Early Childhood Education Journal, Vol. 33, No. 6, June 2006 (© 2006)
DOI: 10.1007/s10643-006-0060-8

Early Mathematical Growth: How to Support Young Children's Mathematical Development

Filiz Varol[1,2] and Dale C. Farran[1]

The purpose of this critical analysis was to investigate the young children's sense of numerical magnitudes and the important attributes of classrooms where children in kindergarten and first grade are exposed to mathematics. This study aimed to offer guidelines that will assist teachers as they teach mathematics to children between the ages of five and seven.

KEY WORDS: mathematics; young children; learning environment; mathematical tools; classroom discourse; mathematical tasks; assessment.

INTRODUCTION

According to the National Council of Teachers of Mathematics (NCTM), educators' goals should be to "create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields" (1995, p. 2). Emphasizing the importance of understanding and using mathematics, the NCTM also notes that "in this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures. A lack of mathematical competence keeps those doors closed" (2000, p. 5). Likewise, the National Association for the Education of Young Children (NAEYC) (2002) states that young children need mathematical understanding and skills not only in mathematics classes but also in science, social studies, and other subjects. Therefore, in order to shape their future, young children should be exposed to mathematics in their early lives. In the first section, the present paper aims to focus on how young children develop mathematical understanding. The following section will discuss the factors that influence the development of students' mathematical understanding in school.

MATHEMATICAL DEVELOPMENT IN EARLY YEARS

A growing body of studies has showed that humans have a sense of numerical magnitudes, which depends on a cognitive system that emerges in humans by five years of age (Antell & Keating, 1983; Gallistel & Gelman, 2000; Huntley-Fenner, 2001; Moore, Benenson, Reznick, Peterson, & Kagan, 1987; Sophian & Adams, 1987; Starkey & Cooper, 1980; Starkey, Spelke, & Gelman, 1983, 1990; Strauss & Curtis, 1981; Xu & Spelke, 2000). Specifically, in the first weeks of their life, infants notice the difference between the small numbers (Antell & Keating, 1983). Five-month-olds (Lipton & Spelke, 2003; Starkey & Cooper, 1980; Xu & Spelke, 2000) and 10-month-olds also demonstrate this skill (Strauss & Curtis, 1981). When children are around 2-years-old, they begin to learn counting; however, they do not necessarily understand the cardinal meanings of

[1]Department of Teaching and Learning, Vanderbilt University, Nashville, TN, USA.
[2]Correspondence should be directed to Filiz Varol, Department of Teaching and Learning, Vanderbilt University, Peabody # 330 230 Appleton Place, Nashville, TN, USA; e-mail: filiz.varol@vanderbilt.edu

1082-3301/06/0600-0381/0

numbers until 4 or 5 years of age (Maxim, 1989; Wynn, 1992). Also, Hunting (2003) summarized the studies that investigated young children's problem solving situations involving quantities in the intuitive range that includes the numbers from 1 to 5. Based on his conclusion, children younger than 5 years can apprehend the numerosity of sets of items up to five without counting.

In addition to numbers and basic number operations, children develop other mathematical concepts informally through discovery. For instance, children are naturally interested in the shapes around them (Clements & Sarama, 2005). They observe and talk about the shapes of signs, buildings, and other objects. Also, when children have, for instance, celery for a snack, they may develop an informal measurement system that helps them compare the length of celery pieces (Maxim, 1989). Moreover, playing with blocks is one of the central activities in preschools and at homes. By using different types of blocks, children have opportunities to classify, measure, order, count, and use fractions (Wolfgang, Stannard, & Jones, 2001).

As it can be seen that human beings begin developing mathematical understanding and reasoning in the first few weeks of their lives. However, according to the joint position statement provided by NCTM and NAEYC (2002), 3–6-year-old children need to be provided high quality, challenging, and accessible mathematics education since children's math related experiences during early years influence their later performance in school as well as their life outside school. In order to provide a high quality math education for young children, educators should consider possible factors that may influence children's mathematical growth. The following section will discuss the potential factors and how those factors influence children's mathematical development.

