Cognition and Instruction

ISSN: 0737-0008 (Print) 1532-690X (Online) Journal homepage: https://www.tandfonline.com/loi/hcgi20

Fostering At-Risk Kindergarten Children's Number Sense

Arthur J. Baroody , Michael D. Eiland , David J. Purpura & Erin E. Reid

COGNITION AND INSTRUCTION, *30*(4), 435–470, 2012

ISSN: 0737-0008 print / 1532-690X online
DOI: 10.1080/07370008.2012.720152

Fostering At-Risk Kindergarten Children's Number Sense

Arthur J. Baroody, Michael D. Eiland, David J. Purpura, and Erin E. Reid
University of Illinois at Urbana-Champaign

A 9-month training experiment evaluated whether computer-assisted discovery learning of arithmetic regularities can facilitate kindergartners' fluency with the easiest sums. After a pretest, kindergartners with at least one risk factor ($n = 28$) were randomly assigned to either a structured add-0/1 training condition, which focused on recognizing the $n + 0/0 + n = n$ and the $n + 1/1 + n =$ the number-after-n rules, or an active control group. Using pretest fluency as the covariate, ANCOVAs revealed that the structured add-0/1 group significantly outperformed the control group on both practiced and unpracticed (transfer) $n + 0/0 + n$ and $n + 1/1 + n$ items at the delayed posttest and had significantly larger gains in mathematics achievement. Key instructional implications include: Early intervention that targets discovering rules for adding with 0 and 1 and *family-specific* developmental prerequisites is feasible and more effective than typical classroom instruction in promoting fluency with such basic sums. Such rules may be a critically important bridge between informal and formal mathematics.

Fluency with the basic addition combinations—efficiently, appropriately, and adaptively generating sums to single-digit items such as $7 + 1$ and $4 + 4$—has long been a central goal of primary-level instruction. Efficiently entails fast and accurate responses. Appropriate use entails selectively applying knowledge. For example, the *add-1 rule* specifies that the sum of $n + 1$ or $1 + n$ ($n + 1/1 + n$)—but not other—items is the number after n in the count sequence (e.g., the sum of $7 + 1$ or $1 + 7$, but not $7 + 0$ or $2 + 7$, is the number after *seven*—*eight*). Adaptive use involves flexibly adjusting or applying knowledge to solve new problems (e.g., transfer of the add-1 rule to previously unpracticed or even multi-digit $n + 1/1 + n$ items).

Fluency with basic sums is widely recognized as a critical expertise *all* children need (e.g., National Research Council [NRC], 2001; National Council of Teachers of Mathematics [NCTM], 2000, 2006). The National Mathematics Advisory Panel (NMAP, 2008), for example, concluded that combination fluency is a foundation for learning and efficiently applying more advanced mathematics skills and critical to long-term mathematics achievement. By freeing cognitive resources, it can expedite inductive reasoning (i.e., discovering new patterns or relations), problem solving, estimating, and other aspects of arithmetic fluency (e.g., efficiently executing mental or written computation with multi-digit and rational numbers).

However, children at risk for academic difficulties often are not developmentally ready for formal arithmetic instruction in general and achieving fluency with basic sums in particular (e.g.,

Correspondence should be addressed to Arthur J. Baroody, College of Education, University of Illinois at Urbana-Champaign, Champaign, IL 61820. E-mail: baroody@illinois.edu

Baroody, Eiland, & Thompson, 2009; Jordan, Kaplan, Olah, & Locuniak, 2006). Moreover, it is not entirely clear how best to help such children achieve fluency with basic combinations. Although numerous drill and practice programs have been developed and some previous research indicates that such instruction can sometimes be more effective than traditional instruction (see review by Kulik & Kulik, 1991; but see Fuchs et al., 2006, and Lin, Podell, & Tournaki-Rein, 1994, for mixed results), particularly unclear is what role computer-based discovery learning might play. Alfieri, Brooks, Aldrich, and Tenenbaum (2011) defined discovery learning as not providing learners with the target information or conceptual information but creating the opportunity to "find it independently . . . with only the provided materials" (p. 2).

RATIONALE FOR THE PRESENT STUDY

The primary aim of the present study was to evaluate the promise or feasibility of two computer programs to foster kindergartners' fluency with basic sums. It also provided the opportunity to evaluate the implications of a hypothetical learning progression of mental addition. The background/justifications for the primary and subsidiary aims are discussed in turn.

Primary Aims: Feasibility of Two Programs for Promoting Fluency With Basic Sums

Reasoning strategies involve using known sums and relations to deduce the sum of an unknown item. For the near double $2 + 3$, for example, the near-doubles strategy entails using the known double ($2 + 2 = 4$) and the relation that 3 is 1 more than 2 to reason that $2 + 3$ must be 5. Addressed, in turn, are why reasoning strategies are key to achieving fluency, why at-risk children may have difficulties learning such strategies, and why meaningful instruction, discovery learning, and game-based computer training may help them learn reasoning strategies.

Developmental Phases: Reasoning Strategies as a Key Transition. The conventional wisdom regarding the role of reasoning strategies has shifted dramatically over the last century (Verschaffel, Greer, & De Corte, 2007). For much of the twentieth century, behavioral/cognitive psychologists and educators typically believed that practice was *the* basis for increasing associative strength and promoting fact recall (Ashcraft, 1992) or was at least the most important factor in memorizing basic facts (Siegler, 1986; Torgeson & Young, 1983). Thorndike's (1922) *law of frequency* (the more two stimuli are presented together, the stronger their association) justified large doses of practice. For example, Siegler and Jenkins (1989) conjectured that each time an expression and its sum (e.g., "$5 + 1 = 6$") is practiced, a memory trace is produced, and thousands of such traces are necessary to achieve efficient fact recall. Moreover, the same was deemed true of related combinations, even a fact's commuted partner. Early theorists regarded both counting and reasoning strategies as crutches or bad habits that interfered with the process of memorizing facts by rote (see, e.g., reviews by Baroody, 1985 and Cowan, 2003). More recently, psychologists have considered counting and reasoning as valuable means of practice and achieving fact recall (Crowley, Shrager, & Siegler 1997; Fuchs et al., 2010; NRC, 2001).

An alternative view is that reasoning strategies are critical to achieving fluency (Baroody, 1985, 1994). Besides providing practice to reinforce the association between a particular expression and its sum, reasoning strategies can serve as a bridge to fluent retrieval in two ways: (a) they "enable pupils to organize and understand relations among facts that aid in [meaningful] memorization and recall" (Rathmell, 1978, p. 16; see also Dowker, 2009, p. 402); and (b) with practice, reasoning strategies can become automatic (Jerman, 1970). Children, then, progress through three overlapping phases in the meaningful learning of a particular basic sum or family of sums: (a) *Phase 1* (*counting strategies*); (b) *Phase 2* (*conscious or deliberate reasoning strategies*); (c) *Phase 3—fluent retrieval* (using an efficient memory network including fact recall and *non-conscious or automatic reasoning strategies* [Baroody, Bajwa, & Eiland, 2009; Fayol & Thevenot, 2012; Verschaffel et al., 2007]).

Indeed, research indicates that teaching children reasoning strategies is more effective than drill in facilitating learning, retention, and transfer of basic combinations (Brownell, 1941; Brownell & Chazel, 1935; Steinberg, 1985; Swenson, 1949; Thiele, 1938; Thornton, 1978). For example, Henry and Brown (2008) found that, whereas the use of textbooks that focused on memorizing all basic addition and subtraction facts by rote and timed tests were *negatively* related to learning basic combinations and the use of flashcards had no positive effect, teaching reasoning strategies was positively correlated with fluency gains at the end of first grade. For these reasons, there is broad agreement that instruction on the basic combinations should focus on fostering relational learning in general and reasoning strategies in particular (NMAP, 2008; NRC, 2001; Rathmell, 1978; Suydam & Weaver, 1975; cf. Katona, 1967).

Developmental Difficulties. Unfortunately, many children have difficulty achieving Phase 2. In fact, a characteristic of pupils with mathematical learning difficulties is that they do not spontaneously invent reasoning strategies and independently achieve Phase 2 (Swanson & Cooney, 1985; Swanson & Rhine, 1985). As a result, achieving fluency with basic sums (Phase 3) is a serious stumbling block for many schoolchildren (e.g., Henry & Brown, 2008), and a lack of fluency is a pervasive characteristic of those who have difficulties learning mathematics (Geary, 1990, 1996, 2003; Jordan, Hanich, & Kaplan, 2003; Jordan, Hanich, & Uberti, 2003).

As achieving Phase 2 and a well-structured Phase 3 seems to stem from number sense (Gersten & Chard, 1999; Jordan, 2007), inadequate informal knowledge and formal instruction that does not build on informal knowledge or otherwise focus on relations may delay or prevent combination fluency. Children at risk for academic failure, in particular, may have critical gaps in informal knowledge—including the developmental prerequisites of mental addition (Baroody et al., 2009). Compounding the problem, teachers in schools with large populations of at-risk children are among the most poorly trained, particularly in regards to mathematics and methods for fostering number sense (Ferguson, 1998; Lee, 2004; Lubienski, 2001; Lubienski & Shelley II, 2003). At-risk children are particularly susceptible, then, to difficulties or delays in achieving fluency with even the most basic sums (Jordan, Huttenlocher, & Levine, 1992, 1994; Jordan et al., 2006). Effective early intervention may help many in this large and growing population of pupils achieve Phases 2 and 3 and avoid debilitating difficulties caused by not achieving fluency with basic sums (Dev, Doyle, & Valente, 2002; Fuchs et al., 2005).

Meaningful Instruction of Reasoning Strategies. In an extensive literature review, the NRC (2001) concluded that Phase 2 can be accelerated by directly teaching reasoning strategies if done conceptually. James (1958) observed that meaningful and secure memorization of new information can be achieved by relating it to what a child already knows (cf. Piaget, 1964). For example, although many kindergartners can efficiently cite the number after another in the count sequence, they do not use this relational knowledge to determine the sum of $n + 1/1 + n$ items and resort to a counting-all strategy. Connecting adding with 1 to extant number-after knowledge yields a general *add-1 rule* that can be applied to any $n + 1/1 + n$ combination for which they know the counting sequence, even those not previously practiced.

The doubles are among the easiest combinations probably because they represent the repetition of common collections or events, such as two hands of five fingers each is ten fingers altogether, and these sums parallel the skip-count-by-two sequence (Baroody & Coslick, 1998; Rathmell, 1978). Moreover, the doubles mimic the first two counts in various skip counts (e.g., $5 + 5$ can be reinforced by knowing the skip-count-by fives: "Five, ten . . . "). Connecting doubles to familiar pairs, the well-known skip-count-by twos, and emerging knowledge of other skip counts should facilitate the learning of these basic combinations.

Direct teaching of reasoning strategies accompanied by explanation of their rationale is often recommended by mathematics educators (Rathmell, 1978; Thornton, 1978, 1990; Thornton & Smith, 1988) and utilized in many elementary curricula. However, not all conceptually based instruction may be equally effective. Chi (2009) hypothesized that interactive learner activities (substantively dialoguing and considering a partner's contribution) are more effective at promoting learning than constructive activities (producing responses that entail ideas that go beyond provided information), which are more effective than active activities ("doing something physically"), which in turn are more effective than passive activities (e.g., listening or watching without using, exploring, or reflecting on the presented material). Direct instruction—even when it attempts to illuminate the rationale for a reasoning strategy—typically embodies passive activities. As a result, such training may be uninteresting, incomprehensible, and merely produce routine expertise, which leads applying a strategy inflexibly and inappropriately. For example, Murata (2004) found that Japanese children taught a make-ten strategy with larger-addend-first items (e.g., $9 + 5 = 9 + [1 + 4] = [9 + 1] + 4 = 10 + 4 = 14$) did not exhibit strategy transfer when smaller-addend-first items were introduced. Torbeyns, Verschaffel, and Ghesquiere (2005) found that children taught the near-doubles strategy sometimes used the strategy accurately but other times inaccurately (e.g., relating $7 + 8$ to $7 + 7 - 1$ or $8 + 8 + 1$ instead of $7 + 7 + 1$ or $8 + 8 - 1$).

Discovery-Learning Approaches. Some mathematics educators (e.g., Swenson, 1949; Thiele, 1938; Wilburn, 1949) have recommended teaching reasoning strategies via discovery learning instead of direct instruction. Alfieri et al. (2011) observed that discovery learning has increasingly supplanted traditional direct instruction in recent decades because "allowing learners to interact with materials [and] explore phenomena . . . affords them . . . opportunities to notice patterns, [relations], and learn in ways that are seemingly more robust" (p. 1). However, discovery learning encompasses a wide range of methods, which may not be equally effective in all cases. At one extreme is structured and explicitly guided practice (e.g., items arranged sequentially to underscore a pattern or relation, explicit scaffolding of questions or hints to guide attention to regularities, explicit feedback that explains why a response is correct or incorrect, and additional hints or scaffolding as needed). At the other extreme is unstructured and unguided practice (e.g., haphazardly presented items, no explicit scaffolding or feedback). Although previous training studies (using an untimed mental-addition task) have found that the latter can help typically developing kindergartners to discover and use the *add-0 rule* (adding with 0 leaves the other number unchanged) and add-1 rule (Baroody, 1989b, 1992), Alfieri et al.'s (2011) meta-analysis of 164 studies revealed that guided discovery learning was more effective than other forms of instruction (e.g., direct instruction or unguided discovery).

