Cognition and Instruction

ISSN: 0737-0008 (Print) 1532-690X (Online) Journal homepage: https://www.tandfonline.com/loi/hcgi20

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COGNITION AND INSTRUCTION, *32*(2), 159–197, 2014

ISSN: 0737-0008 print / 1532-690X online
DOI: 10.1080/07370008.2014.887084

Fostering First Graders' Fluency With Basic Subtraction and Larger Addition Combinations Via Computer-Assisted Instruction

Arthur J. Baroody
University of Illinois at Urbana-Champaign

David J. Purpura
Purdue University

Michael D. Eiland
University of Illinois at Urbana-Champaign

Erin E. Reid
Erikson Institute

Achieving fluency with basic subtraction and add-with-8 or -9 combinations is difficult for primary grade children. A 9-month training experiment entailed evaluating the efficacy of software designed to promote such fluency via guided learning of reasoning strategies. Seventy-five eligible first graders were randomly assigned to one of three conditions: guided subtraction (e.g., If $5 + 3 = 8$, then $8 - 3$ is 5), guided use-a-10 (e.g., If $10 + 7 = 17$, then $9 + 7$ is 16), or an unguided-practice condition for 30-minute sessions twice a week for 12 weeks. An ANCOVA revealed that at the delayed posttest, the guided-subtraction group outperformed both comparison groups on unpracticed subtraction combinations. Analyses of gains in slow but appropriate use of reasoning and decreases in inefficient strategy use indicated that both types of guided training promoted the learning of a targeted reasoning strategy.

Efficiently (quickly and accurately) producing the answer of basic addition combinations (sums of two single-digit addends) and related differences from memory has long been a central goal of primary grade mathematics instruction. The *Common Core State Standards* (Council of Chief State School Officers [CCSSO], 2010), for example, specifies as the fourth goal under the grade 2 operations and algebraic thinking domain (2.OA.4): "By end of Grade 2, know from memory all sums of two 1-digit numbers." With the growing interest in the integrated learning of

Correspondence should be addressed to Arthur J. Baroody, Department of Curriculum and Instruction, College of Education, University of Illinois, Champaign, IL 61820. E-mail: baroody@illinois.edu

concepts, facts, and procedures (e.g., Baroody, Feil, & Johnson, 2007; Hiebert, 1986; Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics [NCTM], 2000, 2006; Rittle-Johnson & Schneider, 2014), *fluency* with basic combinations has received increased attention. Fluency entails applying knowledge appropriately and adaptively, as well as efficiently. *Appropriate use* requires selectively applying knowledge to only suitable cases (e.g., recognizing that the number-after rule, which specifies that the sum of 1 and a number n is the number after n in the counting sequence, applies to $3 + 1$ or $1 + 5$, but not $3 + 5$ or $5 + 3$). *Adaptive use* involves flexibly applying or even adjusting knowledge to solve new problems (transfer).

Although fluency with basic sums and differences is critical to mathematical success (CCSSO, 2010; Kilpatrick et al., 2001; NCTM, 2000, 2006), many pupils struggle to achieve this goal (National Mathematics Advisory Panel [NMAP], 2008). For example, despite using an untimed test of sums and differences and self-reports (methods that can seriously overestimate fluency; Baroody, 1994; Baroody & Tiilikainen, 2003), Henry and Brown (2008) still found that the vast majority of 275 first graders did not achieve the California state standard of fluency with sums to 18 and related differences. Median mastery was only 22% of tested combinations. This was true even for high-performing schools and despite instructional emphasis on such fluency. Children at risk for academic failure (e.g., from low-income families), in particular, may not achieve fluency with basic combinations in a timely manner (Jordan, Kaplan, Olah, & Locuniak, 2006). Furthermore, a lack of fluency is a pervasive characteristic of those with mathematical learning difficulties (Ackerman, Anhalt, & Dykman, 1986; Geary, 1996; Goldman, Pellegrino, & Mertz, 1988; Jordan, Hanich, & Kaplan, 2003; Jordan, Hanich, & Uberti, 2003; Russell & Ginsburg, 1984).

Unfortunately, traditional instruction, which focuses on achieving memorization of basic number facts by rote via repetitive practice, can contribute to children's difficulties with learning the basic combinations (e.g., Baroody, Bajwa, & Eiland, 2009; Brownell, 1935). Such an approach is based, at least implicitly, on Thorndike's (1922) *law of frequency*, which stipulated that the more two stimuli are presented together (e.g., the more frequently a child sees a numerical combination such as "$3 + 1$" and the correct answer "4"), the stronger the association between the two becomes—resulting in efficient recall of the correct answer when the combination is presented. According to the distribution-of-associations model (Siegler & Jenkins, 1989) and its successors (e.g., Shrager & Siegler, 1998), for example, a memory trace is laid down each time an expression and its sum (e.g., $5 + 1 = 6$) is practiced, and thousands of such traces are necessary to achieve efficient fact recall. Moreover, facts are presumed to accumulate associative strength independently of other, even related, combinations (e.g., practice with $3 + 1$ was presumed to have no effect on its commuted partner $1 + 3$). Although such an approach may be effective in promoting the efficient recall of number facts, it is relatively ineffective in promoting fluency for several reasons. The time and effort needed to memorize hundreds of basic combinations by rote is burdensome and even overwhelming for many children. Moreover, memorization by rote may produce associative confusions and a lack of retention (barriers to efficient knowledge use) or a lack of transfer (barriers to appropriate or adaptive use).

Promoting fluency requires *meaningful learning* or *memorization*, which entails linking conceptual, procedural, and factual knowledge (Hiebert & Carpenter, 1992; Hiebert & Lefevre, 1986; Katona, 1967; Mason & Spence, 1999; Moursund, 2002; Resnick & Ford, 1981; Rittle-Johnson, Siegler, & Alibali, 2001). Achieving such learning entails organizing instruction so as to (a) promote children's recognition of regularities (patterns or relations), (b) build on students' existing knowledge, and (c) help pupils take advantage of general principles or concepts. Meaningful learning or memorization better promotes retention and transfer than does learning divorced from organizing and sense-making regularities, extant knowledge, or concepts (Brownell, 1941; Brownell & Chazel, 1935; Gersten & Chard, 1999; Henry & Brown, 2008; James, 1958; Jordan, 2007; Katona, 1967; Kilpatrick et al., 2001; NMAP, 2008; Piaget, 1964; Skemp, 1978, 1979, 1987; Steinberg, 1985; Suydam & Weaver, 1975; Swenson, 1949; Thiele, 1938; Wertheimer, 1959).

The natural course of meaningfully learning a basic combination or a family of combinations typically involves three overlapping phases (Kilpatrick et al., 2001; Steinberg, 1985): In Phase 1 (counting), children use object- or verbal-counting strategies to determine sums or differences. In Phase 2 (reasoning), they use the patterns and relations they discovered in Phase 1 to invent reasoning strategies (e.g., the number-after rule for adding with 1). In Phase 3 (retrieval), children can efficiently, appropriately, and adaptively produce sums and differences from a memory network. Deliberate (conscious and relatively slow) reasoning strategies (Phase 2) serve as a key bridge between using relatively inefficient counting strategies (Phase 1) and efficient retrieval (Phase 3) in two critical ways. One way is that they can provide an organizing framework for learning and storing both practiced and unpracticed combinations (Canobi, Reeve, & Pattison, 1998; Dowker, 2009; Rathmell, 1978; Sarama & Clements, 2009). A second way is that such strategies can become automatic and incorporated into the retrieval system and thus serve as a basis of fluent retrieval that may be superior in efficiency to the recall of a specific basic fact (Baroody, 1985, 1994; Fayol & Thevenot, 2012; Verschaffel, Greer, & De Corte, 2007).

For the reasons outlined in the previous paragraph, reasoning strategies are widely recommended as a goal for primary instruction and commonly incorporated in primary mathematics curricula. For instance, Common Core grade 1 goals include understanding the *subtraction-as-addition strategy* (Goal 1.OA.4): "Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8." Goal 1.OA.6 is: "Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use such strategies as ... making 10 (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$)."

The relative ease of inducing reasoning strategies or learning them via guided instruction differs by strategy. For example, primary grade children who received either unguided or guided instruction discovered and learned the number-after rule for adding with 1, as evidenced by significantly better transfer than an active-control group (Baroody, Eiland, Purpura, & Reid, 2012, 2013). In contrast, although provided equivalent practice, participants who received guided training on the near-doubles strategy (e.g., the sum of a near double such as $5 + 6$ is one more than the sum of its related double $5 + 5 = 10$) did not perform significantly better than active controls on transfer near-double trials.

Although practice frequency may play a role (Ashcraft, 1992), two, often interrelated, factors are perhaps more important. One is the salience of the concepts or relations underlying the meaningful learning of a strategy. Salience, in turn, is determined by (a) the prior conceptual, factual, and procedural knowledge required to discover or induce and understand or assimilate a pattern or relation and (b) the complexity of the regularity. The second factor is prior fluency with component skills needed to execute the reasoning strategy and complexity of these skills.

Children typically learn and apply the number-after rule for adding 1 via guided or unguided instruction because its underlying regularity is highly salient. Specifically, the connection between number-after relations is relatively simple to see. For instance, after using a counting strategy to determine that the sum of 7 + 1 is 8, a child might recall that 8 comes (immediately) after 7 in the counting sequence. Several such experiences may lead the child to conclude that the sum of any number *n* and one is the number after *n*. This is consistent with the active-memory view that practice is not merely a vehicle for strengthening an existing factual association but an opportunity to enrich extant memory by actively creating new memories (Nader & Hardt, 2009). Moreover, even children just beginning school are typically already fluent with the prerequisite skills for discovering and efficiently implementing the number-after rule for adding 1. Preschoolers usually discover the increasing magnitude principle—that numbers further along in the counting sequence represent larger collection than numbers before it—before they turn five years of age (Sarnecka & Carey, 2008; Schaeffer, Eggleston, & Scott, 1974). Moreover, children are so familiar with the counting sequence that they can readily recall and apply number-after relations to new tasks, such as mentally adding with 1 (Fuson, 1988).

As Kirschner, Sweller, and Clark (2006) noted, although guided instruction is generally more effective than unguided instruction, this advantage recedes "when learners have sufficiently high prior knowledge to provide 'internal' guidance" (p. 75). In comparison, the near-doubles strategy may be more difficult to learn, even with guided instruction, because a child needs to know the doubles, the meaningful learning of which depends on relatively unfamiliar and nonfluent prior knowledge (e.g., recognizing that the 4 + 4 can be determined by skip counting "four, eight" or that sum of any double must be an even number).

RATIONALE FOR THE PRESENT STUDY

The purpose of the present study was to evaluate the efficacy of experimental programs designed to foster a modified subtraction-as-addition reasoning strategy and a nonconventional reasoning strategy for adding with 8 or 9, namely use-a-10 (e.g., if 10 + 7 = 17, then the sum of 9 + 7 is one less than 17, because 9 is one less than 10). Extant research suggests that basic differences are more difficult to learn than basic sums (Carpenter & Moser, 1984; Kraner, 1980; Smith, 1921; Woods, Resnick, & Groen, 1975; see Cowan, 2003, for a review). Similarly, larger sums are more challenging to learn than smaller ones (Kraner, 1980; Smith, 1921; see Cowan, 2003, and NMAP, 2008, for reviews). In turn, why previous experimental training efforts to foster fluency with basic differences and larger sums have had little or limited success, the reasons for the features of the present intervention, and research aims and hypotheses are discussed.

Previous Training Efforts

Subtraction. Replicating results originally found with adults (Bajic, Kwak, & Rickard, 2011),[1] Rickard (2012; Walker, Bajic, Mickes, Kwak, & Rickard, 2014) found that unguided practice improved 6- to 11-year-olds' long-delay efficiency with practiced basic sums but not unpracticed subtraction complements. Walker, Mickes, Bajic, Nailon, and Rickard (2013) found

[1]Campbell and Agnew (2009) and Campbell, Fuchs-Lacelle, and Phenix (2006) observed addition–subtraction transfer by adults at posttesting immediately after practice with trained combinations. Posttesting by Bajic et al. (2011) was done one or more days after practice with trained combinations.

that practice translating fact triangles into equations (e.g., $\overset{8}{\underset{3 \quad 5}{\wedge}}$: $3 + 5 = 8$, $5 + 3 = 8$, $8 - 3 = 5$, $8 - 5 = 3$), a method commonly used by elementary curricula to promote recognition of related or complementary addition and subtraction combinations and their fluency, for one 20-minute session a week over 6 weeks did not promote fluency with practiced or transfer items more effectively than unstructured practice. Baroody (1999) found that even explicitly and repeatedly suggesting the subtraction-as-addition strategy (e.g., "5 take away 3 makes what? [$5 - 3 = \square$] can be thought of as 3 added to what makes 5 [$3 + \square = 5$]") over 24 sessions resulted in little improvement of 6.0- to 7.5-year-olds' grasp that addition can serve as a shortcut for determining related differences. Moreover, this guided training did not result in significantly better transfer to unpracticed differences than did the same amount of drill with practiced addition and subtraction combinations. In a follow-up study with second graders, guided instruction that included providing a rationale for the subtraction-as-addition strategy failed to produce transfer to unpracticed subtraction combinations (Baroody, 2013). A follow-up with third graders produced nonsignificant but promising evidence of transfer (i.e., a modest effect size; Baroody, Eiland, & Baroody, 2011).

