Metacognitive Strategy Use of Eighth-Grade Students With and Without Learning Disabilities During Mathematical Problem Solving: A Think-Aloud Analysis

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Abstract

The purpose of the study was to investigate the metacognitive abilities of students with LD as they engage in math problem solving and to determine processing differences between these students and their low- and average-achieving peers (*n* = 73). Students thought out loud as they solved three math problems of increasing difficulty. Protocols were coded and analyzed to determine frequency of cognitive verbalizations and productive and nonproductive metacognitive verbalizations. Results indicated different patterns of metacognitive activity for ability groups when type of metacognitive verbalization and problem difficulty were considered. Implications for instruction are discussed.

Keywords

Metacognition, Math problem solving, Middle school, Cognitive processes

Despite the realization that conceptual understanding of mathematics is critical to mathematical development, most math instruction, particularly for students with learning disabilities (LD), continues to stress rote learning of math facts and procedures (Jordan & Hanich, 2000). In an attempt to transform the way math is taught and learned, the National Council of Teachers of Mathematics' (NCTM; 2000) *Principles and Standards for School Mathematics* advocated an approach to mathematics teaching that centers on conceptual understanding and problem solving and encourages teachers to develop students' ability to understand and make connections across math concepts as well as actively engage students in meaningful communication about math. Schoenfeld (1987) called this type of classroom "a microcosm of mathematical culture" (p. 205). He wanted teachers to convey the meaning of mathematics as an integral part of students' everyday lives. Schoenfeld was pioneering in his emphasis on the importance of metacognition in the process and the disconnect between students' mathematics classroom experiences and their everyday math experiences.

Math problem solving, which presents a challenge for many students, is strongly correlated with success in mathematics (Bryant, Bryant, & Hammill, 2000; Geary, 2003; Lewis, 1989). To solve math problems, students must be able to represent the problem, develop a solution path, and execute the solution (Mayer, 1985). Several cognitive processes and metacognitive strategies (e.g., visualization, estimation, self-questioning) are integral to problem representation and

problem execution and underlie successful problem solving (Mayer, 1985). Problem representation, generally speaking, is a combination of concrete representations (e.g., manipulatives), something written, and/or a carefully constructed mental image (Janvier, 1987). Problem representation processes and strategies are necessary when comprehending the problem, that is, integrating problem information, maintaining mental images of the problem in working memory, and developing a solution plan, frequently by using alternative solutions to the problem (Silver, 1987).

Students with LD are characteristically poor problem solvers who tend to overestimate their math ability (Garrett, Mazzocco, & Baker, 2006; Montague, 1997). They also exhibit working memory, processing speed, and executive function deficits (Fuchs & Fuchs, 2002; Geary, 2004; Gonzalez & Espinel, 2002; Johnson, Humphrey, Mellard, Woods, & Swanson, 2010). These students tend to respond impulsively, use trial and error, and fail to verify solution paths and evaluate answers (Bryant et al., 2000). They have difficulty with multistep problems (Bryant et al., 2000; Fuchs & Fuchs,

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2002), math vocabulary (Bryant et al., 2000), and fact retrieval and computation (Fuchs & Fuchs, 2002; Gonzalez & Espinel, 2002; Montague, 1991). Additionally, they are less likely than typical students to use task-appropriate metacognitive or self-regulation strategies when solving math problems (Montague & Applegate, 1993b; Stone & May, 2002). Consequently, students with LD are at high risk for failure in mathematics.

Research has confirmed that metacognition is vital to academic success (e.g., Pintrich, Anderson, & Klobucar, 1994; Trainin & Swanson, 2005; Wong, Harris, Graham, & Butler, 2006) and is particularly important to successful problem solving (De Corte, Greer, & Verschaffel, 1996; Lucangeli & Cornoldi, 1997; Montague, 2008; Swanson, 1990). Research also suggests that metacognition develops alongside general aptitude and may be even more effective than general aptitude in predicting learning performance (Swanson, 1990; Veenman & Spaans, 2005). General intelligence may contribute as little as 9% to 25% of the explained variance in performance, whereas metacognition may account for as much as 75% (Van Luit & Kroesbergen, 2006).

Metacognitive strategies (e.g., self-instruction, self-questioning, and self-evaluation) are used to monitor and evaluate cognitive progress during task execution (Montague, 2008; Zimmerman, 2002). It has been widely substantiated in reading research that students with LD demonstrate inadequate metacognitive awareness, possess inefficient strategies, and exhibit a lack of control over their academic behavior (e.g., Mason, Meadan, Hedin, & Corso, 2006; Wong et al., 2006). In contrast, there is limited research on the metacognitive functioning of students with LD focusing on math problem solving (e.g., Carr, Alexander, & Folds-Bennet, 1994; Case, Harris, & Graham, 1992; Montague & Applegate, 1993b). The purpose of the present study to investigate the metacognitive abilities of students with LD as they engage in math problem solving. Metacognition and think-aloud protocols as an investigative method are discussed in the following.

