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Extending the regression-discontinuity approach to multiple assignment variables

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ABSTRACT

The recent scholarly attention to the regression-discontinuity design has focused exclusively on the application of a single assignment variable. In many settings, however, exogenously imposed cutoffs on several assignment variables define a set of different treatments. In this paper, we show how to generalize the standard regression-discontinuity approach to include multiple assignment variables simultaneously. We demonstrate that fitting this general, flexible regression-discontinuity model enables us to estimate several treatment effects of interest.

1. Extending the regression-discontinuity approach to multiple assignment variables

Introduced in the early 1960s, the regression-discontinuity design (RDD) has enjoyed a resurgence in popularity during the past decade and is recognized widely as one of the most robust approaches for making causal inferences from natural experiments. In a standard RDD, participants are assigned to the treatment or control group according to an exogenously imposed cutoff on a single predictor, called a "forcing", "running", or "assignment" variable. Researchers can then draw causal inferences by comparing fitted outcomes for individuals on the margin, those who are "local to" the cutoff determined by the assignment rule.

As described by Cook (2008) and van der Klaauw (2008), researchers have applied this approach to estimate causal effects in a variety of disciplines. There is a large – and growing – literature on the RDD, including discussions of how to specify the underlying statistical models, under what conditions causal inferences can be drawn, how to handle situations in which participants are assigned without perfect compliance (i.e., a "fuzzy" discontinuity), and approaches for assessing the sensitivity of results to alternative model specifications. These topics are discussed in detail in a set of papers published in a 2008 special issue of this journal.

Almost all the previous work has focused on the application of a single forcing variable that defines an individual's assignment to treatment. In this paper, we generalize the standard RDD to include multiple forcing variables, modeling simultaneously discontinuities that arise when multiple criteria determine placement into several different treatment conditions. Such situations arise in many settings. We argue that researchers can better understand the complex relationships at play by incorporating the multiple discontinuities simultaneously into a single multi-dimensional regression-discontinuity model.

2. Examples of regression discontinuity with multiple assignment variables

There are many situations in which values on multiple forcing variables assign participants to a set of different treatment conditions. For example, such practices occur regularly in public education because students often take tests with clear cutoffs in several different subject areas. Many school districts reward teachers for improving student test scores in both mathematics

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and English Language Arts (ELA), so teachers who are judged to be effective in one subject earn a bonus while those who raise test scores in two subjects earn a greater bonus. In other districts, students must pass externally defined standards on tests in several subjects to avoid summer school, to be promoted in grade, or to graduate from high school; failing any of these tests defines different treatments that students must undergo (e.g., summer school in mathematics or summer school in ELA).

In public finance, state and local funding formulas can also have multiple criteria that determine the type or level of funding. For example, Leuven et al. (2007) examine the effects of a policy in the Netherlands in which schools receive extra personnel funding for having at least 70% minority students and extra computer funds for having at least 70% of students from any single minority group. Eligibility for different types of insurance or entitlement program eligibility may also be driven by several criteria, such as family size or family income.

Examples of multiple assignment variables are also common in politics. Pettersson-Lidbom (2008) estimates the effect of party control on fiscal policies and economic outcomes by examining localities in which one party received just over 50% of the vote. We could extend this analysis easily to investigate the effects of party control in more than one branch of government. Modeling discontinuities in both executive and legislative elections would enable researchers to draw causal conclusions about the effect of having the same or different parties in the two branches. In the two-party system present in the United States, there would be four different treatment conditions: Democratic control of the both branches, Republican control of both branches, and two cases with each party controlling one branch.

Thus, multiple assignment variables that assign individuals to a range of different treatment conditions abound in public policy settings. To date, however, analysts have reduced these questions to analyses of single discontinuities by ignoring important differences among treatment conditions[1] or by assuming homogeneous treatment effects across levels of the other forcing variable (see, for example, analyses of high school exit examination policies by Martorell (2004), Reardon et al. (2010), Ou (2010) and Papay et al. (2010).

