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Promoting early abstraction to promote early literacy and numeracy

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ABSTRACT

A learning-set procedure was used to teach the oddity principle, insertions into series, and number conservation to 85 kindergarten children who did not grasp these abstractions. Control groups were given lessons in kindergarten literacy, numeracy, or art in sessions matched in timing and extent. The children who were taught the principles of abstraction early in the school year were a match in literacy at the end of the year for those taught literacy and a match in numeracy for those taught numeracy. They surpassed the literacy group in numeracy, surpassed the numeracy group in literacy, and surpassed the art group in both. This indicates that improving the children's mastery of the abstract thinking involved in oddity, insertions, and conservation facilitated a broad improvement in mastery of kindergarten material.

1. Introduction

Psychologists have a great deal to say about cognitive development and how it may be facilitated. The present research tested one practical application of this knowledge. Techniques and principles from developmental psychology were brought to bear on a classic problem: how may we significantly improve academic achievement in an applied setting without a prohibitive investment of resources or costly training?

In this field research, cognitive development was enhanced in the classroom and the subsequent impact on academic performance was assessed. The approach to cognitive development was based on two assumptions. The first is that cognitive development can be meaningfully enhanced without unreasonable expenditure of time, funds, and effort. That may be a tall order, but it has been a perennial goal for cognitive developmental psychologists. The second assumption is that such enhanced cognitive development will subsequently lead to measurable gains in academic performance in non-laboratory situations. This could happen if the students understood classroom instruction better after their thinking had improved — not an unreasonable supposition. However, a true test of the applicability of procedures advocated by a developmental psychologist is more demanding. The real challenge for applied developmental psychologists is to show that their procedures lead to better results than those already developed by educators. Do gains from instruction in cognition actually lead to greater academic gains than would result from an equal investment of classroom time and resources in instruction that educators have already developed? The present research was designed to test whether approaches developed earlier (Ciancio, Rojas, McMahon, & Pasnak, 2001; Pasnak, McCutcheon, Campbell, & Holt, 1991; Pasnak, Madden, Martin, Holt, & Malabonga, 1996) actually produced better results than state of the art instruction on academic content.

The first step was selecting a target population that was likely to be maximally responsive. The age and initial state of cognitive development of the children were key considerations. Studies of cognitive development (e.g., Capron & Duyme, 1989; Weinberg, Scarr, & Waldeman, 1992) indicate that an individual's cognitive performance depends in part upon that person's genetic potential and in part upon where a person is functioning in the range defined by that potential. Children who receive much stimulus, challenge, and nurture might be functioning near the upper limit of their genetic potential; children who had not had such a

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propitious environment might function closer to the middle of their potential range, and those who were in a less stimulating environment might be functioning closer to the lower limit of their potential.

The second step in choosing a population maximally likely to respond to a cognitive intervention was to select children at an age when cognitive abilities were already changing. It is easier to enhance a developmental process already underway than to instigate development when a child is in a relatively stable state. Early longitudinal research by McCall, Appelbaum, and Hogarty (1973) indicated that changes in psychometric IQ are especially likely at about ages 2, 5–7, and 12–15 years. These coincide with the ages at which Piaget and Inhelder (1973/1968) observed major changes in the nature of children's thought. The secular trend has downshifted the ages of cognitive transitions from Piaget's day, and a shift from thought which is heavily influenced by perception to consideration of abstract relations between concrete objects is usually underway at age 5 for American middle class children. This is also the age at which children now meet the stimulus and demands of formal schooling — a challenge that used to be deferred until age 6 or 7. This could be a propitious age for a cognitive intervention. Thinking is changing, and children are entering a new environment that demands it change.

The performance of children in kindergarten and the early grades is predictable from the extent to which their thinking has progressed to mastery of certain principles of abstract thought (Freyberg, 1966; Kingma & Koops, 1983; Silliphant, 1983). Conservation of mass and number is a hallmark of such progress. Conservation requires the development and integration of many cognitive abilities, including at least decentration, more exhaustive encoding, reversibility, and appreciation of the centrality of the addition/subtraction operation, in addition to an increase in cognitive capacity (Case, 1985; Flavell, 1985). Most kindergarten curricula have some lessons on conservation, but these are too brief to produce mastery of the abstractions involved unless children are already strongly emergent. Pasnak et al. (1991) and Pasnak, Hansbarger, Dodson, Hart, and Blaha (1996) developed preliminary evidence that helping 5-year-olds to master number conservation, in combination with mastery of other forms of early abstraction, led to subsequent improvements in number concepts and verbal comprehension. Some training programs for children who have mild mental disabilities have often shown that helping the children to master some form of conservation led to improved performance on psychometric tests (Kern, 1983; Moreno & Sastre, 1971; Richards & Stone, 1970). Hence, a cognitive intervention that emphasized thorough mastery of some form of conservation might be likely to produce subsequent academic gains.

Mastery of the oddity principle and mastery of insertions into series also seem to be early forms of abstraction that, if adequately developed, facilitate a 5-year-old's ability to profit from kindergarten curricula (Pasnak et al., 1991; Pasnak, Hansbarger, et al., 1996). The oddity principle is the understanding that when all but one item in a group are similar and one is different, that one is "odd," in the sense that it is a member of a different class. Oddity problems demand a purely abstract solution, as the concrete properties that make an object odd in one group may make it non-odd in another. Thus, if there are three oval beads and a spherical bead, the spherical one is odd; but if there are three spherical beads and an oval one, the oval one is odd. Study of children who have mental disabilities show that children develop this form of abstraction just prior to that involved in insertions into series and even more in advance of those required for conservation (McCormick, Campbell, Pasnak, & Perry, 1990). Piaget appears to have overlooked the importance of the oddity principle; he and his colleagues never addressed it. It has not received much study from psychologists, who have concentrated on simpler (sorting or hierarchical classification) or more advanced forms of classification (class inclusion or multiple classification).

Perhaps the most important landmark is the demonstration by Chalmers and Halford (2003) that initial understandings of oddity involve abstracting the number of dimensions on which objects differ. This is clearly part of the process of developing an understanding of the oddity principle. However, not all dimensions are equal; Pasnak et al. (1991) found a horizontal decalage in which shape oddity was much easier than size oddity and that orientation oddity was much harder than either. There are also decalages within the former and latter dimensions. The principal difficulty seems to be that young children are perceptually oriented when they categorize or classify objects (Namy & Gentner, 2002). They prefer to understand the meaningfulness of stimuli by responding to one or more perceptually distinctive characteristics of the objects. Their ability to conceptualize relations between objects requires replacing their tendency to respond perceptually with the ability to respond abstractly (Baldwin, 1989; Case, 1985; Gentner, 1978; Imai, Gentner, & Uchida, 1994). This process emerges earlier for some dimensions than others, and is also affected by the number of dimensions involved. Children younger than 4 years cannot shift from one dimension to another which they know is now pertinent (Zelazo, Frye, & Rapus, 1996), even though they can categorize objects accurately on either of the dimensions involved (Jacques, Zelazo, Kirkham, & Semcesen, 1999). These researchers found that the kind of abstract thinking (i.e., representational flexibility) needed to do so becomes commonplace by age 4 years and that nearly all kindergarteners can think this abstractly.

