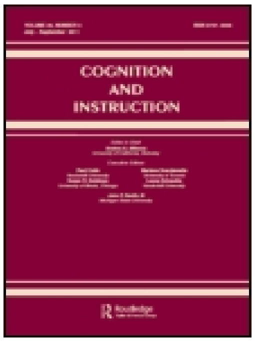

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Blake's Development of the Number Words "One," "Two," and "Three"

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A mother tracked her preschooler's number word development daily from 18 to 49 months of age. Naturalistic observations were supplemented with observations during structured (Kumon) training and microgenetic testing. The boy's everyday use of "two" did not become highly reliable and selective for 10 months (at 28 months), emerged later than that of words representing less abstract concepts, and was used in a relatively abstract manner to describe various visible pairs of items. He quickly generalized "two" to partially visible collections and then those that were not visible. Highly reliable use of "one" and "two" appeared to develop simultaneously, before he started using a plural rule, and before he could put out two items upon request. Reliable and accurate use of number words in everyday situations, particularly child-initiated efforts, preceded such use in the contexts of the Kumon training and microgenetic testing, both of which involved adult-initiated tasks. Educational implications include underscoring differences among the first number words by contrasting, for instance, one with two, and pointing out non-examples of a number ("not two") as well as a wide variety of examples, such as "two blocks, two hands, two socks, two airplanes."

A cardinal concept of number entails simultaneously viewing a collection as composed of individual items or units and as a whole or unit total (Spelke, 2003; von Glasersfeld, 1982). Such a concept also involves recognizing that the total is a commonality shared with other (different) examples of the number (Piaget, 1965). Although there is broad agreement that the cardinal concepts of *one*, *two*, and *three* provide a foundation for the conceptual understanding of one-to-one object counting, larger numbers, and simple arithmetic (Klahr & Wallace, 1973, 1976; Le Corre & Carey, 2007; Sarnecka & Carey, 2008; Schaeffer, Eggleston, & Scott, 1974; Sophian, 1992; Starkey & Cooper, 1995; Wagner & Walters, 1982; Wynn, 1990, 1992), how these concepts are formed remains unclear. In particular, because number research has focused on preverbal children's ability to discriminate among small collections and preschoolers' learning of counting skills, relatively little research has focused on how pre-counting children learn the first few number words and connect these words to cardinal concepts of the small numbers (Mix, 2009; Mix, Sandhofer, & Baroody, 2005).

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METHODOLOGICAL APPROACH

Mix (2009) noted that the methodology of the research that has been done on the emergence of verbal number knowledge has limitations. One-time, cross-sectional, and longitudinal studies have typically used de-contextualized and abstract tasks, which may underestimate number competence. Such methods and even naturalistic observations and speech transcript studies do not sample development continuously or even densely and can miss important transitions.

Although all research has limited external validity due to restricted samples, tasks, or testing conditions, a diary-based case study has particularly meager generalizability, because every child is a special case. Nevertheless, it has some key advantages and can serve a number of important roles. Diary studies that entail observing children in their homes or other natural settings by a parent can provide a rich basis for revealing the intricacies of number development (Blevins-Knabe & Musun-Miller, 1996). Such studies can provide relatively continuous or dense data sampling and may capture developmental transitions missed by other methods. They can capture spontaneous behaviors, particularly those initiated by a child, in real and complex situations. That is, such research has relatively strong ecological validity. Diary-based case studies can, taken together, provide converging evidence with relatively strong ecological validity or can corroborate findings by other types of research, which may have limited ecological validity. Finally, among the most useful purposes of a case study are establishing an existence theorem (a possibility) or raising questions for further, more systematic research.

Unfortunately, relatively few studies (e.g., Anderson, 1991; Baroody, 1987; Baroody & Price, 1983; Durkin, Shire, Riem, Crowther, & Rutter, 1986; Fuson, 1988; Lawler, 1981; Mix, 2009; Saxe, Guberman, & Gearhart, 1987; Walkerdine, 1988) have involved examining preschool number and number-word development in their natural environment or using parental observations. The present study used a combination of methods, including a mother's relatively continuous observations of her child's behavior in natural settings, to examine the long-term development of functional *verbal number recognition* (selectively and reliably using a number word to identify the total of a small collection without counting). Everyday observations were supplemented with observations in a formal instructional setting and intermittent formal testing sessions. This was done, in part, because the child's everyday use of number words or skills was not frequent, to ensure experience and practice with collections and number words up to five, and to provide converging evidence for conclusions based on everyday observations. An unanticipated purpose was to provide a basis for comparing informal and spontaneous behavior with performance in formal and adult-initiated situations.

ISSUES

The present report bears on five issues, which are summarized in Table 1—along with the predictions of various views.

Issue 1: Are the earliest uses of the number word "two" functional (reliably and selectively associated with collections of two) and achieved relatively early because number words map onto pre-existing concepts or categories? Number words are a part of children's vocabularies almost from the time they begin to speak at 18 to 24 months (Fuson, 1988). Existing evidence, though, is mixed on whether the earliest uses of these words are functional.

TABLE 1

Issues Examined, Predictions of Different Theoretical Perspectives, and Views Supported by the Present Results (√)

	Theoretical Perspective				
	Cardinal Concepts Constructed Solely From Experience With Symbols	*Nonverbal Proto-Number Categories Provide a Basis for Constructing Verbal Cardinal Concepts*			*Innate Nonverbal Cardinal Concepts*
		Conceptual Categories	*Object-Tracking Models (Perceptually Based Categories)*		
Issue	*Simultaneous View (Baroody et al.,2006)*	*Mental Models View (Mix et al., 2002)*	*U-Shaped Development Hypothesis (Gibson & Shusterman, 2009)*	*Grammatical-Number or n-Knower View*	*Continuity or Privileged-Domain Hypothesis*
1: Are the earliest uses of "two" functional (and relatively early because number words readily map onto pre-existing concepts or categories)?	No (No).√	No (No).√	Yes [then becomes less so because of interference before becoming fully functional again] (Yes).	No [It may take time to map number words to existing number-like categories, and then "two" indicates any plurality] (Yes).	No [It may take time to map number words to existing nonverbal cardinal concepts] (Yes).
2: Is early functional use of a number word relatively abstract or concrete over—					
(a) various homogeneous collections	(a) No.	(a) No.	(a) Yes.√	(a) N/A[a]	(a) Yes.√
(b) and heterogeneous collections or modalities?	(b) No.√	(b) No.√	(b) Yes.	(b) N/A.	(b) Yes.
3: What is the relation between the functional development of "one" and "two"?	Simultaneous development possible depending on number training, because contrasting terms & examples important. √	Sequential development.	Simultaneous development as both words build readily on a pre-existing category	Sequential development.	Simultaneous development possible if verbal number training builds on both pre-existing concepts simultaneously. √
4: (a) Is a plural rule necessary for functional verbal number recognition of two?	(a) No.√		(a) No.√	(a) Yes.	
(b) Does the development of this rule then interfere with functional use of "two"?	(b) No.√		(b) Yes.	(b) N/A.	
5: Does functional verbal number recognition of two coincide with the two-knower stage (verbal set production of two)?	No.√ [Functional VNR is a prerequisite of may precede verbal set production of 2.]	No.√	No.√	Yes.	

Note. N/A indicates that a view is agnostic with regard to the issue.

[a]Issue #2 is not explicitly addressed by a grammatical-number or *n*-knower view, but extensive use of the verbal set-production task has revealed no differences across various homogeneous collections.

As with other words, children seem to first use "two" and "three" with little specific meaning or to indicate "many" instead of a specific quantity (Ginsburg, 1977; Mix, Huttenlocher, & Levine, 2002; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007; von Glasersfeld, 1982; Wagner & Walters, 1982; Wynn, 1992). One possible way children might construct functional knowledge of small numbers is suggested by the simultaneous view (Baroody, Benson, & Lai, 2003; Baroody, Lai, & Mix, 2006). Specifically, as with many other concepts, children may construct a number concept via an inductive process involving visual examples and non-examples. Words can facilitate infants' and toddlers' formulation of kind concepts—conceptual classes based on causal relations or functions. Common labels (e.g., the same noun used to name different examples of a class) appear to support kind categorization by infants (Balaban & Waxman, 1997; Dueker & Needham, 2003; Waxman & Markow, 1995) or toddlers (e.g., Sandhofer & Smith, 1999).

Similarly, as children see a number word used in conjunction with various visual examples, are corrected for misapplying the term to non-examples, and observe other number words used with non-examples (e.g., "that's three, not two, cookies"), they can construct a well-defined cardinal number concept. For example, seeing different pairs of items labeled "two" can prompt the discovery that appearances (physical characteristics such as shape, color, or arrangement) are not relevant and reveal the commonality that all such examples involve more than one (a plurality). Recognizing that "two" does not apply to collections of three or more items can further prompt them to induce that "two" applies only to couples or pairs. As a result of piecemeal context-specific mappings between concrete instances and a number word (or other symbol such as a numeral), children may simultaneously develop an accurate and general cardinal number concept and functional verbal number recognition (Mix, 2009).

In contrast, Gibson and Shusterman (2009) hypothesized that number words map directly on the object-file representations of the individual elements in a collection. Such nonverbal, non-cardinal representations effectively preserve total number while ignoring the differences among collections and, thus, provide ideal pre-existing equivalence classes that can (a) serve as referents for small number words and (b) facilitate the development of verbal number recognition (Mix, 2009).

Similarly, the continuity hypothesis suggests a direct mapping process (Gallistel, 2007; Gelman & Gallistel, 1978). On this view, number is a biologically primary cognitive domain (Geary, 1994). That is, innate cardinal number concepts help toddlers to focus on numerical aspects of their environment and support the ready learning of the first few number words.[1] Although it might take some time to map specific number words to specific pre-existing number concepts, functional use of a first number word should emerge in relatively short order and at about the same time or soon after the acquisition of other first (biologically supported) functional words.

Indeed, Fuson (1988) reported that her daughters first used cardinal number words functionally. For example, the earliest entry for Adrienne (1 year, 8 months) included: "You know *two* and use it correctly in new situations; *two* means one in each hand" (Table 1–2 on p. 18), not simply "more than one" or "many." Using CHILDES data that excluded children's references

[1]Gallistel and Gelman (1992) hypothesize an innate, nonverbal counting system governed by the same counting principles as govern verbal counting. Moreover, this innate system involves a sequential list of nonverbal symbols that results in reasonably accurate cardinal representations of small numbers and permits nonverbal infants to discriminate among small even heterogeneous collections or across different modalities.

to age, ritual counting, or object counting, Gibson and Shusterman (2009) adduced evidence of U-shaped development: 18- to 24-month-olds used “two” to describe pairs of items exclusively, 25- to 36-month-olds increasingly over-generalized its use to mean “more than one,” and 37- to 39- month-olds increasingly used “two” exactly. They concluded other learning interferes with children’s initial exact understanding of “two,” which then is regained. In a case study of her son Spencer from 12 to 38 months of age, Mix (2009) found that the toddler’s use of “two” paralleled Gibson and Shusterman’s results. The boy’s first cardinal use of number words appeared at 23 months of age and, for 3 months, the child nearly always applied it to pairs only—indicating he was not implying “many.” Between 26 and 30 months, Spencer’s use of “two” became less reliable (was over applied to larger collections in 5 of 23 efforts). By 35 months, he achieved 100% accuracy when spontaneously using “two.”