The interventions to promote reasoning strategies in the present study involved an intermediate type of discovery learning. In an effort to move beyond passive learning activities (Chi, 2009) and provide some guidance (Alfieri et al., 2011), the interventions were structured in the sense that problems were sequentially arranged to highlight a relation and where it was applicable. For example, for the add-0/1 training, answering a number-after-n question (e.g., "What number comes after 3 when we count?") was immediately followed by answering a related $n + 1$ item (e.g., "$3 + 1 = ?$"). In theory, this sequence of questions may result in considering the $n + 1$ expression, its sum, n, and the number after n simultaneously in working memory and, thus, increase the chance of inducing a localized add-1 rule. An $n + 1$ item was then followed by a $1 + n$ item to prompt recognition of additive commutativity and that the add-1 rule applied to $1 + n$ items as well. An $n + 0/0 + n$ item and an $n + m$ item (where n *and* $m > 1$) served as non-examples of the add-1 rule to discourage over-generalizing the rule (i.e., to instill its flexible use). The instruction involved minimal guidance and implicit discovery in the sense that children actively determined sums, reconsidered answers in the face of feedback regarding correctness only, and detected a mathematical regularity and used this regularity to construct strategies for themselves.

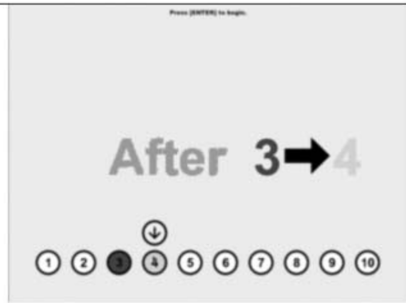
Computer-Assisted, Game-Based Training. Although little software exists to help young children discover relations or invent reasoning strategies, such software might be particularly valuable for early intervention with at-risk children (Rasanen, Salminen, Wilson, Aunio, & Dehaene, 2009). Well-designed computer programs and properly chosen computer games can enhance motivation and provide effective instruction and practice even for young children (Bright, Harvey, & Wheeler, 1985; Clements & Sarama, 2012; NRC, 2009; Ernest, 1986; Sarama & Clements, 2009). A program can provide the scaffolding for structured discovery learning that most teachers cannot. Specifically, it can underscore relations, such as the connection between number-after relations and adding 1 (e.g., see Figure 1 for details), a relation probably unknown by most teachers. Moreover, it can embody a learning trajectory, such as how number-after-n proficiency and mental addition with 1 unfold developmentally (e.g., see Figures 1 to 3), also probably unknown by most teachers. The training also provided immediate feedback on correctness on each of a child's responses, which is difficult to accomplish for a teacher responsible for classes of 20 or more children. The game context provided motivation to learn for children accustomed to being entertained by television and online/mobile/computer games, and games appear to be effective in promoting early mathematics learning.

Subsidiary Aims: Implications of a Hypothetical Learning Progression

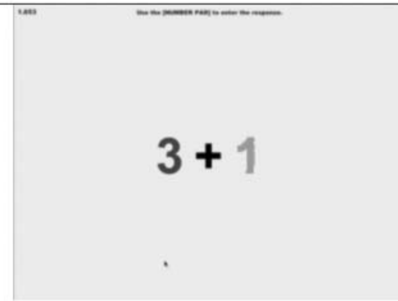
Proposed is a hypothetical model of early mental-addition development based on the following two suggestions: (a) learning trajectories and big ideas can be invaluable in guiding assessment and instructional efforts (Clements & Sarama, 2004, 2009), and (b) "future models of arithmetic [might] benefit from including retrieval structures or other mechanisms that embody numerical

Frame A: Student is asked: "When we count, what number comes after 3?" The 3 is shown in blue.

Frame B: A 1-to-10 number list with a blue-shaded 3 appeared. Child entered his/her answer by clicking the mouse on the number list. The arrow then moved over to the child's answer

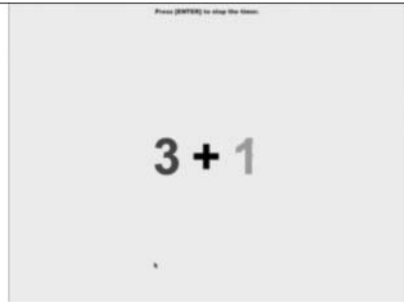


Frame C: If the student was correct, the answer appeared in green. If incorrect, the circle with the corresponding answer became red, and the arrow moved back above the numeral 3. The student was instructed to use a method that would help him/her determine the answer.

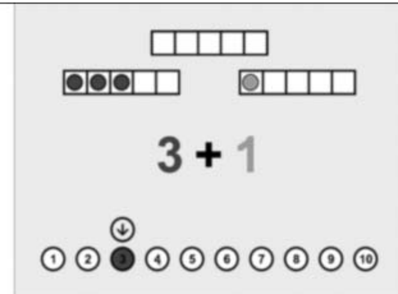


Frame D: After completing the number-after task, a corresponding $n+1$ item appeared featuring the number from the number-after trial.

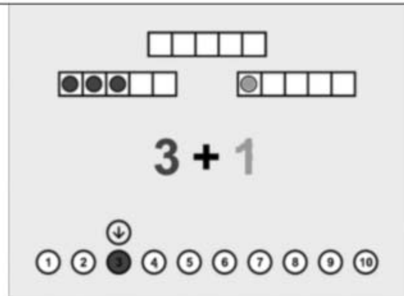
FIGURE 1 The initial add-0/1 training involving a complete bubble line. *(Continued)*



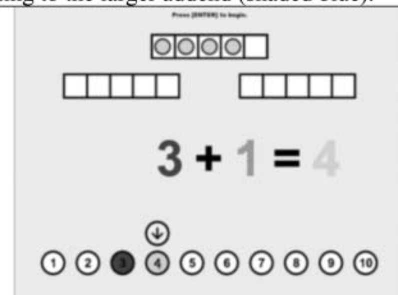
Frame E: The first addend (3) was displayed in blue and the second addend (1) was displayed in yellow. The student was instructed to answer as quickly as possible.



Frame F: After the trainer input the answer using a number pad, the addends were displayed under color-coded 5-frame (or 10-frame) representations and above a 1 to 10 number list with an arrow pointing to the larger addend (shaded blue).

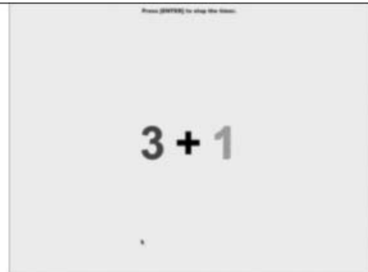


Frame G: A student was asked to provide an answer to the problem and prompted to use any aid (e.g., 5-frames, fingers) to help him/her determine the sum.

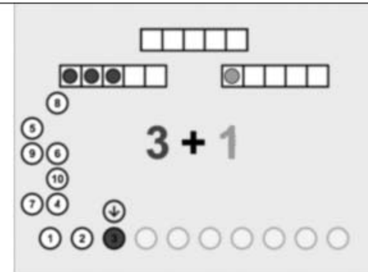


Frame H: The sum was displayed in a single 5-frame (or 10-frame) at the top of the screen in green to highlight the relationship blue (first addend) and yellow (second addend) together are green (sum).

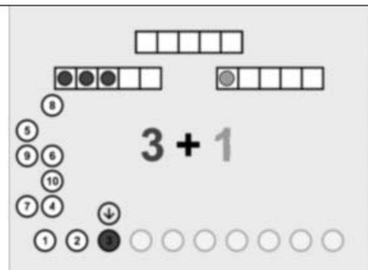
FIGURE 1 (Continued)



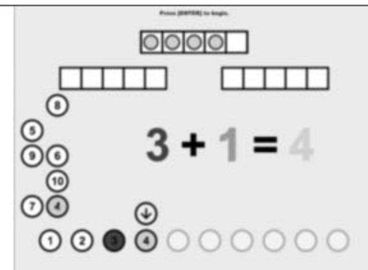
Frame A: Identical to Frame E in Figure 1.



Frame B: Identical to Frame F of Figure 1, except that all numerals greater than the larger addend appear jumbled on the left while their positions on the number list appear shaded.

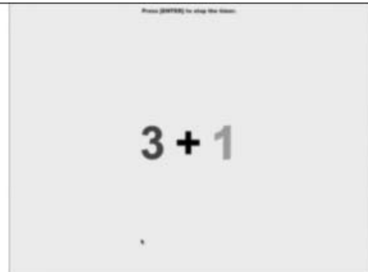


Frame C: Identical to Frame G of Figure 1, except that all numerals greater than the larger addend appear jumbled on the left while their positions on the number line appear shaded.

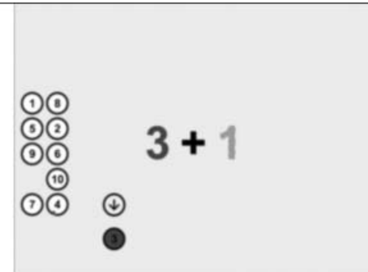


Frame D: The sum is displayed in a single 5-frame (or 10-frame) at the top of the screen in green to highlight the relationship blue (first addend) and yellow (second addend) together are green (sum).

FIGURE 2 Intermediate add-0/1 training with a partial bubble line.



Frame A: Identical to Frame E of Figure 1.



Frame B: Identical to Frame F of Figure 1, except that all other numerals appear jumbled on the left.

Frame C: Students were prompted to provide an answer as quickly as possible by clicking on the appropriate numeral representation.

Frame D: The yellow arrow hopped the corresponding number of places to the sum and displayed its relative position on the number list. The correct answer appears in green.

FIGURE 3 Advanced add-0/1 training with n only and a number jumble (to the left).

magnitude representations" (Siegler & Ramani, 2009, p. 556). The hypothetical learning trajectory and key implications are discussed next.

The Hypothetical Learning Trajectory. The *Common Core State Standards* (CCSS, 2010) includes as a goal for kindergarten to "understand that each successive number name refers to a quantity that is exactly one larger." This goal refers to the big idea described hereafter as the *successor principle*. This principle may be the conceptual basis for (a) re-representing the counting sequence as the (positive) integer sequence (n, $n + 1$, $[n + 1] + 1 \ldots$) and in a linear manner; (b) informal mathematical induction,[1] which permits using a discovered pattern between small successive positive integers to draw a new conclusion about all counting numbers; and (c) the add-1 rule.[2] The add-1 rule may be the retrieval structure that connects mental arithmetic with children's mental addition with their representation of counting and magnitude (e.g., the successor principle).

Implications. Two implications follow from the model:

1. *The meaningful development of fluency with a family of combinations depends on family-specific developmental prerequisites*. A developmental prerequisite for both the successor principle and add-1 rule (or at least their fluent application) is fluency with number-after relations. All of these aspects of knowledge plus automatic knowledge of the doubles are developmental prerequisites for achieving fluent near-doubles reasoning strategy.
2. *Learning the add-1 rule may have a broad impact.* A fluent add-1 rule may be the bridge between computing the sums concretely with objects and doing so mentally. Kindergartners' mental addition, in turn, is predictive of later mathematics achievement or learning difficulties (Jordan & Levine, 2009; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Mazzocco & Thompson, 2005). Specifically, the add-1 rule provides a scaffold for the relatively advanced "abstract counting-on" strategy (Baroody, 1995; Bråten, 1996; Clements & Sarama, 2012), a basis for inducing commutativity (Resnick, 1983), and a foundation for inventing reasoning strategies for larger combinations such as the near doubles. Moreover, it may help children discover, deepen, make explicit, or otherwise reinforce the successor principle and bring into play the benefits of this big idea. In brief, learning the add-1 rule might be expected to substantially improve achievement.

Specific Questions Addressed

The present research addressed four main questions. The first two involve the primary aims (the feasibility of two interventions for promoting fluency with the most basic sums); the last two, the subsidiary aims (implications of the model of early mental addition just discussed).

[1]Unlike empirical induction (merely discovering a pattern) and like deductive reasoning, mathematical induction entails using what is known (the discovered pattern) *and* logical reasoning to create new knowledge. Unlike *formal mathematical induction,* its informal counterpart does not involves reasoning about all integers, formulating a logical proof, or require formal training.

[2]The successor principle and a number-after rule for mentally adding 1 are logically related but reverse operations. The former entails recognizing that 1 must be added to a number to obtain the number after it; the latter involves recognizing that the number after is the result of adding 1 to a number.

