHEMATICS TEACHING?

Quality teaching in preschool resembles good teaching in the early elementary grades. The best preschool programs are those in which children are systematically, regularly, and frequently engaged in a mix of teacher-led and child-initiated activities that enhance the development of knowledge and skills (Barnett & Belfield, 2006). Also, teachers in quality classrooms show familiarity with children's academic needs, are sensitive toward individual children, and modify lessons and activities to fit the emotional and the academic needs of their students (Rimm-Kaufman et al., 2005). They also tend to promote children's learning through scaffolding and support and offer appropriate questioning and feedback (Rimm-Kaufman et al., 2005). Good teachers also prepare carefully, understand the mathematics they teach (Ma, 1999), motivate their students with games and interesting activities, employ useful materials and manipulatives and exploit their potentials for teaching ideas (Williams & Kamii, 1986), possess the pedagogical content knowledge necessary for implementing lessons effectively (Shulman, 2000), encourage language (Klibanoff et al., 2006) and metacognition (Bodrova & Leong, 1996), and blend formative assessment with teaching (Bransford et al., 1998). Good teachers also assign children to appropriate size groups – whole class, small group, and individual – depending on the nature of the activity.

Although early mathematics teaching has seldom been studied, perhaps because it is seldom done, quality teaching of young children is likely to have many of the characteristics identified in the context of elementary education.

TEACHERS' CONCEPTIONS ABOUT EME

Given that ECE has not until recently emphasized the systematic teaching of mathematics, many teachers, administrators, and policymakers have not had to think about what it means to teach mathematical content. Perhaps this explains why many EC teachers believe that focusing on social and emotional development is most appropriate for young children and that academic subjects, such as mathematics, are less important (Ackerman & Barnett, 2005; Graham et al., 1997; Kowalski et al., 2001; Lee & Ginsburg, 2007; Wesley & Buysse, 2003). Many early childhood educators think that it is developmentally inappropriate to teach children numbers and alphabet, to engage them in direct instruction, or to require them to participate in an activity. Play and self-directed activities are emphasized as the overarching principles of ECE. Many early childhood practitioners believe that teachers should intervene as little as possible and allow children to explore and play without adult guidance or support (Bowman, 2001). This view may have resulted in part from overly enthusiastic acceptance of the early NAEYC position paper (Bredekamp & Copple, 1986).

But other beliefs not necessarily related to the issue of DAP also play a role in teachers' conceptions and attitudes toward early mathematics teaching. For example, we have observed many pre-k teachers and administrators to believe that their role is simply to 'prepare' children to learn math but not to really teach math. They believe that 'real' learning begins in elementary school. Beliefs like these have a strong influence on EME.

Teachers' content knowledge also plays a role in their beliefs. Early elementary teachers who seem to understand the mathematics they teach tend to have more optimistic expectations of their students' knowledge of mathematics than do teachers who are not comfortable teaching mathematics (White et al., 2004). And it appears that many early childhood teachers are quite uncomfortable teaching early mathematics.

Socioeconomic status of teacher and child also plays a role in teacher beliefs. Pre-k teachers of low-income children are more likely to place a high priority on academic learning and working toward math and literacy goals whereas teachers of middle-SES children are more likely to value social development (Lee & Ginsburg, 2007). Many early childhood teachers, especially those from middle-SES backgrounds, have been trained *not* to teach content in explicit ways, but rather to focus on the social–emotional functioning of the children (Kowalski et al., 2001).

Teachers' beliefs about teaching practices are also influenced by their experiences working with children and their families (Williams, 1996). In this context, it does not seem surprising that the teachers of low-SES children favor structured and deliberate mathematics instruction (Lee & Ginsburg, 2007). 'Like it or not, their parents insist on this and so do many politicians' (Ginsburg & Ertle, 2008, p. 3). Children from low-income families begin kindergarten a year to a year and a half behind their middle-class peers so there is more pressure on low-SES teachers to transform their students' education and improve their knowledge (Stipek, 2006).

In summary, many early childhood teachers believe that teaching mathematics is developmentally inappropriate for various reasons. This is an important obstacle to overcome in implementing effective EME.

HOW GOOD IS EARLY MATHEMATICS TEACHING? ARE TEACHERS READY FOR IT?

Although we have some ideas about what EME should entail in terms of content and components, little is known about how mathematics is actually being taught in preschool classrooms. The small amount of available research suggests that current preschool mathematics teaching is not at the level needed.