However, existing empirical support for the initial exact use of “two” is not conclusive. As Mix (2009) noted, Fuson’s (1988) observations were not dense or comprehensive. Indeed, Fuson cautioned, “During much of the diary period, I was not engaged in research on young children’s number concepts. Thus, the entries were not systematic or even focused on what I might consider crucial if I were keeping such diaries” (p. 18).

The basis of Gibson and Shusterman’s (2009) conclusion (the CHILDES data matrix) is a compilation of data from a number of sources and samples of children across ages (i.e., is cross-sectional in nature). Another potential limitation is number word utterances may not have been systematically checked for overly broad use. For example, the wife of the second author happily informed him that their 22-month-old daughter Arianne had learned the word “two” because she had correctly applied it to several pairs. The father held up two fingers and asked, “How many fingers Arianne?” The child gleefully replied, “T-w-o daddy!” Sadly, further checking with non-examples of two (holding up three, four, five, and ten fingers) elicited the same gleeful response.

Finally, Mix’s (2009) results may have been the byproduct of how Spencer learned a cardinal use of “two.” Specifically, at 23 months the toddler began to mimic his nanny’s number frame: “Two [noun]. One, two.” The first and possibly subsequent uses of this frame involved an example previously used by the nanny. It is not known whether the nanny used the number frame with collections larger than two. Between 23.5 and 24.5 months, Spencer used a simpler frame “Two [plural noun]” with naturally occurring pairs (two ducks on the television, his eyes, his feet, and two balls on a couch). He began using a number frame that was a cross between his nanny’s and his simpler frame (e.g., “Feet, feet. Two feets”) at 24.5 months and abandoned it at 26 months. The increase in his error rate (between 26 and 30 months), then, corresponds with the use of a strategy three times removed from an imitation of his nanny’s number frame and bearing no resemblance to it.

The present research involved examining longitudinally and continuously whether the earliest use of “two” is exact and when this functional use of a child’s first number word(s) emerged compared to a functional use of other words.

Issue 2: Is early functional use of a number word relatively abstract or concrete? Proponents of the continuity hypothesis have concluded that infants’ ability to discriminate among small numbers over different homogeneous/heterogeneous collections or even different modalities indicates a relatively general concept of the small number (Starkey, Spelke, & Gelman, 1990; but cf. Mix, Huttenlocher, & Levine, 1996). According to object-tracking models, the preverbal categories that provide a basis for cardinal number concepts are similarly abstract (Mix et al., 2002). According to the continuity hypothesis and object-tracking models, nonverbal number or number-like representations are also abstract in the sense that they might serve mental representations of non-visible collections (e.g., comparing a visible collection with a previously viewed but now hidden collection). After a mapping phase, abstract nonverbal number concepts or categories should enable children's first functional use of number words to be relatively general—to encompass differently composed homogeneous collections, different modalities, mentally represented items, or heterogeneous collections.

In contrast, the simultaneous view (Baroody et al., 2006) suggests that a child's initial functional use of number words may be relatively concrete in nature and only gradually become more abstract. Concrete-to-abstract development can involve moving from local to general concepts (Lawler, 1981). In this vein, Resnick (1992) proposed that mathematical development evolves from context-specific thinking to increasingly context-free thinking (e.g., from thinking about specific cases to using generalizations). Concrete-to-abstract number development can also involve moving from concepts that can be triggered only by a perceptual input such as a visible collection to concepts that are relatively independent of what is immediately perceived. According to a constructivist theory of abstraction (Steffe & Cobb, 1988; von Glasersfeld, 1991), an initial *perceptual abstraction* level requires perceptual input, such as seeing a pair of items, to activate memory of *two.* In time, a child increasingly internalizes a concept of two. At a relatively deep level of internalization, two can be called up in the absence of perceptual input and even re-represented (i.e., applied to previously unseen or unfamiliar examples). Yet other possible dimensions of concrete-to-abstract development might entail moving from perceiving number visually and to doing so then through other modalities and moving from recognizing as "two" only collections of naturally occurring pairs (homogeneous collections) to doing so with heterogeneous collections as well.

The mental models view similarly suggests increasing abstraction of the mental models and perhaps the cardinal number concepts based on them. For example, Mix et al. (2002) noted that because the symbols or pointers of a mental model are grounded in object individuation and applied in parallel, they are applied to collections of objects presented simultaneously but not to sequential and spatially non-distinct events or sounds. The model also predicts recognizing equivalencies with homogeneous collections before those involving heterogeneous collections.

Mix (2009) found that Spencer's first use of number words was a non-quantitative identification of written numbers (promoted by his parents) and his subsequent first cardinal use of "two" was applied exclusively to a pair of shoes before being applied to three other kinds of natural pairs a week later. She also found that Spencer engaged in the relatively abstract process of labeling mentally represented collections only rarely but did so throughout the study. Specifically, she concluded that Spencer used "two" in relatively abstract (as well as concrete) ways throughout his development.

However, Mix's (2009) observations regarding the local application of the cardinal term "two" and its subsequent application to represented collections may have occurred during the mapping phase—before Spencer achieved highly selective or functional use of the number word on a permanent basis. Moreover, she observed that the toddler used the word "too." So, it is not clear that early abstract uses of "two"/"too" referred to a cardinal number or simply indicated "another." Mix also noted that Spencer spent much time with a nanny who regularly labeled pairs of things "two," and these examples were not necessarily included in the researcher's diary.

Thus, it is unclear whether Spencer's abstract uses of "two" were spontaneous generalizations or merely imitations of the nanny. Finally, the case of Adrienne (Fuson, 1988) is inconsistent with that of Spencer in two respects. Although the former (at age 1 year, 8 months) apparently used "two" in the single context where she could hold each like item in a hand, the girl did so with various nouns (e.g., "two cups, two cookies"). Moreover, "two" was used in this manner (to refer to simultaneously present items) *before* mentally representing an entity not physically present (sequentially presented) at 1 year, 10 months: "Sam [cat] tape [tail]. Tshad [a dog not in the room] tape. Two tapes" (Table 1–2, p. 18). However, as previously noted, Fuson's observation were not dense or comprehensive. Also, "there has been relatively little [other evidence of] young preschoolers [successfully representing] sequential entities" (Fuson, 1988, p. 28).

The present research entailed evaluating systematically the generality of a child's first *functional* use of a number word and whether subsequent use became increasingly abstract, including sequential or remembered entities. Importantly, the child had not yet learned the word "too" to indicate "another" or "also" before, during, or immediately after such functional use, did not have a surrogate caretaker who actively taught number word skills, and instructional efforts by the parents were noted in the mother-researcher's diary. These constrained and dense observations permitted identification of spontaneous generalizations of "two."

Issue 3: What is the relation between the functional development of "one" and "two"? Logically, the functional use of "one" should precede that of "two." As recognized by the ancient Greeks, one is the unit from which all natural numbers can be composed (e.g., $1 + 1 = 2$, $1 + 1 + 1 = 3$). Furthermore, taking "only one" of something is often a point of emphasis by many parents. Wynn (1990, 1992) and others (e.g., Carey & Sarnecka, 2006; Condry & Spelke, 2008; Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006) have found that children learn the cardinal meaning of the numbers words "one" and "two" in order.

In contrast, Durkin et al. (1986) found that "two" was not only preschoolers' most frequently used number word but also often their only number word for a while. Likewise, Wagner and Walters (1982) found that "two" was children's first number word. These findings make sense for three reasons. (a) "One" is used in a variety of ways, including meanings that are only indirectly or ambiguously related to a cardinal meaning of number, as in "What is one to do" or "On the one hand" (Beilin, 1975; Fuson, 1988). (b) Adults may tend to use a noun to identify a single case, because using the cardinal term "one" to specify exactly one item is often unnecessary or redundant. Although there are other ways of specifying exactly two (e.g., pair, couple, twin, brace; Dantzig, 1954), the cardinal number term "two" is probably the most common. (c) An explicit understanding of oneness and, hence, the need for a word to label this numerical commonality among various single instances of objects, may not emerge until children are required to distinguish between singularities and pluralities, which are highlighted by "two."

Mix (2009) found evidence for a third possibility—the more or less simultaneous emergence of the cardinal terms "one" and "two." She concluded this was evidence that Spencer isolated the "meaning of 'two' by contrasting it with 'one,' as others have argued children do (Bloom & Wynn, 1997; Sarnecka & Carey, 2008; Wagner & Walters, 1982)" (p. 437).

However, Wynn (1990, 1992) and Wagner and Walters' (1982) longitudinal data were laboratory based or included relatively sparse observations. Mix's (2009) longitudinal results were relatively dense but not continuous and may have been the byproduct of the nanny's number frame, which involved simultaneously introducing "one" and "two." The present research served to gauge the developmental order of "one" and "two," which were introduced sequentially, more or less continuously in a child's natural environment.

Issue 4: Is a plural rule necessary for functional verbal number recognition of two? Bloom and Wynn (1997) hypothesized that hearing "one" associated with only singular nouns and "two" with only plural nouns (e.g., "You may take just one cookie, not two cookies") may underscore their meaning as indicating a unit and a plurality, respectively. Indeed, Sarnecka et al. (2007) found that 2- and 3-year-olds whose language (English and Russian) involved plural markers were more successful in understanding "one" and "two" than those whose language (Japanese) did not. Mix (2009) concluded that Spencer's efforts to contrast "two" with "one" may have been aided by his nanny's number frame, which tied "two" to plural nouns.

Some evidence indicates that children even acquire a plural rule before functional use of "two." Wagner and Walters (1982) found that toddlers verbally designated pluralities in a manner distinct from that for single instances (e.g., used "boys" for more than one boy and "boy" for a single boy) at 18 months. Only later did they use "two" to designate pluralities (24 months) and even later to label pairs of objects precisely (26 months). Both Fuson (1988) and Mix (2009) reported that a child typically used a plural noun with "two"—indicating that the plural rule developed before or perhaps simultaneously with functional use of "two."

However, Beilin and Kagan (1969) argued that children must first acquire a conceptual basis in the form of a cardinal concept of two for recognizing, understanding, and acquiring a plural rule. Similarly, Gibson and Shusterman (2009) suggested that an exact use of "two" develops before a plural rule. They further noted that, "when children acquire the plural –s marker, they may reinterpret 'two' from its initial limited meaning to an over-generalized plural." The present study evaluated this possibility—whether functional verbal number recognition might be achieved before learning a plural rule by continuously and exhaustively studying a child just acquiring language in his natural environment.