FACTORS THAT INFLUENCE CHILDREN'S MATHEMATICAL GROWTH

Learning Environment

It is important for mathematics teachers to provide a classroom environment that does not limit any child's access to challenging mathematics instruction. There is a growing body of studies that finds that alterable educational variables that are directly related to improved students' outcomes. One of the variables is the quality of the classroom-learning environment (Padrón, 1992; Pierce, 1994; Wang, Haertel, & Walberg, 1993; Waxman & Huang, 1997). According to Chapin and Eastman (1996), there are two types of learning environment characteristics: external characteristics and internal characteristics. By external characteristics of learning environment, the authors mean, for instance, the arrangement of the furniture in the classroom and additional materials to maintain students' interest. Adequate materials in classrooms may have significant effects on students' behavior and academic success. Bennett, Elliot, and Peters (2005) analyzed the characteristics of kindergarten classrooms and its effects on students' behavioral development. The authors found that classroom resources may improve children's social and behavioral health. They also found that the adequacy of classroom supplies corresponds with parental involvement.

Chapin and Eastman (1996) noted that external characteristics of classrooms do not necessarily create effective learning environments that support students' mathematical dispositions. They emphasized the power of internal characteristics of teachers and noted that personal qualities of teachers, including their beliefs, attitudes toward mathematics, attitudes toward students, and his/her knowledge of mathematics, have the power to transform the classroom into a learning environment that advances students' mathematical ability. Studies showed that teachers' self efficacy beliefs are linked to improved students' academic achievement (Goddard, Hoy, & Woolfolk-Hoy, 2000; Rimm-Kaufman & Sawyer, 2004). Emenaker (1996) synthesized the studies that focused on teachers' beliefs and their effects on teaching mathematics. She noted that elementary school teachers' beliefs toward mathematics have an effect on their approaches in teaching mathematics. Therefore, it is important for elementary mathematics teachers to hold positive beliefs in order to teach more effectively.

Mathematical Tools

Historical voices in the field of early childhood education, including Montessori, Plato, Pestalozzi, Froebel, Owen, and Dewey, believed that mathematics should be introduced to young children through objects (Wolfe, 2002). Today, many early educators are in agreement with these theorists. Scholars agreed that effective mathematics instruction in the elementary grades incorporates use of concrete materials (Ball, 1992; Fuson & Briars, 1990; Marsh & Cooke, 1996; Thompson, 1992; Uttal, Scudder, & Deloache, 1997). Concrete materials make learning an engaging and fun activity. Also, they enable students to develop imaginary pictures of numerals in their mind, which leads students to increase their computational fluency in the future (Russell, 2000). The human body is the most natural and accessible tool for young children to develop an understanding of the numeration system (Hunting, 2003; Maxim, 1989). The body can be used to represent cardinal numbers. For instance, children can learn the number two easily because they have two hands, two feet, two ears, and two eyes. In addition to the human body, teachers may use base-10 blocks, Unifix cubes, playing cards, or even real objects, such as cookies and pies. For instance, children may acquire an informal understanding of fractions if they divide a pie for their friends.

However, research on the value of concrete objects has failed to demonstrate a clear, consistent benefit for instruction using manipulatives over more traditional methods. Although the main purpose of using manipulatives is to help children grasp abstract concepts and written mathematical symbols, children may not establish the necessary connections between manipulatives and mathematical expressions (Bruner, 1966; Fuson & Briars, 1990; Hiebert & Carpenter, 1992). A series of studies conducted by Resnick and Omanson (1987) showed that a child who could solve a problem such as $117+62$ with blocks might have difficulty solving written problems, such as $12+14$. Also, Resnick and Omanson pointed out that young children's understanding of mathematics can be quite weak and may be easily lost. In other words, although a child may establish the relationship between the concrete objects and a mathematical expression, he/she may not retrieve the same knowledge on a different day. Manipulatives can be considered as tools that lead children to learn mathematics by playing with them (Ball, 1992); however, as noted above, children may not be able to directly perceive the relation between manipulatives and abstract mathematical symbols. Therefore, it can be concluded that children's understanding of this concept also depends on classroom discourse.