Childcare teachers teach mathematics in minimal, unconnected, and sporadic ways (Graham et al., 1997). Preschool classrooms generally provide moderately high levels of emotional support but fairly low levels of instructional support, especially with regard to concept development and feedback (La Paro et al., 2004). In fact, Pianta and La Paro (2003) characterize EC environments as being 'socially positive but instructionally passive' (p. 28). We have already shown that preschool teachers seldom acknowledge, let alone build on, mathematical 'teachable moments' that come up during their interactions with children (Kontos, 1999; Seo & Ginsburg, 2004).

In addition, mathematics may be taught differently in classrooms of low-SES children than that of higher-SES children. For example, teachers serving economically disadvantaged students often devote less time and emphasis to higher-level thinking skills important in learning mathematics than do teachers serving more advantaged students (Copley and Padron, 1998).

Given that early mathematics teaching does not appear to meet the current recommendations, though, we must examine whether teachers are actually ready and able to meet the recommendations. In other words, do they have the necessary knowledge, skills, and resources with which to teach mathematics?

The answer to this question is also deeply entangled with SES. For example, in addition to the SES role in prioritization of content versus social–emotional foci, teachers of low- and middle-SES children differ in terms of their educational backgrounds and experiences (Lee & Ginsburg, 2007). Teachers of low-SES children tend to have less teaching experience and lower levels of education than teachers of middle-SES children. Therefore, although teachers of low-income children tend to place greater importance on teaching math, ironically they tend to lack the training and experience that might help them to teach it. Also, classrooms for low-SES children are generally of poorer quality than those for middle-SES children (Pianta et al., 2005), suggesting inadequate resources with which to teach early mathematics.

Research on the teaching of mathematics at the elementary level may also provide insight into preschool teachers' readiness to teach mathematics. Despite requisite schooling and credentialing, elementary teachers often do not have the deep knowledge necessary for teaching mathematics (Ma, 1999). Given that the ECE credentialing requirements are even lower than those for elementary school teachers, and given prior lack of attention to mathematics education in ECE, it seems likely that EC teachers would be less prepared than elementary teachers to teach mathematics.

Further, Copley and Padron (1998) found that although EC teachers generally like teaching reading and other language-oriented skills, they find math and/or science to be difficult subjects, ones they feel unable to teach. They are likely disinclined to give it the time and attention it needs. We have encountered this phenomenon in the teachers with whom we have worked. One teacher, in fact, felt so intimidated by the prospect of teaching mathematics that she turned the task over to her assistant teacher for much of the first year of our work together. And this occurred despite the fact that she attended our professional development workshops to develop her mathematical understanding and readiness to teach. This comes in sharp contrast to her later testimonial – after a year's experience with the program and her gradual increase in understanding and confidence – of how much she loved teaching math, how much her students loved math, and how math had become a part of each day in her pre-k classroom. This provides a promising indication that, with appropriate professional development, preschool teachers' beliefs and practices regarding mathematics teaching can change as their understanding and confidence is increased.

THE STATUS OF EME PROFESSIONAL DEVELOPMENT

Just as with later schooling, successful preschool programs offer ongoing professional development (Bowman et al., 2001; Klein & Knitzer, 2006; Pianta, et. al., 2005). Despite this, little is being done to better prepare preschool teachers to teach mathematics. Most EC teacher preparation programs require just one course in mathematics education, and, more often than not, such courses do not even focus on early childhood (Ginsburg et al., 2006; Sarama & DiBiase, 2004). Further, few early childhood professional development programs focus on mathematics at all (Copley & Padron, 1998; Martinez-Beck & Zaslow, 2006). Instead, most professional development programs aim to give general characterizations of developmentally appropriate curricula, to help build pre-literacy skills, to share classroom management techniques, and to improve children's social and emotional development (Copley and Padron, 1998).

The Good Start Grow Smart Early Childhood Initiative of the No Child Left Behind Act of 2001 also neglected mathematics professional development. Although the initiative states the importance of teaching and assessing 'numeracy' skills, it only provides Head Start teachers with opportunities to receive professional development aimed at improving early literacy. Therefore, despite the current demands for implementing and improving EME, little support is being provided to ensure its success.