It is important to differentiate the cases described above from the case in which multiple variables assign individuals to a single treatment. For example, some school districts assign students to a particular homogeneous summer school program if they fail to achieve benchmark scores on either the end-of-school-year mathematics or ELA test (see Matsudaira, 2008; Jacob and Lefgren, 2004). Thus, performance on two forcing variables – the two tests – determines assignment to one treatment-control contrast. The method described in this paper does not pertain to such situations. However, it would pertain if the content of a mandatory summer school program (i.e., the treatment) depended on which of the two tests a child failed.

3. Incorporating exogenous discontinuities on multiple forcing variables

In a basic regression-discontinuity set-up with standard notation, we observe a binary treatment indicator (W_i) and a forcing variable (X_i) with a related cutoff (c) that describes two treatment conditions ($W_i = 0$ and $W_i = 1$) as follows[2]:

$$W_i = 1\{X_i \geq c\}. \tag{1}$$

We observe individual outcomes (Y_i) and seek to estimate the conditional mean of Y at the cut score for individuals in the treatment ($W_i = 1$) and control ($W_i = 0$) groups. In other words, our primary parameters of interest are

$$\mu_l(c) = \lim_{x \to c^-} E[Y_i|X_i = x] \quad \text{and} \quad \mu_r(c) = \lim_{x \to c^+} E[Y_i|X_i = x]. \tag{2}$$

The causal effect of treatment (τ), for students at the cutoff, is the difference in these means:

$$\tau = \mu_r(c) - \mu_l(c). \tag{3}$$

The same logic applies when exogenously assigned cutoffs exist on multiple forcing variables. With J forcing variables (such that $W_{ji} = 1\{X_{ji} \geq c_j\}\ \forall j = 1, \dots, J$), we define 2^J treatment conditions. For any contrast between these treatment conditions, our primary parameters of interest are again the right-hand and left-hand limits on either side of the relevant cutoff. As the dimensionality increases, the challenges of interpretation and implementation become more complicated. As a result, we focus on the case with two forcing variables.

Having two forcing variables defines four different "treatment" conditions. Here, let X_1 and X_2 define the two assignment variables and c_1 and c_2 define the respective cutoffs. For each individual, we define W_{1i} and W_{2i} as follows:

$$W_{1i} = 1\{X_{1i} \geq c_1\} \quad \text{and} \quad W_{2i} = 1\{X_{2i} \geq c_2\}. \tag{4}$$

Thus, individuals can fall into one of four possible conditions:

Condition A: $W_{1i} = 0$ and $W_{2i} = 0$;
Condition B: $W_{1i} = 1$ and $W_{2i} = 0$;
Condition C: $W_{1i} = 0$ and $W_{2i} = 1$; and
Condition D: $W_{1i} = 1$ and $W_{2i} = 1$.

These conditions define four separate regions in the space spanned by the forcing variables, (X_1, X_2). Again, our parameters of interest are the conditional mean outcomes at the cutoff for individuals in each treatment condition. For example, the causal effect of $W_{1i} = 0$ instead of $W_{1i} = 1$ for individuals with $X_{2i} = c_2$ would be the difference between:

$$\begin{aligned} \mu_l(c) &= \lim_{x_1 \to c_1^-} E[Y_i|X_{1i} = x_1, X_{2i} = c_2] \quad \text{and} \\ \mu_r(c) &= \lim_{x_1 \to c_1^+} E[Y_i|X_{1i} = x_1, X_{2i} = c_2]. \end{aligned} \tag{5}$$

4. Estimating a regression-discontinuity model with two forcing variables

In any regression-discontinuity design, estimating these conditional means relies on assumptions about the relationship between Y and X near c. In the single-variable case, researchers often specify regression models with a variety of functional forms in a certain "window" around the cutoff. Recently, researchers have begun to relax these functional form assumptions by using "nonparametric" or "semi-parametric" approaches (e.g. Ludwig and Miller, 2007; Lee and Lemieux, 2010). As our parameters of interest are boundary objects and standard nonparametric smoothing strategies have poor boundary properties, we can estimate these limits with local linear regression (Fan, 1992; Hahn et al., 2001; Porter, 2003).

Imbens and Lemieux (2008) formalize this nonparametric approach and delineate its implementation. They describe a two-step process in which analysts: (1) choose an "optimal" bandwidth around the cutoff (labeled h^*) that minimizes a clearly

[1] For example, by assuming that funding for computers is interchangeable with funding for personnel.

