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The Transition From Informal to Formal Mathematical Knowledge: Mediation by Numeral Knowledge

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The purpose of the present study was to determine if numeral knowledge—the ability to identify Arabic numerals and connect Arabic numerals to their respective quantities—mediates the relation between informal and formal mathematical knowledge. A total of 206 3- to 5-year-old preschool children were assessed on 6 informal mathematics tasks and 2 numeral knowledge tasks. A year later, these children were assessed on 2 measures of formal mathematical knowledge, namely, the Woodcock-Johnson III Calculation Subtest and a formal number combinations task. Mediation analyses revealed that the relation between informal and formal mathematical knowledge is fully mediated by numeral knowledge, but only when both the skill of numeral identification and an understanding of numeral to quantity relations are considered.

Keywords: mathematics, preschool, learning trajectory, informal, formal

Mathematical knowledge is a critical aspect of early development (Baroody, Lai, & Mix, 2006; Jordan, Hanich, & Uberti, 2003). It provides a foundation for other academic abilities, as indicated by the strong predictive relation between early mathematics achievement and a broad range of later academic abilities (Duncan et al., 2007; Geary, 1994; Jordan, Kaplan, Ramineni, & Locuniak, 2009; National Mathematics Advisory Panel, 2008). Unfortunately, children who fall behind their peers early in mathematics usually continue to develop at a slower rate than more advanced peers and are likely to remain behind them (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). The cumulative nature of early mathematics development—later competencies building on earlier ones—underscores the need to identify key early concepts and skills, determine how they develop, and elucidate their developmental relations. By understanding how these key mathematical skills and concepts are developmentally interrelated, effective classroom curricula and interventions can be constructed that might aid in reducing, or even eliminating, later mathematics difficulties.

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Early Mathematics Learning Trajectories

The meaningful development of mathematical knowledge stems from constructing a systematic and well-interconnected web of mathematical concepts and skills (Baroody, 2003; Gersten & Chard, 1999; National Mathematics Advisory Panel, 2008). By connecting new information to previously learned knowledge, children are able to develop deep and flexible mathematical understanding (Hatano, 2003; James, 1958; Piaget, 1964). This often entails learning mathematical concepts and skills in an empirically delineated sequence. Such a sequence of the development of mathematical concepts and skills is called a learning trajectory (Clements, 2007; Clements & Sarama, 2004; Gravemeijer, 2002). Although significant work has been conducted to assess the developmental sequence of individual skills or concepts (Fuson, 1988; Gelman & Gallistel, 1978) and to construct learning trajectories of early mathematical knowledge (Clements & Sarama, 2009; Sarama & Clements, 2009), prior work has been primarily focused on the development within specific constructs (e.g., how children's counting ability develops across time) or on relating individual early skills to later skills (Aunio & Niemivirta, 2010; Clarke & Shinn, 2004; Martinez, Missall, Graney, Aricak, & Clarke, 2008; Muldoon, Lewis, & Freeman, 2009). What is needed is an empirically validated structure of how mathematics skills or concepts are related to broader transitions or phases of mathematical development.

A Key Transition in Early Mathematical Knowledge

The transition from informal everyday mathematical knowledge to formal school-taught mathematical knowledge is a particularly important juncture in mathematical development (Ginsburg, 1975; Greenes, Ginsburg, & Balfanz, 2004; Starkey, Klein, & Wakeley, 2004). This transition begins in preschool and kindergarten as children begin to learn the written numbering system and leads to the eventual use of mathematics as a vehicle for acquiring and expressing knowledge in other domains such as science and engineering (Basista & Matthews, 2002). Discussed in turn are definitions of informal and formal mathematics, why this transition is critically important, how numeral knowledge develops, the role such development plays in formal mathematical development, and the rationale for the present study.

Informal Mathematics

Informal mathematical knowledge is composed of those competencies generally learned before or outside of school, often in spontaneous but meaningful everyday situations including play, and is characterized by the use of nonconventional and even self-invented symbols, strategies, or procedures rather than conventional written symbols or algorithms (Ginsburg, 1977). Although aspects of informal mathematical knowledge often do not require specific school-based instruction, these skills are malleable and can be enhanced through appropriate and targeted instruction (Arnold, Fisher, Doctoroff, & Dobbs, 2002; Baroody, Eiland, & Thompson, 2009; Clements & Sarama, 2007; Frye et al., 2013; Siegler & Ramani, 2008, 2009).

The central aspects of informal mathematics in the domain of number and operations are flexibly connecting quantities to number words and understanding the relations among these quantities. It has been hypothesized that children go through three overlapping levels of informal mathematics development (Krajewski, 2008; Krajewski & Schneider, 2009): Level 1 = *foundational skills*—distinguishing among quantities (comparison of sets of quantities) and learning to recite the verbal number word sequence (verbal counting) are foundational skills that develop separately; Level 2 = *meaningful numbering skills*—applying the count sequence to fixed sets via one-to-one counting and linking specific number words and quantities via cardinal number knowledge and verbal subitizing (understanding and representing that each number word represents a distinct quantity; e.g., "three" indicates ●●●); and Level 3 = *operations on verbal numbers*—understanding how actions on verbally represented numbers affect them (e.g., recognizing that the outcome of an addition word problem—not involving 0—is larger than either addend). The informal concepts and skills from each of these levels are central to many state and national standards for early mathematics (Common Core State Standards, 2011; National Council of Teachers of Mathematics, 2000, 2006), because they are the developmental precursors to understanding and learning formal mathematics (Bryant, Bryant, Kim, & Gersten, 2006; Chard et al., 2005; Cross, Woods, & Schweingruber, 2009; Frye et al., 2013; Geary, 1994; Ginsburg, Klein, & Starkey, 1998; Griffin & Case, 1997; Jordan et al., 2009; National Mathematics Advisory Panel, 2008).

