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The Effects of Cognitive Strategy Instruction on Knowledge of Math Problem-Solving Processes of Middle School Students With Learning Disabilities

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Abstract

This study investigated the effectiveness of *Solve It!* instruction on students' knowledge of math problem-solving strategies. *Solve It!* is a cognitive strategy intervention designed to improve the math problem solving of middle school students with learning disabilities (LD). Participants included seventh- and eighth-grade students with LD (n = 77) and average-achieving students (n = 77). We examined treatment effects of the intervention, as well as differential effects of treatment across ability levels, on students' knowledge of problem-solving strategies using the Math Problem-Solving Assessment. Results showed that students across ability levels who received *Solve It!* instruction reported using significantly more strategies than students in the comparison group. Implications for instruction are discussed as well as directions for future research.

Keywords

instructional strategies, thinking/cognition, mathematics, learning disabilities

Math problem solving is an increasingly critical skill in today's mathematics curriculum. Success in math problem solving is highly correlated with overall math achievement (Bryant, Bryant, & Hammill, 2000), and the need to develop proficiency in this domain is relevant to students' success in school and beyond. Problem-solving skills span the five curricular content standards and are a means and a goal of learning mathematics (National Council of Teachers of Mathematics, 2000); furthermore, they comprise a skill set that has become central to success in today's workplaces (Hudson & Miller, 2006).

To address the necessity of problem-solving proficiency, major strides have been made to reform the math curriculum from an emphasis on rote skills and procedural knowledge to problem analysis, interpretation, and conceptual understanding (National Council of Teachers of Mathematics, 2000). Pedagogical changes stress student engagement through investigations, multiple representations, and discussion, primarily through problem-solving activities (Goldsmith & Mark, 1999). Yet, despite the increased interest given to math problem solving by researchers and practitioners, students in general, but particularly students with learning disabilities (LD), continue to struggle. Difficulties in working memory and processing speed (Fuchs & Fuchs, 2002), identifying the correct operation and performing the computation (Huinker, 1989; Montague & Applegate, 1993a), higher order reasoning (Maccini & Ruhl, 2001), and the comprehension demands inherent in word problems combine to make math problem solving one of the most challenging parts of the curriculum for this group (Lerner, 2000).

Similar to reading comprehension, math problem solving is a complex skill that requires students not only to calculate an answer but also to comprehend and integrate the problem information, generate and maintain mental images of the problem, and develop a viable solution path (Montague, Warger, & Morgan, 2000). These skills often require high-level thinking and a strategic approach (Hudson & Miller, 2006). Mayer's (1985) model of the problem-solving process identifies four sequential phases: *problem translation* (i.e., utilizing linguistic skills to comprehend what the problem is saying), *problem integration*

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(i.e., mathematically interpreting the relationships among the problem parts to form a structural representation), *solution planning* (i.e., determining which operations to use and the order in which to use them), and *solution execution* (i.e., carrying out the planned computations to solve the problem). Mayer's model illustrates why mathematical word problems are such a struggle for students of all ages; that is, each phase of the problem-solving process is complex, and the correct solution depends on the accuracy of each of the preceding phases (Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007).

Research across academic domains has consistently demonstrated the inability of students with LD to successfully complete academic tasks requiring the use of cognitive and metacognitive skills. Kraai (2011) utilized interview data to determine elementary students' processes during a spelling test and found that the students with LD had difficulty identifying effective strategies to use and had limited ability to monitor, regulate, or correct their performance. The data also revealed inconsistency and ineffectiveness in applying the strategies they did know. Roberts, Torgesen, Boardman, and Scammacca (2008) identified deficiencies in the ability of students with LD to monitor their comprehension on reading passages, and Chalk, Hagan-Burke, and Burke (2005) found similar weaknesses in students with LD during the writing process. Finally, in math problem solving, Montague and Applegate's (1993a) study on middle school students with LD revealed an inability of the participants to accurately solve word problems because they were unaware of effective strategies that would facilitate the task. Furthermore, even with this knowledge, some students seemed to lack the self-regulatory tools necessary to monitor and evaluate the use of those strategies.

In addition to investigating specific deficits in strategy knowledge and use and the complex nature of math problem solving, research has also investigated explicit teaching of cognitive procedures to facilitate math problem solving (Fleischner & Manheimer, 1997; Hutchinson, 1993; Maccini & Hughes, 2000; Montague, Enders, & Dietz, 2011). One approach, cognitive strategy instruction (CSI), has been shown to improve the knowledge and application of effective processes and strategies to increase problem-solving performance (Case, Harris, & Graham, 1992; Montague, 2008; Montague et al., 2011). CSI utilizes components of explicit instruction such as modeling, verbal rehearsal, and scaffolded instruction to help students memorize, apply, and internalize a cognitive routine to improve performance (Harris & Graham, 2009; Krawec & Montague, 2012; Montague & Dietz, 2009; Swanson, 1999). Both knowledge and application of these processes and skills place a high demand on students' metacognitive abilities, which research has shown to be a specific area of deficit for students with LD (Montague & Applegate, 1993b; Roberts et al., 2008; Rosenzweig, Krawec, & Montague, 2011). Thus, CSI emphasizes teaching cognitive processes and metacognitive skills, where students are taught to select and apply them in the context of the task while monitoring their execution (Montague, 2008).