Regardless of the lack of support for developing and providing professional development in EME, efforts have been made to define the requirements of effective mathematics professional development, primarily at the elementary level. Various academics, educators, and policymakers have created or assembled standards and guiding principles for effective professional development in mathematics (Lee, 2001; Loucks-Horsley et al., 1996; NAEYC/NCTM, 2002; NCTM, 1991). For example, NCTM (1991) defined a set of six standards for the professional development of teachers of mathematics. These standards include experiencing good mathematics teaching, knowing mathematics and school mathematics, knowing students as learners of mathematics, knowing mathematical pedagogy, developing as a teacher of mathematics, and the teacher's role in professional development. The joint NAEYC/NCTM (2002) position statement further attempted to clarify the characteristics of effective EME professional development. It states that in-service professional development has the most impact on teacher learning if it includes opportunities for teachers to network or form study groups, sustained and focused opportunities for learning, collective participation of staff who work in similar settings, content focused both on what and on how to teach, active learning techniques, and opportunity for professional development to be seen as a part of a coherent program of teacher learning.

One might believe that effective teacher training will simply result from school districts, government agencies, and policymakers using such standards or guidelines to create and implement professional development programs. However, this may be wishful thinking. After attending 13 professional development sessions and relating the workshops' content to eight common and measurable standards, Hill (2004) noticed that some of the workshops that met many of the standards lacked substance. They only briefly examined the mathematics content and student learning. Other workshops that met fewer standards actually delved more deeply into the math content and student learning. She concluded that a professional development program's adherence to the standards does not ensure workshop quality. Moreover, the current standards lack the necessary substance to help teachers, school districts, or policymakers to create, identify, or implement effective professional development.

We agree. Effective workshops need to offer more substance than the various Standards seem to propose. Preparing teachers and their supervisors for EME requires helping them to understand the necessary mathematics, children's learning and thinking, and pedagogical principles, all in the context of *specific* curriculum activities. It is to this topic that we turn next.

AN EXAMPLE: BIG MATH FOR LITTLE KIDS

It is within this interesting and challenging context that the *Big Math for Little Kids* (BMLK) curriculum and professional development program was introduced to a group of teachers from the New York City Administration for Children's Services Division of Child Care and Head Start (ACS DCCHS) centers. When the program first began, many of the teachers admitted to suffering from 'math phobia.' But they had also learned from academic test results that their children's mathematical competencies were lagging, and they passionately wanted to improve their chances for later school success (Cordero, 2004). At the same time, despite their desire to embrace a math program that would benefit their children, they questioned whether such a program was developmentally appropriate. They questioned whether their 4-year-old children were capable of learning the mathematics covered in the curriculum.

Our work with these teachers resulted in many surprises. As expected, we encountered resistance, but we also encountered enthusiasm, a willingness to learn, great learning potential, and a deep desire to help their children succeed in school. We witnessed amazing changes in attitudes, beliefs, and behaviors. In fact, it has been our experience that teachers who complete the BMLK professional development program become strong advocates for introducing content-oriented mathematics into the ECE. But before describing the program of professional development, we provide background on BMLK.

BIG MATH FOR LITTLE KIDS – AN OVERVIEW

BMLK is a pre-k and kindergarten curriculum developed with funding from the National Science Foundation (Ginsburg et al., 2003). It was developed to provide children with a developmentally appropriate and research-based curriculum that would help prepare them for elementary school. It was also developed with the premise that although mathematics learning in early childhood is different from what it is during later years, it still can engage children in deep thought.

BMLK provides teachers with many different opportunities to help their children learn 'Big Math' concepts. First, the curriculum offers teachers a sequenced, extensive, and in-depth coverage of various mathematical concepts. Second, the curriculum provides teachers direct ways to connect literacy, language, and mathematics. Finally, the curriculum presents opportunities for math learning to directly connect from the classroom to the home.

The BMLK curriculum covers six units: number, shape, patterns and logic, measurement, number operations, and space. Each of these math concepts is introduced in the pre-k curriculum and further developed in the kindergarten curriculum. In the *number* unit children learn to say the counting sequence, to use a number to tell how many (cardinality) and to use ordinal numbers to identify positions in a line (ordinality). In the *shape* unit children learn the names and important attributes of two- and three-dimensional shapes as well as the concept of symmetry. The *patterns and logic* unit gives children experience with patterns involving sound, color, shape, letters, and numbers. Children also learn to reason logically through the use of clues. In the *measurement* unit children develop basic measurement principles as they investigate length, weight, capacity, temperature, time, and money. The number *operations* unit extends children's understanding of number by introducing addition, subtraction, multiplication, and division concepts through manipulatives, stories, and games. In the *spatial relations* unit children learn to identify positions in space, navigate through space, and represent space using maps.