Issue 5: Does functional verbal number recognition of two coincide with the two-knower stage (verbal set production of two)? Some scholars have hypothesized that English-speaking children first equate "two" with the grammatical role of dual marker (to mean a "pair") at the "two-knower" stage (Carey, 2004; Sarnecka et al., 2007). The two-knower stage is typically defined as correctly responding to the "give-me-two" request (verbal set-production task) and refraining from producing two items to other give-me-*n* requests at a somewhat reliable or semi-functional level. However, Gibson and Shusterman (2009) concluded that their data indicate preschoolers treat "two" as a dual marker (and then a plural marker) before becoming a two-knower and recover an exact meaning of "two" about the time of two-knower stage.

Previous research either evaluated the development of verbal set production competence but not verbal number recognition performance (Carey, 2004; Sarnecka et al., 2007), the reverse (Gibson & Shusterman, 2009), or—in the case of Le Corre et al. (2006)—evaluated both skills but did not report a developmental order. Le Corre et al. found that children classified as *n*-knowers on the basis of the verbal set-production task were also classified as *n*-knowers on the basis of "How many?" tasks (via verbal number recognition and often one-to-one counting). However, as these researchers focused on counting principles, only relatively advanced children—those who had memorized a stable count list of at least six number words—were included in their study. Ceiling effects precluded identifying the developmental order of verbal set production and verbal number recognition. The present study directly evaluated this relation vis-à-vis the emergence of a functional use of "two."

METHOD

Participant

Blake was 18 months old when the study began and 49 months old when it ended. Both parents had master's of education degrees. The boy and his family lived in the suburb of a large Midwestern city.

Blake was a healthy and typically developing child, except for a speech delay due to a series of ear infections that began when he was 7 months old. Fluid in his ears caused a slight hearing loss—the inability to hear some tones. The boy received speech therapy throughout the study. It is unlikely, however, that Blake's minor hearing loss and speech delay affected the development of his first few number words. Ansari, Donlan, Thomas, Ewing, and Karmiloff-Smith (2003) found that an understanding of number concepts was predicted by visual-spatial competence and not related to overall language development. More specifically, Blake's physical limitations did not prevent him from distinguishing among the words "one," "two," and "three" and learning to speak these and other words at about the same time as other children.

Sources of Data

As a combination of methods can provide richer data on development than using any single method, naturalistic observations were supplemented with observations during structured (Kumon) training and microgenetic testing.

Naturalistic Observations of Everyday Activities. The first author, Blake's mother, conducted the naturalistic observations. A diary study by the full-time homemaker permitted observing the participant during most of his waking hours and thus relatively continuous and dense sampling. Another advantage was ecological validity. Many of the observations arose from authentic opportunities for Blake to exhibit his number knowledge, including spontaneous or self-initiated applications. Parental feedback was provided.

Blake's mother recorded her observations in real time on a personal digital assistant and later transferred these notes to a computer file. She noted any behavior related to numbers or operations on them, however minor.

Kumon Observations. The Kumon curriculum involves repeated, adult-directed drill. Although this program focuses on the memorization of number skills by rote, Blake's parents felt that it could provide additional opportunities to practice communication skills in general and to identify example collections with an appropriate number word in particular. The Kumon training did not begin until 27 months because his parents did not decide until then that extra practice with number words, especially the number words "three" to "five," and general vocabulary was needed. He completed between one to five (three on average) worksheets per day (per 10-minute session), half of which were reading-related. The mathematics sheets included enumerating collections of dots or pictures (via verbal number recognition or one-to-one counting).[2] A typical worksheet that involved number recognition included two rows of dots (e.g., 5 dots and 1 dot or 4 dots and 2 dots) separated by a line and the instructions (which were read by his mother): How many dots (•) are there? As he progressed through the program, the sheets expanded the number of items presented. Within the first month of the program, though, Blake worked with collections of up to four. Feedback was provided. Blake's mother took observational notes on his progress. The Kumon training continued until the end of the case study and provided extensive practice identifying small numbers, particularly "three," "four," and "five," the participant would have not otherwise received.

Microgenetic Testing. The microgenetic testing was done on 11 occasions at 26, 29, 30, 32, 36, 39, to 41, and 47 to 49 months. The microgenetic testing did not begin until 26 months for the following two reasons: (a) The original plans for the case study did not include such testing. However, natural, spontaneous use of number words was relatively rare, so microgenetic testing was introduced as another way of checking on the child's number knowledge (and providing supporting data for the naturalistic observations). (b) Blake's performance in everyday situations before 26 months suggested that formal testing would be fruitless. He was tested at 26 months to provide baseline data.

The microgenetic testing was planned for every third month. For the most part, a shorter interval did not seem warranted because of the child's relatively slow progress in everyday situations. On several occasions, microgenetic testing was done at 1- or 2-month intervals because data analyses indicated that progress on one or more microgenetic tasks might be expected. For example, Blake started using "one" and "two" functionally in everyday situations when self-initiated and did so with "two" even in adult-initiated situations. At 29 months, the microgenetic testing confirmed successful use of "two" but not "one" with adult-initiated tasks. Microgenetic testing was done a month later to see if he had begun using "one" accurately in adult-initiated situations. On several occasions, the interval between testing sessions was longer than 3 months either because there was reason to believe that the testing would not reveal a change in performance or family circumstances did not permit testing.

Unlike the naturalistic and Kumon observations, no feedback was provided during this formal testing. Blake's mother took notes during these sessions and videotaped them. Repeated testing with conventional tasks is useful for assessing the development of skills over time and provided additional exposure to the numbers three to five.

Trials were presented in the order of size (difficulty) or in a semi-random order. A more difficult trial first often resulted in a participant's "meltdown" (off-task behavior such as unreasonable or unrelated responses, a decrease in attentiveness, a request to stop or play another game, crying, aggressive behavior such as pushing all the tester's materials away, or leaving the testing room). Based on pilot work, trials involving one and two were of equal difficulty, and the same was basically true for those involving three and four. For this reason, trials involving one and two or three and four were sometimes interchanged to create a semi-random order.

[2]Other enumeration activities involved counting collections of objects, such as seven turtles in side-by-side rows of four and three, and the instructions to count the turtles. Other mathematical worksheets involved identifying the missing numeral in a counting sequence.

Microgenetic Tasks

The first author administered six conventional tasks in the format of a game in order to foster motivation. These included 2 nonverbal equivalence, 2 systematic verbal number recognition, and 2 object-counting tasks.

Nonverbal Equivalence Tasks.

An ability to recreate a collection was tested at two levels with tasks adopted from Huttenlocher, Jordan, and Levine (1994): (a) *nonverbal matching task* and (b) *nonverbal set production*.

- *Nonverbal matching task*. The purpose of "Fair Game" was to test simple one-to-one correspondence—whether the participant could re-create collections of one to four items while a model collection remained visible. To prepare for a game, the tester put out a collection on a mat and asked the participant to make his mat the same, so that the game was fair. Success was defined as an exact match (exact matching). Blake's mother demonstrated the matching procedure with one item and then with two. Next, she requested that Blake attempt to match one to four items in that order. Blake's mother instructed him to make his mat (in the shape of a basket, snowman, fish, or truck) just like hers, where quarter-sized checkers represented eggs, buttons, bubbles, and cans.
- *Nonverbal set-production task*. The "Hiding Game" required the participant to create a collection equivalent to a previously seen but then hidden collection, a process that entailed creating and using a mental representation of the model collection. After a collection of one to four white poker chips was presented on the tester's mat for 3 seconds, and then covered by a flap held, the tester said, "Make your mat just like mine." The test trials were administered in the following order: 1, 2, 4, and 3 items.

For both types of equivalence tasks, responses were recorded as either successful or not successful, and if the latter, how many items were put out. Noted were number-related behaviors, such as using verbal number recognition or one-to-one counting to label the cardinal value of the target collection beforehand or the child's collection while or after it was created. Strategy was also noted: (a) *Nonverbal putting out* involved creating or attempting to create an equivalent collection without any apparent verbalizations of number or counting—that is, presumably by a nonverbal matching process. (b) *Verbal putting out* involved spontaneously using verbal number recognition to label a model collection beforehand and to produce (more or less simultaneously) an equivalent collection. (c) *Counting out* involved labeling items with a number word as they were sequentially set out—whether the model collection was labeled with a number word or counted beforehand. These tasks, then, provided opportunities for the participant spontaneously to apply verbal number recognition, one-to-one counting, or verbal set production skills. Moreover, failure on nonverbal equivalence tasks provided additional evidence that a child is unsuccessful with the developmentally more advanced verbal set production skill.

Verbal Number Recognition Tasks.

Two tasks were used to gauge systematically the extent to which the participant could reliably and discriminately label collections with a number word.

- *Task 1*. The Smoothie task involved 12 cards (3 cards of each number from one to four), each with a different fruit on it. Each collection of a number was arranged in different patterns. The participant was told that he was helping to make a smoothie, that the cards were the recipe, and that he needed to tell the tester what was on the card so that she could place the right amount of fruit in a blender to make the smoothie.
- *Task 2*. The Ocean Scene task also involved 12 cards, each with a different ocean component. Blake was told that he needed to tell the tester how many were on the card so that he could receive that many stickers to make an ocean scene.

For both tasks, responses were coded as either successful or not. The strategy used (e.g., answered verbally with a number, showed fingers for the number, or counted) was also noted.

Object-Counting Tasks. The *Animal Spots* (Wynroth, 1986) game served as the pretext for two tasks, namely the *enumeration* and *give-me-n tasks*.

- *Enumeration task.* The first phase of the game involved enumerating a collection via one-to-one counting or verbal number recognition. On his turn, the participant was instructed to draw a [5- × 8-inch] card from a deck of cards, turn it over, and count the [1/2-inch diameter] dots on the card: "Count the dots to see how many spots you can take for your cat" (a leopard-shape cut from a countertop with holes drilled for pegs (1/4-inch × 1-inch pieces of doweling). After the child finished, the tester asked, "How many spots can you take for your cat?" On the tester's turn, the child was encouraged to count the next card turned up to see how many spots the tester could take for her spotted cat.
- *Give-me-n (verbal set-production) task*. The task entailed taking or counting out a verbally specified number of items from a much larger set of the items by using verbal number recognition or counting out. Once the child had enumerated the dots, the tester said: "You may take *n* spots for my [your] cat." The child retrieved the spots from a small plastic container of at least 30 pieces of doweling. If the child was incorrect and ended the tagging process with a number other than the target number, the tester asked, "How many spots were you supposed to take?" This served to check whether a child remembered the number of spots requested (to discount memory failure).

For each task, eight trials (1 to 4, each repeated twice) were presented in semi-random order. The dots were arranged haphazardly.

Procedure

Starting with "one," then "two," and later "three," "four," and "five," Blake's parents labeled a variety of visible, homogenous collections with number words. They routinely provided corrective feedback (i.e., the correct number word) when the boy mislabeled a set.

As observations from these three data sources were recorded, the authors discussed the mathematical significance of the events and related them to research in the field. As children might be more likely to exhibit number competencies in contexts of their own making than when tested by adults, we distinguished between child-initiated and adult-initiated tasks.