Mastery of the oddity principle per se usually begins at age 4 (Ciancio et al., 2001; Malabonga, Pasnak, & Hendricks, 1994) for some dimensions but may not be complete until age 5 or later for others (Waiss & Pasnak, 1993). The decalage is most easily revealed by the slower and more detectable progress of children with mental disabilities. Pasnak, Maccubbin, Campbell, and Gadzichowski (2004) found that such children were first successful with object oddity, wherein the odd object differs from the other objects in all dimensions, whereas the non-odd objects are all identical. This is a simple binary discrimination; *any* difference the child notices provides a basis for correct solutions, making the problem a very easy one (Chalmers & Halford, 2003). Oddity problems that depend upon recognizing a superordinate category (a necklace, earring, bracelet, and a screwdriver) are also solved easily. Such problems are not amenable to the representational complexity interpretation advanced by Chalmers & Halford (2003). Whether the child knows the superordinate category to which the non-odd objects belong is decisive. Recognizing color oddity, when there are no distracting differences in shape or any other dimension is next, followed quickly by solution of color oddity problems even when all objects differ in shape. The first is a binary problem because only one dimension has to be processed, whereas the second is a ternary problem, according to relational complexity theory (Chalmers & Halford, 2003; Halford, Wilson, & Phillips, 1997), which adequately explains their relative difficulty. However, application of the oddity principle to shape problems, wherein all but one object is alike in shape, is significantly more difficult, oddity by size more difficult yet, and oddity of orientation varies, depending on whether objects are placed in vertical–horizontal, slanted, or left–right orientations. A theoretical account of this decalage has yet to be advanced. It is clear that the development of what may seem to be a simple abstraction takes at least a year under normal conditions and involves an incremental marshalling of a child's abstractive abilities.

The relevance to *applied* developmental psychologists is that speeding this process has been shown to improve children's academic performance. Ciancio et al. (2001) showed that improved understanding of the oddity principle improved preschooler's understanding of numeracy. Speeding mastery of this early form of abstraction has also been productive in preliminary studies of children who struggle in kindergarten (Pasnak et al., 1991; Pasnak, Hansbarger, et al., 1996; Pasnak, Madden, et al., 1996). Hence, a focus on the oddity principle seemed likely to be productive in the current intervention.

Insertions into a unidimensionally increasing or decreasing series are *much* harder for a young child than forming a series from scratch (Leiser & Gillieron, 1990; Southard & Pasnak, 1997; Young, 1976). Children are usually able to construct series by responding to absolute characteristics of the items — the method of extremum — long before they can accurately relate one item to others (Inhelder & Piaget, 1964/1959; Piaget & Inhelder, 1969/1966). Piaget repeatedly changed his interpretation of the role of insertions in the development of abstract thought (Leiser & Gillieron, 1990) and never reached a sound resolution. The importance of insertions is that they require abstractions of relations between items in the series (Piaget & Inhelder, 1969/1966). This form of early abstraction, which emerges just prior to conservation (McCormick et al., 1990), has also been shown to contribute to the academic achievement when enhanced in preliminary studies. The approach taken here was to help children become proficient in all three forms of early abstract thought — insertions, oddity, and conservation — in the expectation that they would subsequently function at a level more equivalent to that of their peers and better understand classroom instruction that was designed for ordinary 5-year-olds.

Helping children to think at a level more abstract than their present level is no small task, especially if the thinking is to become routine and be done in many contexts. Many instructional methods can be successful if the goal is to have children apply current cognitive competencies to new situations. Success in developing abstractions of a higher order than those currently possessed is not often achieved. Most successes result from the presentation of numerous and highly variable exemplars of the abstractions to be mastered (Gelman, 1969; Harlow, 1949, 1959; Pasnak et al., 1986; Soraci et al., 1991). Children slowly but surely shift from responding to perceptual characteristics of stimuli that are only sufficient for solution of one problem to responding to abstractions that provide solutions for all problems *if* there are enough problems of sufficient variety. In the words of Halford, Wilson, and Phillips (2006, p. 5), “Just how this occurs has never been satisfactorily explained.” Harlow (1959) theorized that error tendencies are gradually reduced. Reliance on perceptual cues to solve problems that can only be solved abstractly is a prevalent error-producing tendency of young children (Namy & Gentner, 2002). Zelazo, Resnick, and Pinon (1995) and Jacques et al. (1999) showed that children who knew the basis by which they were supposed to respond persistently responded in terms of another. The problem after problem requirement that choices be made on the only correct basis imposed by learning-sets inevitably extinguishes even the most resistant error-producing tendencies.

Gagné (1968) posited that the success of learning-sets rested in large part on their role in the reinforcement of proper observation habits and strengthening of concepts subordinate to that which was to be learned. A more complete explanation was provided by Halford (1993), who observed that learning-sets facilitate learning both the structure of the task and the relation between objects on different trials. The process that underlies the transfer of learning between problems entails forming a schema that incorporates this relation, and transferring it between problems by analogical mapping. This is in Halford's (1993, p.223) words, “a case of transfer based on structure mapping par excellence” and a form of abstraction that depends upon relational knowledge.

The learning-set approach was adopted in the current intervention. Learning-sets require little training of instructors because the power of the set rests on the number and variability of the exemplars of the abstraction, not the technique or explanatory skill of the teacher. This is, of course, a desirable characteristic for a method that is to have wide application.

Our hope that the children would subsequently apply their new abilities to abstract to classroom instruction rested on two suppositions. The safer of these was that the abstractions learned were natural kinds of thought that all children ordinarily develop unaided. The instruction they had received would be reinforced in the normal course of experience and maturation. Hence, it would be normal for children to retain, elaborate and employ them in their everyday life, including their classroom environment. The riskier supposition was that the abstraction abilities taught would be central to the development of thinking, rather than peripheral. This was an important assumption because the mechanisms by which oddity, insertions, and conservation are related to the development of kindergarten academic achievement are not clear. However, preliminary studies have indicated that strengthening children's understanding of the abstractions of oddity, insertions, and conservation has led to improved academic achievement for a variety of children (Lebron-Rodriguez & Pasnak, 1977; Pasnak, Hansbarger, et al., 1996; Pasnak, Madden, et al., 1996; Pasnak et al., 1991; Ciancio et al., 2001). Hence the present study was undertaken to determine, on a very large scale, whether strengthening these abilities for 5-year-olds who lagged in their development would lead to better numeracy and literacy in kindergarten than equivalent investment of instructional time in lessons aimed directly at those two domains.