1. Is computer-assisted structured discovery of the add-0 and add-1 reasoning strategies/rules a feasible approach for improving at-risk kindergartners' fluency? Specifically, (a) consistent with Alfieri et al.'s (2011) meta-analysis, can such training be more effective in promoting fluency with practiced $n + 1/1 + n$ combinations than regular classroom instruction and practice? (b) Might the same be true with unpracticed $n + 1/1 + n$ items—a result that would indicate recognition of general add-0 and add-1 rules?
2. Is computer-assisted structured discovery of the connection between (a) the doubles and real-world analogies and skip counting (e.g., three legs on each side of an insect, counting "3, 6," and $3 + 3 = 6$) and (b) the near doubles and doubles feasible in promoting their fluency with practiced or unpracticed doubles and near doubles?
3. Might achieving fluency depend on mastering the developmental prerequisite(s) for a particular combination family? Specifically, (a) is knowledge of a specific number-after relation such as "after seven comes eight" a necessary condition for fluency with a related $n + 1/1 + n$ items such as $7 + 1$ and $1 + 7$ and (b) is fluency with the doubles and $n + 1/1 + n$ combinations a necessary condition for achieving fluency with a related near double?
4. Might achieving fluency with the add-1 rule have broader effects or transfer? Specifically, might there be carryover effect to general number sense as measured by an achievement test?

METHODS

Participants

The original sample consisted of 30 kindergartners from four classes in one school and two classes from a school in another school district. All schools served a mid-sized mid-western community. Two children in the doubles condition moved before they completed the training. The final sample consisted of 28 kindergartners (5.14 to 6.02 years of age; mean = 5.58 years). All were in the bottom 25th percentile in mathematics achievement, as measured by the Test of Early Mathematics Ability, 3rd ed. (TEMA-3; Ginsburg & Baroody, 2003) or met at least one criterion for academic risk used by their school district and detailed in Table 1. This population was targeted because of the likelihood they would not be fluent with even the easiest basic sums and might profit from the interventions. Indeed, mental-addition pretesting identified that participants had not mastered more than 67% of the $n + 1/1 + n$ items (range = 0 to 64%; median = 0%) or the near doubles (range = 0 to 17%, median = 0%). The corresponding data for the $n + 0/0 + n$ items, which served as a contrast (non-example) for the add-1 rule, and the doubles, which are a developmental prerequisite for the near doubles, had a range = 0 to 78%, median = 0%, and range = 0 to 38%, median = 0%, respectively. Although a diverse sample, it is reasonably representative of the extensive and growing national population of kindergartners with a risk factor and sub-typical fluency. For the purposes of demonstrating the feasibility of the experimental interventions, the sample provides reasonable ecological and external validity (generalizability).

The four kindergarten classes in School 1 ($n = 17$) used Kathy Richardson's (1998) *Developing Number Concepts* program; the two kindergarten classes in School 2 ($n = 11$) used the *Beginning School Mathematics Project* (Miller & McKinnon, 1995). Each program includes

TABLE 1
Participant Characteristic by Condition

Training Condition:	*Add-0/1 (n = 15)*	*Doubles (n = 13)[a]*
Age range/median age	5.1 to 6.0 / 5.5	5.3 to 6.0 / 5.7
Males	7	5
TEMA-3[b]		
Pretest *SD* (Range)	11.2 (73 to 110)	15.4 (60 to 117)
Pretest Mean (Median)	86.5 (82)	82.5 (80)
Posttest *SD* (Range)	5.3 (90 to 112)	12.4 (79 to 125)
Posttest Mean (Median)	103.9 (104)	96.2 (98)
Risk Factors		
Free/reduced lunch	9	8
Minority status	13	8
English Language Learner	2	2
Low birth weight	1	0
Fetal alcohol/drug syndrome	1	0
Speech services	1	3
ADHD	2	1

[a]Original $n = 15$; data reported do not include two participants (both on free/reduced lunch and of minority status) who moved before completing the study.

[b]Because of the skewed distribution, the median is a more accurate average (indication of the typical score) on both the pretest and the posttest than the mean.

activities for both group and individual work with manipulatives and materials common to many early childhood classrooms. Analysis of the curricula revealed that neither program included instructional software. Although teachers provided computer time to play math games, given the scarcity of structured discovery software for young children, it is likely these programs focused on drill of number skills. Both schools were committed to achieving the state's grade K objectives (http://www.isbe.net/earlychi/ pdf/iel_standards.pdf). These objectives included verbal and object counting, comparing collections and numerals, and change concepts of addition and subtraction but not mental addition in general or number-after relations and relating them to adding with 1, skip counting by 2 to 9 and relating such counts to the doubles, and relating the doubles to the near-doubles, in particular. This information was confirmed by informal discussions with the teachers and informal observations of their mathematics instruction.

Project hired staff—two male research assistants and two female academic professionals—were highly qualified to teach and test young children and to observe the learning process. The latter provided qualitative data for formative assessment of the experimental programs. All had a master's degree in education and at least two years of previous teaching experience. All but one research assistant were veterans of the project and had worked with pre-K and grade 1 children the previous 2 years of the project (for details of the training and the pre-K study, see Baroody et al., 2009). One of the academic professionals was a certified teacher. Project staff participated in six 3-hour training sessions on testing and training procedures in the weeks prior to the study.

Measures

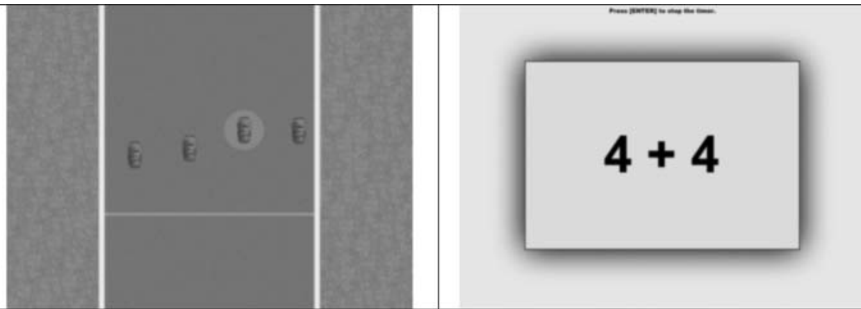
The achievement/diagnostic test and the mental-addition test used are described in turn.

Achievement and Diagnostic Testing. The TEMA-3 (Ginsburg & Baroody, 2003) is a manually and individually administered, nationally standardized test of mathematics achievement for 3- to 8-year-olds. The test is used to measure informal/formal concepts/skills in the following domains: numbering, number comparisons, numeral literacy, combination fluency, calculation, and concepts. The test is highly reliable. Cronbach's alphas for 5-year-olds, 6-year-olds, and overall for the form used (Form A) are .94, .95, and .94, respectively; its test–retest reliability is .83. Form A is equally reliable for the following subgroups: males, females, European Americans, African Americans, Hispanic Americans, Asian Americans, and low mathematics achievers (all reliabilities $\geq$ .98). In terms of criterion-prediction validity, correlations between the TEMA-3 and similar measures (the KeyMath-R/NU, Woodcock-Johnson III, Diagnostic Achievement Battery, and Young Children's Achievement Test) range from .54 to .91.

The following TEMA-3 items were administered to all participants for diagnostic and evaluation purposes: #11 (produce fingers to 5), #13 (number after to 9) including extra trials of involving 4 and 6, #14 (reading numerals to 10), #16 (concrete modeling of addition and subtraction), #17 (part-part-whole), #19 and #20 (number comparisons to 5 and 10, respectively), #23 (counting sets to 10), #24 (verbally count back from 10), #26 (mental addition: sums 5 to 9), #28 (produce sets up to 19), #29 (reading teen numerals), and #31 (verbal counting by ones).

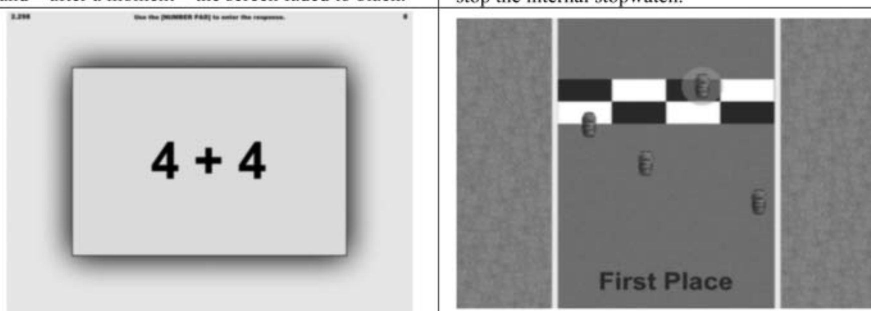
Mental-Addition Test. The testing was done in the context of computer games developed for the project. For the *Race Car Game* (see Figure 4), for example, a tester explained to the child: "We are going to play a game where we pretend you are driving a race car. In order to drive your car, you will need to keep both of your hands on the car's steering wheel at all times. [The tester then encouraged the child to grip a steering wheel tightly with both hands, so as to discourage finger counting or at least make it difficult or obvious.] Your success ... is determined by answering addition problems accurately and quickly. If you know the answer, tell me as quickly as you can. If you are not sure of the answer, make a good guess as quickly as you can." See Figure 4 for details on the testing procedure. In order to gauge fluency or estimation strategies, children were not provided objects, and counting was discouraged. If a child did not respond within 5 seconds, the tester prompted: "What do you think would be a good guess?" and the problem was placed in context (e.g., "Three cookies and one more cookie is about how many cookies altogether?").

Fluency was checked for four categories of items: (a) Practiced in the add-0/1condition (0 + 4, 0 + 7, 3 + 0, 8 + 0, 1 + 3, 1 + 4, 1 + 7, 1 + 8, 3 + 1, 4 + 1, 7 + 1, 8 + 1, 2 + 4, 5 + 3); (b) add-0/1 training transfer items (0 + 5, 0 + 9, 5 + 0, 6 + 0, 7 + 0, 1 + 5, 1 + 6, 1 + 9, 5 + 1, 6 + 1, 9 + 1, 4 + 2); (c) practiced by the doubles group (1 + 1, 2 + 2, 3 + 3, 5 + 5, 6 + 6, 2 + 3, 3 + 4); and (d) transfer for the doubles group (4 + 4, 7 + 7, 8 + 8, 3 + 2, 4 + 3, 4 + 5, 5 + 4). At pretest and posttest, the Cronbach's alphas for practiced $n + 0/0 + n$, unpracticed $n + 0/0 + n$, practiced $n + 1/1 + n$, unpracticed $n + 1/1 + n$, and the practiced double items were .73 & .90, .75 & .87, .91 & .88, .87 & .88, and .41 & .83, respectively. Insufficient variance prevented gauging reliability for other categories. Each testing session consisted of a set of 10 items, a computer reward game, a second set of 10 items, and a final reward game. Items were presented in a haphazard order, except that two items with the same addends or sum were not presented one after another, commuted items were not presented in the same session, and the types of items were evenly distributed across the four sets.



a. The computer highlighted a child's choice of a racecar. The game provided a pretext for tightly holding onto a steering wheel to eliminate or at least suppress finger counting. The child was encouraged to respond quickly and, if not sure of an answer, to quickly make a good guess To begin testing, the tester pressed the ENTER key to start the race. The racecars sped across the starting line, and—after a moment—the screen faded to black.

b. After a blank block appeared to provide a focus for the child's attention, the tester keyed in ENTER on the number pad to initiate a trial and to start the computer's internal stopwatch. Children were not provided objects or encouraged to use objects during the mental-addition testing. As soon as the child responded, the tester (who had his/her finger posed on the ENTER key) depressed the key to stop the internal stopwatch.



c. The child's reaction time (RT) was displayed in the upper left-hand corner and filed electronically. Using the number pad, the tester then entered the child's response and hit ENTER to record the answer electronically. The entered response appeared on the screen in the upper right-hand corner. The tester also recorded the RT and answer manually as a backup and any relevant information (e.g., a response before the trial was displayed, counted objects, finger counting, verbal counting, reasoning such the number after 7 is 8 or 4 and 4 is 8 and 1 more 9).

d. After the testing, the race picked up where it left off and continued for about 15 s. Where the child's car finished was determined by the quality (accuracy and speed) of his/her responses during the testing session. For the purposes of this feedback (but not the scoring of fluency), items were scored on accuracy—correct, close (within 25% of the correct sum but > larger addend), reasonable (within 150% of the correct sum and larger than the addends), or unreasonable—and speed (fast or slow). The average score determined the outcome of the game (e.g., the place the child's car finished in the race).

FIGURE 4 Example of mental-addition testing game (Race Car).

Fluency Scoring

Rationale. Equating correct responses with fluency without regard to a child's reaction time—even if there is a self-report of "I just remembered it" or no overt computing (e.g., Henry & Brown, 2008; Siegler, 1986)—can overestimate fluency. Self-reports may be self-serving or otherwise unreliable, and even some kindergartners are capable of covert computing.