Metacognition

The research in metacognition suggests that it is an important predictor of learning and academic success (Flavell, 1979; Montague & Applegate, 1993a; Veenman, Van Hout-Wolters, & Afflerbach, 2006; Zimmerman, 2002). Common to this research is the notion that students who are active participants in the attainment of personal goals through the use of metacognitive strategies are likely to have greater academic gains than students who are limited in their metacognitive skills. Metacognition, that is, higher-order thinking strategies that serve to control cognitive processes, consists of three primary components: metacognitive knowledge, metacognitive experience, and metacognitive skills (Flavell, 1979; Lucangeli & Cabrele, 2006).

Metacognitive knowledge refers to the interaction of beliefs and knowledge stored in one's memory regarding personal functioning, task execution, and strategy selection (Flavell, Miller, & Miller, 1993; Sperling, Howard, Staley, & DuBois, 2004). Metacognitive experience refers to the conscious reactions and self-judgments regarding personal performance before, during, and after task execution. These beliefs are the result of the self-interpretation or self-evaluation of one's familiarity with the task, comprehension of the task, perception of task difficulty, effort needed to complete the task, and confidence in the ability to complete it (Efklides, Kiorpelisou, & Kiosseoglou, 2006). Metacognitive experience can affect one's knowledge base by assimilating and accommodating new information into stored long-term memory. Metacognitive knowledge and metacognitive experience overlap in that task-specific experiences will likely influence a person's more stable self-beliefs and knowledge (Flavell, 1979). Research in this area has predominantly investigated individuals' evaluation and prediction of their own performance and is usually assessed with questionnaires and interviews (Desoete, Roeyers, & Buysse, 2001; Nussinson & Koriat, 2008; Stone & May, 2002).

In contrast, metacognitive skills refer to the authentic procedures and strategies used during task execution to monitor and control one's cognition (Efklides et al., 2006). These strategies may include self-observation, self-evaluation, self-control, self-monitoring, self-instruction, and self-questioning and are particularly useful in monitoring comprehension when solving novel or challenging problems (Lucangeli & Cabrele, 2006). Metacognitive skills are usually assessed in actual, real-time, concurrent, or on-line activities using think-aloud protocols rather than retrospective self-reports of strategy use. Individuals verbalize their thoughts and cognitive activities while engaged in task execution (Malone & Mastropieri, 1991; Montague & Applegate, 1993b; Ostad & Sorenson, 2007). Think-aloud protocols provide rich verbal data regarding a student's reasoning abilities during problem-solving activities (Fonteyn, Kuipers, & Grobe, 1993) and primarily are used to obtain information on cognitive processing (Ericsson & Simon, 1980). This can be done either concurrently, while the participant is engaged in problem solving, or retrospectively, after the participant has finished the task. According to Ericsson and Simon (1980), thinking out loud does not affect cognitive processes or performance speed.

Research Using Think-Aloud Methods

Three studies were selected purposefully to illustrate how metacognition has been investigated using concurrent think-aloud methods. In the first study, Ostad and Sorenson (2007) examined patterns of private speech, strategy use, and their interaction in children with and without mathematical difficulties (MD) in Grades 2-3, 4-5, and 6-7 ($n = 134$) as they solved math computations. Participants were instructed to think out

loud and were observed individually during two laboratory sessions. Results showed that task-relevant speech was positively correlated with metacognition and successful task completion. Additionally, the children with MD consistently used more backup strategies (e.g., counting on one's fingers) whereas students without MD used more retrieval strategies (i.e., accessing information from one's memory). These findings suggest that the poor metacognitive skills of children may be associated with immature, rather than absent, metacognitive skills.

The second study (Swanson, 1990) examined the relationship between general academic aptitude and metacognition by analyzing think-aloud protocols of 56 students in Grades 4 and 5 for differences in strategy use and problem-solving processes among ability groups. Students were separated into high and low ability groups based on their performance on a cognitive ability assessment and were then administered a 17-item questionnaire to assess metacognition in the general domain of problem solving. Four groups were formed: high aptitude–high metacognition (HA/HM), high aptitude–low metacognition (HA/LM), low aptitude–high metacognition (LA/HM), and low aptitude–low metacognition (LA/LM). Students were audio-recorded solving a pendulum task and a combinatorial task. The think-aloud protocols were transcribed and coded according to 24 mental components. Results indicated that students high in metacognition outperformed low metacognition students regardless of aptitude. Furthermore, the LA/HM group performed significantly better than HA/LM students. However, only the HA/HM group used more heuristic and strategy subroutines than the other groups and consistently used hypothetico-deductive reasoning to work through the problem. These findings suggest that metacognition may be related to but independent of general aptitude.