Formal Mathematics

Formal mathematical knowledge consists of those skills and concepts taught in school and include the use of conventional written numerical notation (e.g., Arabic numerals and operation/equality signs) and written algorithms (e.g., multidigit addition with renaming; Ginsburg, 1977). One of the first—and a particularly important—formal school mathematics skill is fluency with verbally or graphically presented basic number combinations (i.e., addition and subtraction problems presented as a verbal or written expression or equation, such as "Two plus three," "Three and two equals what?" and $2 + 3$ or $3 + 2 = \square$). The Common Core State Standards (2011)—and the curricula that have been aligned with these standards—specify that kindergarteners should be fluent with formal addition and subtraction combinations up to five. Fluency with these formal combinations is one of the first aspects of formal knowledge that children develop—typically beginning at the end of preschool and into kindergarten (National Mathematics Advisory Panel, 2008). The National Mathematics Advisory Panel (2008) indicated that children's fluency with basic formal number combinations is critical to long-term mathematics achievement. Kindergartners' fluency with formal number combinations is predictive of later mathematics achievement and learning difficulties (Jordan et al., 2009; Jordan & Levine, 2009; Mazzocco & Thompson, 2005). Children's formal mathematics skills have also been shown to be malleable through targeted interventions (Baroody, Eiland, Purpura, & Reid, 2012, in press; Clarke et al., 2011; Fuchs et al., 2009). However, the mechanism by which the transition from informal to formal knowledge occurs is unclear. To design and implement targeted interventions for formal mathematical knowledge, it is necessary to know, not only what the key developmental precursors are, but how the developmental precursors connect in their development to formal mathematical knowledge.

The Relation Between Informal and Formal Mathematics

A sizeable body of research has found that different aspects of informal mathematics are important precursors to the acquisition of formal mathematics. For example, longitudinal work by Aunola et al. (2004) found that counting skills in preschool were predictive of both performance level and growth of mathematics skills through second grade. Similarly, research by Aunio and Niemivirta (2010) found that both early counting skills and quantity comparison skills in kindergarten were predictive of first grade general mathematics. Stock, Desoete, and Royers (2007) found that one-to-one counting was a significant marker of later mathematics difficulties, and VanDerHeyden, Broussard, and Cooley (2006) identified one-to-one counting in preschool as a strong predictor of kindergarten mathematics skills. Other work has found that the connection between number words and specific quantities (e.g., subitizing and cardinal number knowledge) is necessary for the attainment of more advanced mathematics (Kroesbergen, Van Luit, Van Lieshout, Loosbroek, & Van de Rijt, 2009; Palmer & Baroody, 2011; Sarnecka & Carey, 2008). Finally, Jordan, Kaplan, Locuniak, and Ramineni (2007) found that, among other variables, verbal story problems and number combinations at kindergarten accounted for a large amount of variance in predicting children's mathematics skills as late as second grade. One concern with existing research is the assumption that informal skills have a direct effect on the development of formal mathematics skills. However, it is likely that another step in development, connecting informal knowledge to the written symbols, is necessary for the acquisition of formal knowledge because it provides a bridge between informal number and arithmetic knowledge and formal representations and procedures.

The Development of Numeral Knowledge

Although the acquisition of numeral names is a central part of the development of numeral knowledge—and presumably one of the more common "mathematical" activities done in school and home—it is critically important to recognize that the development of numeral knowledge extends beyond simply identifying or naming Arabic numerals. This development also involves connecting the written symbols to distinct quantities (e.g., the numeral "4" is a way to represent the quantity ●●●●). Children's ability to identify written numerals and to connect written numerals with number words and quantities have been found to be strong, if not the strongest, predictors of later formal mathematics ability (Bryant et al., 2006; Clarke & Shinn, 2004; Chard et al., 2005; Griffin, Case, & Siegler, 1994; Lembke & Foegen, 2006; Lembke & Foegen, 2009). However, Baroody and Wilkins (1999) indicated that even though one of the first steps toward the development of formal knowledge is learning how to read and write numerals, numeral knowledge could not be classified as either formal or informal knowledge because it does not strictly meet the definition of either.

The development of aspects of numeral knowledge typically begins shortly after children start to develop aspects of their informal mathematical abilities such as knowledge of the counting sequence and the mapping of quantities onto number words (Krajewski & Schneider, 2009; Sarama & Clements, 2009). Once children understand and recognize that numerals are distinct representations from other symbols (e.g., numerals and letters are different), they are able to begin to connect numeral names with the written symbols. Approximately one-quarter of children can identify the numerals 1 to 9 by the time they turn 4 years old (Ginsburg & Baroody, 2003), and some children even begin to identify the first numerals (e.g., 1 and 2) when they are as young as 18 months (Fuson, 1988; Mix, 2009; Sarama & Clements, 2009).