2. Method

2.1. Participants

Over the course of 3 years, 380 kindergartners, 42% female and 58% male, were selected from 61 elementary school classrooms from a school district in a metropolitan area. Selection was based on screening tests demonstrating they did not show an understanding of the oddity principle and were unable to insert objects into an already existing sequence. These children were from an economically diverse school district where 51% of the students are eligible for free or reduced lunch. The children in the study were also culturally diverse: 24% African American, 27% Hispanic/Latino, 15% Mideast Asian, 26% other Caucasian, and 7% East Asian, Southeast Asian, Asian Indian, East African, or West African.

2.2. Design

Children were assigned to yoked quartets. The members of each quartet were assigned randomly to the experimental group or one of three active control groups. The experimental group received instruction in oddity, insertion, and number conservation. One active control group received instruction in literacy activities. A second active control group received instruction in numeracy activities. The final active control group received instruction in art activities.

One hypothesis was that the cognitive (experimental) group would exceed the literacy, numeracy, and art groups on cognitive measures. A second hypothesis was that the cognitive group would do as well as the numeracy group and better than both the literacy and art groups on measures of numeracy. A third hypothesis was that the cognitive group would do as well as the literacy group and better than the numeracy group and art group on measures of literacy.

2.3. Materials

The instructional materials for the experimental group were 60 oddity problems, 65 insertion problems, and 15 conservation problems. The problems were constructed from various items, such as paper clips, bolts, beans, beads, pennies, stars, play money, and wooden hearts that could be found in households or purchased in grocery, drug, hardware, or craft stores.

Each oddity set contained three objects that were identical and one object that was different in one dimension. In 20 of these problems the odd object differed from the other three only in shape. Twenty of the oddity problems contained odd objects that differed from the other objects by size only. In 10 of these problems the odd object was smaller than the other three objects and in 10 problems the odd object was larger than the other three objects. There were an additional 20 oddity by orientation problems. These contained four identical objects, but three of the objects would be presented vertically whereas the odd object was presented horizontally, or one was slanted 45° in one direction whereas the other three were slanted 45° in the opposite direction, or one faced right and the other three faced left.

There were 15 insertion problems that consisted of three objects, 20 problems with four objects, 15 with five, 5 with six, 5 with seven and 5 with eight. The objects differed in length, width or overall size. The conservation problems had 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, and 35 objects.

Large, brightly colored foam letters and Zip-loc sandwich bags, each labeled with a letter and containing items that all began with the same letter were used for the literacy control group. For example, one bag would contain a card with the letter “B” as well as miniaturized items such as a bee, a ball, etc. Large, brightly colored foam numbers and small items such as pennies and marbles were used in the numeracy instruction. Activity packets that included coloring, cutting, pasting, and other art-related skills were used for the art group.

2.4. Tests

Objects other than those used in the learning-sets were used to make 12-problem oddity, 10-problem insertion and 10-problem conservation tests. The classification scale from the Otis–Lennon School Ability Test (O–LSAT) Primary I, the Phonemic Awareness and Literacy Screening (PALS) measure of early literacy, the Stanford Early School Achievement Test (SESAT) Listening to Stories Scale, the Kauffman Test of Early Achievement III (K-TEA), and the Woodcock–Johnson III Problem-Solving scale (WJ III) test of numeracy were also used.

2.5. Procedure

Each participant assigned to the experimental (cognitive) group received instruction on the oddity principle, insertions into a sequence, and number conservation for 15 min/day. The participants assigned to the three control groups received instruction that matched the instruction given to those in the experimental group in timing, duration, and frequency. One control group received instruction in literacy activities, the second control group received instruction in numeracy activities, and the third control group received instruction in art activities. All activities for the control groups were based upon the traditional kindergarten curriculum.

The instruction was conducted in the child's classroom during “centers” time, when children were routinely assigned to small groups for diverse instructional activities. In order to motivate the children and keep them focused on the tasks, instructors used stuffed animals and made the task into a game. When working with the oddity problems, the adult's animal would announce that he was hungry and asked the child to feed him the thing that was different. If the child selected the correct item, then the animal pretended to gobble it up quickly. If however, the child had made an incorrect choice the animal would decline to eat it and say, “This one is the same as the others, they are all ___; feed me the one that is not the same.” The children readily made additional attempts at a problem if they did not succeed at during the first trial. The adult changed the position of the odd object for each trial. Often, after children felt comfortable with the task, they would “teach” their toy animal how to solve the problem by telling it how the correct choice was different from the others.

The shape oddity problems were the first type of oddity game the children played. In order to move on to the next type of oddity, children had to solve each shape oddity problem correctly on their first attempt for 3 days in a row. Following shape oddity, the children played size oddity games. Problems consisted of three large objects and one smaller object or three small objects and one large object, in an alternating order. After the children mastered those problems to the same criterion as the shape problems, they played oddity by orientation games. All of the objects in these problems appeared identical. The odd object was simply the object that was facing a different direction than the other three objects in the group. The first presentations of this type of oddity were problems wherein three objects would be placed horizontally and the odd object was placed vertically, or vice versa. When these games were mastered, new problems were introduced. These problems consisted of three objects slanted at 45° in one direction and the odd object was slanted 45° in the opposite direction. The last variety of oddity by orientation problems had three objects faced right and one object faced left. Each type of orientation problem had to be solved correctly on the first attempt for three consecutive days.

Once criterion had been met on all oddity games, the children began instruction on insertion games. The adult would begin these games by taking the three objects in the problem and, using a toy animal, putting them in order from smallest to largest, saying “Look what I can do. I can line these ___ up — small, medium, big.” At this point the adult's toy animal would push the objects toward the child and say, “Can your animal do that too?” The children met criterion for the 15 three-object games when they could line up the three objects in the problems on the first try for 3 days in a row. This exercise was then repeated using 20 sets of four object problems.

After the children mastered seriating four object problems, they were repeated. However this time only three of the objects were given to the children and they were instructed to line them up by size. The adult held back one of the mid-sized objects and after a child had lined up the first three objects the adult would then give the child the fourth and asked “Where does this one go?” After these four-object insertion problems were mastered, problems containing five objects were introduced. The child was always given four of the objects and the adult held back one object. The one held back was always one that would go in the interior of the series. After the child had lined up four objects, he or she was given the fifth object and was asked where it went. If a child had incorrectly lined up the first four objects, then the adult would use his toy animal to nudge the objects into the correct order before giving the child the fifth one. When criterion was reached on the five-object problems, the adult followed up with problems containing 6, 7, and 8 objects, following the protocol described previously.