Finally, Montague and Applegate (1993b) used think-aloud protocols to investigate self-regulation and strategy use with students with LD (*n* = 28), average achievers (*n* = 25), and gifted students (*n* = 28). The students were provided think-aloud training using two verbal reasoning problems. They were then told to solve three problems (i.e., a one-step, a two-step, and a three-step math word problem) while thinking aloud. On the one-step problem, no differences were found in cognitive or metacognitive verbalizations among the groups. On the two-step problem, gifted students made significantly more cognitive verbalizations than students with LD. Few metacognitive verbalizations were evident across groups. On the three-step problem, gifted students made significantly more cognitive and metacognitive verbalizations than both of the other groups. Thus, as problem difficulty increased, the high ability students verbalized more strategies. These findings support the hypothesis that metacognition is activated when individuals are faced with challenging problems. The authors speculated that an individual's perception of the difficulty of the problem also may have

an effect on persistence in solving the problem and thus, activation of metacognitive strategies. Students with LD may not have the metacognitive resources compared with their higher ability peers and may actually "shut down" cognitively when confronted with problems that are difficult or that they perceive as difficult.

The present study contributes to the limited research in this area by using the think-aloud method to study eighth-grade students' metacognition during math problem solving. Think-aloud protocols provide access to students' short-term memory, which reflects cognitive processing during task completion (Young, 2005). This approach allows the researcher to capture a fairly accurate account of applied metacognitive knowledge or metacognitive skills since it does not require the students to remember what they were thinking. Students are expected to simply verbalize what they are thinking as they are thinking rather than reflect on or interpret the task. Specifically, the study investigated differences in on-line, concurrent metacognitive strategy use among students with LD, low-achieving (LA) students, and average-achieving (AA) students. The present study extends previous think-aloud research by including a group of LA students specifically to investigate differences between these students and students with LD as both groups tend to perform similarly on academic tasks but may have different underlying metacognitive processing differences. Two research questions guided our study: (a) Are there differences in metacognitive verbalizations among students with LD, LA students, and AA students during math problem solving? (b) Do students' metacognitive verbalizations vary as a function of word problem solving complexity? We were particularly interested in differences between students with LD and LA students with respect to metacognition and performance as problem difficulty increased. However, we hypothesized that these students would perform more poorly than AA students and engage in more nonproductive metacognitive activity. Additionally, we thought that students would verbalize more as the problems became more difficult.

Method

Participants

Participants were eighth-grade middle school students in a large metropolitan school district in the southeastern United States (*n* = 73). Participants were drawn from a large intervention study directed by the third author (see acknowledgement). The purpose of the larger study was to investigate the effects of *Solve It!*, a cognitive-metacognitive intervention, on the math problem solving of middle school students who were instructed in general education math classrooms (*n* = 779; Montague, Enders, & Dietz, 2010). Students were separated into three groups according to their performance on the math

Table 1. Student Demographic Data

Variable	Learning Disabilities (n = 14)		Low Achieving (n = 34)		Average Achieving (n = 25)	
	n	%	n	%	n	%
Gender						
Males	8	57	17	50	15	60
Females	6	43	17	50	10	40
Ethnicity						
White	6	43	10	30	5	20
Hispanic	8	57	23	68	18	72
Black	0	0	1	2	1	4
Asian	0	0	0	0	1	4
Free/reduced lunch						
Yes	12	86	27	79	19	76
No	2	14	7	21	6	24

section of *Florida's Comprehensive Assessment Test* (FCAT). The FCAT is administered to students in Grades 3 through 10 and consists of criterion-referenced tests that measure selected benchmarks in mathematics, reading, science, and writing from the Sunshine State Standards. Students are assigned one of five achievement levels based on their scale scores. Levels 1 and 2 indicate below grade-level performance, Level 3 indicates grade-level performance, and Levels 4 and 5 indicate above grade-level performance. Students with LD (*n* = 14) were system-identified by the district and had a current individual education plan (IEP). The school district uses an IQ-achievement discrepancy model to identify students with LD (i.e., ≥ 1.5 *SD* difference). Additionally, for the purposes of this study, to be eligible for placement in the LD group, students also scored in the Level 1 or 2 range on the previous year's math FCAT. LA students (*n* = 34) also scored in the math FCAT Level 1 and 2 range, and AA students (*n* = 25) had a math FCAT level of 3 or 4. Another criterion for participation was a level score of 5 (range 1 to 5) on the district-administered eligibility test for services for English Language Learners. Students scoring at Levels 5 were considered English proficient by the district. Students who met the aforementioned criteria were enrolled in the study on a first-come, first-served basis and were assessed prior to the implementation of the intervention. See Table 1 for participant demographic information.

Measures

Think-aloud protocol. Evidence of students' metacognitive skills was obtained using a think-aloud protocol. Students were audio-taped as they thought out loud while solving the following one-step, two-step, and three-step problems:

- Bill and Shirley are arranging the chairs for a class play. They brought 252 chairs from the storeroom to the auditorium. Their teacher told them to make rows of 12 chairs each. How many rows will they have?
- Four friends have decided they want to go to the movies on Saturday. Tickets are $2.75 each for students. Altogether they have $8.40. How much more money do they need?
- Chain sells for $1.23 a foot. How much will Farmer Jones have to spend for chain in order to enclose a 70 foot by 30 foot patch of ground leaving a 4 foot entrance in the middle of each of the 30 foot sides?