Theoretical Role of Numeral Knowledge in Formal Mathematics Development

Both informal knowledge (connecting number words and quantities) and numeral knowledge (connecting number words and quantities to written symbols) have been shown to be independently related to formal mathematics knowledge (Aunola et al., 2004; Bryant et al., 2006; Clarke & Shinn, 2004). However, some research has indicated that numeral knowledge acts as a ceiling on general mathematical knowledge, preventing children from completing mathematical tasks above their level of numeral knowledge (Sinclair, Siegrist, & Sinclair, 1983). It also has been found that many children with mathematics disabilities tend to have specific difficulties with the symbolic numeral system (Rousselle & Noel, 2007), rather than with informal knowledge (Song & Ginsburg, 1987) or with other cognitive domains (Butterworth & Reigosa, 2007)—indicating that a deficit in an aspect of numeral knowledge development (or deficits in both aspects of numeral knowledge) may inhibit children's successful acquisition of formal mathematics. As such, numeral knowledge may act as a mediator in the development from informal to formal mathematical knowledge.

Rationale for the Present Study

Both informal knowledge and numeral knowledge have been found to be strong predictors of later mathematics achievement and appear to play key roles in the development of formal mathematics. Baroody and Ginsburg (1990) suggested that children who fail to successfully make this transition are at significant risk for later learning difficulties, even if their informal mathematical knowledge had been developing typically. However, the means by which children's informal knowledge and numeral knowledge contribute to the development of formal mathematics is not entirely clear. Thus, the goal of the present study was to determine if informal knowledge directly contributes to the development of formal knowledge or if this relation is mediated in some fashion by numeral knowledge. Further, a secondary goal was to determine the nature of such mediation by testing to see whether (a) the mapping of number word names (which themselves have already been connected to quantities) onto the symbols alone was sufficient to make the connection between informal and formal mathematics, (b) the connection between the quantities (which have already connected with the number words) and the symbols is sufficient to make the connection between informal and formal skills, or (c) both are necessary to make the connection between informal and formal mathematics. In aligning with the theoretical development presented earlier, it was hypothesized that numeral knowledge would fully mediate the relation between informal and formal mathematics and that the combination of both of the numeral knowledge components would be necessary to achieve full mediation.

Method

Participants

In the first year of this study, data were collected from 393 preschool children in 44 public and private preschools serving children from families of low to middle socioeconomic status living in Northern Florida. In the second year of the study, 206 of the original children were tested again. Of these children, 112 had moved on to kindergarten and attended 28 different public or private elementary schools in two counties. The other 94 children moved on to their second year in preschool at 20 different public (Head Start) or private preschools in two counties. The 206 children who completed the assessments at both time points were evenly split by sex (51.9% female) and approximately representative of the demographics of Northern Florida (60.2% Caucasian, 28.2% African American, and 11.6% other race/ethnicity). At Time 1, children ranged in age from 3.18 years to 5.88 years (M = 4.66 years, SD = 0.69 years). The children were primarily English speaking and had no known developmental disorders. The children who completed both testing points were not significantly different on any of the Time 1 mathematics variables than the children who did not complete the Time 2 assessment. Parental consent was obtained for each participating child.

Materials

Children were assessed on informal, numeral, and formal knowledge tasks. These tasks were assessed as a part of a larger battery of tests that took approximately three 20- to 30-min sessions at each time point. All tasks (except an achievement test that was one of the two tasks used to assess formal knowledge) were developed as part of a broader measure (Purpura, 2010). Items on each of the tasks from the broader measure were derived by a process using item response theory that ensured that each item was related to its intended construct (e.g., set comparison), had adequate discrimination (*a* parameter), and did not duplicate the difficulty level (*b* parameter) of other items on the same task. Raw total scores were used for each measure.

Tasks Administered at Time 1

Informal mathematics tasks. Six tasks served to measure informal knowledge.

Verbal counting. Children were asked to count as high as possible. When a child made a mistake, or correctly counted to 100 without making a mistake, the task was stopped. Spontaneous self-corrections were not scored as incorrect, and the child was allowed to continue counting. The highest number counted to was converted to a score based on a 7-point scale. Children were awarded one point each for correctly counting to 5, 10, 15, 20, 25, 40, and 100.

One-to-one counting. Children were presented with a set of three, six, 11, 14, or 16 dots on a page and asked to count the set. Children were awarded one point for each set if they correctly counted each dot only once. This task had an internal consistency (Chronbach's alpha) of .79.

Cardinality. This task was assessed in the context of the one-to-one counting task. At the completion of the counting three, six, and 11 one-to-one counting items, children were asked to indicate how many dots there were in all. Children were awarded one point if they restated the last number counted ("how many?"). This task had an internal consistency of .75.

Subitizing. Children were briefly presented (2 s) a set of pictures (set sizes from one to seven presented in a linear fashion; e.g., ●●●●) and instructed to say how many dots or pictures were presented. For each correct response, children were awarded one point. This task had an internal consistency of .69.

Set comparison. For each of the six items, children were presented with four sets of dots on a page representing different quantities (e.g., | ●●● | ●● | ●●●● | ● |). They were then asked which set had the most (three items) or fewest dots (three items). Children received one point for pointing to the correct set. This task had an internal consistency of .77.

Story problems. Children were presented verbally with story problems that did not contain distracters (e.g., irrelevant information). These story problems were simple addition (three items) or subtraction problems (four items) that were appealing to children. For example, one question was, "Johnny had one cookie and his mother gave him one more cookie, how many cookies did he have now?" Children were awarded one point for each correct response. This task had an internal consistency of .71.