[2] Throughout this paper, we focus on sharp regression discontinuities.

defined cross-validation criterion[3] by conducting a series of local linear regression analyses and (2) estimate the causal effect by conducting a local linear regression analysis using this optimal bandwidth. Given that causal inferences about the treatment effect focus on the cut score, this approach is identical in practice to fitting a single OLS model centered at the cut score, using observations within $\pm h^*$ of the cut score.

With multiple forcing variables, we need to model the outcome as a function of both forcing variables at the boundaries of these regions in order to estimate the effect of different treatment conditions at the cut scores. We can again estimate the relevant conditional expectations (such as those from (5)) using any standard technique. For example, we could take a parametric approach and use multiple regression analysis with higher-order polynomials to estimate the relationships between X_1, X_2, and Y near the joint cutoff. Given the advantages of the nonparametric methods, however, we generalize Imbens and Lemieux's (2008) approach.

As in the single-variable case, we can fit the requisite regression models in each region simultaneously, by specifying a single statistical model with 16 parameters—an intercept and slope parameters to accompany all 15 possible interactions among W_1, W_2, X_1, and X_2. We write the model with the two forcing variables (X_1^c and X_2^c) centered on their respective cut-points:

$$\begin{aligned} E[Y_i] = {} & \beta_0 + \beta_1 W_{1i} + \beta_2 W_{2i} + \beta_3 (W_{1i} \times W_{2i}) + \beta_4 X_{1i}^c + \beta_5 X_{2i}^c \\ & + \beta_6 (X_{1i}^c \times X_{2i}^c) + \beta_7 (X_{1i}^c \times W_{1i}) + \beta_8 (X_{2i}^c \times W_{2i}) \\ & + \beta_9 (X_{1i}^c \times W_{2i}) + \beta_{10} (X_{2i}^c \times W_{1i}) + \beta_{11} (X_{1i}^c \times X_{2i}^c \times W_{1i}) \\ & + \beta_{12} (X_{1i}^c \times X_{2i}^c \times W_{2i}) + \beta_{13} (X_{1i}^c \times W_{1i} \times W_{2i}) \\ & + \beta_{14} (X_{2i}^c \times W_{1i} \times W_{2i}) \\ & + \beta_{15} (X_{1i}^c \times X_{2i}^c \times W_{1i} \times W_{2i}). \end{aligned} \tag{6}$$

This model defines four surfaces in three dimensions, with intercepts at (c_1, c_2). Each surface lies in one of the four regions defined by the intersection of the values of W_1 and W_2, as follows:

$$\begin{aligned} & \text{(a) } E[Y_i | W_1 = 0; W_2 = 0] = \beta_0 + \beta_4 X_{1i}^c + \beta_5 X_{2i}^c + \beta_6 (X_{1i}^c \times X_{2i}^c) \\ & \text{(b) } E[Y_i | W_1 = 1; W_2 = 0] \\ & \quad = (\beta_0 + \beta_1) + (\beta_4 + \beta_7) X_{1i}^c + (\beta_5 + \beta_{10}) X_{2i}^c \\ & \qquad + (\beta_6 + \beta_{11})(X_{1i}^c \times X_{2i}^c) \\ & \text{(c) } E[Y_i | W_1 = 0; W_2 = 1] \\ & \quad = (\beta_0 + \beta_2) + (\beta_4 + \beta_9) X_{1i} + (\beta_5 + \beta_8) X_{2i}^c \\ & \qquad + (\beta_6 + \beta_{12})(X_{1i}^c \times X_{2i}^c) \\ & \text{(d) } E[Y_i | W_1 = 1; W_2 = 1] \\ & \quad = (\beta_0 + \beta_1 + \beta_2 + \beta_3) + (\beta_4 + \beta_7 + \beta_9 + \beta_{13}) X_{1i}^c \\ & \qquad + (\beta_5 + \beta_8 + \beta_{10} + \beta_{14}) X_{2i}^c \\ & \qquad + (\beta_6 + \beta_{11} + \beta_{12} + \beta_{15})(X_{1i}^c \times X_{2i}^c) \end{aligned} \tag{7}$$

In Fig. 1, we present a graphical representation of the four hypothetical surfaces. Note that although we depict these surfaces for many values of X_1 and X_2, our parameters of interest are only defined at the cut scores. In Fig. 2 (top panel), we show the relevant edges of these hypothetical surfaces in two dimensions by taking cross-sections of Fig. 1 when $X_1 = c_1$.

