Hammill Institute on Disabilities

Intervention in School and Clinic 48(5) 276–285

DOI: 10.1177/1053451212473151
isc.sagepub.com hosted at http://online.sagepub.com
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Explicit Mathematics Instruction: What Teachers Can Do for Teaching Students With Mathematics Difficulties

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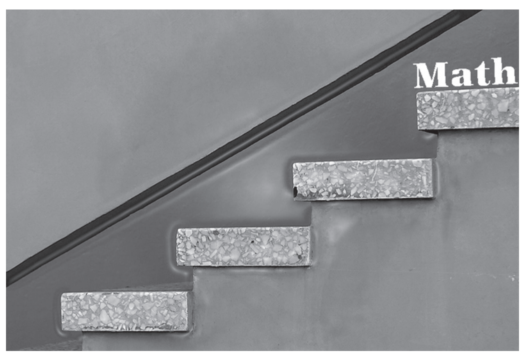

Abstract

This article describes the essential instructional elements necessary for delivering explicit mathematics instruction to students with mathematics difficulties. Mathematics intervention research indicates that explicit instruction is one of the most effective instructional approaches for teaching students with or at risk for math difficulties. Explicit instruction is a systematic approach that facilitates important instructional interactions between teachers and students around critical math content. This article describes a framework for delivering explicit math instruction in the early grades. Although the explicit framework is relevant across the range of early math content (i.e., measurement, geometry), the focus is on explicit instruction in the context of teaching place value concepts in kindergarten and first grade classrooms. Place value is a critical component of whole-number understanding and a necessary concept for students to develop mathematical proficiency. Key questions associated with explicit math instruction are also addressed.

Keywords

mathematics instruction, explicit and systematic instruction, math difficulties

Teaching for mathematical proficiency is a responsibility that all schools bear (Kilpatrick, Swafford, & Findell, 2001). Critical to that effort is ensuring that early mathematics instruction (kindergarten and first grade) is intense enough to meet the needs of students with or at risk for mathematics difficulties (MD). How can schools

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meet this expectation? One solution is to use an explicit instructional approach.

Explicit instruction is an evidence-based practice that provides elementary teachers with a practical and feasible framework for delivering effective and systematic instruction (Archer & Hughes, 2010; Baker, Gersten, & Lee, 2002; Gersten, Chard, et al., 2009). In particular, explicit instruction provides a format to facilitate high-quality instructional interactions among teachers and students around critical math content (Doabler et al., 2012). Moreover, it serves as a method for increasing the number of instructional opportunities that at-risk learners receive both in small-group interventions and in core instruction (Baker, Fien, & Baker, 2010).

This article should help teachers better meet the needs of students with or at risk for math difficulties by promoting the incorporation of explicit instruction elements into their daily math teaching. To do this, the article describes three elements of explicit instruction derived from three sources: (a) the final report of the National Mathematics Advisory Panel (NMAP, 2008), (b) the converging knowledge base of effective instruction (Coyne, Kame'enui, & Carnine, 2011), and (c) the research literature on explicit math instruction (Baker et al., 2002; Gersten, Chard, et al., 2009). These elements, illustrated in Figure 1, include, (a) clear teacher models of math concepts and skills, (b) guided practice opportunities, and (c) academic feedback.

This article describes these elements and provides examples of their role in an explicit instructional approach that supports place value understanding for at-risk learners in kindergarten and first grade (K-1). The examples presented in the article are based on the instructional design framework used in our work at the University of Oregon's Center on Teaching and Learning for developing and efficacy testing early mathematics interventions and curricular programs. Although this article focuses on place value ideas, explicit instruction can also be applied to other concepts and skills in early mathematics. And although the importance of explicit math instruction has been reviewed previously in the literature (Baker et al., 2002; Gersten, Chard, et al., 2009), there is a need to practically address how to translate this research base into the daily practice of elementary teachers. Federal legislation (i.e., Individuals With Disabilities Education Improvement Act, 2004) now requires classrooms to provide high-quality math instruction to students experiencing math difficulties. When used interdependently, these elements (i.e., clear teacher models, guided practice, and academic feedback) may equip teachers who work with students with or at risk for MD with an effective instructional tool for meeting these heightened expectations.