Classroom Discourse

In teaching young children mathematics, it is critical for teachers to understand children's thinking, to compare their strategies, and to resolve their disagreements, which can be done through effective classroom discourse. In traditional classrooms, discourse typically occurs through spoken language. The role of the classroom teacher is to present new information (Sherin, 2002) and the role of the student is to listen to the teacher, to observe, and to evaluate how new knowledge relates to their prior knowledge (Bruner, 1996). On the other hand, in student-centered classrooms the main focus is on teachers and students working cooperatively in order to develop classroom learning environments that support doing and talking about mathematics (Ball, 1991; NCTM, 1991, 2000).

In student-centered classrooms, one strategy that can be useful in helping students learn mathematics is engaging them with interesting classroom discussions. Effective classroom discourse should lead students to share their ideas and solutions about given problems as well as to respond to their classmates' solutions (Sherin, 2002). Also, classroom discourse influences students' reasoning, problem solving competence, self-confidence, and social skills acquisition (Lappan & Schram, 1989). In order to have students share their ideas and actively involve in classroom discussions, classroom teachers should create an environment where students feel secure and comfortable enough to share their beliefs, ask questions, hypothesize, and make mistakes (Ball, 1991; Cobb, Yackel, Wood, & Wheatley, 1988; Vacc, 1993). According to Vacc (1993), classroom teachers may reduce students' anxiety and increase their willingness to participate in classroom discussions by asking questions that have no incorrect answers, arranging seats in a circle so that students can easily see classmates as they speak, letting students discuss their ideas with a partner before sharing them with the whole group, and giving students an opportunity to think about the problem before sharing their thoughts about the problem with their classmates.

Another strategy that teachers can use to increase the effectiveness of classroom discourse is to ask students to work in groups where many different points of view are represented. Marzano, Pickering, and Pollock (2001) pointed out how cooperative grouping strategies can be effective in promoting classroom discourse. Although group discussions may allow students to apply previously learned knowledge to problem-solving situations while receiving feedback from their peers and the classroom teacher (Vacc, 1993), teachers should be careful as they organize the groups. Marzano and his colleagues (2001) mentioned different ways of grouping: according to students' interests, the color that they are wearing, their birthdays, or their ability level. Among the grouping choices, today about three-quarters of the school districts in the United States use ability grouping (Ansalone & Biafora, 2004). However, the National Association of School Psychologists' position statement on ability grouping (2005) demonstrated that the use of ability grouping disproportionately impacts minority students, economically disadvantaged students, and students with lower ability. Therefore, it is important for the classroom teacher to decide how a grouping strategy can be most appropriately handled and how it will support classroom discourse.

In order to provide opportunities for children to engage meaningfully in mathematics, the teachers are responsible for shaping and directing students' discussions. In order to enhance classroom discourse, the teacher should use computers, calculators, and other technology, concrete materials, such as models, pictures, diagrams, tables, and graphs, invented and conventional terms and symbols, metaphors, analogies, stories, written hypotheses, explanations, arguments, oral presentations and dramatizations (NCTM, 1991). However, this does not mean that if the teacher uses the materials given above, he/she will be able to create effective classroom discourse. Therefore, it is crucial for teachers to connect those materials with effective and developmentally appropriate mathematical tasks in which students can engage.

Mathematical Tasks

In order to increase the effectiveness of classroom discourse, teachers should use developmentally appropriate mathematical tasks. Meaningful tasks can increase students' motivation and enable them to think about particular concepts and procedures, their connections with other mathematical ideas, and their applications to real-world contexts (NCTM, 1991). Hiebert and colleagues (1997) described a meaningful task as one that encourages reflection and communication to build mathematical understanding by enabling children to use skills and knowledge they already possess.