For the present study, the operational definition of combination fluency included a 3-second time limit to minimize false positives due to counting. Fluency with a strategy/rule was further evaluated by checking for transfer to unpracticed items. The accurate, appropriate, and flexible application of learning to new cases is strong indication of a general rule or adaptive expertise (Hatano, 2003).

The present study also included a nonconventional measure for identifying combination fluency, namely scoring in context. This scoring method entails taking into consideration responses to other items to ensure that a mental-addition strategy/rule generates reasonable answers and is used selectively or appropriately. Scoring in context is particularly important when the focus is on rule learning, such as the add-0 or add-1 rule. Such learning is often not all-or-nothing, and partially understood or incomplete rules can be misapplied (e.g., over-applied) and result in systematic errors. Indeed, scoring in context reveals that some mental-addition novices resort to mechanical strategies (response biases) that produce systematic errors, which can accidently result in false positives (Baroody, 1998b, 1992; Dowker, 1997, 2003). For example, some novices consistently and inflexibly *state an addend* or *the larger addend*, which can create the appearance they know (some) $n + 0/0 + n$ items. Baroody, Purpura, Reid, and Eiland (2011) found that, with at-risk pre-K children, scoring accuracy on a trial-by-trial basis (accuracy by any means) yielded significantly higher success rates than scoring accuracy in context (with response biases scored as incorrect).

Scoring Procedure. A child was considered fluent on an item if s/he responded correctly in less than 3 seconds and the correct response was not a false positive due to a response bias (i.e., over-applying the state-the-addend or the number-after-*n* strategy)[3] or the result of a counting strategy. Testers identified whether a child used a counting strategy, a reasoning strategy, or other strategy. A counting strategy entails either representing one or both addends before the sum count or representing *both* addends during the sum count (Fuson, 1992). This determination was based on evidence of counting objects (e.g., fingers), verbally citing a (portion) of the counting sequence out loud (even if whispered), or sub-vocally counting accompanied by successive movement of fingers or the eyes. A reasoning strategy was scored if a child exhibited evidence of using deductive reasoning (for $2 + 3$, e.g., stating: "Two and two is four one more is five").

[3]A response bias determination was done separately for each session and based on all 20 items given in a session. A scorer scored the response to each item in one of five ways: possible inappropriate use of a strategy (resulting in an incorrect answer); possible appropriate use of a strategy (resulting in a correct answer); other incorrect estimates (error not attributable to the strategy); other correct estimates (correct response not attributable to the strategy); and use of concrete or abstract counting strategy or no response. In order to rise to the level of a response bias, four criteria had to be met: (a) total inappropriate uses is half or more of all estimation errors: $a_{TOT} \div (a_{TOT} + c_{TOT}) \geq .50$; (b) total inappropriate uses is more than 25% of all trials: $a_{TOT} \div 20 > .25$ (MINIMUM of 6 trials); (c) total uses of strategy is half or more of all estimates: $(a_{TOT} + b_{TOT}) \div (a_{TOT} + b_{TOT} + c_{TOT} + d_{TOT}) \geq .50$; and (d) total uses of strategy is half or more of all trials: $(a_{TOT} + b_{TOT}) \div 20 \geq .50$ (MINIMUM of 10 trials). This straightforward scoring procedure yielded 99% interrater for agreement on whether a response bias was used and, if so, the specific strategy used in (28 participants $\times$ 2 test sessions $\times$ 2 sets/session) 112 possible cases. A more complete description of the scoring process is available upon request from the first author.

Interventions

Bruner (1961) cautioned that discovery learning requires base knowledge or developmental readiness. Consistent with the number sense view (Brownell, 1935; Gersten & Chard, 1999; Jordan, 2007) and recommendations stemming from research reviews (e.g., Clements & Sarama, 2012; NMAP, 2008; NRC, 2001; see also Hasselbring, Goin, & Bransford, 1988), the present study involved an initial training stage (Stage I) that included Phase 1 learning experiences (using counting strategies to solve word problems) to ensure adequate base knowledge for both experimental programs (Stage II): recognizing relations and constructing and automating reasoning strategies (Phase 2 and 3 learning experiences). Stage-I training and Stage-II interventions supplemented regular classroom mathematics instruction and are described in turn.

Stage I. The goals and sequence of the number sense instruction are outlined in Table 2 of Baroody and colleagues (2009) or are available from the first author on request. The sequence starting point depended on the developmental needs of a child. Stage I occurred in the context of manual games, which typically involved several goals. For example, *Animal Spots* involved throwing a die or dice of various types to determine how many pegs (spots) a player could take on his/her turn to fill holes in a wooden cutout of a leopard or giraffe. (The player who filled his/her animal with "spots" first was the winner.) A dot die was used to practice verbal number recognition (subitizing) or enumeration; a dot-and-numeral die was used to connect numerals to concrete collections, the numeral die served to practice numeral recognition; two dot dice were used to introduce addition and practice a counting-all strategy; two dot-plus-numeral dice were used to introduce symbolic addition; and two numeral-only dice were used to practice symbolic addition. *Animal Spots* also entailed practicing the cardinality rule (the last number used in counting a collection represents the total) and verbal production (counting out a specified number of pegs). *Car Race*, which entailed rolling one die or two dice or drawing a card, had the same goals as those just described. The *Number-After* version of this game served to practice the number-after skill. The primary sources of the games were Baroody (1987, 1989a; Baroody & Coslick, 1998) and Wynroth (1986). A description of the games and their sequence is available from the first author.

Stage I training focused on the prerequisites for mental addition needed by both groups. Although knowledge of number-after relations is a prerequisite for the add-1 rule, the ability to enter the counting sequence at any point and continue a count can also facilitate learning the count-by-two and other skip counts.[4] The add-1 rule and skip counting are both needed for the near-doubles reasoning strategy. Moreover, whereas fluency with verbal counting to 10 is a prerequisite for number-after fluency needed by both groups, fluency with counting to 12 and 20 was needed by only the doubles group for learning skip counts.

[4]Logically, skip counting by *n* is not possible if a child cannot start counting with *n* (instead of "one"). Skip counting also logically requires fluent number-after knowledge for continuing a skip count. For example, skip counting by two requires entering the count sequence at "two" and recognizing that the next number in the skip count is two numbers after *two*: after two is three and after three is four.

TABLE 2
Mean Proportion Fluent (and Standard Deviation) by Type of Item, Test, and Training Condition

	Pretest		Delayed Posttest				Statistics		
			Unadjusted Means		*Adjusted Means*		*ANCOVA (ANOVA)*		
Type of Item	*Add 0/1 group*	*Doubles group*	*Add 0/1 group*	*Doubles group*	*Add 0/1 group*	*Doubles group*	$F[1, 25]$	*p*	*Cohen's d*
$n + 0/0 + n$ practiced	.10 (.16)	.23 (.36)	.85 (.30)	.48 (.45)	.90	.42	15.19	<.001*	**1.56**
$n + 0/0 + n$ transfer	.08 (.15)	.22 (.32)	.76 (.32)	.54 (.44)	.81	.48	5.65	.013*	**0.95**
$n + 1/1 + n$ practiced	.06 (.19)	.13 (.27)	.38 (.39)	.11 (.14)	.40	.08	9.26	.025*	**1.22**
$n + 1/1 + n$ transfer	.02 (.09)	.12 (.25)	.34 (.38)	.15 (.29)	.39	.10	7.26	.006*	**1.08**
Practiced n + 1 contrasts (2 + 4, 5 + 3)	.03 (.13)	0 (0)	.10 (.21)	.04 (.14)	.09	.05	0.28	.302	0.21
Unpracticed $n + 1$ contrast (4 + 2)	0 (0)	0 (0)	.07 (.26)	.15 (.38)	N/A	N/A	(0.52)	(.238)	–**0.29**
Practiced small doubles (2 + 2, 3 + 3, 5 + 5)	.13 (.25)	.13 (.22)	.40 (.44)	.46 (.42)	.40	.46	0.23	.318	0.19
Practiced large double (6 + 6)	0 (0)	0 (0)	.07 (.29)	.38 (.51)	N/A	N/A	(4.56)	(.021*)	**0.85**
Transfer small double (4 + 4)	.07 (.26)	.08 (.28)	.13 (.35)	.31 (.48)	.14	.30	1.497	.117	**0.49**
Transfer large doubles (7 + 7, 8 + 8)	.03 (.13)	0 (0)	.03 (.13)	0 (0)	.02	.02	0	1.000	—
Near doubles practiced	0 (0)	0 (0)	.10 (.21)	.08 (.19)	N/A	N/A	(0.09)	(.381)	–0.12
Near doubles transfer	.02 (.06)	0 (0)	.03 (.09)	.04 (.09)	N/A	N/A	(0.01)	(.470)	0.03

One-tailed significance levels are reported because the comparisons were planned (tested directional hypotheses). An asterisk (*) indicates significant after using the Benjamini-Hochberg (1995) correction to account for multiple comparisons. **Bold** indicates effect size exceeds What Works Clearinghouse standard of effective practice (Institute of Education Sciences, 2011).

Stage II. The experimental mental-addition interventions had parallel structures, had identical dosage, and involved the same computer reward games. In both conditions, participants were initially encouraged to solve addition items independently by using mental arithmetic. Each practice item was administered and a correct sum was practiced once in each of 20 sessions. Both the computer and the trainer provided feedback regarding correctness. If a child responded incorrectly to an item, s/he was instructed to try again and, if incorrect again, encouraged or helped to use physical or virtual manipulatives (five frames) to determine the correct sum. In each session, the child completed a subset of 10 items, took a break usually in the form of a brief manual game, and completed another subset of 10 items. A computer reward game was played at the end of a session.

The add-0/1 condition is illustrated in Figures 1 to 3. For session 1, Subset A involved the following trials in order: after 3, 3 + 1, 1 + 3, 3 + 0, 2 + 4; after 4, 4 + 1, 1 + 4, 0 + 4: after 6. Subset B involved the following trials in order: after 7, 7 + 1, 1 + 7, 0 + 7, 5 + 3; after 8, 8 + 1, 1 + 8, 8 + 0; after 9. (In addition to the $n + 0/0 + n$, 2 + 4 and 3 + 5 served as non-examples of the add-1 rule.) Neither the trainers nor the program explicitly pointed out the connection between adding with 1 and number-after relations. To prompt discovery of this relation, the add-0/1 program used the same format for presenting number-after-n and $n + 1$ questions, namely highlighting n on a number list (see Frames B and F in Figure 1, respectively) and providing feedback on correctness, namely highlighting the correct answer on a number list in green (see Frames C and H in Figure 1, respectively).

Moreover, as shown in Figure 1, the initial training provided a complete number list from 1 to 10, so that a child could simply count from one to the highlighted n and then move or count one more to determine the answer for a number-after-n or $n + 1$ question. That is, a child can use the developmentally basic *running-start* strategy to determine the number after n and *counting-from-one* strategy to determine the sum of $n + 1$. As Figure 2 illustrates, the intermediate training involved a partial number list stopped at n and, thus, required children to use these basic strategies in conjunction with their mental representation of the counting sequence to determine an answer. As Figure 3 illustrates, the advanced training entailed showing only n, and children chose their answer from scrambled number bubbles. This supported relying entirely on a mental representation of the counting sequence to determine a number-after or the sum of $n + 1$.

For sessions 1 to 10 of the doubles training, Subset A began with skip counting by 2 to 20, which resulted in highlighting all the even numbers from 1 to 20 on a counting list. A child then entered a response to addition items on this counting list (e.g., Session 1/Set 1A: 2 + 2, 3 + 3 5 + 5, 3 + 4; Session 2/Set 2A: 6 + 6, 1 + 1, 3 + 4). The aim was to highlight that the sums of all doubles, but not near doubles, are even. For sessions 11 to 20, Set A involved a (skip) counts by n, a related double, and near double (e.g., Set 13A: count by 3s to 9, 3 + 3, count by 2s to 6, 2 + 2, 2 + 3, count by 5s to 15, 5 + 5; Set 14A: count by 6s to 12, 6 + 6, count by 1s to 3s, 1 + 1). The aim was to show that (except for 1 + 1) an $n + n$ item, but not near doubles, could be determined by skip counting by n twice. Subset B of all sessions, involved practicing a meaningful analogy for a double, the doubles, and a non-example (near double). For example, for Session 1: one row of eggs and another = ?, 6 + 6, a left foot and a right foot = ?, 1 + 1, and 2 + 3; Session 2: front legs of a dog and back legs of a dog = ?, 2 + 2, insect legs on the left and insect legs on the right = ?, 3 + 3, spider legs on one side and on the other = ?, finger left hand and fingers right hand = ?, 5 + 5, and 2 + 3). Note that, although the analogy for 4 + 4 (spider legs) was practiced, it was not related to the symbolic expression 4 + 4, as was done for the practiced doubles. (Screen shots of the doubles training are available upon request to the first author.)