The purpose of this study was to examine the effectiveness of the *Solve It!* (Montague, 2003) cognitive strategy routine to improve the strategic knowledge and, consequently, the math problem solving of middle school students with LD. *Solve It!* is a researcher-developed intervention to improve the problem-solving performance of students with LD by explicitly teaching the cognitive processes and metacognitive strategies that proficient problem solvers use to solve math word problems (Montague et al., 2000). The four phases of Mayer's (1985) problem-solving model (i.e., translation, integration, planning, and execution) provide the framework for the seven cognitive processes emphasized in the routine (see Figure 1). Thus, in the translation phase, students are taught to *read* the problem for understanding and then *paraphrase* the problem by putting the problem into their own words. Next, students *visualize* the problem in the integration phase by creating a representation that depicts the relationships among the problem parts. After a schematically appropriate representation is made, students enter the planning phase, where they *hypothesize* about problem solutions by selecting the appropriate operations/equations needed to solve the problem. Here, students also *estimate* the answer as a means of later confirming the solution outcome. Finally, in the execution phase, students compute the answer following the steps previously determined, and then they *check* the accuracy of their solution, considering the process and the product. Within each of the seven cognitive processes, students are taught metacognitive strategies whereby they give themselves instructions, ask themselves questions, and evaluate their performance.

It is emphasized to students that these self-regulatory tools require reflectivity, thus making the problem-solving process a recursive activity. In other words, although the process is sequential, metacognitive cues may signal students to go back to previous phases to self-correct or reaffirm their progress. In following the CSI method, students introduced to *Solve It!* are required to reach 100% mastery in reciting the seven cognitive processes and their meanings (i.e., read *for understanding*, paraphrase *your own words*, visualize *a picture or a diagram*, hypothesize *a plan to solve the problem*, estimate *predict the answer*, compute *do the arithmetic*, and check *make sure everything is right*). Once mastery is reached, the routine is modeled through think-alouds to demonstrate how the metacognitive strategies support the processes by guiding and regulating performance. Using scaffolded instruction and distributed practice, students become increasingly independent in their application of the routine, ultimately internalizing the processes in a flexible way based on task demands.