The failure of previous training efforts may be due to five interrelated reasons:

1. The relations between subtraction and addition in general and the *complement principle* that connects unknown subtraction combinations to known addition combinations (e.g., if $5 + 3 = 8$, then $8 – 5 = 3$)[2] in particular are not highly salient and thus relatively difficult for children to learn or apply (Baroody, Ginsburg, & Waxman, 1983; Canobi, 2004, 2005, 2009; Henry & Brown, 2008; Putnam, deBettencourt, & Leinhardt, 1990). As a result, simply practicing basic addition and subtraction combinations, even together (as was done in the study by Rickard, 2012, or Walker et al., 2013; 2014) or telling children to use the subtraction-as-addition strategy (as was done in Baroody's 1999 study) may not help them understand the connections between addition and subtraction that would allow them to invent or comprehend and reliably use and adapt the subtraction-as-addition strategy.
2. The training provided was not (sufficiently) meaningful to participants. In an extensive review of the literature, Kilpatrick et al. (2001) concluded that directly teaching reasoning strategies, *if* done conceptually, could accelerate Phase 2. The unguided training in Rickard's (2012; Walker et al., 2014) study and the guided but misconceived training (prompting use of the strategy without helping participants understand its rationale) in the Baroody (1999) study were without a conceptual connection. Even Baroody's (2013; Baroody et al., 2011) follow-up studies, which involved an attempt to help participants understand the rationale of the subtraction-as-addition strategy, may have fallen short of achieving a conceptual understanding of the strategy (e.g., the feedback provided focused on correctness but not the reasons why a particular response was correct or incorrect).
3. As the relations between subtraction-as-addition are not salient, it is essential to explicitly connect the procedure to its rationale to make instruction of the subtraction-as-addition

[2]This principle is widely, but not universally, called the *inverse principle* or *inversion*. Following Baroody, Torbeyns, and Verschaffel's (2009) suggestion, we will not use these terms interchangeably with the complement principle but reserve them for a different but related aspect of relational knowledge, namely an undoing process: $a + b – b = a$ or $a – b + b = a$.

strategy meaningful. Previous research, including Walker et al. (2013) and Baroody (2013; Baroody et al., 2011), did not explicitly relate the strategy to its conceptual bases.

4. Perhaps relevant to many participants in all previous training studies, children must first become fluent with related sums before they can connect addition to subtraction and use this knowledge efficiently to reason out differences.
5. Even if training is successful in helping children understand and learn the subtraction-as-addition strategy, it may take distributed practice over many months, rather than weeks as with the Walker et al. (2013) study, to automatize the strategy (Jerman, 1970).

Add-With-8 or -9 Combinations. Previous efforts to teach a making-10 strategy (e.g., for determining the sum of 9 + 8: 9 + [1 + 7] = [9 + 1] + 7 = 10 + 7), even if somewhat supported with an effort to make the rationale known, failed to promote fluency with the strategy. Murata (2004) found that Japanese children taught the making-10 strategy with larger-addend-first combinations did not apply the strategy when smaller-addend-first combinations were introduced. Three previous efforts by Baroody (2013; Baroody, Eiland, Bajwa, & Baroody, 2009; Baroody, Eiland, & Baroody, 2011) to promote a making-10 strategy also failed to produce significant transfer to unpracticed combinations.

The making-10 strategy may be relatively difficult to teach for several reasons. For 9 + 7, for example, this strategy entails decomposing the 7 into 1 and 6 (fluency with decomposing numbers to 9), adding the 1 to 9 to make 10 (the associative law of addition and application of the number-after rule), and then adding 10 and 6 (knowing that any single-digit n added to 10 results in the sum n + teen). Children need to be thoroughly grounded conceptually and procedurally in all these prerequisites components to effectively integrate them into a comprehensible and fluent making-10 strategy. However, Canobi et al. (1998) found that first and second graders reported using the relatively arcane associative strategy as a computational shortcut on only 11% of the applicable problems. Pupils up through grade 8 do not do much better (Robinson & Dubé, 2009; Robinson, Ninowski, & Gray, 2006).

As many primary-level children struggle to understand and learn the making-10 strategy, the authors developed a use-a-10 strategy program. Unlike the making-10 strategy, the use-a-10 strategy does not require decomposition, associativity, recomposition to ten, or retaining the remaining portion of decomposed number in working memory. For example, none of these processes is needed to deduce the sum for 9 + 7: (Premise 1) 10 + 7 = 17; (Premise 2) 9 is 1 fewer than 10; (deduction) 9 + 7 is 1 fewer than 10 + 7 or 17 and thus, it necessarily follows that the sum of 9 + 7 must be 16.

Features of the Current Experimental Training

Well designed computer programs and properly chosen computer games might provide an effective and practical means of helping children learn and achieve fluency with the subtraction-as-addition and use-a-10 strategies (cf. Clements & Sarama, 2012; Fuchs et al., 2005, 2006; National Research Council [NRC], 2009; Rasanen, Salminen, Wilson, Aunio, & Dehaene, 2009; Sarama & Clements, 2009). Addressed first is how carefully exploiting the relations between addition and subtraction and extant knowledge of sums may foster the meaningful memorization of the subtraction-as-addition strategy and the otherwise relatively difficult basic subtraction combinations. Addressed second is how carefully building on relatively easy $10 + n$ or $n + 10$ sums and more–less relations may foster fluency with the use-a-10 reasoning strategy and the otherwise relatively hard add-with-8 or -9 combinations. Addressed third are general improvements in both programs.

Subtraction-as-Addition Program. For Piaget (1965), a deep understanding of number entailed understanding *additive composition*—recognizing that addition and subtraction are interdependent operations. Such an understanding is necessary to learn the subtraction-as-addition strategy meaningfully. Additive composition involves three key relations:

- Complement principle and combination families. The *addition–subtraction complement principle* can be summarized in general or algebraic terms as: If $a + b = c$ or $b + a = c$, then $c - b = a$ or $c - a = b$. This principle is the conceptual basis for the subtraction-as-addition reasoning strategy (Common Core Goal 1.OA.4) and implies that certain addition and subtraction combinations (e.g., $3 + 5 = 8$, $5 + 3 = 8$, $8 - 3 = 5$, and $8 - 5 = 3$) belong to the same "family."
- Inversion. Another aspect of recognizing the interdependence of addition and subtraction is the inverse principle, which involves the logical certainty and immediate recognition that adding a number b to a number a can be undone by subtracting the same number b and vice versa ($a + b - b$ or $a - b + b = a$). Piaget (1967, cited in Vilette, 2002) distinguished between this logical knowledge based on true reversibility of thinking and "empirical reversibility" based on unrelated successive actions that only appear to undo or reverse each other (e.g., adding 5 to 2 to make 7 and then taking 5 away from 7 to get 2 again). Although empirical inversion is not the psychological equivalent of the inverse principle, the former may be the basis for discovering the latter and relating it to the complement principle (Baroody, Torbeyns, & Verschaffel, 2009). Put differently, recognizing that adding b to a can be undone by taking away b may help children recognize that if $a + b = c$, then $c - b = a$. Indeed, Canobi (2004) found that children who understood the inverse principle typically also knew the complement principle. Nunes, Bryant, Hallet, Bell, and Evans (2009) found direct causal evidence that inverse knowledge can facilitate learning of the complement principle. Specifically, an intervention designed to foster the inverse relation resulted in improving participants' performance on complement problems.
- Part–part–whole relations. For Piaget and others (Briars & Larkin, 1984; Canobi, 2005; Resnick, 1983; Riley, Greeno, & Heller, 1983), the complement principle, the subtraction-as-addition strategy, and the inverse principle are an outgrowth of part–whole knowledge. Specifically, if Part 1 + Part 2 equals the Whole ($P_1 + P_2 = W$), then taking a part from the whole should leave the other part: $W - P_1 = P_2$ (or $W - P_2 = P_1$). For this reason, curricula introduce "fact triangles" with the whole such as 8 represented at the apex and each part such as 5 and 3 at a base angle. In the case of *Everyday Mathematics* (University of Chicago School Mathematics Project [UCSMP], 2005, see p. 503), the "+, –" are represented in the middle of the triangle and the related equations (e.g., $5 + 3$, $3 + 5$, $8 - 3 = 5$, and $8 - 5 = 3$) to the side.

The computer program for the subtraction-as-addition strategy was designed to have the following five interacting features in order to make the strategy's rationale (the complement principle) more apparent and intelligible, utilize empirical inversion, and highlight part–whole relations between addition and related subtraction combinations.

1. The complement principle and subtraction-as-addition strategy were highlighted as means for solving subtraction in a relatively easy manner via highly guided discovery learning. Research indicates that with less salient relations such as the addition–subtraction complement or less obvious strategies such as subtraction-as-addition, explicit or guided instruction results in more favorable outcomes than implicit or unguided instruction (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011; Clements & Sarama, 2012; Kirschner et al., 2006).
2. Some activities modeled empirical inversion. One activity entailed predicting the result of adding, for instance, 5 to 3, and then immediately afterward predicting the result of taking 5 from 8. Theoretically, modeling the undoing process of subtracting the same number after adding it should help children understand the complement principle and provide a rationale for the subtraction-as-addition strategy.
3. Analogous parts and wholes in complementary equations were color coded and labeled. The total for each related addition and subtraction equation had the same color and was labeled the "whole"; the corresponding part in each equation had its own distinct color and was labeled "part." This underscored the connections among related equations, the idea of combination "families" composed of complementary addition and subtraction combinations, empirical inversion, and the idea of *part–part–whole "triads."*
4. The *known addend* shared by related subtraction and addition combinations appeared in the same position—that is, $c - \boldsymbol{b} = a$ was related to $a + \boldsymbol{b} = c$ instead of $b + a = c$ (e.g., the subtraction combination $8 - \mathbf{5} = ?$ was related to $? + \mathbf{5} = 8$, not $5 + ? = 8$). This feature served to draw a child's attention to the fact that addition and subtraction complements have the same known part (5)—an important start toward recognizing they have identical parts (5 and 3) and share the same whole (8). This notation is also consistent with representing empirical inversion (adding and subtracting the same amount to an initial amount results in the initial amount): $3 + 5 = 8$ followed by $8 - 5 = 3$ once again.
5. Complementary equations were juxtaposed in time and place. A subtraction combination (often) immediately followed a related addition complement, and the feedback for the subtraction item involved displaying the subtraction equation directly below the related addition equation. For instance, the combination $8 - 5 = ?$ was often presented immediately after a child had solved $3 + 5$, and the feedback entailed presenting $8 - 5 = 3$ directly below $3 + 5 = 8$. The aim of this feature was to facilitate the simultaneous representation of the subtraction equation and a related addition equation in working memory. Such a representation affords a child the opportunity to compare and analyze the elements of the equations and discover the complementary relation.

Use-a-10 Program. A new program for guiding the discovery of the less conventional *use-a-10* reasoning strategy was developed. As use-a-10 does not require decomposition, associativity, recomposition to 10, or retaining the remaining portion of decomposed number in working memory, the hope was that the use-a-10 strategy might be more transparent than the often-taught making-10 strategy.

General Program Improvements. A number of refinements were made in both the guided-subtraction and use-a-10 programs to more explicitly and effectively underscore relevant relations and, thus, make them more effective than previous programs (e.g., Baroody, 2013; Baroody et al., 2011):

1. A new "Clue" game replaced a less interesting and effective game. Clue entailed explicitly asking, for instance, which of five addition combinations (e.g., 2 + 5, 4 + 5, 4 + 6, 4 + 7, 5 + 6, or 5 + 7) would help answer 9 – 5 or which of five add-with-10 combinations (e.g., 5 + 10, 6 + 10, 8 + 10, 10 + 4, or 10 + 6) would help answer 9 + 6.
2. Feedback juxtaposed subtraction and related addition equations or add-with-8 or -9 and add-with-10 combinations. For example, for some subtraction programs, the feedback for a subtraction combination involved a subtraction equation and the related addition combinations (with parts and whole labeled and color coded) presented simultaneously. Juxtaposing related equations in space made it more likely that both would be represented in working memory at the same time, in turn making it more likely for a participant to see their connection.
3. For the subtraction program, the elements of the juxtaposed equations were labeled as "part" or "whole" so that corresponding wholes and parts in each equation were more obvious.
4. Feedback, especially when a child was wrong, was significantly improved for all programs. In particular, for the subtraction-as-addition program, feedback explicitly connected why an answer was right or wrong or the application of the strategy to part–whole relations. For example, feedback for missing 9 – 5 included, "What is the whole in 9 – 5 = ? Click on the whole. What is the part? Click on the part. Now look for the same whole on the counting list below (which contained various addition expressions including a 4 + 5 'adding block' located above the 9 in the counting list). Are there any adding blocks above the whole 9 with the same part? Look closer and try again."

Aim of the Research and the Hypotheses Tested

The present research served to evaluate the efficacy of the two programs: highly guided discovery of the subtraction-as-addition strategy (*guided subtraction*) and highly guided discovery of the use-a-10 strategy (*guided use-a-10*). A third program—unguided practice of both subtraction and add-with-8 and -9 combinations (*unguided practice*)—served as a comparison condition. Two main hypotheses were tested.