Teachers are meant to teach BMLK lessons throughout the year on a daily basis for ~15–30 minutes. Lessons involve playing games, reading storybooks, and engaging in activities with children. They include many manipulatives and other 'hands on' materials for children to explore and manipulate, and many opportunities to interact with the other children in the classroom. The curriculum provides explicit learning goals and outcomes, and suggestions for different ways that teachers can assess their children's mathematical understanding.

The curriculum also aids in deepening children's mathematical concepts by providing connections between language, literacy, and mathematics. Every activity has a list of mathematical terms that teachers should use and introduce to children, and each unit's storybook helps to link literacy and mathematics by allowing children to explore mathematical concepts with characters in the story.

The program further attempts to foster language development by encouraging children to explain, justify, and communicate their mathematical ideas. Teachers have reported that this is one of their favorite parts of the curriculum (Cordero, 2004). As one teacher stated 'What's so surprising is that kids use these words now… it's part of their everyday vocabulary.' Comments like these have led us to believe that BMLK is not just a mathematics curriculum but also a literacy curriculum 'in disguise.'

PROFESSIONAL DEVELOPMENT WORKSHOPS FOR BMLK

Our workshops – developed and piloted over a 3-year period – begin in the summer with a one-day intensive introduction and include both teachers and administrators. This meeting is important for several reasons. First and foremost, it describes the overall structure of the program, allowing participants to become acquainted with the scope of the program and the expectations involved as they mentally prepare for the shifts in thinking that will be required. Secondly, some actual activities from each unit are covered, giving participants an idea of the kind of learning and teaching involved. This is particularly important in allowing both teachers and administrators to see how intentional mathematics teaching can be developmentally appropriate and enjoyable at the same time. Finally, it gives the participants an introduction to us, the people involved with creating and implementing the workshops, helping to create a community with a mutual interest in the improvement of mathematics education for young children.

The eight workshops that follow the summer introduction are all conducted in a similar structure. They begin with a short conversation about the previous month's workshop, to find out how the teachers fared with the curriculum thus far. These conversations offer a kind of 'warm-up' for the conversations intended for the rest of the workshop and help to establish the kind of atmosphere we hope to engender. In our experience, we find that treating these workshops less as lectures, and more as 'conversations among colleagues,' helps to make the experience more worthwhile and memorable for everyone involved. Not only do the teachers get a sense of self-importance from these interactions (which is so rare for most inner-city preschool teachers), but also we, the researchers, get a sense of their understanding of the content. In addition, this stylistic choice reflects the BMLK curriculum itself; workshop leaders, in a sense, model for the teachers the kind of respect, group-learning, and open-ended questions with which we hope that they will engage their own students.

Following this 'debriefing,' teachers are presented with a 'challenge question.' Originally initiated as a simple form of evaluation of the teachers' knowledge, each challenge question presents a mathematical scenario with various student responses, which teachers are asked to evaluate. The responses are designed to elicit many possible interpretations and provide insight into teachers' knowledge of the mathematics, children's understanding of the mathematics, and the assessment and teaching of the mathematics. Surprisingly, the first challenge question left a great impression on the workshop attendees. Given their embrace of this unintended learning experience, we received such strong positive feedback from the teachers that we decided to include similar 'challenge questions' followed by discussion as a regular part of the workshop structure.

Next we turn to the content of the current unit. Although each workshop is unique, each begins with a look at what young children already know and understand about the math topics being presented. This is accomplished with examination of video clips of children in naturalistic play and exercises conducted to exemplify these ideas. For example, to demonstrate how difficult it is to memorize the counting numbers from 1 to 20 in English, we ask the workshop attendees to attempt to learn these counting words in a language such as Tagalog, from the Philippines. They fail. This kind of exercise offers insight into the nature of the challenge preschoolers face when they learn counting.