A functional use of a number word was defined as highly reliable and appropriate use—at least 90% accuracy in applying a number word selectively—during a 1-month time period. Semi-functional use was defined as generally reliable and appropriate use—51% to 89% accuracy. For example, if the participant labeled two pairs of items correctly, one pair incorrectly, and misapplied the word "two" twice to a collection of three, then his accuracy rate was scored as 2/5 or 40% for the month. Accuracy rates were scored separately for everyday child-initiated use, everyday adult-initiated use (adult-initiated) microgenetic use, and (adult-initiated) Kumon use.

EVIDENCE AND IMPLICATIONS

The results of the naturalistic observations, the observations during Kumon training, and the microgenetic testing regarding verbal number skills are summarized in Figure 1. The view(s) supported by the present results is indicated in Table 1 by a check.

Issue 1

Whether the use of "two" is functional from the start and emerges relatively early in a child's language development bears on the issues of whether nature in the form of innate concepts or nurture in the form of socially constructed concepts plays the upper hand in shaping key aspects of intelligence ("privileged domains") and whether nonverbal concepts/categories precede and facilitate verbal cardinal number concepts or language in the form the first few number words prompt the construction of such concepts.

Results. Consistent with previous evidence (e.g., Fuson, 1988; Mix, 2009), number words (namely "one" and "two") were part of Blake's vocabulary almost from the time he began to speak. However, their cardinal applications did not become even semi-functional for 8 months. At 19 months, Blake was able to repeat or imitate the number word "one" after spoken by his parents, but he did not spontaneously use the term to quantify collections. For example, his mother said, "You can only have three raisins." Before she put any raisins on a tabletop before him, Blake pointed to the tabletop and said, "One."

The everyday development of Blake's first functional word for a cardinal number occurred after he learned to use numerous other words (e.g., spoon, truck, pretzel, Blake), including color words, in a functional manner and paralleled the development of his color vocabulary in key ways. Consistent with Sandhofer and Smith's (1999) observations (see also review by Rice, 1980), he initially used color words indiscriminately. At 21 months, the child labeled every color *blue* or *yellow*. Soon after, he began using the terms *blue, yellow*, and *purple* somewhat more selectively. After months of inconsistent application, 25-month-old Blake's ability to discriminate among blue, yellow, purple, and seven other colors blossomed in a matter of weeks.

Similarly, as Figure 1 indicates, Blake initially used "two," his most frequently and first spontaneously used cardinal number word, in an unreliable and overly broad manner and only

Age in Mos.	Verbal Number recognition $n = 1$ v	$n = 1$ V	$n = 2$ v	$n = 2$ V	$n = 3$ v	$n = 3$ V	$n = 4$ v	$n = 4$ V	Comments (▼ = verbal recognition of number; Ͼ = 1-to-1 counting; ⦸ = other)
19 to 21	○								▼ Imitates saying, "One." Non-functional spontaneous use of "one" only.
22		○		○		○		○	⦸ Taught to respond "two" (measurement meaning) by rote when asked, "How old are you?" ▼ Responded "Two" to "How many?" questions indiscriminately.
23 to 25									▼ Always identified pluralities as "more."
26*				□		□			▼ "Two" serves to indicate "more." ▼ For microgenetic testing, responded correctly to "How old are you?" but uses "two" unreliably (e.g., identified three checkers as "two"). Matches 1 but not 2 to 4; does not spontaneously label any sets.
27		●	⊙	◇		◇		◇	▼ First functional use of "one" to answer a "How many?" question; still has never spontaneously characterized a singularity as "one." ▼ Spontaneously labels item pairs as "two" in everyday situations more often than not. ▼ Kumon training begins. Learning of "three," then "four," and finally "five" begins. ▼ Uses "two" and then "three" to label 2 to 5 items on Kumon worksheets. Imitates labeling displays of 4 as "four" but does not use "four" either spontaneously or in response to the "How many?" question. ▼ An understanding of the "two"-"three" (i.e., more than two) contrast emerges. For example, shown a Kumon sheet with 3 items, the boy said, "Three." Then—while shaking his head back and forth—he said, "Two" (his way of indicating *not two*). Ͼ Makes one sound for each item in a collection of two or three but does not apply the cardinality principle. For example, counted three items as "two, two, two" and responds "Three" (via verbal number recognition) when asked, "How many?" Ͼ Later strings two number words ("Two, three") together for the first time; can sometimes predict a third object will be labeled "three" when mother counts the first two items.

FIGURE 1 Blake's verbal number recognition and one-to-one counting performance by age and collection size. A lower case v = child-initiated verbal number recognition, a capital V = adult-initiated verbal number recognition. A circle (○) indicates an everyday situation; a square (□) indicates microgenetic testing; and a diamond (◇) indicates a Kumon context. An empty figure (e.g., ○) = 0% to 50% success rate (non-functional use); a partially filled figure (e.g., ⊙) = 51% to 89% success rate (semi-functional); and a completely filled figure (e.g., ●) = a 90% to 100% success rate (functional use). An asterisk (*) = microgenetic testing.

Depicted are initial performance levels and changes in performance level only. Collapsed into a single row are months over which no developmental change or milestone occurred or no new source of training or information was introduced. For example, from 19 to 21 months, Blake's parents focused on modeling and encouraging the imitation of the number word "one" only. All of Blake's spontaneous utterances of "one" did not refer to the cardinal number one. At 22 months, Blake's parents expanded their training efforts to measurement meaning of "two. At 23 months, Blake achieved the milestone of using "more" to indicate "many." As this and the nonfunctional spontaneous use of "one" or "two continued to 25 months, 23 to 25 months are collapsed together.

Age (Mos.)	*Verbal Number recognition*								Comments
	n = 1		*n = 2*		*n = 3*		*n = 4*		
	v	V	v	V	v	V	v	V	(▼ = verbal recognition of number; Ͼ = 1-to-1 counting; ⦸ = other)
28	●		●	●					▼ Almost daily spontaneously uses "two" to label pairs; never over-applies the term to larger collections. ▼ Can hold up one finger and say, "One" or two fingers and say, "Two." ▼ In everyday and Kumon situations, unselectively uses "three (rarely)," "four," "five" (especially), or "a lot" to describe more than two items. Ͼ During Kumon, recited the standard sequence to "five" as mother pointed to items but still does not have a stable counting string. Ͼ During Kumon, accurately used 1-to-1 counting with a collection of three items.
29*				■ ◈					▼ Other than with fingers, first spontaneous use of "one": Indicated he wanted "one, not two" balls — further evidence he distinguishes between 1 and 2 and understands the "one"-"two" contrast. ▼ In everyday situations, uses "five" to indicate a lot (e.g., asked for more mushrooms and when asked to specify how many, responded, "Two," and then quickly, "Five." ▼ During Kumon, often uses "five" to label collections of more than 2 items. Still, rarely uses "three" and never outside Kumon. Ͼ Mother and Blake point to items and recite the standard counting sequence to "six" together, and for the next item, he says, "Seven." Ͼ Merely counts available items when asked for two packs of imitation sugar. Ͼ Accurate 1-to-1 counting of a collection of three sugar packets and later three grapes; responded correctly for the first time to the "How many?" questions (i.e., applies the cardinality principle/rule).
30* to 31		■			○ ◇		○		▼ During microgenetic testing, consistently matched and nonverbally produced 1 correctly after labeling his mother's collection "One." Responded unreliably on (unlabeled) trials involving 2. After asked to put down 2 of the 4 disks he was holding, Blake handed over 2 and said, "Mamma, Two." ▼ In everyday situations, spontaneously identified three coins as "four" three times despite his mother's correction the first two times. ▼ Still seems to view "three" as *many*. Blake asked for "five" nuts. Mother responded, "How about three?" Blake retorted, "Five." Given 3, happily announced, "Five." Also identified 3 sodas as "five." ▼ Consistently identifies collections larger than 2 as "five." Ͼ Does not use a stable counting sequence for counting a collection of four items 1-to-1 ("one, three, four five" twice and "one, two, three"). Ͼ During microgenetic testing, responded correctly to a give-me two request for the first time while saying, "Mama, two."

FIGURE 1 (Continued)

Age (Mos.)	Verbal Number Recognition								Comments
	n = 1		n = 2		n = 3		n = 4		
	v	V	v	V	v	V	v	V	(▼ = verbal recognition of number; ɕ = 1-to-1 counting; ⦸ = other)
32* to 33									▼ While child-initiated use of "two" is still 100% reliable, correctly identifies two items in adult-initiated situations only 5 of 8 times.
34 to 35, 36*, 37 to 39*					⊙	⊙			
40*, 41*, 42, 43					▣	▣	●	□	ɕ At 40 months during microgenetic testing, accurately counts 1-to-1 up to three items; counted four items as "three" while raising three fingers. ɕ At 40 months during microgenetic testing, semi-functional verbal set production of three. ɕ At 41 months during microgenetic testing, accurately counts 1-to-1 up to four items; correctly counts out one or two with perfect accuracy; counts out (verbal set production of) three or four accurately only half the time.
44 to 45								●	
46							○		
47*							●		
48*					■			▣	▼ During the microgenetic testing, self-initiated recognition of "3" finally becomes entirely reliable and selective.
49*									▼ On the micrognetic (adult-imposed) verbal number recognition tasks, achieves an all-time high of 87% correct recognition of 3 but, on one occasion, still spontaneously labeled a collection of 4 as "three."

FIGURE 1 (Continued)

gradually applied it more consistently and selectively. At 22 months, his parents taught him to say, “Two,” when asked, “How old are you?” They wanted to ensure he could specify his age by his second birthday. Although this use of “two” to mean 2 years old technically involves a measurement meaning, a young child might interpret it in terms of the number of birthdays (a cardinal meaning). Blake’s parents also taught him to respond with a cardinal number word to a “How many?” question. However, he typically responded to these questions with “two,” whether a collection involved one, two, or three items. In brief, his initial use of “two” in the measurement and cardinal contexts appeared to be mechanical (i.e., a response learned by rote).

Between 23 and 25 months, he spontaneously labeled collections of more than one as “many.” At 26 months, “two” served the same role. At 27 months, he spontaneously used “two” somewhat more than half of the time to label pairs of items. At 28 months of age, he used the number words “two” functionally in both child- and adult-initiated everyday situations and used other terms to label larger collections.

As Figure 1 illustrates, Blake’s development of “two” in more formal settings (the microgenetic testing and Kumon instruction) paralleled his progress in everyday settings but at a slower pace (see also Mix, 2009).[3] At 27 months (when his self-initiated use in everyday contexts had become semi-functional), his use in formal settings, including self-initiated use during microgenetic testing, remained non-functional. For example, during a Kumon session, his mother corrected him for over applying the label “three” to collections of four or more (“That’s not three”). Subsequently, the child reverted to using “two” unselectively. One month after achieving functional use in everyday situations, Blake exhibited such competence during microgenetic testing and achieved semi-functional use in the Kumon context. However, he never exhibited functional use during formal training.