Design and Procedures

Plan. A training experiment with multiple baselines served to evaluate the feasibility of the two experimental programs. (The Institute of Education Sciences' definition and requirements for feasibility or Goal 2 research can found at http://ies.ed.gov/funding/pdf/2012_84305A.pdf.) Preliminary testing with the TEMA-3 gauged mathematics achievement and identified specific strengths/weaknesses in concepts/skills. So as to ensure readiness for mental addition, Stage I of the intervention focused on remedying knowledge gaps identified by the TEMA-3, particularly the developmental prerequisites for mental addition, such as the number-after relations. It was provided to all participants in pairs.

The computer-based mental-addition pretest was then individually administered to children to gauge baseline fluency. Next, participants were randomly assigned to either the structured add-0/1 or doubles training condition. All children, then, received worthwhile training, which is important for maintaining good working relations with school staff and parents. It also made possible evaluating both programs simultaneously and economically and in a rigorous manner. The computer-assisted Stage II interventions were administered on a one-to-one basis.

Rationale. As the add-0/1 group did not receive experimental training or practice with the doubles/near-doubles items, it served as an active control for the doubles training. Similarly, as the doubles group received only business-as-usual training and practice on the $n + 0/0 + n$ and items, they served as an active control for the add-0/1 group. Thus, a significant posttest difference on, say, the $n + 1/1 + n$ items favoring the add-0/1 group cannot be attributed to history (e.g., classroom instruction or practice), maturation (e.g., natural development of working memory capacity), regression to the mean (unreliability in the mental-addition test), or selection (a systematic difference between the groups) because random assignment theoretically ensures both groups are comparable on these confounding variables. A testing effect (learning due to merely repeated testing) can be discounted because both groups received the identical tests the same number of times. As both groups received training with a similarly structured computer program and reward games, the design also controls for a novelty or Hawthorne effect. This also eliminates as a plausible alternative explanation that any effect is merely due to computer-based delivery and not the structured training or practice. Any contamination or diffusions effect, which would facilitate the learning of the $n + 0/0 + n$ $n + 1/1 + n$ items (e.g., sensitizing a child to mathematical regularities in general or the number-after-n rule for adding 1 in particular) adds to measurement error and makes it more difficult to obtain significant results (i.e., effectively stacks the deck against corroborating a feasibility hypothesis).

Stages I and II were each administered for 30 minutes about twice a week for 10 and 9 weeks, respectively, in a hallway outside the children's classroom. Project computer stations were used for Stage II. Pull outs occurred in non-literacy time blocks, including mathematics instruction and playtime. The add-0/1 and doubles training were conducted simultaneously, and project personnel were each involved in both interventions. The TEMA-3 was re-administered immediately after the Stage II training; mental-addition fluency was individually assessed two weeks after the Stage II training (delayed posttest). Project personnel implemented all testing and training procedures. Positive assent was obtained for all testing and training sessions.

Fidelity. Developmentally appropriate and needed Stage I instruction was assured by a checklist of TEMA-identified strengths and weaknesses for each child and a manual of specific games or game variations for each goal. Project staff updated the checklist as a child progressed. Fidelity of the Stage I and II training was ensured by (a) devoting 10 of the 18 hours of prior staff training to a learning trajectory of mental addition and its developmental prerequisites (Baroody et al., 2009), the rationale and rules for the Stage I manual games, the rationale for the Stage II programs, procedures for implementing the computer-assisted Stage II training and providing feedback (the *Trainer Guidelines for Mental Arithmetic Computer-Assisted Training* is available upon request from the first author), and behavioral management techniques; (b) oversight (on average a session every other week for each staff member) by the second author or project coordinator; (c) brief (10 to 30 minute) staff meetings during the training to review procedures and address training issues as needed; and (d) a lesson log sheet for keeping track of which lessons each participant completed. Stage II fidelity was further ensured by keeping a copy of the *Trainer Guidelines* at each computer station and, most importantly, the use of the computer programs. The programs ensured that each child received (a) the assigned intervention (a child's log-in automatically connected to his/her treatment), (b) the items in the order specified by an intervention, and (c) immediate feedback on correctness. In the add-1 and doubles conditions, the programs also ensured that the child saw feedback that juxtaposed elements of a relation (e.g., $3 + 1 = 4$ and After 3 is 4). In regard to implementation fidelity, all participants completed 100% of the lessons in their assigned Stage II training before posttesting.

Analyses

Analyses of fluency were done using the proportion correct by a child on a test. ANCOVAs, using pretest fluency as the covariate, were used to compare posttest performance on targeted practiced and unpracticed combinations. Given the directional nature of the contrasts, effects of treatment were tested using one-tailed significance values. A correction (Benjamini & Hochberg, 1995) was applied to correct for Type I error due to multiple comparisons. Pretest comparisons were not included in the total number of comparisons to be adjusted because, as children were randomly assigned to groups, differences between these groups were not predicted. Thus, a total of eight comparisons were included in the correction (i.e., comparison of groups at delayed posttest on practiced items and again on unpracticed items). Additionally, we also examined effect size magnitude (Cohen's d) for all specific contrasts of interest due to the limited power of the study and the importance of evaluating effect sizes (Wilkinson & APA Task Force on Statistical Inference, 1999). A Cohen's d was calculated using adjusted posttest means (accounting for pretest) and the posttest standard deviations. According to Cohen (1992), a d of .20, .50, and .80 indicate small, medium, and large effects, respectively.

RESULTS

Pretest covariates were significant in all cases except (a) the three instances where pretest scores for both groups were zero and (b) the near doubles transfer analysis where pretest and posttest scores were nearly zero for both groups. In these exceptional cases, children in both groups were close to 0, which did not allow much pretest variation. For these cases, ANOVAs were run. All assumptions for the ANCOVAs were met (see, e.g., Maxwell & Delaney, 2004), except for homogeneity of variance for the practiced and unpracticed $n + 1/1 + n$ items and the unpracticed double $4 + 4$. However, ANCOVAs are robust against such violations when the sample sizes of groups are comparable. To further ensure that the initial group differences in variance did not adversely impact the results of the study, we included a pretest $\times$ treatment interaction term in the three analyses where the homogeneity of variance assumption was violated. Consistent with the assumption that ANCOVAs are robust to violations of the homogeneity-of-variance assumption, the interaction term was not significant in any of the analyses and did not significantly change the findings. Although fluency and TEMA-3 pretest results did not have a normal distribution, they did have distributions that were expected. Summarized next are the results by question.

Question 1: Feasibility of the Add-0/1 Training

As Table 2 shows, although the comparison (doubles) group more than doubled their fluency on the practiced and transfer $n + 0/0 + n$ combinations during the school year, the add-0/1 group improved substantially more—increasing their fluency by about six times. Although the doubles group made essentially no gains on the practiced and transfer $n + 1/1 + n$ items overall, the add-0/1 group improved from a fluency rate of about 5% on the pretest to about 45% on the delayed posttest. Indeed, using the kindergartners' mean proportion of practiced and transfer items mastered on the delayed posttest as the dependent measure and their pretest score as the covariate, ANCOVAs indicated that participants receiving add-0/1 training mastered significantly more $n + 0/0 + n$ and $n + 1/1 + n$ practiced items than participants who received training on the doubles. More importantly, the same was true of unpracticed $n + 0/0 + n$ and $n + 1/1 + n$ items.

As random assignment was not done within school or class, the results were broken down by school and classroom (see Table 3), and the effects of school and class were checked. At pretest, the schools did not differ significantly on the practiced and transfer $n + 1/1 + n$ items (School 1: $M = .08$, $SD = .21$; School 2: $M = .08$, $SD = .20$; $F[1, 26] = 0.003$, $p = .954$) or the TEMA-3 (School 1: $M = 84.0$, $SD = 14.5$; School 2: $M = 85.6$, $SD = 11.6$; $F[1, 26] = 0.099$, $p = .755$). ANCOVAs with pretest score as the covariate and school as the independent variable did not reveal significant posttest differences among add-0/1 children for the practiced and unpracticed $n + 0/0 + n$ items and $n + 1/1 + n$ items, $F(1, 12) = 1.92$, $p = .191$; $F(1, 12) = 3.18$, $p = .100$; $F(1, 12) = 1.35$, $p = .269$; and $F(1, 12) = 0.67$, $p = .430$, respectively. When class was included as fixed-effect variable, all primary contrasts failed to show significant variance at the class level. That is, there were no significant class-level effects on children's performance.

In terms of posttest fluency on practiced and unpracticed $n + 1/1 + n$ items (again see Table 3), the three add-0/1 participants in Class 4 of School 1 outperformed their seven counterparts in the other classes of School 1 and five counterparts in School 2. This appreciable difference was not

TABLE 3
Mean Mental Addition Pretest–Posttest Performance on Classes of Combination by Condition, School, and Class

	School 1															
	Class 1				*Class 2*				*Class 3*				*Class 4*			
Item(s) or	*Add-0/1 (n = 0)*		*Doubles (n = 1)*		*Add-0/1 (n = 3)*		*Doubles (n = 3)*		*Add-0/1 (n = 4)*		*Doubles (n = 0)*		*Add-0/1 (n = 3)*		*Doubles (n = 3)*	
Item Family	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*	*Pre*	*Post*
Practiced combinations																
$n + 0/0 + n$	—	—	0	0	.17	.83	.25	.42	0	1.00	—	—	.17	.92	.33	.50
					(.29)	(.29)	(.43)	(.52)		(0)			(.14)	(.14)	(.58)	(.50)
$n + 1/1 + n$	—	—	0	0	0	.46	.25	.17	0	.38	—	—	.29	.63	0	.04
						(.51)	(.43)	(.19)		(.37)			(.40)	(.54)		(.07)
Small doubles	—	—	0	0	.11	.67	.22	.56	.08	.33	—	—	.22	.67	0	.44
					(.19)	(.58)	(.39)	(.51)	(.17)	(.27)			(.39)	(.58)		(.39)
$6 + 6$	—	—	0	0	0	.33	0	.67	0	0	—	—	0	0	0	0
						(.58)		(.58)								
Near doubles	—	—	0	0	0	.17	0	.17	0	.25	—	—	0	0	0	0
						(.29)		(.29)		(.29)						
Unpracticed (transfer) combinations																
$n + 0/0 + n$	—	—	0	0	.13	.87	.33	.67	.10	.80	—	—	0	.93	.22	.53
					(.23)	(.12)	(.42)	(.42)	(.20)	(.40)				(.12)	(.39)	(.50)
$n + 1/1 + n$	—	—	0	0	0	.39	.22	.33	0	.33	—	—	.11	.56	0	0
						(.54)	(.39)	(.58)		(.36)			(.19)	(.48)		
$4 + 4$	—	—	0	0	.33	.33	0	.67	0	0	—	—	0	0	0	0
					(.58)	(.58)		(.58)								
$7 + 7$ & $8 + 8$	—	—	0	0	.17	.17	0	0	0	0	—	—	0	0	0	0
					(.29)	(.29)										
TEMA																
	—	—	65	82	90.3	106	84	104.5	79.5	103	—	—	92.3	99.7	81.7	97
					(11.5)	(1.7)	(29.5)	(18.2)	(4.4)	(4.2)			(15.3)	(9.5)	(2.9)	(7.6)

				Practiced combinations				
$n + 0/0 + n$	.06 (.13)	.63 (.48)	.50 (.35)	.88 (.18)	.25	1	.06 (.13)	.44 (.52)
$n + 1/1 + n$	0	.16 (.24)	.19 (.27)	.19 (.27)	0	.25	.16 (.31)	.09 (.12)
Small doubles	0	0	.17 (.24)	.50 (.24)	.67	.67	.17 (.19)	.50 (.58)
$6 + 6$	0	0	0	.50 (.71)	0	0	0	.50 (.58)
Near doubles	0	0	0	.25 (.35)	0	0	0	0
				Unpracticed (transfer) combinations				
$n + 0/0 + n$	.05 (.10)	.45 (.34)	.50 (.42)	.90 (.14)	.20	1	.05 (.10)	.40 (.49)
$n + 1/1 + n$	0	.17 (.33)	.08 (.12)	.17 (0)	0	.33	.17 (.33)	.17 (.24)
$4 + 4$	0	.25 (.50)	0	0	0	0	.25 (.50)	.50 (.58)
$7 + 7, 8 + 8$	0	0	0	0	0	0	0	0
Near doubles	0	.06 (.13)	0	.13 (.18)	0	0	0	0
				TEMA				
	83.5 (12.5)	104.5 (2.1)	97 (0)	101.5 (5)	97	112	79.3 (10.2)	90 (11.8)

due to the classroom instruction in Class 4. Like the control participants in other classes, the three doubles participants in Class 4 did not improve on either the practiced and unpracticed $n + 1/1 + n$ items. Moreover, add-0/1 children from Class 4 of School 1 did not consistently outperform their counterparts on the TEMA-3 pretest or posttest. They did differ, though, in one key respect: Unlike their add-0/1 counterparts, these participants evidenced some fluency on the practiced and unpracticed $n + 1/1 + n$ items at pretest).[5]

Question 2: Feasibility of the Doubles Training

As Tables 2 and 3 reveal, participants in both conditions exhibited substantial improvement on (ultimately achieved fluency on about half of) the three small practiced doubles. However, only the doubles group improved appreciably on the practiced large double $6 + 6$ and the unpracticed small double $4 + 4$. Although the main effects of condition were non-significant, the effect size for each of these items exceeded the 0.25 criterion for effective practice set by the federal What Works Clearinghouse (Institute of Education Sciences, 2011). As indicated by Table 2, neither group made much progress on the practiced or unpracticed near doubles.