Hypothesis 1 (H1): The Efficacy of the Guided-Subtraction Training. The guided-subtraction training should facilitate the meaningful learning of a general subtraction-as-addition reasoning strategy above and beyond students' regular classroom instruction on subtraction (as represented by the guided use-a-10 condition) or even regular classroom instruction plus extra subtraction practice (as represented by the unguided-practice condition). This learning should be manifested in the following three ways (see also Table 1):

H1a. Relatively greater fluency (achievement of Phase 3) with practiced and, more importantly, unpracticed subtraction combinations. Meaningful learning of a general subtraction-as-addition strategy should improve both retention of practiced subtraction combinations and transfer to unpracticed subtraction combinations (Buckingham, 1927; Olander, 1931). In contrast, the

TABLE 1
Predicted Impact of an Intervention Relative to the Other Conditions

Condition	Strategy Taught + Items Practiced	Type of Response	Subtraction Combinations: Practiced	Subtraction Combinations: Transfer	Add-with-8 or -9 Combinations: Practiced	Add-with-8 or -9 Combinations: Transfer
Guided Subtraction	Subtraction-as-addition strategy + Practiced subtraction	(a) fluent (b) reasoned (c) inefficient	Very high gain Very high gain Very high drop	High gain High gain High drop	Little gain Little gain Little drop	Little gain Little gain Little drop
Guided Use-a-10	Use-a-10 strategy + Practiced add-with-8- or -9	(a) fluent (b) reasoned (c) inefficient	Little gain Little gain Little drop	Little gain Little gain Little drop	Very high gain Very high gain Very high drop	High gain High gain High drop
Unguided Practice	None (drill only) + Practiced subtraction & Practiced add-with-8- or -9	(a) fluent (b) reasoned (c) inefficient	Modest gain Little gain Modest drop	Little gain Little gain Little drop	Modest gain Little gain Modest drop	Little gain Little gain Little drop

Note. As a result of maturation (e.g., spontaneous invention of a reasoning strategy) and history effects (e.g., classroom instruction involved practice with subtraction and add-with-8 or -9 combinations, the teaching and practice of the subtraction-as-addition strategy, and—for some participants—the teaching and practice of the make-10 strategy), some gain in fluency or reasoning use can be expected of all participants. This is indicated in the table as "little gain" or the case of inefficient responses as "little drop."

regular first-grade subtraction training experienced by the guided use-a-10 group should yield relatively little improvement on both (e.g., Henry & Brown, 2008). Consistent with Rickard's (2012) results, the relatively small dose of nonconceptual practice provided by the unguided-practice training should improve fluency with the practiced, but not transfer to unpracticed, subtraction combinations above and beyond that produced by external experiences such as classroom training. As some models suggest that hundreds, if not thousands, of repetitions per combination are necessary for efficient basic fact recall (e.g., Ashcraft, 1992; Shrager & Siegler, 1998; Siegler & Jenkins, 1989; Thorndike, 1922), this relatively small dose of extra nonconceptual practice should produce only somewhat greater fluency with practiced subtraction combinations. As a result, at the delayed posttest, the guided-subtraction group should outperform both the guided use-a-10 and unguided-practice groups on both practiced and unpracticed subtraction combinations, and the unguided-practice group should outperform the guided use-a-10 group on practiced, but not unpracticed, subtraction combinations.

H1b. Relatively greater conspicuous use of the subtraction-as-addition strategy (overt Phase 2 strategy use) on practiced and, more importantly, unpracticed subtraction combinations. As the amount of practice provided by the guided subtraction may not be sufficient to automatize this strategy, many participants in the guided-subtraction group may achieve Phase 2 (deliberate strategy use) but not Phase 3 (fluent strategy use). As a result, at the delayed posttest, the guided-subtraction group should exhibit greater gains in the overt and appropriate use of the subtraction-as-addition reasoning strategy on subtraction trials, particularly on unpracticed subtraction trials, than both the comparison groups.

H1c. Relatively greater drop in the use of inefficient strategies characteristic of Phase 1 on practiced and unpracticed subtraction trials. The greater use of fluent retrieval (Phase 3) strategies and slow overt or covert subtraction-as-addition (all Phase 2) strategies should produce a significant drop in behavior characteristic of Phase 1—correct but very slow responses, correct answers determined by counting, and incorrect responses—on practiced and unpracticed subtraction, but not other, trials. (If the reduction in efficient responses was due to efficient covert counting, then the improvement should be across the board and should impact practiced and unpracticed add-with-8 or -9 combinations as well as practiced and unpracticed subtraction combinations.

Hypothesis 2 (H2): The Efficacy of the Guided Use-a-10 Training. For reasons similar to those given for H1, the guided use-a-10 group should exhibit the parallel results with practiced and unpracticed add-with-8 and -9 combinations as summarized in Table 1.

METHOD

Participants

Parent permission letters were received for a total of 164 first graders. A preliminary mental-addition screening test served to check fluency with most basic sums (adding with 0, adding with 1, and the doubles such as $3 + 3$ and $8 + 8$). A total of 87 failed this preliminary screening test and were assigned to a teaching experiment involving the most basic addition combinations. The remaining 77 children were at least reasonably successful on the preliminary screening test (i.e., were fluent on at least 50% of each of the following combination families: add-with-0, add-with-1, and doubles) and pretested for the present study. A total of 75 pupils (6.1 to 7.6 years of age; $M = 6.65$ years old, $SD = 0.30$) were fluent on less than 50% of subtraction and add-with-8 and -9 combinations on the pretest and included in the present study. (Two other children were fluent on more than half of the add-with-8 or -9 combinations and not included in the study.) Participants for the present study were drawn from 12 classes in five schools in two school districts serving a midsized Midwestern community. All 75 qualifying participants completed the study.

Among participating children, 57.3% were male. The majority of children were Caucasian (70.7%). African-American, Hispanic, and mixed, unknown, or other race children comprised 9.3%, 5.3%, and 14.7% of the sample, respectively. Additionally, 30.6% of participants were eligible for free or reduced-price lunch. See Table 2 for the demographic information broken down by intervention group.

As Table 2 also shows, two classes used *Math Expressions* (Fuson, 2006), one class used *Saxon Math 1* (Larson, 2008), and nine classes used *Everyday Mathematics* (UCSMP, 2005). *Math Expressions* uses "Mountain Math" in Unit 3 to introduce whole–part–part triads as a vehicle for introducing missing-addend addition and subtraction (e.g., $\begin{smallmatrix} & 8 & \\ & \wedge & \\ 5 & & \square \end{smallmatrix}$) $= 5 + \square = 8$). Subtraction is then related to missing-second-part triads (e.g., $\begin{smallmatrix} & 8 & \\ & \wedge & \\ 5 & & \square \end{smallmatrix}$) $= 8 - 5 = \square$). The curriculum does not include lessons that entail explicitly teaching a subtraction-as-addition strategy. Unit 4 introduces "make a 10" by representing the 9 in $9 + 5$, for example, as the numeral 9 and the 5 as five circles. The 9

TABLE 2
Participant Characteristics and Classroom Curricula by Condition

		Training Condition		
		Guided Subtraction	*Guided Use-a-10*	*Unguided Practice*
Age range		6.1 to 7.1	6.2 to 7.2	6.2 to 7.6
Mean (SD)		6.6 (0.3)	6.6 (0.3)	6.7 (0.3)
Median age		6.6	6.6	6.8
Number of boys/girls		14 / 11	17 / 8	12 / 13
TEMA-3 range		92 to 137	89 to 142	75 to 125
TEMA-3 mean (SD)/median		107.2 (11.7)/103	106.8 (13.2)/105	103.5 (12.1)/105
Free/Reduced lunch eligible		8	10	5
Black/Hispanic/Multiracial		8	7	7
Family History				
Single-parent		3	3	1
Parent under 18		0	1	0
ESL		4	2	3
Medical/Developmental Condition				
Visual impairment		0	1	1
Speech services		1	0	0
Behavioral Condition				
Aggressive		1	2	0
Passive/withdrawn		1	1	3
Class Curricula				
School 1/class 1	Math Expressions	0	0	1
School 2/class 1	Math Expressions	2	4	4
/class 2	Saxon Math 1	3	1	1
School 3/class 1	Everyday Math	2	3	2
/class 2	Everyday Math	2	2	1
/class 3	Everyday Math	4	3	4
School 4/class 1	Everyday Math	1	1	1
/class 2	Everyday Math	3	3	2
/class 3	Everyday Math	3	3	3
School 5/class 1	Everyday Math	2	2	3
/class 2	Everyday Math	0	1	1
/class 3	Everyday Math	3	2	2

and one circle are enclosed to make a 10; the result is 10 and four remaining circles, which is also represented as $10 + 4 = 14$. Unit 14 (“Visualize Teen Addition”) uses several representations to help equate, for instance $9 + 4 = 13$ with $10 + 3$ and $8 + 6$ with $10 + 4$. The 9 in $9 + 4$ is represented by drawing 9 circles into the first column of a 10×10 grid and then drawing the first of four triangles in the first column and the remaining three in the second column. The 8 in $8 + 6$ is represented by a five-step (a bar subdivided into 5 segments, each with a dot) and a three-step; the 6 by a five-step and a one-step. Combining the two five-steps and the remaining bars results in a ten-step and a four-step. *Saxon Math 1* neither explicitly teaches the subtraction-as-addition strategy nor consistently relates subtraction to addition but does encourage a use-a-10 strategy for adding-with-9 combinations. *Everyday Mathematics* explicitly teaches a subtraction-as-addition strategy, but does not focus on a reasoning strategy for adding with 8 or 9 (at Grade 1).

All programs included activities for both group and individual work with manipulatives and materials common to primary classrooms. No program included instructional software. Half of the teachers reported providing computer time to play math games. Of these, one teacher reported using a single drill and practice game. The others used multiple games, which were basically drill and practice, except for the teacher of Class 2 in School 2, who reported using Brainpop Jr. Math (BrainPOP UK, 1999–2014), which included a making-10 tutorial. All schools were committed to achieving the state's Grade 1 objectives that included operations on whole numbers such as solving one- and two-step problems and performing computational procedures with addition and subtraction (Illinois State Board of Education, 2014).

The project hired personnel—10 female and one male—who had previous teaching experience, training in education, or, in most cases, both. Seven had previous teaching experience (1 to 14 years with a median of 2 years), one homeschooled her three children for grades K to 3, and seven had parenting experience. Each had a bachelor of science or bachelor of arts; seven majored in education and two in psychology, or counseling. Eight had masters of education. Six had teaching certificates. Nine had previous experience on the project (1 to 5 years with a median of 4 years). All staff members had six 3-hour training sessions on the rationale of testing and training procedures; a staff member's role (do's and don'ts) in how to access and navigate the computer programs; the rationale for scoring the mental-arithmetic tests, including the operational definitions of each scoring category or strategy and testing contingencies (e.g., how to respond if a child tried to count or did not respond on a trial); and discipline procedures. The staff-training manual is available upon request.

Measures

Test of Early Mathematics Ability—Third Edition (TEMA-3). The TEMA-3 (Ginsburg & Baroody, 2003) is a manually and individually administered, nationally standardized mathematics achievement test for children 3 to 8 years old. The test measures informal and formal concepts and skills in the following domains: numbering skills, number-comparison facility, numeral literacy, mastery of number combinations, calculation skills, and understanding of concepts. Cronbach's alphas are as follows: (a) .95 for each of the two age intervals represented by the sample (6- and 7-year-olds); (b) .94 for the form used (Form A) overall; (c) .98 for all males, females, European, African, Hispanic, and Asian Americans; and (d) .99 for low mathematics achievers. Test–retest reliability for Form A is .83. In terms of criterion (concurrent) validity, correlations between the TEMA-3 and similar measures (the KeyMath-R/NU, Woodcock-Johnson III, Diagnostic Achievement Battery, and Young Children's Achievement Test) range from .54 to .91.