As Figure 1 summarizes, the selective use of “three” developed appreciably later than that of “two” and functional use of other words. Semi-functional use in everyday situations and microgenetic testing did not occur until 34 and 40 months, respectively, and was never achieved in the Kumon context. Evidence of functional use of “three” did not occur until 48 months.

Conclusions. Blake’s cardinal use of “two” did not exhibit the U-shaped development observed by Gibson and Shusterman (2009) and Mix (2009). Seven months after the onset of language, three months after the first functional use of “more,” and a month after using color words functionally, the child first used “two” to distinguish a *plurality* from a *singularity*. Semi-functional and functional everyday use of “two” required another month and two months, respectively. In comparison to the functional use of other words, functional use of the first three cardinal words emerged relatively late, not early.

[3]Note that Blake did not develop functional self-initiated one-to-one counting skill (use counting to quantify a collection) until 29 months—after recognition of “two” had become highly reliable—and, as with many newly learned skills, did so deliberately (e.g., started the counting with “one,” pointed with his fingers) for many months afterward. Functional one-to-one counting in adult-initiated situations did not begin for many months later. Consistent with most previous research (Klahr & Wallace, 1973; Sarnecka et al., 2007; Schaeffer et al., 1974; Sophian, 1992; Starkey & Cooper, 1995; von Glasersfeld, 1982; but cf. Fuson, 1988), the functional development of two in self-initiated situations particularly, then, was probably not aided by experiences with one-to-one counting. Moreover, many of the more abstract uses of two discussed in the next subsection would not lend themselves to one-to-one object counting.

It could be argued that Blake's results are unrepresentative of typical development because of his hearing and speaking difficulties or the learning effects from the intermittent microgenetic testing and his daily Kumon training. However, as previously noted, his difficulties did not prevent him from achieving functional use of other words or learning to speak the small number words at about the same time as other children. Importantly, contrary to the U-shaped development hypothesis, Blake did not use "two" in a functional manner in earliest everyday efforts from 22 to 26 months, a period before any formal testing or training began. After 26 months, any learning effect of the formal testing and training should have fostered, rather than delayed, his development. In brief, Blake's unselective use of "one" and "two" for so long and his even longer nonfunctional use of "three" cannot be explained by his hearing loss and occurred despite possible learning effects from interventions in the form of parents interested in fostering number knowledge, microgenetic testing, and Kumon training.[4]

Whether a different and more systematic training would have accelerated Blake's functional use of "two" and "three" is, of course, an open question and in need of further research. In everyday situations, Blake's spontaneous and prompted use of number words, his parents' use of examples, and their feedback for incorrect applications of a number word may have been too infrequent. Although he regularly received feedback in Kumon about whether his number identification was correct or not, this source of information about examples and non-examples was often in the form a correction, which the child may have interpreted as disapproval. Neither Blake's parents nor Kumon training systematically compared and contrasted examples and non-examples of "one," "two," and "three" (e.g., "This is one kitten; this is two kittens" or "This is two blocks, this is not two blocks")—an instructive process that does not involve the possibly sensitive action of correcting a child's mistake.

Nonfunctional use of "two" was somewhat more protracted in the formal settings of the microgenetic testing and Kumon training, and its highly reliable or functional use was not achieved in the Kumon training. Spontaneous or self-initiated correct use of the number word requires recognizing that it represents a specific total, but—unlike adult-initiated use—it does not entail understanding the "How many?" question (e.g., connecting this question to cardinal number knowledge) or determining the set referenced or implied by a questioner. This may explain why Blake's semi-functional use of "two" emerged first in self-initiated everyday situations at 27 months. Task differences do not seem to explain why a month later, Blake used the number word in a functional manner in adult-initiated everyday situations but non-functionally during the Kumon. In both cases, the child was responding to essentially the same "How many?" question. Similarly with the use of "three," semi-functional use appeared in both self-initiated and adult-initiated everyday situations six months before doing so in the microgenetic testing and never emerged in the Kumon context.

[4]For Issues #2 to #4, it is possible that the feedback Blake received in the Kumon training had a positive effect on his understanding and recognition of two. However, there are reasons to believe otherwise. Blake's progress in selectively using "two" in purposeful and engaging everyday situations ran well ahead of such progress in the sterile or purposeless Kumon context (as far as the child is concerned). Indeed, he never achieved functional use of "two" in the Kumon context and did not achieve even semi-functional use until 29 months, one month after achieving functional use in everyday situations. This evidence suggests that the training probably did not interest or motivate Blake. Given his relatively poor performance and apparent disinterest over the entire course of Kumon, it not clear that he attended carefully and benefitted significantly from the feedback provided in this training. In regard to Issue #5, the Kumon training did not involve counting out collections of items.

The key difference among the three contexts appeared to be differences in the relative purposefulness of the self-initiated use or an adult's task and, hence, the child's intrinsic motivation to respond thoughtfully. Everyday situations involved real and engaging tasks. The microgenetic tasks were embedded in the context of a game, which may or not may have been engaging to the participant. The Kumon tasks were sterile (without context or motivation) and apparently of little intrinsic interest to the participant. Such a hypothesis (and that regarding task differences in the previous paragraph) is consistent with Piaget and Inhelder's (1969) observation that cognition and affect ("energetics") are "inseparable and complementary" (p. 21).

Issue 2

When and how functional use of a number word becomes relatively abstract remains unclear and bears on central questions about children's number and general concept development.

Results. Nearly all of Blake's use of cardinal number words during the study was context-specific (tied to real collections). In contrast to previous observations (e.g., Lawler, 1981; Mix, 2009), Blake did not first use a cardinal number word appropriately and semi-reliably in a single isolated everyday context (to label a particular collection). Instead, at 27 months, he began using the number word "two" semi-functionally to label many different everyday examples of homogenous pairs, including a couple of dogs, two newspapers, and a pair of tractors. A month later, Blake used "two" functionally in everyday situations to enumerate pairs of visible and simultaneously presented items that (a) were not touchable (e.g., smokestacks or airplanes in the sky), (b) were not previously labeled "two" by his parents (e.g., smokestacks or remote controls), and (c) possessed different physical properties such as different shapes or lengths (smokestacks vs. fingers; Level A in Figure 2).

At 29 months, Blake began to label pairs of successively presented items functionally, including those presented minutes apart (see Level B in Figure 2). For example, his mother forgot that she had a can of soda on the table (in view of Blake) and took another out of the refrigerator. Looking at the can in her hand but not the one on the table, he excitedly proclaimed, "Two!" Similarly, as Blake and his mother were driving home, he saw a lawn crane. After several turns, the child saw a second lawn crane. With the first crane now out of sight, he said, "Two." His mother asked him to what was he referring, and he said, "Cranes."

At 30 months, the child correctly used the label "two" for previously seen pairs when one remained hidden and one returned to view (see Level C in Figure 2). For instance, Blake inserted two coins but got back only one. Looking discouraged and apparently wanting both coins back, he said, "Two money? Two money?" A month later, Blake reliably used the label "two" for pairs of items previously seen but out of view (see Level D in Figure 2). For example, the boy and his mother were looking for his lost sandals in the family van. His mother told him that she found one but could not find the other. Hours later when his mother told him she found the other sandal, Blake said (without seeing either sandal), "Two."

Soon afterward, he used the label "two" for items never previously paired, one of which he had never seen but which had the same name as a previously seen item (see Level E in Figure 2). Overhearing a mention of Wanda, Blake asked, "Billy's Wanda?" His mother answered, "Another

Abstract

F. *Sequential events discriminately quantified* (34 months). Blake sneezed two times and said, "I sneezed two times."

E. *Discriminate labeling of unseen items never previously paired* (31 to 32 months). After mother said, "Mommy has to go potty; Daddy has to go potty," spontaneously said, "Two potties."

D. *Discriminate labeling of two previously viewed but unseen objects or one visible item and a second anticipated item* (31 months). After playing with two pieces of chalk, places each in a separate cubby. With his back to the cubbies, comments, "Two homes." With a cup of milk in front of him, he asked for water. Mother responded, "Let me put it in a cup for you." As she went to the cabinet to get the cup, Blake said, "Two cups."

C. *Discriminate labeling of two previously viewed objects after both are hidden and one object is returned to view* (30 months). Looking discouraged and apparently wanting both coins back, he said, "Two money? Two money?"

B. *Sequentially presented items discriminately quantified* (29 months). Labeling a previously seen item (soda, crane, or block) and second visible example as "two." Blake had one block and turned around and grabbed a second block from a basket behind him. With the first block still out of sight, he said, "Two."

A. *Discriminate labeling of visible pairs* (28 months). Blake appeared to have a strong understanding of *two objects, when he defined them*. He identified two objects in his world almost on a daily basis. During a car trip, for example, Blake saw the two smokestacks and noted, "Two." He also consistently asked for his two blankets. He grabbed two remotes and said, "Two." He saw two jets and said, "Two!" He also became able to hold up two fingers and say, "Two."

Concrete

FIGURE 2 The development of Blake's functional use of "two."

Wanda." The boy said, "Two Wanda." At 34 months, Blake could reliably label pairs of sequential events (see Level F in Figure 2). For instance, he sneezed twice and said, "I sneezed two times."

Inconsistent with Starkey et al. (1990), Blake was unable to enumerate the number of sounds. For example, at 28 months, he could imitate two or three (but not more) consecutive sounds (e.g., "Ba, ba" or "Ba ba, ba") but looked perplexed when asked how many sounds his mother made.

At 29 months, he was able to enumerate touches (tactile inputs). Blake had a book on one side of his pillow and a ball on the other side. He put a hand on each and said, “Two.”

In addition to the example just cited, there was only one other spontaneous effort to label a heterogeneous collection with a number. At 27 months, asked if he wanted milk or water, Blake said, in turn, “milk”; “water.” His mother replied, “Which do you want, milk or water?” The child then answered, “Two” (apparently indicating both).

Conclusions. The evidence for concrete to abstract development of the cardinal term “two” was mixed. Blake did not exhibit evidence of the local (context-specific) to general (context-free) development hypothesized by Resnick (1992) or found by Mix (2009). Also unlike Mix’s finding, he (spontaneously) used “two” semi-functionally and then functionally to label numerous pairs that shared no obvious commonality (Level A in Figure 2) and, over the next three months, labeled increasingly less visible collections (Levels B to E) and then re-occurrence of an event (Level F).

Blake’s early indiscriminate or non-functional application of “two” and feedback from parents with a variety of homogeneous collections may have set the stage for a broad semi-functional/functional application of this number word. Specifically, it may have helped him recognize that numbers are independent of superficial physical appearances and that the number word “two” applies only to pairs of items.