Numeral knowledge skill tasks. Two tasks served to measure numeral knowledge.

Numeral identification. Children were presented with flashcards of nine numbers (1, 2, 3, 7, 8, 10, 12, 14, and 18). They were shown the flashcards one at a time and asked, "What number is this?" For each correct response children were awarded one point. This task had an internal consistency of .90.

Set to numerals. On the first three items in this task, children were presented with a numeral at the top of the page (e.g., 3) and five sets of dots below (e.g., | ●●●● | ● | ●●● | ●● | ●●●●● |). They were instructed to identify which of the sets meant the same thing as the number at the top of the page. On the last two items of this task, children were presented with a set of dots at the top of the page (e.g., ●●●●) and five numerals at the bottom (e.g., 4, 2, 3, 1, 5). They were instructed to identify which of the numerals meant the same thing as the set of dots at the top of the page. Children were awarded one point for each correct response. This task had an internal consistency of .80.

Tasks Administered at Time 2

Formal mathematics skills tasks. Two tasks were used to gauge basic formal knowledge. These tasks were selected because they could be used to assess the most basic addition and subtraction combinations. The primary differences between these tasks are in the presentation of the items and the method of response. In the first task, children are both shown and told the problem, for which they give a verbal answer. In the second task, children are just shown the problem, and they are asked to give a written response.

Number combinations. Children were presented with a formal addition problem (e.g., $1 + 1 =$) and asked, "How much is . . . [stated the problem]." There were five total problems: $0 + 2 =$, $1 + 1 =$, $1 + 2 =$, $2 + 2 =$, $1 + 3 =$. For each correct response children were awarded one point. This task had an internal consistency of .77. This measure was also administered in the first year of the study; however, overall performance on this task was low ($M = 1.19$, $SD = 1.50$) suggesting that the majority of children had little to no formal knowledge at Time 1.

Woodcock-Johnson III Calculation subtest (WJ-III Calc). The WJ-III Calc subtest is a paper-and-pencil arithmetic test where children are asked to solve addition and subtraction problems and has been shown to have a median reliability of .92 for children 5–19 years old (Woodcock, McGrew, & Mather, 2001). Children were awarded one point for each correct answer.

Procedure

Assessment procedure. Preschoolers were assessed on the informal mathematics tasks and numeral knowledge tasks in the spring of Year 1. Participants were assessed a year later (spring of Year 2) on their formal knowledge when slightly over half of the children had advanced to kindergarten. Individuals who had either completed or were working toward completion of a bachelor's degree conducted the assessments. The assessors each completed a 2- to 3-hr training on the measures prior to each testing point (Time 1 and Time 2) and completed an extensive testing out process to ensure accuracy of administration. Assessments took place in the local preschools or kindergarten classrooms during noninstructional time in a quiet room designated by the individual school directors or teachers.

Analytic procedure. As the primary analytic method was to conduct mediation analyses, data analysis was conducted in five steps based on the recommendations of Baron and Kenny (1986) in conjunction with updated recommendations by Zhao, Lynch, and Chen (2010). The first two steps were analyses of the direct effects of informal mathematical knowledge on (Step 1) numeral knowledge and (Step 2) formal mathematical knowledge. The third step was an analysis of the direct effects of numeral knowledge on formal knowledge when controlling for informal knowledge. The fourth step was an evaluation of the mediation effects of numeral knowledge on the relation between informal and formal mathematical knowledge using the percentile bootstrap approach (recommended by Zhao et al., 2010) rather than the Sobel test (recommended by Baron & Kenny, 1986), because the percentile bootstrap approach is more powerful in detecting mediation effects than the Sobel test (Preacher & Hayes, 2004). The fifth step was a comparison between the baseline model from step four and a simpler model that did not include the direct effects of informal mathematical knowledge on formal mathematical knowledge. To evaluate whether only one of the numeral knowledge variables by itself was sufficient to mediate the relation between informal and formal knowledge, these same steps were repeated two additional times, replacing the numeral knowledge latent factor with the individual numeral knowledge variables. All models in the analyses were logically identified.

Results

Descriptive Statistics

Means, standard deviations, skewness, and kurtosis for all variables are included in Table 1 and are presented by age group (younger children are those children who were still in preschool at Time 2, and older children are those children who were in kindergarten at Time 2) in Table 2. All data are presented as raw scores. The distributions of scores for all variables in this study were normal.[1] Correlations between the mathematics tasks that were assessed are presented in Table 3. No significant gender differences in preschool mathematics scores were found. When analyses were conducted using age-regressed standard scores, the results were comparable to the analyses conducted with raw scores. As such, the results using raw scores are reported because the scores are more interpretable.

Primary Analysis

Step 1: Direct effects of informal mathematical knowledge on numeral knowledge. Analyses of the relation between informal mathematical knowledge and numeral knowledge indicated that informal mathematical knowledge significantly predicted numeral knowledge ($\beta = 0.94$, $p < .001$).

Step 2: Direct effects of informal mathematical knowledge on formal mathematical knowledge. Analyses of the relation between informal mathematical knowledge and formal mathematical knowledge indicated that informal mathematical knowledge significantly predicted formal mathematical knowledge ($\beta = .84$, $p < .001$).

Step 3: Direct effects of numeral knowledge on formal mathematical knowledge. Analysis of the relation between numeral knowledge and formal mathematical knowledge, when controlling for the effects of informal mathematical knowledge, indicated that numeral knowledge significantly predicted formal mathematical knowledge ($\beta = .86$, $p = .038$).