Mental-Arithmetic Test. The mental-arithmetic test included five categories of trials: (a) seven subtraction combinations practiced by the guided-subtraction and unguided-practiced groups (*practiced subtraction*); (b) seven subtraction combinations not practiced by any of the three groups (*unpracticed subtraction*), (c) 10 *addition complements* of practiced and unpracticed subtraction combinations that were practiced by only the guided-subtraction group, (d) nine sums with an addend of 8 or 9 practiced by the guided use-a-10 and unguided-practice groups (*practiced add-with-8 or -9*); (e) eight sums with an addend of 8 or 9 not practiced by any group

TABLE 3
Combinations on the Mental Arithmetic Test and Practiced by Training Condition

			Practiced in Stages III to V by—		
Combination Family	*Combinations*	*Pretest/Posttest Mental-Arithmetic Test Category* [b]	*Guided Subtraction Group*	*Guided Use-a-10 Group*	*Unguided Practice Group*
Practiced subtraction	7 – 5, 9 – 5, 10 – 7, 11 – 6, 11 – 7, 12 – 9, 14 – 7	a	Yes	No	Yes
	13 – 6	(not tested)	Yes	No	Yes
Transfer subtraction	9 – 6, 10 – 6, 11 – 5, 11 – 8, 12 – 7, 13 – 7, 16 – 8	b	No	No	No
Addition Complements (subtraction helpers)	3 + 9, 4 + 5, 4 + 6, 5 + 6, 5 + 7, 6 + 5, 6 + 7, 7 + 6, 7 + 7, 8 + 8	c	Yes	No	No
	2 + 5, 3 + 6, 3 + 7, 3 + 8, 4 + 7	(not tested)	Yes	No	No
Practiced add with 8 or 9	5 + 8, 6 + 9, 7 + 8, 7 + 9, 8 + 4, 8 + 7, 9 + 5, 9 + 6, 9 + 9	d	No	Yes	Yes
Transfer add with 8 or 9	4 + 8, 5 + 9, 6 + 8, 8 + 5, 8 + 6, 8 + 9, 9 + 7, 9 + 8	e	No	No	No
Practiced add with 10 (add with 8 or 9 helpers)	2 + 10 to 8 + 10; 10 + 3 to 10 + 9	(not tested)	No	Yes	No
Miscellaneous items[a]	2 + 6, 3 + 5, 10 – 3, 13 – 2	(not tested)	No	No	Yes

Note. [a]Miscellaneous items were added to the unguided practiced training so that each group would have approximately the same number of combinations to practice.
[b] The mental-arithmetic pretest/posttest was composed of four sets of 10 combinations each.
Set 1 in order was: 12 – 9, 8 + 8, 4 + 6, 9 + 8, 11 – 6, 4 + 8, 9 + 6, 11 – 8, 7 + 9, and 13 – 7.
Set 2 in order was: 5 + 8, 14 – 7, 8 + 6, 9 + 9, 6 + 7, 9 – 5, 5 + 6, 8 + 7, 10 – 6, and 5 + 9.
Set 3 in order was: 6 + 9, 4 + 5, 9 – 6, 8 + 4, 10 – 7, 8 + 9, 12 – 7, 6 + 5, 9 + 7, and 7 – 5.
Set 4 in order was: 16 – 8, 3 + 9, 6 + 8, 11 – 5, 7 + 8, 9 + 5, 5 + 7, 11 – 7, 8 + 5, and 7 + 7.

(*unpracticed add-with-8 or -9*). The specific combinations used for each category are delineated in Table 3.

The testing was done in the context of a computer game. For the Race Car Game, for example, a tester explained to the child:

> We are going to play a game where we pretend you are driving a race car. In order to drive your car, you will need to keep both of your hands on the car's steering wheel at all times. [The tester then encouraged the child to grip a steering wheel tightly with both hands so as to discourage finger counting or at least make it difficult or obvious.] Your success driving your car and doing well in the race is determined by answering addition problems accurately and quickly. If you know the answer, tell me as quickly as you can. If you are not sure of the answer, make a good guess as quickly as you can.

Children were not provided objects or encouraged to use objects during the mental-arithmetic test. See Figure 1 in a previous report by the Baroody et al. (2013) or Appendix A (Appendices mentioned in this article can be found on the author's website: http://edwebs1.education.illinois.edu/frp/ci/baroody.html) for illustrations and details on the computer-assisted testing procedure.

Each testing session consisted of a test set of 10 combinations, a computer reward game, a second test set of 10 combinations, and a final reward game (see Footnote b of Table 3 for details). The mental-addition combinations were presented in a semirandom order: They were randomly ordered with the constraints that two combinations with the same addends or sum were not presented one after another, commuted combinations were not presented in the same session, and the types of combinations were evenly distributed across the four sets. Once a child completed a set of 10 combinations, the computer provided overall feedback.[3]

Data were gathered on strategy, response time, and response accuracy. Testers identified whether a child used a counting strategy, a reasoning strategy, or an undetermined strategy. A counting strategy entailed either representing one or both addends before the sum count (*concrete counting strategies*) or representing both addends during the sum count (*abstract counting strategies*; Baroody, Tiilikainen, & Tai, 2006; Fuson, 1992). This determination was based on evidence of counting objects (e.g., fingers), verbally citing a (portion) of the counting sequence out loud (even if whispered), or subvocally counting accompanied by successive movement of a finger or the eyes. A reasoning strategy was scored if a child spontaneously exhibited evidence of using deductive reasoning (for $9 + 7$, e.g., stating, "10 and 7 is 17, so 1 smaller is 16"), and the type of reasoning was noted in order to identify whether it was consistent with the experimental interventions. An internal clock started when a trial was presented. As soon as the child responded, the tester (who had his/her finger posed on the "Enter" key) depressed the key to stop the internal stopwatch. The computer then displayed the response time. The tester entered the child's answer via a number pad, and the tester scored the response as correct or incorrect.

In addition the tester's scoring, another member of the project independently scored a child's response and strategy for each item on the pretest and the delayed posttest videotape for each of the seven participants randomly chosen from each of the three conditions. Interrater agreement with a tester's scoring for 834 pretest and 840 posttest responses was 93.6% and 97.3%, respectively, for accuracy and 90.3% and 96%, respectively, for strategy.

Three variables were derived from the data:

1. A *fluent* (Phase 3) response was defined operationally as an efficient reply (i.e., the correct answer via an underdetermined or reasoning strategy in less than 3 s), not due to counting or a response bias. For instance, the response bias of consistently "stating the subtrahend for subtraction" produces false positive on complements of the doubles such as 14 – 7. A child was deemed as using a response bias during a session (over two sets) if a potential mechanical strategy was overapplied to inappropriate cases more than 50% the time,

[3]For the purposes of providing feedback to a child (but not scoring the child's fluency), combinations were scored on accuracy—correct, close (within 25% of the correct sum but > larger addend), reasonable (within 150% of the correct sum and larger than the addends), or unreasonable—regardless of speed (fast or slow). The average score determined the outcome of the game (e.g., the place the child's car finished in the race).

applied to more than 50% of the trials where it could lead to an apparently correct answer, and used on more than 50% of the trials involving an operation. (Addition and subtraction combinations were considered separately because some children do not use response biases for the relatively familiar operation of mental addition but do use such mechanical strategies for the relatively unfamiliar operation of mental subtraction.) Consider a case from the first pretest session (Sets 1 and 2): A guided-subtraction participant responded with the subtrahend to 11 – 8, 10 – 6, 12 – 9, and 11 – 6 (but not 13 – 7 or 9 – 5) inappropriately (67%) and to 14 – 7 appropriately (100%), for a total of 71% over all trials. Subsequently, her response of 7 to 14 – 7 was scored as incorrect. Only two other response biases were detected on the pretest, but neither led to false positives. Another guided-subtraction participant used a unpracticed-an-addend-into-a-teen response bias (e.g., $6 + 9 = 16$, $8 + 5 = 18$, $8 + 4 = 18$) on 62.5% of the eight addition trials in second testing session. There was no evidence of responses biases on the delayed posttest. Two authors independently assessed all participants' pretest and posttest data for response biases using the straightforward criteria. The rare disagreements were resolved by discussing how one or the other raters incorrectly applied a criterion.

2. A *nonfluent reasoning* response was defined as the use of an overt reasoning strategy in 3 s or more.
3. An *inefficient* response consisted of a very slow (>6 s) correct answer using an undetermined strategy, an answer generated by a counting strategy, or an incorrect answer.

The variables were used to determine three scores for each child at pretest and at delayed posttest:

1. *Fluency rate* was the proportion of a combination type scored as fluent by the child. This served to gauge achievement of Phase 3.
2. *Net reasoning* was the number of a child's correct nonfluent reasoning responses minus the number of incorrect nonfluent reasoning cases for a type of combination. This provided an indication of the appropriate use of a deliberate reasoning strategy or obvious Phase 2 strategy use.
3. *Inefficiency rate* was the proportion of inefficient responses by a child for a combination type and includes responses apparently, or obviously, due to Phase 1 strategies.

Training

Discussed, in turn, are the preparatory training, the experimental interventions, the role of the trainers, and the fidelity of the interventions.

Preparatory Training. The first two of the five stages of training described in Table 4 were common to all participants. The aim of this preparatory training was to prepare all children for the computer-assisted mental-arithmetic testing and Stages III to V training. Specifically, it ensured that all children had the necessary experiences to benefit from the computerized programs (e.g., how to use the virtual manipulatives and enter answers) and the developmental prerequisites to benefit from the experimental interventions (including solving meaningful word problems concretely). Details regarding Stages I and II can be found in Appendices B and C.

TABLE 4
Five Stages of Computer-Assisted Mental-Arithmetic Training

Stage Name	*Rationale*
Stage I: Preparatory Concrete Training (7 sessions; ∼3.5 weeks)	**Aim:** Ensure recognition and understanding of the formal symbolism for addition and subtraction (e.g., 7+1 or 8–2) by connecting it to concrete or meaningful situations and their own informal solutions. (Items presented as meaningful word problems and symbolic expressions. Strategy choice, including informal counting-based strategies, encouraged. To this end, virtual manipulatives, such as 10 frames and number sticks, presented as an option.) **Plan:** For each of 7 sessions, present three sets. Set 1A is a vehicle for learning how to navigate the program (e.g., use the mouse). Sets 1B and 2A to 7A introduce virtual manipulatives (e.g., record a score using a ten frames and dots). Sets 2B to 7B involve solving word problems (relating expressions or equations to a concrete model). Set 1C to 7C focused on relating part-whole terminology to equations and composition and decomposition.
Stage II: Preparatory Mental Arithmetic (Estimation) Training (8 sessions; ∼4 weeks)	**Aim:** Serve as a bridge between Stage I (informal counting-based strategies with objects) and Stage III (using mental-arithmetic strategies involving reasoning or retrieval to determine exact answers). Help children identify and define a good or SMART GUESS. **Plan:** First introduce numerical estimation (approximating the size of a single collection) with Sets 8 and 9 and then arithmetic estimation (gauging the size of sums and differences) with sets 10 to 12. The stage begins with visually estimating the number of carrots or frogs. (About how many carrots or frogs did you see)? This provides a basis for estimating the answers to addition and subtraction problems in sets 10–12.
Stage III: Strategy Training (8 sessions; ∼4 weeks)	**Aim:** Except for the control condition, help a child discover the relations that underlie a reasoning strategy and thus understand and effectively use a reasoning strategy. **Plan:** For the subtraction condition, subtraction items always follow related addition item. Present items concretely (bubble lines and 10 frames) and symbolically. Encouraged a child to determine an answer in manner of his/her choice. No time limit. For incorrect responses, child uses virtual manipulatives to redo trial concretely. Feedback involves showing subtraction equation immediately below addition equation.
Stage IV: Strategy Practice (8 sessions; ∼4 weeks)	**Aim:** Promote efficient use of reasoning strategies. **Plan:** For the subtraction condition, subtraction items sometimes follow related addition item. Present items symbolically. Initially encourage child to respond mentally and quickly ("make a smart guess as quickly as you can"). Use concrete solutions (bubble lines and ten frames) only as a backup for determining the exact answer (correcting an incorrect response) on re-dos. Using concrete means of solving the problems are only for backup; second attempts. Hints in the feedback to use the relationship underscore the need to solve problems in a timely fashion.
Stage V: Strategy Fluency (8 sessions; ∼4 weeks)	**Aim:** Cement efficient use of reasoning strategies. **Plan:** For the subtraction condition, subtraction items never follow related addition item. Encourage child to make a good guess ("smart guess") as accurately and quickly as possible. For an initial incorrect response, give child a second chance to revise her answer mentally (a second-chance guess). Emphasize mental arithmetic ("Give us your answer right away. If unsure of an answer, make a SMART GUESS FAST.") Games underscores the importance of a fast response and smart guess and the disadvantages of counting. Second-chance guess, no hints, no manipulates.

Note. A child's solutions in Stages I and II were untimed; solutions in Stages III, IV, and V were timed. The preparatory (Stages I and II) training was common to all three conditions. Stages III, IV, V were components of the experimental training that differed by condition.

Experimental Interventions. Although the content of the last three of the five training stages described in Table 4 varied by condition, all three programs had five commonalities:

1. Each involved three stages (Stage III had no time limit; Stage IV, a generous time limit; and Stage V, a stringent time limit).
2. Each entailed 22 sessions and practice with 20 to 24 different combinations per session.
3. The same computer games were used as a learning context (see, e.g., Table 5).
4. The dosage for targeted or tested practice combinations was identical (e.g., the guided-subtraction and the unguided-practice groups both practiced 10 – 7 ten times during Stage III, and the guided use-a-10 and unguided-practice groups both practiced 8 + 7 ten times during Stage III.
5. The same choice of computer reward games was available to all groups, and the child's choice of game was played at the end of a session.

The two guided programs involved discovery learning in the sense that a strategy or what to do was not specified (e.g., guided-subtraction participants were never directly told that to answer 8 – 5 = □, they should determine what must be added to 5 to make 8 or chose the number from 3 + 5 = 8 that does not appear in 8 – 5 = □). These two programs had parallel structures. As Table 4 summarizes, Stage III served to teach a reasoning strategy; Stage IV, to practice using a strategy with generous time limits; and Stage V, to practice using a strategy with a stringent time limit to encourage efficient application. The order of combinations was standardized within an instructional condition. Specific mechanisms of instruction were individualized for each condition. The differences between conditions are discussed below.