Blake’s functional development of “two” illustrated in Figure 2 shares similarities with that which Mix (1999a) found with older children on equivalence tasks and is consonant with a constructivist theory of abstraction (Steffe & Cobb, 1988; von Glasersfeld, 1991). His non-functional use of the number word (Pre-Level A) was merely a conditioned response and is consistent with the initial *perceptual abstraction* level in the sense that a stimulus (e.g., “How old are you?” or “How many?”) was necessary to prompt a learned response (“two”). Similarly, at Level A, a perceptual input (a visible pair of items) had to be available to activate memory of *two* as a pair and the associated word “two.” At Level B, the child apparently could view a visible item and a recalled (mentally represented similar) item as “two.” Levels C to F represent increasingly deep *internalization*. In these levels, *two* was sufficiently abstract and stable; it could be re-presented (recreated) in the absence of perceptual input. Unlike Pre-Level A (proto-conceptual knowledge) and Level A (transitional knowledge), Levels C to F clearly involve a *concept*.

Blake’s use of “two” at 27 months to indicate he wanted both milk and water appears to be exceptionally abstract in two senses. One is that the child labeled previously viewed but as yet unseen (but anticipated) items as “two”—Level D in Figure 2, a level of abstraction not revisited again for four months. Another is that he treated drinks with a few different perceptual qualities (e.g., different colors and tastes) as elements of the same (somewhat heterogeneous) collection (drinks). These two abstractions may have been made possible because of his daily experience (familiarity) with these anticipated items and his ready recognition that both were liquids that he drank (i.e., that belong to the same functional category requiring a cup). This exception raises the possibility that the concrete to abstract progression illustrated in Figure 2 may be more, or only, applicable to novel or relatively new cases involving a pair.

Labeling two simultaneously touched objects with numerous different perceptual characteristics (a tactile instance of Level A in Figure 2) at 29 months was Blake’s only clear-cut application of “two” to a heterogeneous collection during the case study and came one month after he could reliably apply the term to homogeneous arrays. Labeling a heterogeneous collection with a number represents a relatively abstract understanding for two reasons (Mix, 1999b). One is that homogeneous pairs represent relatively obvious or concrete examples of a number, whereas heterogeneous collections represent relatively abstract examples. Indeed, for other than highly familiar and clearly related items such as milk and water, the latter entails understanding part–whole relations or hierarchical classification (e.g., recognizing that the classes "cat" and "dog" can belong to a broader class called "pets" or "animals"; Piaget, 1965). A second is that, as with Blake's parents, parents or other caregivers may be more likely to label as "two" more obvious pairs (homogeneous arrays such as two shoes or two cookies) than less obvious pairs (heterogeneous arrays such as calling a cat and a dog "two pets" or "two animals"). Parents may refer to heterogeneous collections on an item-by-item basis (e.g., "Look, a cat and a dog"). For both reasons, children's concept of *two* might include only homogeneous arrays initially and then generalize to heterogeneous collections. Future research is needed to examine when and how children generalize number to heterogeneous arrays.

Issue 3

The relative developmental order of "one" and "two" has implications for teaching concepts in general and the meaning of number words in particular.

Results. Excluding imitation efforts, Blake first correctly used "one" to answer an adult's "How many?" question at 27 months—several days *after* he began to use "two" correctly with some consistency and selectively (semi-functionally). This single instance occurred while playing with straws in a restaurant. His mother asked him, "How many straws?" Blake correctly replied, "Two." She took one away and asked again, "How many straws?" He responded, "One." By 28 months—at the same time his child-initiated use of "two" became functional, Blake was regularly and spontaneously holding up one finger and saying, "One."

Conclusions. Blake's early and isolated training on "one" was ineffective, and—unlike nearly all previous research (e.g., Wynn, 1990, 1992, but see Mix, 2009)—functional use of cardinal term "one" basically developed simultaneously with that of "two." These facts raise the previously overlooked possibility that the cardinal term "one" takes on meaning because of contrasts with the cardinal word "two" (as well as the reverse). Teaching terms that are close in meaning together may help children to contrast their meaning and differentiate the terms.

Issue 4

Whether construction of a plural rule precedes or follows a cardinal understanding of two bears on the issue of how language development and quantitative/number concepts are related.

Results. There was some evidence that Blake's reliable and selective understanding of "two" (28 months) preceded his *consistent* use of the plural rule. At 24 months, the child grabbed two spoons and said, "Spoon." His mother corrected, "Spoons." Blake responded, "More." In brief, the boy appeared to distinguish between single items and multiple items but did not use the number word or plural –s to do so.

At 28 months, he accurately labeled various collections as "two" but without a noun or its plural form. At 29 months, he labeled two successively seen lawn cranes as "two" and, in a separate statement, correctly used the plural "cranes" to clarify what he was enumerating. A month later he used the number word "two" but the singular form of the referent noun ("Two money" and "Two Wanda") to identify pairs of items. At 31 months, the boy consistently used plural forms of nouns, sometimes in conjunction with the number word "two": "two homes" (31 months), "potties" and "two cups" (32 months), and "sneezed two times" (34 months).

Conclusions. The evidence supports a portion of Gibson and Shusterman's (2009) proposition. Consistent also with Beilin and Kagan's (1969) view that a cardinal concept of two is necessary for constructing the plural rule, Blake's functional use of "two" preceded a reliable use of the plural rule. However, learning this rule did not subsequently interfere with his exact understanding of "two" (prompt its over-generalized use) as Gibson and Shusterman proposed. There was a hint that—after apparently learning the plural rule at 29 months—using "two" in conjunction with a noun seemed to relieve the child, at least temporarily (at 30 months), from applying known plural rules. However, the relation between reliable use of "two" and a plural rule clearly needs further fine-tuned research.

The results of the present case study do not refute the claim that hearing "two" in conjunction with plural nouns facilitates the construction of a cardinal concept of two. Blake regularly heard singular nouns associated with "one" and plural nouns associated with "two" or other number words. Furthermore, he was occasionally corrected for using non-plural nouns with multiple items. Blake's response of "more" to his mother's correction of "spoons" may have indicated his recognition that the plural form and his term for a plurality ("more") were equivalent or at least provided one of many opportunities to make this association.

Issue 5

Whether functional verbal number recognition of two coincides with the two-knower stage (verbal set production of two).

Results. Blake's first correct response to a give-me-*n* request occurred in an everyday situation at 29 months. At a restaurant, his mother asked him for one packet of Equal. He grabbed two packets of the sugar substitute. Responding to a second request he said, "Uh huh (Yes)," put one packet back, and handed the remaining packet to his mother. Later Blake grabbed three packets of Sweet-N-Low, and his father instructed, "Give Mama two." He handed her three packets one at a time while counting one-to-one: "One, two, t-h-r-e-e [in a higher pitch]." Asked "How many?" by his father, Blake answered, "Three."

Conclusions. One month after Blake achieved functional verbal number recognition of one and two in everyday situations, he correctly responded to a request for “one” on only one of two tries and incorrectly responded to a request for “two.” These results were obtained despite the stringent criterion for functional verbal number recognition. Consistent with Gibson and Shusterman’s (2009) conjecture, these data indicate that reliable recognition of two may precede functional or even semi-functional verbal set production of two and that this developmental relation bears further examination with a large sample.

The conjecture and results make sense whether a child uses a putting-out or counting-out strategy to produce a requested number of items. In order to put out two items, a child must remember the requested number (i.e., store it in working memory) and either use verbal number recognition to identify a subset of two in a larger set or identify when two has been reached as items are taken from a larger collection one at time. In either case, verbal number recognition of two is a necessary, but not a sufficient, condition for putting out two.

In addition to registering the requested number in working memory, counting out a specified number of items requires children to extend their understanding of cardinality to object counting in the form of two counting principles that typically develop later than the ability to verbally recognize small numbers. Fuson (1988) hypothesized that a necessary condition for successfully counting out a collection is the relatively advanced cardinal-count principle—recognizing that the requested number determines the last number word used in the counting-out process. This principle is, in effect, the inverse of the more basic count-cardinal principle (often described as the cardinality principle of counting): The last number word in the one-to-one counting process represents the total.

The theoretical framework discussed in the previous paragraph suggests an additional explanation for verbal set-production errors to those often given (Baroody, 1987). Blake’s counting out all three available items in response to a request for “two” is a classic “no-stop” error. Resnick and Ford (1981) attributed such errors to a memory problem such as forgetting the requested number or failing to compare the count to the requested amount because of an overload on working memory due, perhaps, to the need to attend carefully to the counting process. Sarnecka and Lee (2009) hypothesized that such errors occur for one of two reasons:

> The first reason is performance error. If the child knows that “three” means 3, but gives some other number of bananas, it is because he or she made a mistake in counting or estimation. The second source of wrong answers is ignorance, which results in guessing. If the child does not know that “three” means 3, then he or she simply needs to guess how many items to give. (p. 327)

It is possible but not likely that Blake’s no-stop error was due to a memory difficulty or ignorance of what “two,” means. Consistent with other case study evidence (Baroody, 1987, 1999; Baroody & Mason, 1984; Wilkins & Baroody, 2000), his no-stop error may well have been due to not knowing the cardinal-count principle—even though he possibly may have known the more basic count-cardinal principle (as indicated by a higher pitch “three” and correctly responding to his father’s “How many?” question). Parenthetically, this hypothesis would explain why all of Le Corre et al.’s (2006) participants who produced sets of a specified number also responded effectively to “How many?” tasks.

GENERAL CONCLUSIONS

Theoretical, methodological, and education implications of the present study are discussed in turn.

Theoretical Issues

The present research involving continuous and dense observations of a child yielded several new findings that need further systematic exploration. It also provided mixed support for the various views of early number development and underscores the need for further systematic research.

Inconsistent with views that suggest small number words map directly and readily to pre-existing number concepts or de facto number categories (e.g., Geary, 1994; Gibson & Shusterman, 2009) and consonant with previous non-dense research and views that suggest children only gradually construct an exact meaning of the small numbers (see, e.g., Le Corre & Carey's 2008 review), a cardinal use of "two" was initially inexact, then served to indicate *many*, and finally indicated *a pair*. The long delays in learning the "one"–"two" and "two"–"three" contrast and especially the "three"–"four" contrast—despite informal training by concerned parents and repeated exposure in the form of formal training and testing—underscore the difficulty of constructing a cardinal meaning of even the simplest number words (Beilin, 1975; Dantzig, 1954; Durkin et al., 1986; Fuson, 1988). The relatively late appearance of functional number words in this case adds to the mounting evidence inconsistent with the privileged-domain hypothesis—that innate concepts of the intuitive numbers direct a toddler's attention to numerical situations and facilitate the relatively rapid development of number skills, including verbal number recognition (Baroody et al., 2006; Baroody, Li, & Lai, 2008; Langer, 2000; Le Corre & Carey, 2007; Le Corre et al., 2006; Simon, 1997).

Consistent with the continuity and object-tracking views, semi-functional and functional cardinal use of "two" was applied generally to a wide variety of pairs. Compatible with the mental models and simultaneous views, however, such applications were—but with one exception (with familiar items)—first restricted to collections in which all items were visible, only gradually extended to more abstract collections that required a mental representation, were not used with a succession of sounds, and only rarely used with heterogeneous collections. This first evidence of an increasingly abstract initial number concept, though, clearly needs to be confirmed by other dense or relatively dense research.