These typical textbook-type problems had been used in previous research (e.g., Montague, 1991; Montague & Applegate, 1993a, 1993b) and had established discriminant validity. They required students to have knowledge only of whole numbers and decimals and the four basic operations, thereby eliminating confounds associated with specialized knowledge of problem types or formulae.

Procedures

All participants were tested individually during an elective class period in a quiet setting that facilitated audio-recording as they thought out loud while solving the math problems. Prior to solving the problems, students were trained in the think-aloud method by the researchers in the following manner. First, the researcher explained the purpose of the study and the reason why a think aloud is a good way to understand how people solve math problems. The researcher read the following script adapted from Johnstone, Bottsford-Miller, and Thompson (2006):

I am interested in how students solve problems, so I want to ask you to solve three problems for me and let me listen to how you solve them. I am not interested in the answer you come up with as much as how you are thinking about the problem. What you say is really important, so I am going to use a tape recorder to make sure I don't forget anything.

Second, the researcher modeled thinking out loud using a logical reasoning problem demonstrating processes such as self-questioning, checking back, and monitoring progress as well as affective statements related to the problem. Then the students were given an opportunity to practice thinking out loud while working through two different logical reasoning problems. The researcher encouraged students to speak with appropriate volume and clarity. Participants were then asked to solve the three math word problems while thinking out loud. The session was audio-recorded and subsequently transcribed to produce verbal protocols. The students were told to constantly think out loud, and, if they were silent for longer than 5 seconds, the researcher reminded them to say everything they were thinking, feeling, and doing while solving the problem. With the exception of the reminders, interactions between the researcher and student were minimal. Students did not vary substantially with respect to the time required to solve the three problems while thinking aloud (i.e., students with LD, 6 minutes, 49 seconds; LA students, 7 minutes, 47 seconds; AA students, 6 minutes, 22 seconds).

Transcription and coding. The audio-recordings were transcribed verbatim by the researchers over a 1-month period. Of the transcriptions, 25% were cross-checked for accuracy with the original recording with a 100% transcription accuracy score. Montague's (2003) model of math problem solving, which includes seven cognitive processes (i.e., reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking) and three metacognitive strategies (i.e., self-instruction, self-questioning, and self-monitoring), served as a base for the researcher-developed coding system. Metacognitive strategies were conceptualized as either domain-general (e.g., "Hmm. What should I do? Oh, okay. I get it.") or domain-specific (e.g., "I need to subtract $8.40 from $11.00 to find out how much more money they need to go to the movies.") and included additional metacognitive strategies to the three represented in the model. All verbalizations were first coded as being metacognitive in nature without assigning them to a specific category. They were then entered into Atlas ti (Muhr, 2004), a computer software program used to systematically code and analyze qualitative data, and coded using emergent coding. This produced 14 metacognitive codes. For parsimony, these were combined and reduced to 7 codes. Finally, the 7 codes were separated into two groups, productive and nonproductive metacognitive verbalizations. Productive verbalizations included self-monitoring, self-instruction,

self-questioning, and self-correction statements/questions directly related to solving the problem such as "I need to re-read the question," "That's not possible. It can't be division," and "What am I doing?" Nonproductive metacognitive verbalizations were more affective in nature such as "I don't know what to do," "I'm confused," and "I need a calculator." Overall, the coding system included 7 cognitive and 7 metacognitive codes (see Figure 1). Verbalizations were coded as cognitive verbalizations (CV; i.e., verbalizations related to specific cognitive processes), productive metacognitive (PM) verbalizations (i.e., verbalizations related to finding a solution), or nonproductive metacognitive (NPM) verbalizations (i.e., verbalizations that have no bearing on progress toward problem solution).

The cognitive, productive metacognitive, and nonproductive metacognitive verbalizations were tallied to provide three frequency counts within each problem type. These frequency counts were then transformed into percentages. To illustrate, the percentage of productive metacognitive verbalizations in the one-step problem was calculated by dividing the frequency count of productive metacognitive verbalizations by the total number of verbalizations across categories. The quotient was multiplied by 100 to produce the percentage of productive metacognitive verbalizations for the one-step problem. Analyzing the percentage of metacognitive verbalizations based on total verbalizations allowed a more accurate representation of the students' metacognitive functioning. For instance, Student A had 5 verbalizations, 1 of which was coded as productive. Student B had 10 verbalizations and also had 1 that was coded as productive. By using the percentage rather than frequency of verbalization type, it is evident that Student A had a higher percentage of productive metacognitive verbalizations than Student B.

To determine interrater agreement, two researchers each coded and then compared 10 think-aloud protocols to establish initial agreement. An iterative process was used until agreement was reached regarding the coding by discussing and resolving disagreements. All protocols were returned to the pool. All protocols were then rated by both researchers. Interrater agreement was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100. Interrater agreement was 92%.