Guided Subtraction. Stage III is illustrated in Appendix D; Stage IV, in Appendix E, and Stage V, in Appendix F. As is illustrated in Table 5, the guided-subtraction condition involved a sequence of combinations designed to facilitate learning of the subtraction-as-addition strategy and the relational concepts underlying the strategy. Stages III and IV had the following six characteristics:

1. Training involved both more implicit discovery of this knowledge and more explicit instruction.
2. Inversion was related to the complement principle and partial addition and subtraction families (2 + 5 = 7 and 7 – 5 = 2).
3. In the Train Game, "track numbers" were depicted as fact triangles with the whole ("route destination") at the apex and the parts at each base corner ("route numbers").
4. Analogous parts and wholes in complementary equations were color coded and labeled. Another method for related complementary combinations to the same part-–part–whole family.
5. The known addends in related subtraction and addition equations were placed in the same position.
6. Related combinations were juxtaposed in time (an addition question followed by a complementary subtraction problem). Feedback often involved juxtaposing related equations in position (one above the other) and place (e.g., 10 – 7 = 3).

TABLE 5
Experimental Mental-Arithmetic (Stages III to V) by Condition

Stage/Set	*Computer Game*[a]	*Guided Subtraction*	*Guided Use-a-10*	*Unguided Practice*
III / A	Castle Wall (feedback on horizontal # line) Train Game (feedback on vertical # line)	Solve addition item such as 3+9; usually followed by solving a related subtraction item 12–9.	Solve add-with-10 item such as 7+10; usually followed by a related add-with-8 or -9 such as 7+8 or 7+9.	Subtraction items such as 12–9 and add-with-8 or -9 solved in haphazard order.
III / B	Does It Help? (Possible helper and target items presented successively)	Solve an addition item such as 2+5 (or 5+7); then asked if it helps solve a subtraction item such as 7–5 (yes for 2+5 and no for 5+7).	Solve a 10+*n*/*n*+10 item such as 6+10; then asked if it helps solve 8+*n*/*n*+8 or 9+*n*/*n*+9 such as 6+9 (yes) or 7+9 (no).	Solve one item and asked if a second had the same answer—e.g., 11–6 & 9–5 (no), 8+7 & 7+8 or 12–9 & 10–7 (yes).
	Wall Help? ? (All possible helper items presented first block; all target items, presented in a second block)	Addition items such as 4+5 and 5+7 solved first and sums arranged sequentially as part of a wall. Subtraction items such as 9–5 presented; child asked which sum in the wall helps.	A block of 10+*n*/*n*+10 items is solved first and sums arranged sequentially as part of a wall. 8+*n*/*n*+8 or 9+*n*/*n*+9 items such as presented; child asked which sum in the wall helps.	A block of items is solved in haphazard order first and sums arranged sequentially as part of a wall. Child asked if item in the wall helps (has the same answer as) an item from second block.
IV / A	Timed Monkey? (Possible helper and target items presented successively)	For 7+6, e.g., a monkey swings to branch 7, child asked to predict out-come of swinging 6 more. Related subtraction follows: If at 13, where will monkey be if swings back 6.	Mocha Monkey swings 7 branches and then 10 more, where will she land? Cocoa Monkey swings 7 branches and then 9 (or 8) more, where will she land?	Mocha Monkey swings 7 branches and then 9 more, where will she land? Cocoa Monkey swings 9 branches and then 9 more, where will she land?
	Clocked Choice (timed version of Does It Help?)	After determining sum (e.g., 4+7 = 11), asked if, helps answer 11–7, 10–7, 11–6, or None. Feedback indicated that both 4+7 and 11–7 have the same whole 11 and same part 7 or that an incorrect choice did not. Then asked to answer the subtraction item. Feedback high-lighted parallel part-whole aspects.	Solved a 10+*n*/*n*+10 item such as 10+7 and provided feedback. Then asked what which 8+*n*/*n*+8 or 9+*n*/*n*+9 was 1 (or 2) smaller than the 10+*n*/*n*+10 item (e.g., The answer to which problem below 1 smaller than 10+7 = 17: 7+8, 7+9, 9+9, or None?)	Solve one item and asked which of three choices (or none) had the same answer.

(Continued on next page)

TABLE 5
Experimental Mental-Arithmetic (Stages III to V) by Condition *(Continued)*

Stage/Set	*Computer Game*[a]	*Guided Subtraction*	*Guided Use-a-10*	*Unguided Practice*
IV / B	Lost Puppies	Asked if a puppy with a dog tag consisting of an addition item was such as 2+5 belonged to part–part family (one part 2; other part 5). Determined sum; feedback indicated that 7 was correct because the whole 7 has parts 2 and 5. Asked if a subtraction item such as 7–5 belongs to the same family with a part of 2 and part of 7. After feedback, child solved for difference.	Asked if a puppy with a dog tag consisting of a 10+*n*/*n*+10 item such as 7+10 belonged to the 16 family. Feedback for a correct answer of "No" indicated that it was not the lost puppy and congratulated the child for not taking someone else's puppy. Then asked to indicate the sum of 7+10. The same procedure was followed for 7+9.	Asked if a puppy with a dog tag consisting of an addition or a subtraction item had a particular answer
	Timed Train	Same as Train Game, except with a clock and increasingly restrictive time limits.		
V / A	Dirt Bike	Block of practiced addition items and then block of practiced subtraction items solved.	Block of practiced 10+*n*/*n*+10 items and then block of practiced 8+*n*/*n*+8 or 9+*n*/*n*+9 items solved.	Practiced addition and subtraction items practiced in haphazard order.
	Long Jump	Practiced addition and subtraction items practiced in haphazard order.	Practiced 10+*n*/*n*+10, 8+*n*/*n*+8, and 9+*n*/*n*+9 items practiced in haphazard order.	Practiced addition and subtraction items practiced in haphazard order.
V / B	Puppy Choice Treasure Hunt *Car Race or Fire truck*	Practiced addition and subtraction items practiced in haphazard order.	Practiced 10+*n*/*n*+10, 8+*n*/*n*+8, and 9+*n*/*n*+9 items practiced in haphazard order.	Practiced addition and subtraction items practiced in haphazard order.

Note. [a]Each game was played 4 times, except the Stage V / Set B games, which were each played twice.

Guided Use-a-10. As indicated in Table 5, the guided use-a-10 condition was analogous to the guided-subtraction training, except that it involved relating add-with-8 or -9 combinations to the easier add-with-10 sums.

Unguided Practice. Participants practiced the target combinations of the two other groups (i.e., the practiced subtraction and add-with-8 or -9 combinations) but not the "helper" combinations—the addition complements or the add-with-10 combinations. Moreover, the presentation order of the combinations was not designed to facilitate the discovery of either the subtraction-as-addition or the use-a-10 strategy. For example, in one set of combinations, children would first receive and addition problem (e.g., $9 + 5 = 14$) and then would receive an unrelated subtraction problem (e.g., $7 - 5 = 2$).

Role of the Project C trainers. Administrative duties included completing lesson log sheets to ensure each participant completed each lesson without duplication and noting where and why children had difficulty with the programs, so they could be improved for the following year. Logistical duties included picking up children from their classrooms, escorting them to a project computer station, logging them into the appropriate computer program, and encouraging on-task behavior. The trainer's primary instructional role was to give voice to the scripted instructions and to the feedback graphically displayed by the computer screen.

Fidelity of Intervention. Fidelity of all training stages was ensured by (a) 10 hours of staff training before the Stage I training on the rationale for the programs, procedures for implementing the computer-assisted training, and behavioral management techniques; (b) a copy of the *Trainer Guidelines* available at each computer station; (c) oversight (on average, a session every other week for each staff member) by the Head Project Teacher or Project Coordinator; (d) brief (10 to 30 minute) staff meetings during the training to review procedures and address training issues as needed; (e) a lesson log sheet for keeping track of which lessons each participant completed; and—most importantly—(e) the design of the computer programs. Computer programs provided identical instruction for each child in Stages I and II and for each child within a training condition in Stages III to V. The programs ensured that each child received (a) the assigned intervention (a child's log-in automatically connected to his or her treatment), (b) the combinations in the order specified by an intervention, (c) immediate feedback on correctness, and in the guided conditions, (d) seeing feedback that juxtaposed elements of a relation. In regard to implementation fidelity, all participants completed 100% of the lessons in their Stage I and II and assigned Stage III to V training before posttesting.

Research Design. A training experiment with multiple baselines served to evaluate the efficacy of the three programs. Discussed in turn are the research procedures and the rationale for the research design.

Procedures. Project personnel implemented all testing and training procedures. Positive assent was obtained for each testing and training session. All project testing and training was conducted at project computer stations in a hallway outside a child's classroom or in a room dedicated to the project. Pull outs occurred in nonliteracy time blocks, including mathematics instruction and play time.

All children in the sample pool simultaneously received the 7.5-week long preparatory (Stage I and II) training. During the entirety of this time, children's mathematical achievement was gauged using the TEMA-3. After the completion of the preparatory training, participants were individually administered a preliminary computer-based mental-addition screening test that served to gauge fluency with the easiest sums: adding with 0 and 1 and the doubles (see Baroody et al., 2012, for details). Participants who demonstrated fluency on most of these combinations were eligible for the present study and were administered a computer-based mental-arithmetic pretest to gauge fluency with subtraction and more difficult addition combinations. Participants who were fluent on less than half of the subtraction and adding with 8 or 9 combinations were then randomly assigned to the guided-subtraction, guided use-a-10, or unguided-practice training condition.[4] The computer-assisted experimental interventions were conducted simultaneously, and each lasted 12 weeks. Each project staff member carried out each of these three interventions so as to control for trainer effects. Both preparatory training and experimental interventions involved one-to-one, 30-minute sessions twice per week. All participants were readministered the mental-arithmetic test two weeks after the training to gauge retention.

Design Rationale. The two guided groups each served as an active control for the other because they practiced mutually exclusive combinations. For instance, the subtraction group, which received regular classroom, but not experimental, training with add-with-8 or -9 combinations, served to control for the impact of external (e.g., classroom) training on these combinations for the use-a-10 group. The unguided-practice group practiced in a semirandom order the subtraction combinations practiced by the guided-subtraction group and the add-with-8 and -9 combinations practiced by the use-a-10 group. This condition served to evaluate whether supplemental practice alone was sufficient to promote fluency with subtraction or add-with-8 or -9 items or whether some scaffolding was necessary to ensure supplemental practice was effective.

Analytic Procedure

For Hypothesis 1a (H1a) and Hypothesis 2a (H2a), the three groups were compared for fluency on each type of combination using ANCOVAs, involving pretest fluency rate, pretest TEMA-3 achievement, and age as the covariates. Posttest fluency rate was the dependent measure. For Hypothesis 1b (H1b) and Hypothesis 2b (H2b) regarding *gain in net reasoning*—the difference between the posttest and the pretest s reasoning—the three groups were compared on four key types of combinations (practiced and unpracticed subtraction and practiced and unpracticed add-with-8 or -9) using four separate ANOVAs. The analyses for Hypothesis 1c (H1c) and Hypothesis 2c (H2c) regarding *drop in inefficiency rate*—the difference between the posttest and the pretest inefficiency rate—was similar to that used for H1b and H2b. Although the assumption of homogeneity of variances was violated in the H1c and H2c analyses, ANOVA is generally robust to violations of assumptions when group sizes are equal. Nevertheless, the degrees of freedom of the F statistics were adjusted using the Brown-Forsythe correction.

[4]Random assignment was done within classroom for Schools 3 to 5 in Table 2 but was not practical for two schools. The single participant from School 1 was randomly assigned to a condition. At School 2, random assignment within classroom resulted in an imbalance in mathematics achievement scores and pretest results for subtraction and add-with-8 or -9 combinations. So as to ensure balance for these two factors, which had more impact on the dependent variables than classroom (as a factor) in the authors' previous research, 10 participants in one class and five participants in another class were randomly assigned within the school. The remaining 56 participants from three classes in each of the other three schools were randomly assigned within classroom.

For all analyses, the intervention group of interest was compared to the unguided-practice group and its active-control group, and the unguided-practice group and active-control group were also compared. A correction (Benjamini & Hochberg, 1995) was applied to adjust for Type I error due to multiple comparisons. Pretest comparisons were not included in the total number of comparisons to be adjusted because, as children were randomly assigned to groups, differences between these groups were not predicted. Thus, a total of nine comparisons were included in the correction for the subtraction-as-addition analyses (i.e., three comparisons on practiced subtraction combinations, three comparisons on unpracticed subtraction combinations, and three comparisons on practiced addition complement combinations), and six comparisons were included in the correction for the use-a-10 analyses (i.e., three comparisons on the practiced add-with-8 or -9 combinations and three comparisons on the unpracticed add-with-8 or -9 combinations). Additionally, effect size magnitude (Hedges' g) was examined for all specific contrasts of interest due to the limited power of the study and the importance of evaluating effect sizes (Lipsey et al., 2012; Wilkinson & APA Task Force on Statistical Inference, 1999).[5] Effect sizes were calculated using posttest means that were adjusted to account for pretest and the other covariates. A Hedges' g that exceeds 0.25 meets the criterion for effective practice set by the federal *What Works Clearinghouse* handbook (Institute of Education Sciences [IES], 2011).

RESULTS

Descriptive statistics for children's performance on each of the combination categories are presented in Table 6. Pretest results, the delayed posttest regarding H1, and the delayed posttest regarding H2 are discussed in turn.