Who Can Benefit From Explicit Math Instruction?

The research literature on math instruction uses the term *math difficulties* to refer to students who are currently identified as having a math disability, as well as those students at risk for math disabilities (Gersten, Jordan, & Flojo, 2005; Fuchs et al., 2010). This same general definition is applied here because it encompasses a broader range of students. For example, Gersten et al. (2005) defined students with MD as those students "performing in the low average range (e.g., at or below the 35th percentile) as well as those performing well below average" (p. 294). Among the considerable number of students who face difficulties with mathematics, many come from economically and educationally disadvantaged backgrounds (Aud et al., 2011). For these students, their MD manifest early and remain persistent throughout the later grades. Typically, they enter school at an elevated risk for math failure because they receive few informal experiences in early mathematics, such as counting and use of math vocabulary (Klein, Starkey, Clements, Sarama, & Iyer, 2008; Starkey, Klein, & Wakeley, 2004).

Generally speaking, students without MD are able to solve math problems fluently and accurately. For students with MD, becoming mathematically proficient can be challenging. Research has shown that students with MD have trouble acquiring a deep understanding of mathematics, both conceptually and procedurally. Evidence suggests these difficulties may stem from poor instruction and a variety of cognitive correlates, including processing speed, working memory, and attention (Fuchs et al., 2005; Gersten et al., 2005).

Among the difficulties students with MD tend to experience, one is most prominent: developing number sense (Gersten & Chard, 1999). Number sense refers to a child's awareness of and fluidity with numbers (Gersten & Chard, 1999; Hudson & Miller, 2006). As described by Berch (2005), "Possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems" (p. 333). Difficulties with number sense typically cut across the conceptual and procedural aspects of math. Students who experience procedural difficulties in K-1 struggle to use efficient counting strategies (e.g., *min* strategy), recall answers to basic number combinations, such as 5 + 2, and solve computational problems involving multidigit addition and subtraction (Bryant & Bryant, 2008; Fuchs et al., 2010; Gersten et al., 2005).

When it comes to whole numbers, research also indicates that many students with MD face difficulties in building conceptual understanding (Gersten, Beckmann, et al., 2009; Kilpatrick et al., 2001). Conceptual understanding in the early grades (K-1) occurs when a student comprehends the importance of math ideas, such as place value (e.g., groupings of ones and tens). A solid understanding of place value is critical for achieving proficient levels in more sophisticated mathematics, such as solving multidigit addition and subtraction (Common Core State Standards Initiative, 2010; Fuson et al., 1997; Van de Walle, 2001). Students with a deep understanding of place value recognize the relationship