Step 4: Mediation effects of numeral knowledge on the relation between informal and formal mathematical knowledge. Significant mediation effects were determined through use of the percentile bootstrap approach. This method utilizes a random sampling with replacement approach to calculate a sampling of indirect effects. Indirect effects from each sampling were then sorted from low to high and the highest and lowest 2.5% (when using a 95% confidence interval) were removed. Significant mediation effects are present if the confidence interval does not contain 0. Mediation analyses showed significant mediation effects of numeral knowledge on the relation between informal and formal mathematical knowledge. In Figure 1, the mediation model that includes a direct—but nonsignificant—effect of informal mathematical knowledge on formal mathematical knowledge is presented. Overall, the magnitude of the indirect effect of the mediation model (or amount of variance accounted for in formal mathematical knowledge by the indirect effect) was large ($R^2 = .81$) and accounted for 98% of the total variance.

Step 5: Comparison of models to determine full or partial mediation. The model with the direct effect of informal mathematical knowledge on formal mathematical knowledge was then compared to the same model without the direct effect of informal mathematical knowledge on formal mathematical knowledge included. In Table 4, the tests of model fits for both models are presented. Test of chi-square differences indicated no significant difference between the models and all other fit indices were nearly identical between the models. Given that no differences were found between the model fits, the more parsimonious model (the full mediation model—the one with no direct effect of informal knowledge on formal knowledge) was selected as the preferred model. In general, the model fit indices provide evidence that the selected model provides a good fit to the data (Brown, 2006; Hu & Bentler, 1999; Mueller & Hancock, 2010).

Table 1
Means, Standard Deviations, Range, Skewness, and Kurtosis of the Sum Scores of the Mathematics Tasks

Task	*M*	*SD*	Range[a]	Skew	Kurtosis
Testing Time 1					
Informal mathematics					
Verbal counting	3.43	1.83	0–7	.27	−1.01
One-to-one counting	3.33	1.50	0–5	−.41	−0.96
Cardinality	2.18	1.02	0–3	−.96	−0.34
Subitizing	3.86	1.61	0–7	−.26	0.01
Set comparison	3.92	1.89	0–6	−.44	−1.04
Story problems	3.36	1.96	0–7	.11	−0.90
Numeral knowledge					
Number identification	5.35	3.06	0–9	−.36	−1.17
Set to numerals	2.94	1.67	0–5	−.36	−1.10
Testing Time 2					
Formal mathematics					
WJ-III calculation	3.62	3.72	0–16	.87	0.15
Number combinations	2.87	1.96	0–5	−.27	−1.51

Note. $N = 206$. WJ-III = Woodcock-Johnson Tests of Achievement (3rd ed., Woodcock, McGrew, & Mather, 2001).
[a] The range indicates both the possible and actual range of scores for all tasks other than the WJ-III Calculation task. For the latter task, only the actual range is presented.

[1] All children were able to attempt the items on the WJ-III Calculation test. Of the 206 children who participated in the study, 72 obtained a raw score of 0 on the WJ-III Calculation test and 40 obtained a score of 0 on the number combinations task. Importantly, only 27 total children (13%) obtained scores of zero on *both* tasks.

Table 2

Means, Standard Deviations, and Ranges of the Sum Scores of the Mathematics Tasks by Age Group

	Younger children			Older children		
Task	*M*	*SD*	Range[a]	*M*	*SD*	Range[a]
Informal mathematics						
Verbal counting	2.49	1.35	0–7	4.20	1.81	0–7
One-to-one counting	2.81	1.49	0–5	3.75	1.37	0–5
Cardinality	1.78	1.11	0–3	2.51	0.80	0–3
Subitizing	3.33	1.58	0–7	4.30	1.52	0–7
Set comparison	3.12	1.78	0–6	4.58	1.73	0–6
Story problems	2.52	1.66	0–6	4.05	1.93	0–7
Numeral knowledge						
Number identification	4.06	3.01	0–9	6.41	2.68	0–9
Set to numerals	2.05	1.56	0–5	3.66	1.39	0–5
Formal mathematics						
WJ-III calculation	1.13	2.08	0–10	5.67	3.53	0–16
Number combinations	1.65	1.60	0–5	3.88	1.64	0–5

Note. $N = 206$. WJ-III = Woodcock-Johnson Tests of Achievement (3rd ed., Woodcock, McGrew, & Mather, 2001). Younger children $n = 93$, older children $n = 113$.

[a] The range indicates both the possible and actual range of scores for all tasks except the story problems task for the younger children and the WJ-III calculation task for both the younger and older children. For the story problems task, the maximum possible correct was seven. The WJ-III calculation subtest is design for individuals of all ages and thus, the scores presented only represent the actual range attained in this sample.

Mediation by Individual Numeral Knowledge Variables

The same series of mediation analyses were conducted using each of the numeral knowledge variables separately to determine if one aspect of numeral knowledge accounted for the mediation findings.

Step 1: Direct effects of informal mathematical knowledge on numeral knowledge variables. Analyses of the relation between informal mathematical knowledge and performance on the numeral identification task and performance on the set-to-numerals task indicated that informal mathematical knowledge significantly predicted performance on the numeral identification task ($\beta = .74$, $p < .001$) and significantly predicted performance on the set-to-numeral task ($\beta = .76$, $p < .001$).