Analyses

To investigate differences among the ability groups, a 3 (group: LD, LA, AA) × 2 (metacognitive verbalization: PM, NPM) × 3 (problem difficulty: one-step, two-step, three-step) factorial ANOVA was used. Thus, we investigated differences in metacognitive verbalizations among ability groups as a function of problem difficulty. We were particularly interested in the interactions between metacognitive verbalizations and problem difficulty as we hypothesized that as problems

SUBCODE _____-_____-_____- **PRE / POST**

Think Aloud Coding Sheet

		P1	P2	P3	
Answered correctly: Yes (2) - No (1) - DNS (0)					
Error Type: Correct (3) - Product (2) - Process (1)					
Time					

Cognitive								
Category	**Operational Definition**	**Code**	**P1**		**P2**		**P3**	
			F	%	F	%	F	%
Reading	Reads the problem in its entirety	R						
Paraphrasing	Restates the problem in own words	P						
Visualize	Use of images (diagrams, pictures, mental imagery) to understanding task	V						
Hypothesizing	Sets up a plan, decides on a solution path, sets up a goal identifying operations to use	H						
Estimating	Predicts an answer	E						
Computing	Verbalizes computation	C						
Checking	Checks steps are completed, information is used, computations are accurate	CH						
Total								

Metacognitive								
Category	**Operational Definition**	**Code**	**P1**		**P2**		**P3**	
			F	%	F	%	F	%
Non productive								
Calculator	Requests the use of a calculator	Cal						
Comment	Statements of personal functioning during task execution	Com						
Affect	Statements concerning emotional disposition	NF						
Total								
Productive								
Self-Correct	Corrects products of process errors	SC						
Self-Instruct	Statements regarding procedural control	SI						
Self-Monitor	Observes performance and progress	SM						
Self-Question	Considers problem and solution path	SQ						
Total								
Grand Total								

Figure 1. Think-aloud coding sheet.

increased in difficulty, metacognitive verbalizations would increase and also that the increase in metacognitive verbalizations would be greater for AA students compared with students with LD and low achievers.

Results

Tables 2 and 3 display the summary statistics and the means and standard deviations for the three main effects and four

Table 2. Three-Way Factorial ANOVA Summary Statistics

	F	df	η^2	p
Ability	2.97	2, 70	.078	.058
Problem difficulty (PD)	14.18	2, 140	.168	< .001***
Metacognition type (MT)	4.11	1, 70	.056	.046*
Ability × PD	3.32	4, 140	.087	.013*
Ability × MT	1.79	2, 70	.049	.175
PD × MT	9.12	2, 140	.115	< .001***
Ability × PD × MT	3.27	2, 140	.085	.013*

$*p < .05.$ $***p < .001.$

interaction effects of the analysis. Because the main effect for ability was close to significant, $F(2, 70) = 2.968$, $p = .058$, $\eta^2 = .078$, we conducted pairwise comparisons indicating that students with LD produced significantly more overall metacognitive verbalizations than average achieving students, $p = .018$. The main effect for problem difficulty was significant, $F(2, 140) = 14.181$, $p \leq .001$, $\eta^2 = .168$. Pairwise comparisons revealed that students produced significantly more metacognitive verbalizations on the three-step problem compared with the one-step problem, $p < .001$, and the two-step problem, $p < .001$. The main effect for metacognitive verbalization type also was significant, $F(1, 70) = 4.113$, $p = .046$, $\eta^2 = .054$. Overall, students produced significantly more productive metacognitive than nonproductive verbalizations.

Three significant interaction effects were found. First, the interaction between ability and problem difficulty was significant, $F(4, 140) = 3.315$, $p = .013$, $\eta^2 = .087$. Low-achieving students produced fewer metacognitive verbalizations for the two-step problem compared with the one-step problem, $p = .018$. A significant increase in metacognitive verbalizations from the two-step to the three-step problem was found for all three ability groups: LD, $p = .008$; LA, $p = .012$; AA, $p = .004$. There also was a significant increase in metacognitive verbalizations for students with LD and AA students from the one-step to the three-step problem, $p = .029$ and $p < .001$, respectively.

Second, the interaction between metacognitive verbalization type and problem difficulty also was significant, $F(2, 140) = 9.12$, $p < .001$, $\eta^2 = .115$. As problems became more difficult, significant differences in nonproductive verbalizations were found. That is, NPM verbalizations decreased from the one-step to the two-step problem, $p < .001$. However, NPM verbalizations increased from the two-step to the three-step problem, $p < .001$. There was also a significant increase in NPM verbalizations from the one-step to the three-step problem, $p = .020$.