The number conservation games initially had two rows of three equally spaced items. The adult's animal would explain, “This is your row and this is my row” and point to the rows respectively. Then the animal would ask the child, “Do we have the same amount of __ (e.g., stars, coins, hearts), do you have more or do I have more?” This first question was important because the child had to agree that each row did in fact have the same number of items in it in order to move on to the next part of the problem. At this point, the adult would expand one row, contract the other row, and ask the child, “Do we still have the same number of hearts in our rows or are there more in your row now or more in my row now?” Once a child mastered this version of the number conservation problem, more items were added to the rows. The problems became more complex when the rows were unequal to begin with and the addition or subtraction of items from a row might or might not make them equal. Additionally, in the later problems items might be subtracted from one row while objects were added to the other row.

The literacy group worked with foam letters to both recognize and identify all of the letters in the alphabet. Much the same way toy animals were used with the experimental group, the adults used toy animals to create games when working with the children in the literacy group. Children with minimal knowledge of the alphabet would be shown only three or four letters at a time. The adult's animal would tell the child that he was very hungry but he only want to eat the letter B (or x, p, c, etc.). If the child selected the correct letter from the pile, then the animal ate it quickly and thanked the child for feeding him. If the child offered the animal the incorrect letter, the animal would decline to eat it and ask the child to try again. As the child learned more letters, the selection of letters he had available to choose from increased. A second type of game involved the animal showing the child different letters and having the child name those letters. When a child incorrectly named a letter, the adult used his toy animal to show the child the correct choice, had him trace the letter with his finger, and paid special attention to how the letter sounded. The child was also encouraged to repeat that sound. In order to make this instruction more interesting for the children, letter bags were assembled and used during instruction. Letter bags contained small items or pictures with the same beginning sound, e.g. bee, book, and ball. The adults would show the child one letter bag at a time and ask the child to name each item as it was withdrawn from the bag. If the child did not know what it was called, the adult told him, emphasizing the beginning letter sound. The items from two bags were mixed together and the child would help his animal separate the items into their appropriate piles. The letter bags also included letter blends, e.g., bl, br, pr, pl, and the same procedure. Two short rhyming games completed the instruction for this group.

The numeracy group received instruction in number recognition and identification. Foam numbers and toy animals were used in these games. Hungry toy animals would request to be fed specific numbers and would eat only the correct choice and help steer a child in the direction of the correct number if needed. To begin with, the children were asked to feed the animal numbers 0–10. In addition, the children were asked to name specific numbers. Once a child could recognize as well as identify the numbers 0–10, the

Table 1
Means (and *SD*) for each group on six dependent variables.

	Group							
	Cognitive		Numeracy		Literacy		Art	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Oddity	10.74	1.62	9.89	2.07	9.53	2.25	9.54	2.42
Insertions	8.80	1.78	7.08	2.71	6.47	3.14	6.92	3.03
Conservation	7.32	2.57	4.48	3.60	5.03	3.50	5.09	3.41
O–LSAT	7.18	2.37	6.49	2.64	6.57	2.67	6.60	2.65
Numeracy (WJ)	19.07	3.73	18.94	3.55	17.60	3.94	17.71	3.52
Literacy (z)	.23	.83	− .20	1.08	.09	1.02	− .11	.93

adult would pick a number and ask the child to help him count out the appropriate number of pennies, stars, hearts, etc. from a pile of items with one or two extra items. Instruction progressed with numbers greater than 10 as the children progressed. To avoid having the activities become mundane, number bingo was played periodically. Bingo cards had the numbers through 5, 10, 15, or 20 on them, and children played with only the bingo cards containing the numbers they already knew.

When the children had learned the numbers up to 30, small blocks or stars in lines of five were used as a visual aid to teach the children to count by fives. The adult would put 5 blocks in a row and ask how many blocks there were. Then the adult would place another row of 5 blocks below the first one so that they lined up and ask how many there were. Once children understood that there were 10 blocks there, they were encouraged to point to the first row and say "5" and then at the second row and say "10." Finally, the children were shown numbers and asked to pick a number that was higher than or less than it. These numeracy activities were derived from instructional methods used to meet state standards.

The children in the art control group each had their own folders containing activities that went along with the art curriculum being used in the kindergarten classrooms. Some of the activities had the children cutting out pictures and arranging them to make a story. Other activities required that the children color in pictures or draw pictures and talk about what they drew or colored. There were also activities that enabled the children to cut out pictures and shapes and paste them on another sheet of paper.

When a child in the experimental group met criterion on oddity, insertion, and number conservation, instruction was terminated for that child and the yoked children in each of the three control groups. Because the children were yoked in this manner, the average number of sessions was the same across the groups, even though the children progressed at different rates — the instruction for some quartets ended in late December and for others in late March. In late May and early June testers who were blind to which type of instruction each child had received administered post tests to assess them on insertion, oddity, conservation, the oddity scale from the O–LSAT, the W–J III Applied Problems (numeracy) scale, and one of the literacy scales (PALS, SESAT, or K-TEA).

3. Results

It required an average of 5.69 ($SD = 3.14$) sessions for the children to learn form oddity, 6.86 ($SD = 2.06$) to learn size oddity, 8.22 ($SD = 3.19$) to learn orientation oddity, 14.28 ($SD = 4.81$) to learn insertions, and 8.65 ($SD = 4.34$) to learn conservation, for an overall average of 43.70 ($SD = 10.52$) sessions.

Complete year-end data were obtained from 339 children; means and standard deviations for all measures are given in Table 1. Scores on the literacy scales were converted to z scores to make them comparable. Note that small differences in z scores may represent large differences in raw scores and that the standard deviations for z scores should be approximately 1.00. In all statistical comparisons, alpha was set at .05.

Table 2
Summary of multivariate analysis of overall differences between groups on six variables.

Source	R^2	*df*	Mean square	*F*	*p*
Between participants					
Oddity	0.52	3	27.12	6.11	.0001
Insertion	0.10	3	91.00	12.39	.0001
Conservation	0.10	3	136.98	12.63	.0001
O-LSAT	0.01	3	8.83	1.32	> .05
Numeracy	0.03	3	53.69	3.94	.009
Literacy (z)	0.03	3	3.39	3.36	.019
Within participants					
Oddity		334	4.44		
Insertion		334	7.35		
Conservation		334	10.85		
O–LSAT		334	6.69		
Numeracy (WJ)		334	13.64		
Literacy (z)		334	.95		

Table 3
Post hoc comparisons of groups on each dependent variable.