Step 2: Direct effects of informal mathematical knowledge on formal mathematical knowledge. The results in this step are the same as were reported in the previous Step 2. Analyses of the relation between informal mathematical knowledge and formal mathematical knowledge indicated that knowledge of informal

Table 3

Correlations Between the Sum Scores of All the Mathematical Knowledge Tasks

Variable	1	2	3	4	5	6	7	8	9	10
Testing Time 1										
Informal mathematics										
1. Verbal counting	—									
2. One-to-one counting	.62	—								
3. Cardinality	.55	.72	—							
4. Subitizing	.41	.49	.43	—						
5. Set comparison	.52	.52	.51	.35	—					
6. Story problems	.52	.46	.49	.46	.62	—				
Numeral knowledge										
7. Numeral identification	.55	.54	.55	.45	.55	.57	—			
8. Set to numerals	.60	.59	.54	.48	.58	.53	.65	—		
Testing Time 2										
Formal mathematics										
9. WJ-III calculation	.53	.46	.47	.39	.52	.60	.54	.52	—	
10. Number combinations	.53	.49	.49	.42	.54	.55	.62	.63	.68	—

Note. $N = 206$. WJ-III = Woodcock-Johnson Tests of Achievement (3rd ed., Woodcock, McGrew, & Mather, 2001). All correlations were significant at $p < .01$.

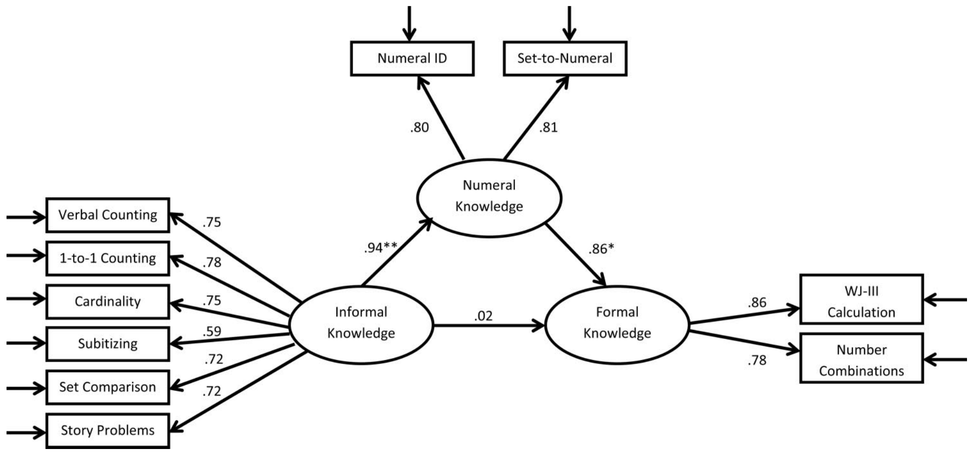

Figure 1. The figure shows the mediation of numeral knowledge in the relation between informal and formal mathematical knowledge. WJ-III = Woodcock-Johnson Tests of Achievement (3rd ed.; Woodcock, McGrew, & Mather, 2001). $^{*}p < .05$. $^{**}p < .01$.

mathematical knowledge significantly predicted formal mathematical knowledge ($\beta = .84$, $p < .001$).

Step 3: Direct effects of individual numeral knowledge variables on formal mathematical knowledge. Analysis of the relation between performance on the numeral identification task and formal mathematical knowledge and between performance on the set-to-numerals task and formal mathematics, when controlling for the effects of informal mathematical knowledge, indicated that performance on the numeral identification task significantly predicted formal mathematical knowledge ($\beta = .20$, $p = .029$) and performance on the set-to-numerals task marginally significantly predicted formal mathematical knowledge ($\beta = .17$, $p = .079$).

Step 4: Mediation effects of individual numeral knowledge variables on the relation between informal and formal mathematical knowledge. Significant mediation effects were determined through use of the percentile bootstrap approach. Mediation analyses showed significant mediation effects of performance on the numeral identification task on the relation between informal and formal mathematical. Overall, the magnitude of the indirect effect of the mediation model was ($R^2 = .15$) and accounted for only 21% of the total variance. Mediation analyses also showed a marginally significant mediation effect of performance on the set-to-numerals task. Overall, the magnitude of the indirect effect of the mediation model was small ($R^2 = .13$) and accounted for only 19% of the total variance. These findings revealed that individually, both performance on the numeral identification task and performance on the set-to-numerals task only partially mediated the relation between informal and formal mathematical knowledge because the direct effect of informal knowledge on formal knowledge remained large and significant (see Figures 2A and 2B). In fact, in both of these analyses, informal knowledge accounted for the majority of the variance in formal knowledge. These findings suggest that the relation between informal and formal mathematical knowledge is fully mediated by numeral knowledge but only when both aspects of numeral knowledge are considered together.