Examining these surfaces at the cut scores, we can describe several treatment effects of interest for individuals who fall near the joint cut scores. We illustrate these effects for individuals with $X_1 = c_1$ (effects (a) and (b)) in the bottom panel of Fig. 2. Here, the vertical distances between the edges represent the estimated causal effects identified by this approach, as follows:

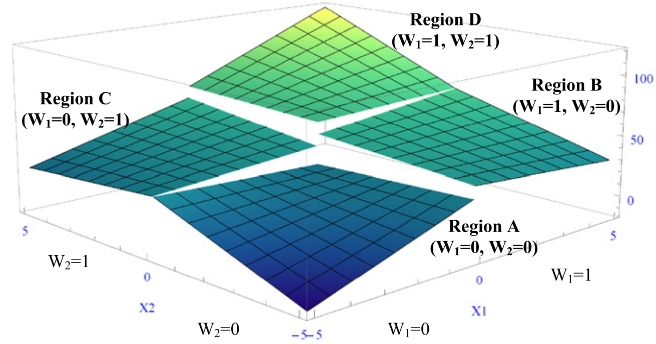

Fig. 1. Hypothetical population representation of the four surfaces (A, B, C, and D) from Eq. (1) defined by the two cut scores, $X_1 = 0$ and $X_2 = 0$.

$$\begin{aligned} & \text{(a) Effect of } W_1 = 1 | W_2 = 0, X_1 = c_1: \\ & \quad \beta_1 + \beta_{10} X_{2i}^c \\ & \text{(b) Effect of } W_1 = 1 | W_2 = 1, X_1 = c_1: \\ & \quad (\beta_1 + \beta_3) + (\beta_{10} + \beta_{14}) X_{2i}^c \\ & \text{(c) Effect of } W_2 = 1 | W_1 = 0, X_2 = c_2: \\ & \quad \beta_2 + \beta_9 X_{1i}^c \\ & \text{(d) Effect of } W_2 = 1 | W_1 = 1, X_2 = c_2: \\ & \quad (\beta_2 + \beta_3) + (\beta_9 + \beta_{13}) X_{1i}^c \end{aligned} \tag{8}$$

Fig. 2 illustrates the flexibility of this approach in representing several treatment effects of interest simultaneously. Note that by setting both $X_1 = c_1$ and $X_2 = c_2$, we can recover the effects of each of the treatment conditions for individuals at the joint cutoff. For example, in the bottom panel of Fig. 2, we can describe the effect of $W_1 = 1$ for individuals at the cutoff on *both* X_1 and X_2. Here, the effect for individuals at the cutoff with $W_2 = 0$ is β_1, the vertical height of the line on the left, while the analogous effect for individuals with $W_2 = 1$ is $\beta_1 + \beta_3$, the vertical height of the line on the right. Our approach enables us not only to describe these effects at the joint cutoff, but also to examine explicitly how the causal impact of assignment by one forcing variable differs by levels of the other. The set of models in (8) describes the more nuanced relationships at different levels of X_1 and X_2, at least local to the cut scores.

4.1. Implementation

To fit our model in a data sample, we must first choose appropriate joint bandwidths (labeled h_1^* and h_2^*) to govern our smoothing for X_1 and X_2 simultaneously. As we have two assignment variables, our bandwidth is actually a two-dimensional area in the (X_1, X_2) plane bounded by (h_1^*, h_2^*). To estimate h_1^* and h_2^*, we generalize the iterative cross-validation procedure described by Imbens and Lemieux (2008). For each observation, at each point on the (X_1, X_2) grid, we use local linear regression analysis[4] – within an arbitrary bandwidth (h_1, h_2) – to estimate a

[3] Imbens and Kalyanaraman (2009) have proposed a plug-in estimator for the optimal bandwidth. The properties of this estimator have not been proven for forcing variables with discrete support, such as test scores often used in regression-discontinuity designs. As a result, we choose to generalize the cross-validation criterion recommended by Imbens and Lemieux (2008).