Question 3: The Necessity of Family-Specific Developmental Prerequisites

Frame A of Table 4 summarizes the relation between knowledge of number-after relations (as gauged by TEMA-3 posttesting) and fluency with related practiced or unpracticed $n + 1/1 + n$ items (for cases where data were available on both). In essence, at posttest, participants were not proficient on the latter without also being proficient on the former. Unlike the doubles children, nearly all add-0/1 participants were proficient on number-after relations. As Frame B of Table 4 indicates, the vast majority of participants in the doubles and comparison group typically had not mastered one or both of the developmental prerequisites necessary to execute the near-doubles reasoning strategy fluently.

Question 4: Broader Effects of Intervention

The TEMA-3 pretest and posttest scores are reported in Tables 1 and 5 by condition and by achievement levels within conditions, respectively. Using the TEMA-3 pretest as the covariate, an ANCOVA revealed that the TEMA-3 participants in the add-0/1 group scored significantly higher at posttest than those in the doubles group, $F[1, 25] = 5.246, p = .031, d = .92$. In particular, the former improved significantly more on TEMA-3 item #22 (Number After: Numbers to 40) than the latter group, $F[1, 25] = 8.738$, $p = .007$, $d = 1.05$. (For item #22, which involved the unpracticed trials “after 24” and “after 33,” Cronbach’s alpha is .93; the trials correlated with the TEMA-3 standard score .49, $p = .009$, and .48, $p = .011$, respectively).

[5]A qualitative analysis of response biases further supported the feasibility of structured add-0/1 training. This analysis can be obtained by contacting the first author.

TABLE 4
Relation Between Performance on Hypothesized Developmental Prerequisites and Mental Addition Posttest Fluency of n + 1/1 + n items (Frame A) and Near Doubles (Frame B) by Condition: Add-1 Condition (bold) vs. Near-Doubles Condition (italics)

Developmental prerequisite	*Mental-addition posttest item*			
Frame A: Number-after items on the TEMA-3 posttext x add-1 items				
	1 + 3		*3 + 1*	
Number After 3	Not	Fluent	Not	Fluent
Fluent	**8** (*8*)	**7** (*1*)	**8** (*8*)	**7** (*1*)
Not	**0** (*4*)	**0** (*0*)	**0** (*4*)	**0** (*0*)
	***p* = .004** (*p = .004*)		***p* = .004** (*p = .004*)	
	1 + 4		*4 + 1*	
Number After 4	Not	Fluent	Not	Fluent
Fluent	**7** (*6*)	**7** (*2*)	**6** (*7*)	**8** (*1*)
Not	**1** (*5*)	**0** (*0*)	**1** (*5*)	**0** (*0*)
	***p* = .008** (*p = .016*)		***p* = .016** (*p < .008*)	
	1 + 7		*7 + 1*	
Number After 7	Not	Fluent	Not	Fluent
Fluent	**7** (*8*)	**6** (*1*)	**5** (*8*)	**8** (*1*)
Not	**2** (*2*)	**0** (*2*)	**2** (*4*)	**0** (*0*)
	***p* = .004** (*p = .055*)		***p* = .031** (*p = .004*)	
Frame B: Doubles + n + 1 x near doubles[1]				
	2 + 3		*3 + 2*	
Both 2 + 2 *and* 4 + 1	Not	Fluent	Not	Fluent
Fluent	(**3**) *0*	(**1**) *1*	(**3**) *1*	(**1**) *0*
Not	(**9**) *11*	(**2**) *1*	(**11**) *12*	(**0**) *0*
	(**p = .500**) *p < .250*		(**p = .125**) *p < .250*	
	3 + 4		*4 + 3*	
Both 3 + 3 *and* 6 + 1	Not	Fluent	Not	Fluent
Fluent	(**3**) *2*	(**0**) *0*	(**3**) *1*	(**0**) *1*
Not	(**12**) *11*	(**0**) *0*	(**11**) *10*	(**1**) *1*
	(p = .**125**) *p < .250*		(p = .**313**) *p < .250*	
	4 + 5		*5 + 4*	
Both 4 + 4 *and* 8 + 1	Not	Fluent	Not	Fluent
Fluent	(**0**) *0*	(**0**) *0*	(**0**) *0*	(**0**) *0*
Not	(**15**) 13	(**0**) *0*	(**15**) 13	(**0**) *0*
	(**NA**) *N/A*		(**NA**) *N/A*	

Note. Data for the Add-1 condition are shown in **bold**; for the Near-doubles condition, in *italics*. Data for the group that received training on the combination family are shown without parenthesis; for the comparison group (which did not work with combination family), with parenthesis. For Frame A, the developmental prerequisite for efficient use of the add-1 rule for a 1 + 3 or a 3 + 1, for example, is fluency with the number-after relation 3 is 4 (NA 3). For Frame B, needed for efficient use of the near-doubles strategy for 2 + 3 and 3 + 2, for instance, are fluency with the double 2 + 2 and 4 + 1. "Fluent" for the developmental prerequisites indicates that a child was fluent on both the related double and $n + 1$ item; the "Not" indicates that a child was not fluent on one or both prerequisites. A significant p value (McNemar test) confirms that a hypothesized prerequisite is a necessary condition for fluency with it.

TABLE 5
Number of Children at TEMA-3 Achievement Levels Before and After Intervention by Condition

	Pretest		Posttest	
Raw Score Range/Achievement Levels	*Add-0/1*	*Doubles*	*Add-0/1*	*Doubles*
Raw score of 55 to 69 = Very Low (0% to 2% nationally)	0	3	0	0
70 to 84 = Low (3% to 16%)	10	5	0	3
85 to 92 = Low Average (17% to 33%)	0	2	1	2
93 to 107 = Average (34% to 67%)	4	2	12	7
108 to 115 = High Average (68% to 84%)	1	0	2	0
116 to 130 = High (85% to 98%)	0	1	0	1

DISCUSSION AND IMPLICATIONS

The results by question and the study's limitations/future directions are discussed in turn.

Question 1: Feasibility of the Add-0/1 Training

Instructional and methodological implications are discussed in turn.

Instructional Implications. The significant gains by the add-0/1 group with the practiced and particularly the unpracticed $n + 0/0 + n$ and $n + 1/1 + n$ items indicate that computer-assisted, structured discovery learning is feasible for helping at-risk kindergartners achieve fluency with the add-0 and add-1 rules. These results were obtained with only 20 repetitions for each practiced $n + 0/0 + n$ and $n + 1/1 + n$ items (and no repetitions for transfer items).[6] This is substantially less practice than the thousands of repetitions necessary to achieve memorization (by rote) of these facts specified by earlier models and computer simulations of arithmetic learning (e.g., Shrager & Siegler, 1998; Siegler & Jenkins, 1989). Moreover, the add-0/1 group improved their fluency with $1 + 1$ slightly more than did the doubles group, despite a practice differential of 0 to 20 repetitions, but they showed little gain on practiced $n + 1$ contrast items. These findings suggest add-0/1 participants learned rules rather than isolated facts. Theses results support the recommendations of both the NMAP (2008) and the number sense view (Brownell, 1935; Gersten & Chard, 1999; Jordan, 2007) that structured discovery of mathematical regularities can be an effective instructional tool in promoting combination fluency. Such instruction is particularly well suited for combinations families that have a relatively apparent or salient underlying rule, such as the $n + 0/0 + n = n$ pattern and number-after/adding-one relation and is fortified by research that suggests such rules can be compiled and become more efficient than recall of specific facts (Fayol & Thevenot, 2012).

[6]For $n + 1$ items, the total number of repetitions would be 60 if practice with the related number-after n item and—as suggested by some (but not all) models of mental addition development—practice of a commuted $1 + n$ partner were counted. This is still relatively little practice.

Sobering, however, is the fact that—at end of grade 1 and *with* extra intervention—the add-0/1 group achieved fluency for only about 80% and 45% of the $n + 0/0 + n$ items and $n + 1/1 + n$ combinations, respectively. With business-as-usual, the comparison group fared far worse ($<55\%$ and 15%, respectively). Clearly, fluency with even the relatively salient add-0 and add-1 rules cannot be taken for granted among kindergartners who are at risk for academic failure and need targeted instruction.

Aside from attention/motivation/behavioral problems that interfered with a few of the participants' progress and might interfere with any intervention regardless of quality, there are clearly a number of ways the present structured add-0/1 training could be improved.

- Some in the add-0/1 condition may have discovered the add-0 and add-1 rules (Phase 2), but the prescribed practice was insufficient for them to achieve automaticity (Phase 3). This includes two children who did not (fluently) know all the number-after relations to 9 (the prerequisite for fluently applying the add-1 rule). Increasing the dosage of the number-after and $n + 1/1 + n$ practice might result in more successful intervention with at-risk kindergartners.
- Although relatively implicit and minimally guided/structured instruction was sufficient for many at-risk kindergartners to discover the salient add-1 rule, simply presenting a $n + 1$ item immediately after solving a related number-after question and providing feedback on only the correctness of each answer was not sufficient for others. To better ensure the simultaneous representation of a number-after relation and a related $n + 1/1 + n$ combination in working memory and, thus, the opportunity to discover the relation between these aspects of knowledge, the feedback for each could be juxtaposed. For example, a square containing $5 + 1 = 6$ could move above a number list until directly above its 6 square and then settle down on top of the previously presented feedback (a square containing After $5 = 6$ already there). Another method for prompting the simultaneous consideration of the relation and more explicitly drawing attention to it is to engage a child in a "Does It Help?" activity. This would entail asking a child to consider whether knowing that the number after five is six would help answer $5 + 1$ and providing implicit feedback ("Yes, it can help") or explicit feedback ("Yes, because the answer to $5 + 1$ is the number after five when count, which is six").
- The use of non-examples to define the limits of a rule's applicability and minimize its over application could be improved by providing sharper distinctions and a wider range of contrast, such as comparing $3 + 1$ with $3 + 2$ (instead of $2 + 4$ as was done in the present study) and $3 - 1$. Although introducing number-before relations and subtraction along with number-after and addition training proved confusing to preschoolers with a risk factor (Baroody et al., 2009), Jordan, Glutting, Dyson, Hassinger-Das, and Irwin (2012) introduced addition and subtraction simultaneously in their successful intervention with kindergartners.

Methodological Implications. Simply scoring accuracy or even accuracy and response time (efficiency) is inadequate for assessing fluency, which entails appropriate and adaptive, as well as efficient, use of a strategy. Scoring fluency—particularly that involving rules—is analogous to scoring toddlers' understanding of small numbers in that both require taking into account responses across different items and response biases (scoring in context). Scoring a preschooler's understanding of "two," for instance, on only trials involving two—without taking into account whether s/he over-generalizes the term to non-examples of the number—could lead to the incorrect conclusion that the child has an exact concept of two when, in fact, s/he equates "two" with "many" (Palmer & Baroody, 2011). Developmental psychologists commonly give a child credit for correctly labeling a collection of two with "two" but then deduct credit for incorrectly applying the term to a collection of three or four. This differentiates between a child who has an imprecise understanding of "two" as "many" and one who has an exact understanding of the concept. Analogously scoring in context is needed to distinguish between a developmental less-advanced child who is using a number-after-n response bias and one who is using Strategy I (the add-1 rule).

Although simply scoring efficiency and scoring in context did not differ substantially in the present study,[7] it does not diminish the need for the latter. This is particularly true with studies involving a larger sample. For example, with a sample five times larger than the small sample used in the present study, scoring method made a difference between significant and non-significant results in the Baroody et al.'s (2011) study.

Question 2: Feasibility of the Doubles/Near-Doubles Training

The doubles training did not have a significant impact above and beyond regular classroom training for the small practiced doubles, which included $2 + 2$ and $5 + 5$—among the easiest combinations for children to learn (see, e.g., reviews by Brownell, 1941, and Cowan, 2003). However, appreciable gains (effect sizes) indicate that this training had some success with $6 + 6$ (the practiced large double) and $4 + 4$ (the unpracticed small double). Excluding the one participant in each condition who was fluent with $4 + 4$ at pretest, the doubles training helped four (31%) and three (25%) at-risk children achieved fluency with $6 + 6$ and $4 + 4$, respectively, whereas regular training helped only one child do so with each of these items (7% and 8%, respectively). In practical terms, the intervention helped at least three times as many children with a risk factor achieve fluency with these two more difficult doubles than did regular training.