Variable	Group	Contrast	Mean difference	*SE*	*p*	*d*
Oddity	Cognitive	Literacy	1.20	.32	.001	.62
		Numeracy	.84	.32	.009	.46
		Art	1.19	.33	.001	.57
	Literacy	Numeracy	− .36	.32	> .05	
		Art	− .02	.33	> .05	
	Numeracy	Art	.34	.33	> .05	
Insertion	Cognitive	Literacy	2.34	.41	.001	.97
		Numeracy	1.72	.41	.001	.76
		Art	1.88	.42	.001	.93
	Literacy	Numeracy	− .62	.41	> .05	
		Art	− .46	.42	> .05	
	Numeracy	Art	.16	.42	> .05	
Conservation	Cognitive	Literacy	2.29	.50	.001	.76
		Numeracy	2.85	.50	.001	.92
		Art	2.26	.51	.001	.76
	Literacy	Numeracy	.56	.50	> .05	
		Art	− .03	.51	> .05	
	Numeracy	Art	.59	.52	> .05	
O–LSAT	Cognitive	Literacy	.62	.39	> .05	
		Numeracy	.70	.40	.045	.28
		Art	.59	.40	> .05	
	Literacy	Numeracy	.08	.39	> .05	
		Art	− .03	.40	> .05	
	Numeracy	Art	− .11	.4	> .05	
Numeracy	Cognitive	Literacy	1.47	.56	.009	.37
		Numeracy	.13	.56	> .05	
	Literacy	Numeracy	− 1.39	.56	.018	.36
		Art	− .06	.57	> .05	
	Numeracy	Art	1.28	.58	.027	.34
Literacy	Cognitive	Literacy	.14	.15	> .05	
		Numeracy	.43	.15	.004	.45
		Art	.33	.15	.031	.38
	Literacy	Numeracy	.30	.15	.047	.28
		Art	.19	.15	> .05	
	Numeracy	Art	− .10	.15	> .05	

A multivariate analysis of the effects of the type of instruction on the six dependent variables showed that *overall* group differences were significant for all variables except the O–LSAT classification scale (see Table 2). However, an ultraconservative (Scheffé) post hoc analysis showed that the cognitive group was superior to all three control groups combined, $S(3, 334) = 3.45$, $p < .005$ on this O–LSAT measure.

To test the experimental hypotheses directly, each of the four groups was compared with each of the other groups on each dependent variable with conservative least significant differences post hoc contrasts (see Table 3). The cognitive group was superior to each of the other groups on the oddity, seriation, and conservation tests. Effect sizes for oddity were medium, those for insertion were large, and those for conservation ranged from medium to large (Cohen, 1992). Although the objects used for these tests were novel, they were similar in format to those used in the instruction. Hence, the learning-set instruction was effective in producing sufficient mastery of the concepts taught to produce generalization to similar types of problems.

Generalization outside the parameters of the instruction was more limited, as would be expected, and the size of effects was correspondingly smaller. The differences on the O–LSAT oddity measure consistently favored the cognitive group over the control groups. Only the contrast with the numeracy group was statistically significant when comparisons were made group by group, but effect sizes are similar for all of the control groups, ranging from .24 to .28. These are, in Cohen's (1992) words, not so small as to be trivial and indicate that there was some generalization to the worksheet presentations common to classrooms and to test items that predict academic performance in kindergarten. The differences between the O–LSAT alternatives are very variable and are in internal details of graphic or pictorial representations, rather than in overall size, shape, or orientation of concrete objects, and often require hierarchical classification.

Differences on the academic measures were roughly similar, with effect sizes generally in the upper .30 s (see Table 3). The cognitive group was superior to the numeracy and art groups in literacy, indicating that the instruction on the abstractions involved in oddity, insertions, and number conservation improved literacy. The literacy scores of the cognitive and literacy groups were about the same, indicating roughly equal outcomes from the cognitive instruction or instruction directly on literacy.

The cognitive group was superior to the literacy and art groups in numeracy. This shows that the cognitive instruction improved numeracy. The numeracy scores of the cognitive and numeracy groups were equivalent. Hence, the cognitive instruction and instruction directly on numeracy both had roughly similar effects on the children's understanding of numbers and number relationships.

The numeracy group was superior to the literacy and art groups on numeracy, indicating that the numeracy instruction was effective. The literacy group made higher scores on the literacy measure than the numeracy or art groups. The literacy group's superiority over the numeracy group was statistically significant, but the difference with the art group was not. In aggregate, these results indicate that the literacy instruction was effective, but that its effect was limited. It appears that children who are weak in abstractive abilities have substantial difficulty in profiting from literacy instruction.

4. Discussion

These children showed the decalage in applying the oddity principle to different dimension of matter that has been observed during instruction in the course of previous research, most recently that of Pasnak et al. (2007). Shape oddity problems are easiest, size oddity next, and orientation oddity next, with further decalage within the last dimension. Vertical–horizontal orientation oddity problems are not as difficult as orientation problems involving slants, even though the disparity in orientation is 90° for both types of orientation problems. Left–right orientation problems are even more difficult. Significant amounts of training on each type of problem were required before the children could generalize the oddity principle from shape to size to orientation or from one type of orientation problem to another, despite supportive and explanatory feedback. The children resembled Zelazo and Frye's (1998) preschoolers, who could not apply the rule they had learned for one dimension to another. This is not what would be predicted from relational complexity theory (Halford, Andrews, Dalton, Boag, & Zielinski, 2002). Shape and size (and also color) oddity problems should be equally difficult in that theory and were treated as such in the oddity problems Chalmers and Halford (2003) posed for children. However, learning derived from reinforced experience did not produce much generalization to new dimensions. This answers in the negative the question Zelazo and Muller (2002, p. 462) posed for relational complexity theory.

Whether this was due to a failure to inhibit unsuccessful response biases (Dempster, 1992; Harnischfeger & Bjorklund, 1993) or to the difficulty predicted from cognitive complexity and control theory (Frye, Zelazo, & Palfai, 1995; Zelazo & Frye, 1997) in developing overarching abstractions (Jacques et al., 1999), remains to be determined. During shape oddity instruction, inhibiting responses based on color was important because the children showed a strong bias to base choices on color. This bias was especially strong when a color that had been coincidentally associated with a correct choice on a previous problem was associated with a wrong alternative in a new problem. For example, if children who had learned to choose a red triangular paper clip instead of a blue, yellow, or green oval paper clip were presented with white, black, and red rectangular beads and a purple spiral bead, they would tend to choose the red rectangular bead. This is not surprising, but should not have occurred if the children were simply choosing the object which was most different (Chalmers & Halford, 2003). When there is a preference for responding to a certain perceptual characteristic or dimension, whether the preference is natural or due to a history of reinforcement, such responses have to be inhibited, and learning-sets are effective in developing such inhibition (Harlow, 1959). When there is no such preference to begin with, the tendency Chalmers and Halford (2003) found most prevalent for their participants — selecting the object which differs on the most dimensions — is more likely to emerge.