Pretest Analyses

Preliminary analyses revealed that children in the three groups did not differ in gender, $\chi^2\,(2, N = 75) = 2.07, p = .355$; ethnicity (comparing Caucasian, African American, and other), $\chi^2\,(4, N = 75) = .72, p = .948$; or age, $F(2, 72) = 0.56, p = .574$. Using nationally normed TEMA-3 standard scores, an ANOVA did not reveal significant group differences in mathematics achievement, $F(2, 72) = 0.66$, $p = .519$. Using the proportion fluent, ANOVAs revealed no significant differences among groups on any of the five categories of combinations: practiced subtraction, $F(2, 72) = 1.86$, $p = .164$; unpracticed subtraction, $F(2, 72) = 1.66$, $p = .197$; addition complements, $F(2, 72) = 0.12$, $p = .883$; practiced add-with-8 or -9, $F(2, 72) = 0.40$, $p = .675$; or unpracticed add-with-8 or -9 combinations, $F(2, 72) = 0.02$, $p = .985$.

[5]Lipsey et al. (2012, p. 3) noted, for example, "The p-values characterize only statistical significance, which bears no necessary relationship to practical significance or even to the statistical magnitude of the effect. Statistical significance is a function of the magnitude of the difference between means, to be sure, but it is also heavily influenced by the sample size, the within samples variance on the outcome variable, the covariates included in the analysis, and the type of statistical test applied. None of the latter is related in any way to the magnitude or importance of the effect."

TABLE 6
Adjusted Mean Proportion Fluent and Standard Deviations for Group Performance on Each Set of Arithmetic Combinations

Combination Family	*Guided Subtraction Condition (n = 25)*				*Guided Use-a-10 Condition (n = 25)*				*Unguided Practice Condition (n = 25)*			
	Pretest		*Adjusted Posttest*		*Pretest*		*Adjusted Posttest*		*Pretest*		*Adjusted Posttest*	
	M	*SD*	*M*[a]	*SD*	*M*	*SD*	*M*[b]	*SD*	*M*	*SD*	*M*[c]	*SD*
Practiced Subtraction	.02	.05	.43/.41	.23	.06	.12	.10/.11	.21	.03	.07	.35/.37	.31
Transfer Subtraction	.01	.06	.20/.18	.24	.05	.10	.10/.11	.19	.02	.05	.08/.09	.14
Addition Comple-ment	.16	.18	.52/.51	.24	.16	.17	.25/.24	.25	.14	.19	.31	.24
Practiced add with 8 or 9	.04	.08	.15/.14	.22	.03	.06	.32	.21	.03	.08	.40/.39	.30
Transfer add with 8 or 9	.03	.05	.10/.09	.21	.03	.06	.13	.22	.03	.06	.15/.14	.18

Note. Posttest means are adjusted for age, TEMA-3, and pretest. Some adjusted posttest means are different for different comparisons because adjustments are based only on participants involved in specific comparisons. [a]The first mean is for Guided Subtraction vs. Guided Use-a-10 and the second is for Guided Subtraction vs. Unguided Practice. [b]The first mean is for Guided Use-a-10 vs. Guided Subtraction and the second mean is for Guided Use-a-10 vs. Unguided Practice. [c]The first mean in for the Guided Subtraction vs. Unguided Practice and the second mean is for the Guided Use-a-10 vs. Unguided Practice.

H1: The Efficacy of the Guided-Subtraction Training

Analyses were initially conducted with classroom and school included as random-effect covariates. However, neither classroom nor school was found to be significant in any of the analysis of subtraction combinations. Thus, only the single-level analyses are presented.

H1a: Fluency.

For subtraction combinations practiced by the guided-subtraction and unguided-practice groups, the guided-subtraction group significantly outperformed the guided use-a-10 group, $F(1, 46) = 31.68$, $p < .001$, Hedges' $g = 1.46$, but not the unguided-practice group, $F(1, 46) = .68$, $p = .414$, Hedges' $g = 0.22$. Additionally, the unguided-practice group significantly outperformed the guided use-a-10 group, $F(1, 46) = 13.21$, $p = .001$, Hedges' $g = 0.95$. When the Benjamini-Hochberg adjustment for multiple comparisons was applied, statistically significant results remained significant.

As predicted for the subtraction combinations not practiced by any of the three groups, the guided-subtraction group significantly outperformed the guided use-a-10 group, $F(1, 46) = 4.07$, $p = .050$, Hedges' $g = 0.50$, and the unguided-practice group, $F(1, 46) = 4.26$, $p = .045$, Hedges' $g = 0.52$. The unguided-practice group did not outperform the guided use-a-10 group, $F(1, 46) = .15$, $p = .704$, Hedges' $g = –0.10$. With the Benjamini-Hochberg adjustment for multiple comparisons, the guided-subtraction group's performance was only marginally statistically significantly better than the other two groups.[6] Even so, these differences well exceeded the 0.25 criterion for effective practice set by the federal *What Works Clearinghouse* (IES, 2011).

In comparison to regular classroom subtraction training received by the guided use-a-10 group, the guided-subtraction intervention produced a net gain of 925% and 280% on the practiced and unpracticed add-1 combinations, respectively.[7] The corresponding figures for net pretest–posttest gains for the unguided-practice training are substantially less (500% and 17%, respectively).

Parenthetically, for the practiced addition complements of the subtraction combinations, ANCOVAs (which included the pretest practiced addition score as one of the covariates) indicated that—as might be expected—the guided-subtraction group significantly outperformed the unguided-practice group, $F(1, 46) = 14.25$, $p < .001$, Hedges' $g = 0.77$, and the guided use-a-10 group, $F(1, 46) = 18.58$, $p < .001$, Hedges' $g = 1.04$. Although the unguided-practice group did not significantly outperform the guided use-a-10 group, $F(1, 46) = 1.33$, $p = .255$, Hedges' $g = 0.26$, the effect size unexpectedly exceeded the 0.25 criterion for effective practice (IES, 2011).

A plausible reason for why the unguided-practice group outperformed the guided use-a-10 group on the addition complements, which neither group practiced, is that the former benefitted from a carryover effect of practicing the related subtraction combinations (which the guided use-a-10 group did not practice). Indeed, follow-up analyses indicated that the unguided-practice group did better on the four addition complements ($3 + 9$, $4 + 5$, $5 + 6$, and $7 + 7$) for which they practiced the related subtraction combinations (mean fluency = 0.38, $SD = 0.49$) than on the four addition complements ($4 + 6$, $5 + 7$, $6 + 5$, $6 + 7$, and $8 + 8$) for which they did not practice a corresponding subtraction combination (mean fluency = 0.23, $SD = 0.42$). Moreover the unguided-practiced group did somewhat better than the guided use-a-10 group on the first set of combinations (mean fluency = 0.31, $SD = 0.47$; mean Hedges' $g = 0.14$) but only equally well on the second set (mean fluency = 0.21, $SD = 0.41$; mean Hedges' $g = 0.05$).

H1b: Gain in Net Reasoning. Although the amount of deliberate (overt, accurate but slow) reasoning (Phase 2 responses) on trials of all types of combinations was small, the trends for unpracticed subtraction trials displayed in Figure 1 were consistent with H1b. Planned comparisons revealed that participants in the guided-subtraction condition exhibited greater gains in deliberate

[6]Although an alpha of .05 or less is usually used as the criterion for assessing "significance," the .05 level is not a fixed cutoff for statistical significance (Wainer & Robinson, 2003).

[7]A difference of differences method (100% x [Treatment net gain – Control net gain] / Control net gain) was used instead of a differential effect method (100% × [Treatment posttest – Control posttest] / Control net gain; Lipsey et al., 2012). Although the two methods yield the same results when the pretest scores of the Treatment and Control groups are identical, the same is not true for nearly identical pretest scores. Moreover, the difference of differences method yields results that are more easily interpreted. For example, the corresponding result for the guided subtraction versus use-a-10 comparison with transfer subtraction combinations differential effect methods is a confusing –900%. That is, it does not clearly indicate that while the guided-subtraction group made substantial gains, the guided use-a-10 group did not. In contrast, the 1,100% figure generated by the difference of differences method is easily interpreted as 11 times more effective. Similarly, the 0% figure for the unguided versus use-a-10 comparison for transfer subtraction clearly indicates that the former did not improve more than the latter, whereas the differential effect method yields the unclear result of –200%.

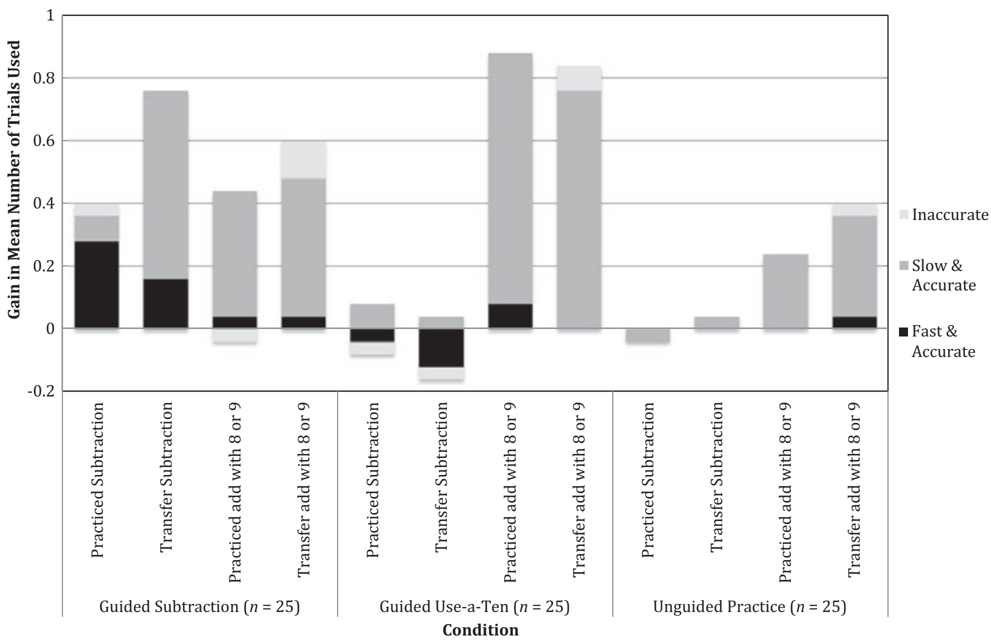

FIGURE 1 Net gain in the number of fluent, accurate but slow, and incorrect reasoning responses by condition and problem type.

and appropriate (nonfluent) reasoning strategy use on unpracticed subtraction trials than their peers in the guided use-a-10 condition, $F(1, 30.01) = 7.23$, $p = .005$ (1-tailed), Hedges' $g = 0.75$, and in the unguided-practice condition, $F(1, 34.52) = 5.82$, $p = .01$ (1-tailed), Hedges' $g = 0.67$. Gains with practiced subtraction trials were not significantly better than that for guided use-a-10 condition, $F(1, 48) = .06$, $p = .400$ (1-tailed), Hedges' $g = -0.07$, or the unguided-practice condition, $F(1, 48) = .28$, $p = .301$ (1-tailed), Hedges' $g = 0.15$.

H1c: Drop in Inefficient Strategies. As Table 7 shows, the rate of counted, correct but very slow, or incorrect responses decreased substantially—from about 50% to 15%—for all groups on all types of combinations between pretest and delayed posttest. As predicted, the decrease in the rate of practiced-subtraction trials scored as relatively inefficient or incorrect was significantly greater for the guided-subtraction group than for the guided use-a-10 group, $F(1, 44.84) = 33.81$, $p < .000$ (1-tailed), Hedges' $g = 1.62$, but only modestly more than the unguided-practice group, $F(1, 48) = 1.78$, $p = .094$ (1-tailed), Hedges' $g = 0.37$. More importantly, the proportion of relatively inefficient or incorrect responses to total responses on unpracticed subtraction trials decreased significantly more for the guided-subtraction group in comparison to both the guided use-a-10 group, $F(1, 41.80) = 9.04$, $p = .002$ (1-tailed), Hedges' $g = .084$, and the unguided-practice group, $F(1, 41.29) = 8.11$, $p = .003$ (1-tailed), Hedges' $g = 0.79$.

TABLE 7
Net Changes in the Proportion of Nonfluent Responses by Condition and Problem Type

	*Guided Subtraction Condition (*n = *25)*	*Guided Use-a-10 Condition (*n = *25)*	*Unguided Practice Condition (*n = *25)*
Practiced Subtraction (175 total responses)	−0.43	−0.15	−0.38
Transfer Subtraction (175 total responses)	−0.36	−0.15	−0.15
Practiced add with 8 or 9 (225 total)	−0.19	−0.48	−0.44
Transfer add with 8 or 9 (200 total)	−0.15	−0.35	−0.28

Note. Nonfluent responses include other correct but very slow ($\geq$ 6 seconds) answers, correct answers via counting, and incorrect answers. The last category, by far, contributed most to the proportions reported. Non-fluent responses do not include correct answers in less than 6 seconds or answers determined by reasoning.

H2: The Efficacy of the Guided Use-a-Ten Training

Analyses were also conducted with school and classroom included as random-effect covariates. As expected, school was not found to be significant in any add-with-8 or -9 analyses. Classroom was a significant factor for the unstructured-practice versus guided-subtraction comparison on the add-with-8 or -9 practiced combinations ($p = .001$), but the main effect of condition was significant with or without including classroom ($p < .001$). Classroom was also a significant factor for guided use-a-10 versus guided-subtraction ($p = .042$) and the unguided-practice versus guided-subtraction comparisons ($p = .040$) for use-a-10 unpracticed combinations. However, with or without including classroom, the main effects of condition were not significant. Since including classroom level effects did not alter any results and classroom was not significant for the majority of analyses, only the single level analyses are presented.