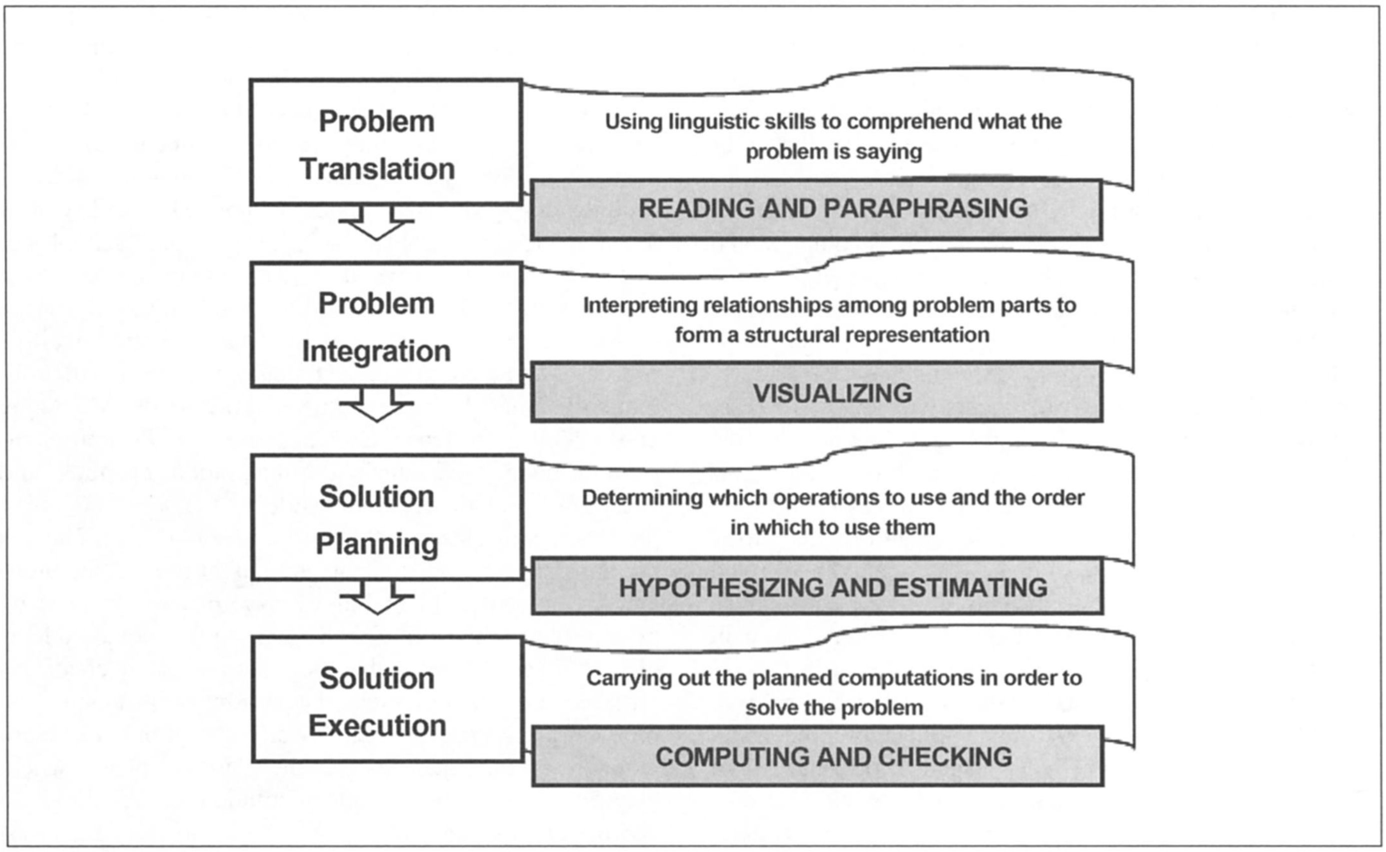

Figure 1. The problem-solving process (Krawec, 2012).
Note. This figure illustrates an integration of the work of Mayer (1985) and Montague (2003).

The effectiveness of *Solve It!* in improving the problem-solving performance of students with LD has been demonstrated through single-subject studies (e.g., Montague, 1992; Montague & Bos, 1986) as well as randomized control trials in inclusive classrooms with teachers delivering instruction (Montague, Enders, & Dietz, 2012; Montague et al., 2011). Results have consistently shown that students with LD increase their problem-solving accuracy following instruction, in some cases commensurate with that of their average-achieving (AA) peers (Montague et al., 2011). Although the primary focus of this 3-year study was to investigate change in students' math problem-solving performance following *Solve It!* instruction, this article specifically focuses on the intervention's effect on students' knowledge of math problem-solving processes. *Solve It!* instruction directly addresses students' knowledge of processes during math problem solving, but previous studies have not analyzed these particular variables, instead focusing solely on problem-solving performance. Thus, built into the design of this study were measures to assess students' development of process knowledge in addition to their problem-solving performance. The focus of this article is on students' process knowledge following *Solve It!* Instruction; therefore, the research questions were the following:

Research Question 1: What are the effects of *Solve It!* instruction on middle school students' knowledge of math problem-solving strategies?

Research Question 2: Are there differential effects of *Solve It!* instruction on students' knowledge of these math problem-solving strategies as a function of ability (i.e., students with LD and AA students)?

Method

Participants

Data analyzed in this study were collected over the course of 2 years with two separate samples: students in Sample 1 were in eighth grade in the 2008–2009 school year and students in Sample 2 were in seventh grade in the 2009–2010 school year. Eligibility criteria and procedures were the same for both samples.

Sample 1. Forty middle schools were initially recruited from the Miami-Dade County Public Schools (M-DCPS) in 2008–2009. M-DCPS is the fourth largest school district in the nation serving approximately 340,000 students (i.e., 9% White, 30% African American, 59% Hispanic, and 2% Other; 60% districtwide qualify for the free/reduced-lunch program). These schools were matched pairs based on the spectrum of state assessment (i.e., *Florida Comprehensive Assessment Test* [FCAT]) performance levels and socioeconomic status (SES). The Florida Department of Education's assigned FCAT school grades (A, B, C, D, or F) and the percentage of students who qualified for free or reduced lunch indicated performance level and school-level SES, respectively. Then, paired schools matched on SES and school grade were randomly assigned to intervention and comparison groups. A general education eighth-grade teacher certified in math and teaching at least two classes, including students with LD from each school, was nominated by an administrator to participate. In all, 24 teachers and their students across 89 classrooms participated in the study.

The teachers who were in the intervention group attended a 3-day *Solve It!* professional development workshop prior to the start of the school year. All students in the inclusion, intensive, general, and prealgebra math class periods had the same chance to participate in the study. Participating teachers taught a range of two to six classes that met study criteria. Participating students signed assent forms and returned consent forms signed by parents or legal guardians that described either the intervention or comparison condition. Teachers in the intervention group provided instruction to all students, but data were collected only from students who returned consent forms. Students with LD were district identified using the following criteria: (a) a deficit in one or more of the basic psychological processes involved in understanding or in using language; (b) academic achievement that differs significantly (i.e., at least 1.5 *SD*) from the student's measured aptitude where at least one of the Wechsler Intelligence Scale for Children (WISC) scale scores is above 85; (c) a learning problem not primarily the result of other disabilities, economic status, or cultural difference; and (d) ineffectiveness of research-based teaching strategies in the general education setting. Furthermore, students in the LD group scored a Level 1 or 2 out of a possible 5 on the previous year's math FCAT. In contrast, AA students had no identified disabilities and had FCAT math levels of 3 or 4. English language learners enrolled as English for Speakers of Other Languages (ESOL) in Levels 1, 2, or 3 were excluded from participation.