It is likely that both response inhibition and formation of higher order rules were involved and both are accomplished by learning-sets. Gagné (1968) and Gagné and Paradise (1961) offered a theoretical account of how the construction of superordinate choice rules are developed via learning-sets, in the context of conservation, and Lopata and Pasnak (1976) provided a practical example of how it can be done.

It is noteworthy that although the control groups were making nearly twice as many errors as the cognitive group on the oddity test in May, the absolute scores for all groups were high. This reflects the natural development of the oddity concept, unaided by interventions, which usually occurs by age five or sooner. The advantage of the cognitive group is as much that they learned it sooner as that it was more fully developed. Test scores on insertions show more of a gap because this abstraction develops somewhat later. Nearly all children could seriate three objects, but forming an ordered line of four objects was more difficult, primarily because many children lacked any method for detecting errors and had no method for correcting them. Such a deficit in the evaluation phase of executive function (Zelazo, Carter, Resnick, & Frye, 1997) is not unusual for young children who attempt to order objects on one dimension (Southard & Pasnak, 1997; Young, 1976).

When children had an idea for correcting errors, it was usually to put one object at the end of the line. Such a correction, observed previously by Leiser and Gillieron (1990), Southard and Pasnak (1997), and Young (1976) is an extension of the method of extremum observed by Inhelder and Piaget (1964/1959) and seldom produced a correct series. This tendency was almost universal when children were asked to insert an object into a series of four, even after they could readily form a 4-object series if all four objects were available from the start. Inserting a new object into a series of three was the most difficult part of the instruction for most children. Once this had been mastered, inserting a fifth object into a series of four was still challenging. But when children had mastered insertions into 4-object series, inserting objects into longer series was not especially difficult. They had mainly to learn to align the bases of objects correctly and to be careful in comparisons — performance issues rather than conceptual ones.

Learning to insert objects into a series did not seem to be much affected by perceptual biases, in contrast to learning the oddity principle, even when objects were different colors or shapes. This was presumably because choices always involved size rather than the irrelevant dimensions. There was also little difficulty with competing response tendencies, except for the tendency to place new objects at the end rather than the interior of a series. The problem was primarily inability to abstract the relations necessary to form a series of four or more objects, to insert an object into one, and to remedy mistakes. Selection between rules (Chen & Siegler, 2000; Zelazo & Muller, 2002) was not the primary issue in mastering the concept of insertion.

Conserving number was by far the most difficult part of the instruction for the children in the cognitive group. The first challenges for the children were to overcome their tendency to rely on perceptual cues — the longest row looks to both children and adults as if it contains more objects — and to grasp the idea that the number of objects changed only if one or more objects were added or subtracted. Conservation makes many demands on children. Overcoming these challenges was accomplished by consistently extinguishing perceptually based choices as the children moved through the conservation problems rewarding attention to and choices based on the addition or subtraction of objects. However, ignoring the deceptive perceptual cues of the number conservation problems and attending to whether an addition or subtraction had occurred was not enough to produce correct solutions of the more advanced problems. The children were required to learn that enough objects must be added or subtracted to either reverse or equalize the disparity between the number of objects in the rows. If one row had 11 objects and the other 9, subtracting an object from the first row left it still having the largest number of objects. Any problem of this variety could only be solved by ignoring the deceptive appearances that resulted from spreading out or pushing together the objects, remembering not just whether the rows initially had equal numbers of objects or which had more, but also how great any disparity was, and whether any subsequent additions or subtractions were to the right row and in the right amount to change number. These demands enlarged their concept of number conservation far beyond that which would have been sufficient to earn them the label "conservers" in nearly all research or instructional paradigms. The children not only had to develop increasingly accurate rules for the determination of number (Siegler, 1996); they also had to improve their executive function until they consistently selected the right rule to solve each type of problem (Zelazo & Frye, 1998; Zelazo & Muller, 2002).

The consequences of this learning-set instruction on conservation, oddity, and insertions into series are relatively close to those predicted from earlier research. The children developed these abstractions and the subordinate abilities involved in them better than the control groups and consequently became better than the control groups in the domain or domains, literacy or numeracy, in which the control children did not receive experimental instruction. This was not because the control instruction was ineffective. The numeracy control group was superior in numeracy to the other control groups, indicating that the control numeracy instruction was helpful to the children. Although the literacy control group was not significantly better than the art group in literacy, the difference was in the right direction, and it *was* better than the numeracy group, which demonstrates that the control literacy instruction did have an effect. The limited sizes of the effects of the control instruction probably derive from two sources. First, it may have been difficult for the control children to comprehend the control instruction because they lacked the thinking abilities normal for their ages. This was a premise of the research. If the abstractions involved in identifying which item in a collection differs from the others, and understanding the relations between a series of items, and what changed the relations between the numbers of items in two collections were not relevant to early literacy and numeracy, there was little reason to expect the cognitive instruction to be effective. However, there is a great deal of research that indicates that these thinking abilities are involved in the development of literacy and numeracy (Freyberg, 1966; Kingma & Koops, 1983; Pasnak et al., 1991; Silliphant, 1983). The precise nature of the relations between these aspects of cognitive development and response to instruction in numeracy and literacy remain to be elucidated, but the current research confirms earlier research in demonstrating that relations do exist. Consequently, the instruction on literacy and numeracy in the control sessions was an uphill battle, with the instructors hampered by the control children's weakness in some of the cognitive ability needed to make large advances in literacy and numeracy.

The second reason is that a large part of the kindergarten day is spent on literacy and numeracy. Because of the school district emphasis on passing state-mandated assessments, teachers often sacrificed other subject matter, when necessary, to give adequate time to literacy and numeracy lessons. The effect of the thrice weekly 15-minute literacy or numeracy instruction in the control sessions was probably dwarfed by the ample literacy and numeracy instruction all children received from their classroom teacher and teaching assistant. Therefore, one limitation of this study was the absence of a control group that received no treatment. This limits the ability to draw conclusions about the effects of being engaged in an additional thrice weekly 15-minute instructional activity regardless of its nature.