Finally, the interaction of ability, problem difficulty, and metacognitive verbalization type was significant, $F(4, 140) = 3.27$, $p = .013$, $\eta^2 = .085$. Post hoc analyses indicated that as problems became more difficult, the type of metacognitive verbalizations varied across ability groups. Students with LD significantly increased their NPM verbalizations from the

one-step problem to the three-step, $p = .003$, and from the two-step to the three-step problem, $p = .003$. Low-achieving students produced significantly fewer NPM verbalizations from the one-step to the two-step problem, $p = .008$, and significantly more NPM verbalizations from the two-step to the three-step problem, $p = .029$. Average-achieving students significantly increased their PM verbalizations from the one-step to the two-step problem, $p = .003$, and the one-step to the three-step problem, $p < .001$. They also produced significantly more NPM verbalizations from the two-step to the three-step problem, $p = .001$. Figure 2 provides a graphic display of the three-way interaction.

Discussion

This study explored differences in metacognitive verbalizations among students with learning disabilities, low-achieving students, and average-achieving students as a function of problem difficulty and also investigated the interactions among metacognitive verbalizations, problem difficulty, and ability group. We operationalized metacognitive verbalizations as either productive or nonproductive. Productive verbalizations direct and help the problem solver move toward a problem solution by using strategies such as self-instruction, self-questioning, and self-monitoring. In contrast, nonproductive verbalizations do not directly contribute to problem solving and generally are affective responses or emotional reactions (e.g., negative self-talk and expressions of confusion or frustration) or incidental asides (e.g., "I need a calculator."). Results of the present study suggested that when one does not discriminate between the types of metacognitive verbalizations, students across ability groups look relatively equivalent in the quantity of verbalizations regardless of the problem difficulty. However, different patterns of metacognitive activity were evident for different ability groups when type of metacognitive verbalization and problem difficulty were considered. Several of the salient findings are described next.

Overall, as hypothesized, students made significantly more metacognitive verbalizations on the three-step problem than on either the one-step or two-step problem. That is, on the most difficult problem, all students behaved more metacognitively. It was encouraging that overall, students made more productive metacognitive verbalizations than nonproductive. However, this finding is most likely attributed to the verbalizations of the average achievers who had significantly more productive than nonproductive verbalizations overall. It appears that students with LD, despite making significantly more metacognitive verbalizations than average achievers, were making more nonproductive verbalizations. Only the average-achieving students significantly increased their productive metacognitive verbalizations as problems became more difficult. In contrast, students with LD and the low-achieving students showed a different but similar

Table 3. Means and Standard Deviations: Ability Group by Problem Difficulty by Type of Metacognition

	One-Step				Two-Step				Three-Step				Mean of Three Problems			
	LD	LA	AA	Total	LD	LA	AA	Total	LD	LA	AA	Total	LD	LA	AA	Total
n	14	34	25	73	14	34	25	73	14	34	25	73	14	34	25	73
Total metacognitive verbalizations																
M	19	22	9	17	18	14	13	14	34	21	24	25	24	19	16	19
SD	23	19	14	19	11	12	13	12	16	14	12	14	12	11	8	11
Productive metacognitive verbalizations																
M	11	19	4	9	14	12	11	12	10	13	14	13	11	12	10	11
SD	17	14	7	13	11	11	10	11	9	10	10	10	9	7	6	7
Nonproductive metacognitive verbalizations																
M	8	9	5	8	4	2	2	3	25	12	10	12	12	7	6	8
SD	14	13	10	12	6	6	7	6	22	15	11	16	12	8	8	8

Note: The values are presented as percentages. LD = learning disabilities; LA = low achieving; AA = average achieving.
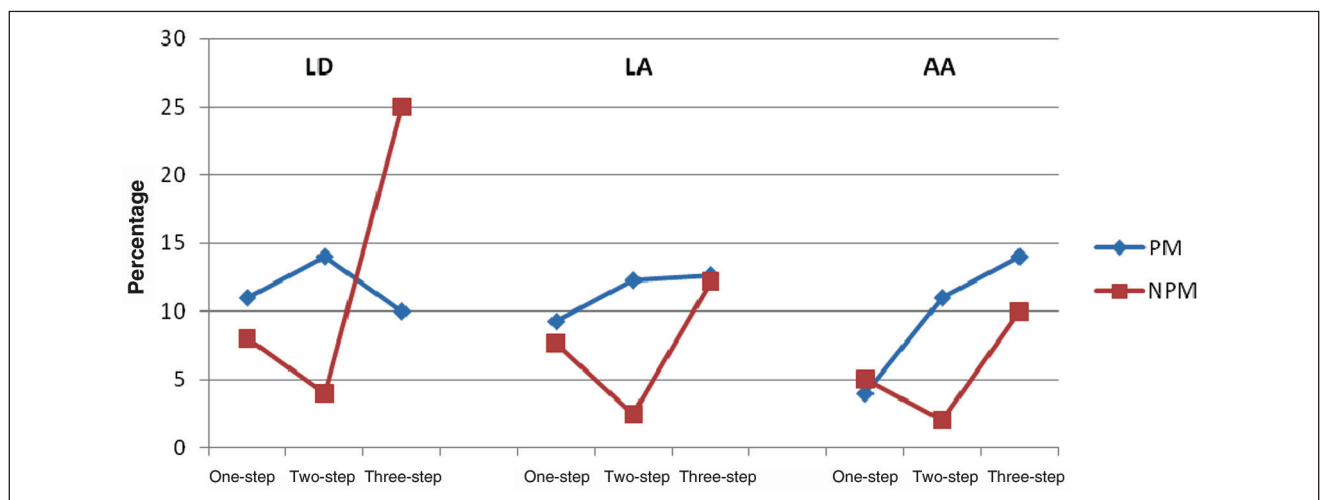
Figure 2. Interaction effects by ability group.
Note: LD = learning disabilities; LA = low achieving; AA = average achieving; NPM = nonproductive metacognitive; PM = productive metacognitive.