Sample 2. The participants in Sample 2 were seventh-grade students in the M-DCPS in 2009–2010. The criteria for screening schools, teachers, and students were the same as those for the Sample 1. Overall, 36 teachers and their students across 111 classrooms participated. Demographic data for all participating students are presented in Table 1.

Table 1. Student Demographic Data.

	Intervention (n = 88)	Comparison (n = 73)
Variable	n (%)	n (%)
Grade		
Seventh	53 (60)	29 (40)
Eighth	35 (40)	44 (60)
Ability level		
AA	46 (52)	37 (51)
LD	42 (48)	36 (49)
Gender		
Male	45 (51)	37 (51)
Female	43 (49)	36 (49)
Ethnicity		
Hispanic	49 (56)	47 (64)
Black	28 (32)	15 (21)
White	9 (10)	8 (11)
Other	2 (02)	3 (04)
Free/reduced lunch		
Yes	70 (80)	41 (56)
No	18 (20)	32 (44)

Note. AA = average achieving; LD = learning disabilities.

Measure

The Math Problem-Solving Assessment (MPSA) is a structured interview that consists of three word problems and 34 items, which were selected from a longer version developed for research purposes (Montague, 1996). Three studies using the original MPSA indicated its discriminant validity by differentiating among students with LD, average achievers, and above-average achievers in mathematics on problem solving and strategy knowledge (Montague & Applegate, 1993a; Montague & Bos, 1990; Montague, Bos, & Doucette, 1991). The six word problems had strong concurrent validity with the Woodcock–Johnson Applied Problems subtest (Woodcock & Johnson, 1977), supporting their validity as a measure of mathematical problem solving. The MPSA measures student perception of math achievement and the importance of math problem solving as well as attitude toward mathematics and the cognitive constructs, that is, students' knowledge, use, and control of the seven problem-solving processes. The MPSA includes three word problems (Steps 1, 2, and 3), 5 Likert-type items, and 29 open-ended items. For the purposes of this study, only data derived from 23 of the open-ended questions were analyzed. These 23 questions follow the seven cognitive processes outlined in *Solve It!* (i.e., read, paraphrase, visualize, hypothesize, estimate, compute, and check), with several questions linked to each of the processes. For example, to probe students' understanding of hypothesizing, students are asked the following questions: "How do you make a plan to solve math word problems? How do you know which operations to use? How do you decide how many steps are needed to solve a math word problem?" It should be noted that the MPSA was designed to be an informal measure of students' knowledge, use, and control of problem-solving strategies and, thus, was evaluated using a researcher-developed coding system. For clarity, a completed MPSA protocol is included in the appendix.

Coding. The coding system for the MPSA utilized responses from the 23 questions assessing students' knowledge, use, and control of the seven cognitive processes. Students' responses to these questions were scored using a dichotomous scale: productive strategies were scored as 1 and nonproductive strategies were scored as 0. If a student reported the use of multiple strategies for one process, each strategy was scored individually. Not all students in the two samples were included in the analyses: 100% of the students with LD who had completed pretest and posttest MPSAs were included; then, from the entirety of AA students in the two samples who completed pretest and posttest MPSAs, 77 were randomly selected to be included in the analyses.

A senior research assistant and a graduate assistant established interscorer agreement by independently scoring and then comparing five of the protocols, resolving all disagreements. After agreement was established on all discrepancies, the senior research assistant scored all protocols independently and the graduate assistant scored 20% of them. We calculated interscorer agreement by dividing the agreements by the agreements plus disagreements and multiplying by 100. In the first sample, the agreement on pretest was 83.7%, with a range of 71% to 95%. The posttest scoring agreement was 82.4%, with a range of 57% to 100%. In the second sample, pretest agreement was 81.2% (range = 62%–100%) and posttest scoring agreement was 83.6% (range = 71%–95%). Eighty-percent agreement is typically considered adequate (Kennedy, 2005).

Procedures

Intervention. The *Solve It!* instructional manual (Montague, 2003) includes scripted lessons and an instructional guide for the teacher as well as class materials such as class charts and student cue cards. Practice problems and outlined solution paths are also included. Teachers participating in the intervention attended a 3-day professional development workshop in August prior to the start of school, which provided comprehensive training on the program. They were given an overview of CSI and the *Solve It!* approach, a description of the assessment tools and treatment fidelity checklists, demonstrations and modeling of the three initial *Solve It!* lessons, and practice using instructional procedures (e.g., verbal rehearsal). There was also a half-day breakout session for teachers to practice process modeling (i.e., thinking aloud) while they solved problems. For both samples, the intervention began in October and continued across the school year. Three days of intensive instruction were implemented and then once-weekly 30-min problem-solving practice sessions followed, with word problems aligned to the week's math content standard. Instruction during all other times of the week was delivered as usual. Therefore, students in the intervention group received *Solve It!* instruction over the course of the year by embedding instruction in the district curriculum once weekly for 30 min, following the 3-day initial instruction. In contrast, the comparison teachers did not attend a workshop. They were instructed to proceed with "business as usual" and were asked to focus on word problem solving during at least one class period per week for the duration of the year. All students in both groups were allowed to use calculators during practice and testing sessions.