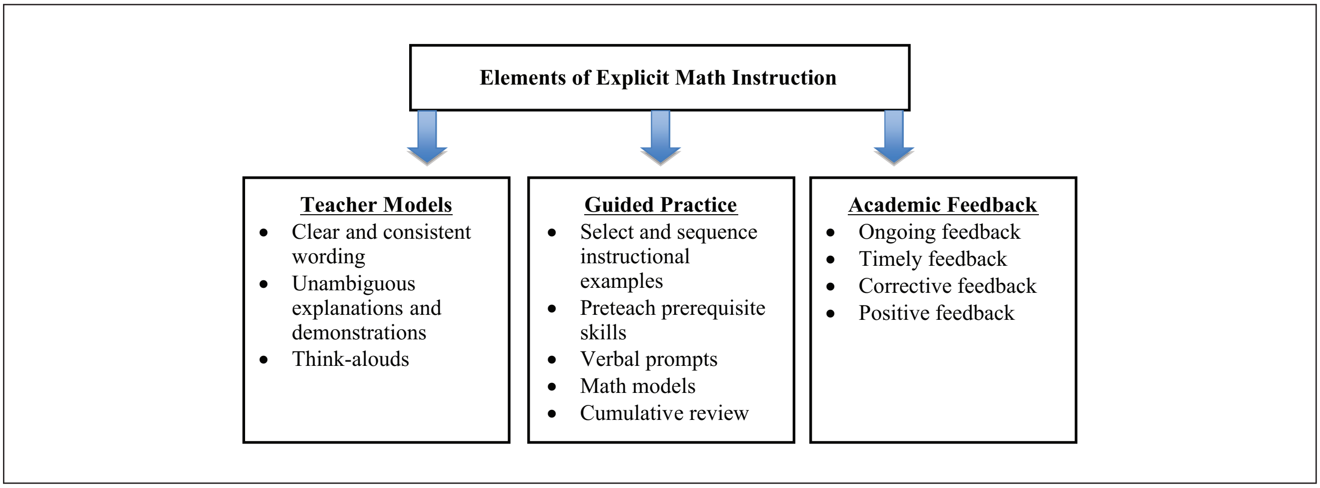


Figure 1. Elements of explicit instruction.

between abstract symbols (i.e., numbers) and math models (e.g., base-10 blocks). They can connect previously learned mathematical knowledge with novel problems and understand the contexts in which particular mathematical skills apply. For example, first grade students with conceptual understanding know the place value of teen numbers and can represent these quantities with base-10 blocks. They also understand that a 10-model is equivalent to 10 ones or a group of 10 units. Conversely, students who have gaps in place value understanding have difficulty grasping the "ten-to-one relationship" (Van de Walle, 2001, p. 153). For example, when asked how many ones make up a 10-model, students with conceptual knowledge difficulties would likely state that it represents one. They do not know that 10 ones is equivalent to 10 as a singular group (Fuson et al., 1997). Although it is difficult to pinpoint the exact cause, conceptual knowledge difficulties, such as procedural knowledge deficits, are likely to be at least partially a function of poorly designed curricula and instruction, and inadequate intervention support (NMAP, 2008).

Explicit Mathematics Instruction: What The Research Says

Explicit instruction has been indentified as one the most effective approaches for teaching students with MD (NMAP, 2008). Studies of mathematics interventions underscore this beneficial impact (Gersten, Chard, et al., 2009). Although most of the research on the application of explicit instruction has been conducted in (Tier 2) small-group instructional formats, recent studies have begun to show that explicit instruction can play a critical role in (Tier 1) whole-class instruction. For example, Clarke et al. (2011) investigated the efficacy of a core math program (Early Learning in Mathematics) with an explicit instructional focus in 65 kindergarten classrooms. Specifically, the yearlong core program included teacher demonstrations to show what students will learn as well as specific wording for providing academic feedback. In addition, the program provided frequent practice opportunities that involve verbal interactions between teachers and students. Clarke et al. found kindergartners at risk for math failure in the treatment classrooms made significant gains relative to their at-risk peers in control classrooms. Of interest, the at-risk students in the treatment classrooms also narrowed the achievement gap with their typically achieving peers.

Bryant and colleagues (2008) explored the impact of a Tier 2 small-group intervention on the math achievement of at-risk first graders. Participants in the study had demonstrated difficulties in their general education classrooms (Tier 1) and thus were deemed as being at risk for MD. The 23-week small-group intervention provided children with explicit instruction that focused on number, whole-number operations, and quantitative reasoning. At the intervention's conclusion, results showed a significant intervention effect on a proximal measure of math achievement. In particular, the findings supported the use of explicit and systematic instruction for improving students' ability to solve place value problems and to answer addition and subtraction facts.

In a more recent Tier 2 study, Dyson, Jordan, and Glutting (2011) tested the effects of an 8-week intervention on kindergarten students' understanding of whole-number concepts. The intervention, which supplemented the students' core math program, provided 30 minutes of small-group instruction, 3 times per week. Dyson and colleagues randomly assigned students to either intervention or control groups and used a combination of student-level covariates to further control for any pretest differences between the two groups. The intervention group showed greater gains over the 8-week intervention than the control group on a standardized measure of math achievement and a measure proximal to the intervention.