The appreciable gains (effect sizes) were probably due to doubles participants' ability to remember the association between the meaningful analogy of "6 eggs in row and 6 eggs in another makes 12 (a dozen) eggs" and the symbolic expression $6 + 6$ and to apply (transfer) the practiced meaningful analogy of "4 legs on the left and 4 legs on the right is a total of 8 legs" to the unpracticed symbolic expression $4 + 4$. The lack of transfer to $7 + 7$ and $8 + 8$ may have been due, in part, to the fact that doubles training did not include training on a real-world analogy for these items. These results support the feasibility of using meaningful real-world analogies help children visualize and learn the doubles (e.g., Rathmell, 1978) in a computer environment.[8]

[7]When simply scoring efficiency, the results for the practiced and unpracticed $n + 0/0 + n$ were the same or similar as those produced by scoring in context. However, the significance level and effect size for the unpracticed $n + 1/1 + n$ were substantially improved ($p = .001$ and $d = 1.22$), and the effect size for the unpracticed $n + 1/1 + n$ was substantially increased ($d = 1.34$).

[8]The appreciable negative effect size for the unpracticed $4 + 2$ may have been due to another a feature—the counting-by-twos practice. This practice may have enabled a couple of doubles participants to use skip counting by two (2, 4, 6) to efficiently determine the sum of $4 + 2$. Clearly, though, such an effect needs to be replicated with a larger sample.

The structured discovery/practice did not help at-risk kindergartners achieve significant improvement in fluency with the near doubles. Further research is needed to determine whether better-designed training might be more effective or if such training should be delayed until first grade for at-risk children.

Question 3: Necessity of Family-Specific Developmental Prerequisites

The success or failure of an intervention in the present study was dependent on children mastering the developmental prerequisites for a *particular* combination family. The success of the add-0/1 training was related to fluency with number-after relations, which some had mastered informally before the study and others appeared to have mastered during Stage I and/or Stage II training. In contrast, the near-doubles training was severely hampered by a lack of fluency with the related doubles (on average only 17% and 50% at pretest and posttest, respectively) and $n + 1$ sums (between 11% and 15% on both tests). Participants in the add-0/1 condition had more opportunities to build on and connect their Stage II training to their prior knowledge than those trained on the near doubles. The implementation of the near-doubles training in the present study mimics the all too common practices of using (a) lockstep instruction—moving on to a new unit before a child has had the opportunity to master previous units in effort to get through a curriculum, and (b) premature drill and practice—combination practice before a child understands the relations they involve (Brownell & Chazal, 1935; NRC, 2001; Rathmell, 1978). Two educational implications follow.

Implication 1. *Educators need to consider carefully whether a child has mastered the prerequisites for a particular combination family, as well as the developmental prerequisites for mental addition in general*. Such diagnostic information is essential for successful differentiated instruction. Such instruction here means diagnosing strengths and weaknesses of competencies on a theory- and research-based learning trajectory, remedying deficits in an individual child's developmental prerequisites for a key grade-level goal, and—consistent with Piaget's (1964; James, 1958) principle of assimilation—then building on existing knowledge to ensure meaningful achievement of key goals.

However, current guidelines and nearly all elementary curricula pay insufficient attention to the developmental trajectories and prerequisites for achieving fluency with the $n + 1/1 + n$ or other combination families. The *Curriculum Focal Points* (NCTM, 2006) do not include quick recall of these or any basic addition facts until grade 2. A grade 1 focal point is developing understandings of addition strategies for basic addition facts, including "the connections between counting and ... addition ... (e.g., adding two is the same as 'counting on' two)" (p. 13). Although on the right track, it does not make clear that fluency with *number-after relations* is critical for fluency with $n + 1/1 + n$ sums. Although the use of reasoning strategies is a central aspect of the current British National Numeracy Strategy, neither the Year 1 objectives for 5- to 6-year-olds nor the Year 2 objectives for 6- to 7-year-olds proposed by the National Centre for Literacy and Numeracy include using knowledge of number-after relations to learn $n + 1/1 + n$ combinations (Dowker, 2005). The CCSS (2010) includes as goals for grade K: (a) understanding the successor principle and (b) "fluently add and subtract within 5" (p. 11). It also includes "reason abstractly and quantitatively" and "look for and express regularity in repeated reasoning" (p. 11). Unfortunately, it does not indicate how these goals and processes are interrelated in achieving the first steps toward mental-addition fluency. Specifically, it does not prescribe that knowledge of number-after-relations is critical to learning the add-1 rule and that instruction should help kindergartners relate these particular aspects of counting and arithmetic knowledge.

Moreover, Ginsburg, Klein, and Starkey (1998) found that both traditional and non-traditional U.S. textbooks provide a weak conceptual basis for mathematics instruction. With the notable exception of the *Building Blocks* curriculum (Clements & Sarama, 2007, 2008), grade K curricula, including those used in the participating schools, do not specifically encourage exploiting the relation between children's number-after knowledge and adding with 1. The *Everyday Math* program at grade 1 makes an ineffectual effort to do so. It involves but a single lesson, practicing number-after relations and $n + 1/1 + n$ items separately or implicitly (e.g., adding 1 using a number line; University of Chicago School Mathematics Project, 2005).

Implication 2. *Practice is an instructional tool that needs to be used judiciously*. Specifically, practice of even the simplest (adding 1 and doubles) combinations is likely to be ineffective before a child has mastered the developmental prerequisites for a combination family. Whereas practice had a beneficial effect when it involved a meaningful relation either to children's existing knowledge of number-after relations (in the case of adding with 1) or a everyday analogies (in the case of the doubles), it had little or no effect in the absence of such relations. Even though doubles participants practiced both types of combinations equally, some improved on $6 + 6$ and $4 + 4$ but not on the small near doubles $2 + 3$ and $3 + 4$.

Question 4: Broader Effects of Intervention

Interestingly, the structured add-0/1 training, which involved training on number-after relations to 10 and structured learning of the simplest rules (involving adding 0 or 1), appeared to promote general number sense or TEMA-3 achievement significantly more than the doubles training. These results have two important educational implications.

Implication 1. *Fluency with number-after relations and add-0 and -1 rules should be key goals or focal points for all kindergartners, because they can have a significant impact on various other aspects of mathematical thinking/knowledge*. Recently, educators have become interested in identifying a few key mathematical goals for a particular age level (NCTM, 2006; CCSS, 2010). Efficient knowledge of number-after relations and the add-0 and -1 rules is important for kindergartners to achieve for several reasons: (a) understanding that $n + 1$ is the number after n can help children recognize that $n + 2$ and $n + 3$ must be two and three numbers after n—the conceptual basis for inventing the relatively advanced "abstract counting-on" strategy (Baroody, 1995; Clements & Sarama, 2012); (b) efficient mental addition involving 0 or 1 is the gateway to efficient mental addition with more difficult combinations (e.g., with adding two or the near doubles); (c) kindergartners' mental addition is an important bridge between informal and formal mathematics and is predictive of later school mathematics achievement and learning difficulties (Jordan et al., 2009; Jordan & Levine, 2009). For example, the ability to perform mental addition with basic combinations in kindergarten is a significant predictor of grade 2 calculation fluency (Jordan & Locuniak, 2008). Lastly, (d) it may prompt, reinforce, or deepen the development of the successor principle and promote mathematical reasoning, as discussed next.

Implication 2. *Practice is not merely a vehicle for strengthening a factual association but an opportunity to actively create new connections, insights, and qualitatively change mental representations* (Nader & Hardt, 2009). The additional practice with number-after relations up to 10 and structured discovery of the add-1 rule may have promoted the mathematical development and thinking of many add-0/1 participants in a number of important ways. For example, recalling that the number after "seven" is "eight" after calculating the sum of $7 + 1 = ?$ may help children to construct or strengthen the successor principle and re-represent the count sequence as an integer sequence and in a linear manner. This hypothesis is an extension of the proposal that practice is effective in strengthening a specific association only when an expression and its answer are simultaneously present in working memory (Fuchs et al., 2006; Geary, 1993).

Representing counting sequence as the integer sequence is a developmental prerequisite for reasoning by informal mathematical induction, which previous research indicates develops as early as 5 to 7 years of age (Smith, 2002). It may have allowed such participants to generalize their training to unpracticed number-after relations for multi-digit numbers between 21 to 40 (TEMA-3 item #22) and unpracticed $n + 1/1 + n$ items. Put differently, the training may have prompted children to use their automatic generative rules for counting to efficiently deduce the number-after any n and the sum of any $n + 1/1 + n$ item for any known part of the counting sequence.

Limitations of the Present Research and Future Research Directions

As the same project personnel conducted both the training and testing, one limitation of the present study was "experimenter bias." Arguments against such bias are the uneven results (i.e., disappointing results with doubles/near-doubles program)—despite the hope by project personnel that both the structured add-0/1 and doubles programs would be successful—and the unexpected result (as far as the project staff were concerned) that add-0/1 participants did at least as well as doubles participants on $1 + 1$. Another limitation of the present research was the absence of an unstructured add-0/1 or doubles practice condition. Thus, it is unclear whether the significant or appreciable gains for each condition is due to the structured training or the additional practice provided by a condition (above and beyond that received in school). A counter argument is that the positive results were achieved with relatively little practice. Still, future research that focuses on the efficacy of the programs should include an unstructured practice condition to control for additional practice provided in the structured conditions.

Future research is also needed to systematically examine the proposed developmental trajectory that ties adding with 1 and mental addition with counting knowledge and representations, including the successor principle and a linear representation. Furthermore, this research needs to explore how these developments are related to the understudied areas of using subtraction as a contrast and informal mathematical induction.

ACKNOWLEDGMENTS

The research reported herein was supported by a grant from the Institute of Education Science, U.S. Department of Education, through Grant R305K050082 ("Developing an Intervention to Foster Early Number Sense and Skill"). Preparation of this article also was supported, in part, by grants from the U.S. Department of Education (R305A080479: "Fostering Fluency with Basic Addition and Subtraction Facts") and (R305B100017: "UIUC Postdoctoral Research Training Program in Mathematics Education"). The opinions expressed are solely those of the authors and do not necessarily reflect the position, policy, or endorsement of the Institute of Education Science or Department of Education.

REFERENCES

Alfieri, L., Brooks, P. J., Aldrich, N. J., & Tenenbaum, H. R. (2011). Does discovery-based instruction enhance learning? *Journal of Educational Psychology*, *103*, 1–18.

Ashcraft, M. H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition*, *44*, 75–106.

Baroody, A. J. (1985). Mastery of the basic number combinations: Internalization of relationships or facts? *Journal for Research in Mathematics Education*, *16*(2), 83–98.

Baroody, A. J. (1987). *Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers*. New York, NY: Teachers College Press.

Baroody, A. J. (1989a). *A guide to teaching mathematics in the primary grades*. Boston, MA: Allyn & Bacon.

Baroody, A. J. (1989b). Kindergartners' mental addition with single-digit combinations. *Journal for Research in Mathematics Education*, *20*, 159–172.

Baroody, A. J. (1992). The development of kindergartners' mental-addition strategies. *Learning and Individual Differences*, *4*, 215–235.

Baroody, A. J. (1994). An evaluation of evidence supporting fact-retrieval models. *Learning and Individual Differences*, *6*, 1–36.

Baroody, A. J. (1995). The role of the number-after rule in the invention of computational short cuts. *Cognition and Instruction*, *13*, 189–219.

Baroody, A. J., Bajwa, N. P., & Eiland, M. D. (2009). Why can't Johnny remember the basic facts? *Developmental Disabilities Research Reviews*, *15*, 69–79.

Baroody, A. J., & Coslick, R. T. (1998). *Fostering children's mathematical power: An investigative approach to K–8 mathematics instruction*. Mahwah, NJ: Erlbaum.

Baroody, A. J., Eiland, M. D., & Thompson, B. (2009). Fostering at-risk preschoolers' number sense. *Early Education and Development*, *20*, 80–120.

Baroody, A. J., Purpura, D. J., Reid, E. E., & Eiland, M. D. (2011, September). *Scoring fluency with basic addition combinations in context*. In A. J. Baroody (Chair), *Issues in the assessment and scoring of early numeracy skills*. Symposium at the annual meeting of the Society for Research on Educational Effectiveness, Washington, DC.

Benjamini, Y., & Hochberg, Y. (1995). Controlling the false discovery rate: A practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society. Series B (Methodological)*, *57*, 289–300.

Bråten, I. (1996). Cognitive strategies in mathematics (Report No. 10). Oslo, Norway: Institute for Educational Research, University of Oslo.

Bright, G. W., Harvey, J. G., & Wheeler, M. M. (1985). *Learning and mathematics games* (JRME Monograph Number 1). Reston, VA: NCTM.

Brownell, W. A. (1935). Psychological considerations in the learning and the teaching of arithmetic. In W. D. Reeve (Ed.), *The teaching of arithmetic* (Tenth yearbook, National Council of Teachers of Mathematics; pp. 1–31). New York, NY: Bureau of Publications, Teachers College, Columbia University.