Inconsistent with the currently popular *n*-knower model that posits children learn the number words in the predictable sequence "one," then "two," and next "three" (e.g., Le Corre & Carey, 2007; Le Corre et al., 2006; Sarnecka & Carey, 2008; Sarnecka et al., 2007; Sarnecka & Gelman, 2004; Wynn 1990, 1992), the present data provide the second dense data point to suggest that functional use of "one" and "two" might develop simultaneously. Clearly, what affects acquisition order and whether a particular order has more benefits need further examination.

The present results support a recent and important proposition by Gibson and Shusterman (2009) but contradict another. The data demonstrate the possibility that an exact use of "two" develops before children acquire the plural –s marker and may support the learning of this rule. The present results, however, did not demonstrate that learning the plural rule subsequently interfered with his exact understanding of "two" (i.e., prompted over-generalized use of this number word). Whether this is true of most or even some other children needs to be studied.

Research indicating that some children construct a plural –s rule before a cardinal understanding of "two" may not be inconsistent with Beilin and Kagan's (1969) hypothesis that children must first acquire a conceptual basis for learning a plural rule. Recall that Blake consistently used "more" to indicate pluralities as early as 23 months—6 months before reliably using the grammatical rule. Although a concept of plurality was not sufficient for him to construct a plural –s rule, it rather than a cardinal concept of two may have served as the necessary conceptual underpinning of this grammatical rule. If so, this might explain the discrepant research findings regarding the acquisition of a plural –s rule and a cardinal concept of two. Systematic research is needed to evaluate whether consistent use of "more," a particular number word such as "three," or other word for a plurality (e.g., "many," "too") is a necessary condition for the plural rule and what other conditions can facilitate or are necessary for the acquisition of this rule.

The present research provides the first direct evidence for the possibility that children may construct a cardinal concept of a number before they are successful on the give-me-*n* task. Furthermore, the evidence points to an important extension of Sarnecka and Lee's (2009) explanation for give-me-*n* errors. Specifically, it relates no-stop errors to what Fuson (1988) called the cardinal-count principle—a concept long overlooked by many cognitive psychologists.

The mixed evidence in this case raises the possibility that the truth about the relation between number and number word language development may not be captured by any one view (Mix et al., 2005). Children may construct proto-numerical categories before language acquisition and may even have an intuitive sense of singularities, "a few," and larger pluralities. However, learning words can facilitate the construction of property and less abstract kind concepts that lay the groundwork for recognizing number as an important property. Furthermore, the application of the number words "one" to "three" to examples, which also serve as non-examples of other small numbers, can be instrumental in constructing the relatively abstract kind concept of the cardinalities one to three.

Given the various non-cardinal uses of "one" (Beilin, 1975; Dantzig, 1954), it makes sense that it took Blake so long to acquire a functional use of the cardinal number word "one" and that this achievement occurred about the same time he started using "two" at least semi-functionally. Various examples of "one" and obvious, contrasting non-examples of "two" may be instrumental in constructing a relatively explicit cardinal concept of one (e.g., a single teddy bear, block, and shoe share commonalities such as all can be held in a single hand).

For Blake, learning explicit terms for one and two may have highlighted the necessity of distinguishing between singular nouns and plural ones and learning the plural rule. Nonetheless, other aspects of language may have been helpful. For example, the use of singular nouns in conjunction with the word "one" and examples of single instances (and non-examples of "two") and plural nouns in conjunction with the word "two" and examples of pairs (and non-examples of "one") may be more helpful together than use of the singular-plural distinction or examples and non-examples alone. Future training studies are needed to explore this possibility.

Methodological Issues

The present evidence raises the possibility that functional verbal number recognition of two can develop before the ability to produce a collection of two upon request. The give-me-*n* task, even when a child does not count out a collection, entails cognitive demands beyond understanding number (e.g., remembering the requested number and then using verbal number recognition to compare the items produced to this representation) and may underestimate number understanding. If further research confirms this finding, then a more accurate operational definition of the two-knower stage will be needed.

Consistent with Mix's (2002) and other case study data, Blake's performance during formal (microgenetic) testing and the Kumon training, in particular, was not consistent with that in informal everyday situations. For example, adult-initiated testing indicated that 2.8-year-old Nathaniel had little nonverbal or verbal number knowledge (Baroody et al., 2003). He did not, for example, respond correctly to a matching or a nonverbal production task involving one to four items, and he declined to play a game that entailed determining the cardinal value of small collections. However, as the tester was cleaning up, Nathaniel asked for a soldier, and she allowed him to keep one until the next session. As the tester was leaving, he became upset and said, "I want two soldiers." He went into her bag and grabbed another soldier and said, "Aha! I got two!" He apparently recognized that the one soldier in his hand was not enough (not "two"); he also appeared to know that he needed another soldier to have two soldiers. Administering adult-imposed psychological tasks might not provide the most accurate picture of young preschoolers' number competence.

Moreover, with adult-imposed tasks, care needs to be taken with the size of the collections involved. Specifically, the case of Blake indicates that even number tasks involving collections smaller than five may overwhelm young preschoolers and cause a meltdown. Surprisingly, we have found such results to be common. In three studies involving 17 2-year-olds and 15 3-year-olds, more than half exhibited evidence of melting down on number tasks used in the present study (Baroody, 2002, 2008; Baroody, Lai, & Mix, 2005, 2006; Benson & Baroody, 2003). These and the present results suggest that developmental readiness and task demands are key factors. Overall, younger 2s are more prone to meltdown than older children, and tasks that require a mental representation (e.g., the nonverbal production task) seem more likely to cause a meltdown than those that do not (e.g., the match task). Clearly, though, this issue needs further systematic examination.

Educational Implications

In recent years, it has become increasingly apparent that preschoolers can acquire substantial informal knowledge of mathematics and that this everyday knowledge is an important basis for understanding and learning school-taught or formal mathematics. Because of serious individual differences in informal knowledge, there is growing interest in early childhood mathematics education to "level the playing field" for less advantaged children. Although further systematic evaluation is needed for the following conclusions, the results of the present case study suggest six implications for getting 2-year-olds or preschoolers operating at a 2-year-old level *started* with numbers.

Implication #1: Meaningful instruction of number words should commence after a child has learned several less abstract kind concepts or property-based concepts such as color. Two reasons are: (a) In typical everyday use, labeling a discrete quantity with a number word, like any form of quantification, entails explicitly or implicitly specifying a unit—what is being quantified (e.g., three *blocks* or two *red ones*). In effect, a child needs to construct other kind concepts or property concepts (a perceptual feature such as color, shape, or texture) to serve as units to be quantified. (b) Whereas property representations (e.g., colors such as *red* and *blue*) and many kind concepts (e.g., *cats* or *sisters)* are relatively concrete (can be defined by readily observable and consistent physical properties), number concepts are relatively abstract in that observable physical properties can be irrelevant and vary greatly across examples.

Implication #2: Verbal number recognition of even the smallest numbers may not unfold spontaneously and quickly but appears to be heavily dependent on socialization over an extended period of time. Parents and preschool teachers must be active and patient agents in fostering concepts of the small numbers and the reliable use of verbal number recognition.

Implication #3: Using a wide variety of examples of homogeneous collections and then heterogeneous collections may help children construct abstract cardinal concepts more quickly. Labeling various examples of single instances "one," different examples of pairs "two," and diverse cases of triplets "three" may help children abstract a concept of one, two, and three. Unlike Kumon (in which number recognition training often involved collections of only dots), using the terms "one" and "two" with myriad *referents* (e.g., "two eyes," "two hands," "two cats," "two blocks," "two of red ones") might further facilitate the abstraction process. Specifically, it can help children understand that a wide variety of physical characteristics are irrelevant to number concepts and prompt their search for a common attribute (numerosity). Consider, for instance, the *Can You Find?* game in which a parent or teacher might put out a large and a small blue, red, and yellow block and point to the two red blocks and announce "two red blocks." Follow up questions for a child or children might include "Can you find and give me two blue blocks?" "Can you find the two blocks on this end [point]," "Can you find and give me two big blocks?" This illustrates that the color, location, or size of an object can define what constitutes a particular collection but that they do not apply to examples of a number and thus are not critical (defining) attributes of a number. Note also that such an activity may help a child to make the transition to using verbal number recognition as a means to verbally producing a collection of a specified size. Once children can reliably recognize homogeneous collections of a particular number, questions about heterogeneous collections (e.g., "How many toys are in your toy box here?") may help deepen or broaden their understanding of a cardinal number.

Implication #4: Explicitly pointing out non-examples of a number may more readily define the boundaries of the concept. (a) One way of comparing and contrasting number words is to introduce them in pairs. Adults might help children construct cardinal concepts by first focusing on one and two, next on two and three, and then on three and four. For instance, reliable identification of one and two can serve as the lower boundary for "three" and examples of "four" can help define its upper boundary. (b) A second method is systematically labeling collections as "*n*" and "not *n.*" For example, after labeling two fingers "two," a parent or teacher could hold up one, three, four, and five fingers, in turn, and label each as "not two." Contrasting examples and non-examples can be done in the context of a game such as *Number—Not the Number.* For example, a child can point to all the collections of two s/he can see and then to all the non-examples of two s/he sees ("Point to something that is not two"). For small groups of children, players can take turns pointing out an example and a non-example of a number. In either case, the game can be made more challenging by putting a time limit on the pointing out process or done in small groups, where children take turns identifying an example of number and a non-example. (c) Children's errors can serve as an opportunity to point out non-examples and provide precise feedback. For instance, if a child misidentifies a picture of three bears as "two," a parent or preschool teacher could say, "That's not two bears; it's *three* bears." (d) Perhaps less discomfiting and often highly enjoyable, adults can use error-detection games, in which children indicate when an adult or muppet makes a number identification error.

Implication #5: Instruction (and testing) should be sensitive to developmental level or individual zones of competence. Preschool teachers of 2s or somewhat older at-risk children should be aware that what constitutes an overwhelmingly large number may vary from child to child and that this key internal factor can involve very small numbers and may be narrow (exceeded quickly). Note that what constitutes an overwhelming task evolves with development. For instance, children who have constructed accurate cardinal concepts of one, two, and three, but who are just learning to count collections in a one-to-one fashion, may perform well on number recognition task involving small numbers but melt down on a task that requires counting ten haphazardly arranged items.

Implication #6: Direct instruction without a purpose, such as the Kumon training, may not kindle intrinsic motivation and, thus, may not efficiently promote functional use of the small numbers. The relative success of adult-initiated number tasks with young children may depend to a significant degree on making such tasks engaging and inviting (e.g., a project, math game, or lessons that build on children's curiosity; Smith, 2002). This is not to say that adults cannot or should not mold children's interests but that attention to affect is critical in implementing meaningful learning (Brownell, 1935; Piaget & Inhelder, 1969). Taking advantage of child-initiated efforts to learn and use numbers, such as building on a child's question, may be particularly important opportunities for promoting early number learning.