metacognitive behavior pattern. Both groups significantly increased their nonproductive metacognitive verbalizations from the two-step to the three-step problem, which suggests that when faced with a very difficult problem, these students did not have or did not use appropriate resources for solving the problem. It should be noted that all ability groups had a significant increase in nonproductive metacognitive verbalizations from the two-step to the three-step problem, but the average-achieving students also significantly increased their productive metacognitive verbalizations, which may have compensated for the parallel increase in nonproductive verbalizations. Additionally,

the students with LD had significantly more nonproductive statements than both the low and average achievers on the three-step problem, indicating their increased frustration with the problem. Interestingly, the students with LD initially increased their productive verbalizations from the first to the second problem, but then decreased these more useful verbalizations on the third problem where, theoretically, they should have used more productive metacognition. The decrease in productive metacognitive strategies may be due to the students' perception that the problem was too difficult and they simply shut down or perhaps they had exhausted their metacognitive resources. However, on

the easier problems, they appeared to activate appropriate metacognitive strategies. The low-achieving students demonstrated almost no variation in productive verbalizations as problems became more difficult, suggesting that they do not discriminate and use the same resources regardless of problem difficulty. It should be noted that none of the students in any of the ability groups correctly solved the three-step problem. For the one-step problem, 42% of students with LD, 41% of low-achieving students, and 76% of average-achieving students solved the problem correctly. For the two-step problem, 7% of students with LD, 32% of low-achieving students, and 52% of average-achieving students solved the problem correctly. As reflected by the greater number of productive metacognitive verbalizations made by the average achievers on the three-step problem, it seems that these students were more productively engaged in solving the problem.

Based on our findings, we believe that more metacognitive activity does not necessarily mean better metacognitive activity or better problem solving. For metacognitive strategies to have a positive impact on problem solving, they need to be anchored in developmentally appropriate cognitive skills. The literature suggests that when solving a novel problem or engaging in a difficult task when the necessary cognitive processes and skills may not be readily available, students rely on their metacognitive skillfulness to work through the problem (Veenman & Spaans, 2005). However, while metacognitive strategy use may help to advance students toward problem completion, appropriate cognitive processes and skills must be present as well. Expert problem solvers generally do not need to activate metacognitive strategies unless the problem presents some difficulty as they have the cognitive processes and skills needed to represent and solve problems without such support (Crowley, Shrager, & Siegler, 1997). This point exemplifies the role that automaticity plays in problem solving. The goal of conscious and deliberate use of metacognitive strategies is to aid in the application of cognitive processes until eventually automaticity is developed. Students who have reached mastery will not need to activate metacognitive strategies to solve math word problems unless the problems are challenging and will therefore produce fewer metacognitive verbalizations than students who have not yet mastered the problem-solving processes and skills. In the present study, students with LD and low-achieving students, who presumably were not as skilled in math problem solving as the average-achieving students, used metacognition but their verbalizations were primarily nonproductive. That is, their metacognitive behavior did little to enhance their problem solving. They may not have developed the self-regulation strategies, or if they had, they may not have had the requisite problem-solving processes and skills in place. To illustrate, Figure 3 presents two think-aloud protocols of students with LD. The first student did not make any metacognitive verbalizations and answered the problem correctly, whereas 33%

of the second student's verbalizations were metacognitive and she answered the problem incorrectly. It is evident that the first student was operating at the automatic level and had the necessary skills in place to solve the problem, whereas the second student did not. There is obviously variability within the ability groups with respect to problem-solving ability and metacognitive functioning. Both need to be addressed to ensure that students acquire the necessary math problem-solving processes, skills, and strategies for successful problem solving.

Limitations

There were two main limitations in the present study that should be considered in future research. First, using think-aloud protocols as a means of data collection rests on the assumption that the student is capable of thinking out loud while engaged in task completion. Inclusion criteria included the ability to think out loud during the training session. Additionally, students who paused for more than 5 seconds three or more times on an individual problem or needed prompting to think out loud more than three times without success during training were excluded. However, it was still possible that students were engaged in metacognitive activity that was not verbalized and therefore was not accessible to the researcher. Second, much of the research using think-aloud methodology utilizes small sample sizes (e.g., Bannert & Mengelkamp, 2008; Swanson, 1990). Thus, while our groups were comparable to or larger than sample sizes in similar studies, by traditional quantitative standards the groups were small. A larger sample would help to describe characteristics of students within groups to generate a more comprehensive profile of students with differing abilities, particularly those with LD. Finally, there was wide variability within groups on all measures of metacognitive activity. Future research might consider more extensive participant selection to reduce this heterogeneity within groups and provide a better opportunity to detect differences.