Nicol and Crespo (2005) summarized that mathematical tasks can be driven from contexts outside of school and the use of subject areas as context. Nicol and Crespo noted that in order to contextualize mathematics in real life, teachers may focus on how carpenters, candy sellers, or fishermen use mathematics in their lives. As a mathematics teacher, Underwood introduced a sequence of activities about her Aunt Mary's candy making (Whitenack, Knipping, Novinger, & Underwood, 2001). According to Underwood, this activity enabled her to produce numerous activities that advanced her students understanding of two-digit numbers. Therefore, using such scenarios as Aunt Mary's candy may allow children to experience mathematics as a human activity rather than as a static body of knowledge. While engaging children with real life activities, it is important for teachers to encourage students to develop their own methods. For instance, relating patterns in number helps children understand connections among other mathematical topics, such as addition and subtraction (NCTM, 1989). According to Hargreaves, Shorrocks-Taylor, and Threlfall (1998), most children preferred to only look for the differences as they tried to find a number pattern. However, there are other strategies that children may use in order to identify patterns in numbers, such as looking for differences between differences, looking for multiplication tables, and combining terms to make other terms. Therefore, the authors suggested that teachers should provide children with activities that require children to work on sequencing, covering a wide range of structures. Also, they noted that children should be encouraged to develop their own strategies.

Assessment

Teachers spend as much as one-third to one-half of their time on assessment related activities in order to obtain information about students' skills and potential, to improve instruction, and to be able to provide useful feedback for students (Baker & Mayer, 1999; Beevers, Goldfinch, & Pitcher, 2002; Gardner, 1991; NCTM, 1995). Neuman, Copple, & Bredekamp (2000) state the benefits of formative assessment by noting six primary purposes in education:

- Monitor and document children's progress over time
- Ensure that instruction is responsive and appropriately matched to what children are and are not able to do
- Customize instruction to meet individual children's strength and needs
- Enable children to observe their own growth and development
- Identify children who might benefit from more intensive levels of instruction, such as individual tutoring, or other interventions (p. 103)

More specifically, according to the NCTM (1995), while assessment may provide for these functions, it should also provide an opportunity for children "to formulate problems, reason mathematically, make connections among mathematical ideas, and communicate about mathematics' (p. 11).

According to NCTM (2003), in grades K-2, students' progress in mathematics should be assessed in order to understand what they can do or cannot do. In order to get more accurate information about what children's ability, the NCTM provides five mathematics assessment standards that are considered as "learning targets" or "benchmarks for learning" (2003, p. 9) that can be used for monitoring students' progress, making instructional decisions, and evaluating students' achievement at the end of each unit of instruction. These include the learning standard, the equity standard, the openness standard, the inferences standard, and the coherence standard. First, effective assessment, which is considered a part of instruction, should provide opportunities for students to illustrate what they know and can do; thus, assessment's contribution to students' learning of mathematics and its relationship to instruction should be determined carefully in order to enhance learning. Second, assessment should promote equity. Each child has unique qualities and experiences, thus equitable assessments would provide opportunities for children to demonstrate their unique ability. Third, assessment should be an open process. According to the NCTM, before being assessed, students and the public (including parents, policymakers, business and industry leaders, members of the mathematics community and interested citizens) should be informed about the nature of the assessment so that everyone who is impacted by the assessment of students' learning has the opportunity to develop a collective understanding of the performance criteria for mathematics. Fourth, assessment should promote valid inferences about students' mathematics learning. Finally, assessment should be a coherent process, meaning that assessment should match the purposes for which it is being done and should be aligned both with the curriculum and with instruction. The five assessment standards provided by NCTM serve as important criteria that can be used to inform and improve the quality of classroom assessment components.

CONCLUSION

As noted above, studies show that children begin learning mathematics when they are a few weeks old. Although children can develop basic math skills through discovery, it is important to provide children with high quality mathematical education in order to enable them to shape their future. There are several factors that influence the quality of math education. The first factor is the classroom environment where children are exposed to math. Children should feel safe and comfortable enough to reason and communicate mathematically. The second factor is the use of mathematical tools that facilitate children's learning. Concrete materials, such as base-10 blocks, Unifix cubes, playing cards, or even real objects, should be used in order to help students grasp abstract concepts and written mathematical symbols. Another factor is the effectiveness of classroom discourse. Children should be guided as they use the concrete materials so that they understand the relationships between the materials and mathematical concepts. The next factor is the use of mathematical tasks that increase students' motivation and enable them to think about particular concepts and procedures. The final factor is the quality of assessment that enables teachers to obtain information about students' skills and potential, to improve instruction, and to provide useful feedback for students. Although the given factors seemed separate from each other, all of them have one common attribute: the quality of each depends on the classroom teacher. In other words, it is the teacher's role to create a safe learning environment, to select developmentally appropriate tools, to provide effective classroom discourse, to plan worthy tasks for students, and to assess students' thinking in order to make necessary changes in the instruction. In conclusion, all of these factors promote a high degree of mathematical competency for young children.