Books and Resources on Explicit Instruction

Highlighted below are four sources of information that are likely to benefit the professional development of teachers who work with students struggling with mathematics.

- *Designing Effective Mathematics Instruction: A Direct Instruction Approach – 4th Edition (Stein, Silbert, & Carnine, 2006)*
- *Designing and Implementing Mathematics Instruction for Students with Diverse Learning Needs (Hudson & Miller, 2006)*
- *Math Instruction for Students with Learning Problems (Gurganus, 2007).*
- *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* (Gersten et al., 2009a), a practice guide offered by the Institute of Education Sciences (IES).

Figure 2. Books and resources on explicit instruction.

Collectively, these studies support the notion that students with MD benefit from instruction that is systematically designed and explicitly delivered. Findings from this literature base have implications for all key personnel that may be responsible for providing tiers of support in math instruction, including general education teachers, mathematics interventionists, and special education teachers. Unless classrooms provide explicit math instruction on a regular basis, it is highly likely that students with MD will continue to struggle to learn mathematics successfully (Gersten, Beckmann, et al., 2009; NMAP, 2008).

Elements of Explicit Mathematics Instruction

Often referred to as direct instruction, explicit instruction draws its early roots from the work of Siegfried Engelmann and colleagues (Becker, Engelmann, Carnine, & Rhine, 1981). Although its meaning varies in the research literature (Gersten, Chard, et al., 2009), most would agree that explicit math instruction involves a series of teaching behaviors that include (a) the teacher modeling a new concept or skill, (b) the teacher providing guided practice opportunities, (c) the teacher checking for student understanding, (d) the teacher providing academic feedback, and (e) the students engaging in independent practice. For specific sources of information on explicit math instruction, see Figure 2.

To provide some context for how explicit math instruction works in the classroom, imagine a first grade teacher teaching students how to recognize the place value of three-digit numbers, such as 117. Because students need a deep and lasting understanding of place value (Common Core State Standards Initiative, 2010; Van de Walle, 2001), explicit instruction is ideal for this type of instructional situation. At a glance, the teacher would complete the following explicit instructional behaviors.

1. The teacher would begin instruction by describing the lesson's purpose and objectives. In this case, the teacher would explain to the students that they are going to learn how to use base-10 language to name the place value of three-digit numbers.
2. The teacher would then model for students several different three-digit numbers. For each three-digit number, the teacher would explicitly show and state how many hundreds, tens, and ones (e.g., "117 has 1 hundred, 1 ten, and 7 ones").
3. The teacher might incorporate base-10 models in the demonstrations to further students' conceptual understanding and include one or two 2-digit numbers to teach students how to recognize place value differences among multidigit numbers.
4. Next, the teacher would have the students (group and individuals) verbally identify the place value of three-digit numbers.
5. For incorrect responses, the teacher would provide immediate corrective feedback. It is critical to address any student misconceptions immediately and to provide follow-up review to assess student accuracy in responding (Stein, Silbert, & Carnine, 2006).

The following section provides deeper information on the three elements of explicit instruction. These elements include teacher modeling, guided practice opportunities, and academic feedback. Table 1 addresses some common questions that arise around explicit math instruction.

Clear Teacher Models

The first element of explicit instruction is teacher models. Teacher models are clear demonstrations and unambiguous explanations, and they may include teacher think-alouds (Coyne et al., 2011). Effective teacher models show students exactly what math content they will learn and how they will apply it. There is strong evidence that supports the use of teacher models when teaching students with MD. Baker et al. (2002) synthesized data from math interventions that targeted low-achieving students. Overall, Baker and colleagues reported moderate effect sizes for studies that provided extensive teacher modeling. In a more recent meta-analysis, Gersten, Chard, et al. (2009) analyzed 42 studies involving students with math disabilities and found that the most effective interventions were ones that provided step-by-step demonstrations for solving math problems. Taken together, these studies suggest that students with or at risk for MD are more successful in acquiring new math knowledge when directly shown what to do prior to receiving guided and/or independent practice of such newly learned skills or concepts.

