

Early Number Skills Gains and Mathematics Achievement: Intervening to Establish Successful Early Mathematics Trajectories

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Lina Shanley, PhD¹, Ben Clarke, PhD¹, Christian T. Doabler, PhD²,
Evangeline Kurtz-Nelson, BA¹, and Hank Fien, PhD¹

Abstract

Early number skills, comprised of both informal and formal skills, are associated with later mathematics achievement. Thus, the development of foundational early number skills is an important aspect of early mathematics instruction. This study explored relations between early number skills gains and mathematics achievement for students at risk for mathematics difficulties in a kindergarten intervention study. Results indicated strong relationships between formal number skills gains and mathematics achievement across kindergarten and Grade 1. Intervention participants demonstrated larger informal and formal early number skills gains compared with their control peers, and relations between early number skills gains and first-grade mathematics achievement were moderated by intervention participation. Importantly, these findings suggest that formal and informal number skills gains may be critical components of later mathematics achievement for at-risk students. Implications for special education and intervention in tiered instructional systems are discussed.

Keywords

mathematics, intervention, early number skills, kindergarten

Early elementary mathematics achievement is an important prerequisite for future math learning (Bodovski & Farkas, 2007; Duncan et al., 2007; Morgan, Farkas, & Wu, 2009), and elementary mathematics concepts are essential building blocks for future academic attainment (Claessens & Engel, 2013). Kindergarten is an especially critical time for establishing strong conceptual foundations and setting students up for success (Duncan et al., 2007; Kurdek & Sinclair, 2001; Schulting, Malone, & Dodge, 2005). Because early number skills, which are often introduced prior to school entry, have been linked to future mathematics achievement (Aunola, Leskinen, Lerkkanen, & Jari-Erik, 2004), and students enter kindergarten with a range of preschool experiences and widely variable academic skills, a focus on early number skills in kindergarten is crucial to closing achievement gaps. Thus, various mathematics interventions have been developed to support early mathematics skill development (Clarke et al., 2014; Dyson, Jordan, & Glutting, 2013; Fuchs et al., 2005; Gersten et al., 2015).

Although many of these interventions show promise for improving mathematics achievement for at-risk students, there are some students who do not respond to these targeted interventions (Fuchs & Vaughn, 2012) and more still who do not maintain their gains in future years (Starkey &

Klein, 2008). Improved screening and increased efforts to provide differentiated and targeted instruction based on each student's individual learning needs are required to improve educational outcomes for students who are likely to fail to respond to otherwise efficacious interventions (Miller, Vaughn, & Freund, 2014). Efforts to improve response and intensify interventions for students with significant learning needs can be informed by improving our understanding of how gains in early number skills, both informal and formal, are related to concurrent and future mathematics achievement. Such investigations into these prerequisite early number skills may provide guidance to support the development of increasingly efficacious Tier 2 (i.e., targeted, small group) and Tier 3 (i.e., intensive, individualized) interventions to ensure that all students acquire the foundational whole number mathematics concepts and

¹University of Oregon, Eugene, USA

²The University of Texas at Austin, USA

Corresponding Author:

Lina Shanley, Center on Teaching and Learning, University of Oregon,
1600 Millrace Dr., Suite 207, Eugene, OR 97403, USA.
E-mail: shanley2@uoregon.edu

understanding necessary for academic success in future years.

Early number skills can be separated into two general domains: informal and formal (Purpura, Baroody, & Lonigan, 2013; Skwarchuk, Sowinski, & LeFevre, 2014). Although perhaps sometimes considered skills that are simply gleaned from everyday experiences and not formally taught (Mazzocco & Thompson, 2005), there is a growing body of literature addressing the development and even teaching of informal skills (Park, Bermudez, Roberts, & Brannon, 2016; Ramani & Siegler, 2015). Thus, in the current study, informal early number skills are defined as those that are devoid of numerals, but include relational phrases, counting processes, quantity comparisons, and number words. In contrast, formal early number skills utilize numerals to describe, compare, and combine quantities.

Formal early number skills are highly correlated with mathematics outcomes (Schneider et al., 2017). For example, numeral knowledge (i.e., number identification and number sequencing) has been shown to mediate the relationship between informal and formal mathematics knowledge for preschool and primary-aged children (Purpura et al., 2013) and is also a strong predictor of achievement in Grades K–6 (Göbel, Watson, Lervåg, & Hulme, 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012). Likewise, relationships between formal early number skills and mathematics achievement are well established (Kolkman, Kroesbergen, & Leseman, 2013; Sasanguie, Defever, Maertens, & Reynvoet, 2014; Toll, Van Viersen, Kroesbergen, & Van Luit, 2015), and students with mathematics learning disabilities tend to struggle with numeral-based tasks (De Smedt & Gilmore, 2011; Rousselle & Noël, 2007). In practice, early numeracy screeners measuring knowledge of numeral based quantities and number combinations are commonly used to quickly determine student mathematics proficiency and have demonstrated strong, relatively consistent relationships with future mathematics performance (i.e., correlations ranging from .50 to .70; Gersten et al., 2012).