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REFERENCES

Anderson, A. (1991). Learning mathematics at home: A case study. *Canadian Children*, *16*(2), 47–58.

Ansari, D., Donlan, C., Thomas, M., Ewing, S., & Karmiloff-Smith, A. (2003). What makes counting count? Verbal and visuo-spatial contributions to typical and atypical number development. *Journal of Experimental Child Psychology*, *85*, 50–62.

Balaban, M. T., & Waxman, S. R. (1997). Do words facilitate object categorization in 9-month-old infants? *Journal of Experimental Child Psychology*, *64*, 3–26.

Baroody, A. J. (1987). *Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers*. New York, NY: Teachers College Press.

Baroody, A. J. (1999). The development of basic counting, number, and arithmetic knowledge among children classified as mentally retarded. In L. M. Glidden (Ed.), *International review of research in mental retardation, vol. 22* (pp. 51–103). New York, NY: Academic Press.

Baroody, A. J. (2002, May). *The developmental foundations of number and operation sense*. Poster presented at the EHR/REC (NSF) Principal Investigators' Meeting ("Learning and Education: Building Knowledge, Understanding Its Implications"), Arlington, VA.

Baroody, A. J. (2008, March). *Fostering early number sense*. Invited keynote speech presented at the Banff International Conference on Behavioural Science, Banff, Alberta.

Baroody, A. J., Benson, A. P., & Lai, M. L. (2003, April). *Early number and arithmetic sense: A summary of three studies*. Paper presented at the biennial meeting of the Society for Research in Child Development, Tampa, FL.

Baroody, A. J., Lai, M., & Mix, K. (2005, December). Changing views of young children's numerical and arithmetic competencies. Paper presented at the annual conference of the National Association for the Education of Young Children, Washington, DC.

Baroody, A. J., Lai, M.-L., & Mix, K. S. (2006). The development of young children's number and operation sense and its implications for early childhood education. In B. Spodek & O. Saracho (Eds.), *Handbook of research on the education of young children* (pp. 187–221). Mahwah, NJ: Erlbaum.

Baroody, A. J., Li, X., & Lai, M.-L. (2008). Toddlers' spontaneous attention to number. *Mathematics Thinking and Learning*, *10*, 1–31.

Baroody, A. J., & Mason, C. A. (1984). The case of Brian: An additional explanation for production deficiencies. In J. Moser (Ed.), *Proceedings of the Sixth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 2–8). Madison, WI: Wisconsin Center for Educational Research.

Baroody, A. J., & Price, J. (1983). The development of number word sequence in the counting of three-year-olds. *Journal for Research in Mathematics Education*, *14*, 361–368.

Beilin, H. (1975). *Studies in the cognitive basis of language development*. New York, NY: Academic Press.

Beilin, H., & Kagan, J. (1969). Pluralization rules and the conceptualization of number. *Developmental Psychology*, *1*, 697–706.

Benson, A. P., & Baroody, A. J. (2003, April). *Where does non-verbal production fit in the evergence of children's mental models*? Paper presented at the annual meeting of the Society for Research in Child Development, Tampa, FL.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting*, *5*(1), 35–45.

Bloom, P., & Wynn, K. (1997). Linguistic cues in the acquisition of number words. *Journal of Child Language*, *24*, 511–533.

Brownell, W. A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In D. W. Reeve (Ed.), *The teaching of arithmetic (Tenth yearbook, National Council of Teachers of Mathematics*, pp. 1–50). New York, NY: Bureau of Publications, Teachers College, Columbia University.

Carey, S. (2004). Bootstrapping and the origin of concepts. *Daedalus*, *133*, 59–68.

Carey, S., & Sarnecka, B.W. (2006). The development of human conceptual representations. In M. Johnson & Y. Munakata (Eds.), *Processes of change in brain and cognitive development: Attention and performance XXI*, 473–496. New York, NY: Oxford Press.

Condry, K. F., & Spelke, E. S. (2008). The development of language and abstract concepts: The case of natural number. *Journal of Experimental Psychology: General*, *137*, 22–38.

Dantzig, T. (1954). *Number: The language of science*. New York, NY: The Free Press. (Original work published in 1933)

Dueker, G., & Needham, A. (2003, April). *Labeling facilitates 5-month-old infants' category formation*. Poster presented at the annual meeting of the Society for Research in Child Development, Tampa, FL.

Durkin, K., Shire, B., Riem, R., Crowther, R. D., & Rutter, D. R. (1986). The social and linguistic context of early number word use. *British Journal of Developmental Psychology*, *4*, 269–288.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York, NY: Springer-Verlag.

Gallistel, C. R. (2007). Commentary on Le Corre & Carey. *Cognition*, *105*, 439–445.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*, 43–74.

Geary, D. C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Gibson, D. J., & Shusterman, A. (2009, April). Evidence for very early understanding that "two" means "a pair." Poster presented at the biennial meeting of the Society for Child Development, Denver, CO.

Ginsburg, H. P. (1977). *Children's arithmetic*. New York, NY: D. Van Nostrand.

Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General*, *123*, 284–296.

Klahr, D., & Wallace, J. G. (1973). The role of quantification operators in the development of conservation of quantity. *Cognitive Psychology*, *4*, 301–327.

Klahr, D., & Wallace, J. G. (1976). *Cognitive development: An information-processing view*. Hillsdale, NJ: Erlbaum.

Langer, J. (2000). Response to peer commentaries. *Developmental Science*, *3*(4), 385–388.

Lawler, R. (1981). The progressive construction of mind. *Cognitive Science*, *5*, 1–30.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*, 395–438.

Le Corre, M., & Carey, S. (2008). Why the verbal counting principles are constructed out of representations of small sets of individuals: A reply to Gallistel. *Cognition*, *107*, 650–662.

Le Corre, M., Van de Walle, G., Brannon, E. M., & Carey, S. (2006). Re-visiting the competence/performance debate in the acquisition of the counting principles. *Cognitive Psychology*, *52*, 130–169.

Mix, K. S. (1999a). Preschoolers' recognition of numerical equivalence: Sequential sets. *Journal of Experimental Child Psychology*, *74*, 309–332.

Mix, K. S. (1999b). Similarity and numerical equivalence: Appearances count. *Cognitive Development*, *14*, 269–297.

Mix, K. S. (2002). The construction of number concepts. *Cognitive Development*, *17*, 1345–1363.

Mix, K. S. (2009). How Spencer made number: First uses of the number words. *Journal of Experimental Child Psychology*, *102*, 427–444.

Mix, K. S., Huttenlocher, J., & Levine, S. C. (1996). Do preschool children recognize auditory-visual numerical correspondences? *Child Development*, *67*, 1592–1608.

Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). *Quantitative development in infancy and early childhood*. New York, NY: Oxford University Press.

Mix, K. S., Sandhofer, C. M., & Baroody, A. J. (2005). Number words and number concepts: The interplay of verbal and nonverbal processes in early quantitative development. In R. Kail (Ed.), *Advances in child development and behavior, Vol. 33*. New York, NY: Academic Press.

Piaget, J. (1965). *The child's conception of number*. New York, NY: Norton.

Piaget, J., & Inhelder, B. (1969). *The psychology of the child*. New York, NY: Basic Books.

Resnick, L. B. (1992). From protoquantities to operators: Building mathematical competence on a foundation of everyday knowledge. In G. Leinhardt, R. Putnam, & R. A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 373–425). Hillsdale, NJ: Erlbaum.

Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction*. Hillsdale, NJ: Erlbaum.

Rice, M. (1980). *Cognition to language: Categories, word meanings, and training*. Baltimore, MD: University Park Press.

Sandhofer, C. M., & Smith, L. B. (1999). Learning color words involves learning a system of mappings. *Developmental Psychology*, *35*, 668–679.

Sarnecka, B. W., & Carey, S. (2008) How counting represents number: What children must learn and when they learn it. *Cognition*, *108*, 662–674.

Sarnecka, B. W., Kamenskaya, V. G., Yamana, Y., Ogura, T., & Yudovina, J. B. (2007). From grammatical number to exact numbers: Early meanings of "one," "two," and "three" in English, Russian, and Japanese. *Cognitive Psychology*, *55*, 136–168.

Sarnecka, B. W., & Lee, M. D. (2009). Levels of number knowledge during early childhood. *Journal of Experimental Child Psychology*, *103*, 325–337.

Saxe, G. B., Guberman, S., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development*, *52* (Serial No. 2).

Schaeffer, B., Eggleston, V. H., & Scott, J. L. (1974). Number development in young children. *Cognitive Psychology*, *6*, 357–379.

Simon, T. J. (1997). Reconceptualizing the origins of numerical knowledge: A "nonnumerical" account. *Cognitive Development*, *12*, 349–372.

Smith, L. (2002). Reasoning by mathematical induction in children's arithmetic. Boston, MA: Pergamon.

Sophian, C. (1992). Learning about numbers: Lessons for mathematics education from preschool number development. In J. Bideaud & C. Meljac (Eds.), *Pathways to number* (pp. 19–40). Hillsdale, NJ: Erlbaum.

Spelke, E. (2003). What makes us smart? Core knowledge and natural language. In D. Genter & S. Goldin-Meadow (Eds.), *Language in mind*. Cambridge, MA: MIT Press.

Starkey, P., & Cooper, R. G. (1995). The development of subitizing in young children. *British Journal of Developmental Psychology*, *13*, 399–420.

Starkey, P., Spelke, E. S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, *36*, 97–127.

Steffe, L., & Cobb, P. (1988). *Construction of arithmetical meanings and strategies*. New York, NY: Springer-Verlag.

von Glasersfeld, E. (1982). Subitizing: The role of figural patterns in the development of numerical concepts. *Archives de Psychologie*, *50*, 191–218.

von Glasersfeld, E. (1991). Abstraction, re-presentation, and reflection: An interpretation of experience and Piaget's approach. In L. P. Steffe (Ed.), *Epistemological foundations of mathematical experience* (pp. 46–67). New York, NY: Springer-Verlag.

Wagner, S. H., & Walters, J. A. (1982). A longitudinal analysis of early number concepts. In G. Forman (Ed.), *Action and thought: From sensorimotor schemes to symbolic operations* (pp. 137–161). New York, NY: Academic Press.

Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. New York, NY: Routledge.

Waxman, S. R., & Markow, D. B. (1995). Words as invitations to form categories: Evidence from 12- to 13-month-old infants. *Cognitive Psychology*, *29*, 257–302.

Wilkins, J. L. M., & Baroody, A. J. (2000). An additional explanation for production deficiencies. In M. L. Fernández (Ed.), *Proceedings of the Twenty-Second Annual Meeting North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, p. 740). Columbus, OH: The Eric Clearinghouse for Science, Mathematics, and Environment Education.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*, 155–193.

Wynn, K. (1992). Children's acquisition of the counting words in the number system. *Cognitive Psychology*, *24*, 220–251.

Wynroth, L. (1986). *Wynroth math program—The natural numbers sequence*. Ithaca, NY: Wynroth Math Program (Original work published in 1975).