Teachers can model a variety of math content including complex concepts (e.g., place value), key vocabulary (e.g.,

Table 1. Frequently Asked Questions About Explicit Math Instruction.

Question	Response
When is it appropriate for teachers to model math content?	Typically, teacher models are delivered at the beginning of instruction when content is first introduced. However, teacher models can occur throughout a lesson. For example, a teacher might demonstrate how to solve a problem on a math worksheet that students are asked to complete at the end of the lesson. Most important, students should be exposed to a sufficient number of demonstrations before being allowed to work independently.
What are some effective ways to increase the number of practice opportunities for students with or at risk for mathematics difficulties?	An effective way to increase the number of practice opportunities is to elicit responses from all students. Group responses allow more than one student to respond at the same time. For example, a teacher might ask an entire class to answer the following question: "Everyone, what does nine plus three equal?" Mixing group and individual responses also helps keep students actively engaged and provides teachers with an efficient way to gauge understanding of individual students and the group at large.
Should academic feedback be the same for all student responses?	No. The breadth and depth of academic feedback will vary based on the type of response and how far students are in the learning process. For instance, procedural errors typically require more detailed feedback than verbal errors. Corrective feedback for verbal errors requires teachers to provide the correct response, as in the following statement: "Jonah, this is the ones column." Procedural errors, on the other hand, require teachers to remodel one or all of the steps involved in solving the problem. For example, a teacher would show the algorithm for multidigit subtraction on the board if a group of students had difficulty regrouping from the tens to the ones. The amount of academic feedback will also differ based on where students are in the stages of learning. If students are in the early stages (i.e., acquisition), teachers will need to be thorough in their academic feedback. For mistakes made in the latter stages of learning, simple reminders will work.

hundreds), and mathematical procedures and strategies (e.g., subtraction with regrouping). A critical feature of teacher modeling is the use of clear, unambiguous teacher language to guide students through the steps required to solve math problems, as found in Gersten, Chard, et al. (2009) and highlighted in Figure 3. In the example, the teacher explains step-by-step how to solve a multidigit problem involving addition without regrouping. Multidigit operations are particularly troublesome for students with MD because they involve multiple steps and require a sound understanding of place value (Bryant & Bryant, 2008; NMAP, 2008; Van de Walle, 2001). The teacher first identifies the problem type and then draws students' attention to how multidigit problems merely consist of simple, single-digit computations (Wu, 2009). Next, the teacher solves the problem in the ones column followed by the problem in the tens column. For each single-digit computation, the teacher emphasizes how to vertically align the numbers in the appropriate column.

Archer and Hughes (2010) offer three strategies for improving the effectiveness of teacher models. First, they recommend that teachers use precise wording to communicate both simple and complex ideas to students. Second, they recommend that teachers structure instructional interactions that emphasize the active participation of students in the learning process. Finally, they suggest that a sufficient amount of teacher demonstrations be provided so that students clearly understand the learning objectives for the lesson and what proficient performance looks like. These three strategies provide teachers with a framework to facilitate access for students with MD to the critical content of beginning mathematics.

Use clear and consistent language. Demonstrations that contain ambiguous language and complex vocabulary are likely to further overwhelm at risk learners who are already struggling (Stein et al., 2006). To reduce student misconceptions, teachers should use clear and consistent language when modeling math content. Precise wording also helps clarify the expectations of students during a particular math activity. For example, when teaching students that two-digit numbers represent so many tens and so many ones, a teacher would state the following: "Thirty is made up of three tens and zero ones." To keep consistent, the teacher would apply the same language with other two-digit numbers, like 47 (e.g., "4 tens and 7 ones").