It is important in this context to keep in mind that the literacy and numeracy gains of the children given cognitive instruction depended on this delivery of conventional classroom literacy and numeracy instruction by their teachers. The improved relational thinking they developed was fruitful only because it could be applied to lesson plans developed for children thinking at the level of abstraction employed by the great majority of children in these classrooms (963/1343) scored too well on the initial screening to be included in the research). Without such regular classroom literacy and numeracy instruction, the children in the cognitive group would have been "ready" in the Piagetian sense to profit from classroom instruction, but would have had no opportunity to do so. Likewise, it is important that their new thinking abilities were developed by midyear. Pasnak (1987) found that beginning instruction in midyear produced a high level of mastery of oddity, insertion, and conservation at year's end, but no literacy or numeracy gains, presumably because the improvements in relational and abstract thought came too late to be applied to classroom lessons.

There is no reason to think that the cognitive gains produced by the learning-set instruction will be temporary. These are natural thinking abilities, which all children eventually develop of their own accord, and they will be constantly reinforced by interaction with a child's environment. The academic advantages to the children probably depend on how long these simple reasoning abilities are relevant to the children's school performance. A limited follow-up study of kindergartners taught oddity, insertions, and conservation indicated that measurable superiority over a passive control group was evident at the end of first grade (Pasnak, Madden, et al., 1996). Eventually, however, higher order thinking abilities must become determinants of how well children fare in the elementary school environment.

The results of this study suggest that teaching kindergarten children these higher order thinking abilities has an effect not only on the development of early abstractions, but also on literacy and numeracy achievement. A larger scale follow-up study that compares children who were engaged in the experimental treatment with those who were not would provide further insights into the long-term effects of teaching kindergarten children early abstractions. Noting the effectiveness of the intervention with kindergartners, it would also be important to determine whether providing instruction on oddity, insertion into series, and number conservation to three- and four-year-old children would have a similar effect on children's development of early abstractions and their achievement on literacy and numeracy measures.

5. Summary and conclusions

This attempt to apply laboratory findings in the field produced three main findings. First, a method requiring little training or skill and no unusual instructional materials can be successful in teaching the oddity principle, insertions into series, and number conservation to low functioning children in the classroom environment. Second, improvement in both numeracy and literacy results from mastery of these concepts. They match, in each domain, those resulting from equal investment of resources in state of the art instruction in that domain alone. The effectiveness of the cognitive intervention in producing gains in both domains makes it of practical utility to educators. One caveat, based on earlier research, is that such results depend on achieving mastery of the concepts taught significantly before the end of the school year. A second is that the effectiveness of the intervention depends on a good match between the cognitive intervention and the children at whom it is directed. Previous research indicates that this type of intervention is not effective if the children are functioning at too low a level when the intervention is attempted (Pasnak, Hansbarger et al., 1996) or too high a level (Waiss & Pasnak, 1993). Within these constraints, the present project demonstrates that cognitive developmentalists can improve academic achievement on a relatively large scale in ordinary classroom settings with an intervention that requires only a few months and does not depend upon much of an investment in materials or instructor training.

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References

Baldwin, D. A. (1989). Priorities in children's expectations about object label reference: Form over color. *Child Development*, *60*, 1289–1306.
Capron, C., & Duyme, M. (1989). Assessment of the effects of socio–economic status on IQ in a full cross-fostering study. *Nature*, *340*, 552–554.
Case, R. (1985). *Intellectual development: Birth to adulthood.* New York: Academic Press.
Chalmers, K. A., & Halford, G. S. (2003). Young children's understanding of oddity: Reducing complexity by simple oddity and "most different" strategies. *Cognitive Development*, *18*, 1–23.
Chen, Z., & Siegler, R. S. (2000). Across the great divide: Bridging the gap between understanding the thought of toddlers and older children. *Monographs of the Society for Research in Child Development*, *65*(2) Serial No. 261.
Ciancio, D. S., Rojas, A. C., McMahon, K., & Pasnak, R. (2001). Teaching oddity and insertion to Head Start children: An economical cognitive intervention. *Journal of Applied Developmental Psychology*, *22*, 603–621.
Cohen, J. (1992). A power primer. *Psychological Bulletin*, *112*, 155–159.
Dempster, F. N. (1992). The rise and fall of the inhibitory mechanism: Toward a unified theory of cognitive development and aging. *Developmental Review*, *12*, 45–75.
Flavell, J. H. (1985). *Cognitive development*, (2nd ed.), Englewood Cliffs, NJ: Prentice-Hall.
Freyberg, P. S. (1966). Concept development in Piagetian terms in relation to school attainment. *Journal of Educational Psychology*, *57*, 164–168.
Frye, D., Zelazo, P. D., & Palfai, T. (1995). Theory of mind and rule-based reasoning. *Cognitive Development*, *10*, 483–527.
Gagné, R. M. (1968). Contributions of learning to human development. *Psychological Review*, *75*, 177–191.
Gagné, R. M., & Paradise, N. E. (1961). Abilities and learning-sets in knowledge acquisition. *Psychological Monographs*, *75* (Whole number 518).
Gelman, R. (1969). Conservation acquisition: A problem of learning to attend to relevant attributes. *Experimental Child Psychology*, *7*, 67–87.
Gentner, D. (1978). What looks like a jiggy but acts like a zimbo? A study of early word meaning using artificial objects. *Papers and Reports on Child Language Development*, *15*, 1–6.
Halford, G. S. (1993). *Children's understanding: The development of mental modes.* Hillsdale, NJ: Lawrence Erlbaum.
Halford, G. S., Andrews, G., Dalton, C., Boag, C., & Zielinski, T. (2002). Young children's performance on the balance scale: The influence of relational complexity. *Journal of Experimental Child Psychology*, *81*, 383–416.
Halford, G. S., Wilson, W. H., & Phillips, S. (1997). Abstraction: Nature, costs, and benefits. *International Journal of Educational Research*, *27*, 21–35.
Halford, G. S., Wilson, W. H., & Phillips, S. (2006). *Abstraction: Nature, costs, and benefits.* http://staff.aist.go.jp/steven.phillips/papers/abstraction.pdf
Harlow, H. (1949). The formation of learning-sets. *Psychological Review*, *56*, 51–65.
Harlow, H. (1959). Learning-set and error factor theory. In S. Koch (Ed.), *Psychology: A study of a science*, *Vol. 2* (pp. 28–49). New York: McGraw-Hill.
Harnischfeger, K. K., & Bjorklund, D. F. (1993). The ontogeny of inhibition mechanisms: A renewed approach to cognitive development. In M. L. Howe & R. Pasnak (Eds.), *Emerging themes in cognitive development*, *Vol. 1* (pp. 28–49). New York: Springer-Verlag.
Imai, M., Gentner, D., & Uchida, N. (1994). Children's theories of word meanings. *Cognitive Development*, *9*, 45–75.
Inhelder, B., & Piaget, J., (1964). *The early growth of logic in the child: Classification and seriation*. (E. A. Lunzer and D. Papert, Trans.) London: Routledge and Kegan Paul (Original work published 1959).
Jacques, S., Zelazo, P. D., Kirkham, N. Z., & Semcesen, T. (1999). Rule selection versus rule execution in preschoolers: An error detection approach. *Developmental Psychology*, *35*, 770–780.
Kern, H. J. (1983). The effects of fading, feedback, and special teacher instruction on number conservation tasks with retarded children. *Psychologie in Erziehung und Unterricht*, *30*, 120–127.
Kingma, J., & Koops, W. (1983). Piagetian tasks, traditional intelligence, and achievement tests. *British Journal of Educational Psychology*, *53*, 278–290.
Lebron-Rodriguez, D. E., & Pasnak, R. (1977). Induction of intellectual gains in blind children. *Journal of Experimental Child Psychology*, *24*, 505–515.
Leiser, D., & Gillieron, C. (1990). *Cognitive science and genetic epistemology: A case study of understanding.* New York: Plenum Press.
Lopata, D., & Pasnak, R. (1976). Accelerated conservation acquisition and IQ gains by blind children. *Genetic Psychology Monographs*, *93*, 3–25.
Malabonga, V., Pasnak, R., & Hendricks, C. (1994). Generalization effects from instruction of kindergartners on classification and seriation. *Perceptual and Motor Skills*, *79*, 545–546.
McCall, R. B., Appelbaum, M. I., & Hogarty, P. S. (1973). Developmental changes in mental performance. *Monographs of the Society for Research in Child Development*, *38*(3) Serial No. 150.
McCormick, P., Campbell, J. W., Pasnak, R., & Perry, P. (1990). Instruction on Piagetian concepts for children with mental retardation. *Mental Retardation*, *28*, 359–366.