H2a: Fluency. For add-with-8 or -9 combinations practiced by the guided use-a-10 and the unguided-practice groups, the guided use-a-10 group significantly outperformed the guided-subtraction group, $F(1, 46) = 9.03$, $p = .004$, Hedges' $g = 0.78$, but not the unguided-practice group, $F(1, 46) = 1.18$, $p = .284$, Hedges' $g = –0.26$. In the latter analysis, it should be noted that the effect size exceeds IES' (2011) 0.25 criterion for effective practice in the direction favoring the unguided training. Additionally, the unguided-practice group significantly outperformed the guided-subtraction group, $F(1, 46) = 18.79$, $p < .001$, Hedges' $g = 0.99$.

For add-with-8 or -9 combinations not practiced by any of the groups, the guided use-a-10 group did not significantly outperform the guided-subtraction group, $F(1, 46) = .39$, $p = .536$, Hedges' $g = 0.16$, or the unguided-practice group, $F(1, 46) = .03$, $p = .855$, Hedges' $g = –0.05$. The unguided-practice group did not significantly outperform the guided-subtraction group, $F(1, 46) = 1.71$, $p = .197$, Hedges' $g = 0.30$.

H2b: Gain in Net Reasoning. As Figure 1 illustrates, the guided use-a-10 training resulted in greater gains in fluent and nonfluent reasoning on both practiced and unpracticed add-with-8 or -9 combinations than did the comparison conditions. Planned comparisons indicated that participants in the guided use-a-10 condition exhibited modestly greater gains (as indicated by effect size) in deliberate and appropriate (nonfluent) reasoning strategy use on practiced add-with-8 or -9 trials than their peers in the guided-subtraction condition, $F(1, 48) = .86$, $p = .180$ (1-tailed), Hedges' $g = 0.26$, and the unguided-practice condition, $F(1, 48) = 2.22$, $p = .071$ (1-tailed), Hedges' $g = 0.42$. More importantly, participants in the guided use-a-10 condition exhibited modestly greater gains (as indicated by effect size) in deliberate and appropriate (nonfluent) reasoning strategy use on unpracticed add-with-8 or -9 trials than their peers in the guided-subtraction condition, $F(1, 48) = 2.32$, $p = .067$ (1-tailed), Hedges' $g = 0.42$, and the unguided-practice condition, $F(1, 48) = 1.90$, $p = .087$ (1-tailed), Hedges' $g = 0.38$.

H2c: Drop in Inefficient Strategies. The drop in the inefficiency rate of practiced add-with-8 or -9 trials was substantially greater for the guided use-a-10 group than for the guided-subtraction group, $F(1, 48) = 17.11$, $p < .001$ (1-tailed), Hedges' $g = 1.15$, but not for the unguided-practice group, $F(1, 48) = .02$, $p = .450$ (1-tailed), Hedges' $g = -0.04$. More importantly, the rate of inefficient responses on unpracticed add-with-8 or -9 trials decreased more for the guided use-a-10 group than both the guided-subtraction group, $F(1, 48) = 1.98$, $p = .083$ (1-tailed), Hedges' $g = 0.39$, and the unguided-practice group, $F(1, 48) = 2.11$, $p = .077$ (1-tailed), Hedges' $g = 0.40$. Note that the effect size in both cases exceeded the IES' (2011) 0.25 criterion for effective practice in the direction favoring of the use-a-10 group.

DISCUSSION

Implications of the Results

Posttest fluency rate, gains in overt use of a reasoning strategy, and reductions in inefficient (correct but very slow, counted, or incorrect) responses varied by treatment group according to the 36 predictions outlined in Table 1, with four exceptions. Two exceptions were that the unguided-practice training had more than the predicted modest positive effect on the fluency of both practiced subtraction and add-with-8 or -9 combinations. The long lasting effect of the small (less than three dozen) extra dose of drill may have been due in part to the combination-specific practice effect observed by Rickard and colleagues (2012; Walker et al. 2013, 2014) and the impact of the program's entertaining computer games. The third exception is that the guided use-a-10 training did not affect the posttest fluency rate with the unpracticed add-with-8 or -9 combinations. However, other evidence, namely the significant increase in overt and nonfluent use of the use-a-10 strategy and the significant decrease in inefficient (incorrect or very slow) responses, indicated that many participants in this condition did learn a general, albeit deliberately executed, use-a-10 reasoning strategy. The fourth exception is that the guided-subtraction group did not exhibit significantly greater overt and deliberate use of the subtraction-as-addition reasoning strategy with practiced subtraction combinations. This was partly due to the fact that a number of participants had achieved fluency with the strategy. More importantly, though, the guided-subtraction group did exhibit significantly greater evidence of deliberate reasoning (Phase 2 behavior) with unpracticed subtraction combinations.

H1: Efficacy of Guided-Subtraction Training. The efficacy of the guided-subtraction intervention was corroborated. As indicated by a moderate effect size, the guided-subtraction

TABLE 8
A Comparison of the Guided Subtraction Program's Features With Those of Published Curricula

	Feature					
Curriculum	*Explicit Complement Principle[1]*	*Explicit Subtract-as-Add Strategy[1]*	*Same Numbers or Fact Families[1]*	*Empirical Inversion[1]*	*Explicit Part–Whole Relations[1]*	*Known Part in the Same Position[2]*
Bridges in Mathematics (Mathematics Learning Center, 2009)	—	—	—	√√[a]	—	N/A
Everyday Mathematics (UCSMP, 2005)	√[b]	√√[c]	√√[d]	—	√[e]	—
Go Math (Houghton Mifflin Harcourt, 2012)	√√	√√	√√	√	√	√
Math Connects (Macmillan/ McGraw Hill, 2009)	√√[f]	√√[f]	√√[f]	-[g]	√	√
Math Expressions (Fuson, 2006)	—	√	√√	–[h]	√	—
Saxon Math 1 (Larson, 2008)	—	√[i]	√√[j]	√[j]	—	N/A

(Continued on next page)

[1]A double check (√√) = clearly a prevalent characteristic and explicit; a single check (√) = an infrequent or implicit characteristic; and a dash (-) = not characteristic.

[2]A double check √√ = common part in the same position for related addition and subtraction equations; a single check (√) = sometimes in the same position, sometimes not; and a dash (-) = principle/strategy always illustrated with common part in a different position. N/A = subtraction-as-addition strategy not explicitly taught.

[a]For example, both operations are introduced as "hopping along a number line." An addition item such as 5 + 3 is represented as hopping to the right 5 and then 3; 5 + 3 = 8 is followed by 8 – 3, which is represented as hopping from 8 in the opposite direction (to the left) three times (8 – 3 = 5).

[b]Lesson 4.11 contains the following general hint (p. 309): "Some facts lead to other facts. There are not really that many different facts to learn or memorize. You will learn some easy 'shortcuts' in later lessons." Lesson 5.10 encourages the if-then shortcut with commuted combinations (p. 379): "If you know one fact, then you also know its tum-around fact." Although implied, an if-then shortcut (the complement principle) is not explicitly specified for subtraction.

[c]The subtraction-as-addition strategy is explicitly recommended in at least one subtraction lessons. For Lesson 6.5 (Using the Addition/Subtraction Facts Table to Solve Subtraction Problems), the instruction specify (p. 510): "To find the answer to 15 – 9, ask yourself: '9 plus what number is 15?"' However, an analogous strategy is not explicitly recommended when working with other models (fact triples such as 3, 5, 8 represented by dominoes or fact triangles). A teacher note in Unit 6 specifies (p. 500): "For many first graders, it is helpful to think about 8 – 5 = ? as 5 + what number? = 8. This approach encourages 'adding up' to subtract, a strategy that also works well with multidigit numbers." Encouraging thinking of subtraction as addition and using a counting-up strategy to solve for the missing addends can be a meaningful and useful for step toward, but not the same as recommending, using known sums as a vehicle for shortcutting subtraction computation—for deducing differences.

[d]Unit 6.3 introduces addition/subtraction fact families (four facts related by the complement principle such as 3 + 5 = 8, 5 + 3 = 8, 8 – 3 = 5, and 8 – 5 = 3) and finding both sums and differences using dominoes (fact triples: 3, 5, 8) and looking for the equivalent names for sums and differences using the same Addition/Subtraction Fact Table.

Unit 6.4 does the same sums using fact triangles. Unit 6.5 uses Addition/Subtraction Fact Table to determine differences and Fact Triangles to generate addition-subtraction fact families.

TABLE 8
A Comparison of the Guided Subtraction Program's Features With Those of Published Curricula *(Continued)*

	Feature					
Curriculum	*Explicit Complement Principle*[1]	*Explicit Subtract-as-Add Strategy*[1]	*Same Numbers or Fact Families*[1]	*Empirical Inversion*[1]	*Explicit Part–Whole Relations*[1]	*Known Part in the Same Position*[2]
Scott Foresman Mathematics (Charles & Hall, 2008)	√√[k]	–[k]	√√[k]	–[l]	√	N/A

[e] For Unit 3, teachers are instructed: "Point out that the domino has a part with 3 dots and a part with 5 dots and that the whole domino has 8 dots" (p. 234), and children practice translating various dominoes into part–part–whole diagrams. However, this is not continued later (e.g., Unit 6.2) when the focus is on addition facts and never related to subtraction facts. Indeed, there is no mention of "parts" and "wholes" in Unit 6.2. In brief, the models used in this unit, such as Fact Triangles only implicitly represent part–part–whole relations of related addition and subtraction combinations.

[f] Exercises include, for instance, (a) "Think $5 + 9 = \square$, so $14 – 9 = \square$"; (b) Circle the addition fact that will help you subtract $12 – 9 = \square$: $5 + 7 = 12$ or $5 + 6 = 11$, and (c) "To find $16 – 7$, I think $\square + 7 = 16$." Related addition and subtraction facts are defined as having the same numbers (e.g., $1 + 6 = 7$ and $7 – 6 = 1$ or $3 + 7 = 10$ and $10 – 3 = 7$).

[g] Exercises include showing two collections (e.g., 6 bees in a circle and 3 bees in a line) and asking a child to write one addition sentence ($6 + 3 = 9$ or $3 + 6 = 9$) and one subtraction sentence ($9 – 3$ or $9 – 6 = 9$). This might implicitly involve empirical inversion if a child thought: "6 and 3 more is 9, and 9 take away the 3 is, oh, 6 again."

[h] For Unit 2, a teacher is encouraged to show that $8 – 3 = 5$ and an iconic representation (~~OOO~~ |OOOOO) are the "reverse story of 5+3 = 8." However, this relation is not shown in the textbook or a (seatwork or homework) worksheet, with one exception (an exercise for students "on [grade] level" in the "Extending the Lesson—Differentiated Instruction/Activities for Individualizing" section).

[i] The program inconsistently related subtraction to addition. On one hand, for example, although adding with 1 is practiced immediately before subtracting by 1 is introduced in Lesson 44, the two are not related. Similarly, the addition facts with sums to 10 are introduced in Lessons 94 and 95. Although Lessons 101 and 102 begin with practicing these sums, they are not related to the focus of the lesson—subtracting a number from 10. Although Lesson 121, which introduces the "Differences of 1 Subtraction Facts," relates $10 – 9 = 1$ to using addition to check a difference ($1 + 9 = 10$), add-with-1 combinations are not recommended as vehicle for determining the differences of such subtraction combinations. In Lesson 68 adding 2 to an odd or an even number with linking cubes is reviewed immediately before subtracting 2 from odd and even numbers is introduced with cubes. The former is summarized by the rule that "adding 2 is like saying the next add even or odd number" and the later is summarized as "subtracting 2 is like saying the even or odd numbers backwards by 2's." Although these rules may be useful, addition again is used as a shortcut for determining differences. On the other hand, in Lesson 129, unknown subtraction combinations are linked to known addition combinations. For "subtracting half of a double" such a 14 – 7, 12 – 6, and 8 – 4, children are asked: "What do you notice about each these problems? [They are the doubles going the other way] ... How can we remember these answers? We will call these problems the 'subtracting half of a double facts."' See Footnote j for additional examples.

[j] In Lesson 132, blue and red linking cubes are used to model $4 + 1 = 5$ and then $5 – 1 = 4$, $1 + 4 = 5$ and then $5 – 4 = 1$ (examples of empirical inversion) to introduce the concept of "addition and subtraction families" and as a method for learning 9 – 4, 9 – 5, 9 – 3, and 9 – 6. Fact families, but not empirical inversion, are used to introduce the 7 – 3, 7 – 4, 8 – 3, and 8 – 5 subtraction facts in Lesson 134.

[k] Children are told: "You can use addition to help you subtract" and are encouraged to "think if $5 + 8 = 13$, then $13 – 8$" and "think $5 + 8 = ?$, so $13 – 8 = ?$ (the complement principle). Although text points out that "the sum of an addition fact is the first number in [a related] subtraction fact" (p. 138), nowhere is it explained that the difference (missing part) in a subtraction equation is an addend (known part) in a related addition equation. Put differently, instruction does not suggest how the complement principle can be used to determine unknown differences by using known addition combinations (e.g., solve $13 – 8 = ?$ By thinking "what do I have to add to 8 to make 8?").