Involve students in the teacher models. When students are actively involved in instructional demonstrations, their motivation to learn new content is likely to increase (Archer & Hughes, 2010; Gersten, Beckmann, et al., 2009). Student participation might include answering math-related questions or playing an active role in the instructional demonstration. For example, a teacher might include students in a demonstration

- **Today, I am going to show you how to solve addition problems that have two-digit numbers.**
- **Solving addition problems that have two-digit numbers is just like answering basic math facts** (the teacher writes 46 + 32 on the board).
- **This problems says, forty-six plus thirty-two** (the teacher points to problem).
- **This problem has two math facts in it.**
- **Here is the first math fact, six plus two** (the teacher covers up the tens column and points to the math fact, 6 + 2, in the ones column).

Tens	Ones
4	6
+ 3	2

- **And here is the second math fact, four plus three** (the teacher covers up the ones column and points to the math fact, 4 + 3, in the tens column).
- **Watch as I solve this problem. I am going to start with the math fact in the ones column first** (the teacher covers up the tens column).

Tens	Ones
4	6
+ 3	2

- **The math fact in the ones column is six plus two. Six plus two equals eight. Since I am working in the ones column, I need to write my answer right here** (the teacher points to the ones column and writes "8").
- **Now, I am going solve the math fact in the tens column** (the teacher covers up the ones column).
- **The math fact in the tens column is four tens plus three tens. Four tens plus three tens equals seven tens. Since I am working in the tens column, I need to write my answer right here** (the teacher points to the tens column and writes "7").
- **I just solved the problem. Forty-six plus thirty-two equals seventy-eight.**
- **Everyone read the problem and answer with me.** (46 + 32 = 78)

Tens	Ones
4	6
+ 3	2
7	8

Figure 3. Example of teacher wording for introducing multidigit addition.
Note. The teacher demonstrates two more problems (35 + 23 and 82 + 17), using the same procedures and teacher wording as described above.

on how to combine 10 unifix cubes to form a 10-model. Initially, the teacher would demonstrate combining the cubes on her or his own. In a following demonstration, she or he would provide 10 children each with cube and then overtly show how to combine the cubes (students) to make a 10.

Provide an appropriate amount of teacher models. When used effectively, teacher models help students better understand new and complex content. But if teachers provide too many models, they can take time away from students' independent practice. In fact, too many teacher models are more akin to a lecture than an interactive instructional environment. Teachers should consider three factors when determining how many models to provide students with MD: (a) task difficulty, (b) the knowledge that students bring to the classroom, and (c) student response to the instruction. If the task is relatively easy and students are readily grasping the content, one or two instructional examples might suffice. However, if the task is complex and students lack prerequisite skills, students will likely need additional support. Also, the percentage of students with MD in a classroom will influence the amount of teacher modeling that may be necessary. Teachers with one or two struggling students per classroom compared to classrooms with 25 struggling students will likely differ in the amount and/or type of modeling they provide. In other words, classrooms with higher percentages of students with MD will require frequent modeling. Finally, the accuracy of student responses will be a critical indicator to gauge the appropriateness of ongoing models and to decide whether additional models are necessary. Multiple demonstrations will be required if students are making a considerable number of errors. For example, if students are misidentifying the ones, tens, and hundreds places of three-digit numbers, the teacher will have to provide further demonstrations. However, if students are responding with 100% accuracy, teachers may not want to overdo the provision of additional demonstrations following the initial model.

Guided Practice

The second element of explicit math instruction is guided practice. In many ways, guided practice resembles the support that one would provide when teaching a child to ride a bicycle. Rather than putting a child on a bike for the first time and yelling, "Pedal, pedal, pedal," one would install training wheels until the child learns how to balance and pedal. Similarly, guided practice in the classroom supports students during the early stages of math learning. Such support is systematically withdrawn as students become more proficient with a particular math concept or skill (e.g., place value understanding).