[4] There is an interesting debate in the literature concerning how to weight different data points in the bandwidth in fitting these local linear regressions. Some analysts recommend simply using OLS regressions (i.e., applying a rectangular kernel) (e.g. Imbens and Lemieux, 2008), while others argue that triangular or other, more complicated, kernel weightings may lead to more appropriate estimation (e.g. Ludwig and Miller, 2007). For simplicity and interpretability, we recommend using OLS regressions. In practice, sensitivity of the results to kernel choice will be reflected in sensitivity to bandwidth choice because bandwidth selection is simply a more extreme version of a choice of weights (see Lee and Lemieux, 2010).

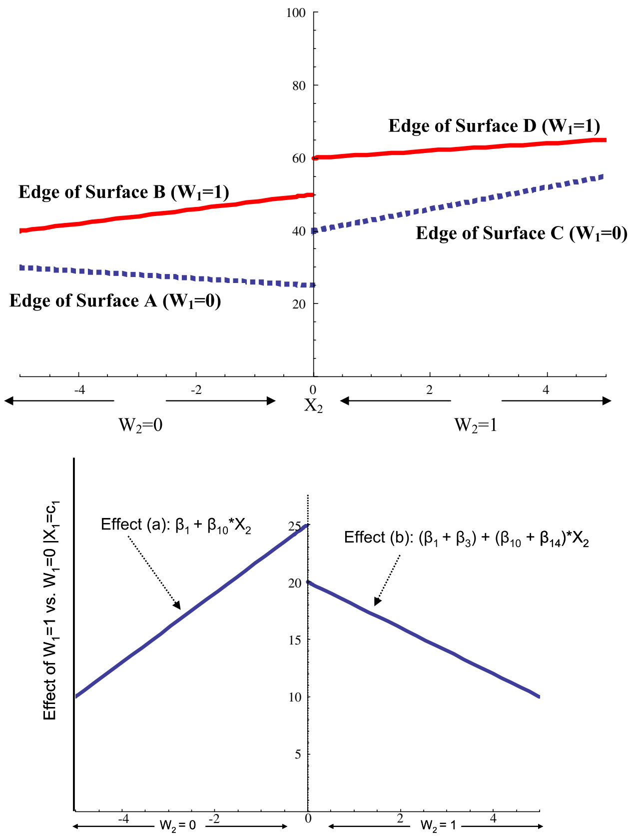

Fig. 2. Cross-section of Fig. 1 at the cut scores, showing the predicted value of Y, when $X_1 = c_1$, at different values of X_2 (top panel) and hypothetical plot showing the causal effect of $W_1 = 1$ at different levels of X_2. Note that the orientation is opposite to the orientation of Fig. 1 (negative values X_2 are to the left here).

fitted value of the outcome at that point:

$$\hat{\mu}(X_{1i}, X_{2i}, h_1, h_2) = \hat{\gamma}_0 + \hat{\gamma}_1 X_{1i} + \hat{\gamma}_2 X_{2i} + \hat{\gamma}_3 (X_{1i} \times X_{2i}). \tag{9}$$

In each case, we limit the observations used to estimate $\hat{\mu}(X_{1i}, X_{2i}, h_1, h_2)$ to the region in which the grid-point falls and estimate $\hat{\mu}(X_{1i}, X_{2i}, h_1, h_2)$ as if it were a boundary point in order to mirror the regression-discontinuity approach that estimates limits defined at the boundary points of the region. Thus, the sample used to estimate $\hat{\mu}(X_{1i}, X_{2i}, h_1, h_2)$ differs in each of the four regions A, B, C, & D defined in Fig. 1. For instance, in region A (where $X_{1i} < c_1$ and $X_{2i} < c_2$), we estimate $\hat{\mu}(X_{1i}, X_{2i}, h_1, h_2)$ at grid-point (X_1, X_2) using only observations in the area $(X_{1i} - h_1 \le x_1 < X_{1i}) \cap (X_{2i} - h_2 \le x_2 < X_{2i})$, for every value of X_1 and X_2 in the region. By contrast, in region D ($X_{1i} \ge c_1$ and $X_{2i} \ge c_2$), we use observations in the area $(X_{1i} \le x_1 \le X_{1i} + h_1) \cap (X_{2i} \le x_2 \le X_{2i} + h_2)$.