Discussion

The results of this study indicate that the relation between informal and formal mathematical knowledge is fully mediated by children's numeral knowledge. Mapping both number-words and quantities to the written symbols are necessary steps for children to apply their formal mathematics knowledge to formal concepts. Although prior research typically tied informal knowledge directly to the development of formal knowledge (Aunola et al., 2004; Jordan, Kaplan, Locuniak, & Ramineni, 2007; Stock, Desoete, & Royers, 2007), the current findings indicate that there is no direct impact of informal mathematical knowledge on formal mathematical knowledge. Rather, children must map their informal knowledge directly onto numeral knowledge, which then must be

Table 4
Fit Indices for the Mediation Models With and Without the Direct Effect of Informal Mathematical Knowledge on Formal Mathematical Knowledge

Model	χ^2	*df*	CFI	TLI	RMSEA	SRMR	χ^2 dif
With direct effect	84.29	32	.95	.94	.09	.04	—
Without direct effect	84.29	33	.96	.94	.09	.04	0.00 (*ns*)

Note. $N = 206$. CFI = comparative fit index; TLI = Tucker-Lewis index; RMSEA = root-mean-square error of approximation; SRMR = standardized root-mean-square residual. The dash indicates that, because the comparison was made between the two models (e.g., the model without the direct effect was compared to the model with the direct effect), there was only one analytic comparison.

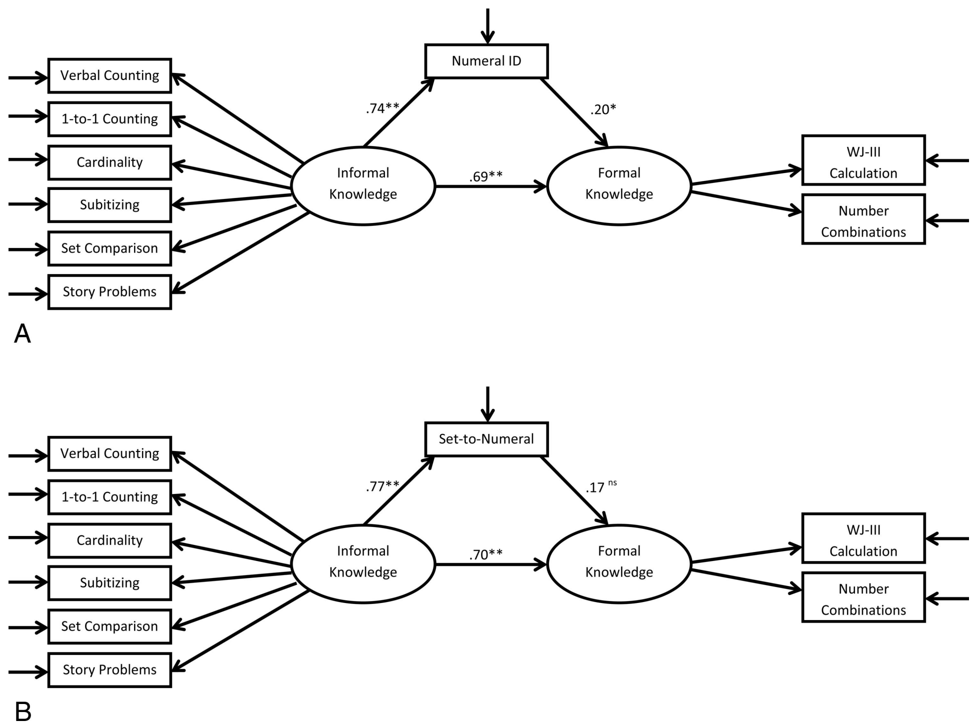

Figure 2. A. The partial mediation of numeral identification on the relation between informal mathematical knowledge and formal mathematical knowledge. B. The partial mediation of set-to-numerals task on the relation between informal mathematical knowledge and formal mathematical knowledge. WJ-III = Woodcock-Johnson Tests of Achievement (3rd ed.; Woodcock, McGrew, & Mather, 2001). $^{*} p < .05$. $^{**} p < .01$.

mapped onto formal knowledge—suggesting that there is a developmental trajectory of these skills/concepts. Further, these results indicate that a depth of numeral knowledge—both the ability to identify numerals and connect numerals to quantities—is necessary to achieve the full mediation. Only partial mediation was found when either the numeral identification task or the set-to-numerals task was included in the model alone. This suggests that both the procedural skill of identifying numerals and the conceptual ability of understanding that each numeral represents a specific quantity appear to represent necessary functions of the numeral knowledge domain—meaning, numeral knowledge may act as a gatekeeper (or barrier) in the development of formal mathematical knowledge.

These findings delineate an important learning trajectory underscoring children's mathematical growth across a critical developmental juncture by building on prior models of informal mathematics development (Krajewski & Schneider, 2009). Notably, these findings support a critical additional step not included in prior models of early mathematics acquisition. The mediational findings also help to explain why numeral knowledge skills are often found to be even more highly correlated with formal mathematics skills than are informal mathematics skills (with *r*s $> .60$; Bryant et al., 2006; Clarke & Shinn, 2004; Lembke & Foegen, 2006, 2009; Purpura & Lonigan, 2013). As children must map informal knowledge onto numeral knowledge and then onto formal knowledge, it appears that numeral knowledge is essentially a necessary precursor for the acquisition of formal mathematical knowledge (Baroody & Wilkins, 1999). The results of this study are not intended to discount the importance of the development of individual informal mathematics skills but, rather, put this development within a broader context because prior research has indicated that individual informal mathematics skills are directly connected to formal mathematical knowledge. For example, Aunola et al. (2004) utilized various measures of counting knowledge to predict later general mathematics knowledge and found large and significant relations between counting skills and general mathematics skills. Such significant effects would be expected given that the different counting skills are *steps* in the development of overall mathematical abilities. However, it is critical to emphasize that development of individual skills are *steps* in the broader acquisition of mathematics and the findings of this study clarify one critical step in the broader context.