Treatment fidelity. Research assistants observed teachers implement the initial three lessons of the intervention and the practice sessions. Observation checklists reflected teacher behaviors directly associated with each scripted lesson. Checklists contained 13 to 16 items and were scored as either "yes" or "no," thus indicating whether or not the lesson component was carried out or displayed. Two research assistants observed all treatment teachers during each of the 3 days of intensive instruction. Comparison teachers' weekly problem-solving lessons were observed 6 times over the course of the year. Each of the subsequent weekly practice sessions was observed by at least one research assistant. Verbal feedback was given to treatment teachers regarding implementation following the observation. We averaged the level of treatment fidelity and interrater agreement across the observations for each group separately. Percentages were calculated by dividing the number of agreements by agreements plus disagreements multiplied by 100 (Kazdin, 1982). Fidelity of implementation for the treatment group averaged 90% and interrater agreement averaged 94% for the initial three lessons; for the weekly practice sessions, treatment fidelity averaged 84% and interrater agreement averaged 98%. For the comparison group, fidelity of implementation averaged 2.8% and interrater agreement averaged 99.5% across the six observations conducted in each of the comparison teachers' classrooms.

Assessment. The pretest/posttest MPSA was individually administered to a randomly selected subset of students in each of the ability groups (i.e., LD and AA) in treatment- and comparison groups before and after the intervention. Research assistants administered the measure individually to students in a quiet location in the school. Students responded to five statements (e.g., It is important to be a good math problem solver) using a 5-point scale and then solved the three math word problems. The rest of the MPSA was conducted as a structured interview, where students responded to the 29 open-ended questions. Research assistants recorded student responses verbatim, and nonspecific probes such as "tell me more" and "describe what you mean" were used as needed. Again, all Sample 1 and Sample 2 participants in the LD group who completed pretest and posttest MPSA protocols were included, which amounted to a total of 77. Then, participants in the AA group who had pretest and posttest measures were randomly selected. These data were then analyzed using the coding system described above.

Data Analysis

First, we conducted a series of chi-square analyses on student grade, ethnicity, gender, ability group status, and SES to

Table 2. Means and *SD*s for Pretest and Posttest Measures by Ability Group Status and Condition.

	AA		Students with LD	
	Treatment (n = 46)	Comparison (n = 37)	Treatment (n = 42)	Comparison (n = 36)
Strategy use measures	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Pretest	15.78 (3.88)	15.19 (4.73)	13.33 (3.71)	12.69 (3.48)
Posttest	17.43 (4.04)	14.16 (4.47)	14.95 (3.12)	12.31 (4.12)

Note. AA = average achieving; LD = learning disabilities.

determine the equivalency of the treatment and comparison groups on demographic characteristics. To determine pre-treatment equivalency on pretest strategy use, we conducted a two-way ANCOVA, with condition and ability group status as the between-group variables and included specific demographic variables (i.e., SES, student grade) found to be significant in the chi-square analysis as covariates.

Next, we used a 2 Time (pretest, posttest) × 2 Condition (treatment, comparison) × 2 Ability (students with LD, AA students) ANOVA with repeated measures on time of testing to determine the effectiveness of *Solve It!* instruction on total strategy use scores and whether effectiveness differed by ability. Effect sizes for all results were calculated using Cohen's d (0.2 = small, 0.5 = medium, 0.8 = large; Cohen, 1988).

Results

Table 2 displays the means and standard deviations by ability group and condition for the MPSA measure over time.

Equivalency of Groups

Results of chi-square analyses indicated no statistically significant differences between treatment and comparison groups on ethnicity, gender, and ability (all $ps > .05$). With regard to SES, results revealed that the treatment group had significantly more students receiving free/reduced lunch, $\chi^2(1) = 24.39$, $p < .001$, than comparison students. In addition, there were statistically significant differences between conditions on student grade, $\chi^2(1) = 12.77$, $p < .001$; students in the treatment group were significantly younger than students in the comparison group. Because of the lack of equivalence between conditions on SES and student grade, these two variables were included as covariates in subsequent analysis.

On the pretest strategy use, results indicated no statistically significant differences between condition, $F(1, 145) = 2.51$, $p > .05$. However, there were statistically significant differences between ability groups, $F(1, 145) = 10.42$, $p = .002$, $d = 0.54$, with AA students outperforming students with LD; this finding was expected. The interaction effect between ability and condition was not statistically significant, $F(1, 145) = 1.10$, $p > .05$. SES, $F(1, 145) = 0.35$, $p >.05$, and student grade, $F(1, 145) = 0.20$, $p > .05$, were not statistically significant.