[l] One enrichment worksheet illustrates, for example, 1 green interlocking cube added to 4 white cubes with equation $4 + 1 = 5$ and the statement, "Add the green cube. 4 and 1 more is 5." Immediately below this is the equation $5 – 1 = 4$ and the statement, "Take away the green cube. 5 take away 1 more is 4." Unfortunately, as an enrichment activity, it may not be used in most classes.

group achieved a greater fluency rate than both the unguided-practice and guided use-a-10 groups on the unpracticed subtraction combinations on the delayed posttest. (In comparison, the extra dose of practice provided by the unguided-practice training did not improve subtraction transfer above and beyond regular classroom instruction on subtraction.) The significant and moderately large gain (as indicated by effect size) in the accurate, appropriate, and adaptive use of reasoning (a Phase 2 strategy) and the decrease in inefficient or incorrect responses (characteristic of Phase 1 strategies) with unpracticed subtraction further indicate that many guided-subtraction participants used a Phase 2, if not a Phase 3, strategy on at least some unpracticed subtraction combinations. The pattern of results indicates that the guided-subtraction training was more effective in promoting the learning of a general subtraction-as-addition reasoning strategy that could be applied appropriately and adaptively, if not always quickly, than either supplemental subtraction practice or typical classroom instruction on subtraction.[8]

Retention and transfer of subtraction-as-addition reasoning strategy was achieved despite only 34 repetitions for each practiced subtraction combination and 26 repetitions for each related addition complement. This is substantially less practice than the hundreds or even thousands of repetitions per combination necessary to achieve memorization (by rote) of a basic fact specified by earlier models and computer simulations of arithmetic learning (e.g., Shrager & Siegler, 1998; Siegler & Araya, 2005; Siegler & Jenkins, 1989).

Not surprisingly, the guided-subtraction group, which practiced the addition complements, was more fluent on such combinations at the delayed posttest than the other groups, which did not practice the combinations. It is unclear whether this superior performance was due to the guided training (e.g., relating addition and subtraction to each other and part–whole relations), practicing related sums and differences together (e.g., the facilitating effect of practicing another related combination—one with same parts and whole), or merely extra addition practice. The latter, however, seems unlikely for two reasons. One is that addition combinations were practiced only 26 times each during the intervention. Another is that the unguided-practice group, which practiced subtraction and unrelated addition combinations, surprisingly became more fluent with the addition complements (which they did not practice) than the guided use-a-10 group, which practiced only combinations involving the addition of 8, 9, or 10. A possible contributing factor is that practicing $14 - 7 = 7$, for instance, facilitated learning the unpracticed addition combination $7 + 7$. Although clearly in need of further systematic research, it may be that learning subtraction combinations can impact learning of addition complements as well as vice versa.

H2: Efficacy of Guided Use-a-10 Training. The results indicated that the guided use-a-10 training was partially successful. Participants who received this training did not improve significantly or (as indicated by effect size) substantially in the fluent transfer of the use-a-10 strategy (i.e., achieve Phase 3). Nevertheless, as indicated by small effect sizes indicative of

[8]It might be argued that participants in the guided-subtraction condition became more adept at using addition knowledge to check computed or retrieved differences afterwards rather than using it to determine the difference itself. This is unlikely in the case of fluent responses because using addition requires three steps: determining the difference, determining the sum of a related sum, and comparing the two. This alternative hypothesis does not explain the changes in the use of overt reasoning strategies. Although it is a possible explanation for the decrease in inefficient and erroneous subtraction responses by the guided-subtraction group, it does not seem the most likely for several reasons. Although children have many informal strengths, there have been no reports of young children spontaneously inventing an add-back procedure for checking subtraction computations. Classroom instruction and curriculum materials did not focus on the inverse principle—the conceptual basis for the add-back checking procedure—or the add-back procedure itself. Many children are loath to do an extra computation to check another computation.

effective practice (IES, 2011), the guided use-a-10 group did outperform both comparison groups in terms of appropriate, overt reasoning and achieving a lower rate of inefficient (very slow unknown, counting-based, or incorrect) responses with add-with-8 or -9 combinations. Both results indicate that many guided use-a-ten participants used a Phase 2 (reasoning), if not a Phase 3, strategy. The achievement of Phase 2 is an important step away from using inefficient counting (Phase 1) strategies and toward using efficient retrieval (Phase 3).

Limitations

Although these findings are promising, seven limitations must be noted. First, the sample size for each of the groups was small (only 25 children per group). Several of the comparisons that resulted in effect sizes indicative of effective practice, but not statistically significant effects, may have been significant with a larger sample. Second, although most first graders in the study had at least one risk factor, 71% of the sample was Caucasian, 69% did not receive assistance for lunch, and 95% scored at the 26th percentile or higher in mathematics achievement. It is not clear if these effects would generalize to other categories of children (e.g., first graders with low mathematics achievement, those who are predominantly minority, or from low-income families). Third, this study was only designed to evaluate the impact of two specific reasoning strategies and thus cannot provide information relative to other such strategies. Fourth, these interventions were administered in a relatively guided research environment and provide evidence of their efficacy, but not their effectiveness in less guided (standard classroom or home) environments. Fifth, within a type of problem, such as subtraction, specific trained combinations and specific unpracticed combinations were not counterbalanced, which limits conclusions about the generality of the transfer effect. Sixth, as the same project personnel conducted both the training and testing, experimenter bias may have been a confounding factor. Arguments against such a bias are the uneven results (i.e., the nonsignificant fluency results with the use-a-10 program)—despite the hope by project personnel that both the guided programs would be successful—and several unexpected results.

Seventh, in the present study, the data needed to evaluate the learning rate (e.g., average length of time a child took complete each lesson) were not collected. Such an analysis might be useful in determining the more efficient of two interventions that produce similar results, such as Baroody et al.'s (2012, 2013) previous research that showed both unguided and guided training promoted fluency with practiced and unpracticed add-1 combinations. However, in the present study, unguided-practice and guided-subtraction programs were both effective in promoting fluency with practiced subtraction combinations, but only the guided-subtraction training was effective in promoting transfer (fluency with unpracticed items). Although it might be useful to evaluate which approach was more efficient in promoting fluency with the practiced subtraction combinations, the issue of comparative learning rate with unpracticed items is moot, as guided practice achieved transfer (was significantly better than either unguided practice or the active control), and the unguided practice did not (was not significantly better than the active control). Thus, even if the unguided practice was more efficient in promoting fluency with practiced items, it failed to promote the learning of a general subtraction-as-addition strategy. In brief, although it requires less effort to give a starving man a fish, in the long run, it is better to take the time to teach him how to fish.

CONCLUSIONS

The results regarding the guided-subtraction and use-a-10 training are consistent with the recommendations of the NRC (Kilpatrick et al., 2001) that the learning of reasoning strategies can be accelerated by conceptually based instruction. Likewise, the present results are consistent the NMAP (2008) conclusion that guided discovery can be an effective instructional tool in promoting the learning of arithmetic relations and promoting arithmetic fluency. Although both guided and unguided practice were more effective in improving fluency with practiced subtraction combinations than typical first-grade subtraction instruction, only the guided training was effective in promoting the learning of a general reasoning strategy that could be applied effectively, and in many cases, efficiently, to unpracticed subtraction combinations.

The present subtraction results diverge from Walker et al.'s (2013) results in three key ways. First, with practiced subtraction combinations, these researchers found that unguided practice was significantly more effective in promoting fluency than guided instruction that involved translating fact triangles into related equations. In the present study, the unguided practice and guided-subtraction training did not differ significantly. Second, with unpracticed subtraction items, Walker et al.'s two conditions did not differ significantly, whereas in the present study, guided-subtraction training produced significantly greater transfer than unguided practice. Third, contrary to the identical elements model (Walker et al., 2013; 2014), practice with one operation did transfer to another: from practiced addition complements to unpracticed but related subtraction items for the subtraction group and from practiced subtraction combinations to unpracticed but related addition complements in the case of the unguided practice group.

Five reasons might account for the three differences noted:

1. The frequency and duration of sessions varied (once a week for six weeks in Walker et al.'s 2013 study versus twice a week for 11 weeks in the present study).
2. Translating fact triangles into related equations may be useful for helping children see that there are families of addition and subtraction combinations, but in contrast to the present training, Walker et al.'s guided training did not involve explicitly relating this connection to the subtraction-as-addition strategy or part–whole relations.
3. Contrary to Brownell and Chazel's (1935) caution regarding "premature drill," the guided (and unguided) training of Walker et al. (2013) apparently provided practice before many participants had learned a general subtraction-as-addition strategy. In contrast, the present training involved three phases: first learning the strategy, then practicing strategy with scaffolding, and finally practicing the strategy in a way that more closely simulated everyday use (semirandom order, with no feedback except for correctness).
4. Walker et al. (2013) used only complete fact triangles that did not entail generating a difference (or sum), whereas the present training used this model in way that may be more likely to promote fluency: presented incomplete fact triangles that required producing the unknown difference and then followed up with complete fact triangles to provide feedback.
5. Whereas Walker et al. (2013) used fact triangles in isolation, the present training used this model—like most curricula—as one of several tools for the fostering the learning and fluency of the subtraction-as-addition strategy.

Specifically, several features of the guided-subtraction training may have contributed to its success and help explain why the present training fostered transfer, and the *Everyday Mathematics* (UCSMP, 2005) training completed by all participants in the Walker et al. (2013) did not foster transfer:

1. The complement principle, subtraction-as-addition strategy (as a means for solving subtraction in a relatively easy manner), and the necessity that related combinations have the same numbers were explicitly highlighted. Children were asked, for instance, "if $7 + 6 = 13$, then $13 – 6 = ?$" or "knowing that $7 + 6 = 13$ helps answer which subtraction combination?" (in which $13 – 6$ was a possible choice). They were told "to be of help an addition combinations needs to have the same part and whole as an unknown subtraction combination" (e.g., $7 + 6 = 13$, but not $4 + 6 = 10$, can help with $13 – 6 = ?$). Our review of seven Grade 1 mathematics curricula summarized in Table 8 indicates such explicit instruction is clearly characteristic of only some current primary curricula.
2. Some activities modeled empirical inversion (e.g., predicting the result of adding 5 to 3 and then immediately afterward predicting the result of taking 5 from 8). Theoretically, modeling the undoing process of subtracting the same number after adding it should help children understand the complement principle (e.g., why if $7 + 6 = 13$, then $13 – 6 = 7$), the addition-as-subtraction strategy (e.g., why $7 + 6 = 13$ can be helpful in answering $13 – 6 = \square$), and why certain combinations belong to the same "family" (e.g., why $7 + 6$, $6 + 7$, $13 – 6$, and $13 – 7$ are related). As Table 8 reveals, although two curricula used empirical inversion on a couple of occasions, only one program focused on using empirical inversion: *Bridges in Mathematics* (The Mathematics Learning Center, 2009).
3. Labeling and color coding the common whole and parts in each juxtaposed complementary equation underscored that complements shared the same parts and whole and the idea of addition-subtraction families. As Table 8 indicates, labeling of common elements is done explicitly but not consistently in most curricula.
4. The subtraction-as-addition strategy was illustrated with equations in which the known addend shared by related subtraction and addition combinations appeared in the same position (e.g., $8 – \mathbf{5} = \square$ was related to $\square + \mathbf{5} = 8$ but not $5 + \square = 8$). As Table 8 shows, some curricula—Macmillan/McGraw Hill (2009) *Math Connects* and *Scott Foresman-Addison Wesley Mathematics* (Charles & Hall, 2008)—present known addends for addition and related subtraction combinations in both the same and different position; some others—such as Houghton Mifflin Harcourt (2012) *Go Math*—present known addends only in different positions.
5. Complementary equations were juxtaposed in time (i.e., presented successively) and place (feedback portrayed addition and related subtraction equations one above the other). This was characteristic of all curricula reviewed in one form or another.

The present results suggest that the meaningful learning of a subtraction-as-addition strategy, which children can apply to unpracticed subtraction combinations, can significantly reduce the amount of time and practice needed to achieve fluency with basic subtraction combinations. What remains unclear and in need of further research is which feature or combination of features can best foster fluency with basic subtraction combinations. Given that there is no consensus about what features should be used or emphasized, such research is critical for increasing the meaningful memorization of subtraction combinations among schoolchildren.

Finally, the greater, if often slow, use of the subtraction-as-addition strategy by the guided-subtraction participants on unpracticed subtraction combinations indicates that Common Core (CCSSO, 2010) goal 1.OA.4 (to understand subtraction as an unknown-addend problem) is clearly grade appropriate. However, the low levels of fluency, the infrequent use of reasoning, and the high levels of inefficient or incorrect responses on unpracticed subtraction combinations by the comparisons groups indicates that typical textbook-based classroom instruction, including that which explicitly focuses on a subtraction-as-addition strategy, is largely ineffective. Clearly, more meaningful means of instruction, such as the computer-delivered guided-subtraction used in the present study, are needed to achieve Common Core goal 1.OA.4.

FUNDING

The research reported herein was supported by a grant from the Institute of Education Science, U.S. Department of Education, through Grant R305A080479 “Fostering Fluency with Basic Addition and Subtraction Facts”). Preparation of this manuscript also was supported in part by another grant from the U.S. Department of Education (R305B100017: “UIUC Postdoctoral Research Training Program in Mathematics Education”). The opinions expressed are solely those of the authors and do not necessarily reflect the position, policy, or endorsement of the Institute of Education Science or Department of Education.

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