Educational Implications and Future Directions

In spite of these limitations, there are some clear implications for instruction. First, the think-aloud procedure may be useful for teachers interested in determining specific areas of weakness in students' processing skills, different types of errors, and strategy use during problem solving. Think-aloud data can provide information inaccessible through paper-and-pencil performance measures. The information can then be used to create processing "profiles" of students, and instruction based on specific deficits can be differentiated to accommodate an individual student's needs. For example, students with productive metacognitive strategies but ineffective utilization due to cognitive limitations would require an instructional focus built around concept and skill development, whereas students who

Participant #: 20443   Ability: LD   Time: 45 seconds   Solved: Correct

R
Bob and Shirley are arranging the chairs for a class play. They brought two fifty two chairs from the

Storeroom to the auditorium. Their teacher told them to make rows of twelve each. How many rows
      H   *C*
Will they have?. So you twelve divided by two hundred and fifty two. And then you start with two, so
                                      A
you can't. Twenty-five, so that's like two and then twelve divided by twelve is one...twenty-one.

Cognitive: 100%   Meta: 0%   Prometa: 0 %   Nonprometa: 0%

Participant #: 20501   Ability: LD   Time: 171 seconds   Solved: Incorrect

R
Bill and Shirley are arranging the chairs for a class play. They brought two hundred and fifty-two chairs

From the storeroom to the auditorium. Their teacher told them to make rows of twelve chairs each.
           SI   *Pr*
How many rows will they have? First, I'm gonna underline the important details, which is Billy and

Shirley are arranging the chairs for a class play. They brough two hundred and fifty two chairs from the

Storeroom...and their teacher told them to make rows of twelve chairs each. And the questions is how
          H   *C*
many rows will they have. So I'm going to divide two hundred and fifty two by twelve. [7sec]. So first
        SC
I'm gonna do two goes into two. Wait (student erases work). *Remember to say everything out loud.* So

Twelve divided by two hundred and fifty two. First I'm going to see how many times can twelve...two

Hundred and fifty-two times...which is zero. And twelve times zero is zero. And I subtract that from

Twenty-five, which gives me twenty-five. And then I add the two. So then I see how many times does

Twelve go into twenty-five, two hundred and twenty-five... so two goes into two once. Two divided by

Five is two. Two goes into two one. One divided by two is two. One divided by five is five, and one

Divided by two is two. Then I add them...one, two plus two is four, five plus one is six. Let me see. And
                                    A
my answer is two, and my answer is two um wait. So I would need to have two hundred, two thousand

six hundred and forty-one rows.

Cognitive: 67%   Meta: 33%   Prometa: 33 %   Nonprometa: 0%

Figure 3. Two coded think-aloud protocols of students with learning disabilities.

demonstrate poor metacognitive skills would benefit from complementing their already competent cognitive skill set with explicit instruction in the application of self-questioning, self-instruction, self-correction, and self-monitoring. For example, *Solve It!* includes a SAY, ASK, CHECK procedure as part of the cognitive-metacognitive routine to explicitly teach students how to tell themselves what to do, ask themselves questions, and check themselves as they solve problems (Montague, 2003). In addition to creating student profiles of problem-solving ability, it can be expected that students who have demonstrated ineffective metacognitive strategy use in math problem solving presumably would also have difficulty in other academic contexts. Thus, generalization of metacognitive strategies should be one of the instructional goals.

Second, our study contributes to the relatively small body of research concerning the metacognitive performance of students with LD as they engage in mathematical problem solving. Understanding what students actually do during problem solving rather than simply reporting what they say they do is important to providing students with the necessary remediation to be successful. Third, since metacognition is activated during challenging and novel tasks, this study contributes to the literature by investigating how students adjust their metacognition when faced with increasingly complex math word problems. Finally, metacognitive research has focused predominantly on differences between students with LD and average- or high-achieving students, largely ignoring more subtle differences between students with LD and low-achieving students.

In summary, the present study investigated differences in metacognitive verbalizations of students with and without learning disabilities. The results suggest that students with LD demonstrate a lack of metacognitive control and have limited metacognitive resources when compared to their average-achieving peers. As problems increase in complexity, students with LD tend to use more nonproductive metacognitive strategies and fail to adjust their productive metacognitive strategies. These students may have either a deficiency in productive metacognitive strategies or simply do not use the strategies they have effectively and efficiently. Thus, it is essential that remediation focus on explicitly teaching students metacognitive strategies that will help them be successful when engaged in tasks that require higher-order thinking.

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