After considering what kind of knowledge the children might already have in the topic under consideration, we look at the mathematical content of the unit. This is where goals for the unit are covered, and we discuss the mathematical ideas involved. While most who are unfamiliar with preschool math might assume that this part of the workshop is unnecessary because the mathematical content is trivial, the discussion often produces profound revelations for the teachers, many of whom have never considered the complexity of what is being taught. For example, many of the workshop attendees have never before considered that a square is actually a special type of rectangle. Content knowledge like this is discussed and debated. In fact, some of these conversations have even forced us, the researchers, to debate and investigate some mathematical quandaries that arise. For example, we found it very difficult to define clearly what a pattern is. Try it.

After an examination of the content, we discuss key unit activities (not all). They are addressed in a variety of ways: through role playing, discussions about various aspects of the activity, and examining video clip examples. Throughout these activities, we discuss issues of pedagogy, methods of assessment, grouping of children, and construction of materials. This allows teachers to get a realistic sense of the key activities, so that they leave with the confidence to return to their classrooms and teach them.

WHAT WE HAVE LEARNED ABOUT PROFESSIONAL DEVELOPMENT

We constructed our workshops by following guidelines and principles of good professional development seen in the field. But in the process of developing and implementing our workshops, we also discovered several additional guiding principles that we feel are key to successful professional development (Morgenlander & Manlapig, 2006).

Principle 1: Teachers are professionals and should be treated as such. In our workshop environment, we try to show professional respect for teachers and their knowledge. By purposefully setting up time for teachers to have conversations with one another regarding how to connect ideas from the presentation and actual classroom practice, we step back as experts and allow the teachers' expertise to come to play.

Principle 2: Teachers may have a fear of math. Many of the teachers attending these workshops have confessed to a fear of teaching math prior to participating in these workshops. It is important to consider such fear when we design our workshops. Our hope is that the workshops give teachers a comfort zone by showing examples on video, by explaining and discussing math concepts, and by having them try out math activities in a peer-group setting. We believe that all this will help them overcome math phobias.

Principle 3: Teacher incentives help. Although the teachers who have attended these workshops have all chosen to be there – they want to learn how to teach mathematics to their preschoolers – we recognize that the time to attend competes with all of the other demands on their time. As such, we want to make attendance as worthwhile as possible by offering incentives. We offer breakfast, 'on-time prize' for those who arrive on time, and 'take-away' items that are useful for teaching the mathematics content of the unit. Offering these incentives serves to boost attendance and bolster relationships between workshop presenters and teachers, both of which are important to the success of a long-term professional development program.

Principle 4: Allowing time for thoughtful discussions is important. Teachers often remark that one of the most helpful portions of each workshop is the discussion they have with other teachers. Our goal is to provide an environment where conversations about mathematics, children's thinking, and the curriculum can occur. These discussions are helpful for working out connections between the workshop topics and the realities of the classroom. In addition, discussions can also serve to encourage teachers who are hesitant to implement change.

Principle 5: Thoughtful discussions result from constant facilitation. Although discussion time is important, it also requires facilitation, which presents many challenges and unknowns into each workshop. On one hand, teachers often have great advice, insight, and knowledge to offer both the workshop presenter and other teachers. On the other hand, some teachers may want to discuss topics that can detract from the primary goals of the workshops. It is the responsibility of the workshop leader to set up and direct the conversation so that deep and meaningful conversation related to the workshop's goals occurs.

Principle 6. The use of video is crucial. Many workshops involve activities with hands-on manipulatives so that mathematics can be made concrete and 'real.' But videos offer other benefits. They can illustrate lessons in classrooms. They can show examples of children's behavior and thinking. They can be played and replayed, argued, and interpreted. They are in effect another kind of 'manipulative' that can be used to promote understanding of teaching, lessons, and children.

Principle 7. Each workshop should cover the mathematics, the child, and the activities to be taught. We have found that our practice of covering each of these topics, and relating them to each other, provides a practical perspective on teaching. Teachers need to understand the mathematics to be taught, but often do not. They need to understand how the child thinks about specific mathematical ideas, but they often do not go beyond vague (and we think virtually useless) platitudes about constructivism or cognitive development, for example that children are 'concrete.' And they need to learn how knowledge of the mathematics and understanding of the children need to be embedded in the teaching of lessons.