Not only do these findings build on prior models of mathematics development and support Baroody and Wilkins' (1999) assertion, the findings also fit the developmental framework and the theoretical structure indicated by theories of meaningful mathematical learning (Baroody, 1987; James, 1958; Piaget, 1964). In such views, it is believed that children must be able to connect each type of new mathematical information to existing knowledge to develop their mathematical competence. For preschool and kindergarten children to develop formal knowledge, they must not only learn the numerical symbols by which the system is structured (a procedural skill) but must actively connect the symbol name information to their informal knowledge of number-words and quantities (a conceptual ability). Essentially, children must develop the connection between number words and quantities, and then connect both the number words and quantities to the written symbols to develop formal knowledge. This supports the growing recognition for the need to integrate procedural and conceptual knowledge (see Baroody, Feil, & Johnson, 2007; Cross et al., 2009; National Mathematics Advisory Panel, 2008). The results may also help to explain Mazzocco and Thompson (2005) findings that a composite of three informal items at kindergarten (cardinality, comparisons of one-digit numbers, and mentally adding one-digit numbers) and one numeral knowledge item (reading numerals) from the TEMA-2 were predictive of which children would be designated mathematically "learning disabled" during both Grades 2 and 3 because the composite included items that tapped informal, formal, and numeral knowledge abilities. Utilizing the developmental framework identified here, may lead to a better and more efficient process for early identification of children at risk of later mathematics difficulties.

Limitations

Two limitations of this study should be noted. First, there was significant attrition across the two time points, primarily due to student mobility. Although these data were assumed to be missing at random, and there were no differences on Time 1 mathematics scores between completers and noncompleters, the level of attrition could have added a level of unknown variation into the findings. Second, this study solely focused on the "exact language-based" number system and does not incorporate measures that assess the approximate (nonverbal) number system (ANS; Dehaene, 1992).

Future Directions

Identifying this developmental sequence of early mathematics skills provides a foundation on which to conduct future research. Specifically, there is a need to expand beyond the broad definitions of "informal" and "formal" mathematics and delineate how the individual informal or formal skills and concepts (e.g., one-to-one counting, comparison, or number combinations, place value) interact in their development to create the web of informal or formal mathematical knowledge. This learning trajectory can also be used to provide information for the development of targeted interventions. By understanding where in the learning trajectory a child's skills are underdeveloped, teachers can provide specific interventions to enhance those skills and hopefully prevent future learning difficulties. Further, delineating such a sequence also would enable teachers to easily identify the next instructional phase for a typically performing or advanced student. A learning trajectory will also enable both teachers and researchers to identify whether children have developed the appropriate developmental prerequisites to benefit from a broader curriculum or specific intervention. For example, teachers may find that children who have not fully developed their understanding of numeral knowledge may not be ready for mathematics interventions involving formal knowledge. Conversely, the may find that a younger child who has developed both informal and numeral knowledge may be ready for more advanced instruction in formal concepts.

The expansion of this learning trajectory beyond the verbal/symbolic mathematics system could be important to developing a broader understanding of early mathematics development. Specifically, identifying the role that the ANS plays—as it is related to informal and formal mathematics development—in early mathematics development may allow for better identification of early mathematics difficulties. The ANS may also play a direct or indirect role in contributing to the development of formal mathematics development (Gilmore, McCarthy, & Spelke, 2007; Libertus, Feigenson, & Halberda, 2011). Prior research has shown that the ANS is correlated with informal and formal mathematics skills, even after controlling for language and intelligence (Libertus et al., 2011); however, these relations were not evaluated controlling for other early mathematics abilities. Thus, it is not clear if the relation between the ANS and formal mathematics is a direct relation or an indirect relation mediated by informal knowledge. Future research should be conducted to evaluate the relation of the ANS to the current learning trajectory to better understand the broader development of children's early mathematical concepts.

An additional direction for future research should be the determination of nonmathematical factors that account for the remaining variance in the developmental model. It is likely that additional variance could be accounted for by including cognitive or behavioral abilities in the model. Prior research has found that working memory (Swanson, 2004; Swanson & Beebe-Frankenberger, 2004; Swanson & Kim, 2007), attention (Fuchs et al., 2005, 2006), and rapid digit naming or processing speed (Cirino, 2011; Krajewski & Schneider, 2009) have an impact on the development of mathematical abilities. Further, children's language and print knowledge skills have also been found to be important factors in mathematics development (Fuchs et al., 2008; Leong & Jerred, 2001; Purpura, Hume, Sims, & Lonigan, 2011). Given that both language and print knowledge have been identified as significant predictors in later reading development (Lonigan, Schatschneider, & Westberg, 2008; Morris et al., 1998; Stanovich, Siegal, & Gottardo, 1997)—and numeral identification is likely to be highly rooted in basic concepts of print knowledge and/or language development (LeFevre et al., 2010)—it is plausible that a child who does not adequately develop print and language skills will also not sufficiently develop their numeral knowledge. The precise stage in the learning trajectory model where cognitive, behavioral, and language/print skills affect development should be identified to determine if such connections plays a role in combined mathematics and reading disorders. Ideally, it should be evaluated whether the impact is primarily found at one developmental level (e.g., informal mathematics), or whether the impact is general to all stages of mathematical development. If one of these specific factors is found to adversely impact the development of mathematics skills at either a specific or general level, it may be prudent to conduct interventions that target that factor (e.g., working memory, attention, language/print knowledge) in conjunction with early mathematics interventions to best improve children's early mathematics skills.

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