REFERENCES

Ansalone, G., & Biafora, F. (2004). Elementary school teachers' perceptions and attitudes to the educational structure of tracking. *Education*, *125*(2), 249–259.

Antell, S., & Keating, D. (1983). Perception of numerical invariance in neonates. *Child Development*, *54*, 695–701.

Ball, D. L. (1991). What's all this talk about "discourse"? *The Arithmetic Teacher*, *39*(3), 44–47.

Ball, D. K. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator*, *16*, 14–18.

Baker, E. L., & Mayer, R. E. (1999). Computer-based assessment of problem solving. *Computers in Human Behavior*, *15*(3/4), 269–282.

Beevers, C., Goldfinch, J., & Pitcher, N. (2002). Aspects of computer-based assessment in mathematics. *Active Learning in Higher Education*, *3*(2), 159–176.

Bennett, P., Elliot, M., & Peters, D. (2005). Classroom and family effects on children's social and behavioral problems. *The Elementary School Journal*, *105*(5), 461–499.

Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge, MA: Belknap Press.

Bruner, J. S. (1996). *The culture of education*. Cambridge, MA: Harvard University Press.

Chapin, S. H., & Eastman, K. E. (1996). External and internal characteristics of learning environments. *The Mathematics Teacher*, *89*(2), 112–115.

Clements, D. H., & Sarama, J. (2005). Think math!. *Scholastic Parent and Child*, *13*(2), 25 .

Cobb, P., Yackel, E., Wood, T., & Wheatley, G. (1988). Creating a problem solving atmosphere. *Arithmetic Teacher*, *36*(1), 46–47.

Emenaker, C. (1996). A problem-solving based mathematics course and elementary teachers' beliefs. *School Science and Mathematics*, *96*(2), 75–85.

Fuson, K. S., & Briars, D. J. (1990). Using a base-ten blocks learning/teaching approach for first and second grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematics Education*, *21*, 180–206.

Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences*, *4*, 59–65.

Gardner, H. (1991). Assessment in context: The alternative to standardized testing. In B. Gifford (Ed.), *Changing assessments: Alternative views of aptitude, achievement and instruction* (pp. 239–252). Boston: Kluwer Academic Press.

Goddard, R. D., Hoy, W. K., & Woolfolk-Hoy, A. W. (2000). Collective teacher efficacy: Its meaning, measure, and impact on student achievement. *American Educational Research Journal*, *37*(2), 479–507.

Hargreaves, M., Shorrocks-Taylor, D., & Threlfall, J. (1998). Children's strategies with number patterns. *Educational Studies*, *24*(3), 315–331.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Oliver, A., & Human, P. (1997). The nature of classroom tasks. In (author) M*aking sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). New York: Macmillan.

Hunting, R. P. (2003). Part-whole number knowledge in preschool children. *Journal of Mathematical Behavior*, *22*, 217–235.

Huntley-Fenner, G. (2001). Children's understanding of number is similar to adults' and rats': Numerical estimation by 5- to 7-year-olds. *Cognition*, *78*, B27–B40.

Lappan, G., & Schram, P.W. (1989). Communication and reasoning: Critical dimensions of sense making in mathematics. In P. Trafton, & A. Shulte (Eds.), *New directions for elementary school mathematics* (pp. 14–30). Reston, VA: NCTM.

Lipton, J. S., & Spelke, E. S. (2003). Origins of number sense: Large number discrimination in human infants. *Psychological Science*, *14*, 396–401.

Marsh, L. G., & Cooke, N. L. (1996). The effects of using manipulatives in teaching math problem solving to students with learning disabilities. *Learning Disabilities Research and Practice*, *11*(1), 58–65.

Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001). *Classroom instructions that works: Research-based strategies for increasing student achievement*. Alexandria, VA: Association for Supervision and Curriculum Development.