We then compare our fitted values to the observed values, across the entire sample, using the generalized Imbens & Lemieux cross-validation criterion:

$$\mathrm{CV}_Y(h_1, h_2) = \frac{1}{N}\sum_{i=1}^{N}(Y_i - \hat{\mu}(X_{1i}, X_{2i}, h_1, h_2))^2. \tag{10}$$

We vary the joint bandwidth dimensions (h_1 and h_2) systematically and, for each bandwidth pair, we obtain a value of this cross-validation criterion. Our optimal joint bandwidth, h_1^* and h_2^*, is the pair of bandwidths that minimizes the CV criterion.

This procedure uses the entire sample to develop optimal bandwidths for an analysis in which the objects of interest are the parameters at the joint cut scores. We then fit our full model from Eq. (6), applying our joint bandwidth. Given that our parameters of interest are defined at the cut scores, we can focus our attention on the single local linear regression analysis that uses only observations with forcing variable values within one optimal bandwidth on either side of the relevant cut scores. As in the single-variable case, we can interpret this single model centered at (c_1, c_2) *parametrically* for observations local to the cut score and use the standard errors to conduct appropriate statistical tests.[5]

One key advantage of the approach that we have laid out is that it provides an externally defined criterion for bandwidth choice, rather than leaving the investigator to make subjective decisions about which bandwidths provide the most credible findings. However, in all cases, we recommend assessing whether findings are robust to bandwidth choice by systematically refitting the primary model using a range of plausible bandwidths. In doing so, we recommend focusing on the magnitudes of parameter estimates in addition to the results of hypothesis tests. As bandwidths grow smaller, estimates will necessarily become less precise, but any convincing substantive story should remain the same.[6]

5. Discussion

Natural experiments in which units of analysis are assigned to several different treatment conditions based on values of multiple forcing variables are quite common. Typically, researchers have analyzed these cases by creating a single composite forcing variable or by focusing on one of the assignment variables and examining separately the effects for individuals on either side of the second cutoff. While sensible, these approaches have important limitations.

The first method implicitly blurs distinctions between different treatments by evaluating them as a single condition. The approach described here permits the articulation of several different treatment conditions and the examination of the effects of different combinations of treatments. The second approach implicitly provides estimates of average effects rather than distinguishing among individuals at different levels of a second assignment variable. Such simplified analyses can be misleading. Our hypothesized model represents a more nuanced way to answer many different questions of interest. Rather than estimate the average effect of a single treatment, we can explore how this effect differs by levels of a second assignment variable.

The approach we describe does have limitations. First, like any regression-discontinuity design, it has limited external validity: causal effects are only identified for observations in the immediate vicinity of the cut scores. Second, statistical power is a key issue. Estimating these effects precisely requires a substantial density of data points near the joint cut scores. Nonetheless, the approach that we describe in this paper has a number of advantages over more conventional methods for addressing causal questions in situations in which multiple forcing variables assign individuals exogenously to different treatments. In particular, with sufficient data, the method provides a more complete picture of the relationship between different combinations of treatments and the outcome of interest.

5 It is worth noting that we can get $E[Y|W_1 = 0, W_2 = 0] = \sum_{W_1=0,W_2=0} \frac{E[Y|W_1=0,W_2=0]}{N_{00}}$ within the bandwidths, where N_{00} is the total number of observations within the bandwidths for which $W_1 = 0$ and $W_2 = 0$. Analogous results exist for the other expressions in Eq. (7) in each of the relevant regions. By comparison, we can get $E[Y|W_1 = 0]$ and $E[Y|W_1 = 1]$ through a standard, univariate regression-discontinuity approach.

6 Furthermore, credible regression-discontinuity designs will demonstrate that key assumptions (including the exogeneity of the assignment cutoff) are satisfied. See the 2008 special issue of the Journal of Econometrics and Lee and Lemieux (2010) for recommended tests.

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