Principle 8. Theoretically grounded specificity is the key. Some workshops we have seen are mere collections of activities. They can be useful if teachers understand how and why to use them. But these 'low-level' workshops seldom explore these matters in any depth; they lack a conceptual framework for understanding the activities to be undertaken. Other, 'high-level' workshops traffic in abstract principles like constructivism or DAP. These principles can be useful if teachers understand how they relate to the teaching of specific activities. Yet the high-level workshops seem disconnected to a significant degree from the nitty-gritty of classroom practice.

We believe that teachers benefit most from a kind of theoretically grounded specificity. When covering the children, the mathematics, and the activities, the workshops need to be theoretically grounded, not intellectually vacuous, and at the same time not mindlessly empirical. For example, in introducing early addition, workshops need to help teachers understand that:

- Children often begin addition by combining two sets, counting them, and learning that order of objects counted and nature of objects counted is irrelevant to the result. In this case, both strategies and ideas are important features of children's behavior. It is inaccurate to say that children's thinking is mainly concrete.
- Addition of whole numbers can be considered to be a process of joining and enumerating elements of separate sets, in any order, regardless of the identity of the elements comprising the sets. These are the ideas and strategies to be learned, and in fact children often try to learn them on their own. Our teaching will be more effective to the extent that it recognizes what needs to be learned and what the children are trying to do.
- A particular activity, as figuring out how many objects are in two cups altogether, tries to capitalize on children's tendency to combine sets and on the mathematical concept that addition can be interpreted as the union of sets. The teacher needs to learn that it would be desirable in the activity to employ different items within each cup, so that children can learn that identity of the items is irrelevant. The teacher needs to learn that children should be encouraged to count the items in many different orders. The teacher needs to learn that systematic counting is to be encouraged, not discouraged in favor of helping the children to memorize the results. And the teacher needs to learn that children – who at this age are abstract thinkers – should not be discouraged from counting 'in their heads,' so long as they have ways to check the results.

All this is what we mean by theoretically grounded specificity. It is a way to give teachers useful practitioner knowledge.

ASSESSING THE SUCCESS OF OUR WORKSHOPS

In the first few years of doing workshops, we gave teachers written surveys at the end of the workshops to evaluate that day's discussion, as well as at the beginning of the next workshop to elicit any reactions the workshop attendees may have had after letting the last workshop 'sink in.' It became clear that all this paperwork revealed very little; most of the responses were flattery, and very little was learned.

It was not until the end of each year, when we held an end-of-the year celebration/debriefing workshop that interesting findings became apparent. Teachers really opened up in these sessions, given that we were primarily there to listen to their reflections on the year of undertaking this new curriculum. Some of these sessions have been videotaped and all analyzed (Cordero, 2004). Here are salient results.

First, we have learned that the workshops help teachers overcome their fears of teaching math. Many teachers recalled feeling apprehension or fear of mathematics at the beginning of the year. By the end of the year, though, the situation had changed. According to one teacher:

> *I had a real math phobia and if it hadn't been for BMLK, I wouldn't know how to begin, where to start teaching my kids math. Now I feel that I'm much more confident and I'm much better at implementing math.*

Secondly, teachers usually began the year with a concern about how to fit the curriculum into their already busy schedules. By the end of the year, teachers (and their students) were enjoying it so much that they had no problem finding the necessary time.

This finding is related to another one – teachers really enjoy teaching the activities by the end of the year. In part, this is because they see that their children are enjoying them too. While we always stressed during workshops that teachers need to have fun while teaching, this is sometimes easier said than done, especially if the teacher comes to the workshops with a negative disposition toward teaching math. But the fears of teaching math tend to dissipate, and what replaces them is a real joy.

Finally, teachers reported that by the end of the year, their students both look smarter and feel smarter. This is a vital point, given that the curriculum is intended to increase the confidence and abilities of some of our nations' highest-risk students. One teacher told us:

> *They want to be smart, they want to know these things and at the end of a lesson, they all get it and are happy about that. That's the best thing about it; it makes their day because they know they are smart!*

Stories like these indicate that not only does the curriculum work well, but so do our workshops for the teachers. We are very proud that our teachers can return to us with such positive results.

As the workshops progressed throughout the year and teachers were encouraged to share, discuss, and reflect on their BMLK-related experiences at the sessions, they grew more enthusiastic about the program. They noted that the activities progress from simple to complex and that these lend themselves to individualization; they also observed that new math ideas build on previous ones and that important mathematical ideas are revisited again and again within different mathematical contexts. The value and significance of a 'sequenced and coherent' math curriculum slowly became obvious. One teacher expressed her appreciation that the curriculum provided her with a sequence of activities that she did not have to try to develop on her own.