Maxim, G. W. (1989). Developing preschool mathematical concepts. *The Arithmetic Teacher*, *37*(4), 36–42.

Moore, D., Benenson, J., Reznick, J. S., Peterson, M., & Kagan, J. (1987). Effect of auditory numerical information on infants' looking behavior: Contradictory evidence. *Developmental Psychology*, *23*, 665–670.

National Association for the Education of Young Children and National Council of Teachers of Mathematics. (2002, April). *Early childhood mathematics: Promoting good beginnings*. Joint position statement. Washington, DC: NAEYC.

National Association of School Psychologists. (2005). *Position Statement on Ability Grouping and Tracking*. Retrieved on 12/03/2005 from http://www.nasponline.org/information/pospaper_ag.html.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Neuman, S. B., Copple, C., & Bredekamp, S. (2000). *Learning to read and write: Developmentally appropriate practices for young children*. Washington, DC: National Association for the Education of Young Children[ED463904].

Nicol, C., & Crespo, S. (2005). Exploring mathematics in imaginary places: Rethinking what counts as meaningful contexts for learning mathematics. *School Science and Mathematics*, *105*(5), 240–251.

Padrón, Y. N. (1992). Comparing bilingual and monolingual students' perceptions of their classroom learning environment. In H. C. Waxman, & C. D. Ellett (Eds.), *The study of learning environments* (pp. 108–113). Houston, TX: University of Houston.

Pierce, C. (1994). Importance of classroom climate for at-risk learners. *Journal of Educational Research*, *88*, 37–42.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (pp. 41–96). Hillsdale, NJ: Erlbaum.

Rimm-Kaufman, S. E., & Sawyer, B. E. (2004). Primary-grade teachers self-efficacy beliefs, attitudes toward teaching, and teaching practice priorities in relation to the responsive classroom approach. *The Elementary School Journal*, *104*(4), 321–341.

Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching Children Mathematics*, *7*(3), 154–158.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in a mathematics community. *Journal of Mathematics Teachers Education*, *5*, 205–233.

Sophian, C., & Adams, N. (1987). Infants' understanding of numerical transformations. *British Journal of Developmental Psychology*, *5*, 257–264.

Starkey, P., & Cooper, R. (1980). Perception of numbers by human infants. *Science*, *210*, 1033–1035.

Starkey, P., Spelke, E., & Gelman, R. (1983). Detection of intermodal numerical correspondences by human infants. *Science*, *222*, 179–181.

Starkey, P., Spelke, E., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, *36*, 97–128.

Strauss, M. S., & Curtis, L. E. (1981). Infant perception of numerosity. *Child Development*, *52*, 1146–1152.

Thompson, P. W. (1992). Notations, conventions, and constraints: Contributions to effective uses of concrete materials in elementary mathematics. *Journal for Research in Mathematics Education*, *23*, 123–147.

Uttal, D. H., Scudder, K. V., & Deloache, J. S. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology*, *18*, 37–54.

Vacc, N. N. (1993). Implementing the professional standards for teaching mathematics: teaching and learning mathematics through classroom discussion. *The Arithmetic Teacher*, *41*(4), 225–227.

Wang, M. C., Haertel, G. D., & Walberg, H. J. (1993). Toward a knowledge base for school learning. *Review of Educational Research*, *63*, 249–294.

Waxman, H. C., & Huang, S. L. (1997). Classroom instruction and learning environment differences between effective and ineffective urban elementary schools for African American students. *Urban Education*, *32*(1), 7–44.

Whitenack, J. W., Knipping, N., Novinger, S., & Underwood, G. (2001). Second graders circumvent addition and subtraction difficulties. *Teaching Children Mathematics*, *8*(4), 228 .

Wolfe, J. (2002). *Learning from the past: Historical voices in early childhood education* (2nd ed.). Alberta, CA: Mayerthorpe, Piney Branch Press.

Wolfgang, C. H., Stannard, L. L., & Jones, I. (2001). Block play performance among preschoolers as a predictor of later school achievement in mathematics. *Journal of Research in Childhood Education*, *15*(2), 173–180.

Wynn, K. (1992). Addition and subtraction by human infants. *Nature*, *358*, 749–750.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*, B1–B11.