Treatment Effects on Strategy Use

Results of the repeated measures ANOVA on strategy use scores indicated statistically significant main effects for condition, $F(1, 157) = 10.75$, $p = .001$, $d = 0.52$, and ability level status, $F(1, 157) = 18.11$, $p < .001$, $d = .68$. Students who received the intervention reported using significantly more strategies than students in the comparison group; AA students reported using significantly more strategies than students with LD. However, the main effect for time was not statistically significant, $F(1, 157) = 2.12$, $p = .147$. The interaction between time and condition was statistically significant, $F(1, 157) = 13.54$, $p < .001$, $d = 0.39$. An analysis of simple effects indicated that students in the treatment group improved significantly from pretest to posttest on their reported strategy use ($p < .001$) but the comparison group did not ($p = .130$; see Figure 2). Furthermore, before *Solve It!* instruction, there were no significant differences between treatment conditions ($p = .325$), but after *Solve It!* instruction, students in the treatment group reported using significantly more strategies ($p < .001$). None of the other interactions were statistically significant: condition by ability level status, $F(1, 157) = .07$, $p = .791$; time by ability, $F(1, 157) = 0.23$, $p = .635$; and time by condition by ability level status, $F(1, 157) = .28$, $p = .599$.

Discussion

The present study investigated the efficacy of the *Solve It!* intervention on students' reported strategy use while solving mathematical word problems. The study also examined specific differences in strategy use for AA students and students with LD and whether *Solve It!* differentially improved students' strategy knowledge by ability. Our findings of students' reported strategy use during math problem solving as measured by the MPSA showed that treatment students outperformed comparison students on the strategies reported from pretest to posttest. That is, students who received *Solve It!* instruction reported using more strategies to solve mathematical word problems than students in the comparison group after the intervention.

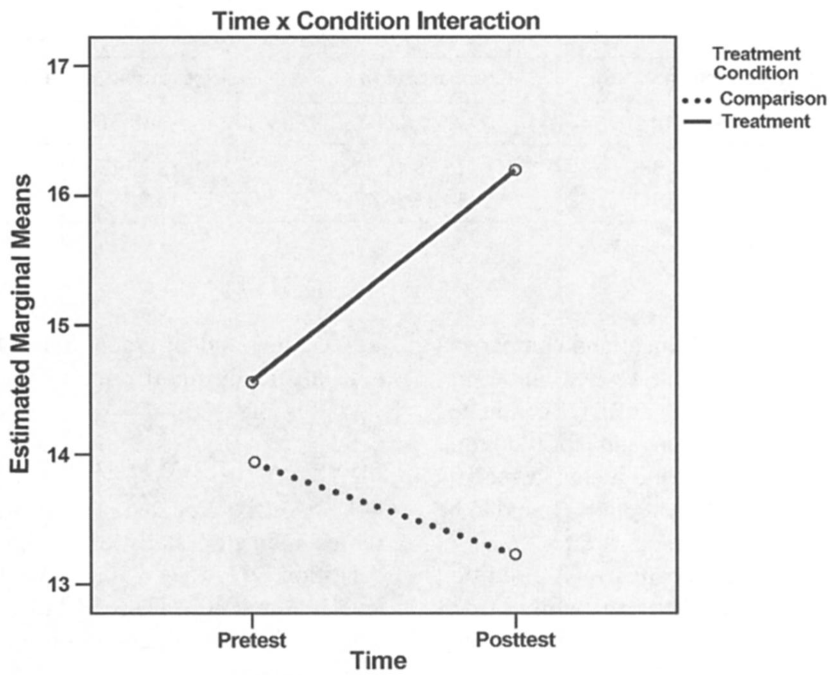

Figure 2. Pretest and posttest strategy use performance of treatment and comparison conditions.

Furthermore, our results showed that *Solve It!* was equally effective for students regardless of ability level (AA students and students with LD).

Previous studies have validated the effectiveness of the *Solve It!* intervention in improving the problem-solving performance of students with LD (e.g., Montague, Applegate, & Marquard, 1993; Montague et al., 2011, 2012). Our finding that students receiving *Solve It!* intervention outperformed control students on reported strategy use regardless of ability level, with a medium effect size of 0.52, is in agreement with these previous studies that emphasized solution accuracy. The present finding, with its emphasis on strategy use, adds to the understanding of *why* the intervention may be effective. The core of the *Solve It!* routine is strategy instruction, whereby students learn specific strategies, as well as when and how to use them. However, because *Solve It!* is a comprehensive instructional routine, there are several components that may contribute to its effectiveness. The results of the present study highlight the critical role of strategy knowledge in students' problem-solving proficiency; that is, *Solve It!* appears to improve students' problem-solving accuracy in part by increasing their repertoire of effective strategies, providing them with the means to successfully solve word problems.

In addition to the finding that students in the treatment group reported using significantly more strategies than those in the comparison group, results showed no significant interaction between condition and ability. The intervention effect was not mediated by ability level, suggesting that it was equally effective for students with LD and AA students. Interestingly, an analysis of the posttest means revealed that students with LD in the treatment group increased their strategy knowledge to a level slightly above that of the AA students in the comparison group (M = 14.95, SD = 3.12, and M = 14.16, SD = 4.47, respectively). Although the intervention did not differentially improve strategy knowledge of students with LD over that of their AA peers who also received the intervention, that their knowledge was raised to a level commensurate with that of the AA comparison group provides support for the benefits of the intervention. The middle school curriculum assumes basic competency in computational skills and so focuses on higher order skills and concepts, making middle school math classes particularly difficult for students with LD (Bryant, Kim, Hartman, & Bryant, 2006). Previous research has identified these years as pivotal for students with LD because the achievement gap between them and their AA peers often widens in math (Kavale & Reese, 1992); this may be due in part to the shift in curricular focus. An intervention that does not close the gap between ability groups but one that improves the skill level of all students by explicitly teaching higher order concepts and skills is noteworthy. At the same time, even though strategy knowledge rather than problem-solving accuracy was of interest in this study, strategy knowledge is possibly a critical prerequisite to eventual problem-solving success.