Brownell, W. A. (1941). *Arithmetic in grades I and II: A critical summary of new and previously reported research*. Durham, NC: Duke University Press.

Brownell, W. A. & Chazal, C. B. (1935). The effects of premature drill in third-grade arithmetic. *Journal of Educational Research*, *29*, 17–28.

Bruner, J. S. (1961). The act of discovery. *Harvard Educational Review*, *31*, 21–32.

Chi, M. T. H. (2009). Active-constructive-interactive: A conceptual framework for differentiating learning activities. *Topics in Cognitive Science*, *1*, 73–105.

Clements, D. H., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, *6*, 81–89.

Clements, D. H., & Sarama, J. (2007). *Building blocks—SRA real math teacher's edition, grade preK*. Columbus, OH: SRA/McGraw-Hill.

Clements, D. H., & Sarama, J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal*, *45*, 443–494.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York, NY: Routledge.

Clements, D. H., & Sarama, J. (2012). Learning and teaching early and elementary mathematics. In J. Carlson & J. Levin (Eds.), *Psychological perspectives on contemporary educational issues: Instructional strategies for improving students' learning* (pp. 107–162). Charlotte, NC: Information Age Publishing.

Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*, 155–159.

Common Core State Standards. (2010). *Common Core State Standards*: Preparing America's students for college and career. Retrieved from http://www.corestandards.org/

Cowan, R. (2003). Does it all add up? Changes in children's knowledge of addition combinations, strategies, and principles. In A. J. Baroody & A. Dowker, A. (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 35–74). Mahwah, NJ: Erlbaum.

Crowley, K., Shrager, J., & Siegler, R. S. (1997). Strategy discovery as a competitive negotiation between metacognitive and associative mechanisms. *Developmental Review*, *17*, 462–489.

Dev, O. C., Doyle, B. A., & Valente, B. (2002). Labels needn't stick: "At-risk" first graders rescued with appropriate intervention. *Journal of Education for Students Placed At Risk*, *7*, 327–332.

Dowker, A. D. (1997). Young children's addition estimates. *Mathematical Cognition*, *3*, 141–154.

Dowker, A. D. (2003). Young children's estimates for addition: The zone of partial knowledge and understanding. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 35–74). Mahwah, NJ: Erlbaum.

Dowker, A. (2005). *Individual differences in arithmetic: Psychology, neuroscience and education*. Hove, UK: Erlbaum.

Dowker, A. (2009). Use of derived fact strategies by children with mathematical difficulties. *Cognitive Development*, *24*, 401–410.

Ernest, P. (1986). Games: A rationale for their use in the teaching of mathematics in schools. *Mathematics in School*, *15*, 2–5.

Fayol, M., & Thevenot, C. (2012). The use of procedural knowledge in simple addition and subtraction. *Cognition*, *123*, 392–403.

Ferguson, R. F. (1998). Can schools narrow the black-white test score gap? In C. Jencks & M. Phillips (Eds.), *The black-white test score gap* (pp. 318–374). Washington, DC: Brookings Institution Press.

Fuchs, L. S., Compton, D., Fuchs, D., Paulsen, K., Bryant, J., & Hamlett, C. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology*, *97*, 493–513.

Fuchs, L. S., Fuchs, D., Hamlet, C. L., Powell, S. R., Capizzi, A. M., & Seethaler, P. M. (2006). The effects of computer-assisted instruction on number combination skill in at-risk first graders. *Journal of Learning Disabilities*, *39*, 467–475.

Fuchs, L. S., Powell, S. R., Seethaler, P. M., Cirino, P. T., Fletcher, J. M., Fuchs, D., & Hamlett, C. L. (2010). The effects of strategic counting instruction, with and without deliberate practice, on number combination skill among students with mathematics difficulties. *Learning and Individual Differences*, *20*, 89–100.

Fuson, K. C. (1992). Research on whole number addition and subtraction. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243–247). New York, NY: Macmillan.

Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology*, *49*, 363–383.

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin*, *114*, 345–362.

Geary, D. C. (1996). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association. Originally published 1994.

Geary, D. C. (2003). Arithmetic development: Commentary on chapters 9 through 15 and future directions. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 453–464). Mahwah, NJ: Erlbaum.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education*, *33*, 18–28.

Ginsburg, H. P., & Baroody, A. J. (2003). *Test of early mathematics ability* (3rd ed.; TEMA-3). Austin, TX: Pro-Ed.

Ginsburg, H. P., Klein, A., & Starkey, P. (1998). The development of children's mathematical knowledge: Connecting research with practice. In I. E. Sigel & K. A. Renninger (Eds.), *Handbook of child psychology: Vol. 4. Child psychology in practice* (5th ed.; pp. 401–476). New York, NY: Wiley & Sons.

Hatano, G. (2003). Forward. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. xi–xiv). Mahwah, NJ: Erlbaum.

Hasselbring, T. S., Goin, L. T., & Bransford, J. D. (1988). Developing math automaticity in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children*, *20*, 1–7.

Henry, V., & Brown, R. (2008). First-grade basic facts. *Journal for Research in Mathematics Education*, *39*, 153–183.

Institute of Education Sciences. (2011). *What works clearinghouse: Procedures and standards handbook (Version 2.1)*. Retrieved from http://ies.ed.gov/ncee/wwc/pdf/reference_ resources/wwc_procedures_v2_1_standards_handbook.pdf

James, W. (1958). *Talks to teachers on psychology: And to students on some of life's ideals*. New York, NY: W. W. Norton & Company. (Talk originally given in 1892.)

Jerman, M. (1970). Some strategies for solving simple multiplication combinations. *Journal for Research in Mathematics Education*, *1*, 95–128.

Jordan, N. C. (2007, October). The need for number sense. *Educational Leadership*, *65*, 63–66.

Jordan, N. C., Glutting, J., Dyson, N., Hassinger-Das, B., & Irwin, C. (2012, April). Building kindergartners' number sense: A randomized controlled study. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, Philadelphia.

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology*, *85*, 103–119.

Jordan, N. C., Hanich, L. B., & Uberti, H. Z. (2003). Mathematical thinking and learning difficulties. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 359–383). Mahwah, NJ: Erlbaum.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology*, *28*, 644–653.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performance of middle- and low-income children. *Learning and Individual Differences*, *6*, 413–432.

Jordan, N. C., Kaplan, D., Olah, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development*, *77*, 153–175.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*, 850–867.

Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities Research Reviews*, *15*, 60–68.

Jordan, N. C., & Locuniak, M. N. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities*, *41*, 451–459.

Katona, G. (1967). *Organizing and memorizing: Studies in the psychology of learning and teaching*. New York, NY: Hafner. (Originally published in 1940).

Kulik, C.-L. C., & Kulik, J. A. (1991). Effectiveness of computer-based instruction: An updated analysis. *Computers in Human Behavior*, *7*, 75–94.

Lee, J. (2004). Multiple facets of inequality in racial and ethnic achievement gaps. *Peabody Journal of Education*, *79*, 51–73.

Lin, A., Podell, D. M., & Tournaki-Rein, N. (1994). CAI and the development of automaticity in mathematics skills in students with and without mild mental handicaps. *Computers in the Schools*, *11*, 43–58.

Lubienski, S. T. (2001). *Are the NCTM standards reaching all students? An examination of race, class, and instructional practices*. Paper presented at the annual meeting of the American Educational Research Association, Seattle.

Lubienski, S. T., & Shelley II, M. C. (2003, April). *A closer look at U.S. mathematics instruction and achievement: Examinations of race and SES in a decade of NAEP data*. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Maxwell, S. E., & Delaney, H. D. (2004). *Designing experiments and analyzing data: A model comparison perspective* (2nd ed.). Mahwah, NJ: Erlbaum.

Mazzocco, M., & Thompson, R. (2005). Kindergarten predictors of math learning disability. *Learning Disabilities Research & Practice*, *20*, 142–155.

Miller, D. & McKinnon, A. (1995). *Beginning school mathematics project:* An ASCD Curriculum Handbook case study section 11H. Alexandria, VA: Association for Supervision and Curriculum Development.

Murata, A. (2004). Paths to learning ten-structured understandings of teen sums: Addition solution methods of Japanese grade 1 students. *Cognition and Instruction*, *22*, 185–218.

Nader, K., & Hardt, O. (2009). A single standard for memory: The case for reconsolidation. *Nature Reviews/Neuroscience*, *10*, 224–234.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics: Standards 2000*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*. Reston, VA: Author.

National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the National Mathematics Advisory Panel. Washington, DC: U.S. Department of Education.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Washington, DC: National Academy Press.

Palmer, A., & Baroody, A. J. (2011). Blake's development of the number words "one," "two," and "three." *Cognition & Instruction*, *29*, 265–296.

Piaget, J. (1964). Development and learning. In R. E. Ripple & V. N. Rockcastle (Eds.), *Piaget rediscovered* (pp. 7–20). Ithaca, NY: Cornell University.

Rasanen, P., Salminen, J., Wilson, A., Pirjo, A., & Dehaene, S. (2009). Computer-assisted intervention for children with low numeracy skills. *Cognitive Development*, *24*, 450–472.

Rathmell, E. C. (1978). Using thinking strategies to teach basic facts. In M. N. Suydam & R. E. Reys (Eds.), *Developing computational skills* (1978 Yearbook, pp. 13–50). Reston, VA: National Council of Teachers of Mathematics.

Resnick, L. B. (1983). A developmental theory of number understanding. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 109–151). New York, NY: Academic Press.

Richardson, K. (1998). *Developing number concepts (Book 1: counting, comparing, and pattern and Book 2: Addition and subtraction)*. Upper Saddle River, NJ: Dale Seymour.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge.

Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science*, *9*, 405–410.

Siegler, R. S. (1986). Unities across domains in children's strategy choices. In M. Perlmutter (Ed.), *Minnesota symposium of child psychology* (Vol. 19, pp. 1–48). Hillsdale, NJ: Erlbaum.

Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies*. Hillsdale, NJ: Erlbaum.

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, *101*, 545–560.

Smith, L. (2002). *Reasoning by mathematical induction in children's arithmetic*. Boston, MA: Pergamon.

Suydam, M. & Weaver, J. F. (1975). Research on mathematics learning. In J. N. Payne (Ed.), *Mathematics learning in early childhood* (pp. 44–67). Reston, VA: National Council of Teachers of Mathematics.

Steinberg, R. M. (1985). Instruction on derived facts strategies in addition and subtraction. *Journal for Research in Mathematics Education*, *16*, 337–355.

Swanson, H. L., & Cooney, J. 1985). Strategy transformation in learning disabled and nondisabled students. *Learning Disability Quarterly*, *8*, 221–230.

Swanson, H. L., & Rhine, B. (1985). Strategy transformation in learning disabled children's math performance: Clues to the development of expertise. *Journal of Learning Disabilities*, *18*, 596–603.

Swenson, E. J. (1949). Organization and generalization as factors in learning, transfer and retroactive inhibition. In *Learning theory in school situations*. University of Minnesota Studies in Education, No. 2. Minneapolis, MN: University of Minnesota Press.

Thiele, C. L. (1938). *The contribution of generalization to the learning of addition facts*. New York, NY: Bureau of Publications, Teachers College, Columbia University.

Thorndike, E. L. (1922). *The psychology of arithmetic*. New York, NY: Macmillan.

Thornton, C. A. (1978). Emphasizing thinking strategies in basic fact instruction. *Journal for Research in Mathematics Education*, *9*, 214–227.

Thornton, C. A. (1990). Solution strategies: Subtraction number facts. *Educational Studies in Mathematics*, *21*, 241–263.

Thornton, C. A., & Smith, P. J. (1988). Action research: Strategies for learning subtraction facts. *Arithmetic Teacher*, *35*, 8–12.

Torbeyns, J., Verschaffel, L., & Ghesquiere, P. (2005). Simple addition strategies in a first-grade class with multiple strategy instruction. *Cognition and Instruction*, *23*, 1–21.

Torgeson, J. K., & Young, K. A. (1983). Priorities for the use of microcomputers with learning disabled children. *Journal of Learning Disabilities*, *16*, 234–237.

University of Chicago School Mathematics Project. (2005). *Everyday mathematics Teacher's Lesson Guide (Volume 1)*. Columbus, OH: McGraw-Hill.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557–628). Charlotte, NC: NCTM & Information Age Publishing.

Wilburn, D. B. (1949). Methods of self instruction for learning arithmetic. *Supplementary Educational Monographs*, No. 70. Chicago, IL: University of Chicago.

Wilkinson, L., & APA Task Force on Statistical Inference. (1999). Statistical methods in psychology journals: Guidelines and explanations. *American Psychologist*, *54*, 594–604.

Wynroth, L. (1986). *Wynroth math program—The natural numbers sequence*. Ithaca, NY: Wynroth Math Program. (Originally published in 1975).