Teachers seemed most convinced by what they observed in the classroom. Their animated discussions were replete with stories about how much their children are enjoying the activities and how quickly they are learning new math skills, ideas, and language. These teachers became convinced of the value of the program and are its strongest advocates.

NEEDED SUPPORT FOR AN EME PROGRAM

Our BMLK professional development efforts at a publicly funded early care and education agency in New York City have proven to be both challenging and encouraging, and we anticipate that many more challenges lie ahead as the various education sectors attempt to close the achievement gap between disadvantaged and advantaged children. Our experience with BMLK also indicates that although essential, teacher training is not the only factor in play if city and state early care and education agencies intend to make a positive, system-wide, and sustainable impact on low-income children's math achievement. We need to obtain support for five constituencies that contribute to sustainable change.

TOP-LEVEL ADMINISTRATION

Top-level administrative officials in city and state early care and education agencies need to have a clear understanding of the ECE field, the significance of new research in education, and the vision, ability, commitment, and courage to make the necessary institutional changes that will support a plan for quality improvement. Not only will agency heads need to find ways of obtaining additional funds and resources, but they may also have to develop an infrastructure that may require a reorganization of their agency. This is much easier said than done. Resistance to change is a ubiquitous force, and top-level city and state officials are not immune to it.

SUPERVISORY STAFF

Supervisory staff in local or regional offices also play a critical role in enabling systemwide, sustainable change in ECE. These supervisors are usually expert teachers who have been in the system for a long time. Their major responsibility consists of providing support and technical assistance to early childhood programs based on agency policies that often have not been updated to reflect recent changes in the field. For example in New York City, regional supervisors provide technical assistance to childcare centers based on outdated program assessment tools. It is not uncommon for teachers to complain that regional supervisors are critical of some of the BMLK activities. Consultants penalize teachers for implementing activities that are considered too 'teacher-directed' in lieu of the more play-oriented, child-initiated approaches recommended by the out-dated program assessment protocol. The same holds true for using activities that involve written material that might superficially seem to resemble worksheets (in that writing is involved!), irrespective of their educational value or intention. And teachers are very much aware of these contradictions. As long as local or regional supervisors are not trained in the relevant mathematics program, it is likely that they will subvert it.

PROGRAM SUPERVISORS

Program supervisors such as directors of centers and education directors also play a critical role in supporting their teachers' professional development in an EME program and in ensuring the success of the program and the benefits for the children. Often, program supervisors are very eager to have their teachers trained in new approaches and evidence-based curricula, but they do not undergo the training themselves. Hence, they are not in the position to supervise, guide, or support their teachers' classroom practices. Furthermore, trained teachers will probably discontinue using BMLK over time if they do not feel guided and supported by a knowledgeable program supervisor. It has been our experience that when capable directors and teachers work in teams during the professional development phase, they report continued implementation of BMLK.

PARENTS

One of the most common concerns that low-income parents voice to preschool program directors and teachers is that they would much rather their children learn to read, write, and do mathematics than play. And although directors and teachers spend much time and energy trying to convince parents of the contrary, the latter are not easily dissuaded. And they have good reason to hold their ground; they know that their children are likely to fail in the schools as currently constituted and suspect that preschools do not offer adequate preparation.

Interestingly, teachers implementing BMLK report that parents very quickly respond with enthusiasm to the program. One program director recently reported that a group of parents decided not to transfer their children to another school at the end of the year when they found out that the children would continue with the BMLK program the following year. As this anecdote suggests, given the opportunity, low-income parents will place their children in high-quality early childhood programs. Parents vehemently want early childhood programs to prepare their children for school. Their voice and support in the pursuit of quality education cannot be underestimated.

CONCLUSION

Early math education is new and challenging. It involves deep mathematics and many different components, ranging from free play to an organized curriculum. Teaching it is not easy and in fact may be similar to teaching mathematics at the elementary level. But we have some idea of how to help; we need to provide teachers with theoretically grounded and specific professional development opportunities. We need to help various constituencies to understand the need for EME and how to support it. The major question is not whether children can learn mathematics. It is whether we can help teachers to teach it. The unresolved issue is whether the political system will support the effort to provide all young children with effective early education.

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