Limitations

Although the results of the present study contribute to the literature on improving the problem-solving skills of students with LD, there are some important limitations that should be considered. First, there is an important distinction between one's knowledge of effective strategies and one's actual application of those strategies during a task; it is an inherent limitation of measuring students' reported strategy use. Thus, it is not known whether students actually utilized those strategies reported while they solved word problems. According to Roberts and colleagues (2008), students with LD struggle with cognitive and metacognitive strategies for one of two reasons: either they are unaware of effective strategies to use or they know of the strategies but do not actively utilize them during task execution. Knowing the effective strategies to use is an obvious prerequisite to using them, and results suggest that *Solve It!* addressed this skill although the study did not measure students' actual use of strategies while solving problems. However, based on students' overall improvement in math problem-solving performance and the important relationship between knowledge and control, it can be inferred that this knowledge indeed transferred to students' problem solving.

Second, it should be noted that although the comparison teachers participating in the study were required to teach a problem-solving lesson once a week for the duration of the study, no data were collected on their actual time spent on the skill. Each comparison teacher was observed for six math problem-solving lessons over the course of the school year and an analysis of those records shows that the primary instructional approach utilized was worked example (i.e., the teacher demonstrated a problem on the board and then let it remain for students to use as a model as they solved similarly structured problems). This method lacks the strategic emphasis foundational to *Solve It!*, but these differences in instructional approach cannot eliminate the possibility that simply time spent problem solving, and not the intervention, improved students' strategy knowledge.

Finally, the study did not qualitatively distinguish between types of strategies reported. All strategies that were scored as 1 were considered productive, but it is known that some strategies are more complex/effective than others (Gersten & Chard, 1999). For example, students who reported using "key words" to remember the problem content received the same score as those who stated that they "put the important information into my own words." Although using key words is a commonly taught strategy, research has demonstrated that using key words encourages a superficial understanding of the problem and also may lead students to select the wrong operation (e.g., "more" may require subtraction; Hudson & Miller, 2006). Thus, the use of key words is a less effective strategy than paraphrasing the important information, and aggregating strategies with qualitative gradations may have masked actual group differences. Although the effectiveness of *Solve It!* was evident through the quantitative analysis conducted in this study, analyzing students' reported strategies qualitatively would further strengthen our understanding of students' knowledge of effective strategies.

Implications for Practice and Future Directions

Findings from this study have several instructional implications for practice. First, the success of *Solve It!* instruction is founded on effective cognitive and metacognitive processes and strategies for math problem solving, and it provides students with a research-validated problem-solving routine, which has demonstrated results. It teaches students the processes and strategies needed to represent mathematical word problems and how to apply those processes and strategies when solving problems. Results of this study suggest that *Solve It!* enhances the strategy knowledge of students across ability levels. Policies at the state and national levels are working to increase the number of students with disabilities receiving instruction in general education classrooms (e.g., Individuals With Disabilities Education Act, 2004). Furthermore, unlike the way reading instruction is typically structured, few math classes place students in flexible skill-based groups and then differentiate instruction accordingly (Fleischner & Manheimer, 1997; Fuchs, Fuchs, Schatschneider, Hollenbeck, & Hamlett, 2008). As such, comprehensive interventions that improve student performance across ability levels support the feasibility of providing effective instruction to academically diverse students. Instructional characteristics imbedded in the *Solve It!* program, such as varied levels of scaffolding, data-based decision making, and mixed groupings for instruction make it well suited to classrooms with diverse ability levels.

Future research is needed to address more specifically the differences in strategy knowledge among students of varying abilities. Although the present study identified differences in the number of strategies students reported, directions for future research include looking more closely at the productive strategies reported and paying particular attention to strategy complexity and effectiveness to identify patterns, and perhaps specific deficits, among ability groups. In addition, future research is needed to shed light on exactly how students utilize these strategies while actively solving problems. Knowledge without application is of no benefit to problem-solving accuracy as both must be present and active. According to Schmitt and Sha (2009), a student "who knows the effectiveness of certain strategies may not actually use them due to . . . any number of reasons" (p. 256). Think-aloud research has been shown to best access students' cognitive processing during a task (e.g., Lau, 2006; Montague & Applegate, 1993b; Rosenzweig et al., 2011; Schellings & Broekkamp, 2011); thus, recording students verbalizing their thinking while solving math words problems will provide a direct measure of their knowledge *and use* of effective strategies. Finally, future research on the relationships among students' strategy knowledge, actual strategy use, and problem-solving accuracy is warranted to further our understanding of the problem-solving process and help to optimize instruction.

Appendix

SOLVE IT!