Over the years, researchers have identified aspects of effective guided math practice (Archer & Hughes, 2010; Carnine, 1997; Chard & Jungjohann, 2006; Doabler et al., 2012; Kame'enui & Simmons, 1999). These aspects can be summarized as the following:

Identify and preteach prerequisite skills. Teachers should ensure that students have the prerequisite skills necessary to be successful in the new content (Kame'enui & Simmons, 1999). To help prepare students, prerequisite skills should be addressed prior to the introduction of more advanced content. For example, a kindergarten teacher might incorporate an activity involving teen numbers to prepare students for working with numbers greater than 19. For each teen number, the teacher would show a number card and then have the students name the number and identify the tens and ones. Teachers may use formal or informal assessments to determine if students, or which students, have the prerequisite skills for new math content or if these skills should be pretaught.

Select and sequence instructional examples. Complex instructional examples at the start of instruction can overwhelm struggling learners (Chard & Jungjohann, 2006). To best support initial instruction, teachers should use instructional examples that are easier for students to solve and understand. Judicious selection of instructional examples will help students ease into new math content. For example, some students experience difficulties when more than one new number is introduced at a time, in particular teen numbers (Stein et al., 2006). Teen numbers can be troublesome because of their irregular pronunciations and the place value of the tens. To avoid this confusion, the teacher would introduce only one number at a time and sequence it with numbers the students already know.

Use verbal prompts. Verbal prompts, such as purposefully designed math questions, present a systematic way for teachers to manage math discourse in the classroom. Math discourse is important because it allows students to share their mathematical thinking and understanding (Kilpatrick et al., 2001). Teachers can prompt students to answer math-related questions and explain solutions to problems. For example, a teacher might ask, "Lucia, tell us how you represented 73 with the place value blocks." Teachers should use verbal prompts to initiate student math reasoning and facilitate richer math-centered discourse. To have students verbally respond together, the teacher would use some type of response signal, such as a clap, finger snap, or verbal cue. For example, the teacher might state, "How many hundreds are there in 120? [signal], How many tens? [signal], How many ones? [signal]." Using signals when soliciting group responses prevents one or two students from answering the question while others parrot the answer.

Use multiple representations of math ideas. Math manipulatives are pictorial and concrete representations of math ideas. Among other things, they include number lines, 10 frames, and place value blocks. As with most instructional tools, when used appropriately, manipulatives can help students develop a fundamental understanding of math concepts and skills (Gersten, Chard, et al., 2009; Hudson & Miller, 2006; Van de Walle, 2001). Teachers can incorporate math models into their teaching routines by using a concrete–representational–abstract (CRA) sequence. Under the CRA sequence, a teacher might use 10 bundles, each with 10 straws, to represent 100. In later examples, the teacher might include a base-10 block of 100 to further promote conceptual understanding. Then as students demonstrate understanding of 100, the teacher would fade out the manipulatives and transition to abstract symbols (i.e., numbers).

Provide cumulative review. Carnine (1997) described cumulative review as a way to help students remember and maintain math skills that have been taught previously. Cumulative review also provides teachers with ongoing information on whether students are retaining previously learned concepts or skills. Cumulative review should include a mixture of math problems that students have recently and previously learned. Including a combination of problems will help students practice determining when to apply particular math skills. For example, if students were learning how to add two two-digit numbers, an effective review would also include addition problems that involve single-digit numbers.

Academic Feedback

The third element of explicit instruction is academic feedback. Academic feedback is used to affirm and, when necessary, correct student responses (Hudson & Miller,