Moreno, M., & Sastre, G. (1971). Evolution of subjective intellectual deficiencies in a learning operation. *Anuario de Psicologia*, *4*, 71–146.
Namy, L. L., & Gentner, D. (2002). Making a silk purse out of two sow's ears: Young children's use of comparisons in category learning. *Journal of Experimental Psychology: General*, *13*, 5–15.
Pasnak, R. (1987). Acceleration of cognitive development of kindergartners. *Psychology in the Schools*, *24*, 358–363.
Pasnak, R., Brown, K., Kurkjian, M., Mattran, K., Triana, E., & Yamamoto, N. (1986). Cognitive gains through training on classification, seriation, and conservation. *Genetic, Social, and General Psychology Monographs*, *113*, 295–321.
Pasnak, R., Hansbarger, A., Dodson, S., Hart, J. B., & Blaha, J. (1996). Differential results of instruction at the preoperational–concrete operational transition. *Psychology in the Schools*, *33*, 70–83.
Pasnak, R., Kidd, J. K., Ferral-Like, M., Gadzichowski, M. K., Gallington, D. A., & Saracina, R. P. (2007). *Journal of Developmental Processes*, *2*, 90–115.
Pasnak, R., Maccubbin, E. M., Campbell, J. L., & Gadzichowski, M. (2004). Learning set instruction in seriation and the oddity principle for a man with severe mental disabilities. *Education and Training in Developmental Disabilities*, *39*, 337–345.
Pasnak, R., Madden, S. E., Martin, J. W., Holt, R., & Malabonga, V. A. (1996). Persistence of gains after instruction in classification, seriation, and conservation. *Journal of Educational Research*, *89*, 1–6.
Pasnak, R., McCutcheon, L., Campbell, J. W., & Holt, R. (1991). Cognitive and achievement gains for kindergartners instructed on Piagetian operations. *Journal of Educational Research*, *85*, 5–13.
Piaget, J., & Inhelder, B., (1969). The Psychology of the Child. (H. Weaver, Trans.). New York: Basic Books. (Original work published 1966).
Piaget, J. & Inhelder, B., (1973). Memory and Intelligence. (A. J. Pomeranz, Trans.) New York: Basic Books. (Original work published 1968).
Richards, H., & Stone, D. (1970). The learning and transference of the Piagetian concept of conservation. *Mental Retardation*, *8*, 34–37.
Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking.* Oxford: Oxford University Press.
Silliphant, V. (1983). Kindergarten reasoning and achievement in grades K-3. *Psychology in the Schools*, *20*, 289–294.
Soraci, S. A., Deckner, C. W., Baumeister, A. A., Bryant, J. T., McKay, H. A., Stoddard, L. T., et al. (1991). Generalized oddity performance in preschool children: A bimodal training procedure. *Journal of Experimental Child Psychology*, *51*, 280–295.
Southard, M., & Pasnak, R. (1997). Effects of maturation on preoperational seriation. *Child Study Journal*, *27*, 255–268.
Waiss, S., & Pasnak, R. (1993). Instruction of young children on number conservation and unidimensional classification. *Bulletin of the Psychonomic Society*, *31*, 205–208.
Weinberg, R. A., Scarr, S., & Waldeman, I. D. (1992). The Minnesota transracial adoption study: A follow-up of test IQ performance. *Intelligence*, *16*, 117–135.
Young, R. M. (1976). *Seriation by children: An artificial intelligence analysis of a Piagetian task.* Basel, Germany: Birkhauser Verlag.
Zelazo, D. P., Resnick, J. S., & Pinon, D. E. (1995). Response control and the execution of verbal rules. *Developmental Psychology*, *31*, 508–517.
Zelazo, P. D., Carter, A., Resnick, J. S., & Frye, D. (1997). Early development of executive function: A problem-solving framework. *Review of General Psychology*, *1*, 198–226.
Zelazo, P. D., & Frye, D. (1997). Cognitive complexity and control: A theory of the development of deliberate reasoning and intentional action. In M. Stamenov (Ed.), *Language structure, discourse, and the access to consciousness* (pp. 113–153). Amsterdam: Benjamins.
Zelazo, P. D., & Frye, D. (1998). Cognitive complexity and control: II. The development of executive function. *Current Directions in Psychological Science*, *7*, 121–126.
Zelazo, P. D., Frye, D., & Rapus, T. (1996). An age-related dissociation between knowing rules and using them. *Cognitive Development*, *11*, 37–63.
Zelazo, P. D., & Muller, U. (2002). The balance beam in the balance: Reflections on rules, relational complexity, and developmental processes. *Journal of Experimental Child Psychology*, *81*, 458–465.