Math Problem Solving Assessment

Name ______________________________ Date ____________________

Grade _____________ Start Time: _____________ End Time: ______________

Part A

Directions

1. Read the three problems to the students as examples of word problems.
2. Read the interview questions.
3. Write the student's response to each item in the space provided beneath the descriptors. If the response is unclear or seems incomplete on the open-ended items (5,7,8,9,10), probe for additional information using nonspecific probes (e.g., Tell me more. Describe what you mean. Give an example. Anything else? What else do you do? Please explain that.)

Script

Examiner: Say, "Here are three examples of math word problems. [Show the problems in student activity sheets that follow.] I will read them to you. You do not need to solve them." Read the mathematical word problems:

- Bill and Shirley are arranging the chairs for a class play. They brought 252 chairs from the storeroom to the auditorium. Their teacher told them to make rows of 12 chairs each. How many rows will they have?
- Four friends have decided they want to go to the movies on Saturday. Tickets are $2.75 each for students. Altogether they have $8.40. How much more money do they need?
- Chain sells for $1.23 a foot. How much will Farmer Jones have to spend for chain in order to enclose a 70 foot by 30 foot patch of ground, leaving a 4 foot entrance in the middle of each of the 30 foot sides?

Say, "Now I would like you to answer the following questions. I will write your answers."

PERCEPTION OF MATH PERFORMANCE

1) Describe your math skills.

____very poor______ poor _______ average ______ good_______ very good

2) Describe your math grades.

____very poor______ poor _______ average ______ good_______ very good

3) Describe how well you solve math word problems.

____very poor______ poor _______ average ______ good_______ very good

(x÷=−+) (x÷=−+) (x÷=−+) (x÷=−+)

(continued)

Appendix (continued)

SOLVE IT!

Math Problem Solving Assessment (cont.)

ATTITUDE TOWARD MATH

4) Do you like math?

___Not at all____¼ of the time_____½ of the time ____¾ of the time ____Always

5) Why or why not?

6) Do you like to solve math word problems?

___Not at all____¼ of the time_____½ of the time ____¾ of the time ____Always

7) Why or why not?

KNOWLEDGE OF MATH PROBLEM SOLVING STRATEGIES

8) Tell me what you remember about being taught how to solve math word problems?

9) What do you do to solve math word problems like the examples I showed you?

10) A strategy is a general plan or a specific activity people use to solve problems. Tell me about any strategies you use to solve math word problems. (List all strategies suggested by the student.)

Part B

This section has two parts:

- The student completes three word problems, one at a time.
- The examiner poses questions to the student.

Part B-1

Examiner: Show the word problems to the student (see attached student activity sheet). Say, "Now I would like you to solve the problems. If you have trouble reading or understanding words, ask me for help. Tell me when you finish the problem."

[Give the problems to the student one at a time. When the student has finished all of the problems, place the problems in front of the student for reference.]

Part B-2

[Begin questioning. Write the student's response to each item.]

11) As you read, how do you help yourself understand math story problems? What else do you do when you read math story problems?

12) How many times do you read math word problems?

13) If you do not understand something about the problem, what do you do?

(x÷=−+) (x÷=−+) (x÷=−+) (x÷=−+)

(continued)

Appendix (continued)

SOLVE IT!

Math Problem Solving Assessment (cont.)

14) When you are finished reading a math word problem, what questions do you ask yourself before, during, and after you solve the problem?
15) How do you help yourself remember what the problem says?
16) Do you put what you read into your own words? If so, how do you do this? Now I would like you to put problem #3 into your own words.
17) When you put the problem into your own words, how do you know what you said is correct?
18) What do you do to make a picture in your mind? Is there anything else you do when you visualize?
19) Do you ever make a drawing of the problem or see a picture of the problem in your mind? [Have the student clarify by using the following probes: What kind of picture? How often do you use drawings or pictures? When do you make drawings of problems? Under what conditions do you make drawings or see pictures in your mind? Which problems?]
20) Draw a picture of problem #3.
21) How do your pictures help you solve math word problems?
22) How do you make a plan to solve math word problems?
23) How do you use your plan to help you solve math word problems?
24) How do you know which operations to use (such as adding, subtracting, multiplying, and dividing)?
25) How do you decide how many steps are needed to solve a math word problem?
26) What is estimation?
27) Estimation is making a prediction about the answer using the information in the problem. How does estimation help in solving math word problems?
28) How do you estimate, imagine, or predict the answer before you complete the operations for a math word problem?
29) How do you compare your estimate to your answer?
30) What do you do when you compute answers to word problems? What goes on in your head while you are computing?
31) How do you know your computation is correct?
32) What is checking?
33) How do you check that you have correctly completed a math word problem?
34) Examiner: Say, "I have one more question for you. Now that you have thought about what you do when you solve math word problems, tell me about the problem-solving strategies you use when you solve math word problems."

(x÷=−+) (x÷=−+) (x÷=−+) (x÷=−+)

Authors' Note

Dr. Marjorie Montague passed away on May 13, 2012. She is sincerely missed but will be remembered fondly for her important contributions to the field of special education.

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