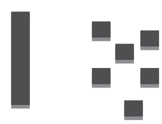
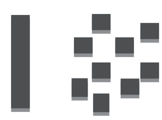
Instructional Day	Instructional Wording
Day 1 tens | ones 1 | 2 	• **Today we are going to learn about place value and teen numbers.** • Teacher writes 12 on the board • **What number?** (12) • **Twelve is made up of one 1 ten** (teacher places 1 ten-stick in tens column) **and two ones** (teacher places 2 cubes in ones column). • **How many tens are in 12?** (1) **How many ones are in 12?** (2) **Nice work!** • Teacher writes 12 under the model in the correct place value columns • Teacher repeats with numbers: 14, 17, 13
Day 2 tens | ones 1 | 6 	• **Today we are going to continue to learn about place value and teen numbers.** • Teacher writes 16 on the board • **What number?** (16) • **How many tens are in 16?** (1) **Yes, 1 ten.** (teacher places 1 ten-stick in tens column) • **How many ones are in 16?** (6) **Yes, 6 ones.** (teacher places 6 cubes in ones column) • **Everyone count the base-10 blocks to see if we have 16.** (teacher touches the tens column as student count on to 16). • **So, 16 is made up of 1 ten and 6 ones. What makes up 16?** (1 ten and 6 ones) **Yes, 1 ten and 6 ones.** • Teacher tells one student to write 16 in the correct place value columns • Teacher repeats with numbers: 1 0, 12, 18, 19
Day 3 tens | ones 1 | 9 	• **Today we are going to learn how to show teen numbers with our base-10 blocks and place value mats** • Teacher writes 19 on the board • **What number?** (19) • **Nineteen has how many tens?** (1) **Yes, 1 ten.** • **Nineteen has how many ones?** (9) **Yes, 9 ones.** • **Use your ten sticks and cubes to show 19** (teacher monitors as student show 19 in the correct place value columns). • **Everyone, count the ten sticks and cubes to make sure we have 19. How many?** (19) **Yes, 19.** • **Now I want everyone to write 19 on the place value mat.** • Remaining practice opportunities include teen numbers and single-digit numbers (e.g., 15, 16, 8, 13, 9, 2)

Figure 4. Using explicit instruction to teach place value across 3 instructional days.

2006). Consistent academic feedback reduces the potential for misunderstandings and helps deepen student understanding of math concepts and skills (Doabler et al., in press). Teachers should provide timely feedback since errors are easier to repair the earlier they are acknowledged (Stein et al., 2006). When teachers correct student errors, they should use language that is positive in nature and specific to the mistake. In particular, they should state the correct response and then provide a second practice opportunity to the student or group of students who made the mistake. For example, when students misidentify the number of tens in 83, the teacher would state, "Eighty-three has eight tens and three ones. How many tens are in 83? [eight] Yes, there are eight tens. How many ones are in 83? [three] Yes, there are three ones." Correct responses also require teachers' attention. Teachers can enhance the instructional learning opportunity and increase student motivation when they affirm a correct response. Positive academic feedback also lets students know that they are on track and can be successful in doing mathematics (Kilpatrick et al., 2001).

Putting the Elements of Explicit Instruction in Practice

Figure 4 shows an example of explicit instruction when teaching place value to first grade students. The example includes activities that span across 3 instructional days. Highlights of the instructional sequence can be briefly summarized as the following:

1. States clear expectations at the start of instruction
2. Starts instruction with a relatively easy instructional example
3. Limits the number of instructional examples
4. Uses consistent wording throughout the activities
5. Provides clear demonstrations and step-by-step explanations
6. Provides frequent practice opportunities
7. Uses math manipulatives to build conceptual understanding
8. Offers ongoing academic feedback
9. Provides cumulative review at the end of the third activity

Conclusion

By drawing from the final report of the NMAP (2008) and the research literature on effective math instruction, this article provides guidelines that teachers can use to improve the quality of math instruction for students at risk for math failure by making daily math instruction more explicit and systematic.

Mathematical proficiency is essential for student success in school and in postsecondary experiences. Despite increased interest in supporting the development of math proficiency for all students, compelling evidence indicates many children experience an early onset of MD that last across the academic years (Morgan, Farkas, & Wu, 2009). To meet the needs of students struggling with mathematics, efforts will require the delivery of evidence-based math instruction in general education classrooms and small-group interventions. The use of explicit math instruction in both educational settings will likely strengthen these efforts.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through grants R305A080114 and R324A090341 to the Center on Teaching and Learning at the University of Oregon.

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