Explorations of early number skills have also revealed the importance of informal early number skills including verbal number knowledge and counting using number words as an essential aspect of preschool mathematics achievement (Chu, vanMarle, & Geary, 2015). Tasks that require the manipulation of exact quantities using verbal representations (LeFevre et al., 2010; Noël & Rousselle, 2011) and verbal number-based quantity comparison skills (Desoete, Ceulemans, De Weerd, & Pieters, 2012) are also critical to early numeracy development. Verbal number estimation and relational skills may serve as important intermediaries between informal and formal demonstrations of number knowledge (Aunio & Niemivirta, 2010; Libertus, Odic, Feigenson, & Halberda, 2016), and verbal number

skills support mapping or translating between nonsymbolic (i.e., arrays) and symbolic (i.e., both number words and digits) representations of numbers (Benoit, Lehalle, Molina, Tijus, & Jouen, 2013). These mapping skills may be particularly important for at-risk students. Recent research suggests that mapping between number words and approximate number systems (i.e., nonsymbolic number skills) may be a key aspect of mathematics development (Libertus et al., 2016; Mundy & Gilmore, 2009), and students with math learning disabilities often struggle with tasks that require mapping between various number representations (Mejias, Mussolin, Rousselle, Grégoire, & Noël, 2012).

Given the critical role early number skills play in mathematics development, this study sought to (a) measure gains in early number skills and (b) explore relationships between gains in early number skills and global mathematics achievement for students identified as at risk for mathematics difficulties in kindergarten. Because this study was conducted within the context of an intervention study, we also investigated (a) how gains in early number skills differed for students who participated in a targeted whole number intervention and (b) the extent to which relationships between early number skills and mathematics achievement differed based on intervention participation. These study aims were addressed by the following research questions:

Research Question 1: To what extent do kindergarten students who participate in an evidence-based whole number intervention demonstrate differential gains on early number skills?

Research Question 2: How are kindergarten gains in early number skills related to kindergarten and first-grade gains on summative mathematics achievement measures for at-risk students?

Research Question 3: How do the relationships between early number skill gains and mathematics achievement differ for intervention and control participants?

It was hypothesized that kindergarten gains in both informal and formal early number skills would be associated with summative measures of mathematics achievement. Given previously documented relationships among informal early number skills, formal early number skills, and later mathematics achievement (Kolkman et al., 2013; Libertus et al., 2016), it was hypothesized that gains in early number skills would be related to first-grade mathematics achievement. In addition, based on the importance of teacher-directed mathematics instruction in the primary grades (de Haan, Elbers, & Leseman, 2014; Morgan, Farkas, & Maczuga, 2015) and because the intervention studied here is intensive and supports the development of informal number skills through the use of verbal- and nonnumeral-based representations and models, it was hypothesized that intervention participants

would demonstrate greater gains in both informal and formal early number skills. Last, it was hypothesized that intervention participants would demonstrate differential associations between early number skills gains and general mathematics achievement, based on previous research findings suggesting that relations between approximate number skills, mathematics language, and mathematics performance in preschool differed based on developmental abilities (Purpura & Logan, 2015).

Method

Data analyzed here represent two cohorts of kindergarten students who participated in the first 2 years of a 4-year efficacy trial of the ROOTS intervention funded by the Institute of Education Sciences. Prior to identifying potential research participants, human subject approval was obtained from Research Compliance Services in accordance with institutional guidelines.

Setting and Participants

This study took place in 73 kindergarten classrooms in two regions of the United States. Thirty-seven classrooms were from four school districts in the Pacific Northwest. Of the Pacific Northwest districts, one school district was in a metropolitan area, whereas the remaining three districts were in suburban and rural areas. Across the four districts, student enrollment ranged from 2,736 to 38,557 students. Within the participating schools, which ranged from satisfactory to outstanding (Oregon Department of Education, 2016), 8% to 23% of the students received special education services, 5% to 68% were English language learners, and 17% to 86% were eligible for free or reduced lunch. The remaining 36 classrooms were in two school districts in the metropolitan Boston area. District enrollment ranged from 6,118 to 6,843 students, and both districts were classified as Level 3 districts indicating that one or more schools were underperforming relative to other schools statewide (Massachusetts Department of Elementary and Secondary Education, 2016). Within the participating schools, of which two of nine were Title 1 eligible, 8% to 23% of the students received special education services, <1% to 27% of students were English language learners, and 21% to 86% of students received free or reduced lunch.

In all, 1,728 kindergarten students were screened in late fall of 2012 and 2013 to determine eligibility for the intervention study. In all, 609 students met the inclusion criteria and were assigned to an intervention or control condition. In the first 2 years of the ROOTS study between 2012 and 2014, 27 students moved or left the study, so the final analytic sample for the current study included 582 students. See Table 1 for sample demographics.

Table 1. Descriptive Statistics for Student Characteristics by Condition.

| Student characteristic | ROOTS | Control |
|--|-----------|-----------|
| Age at pretest, <i>M</i> (<i>SD</i>) | 5.2 (0.4) | 5.2 (0.4) |
| Male | 45% | 49% |
| Race | | |
| American Indian/Alaskan Native | 2% | 2% |
| Asian | 2% | 3% |
| Black | 5% | 5% |
| Native Hawaiian/Pacific Islander | 0% | 0% |
| White | 76% | 72% |
| More than one race | 3% | 4% |
| Unknown | 13% | 13% |
| Hispanic | 43% | 42% |
| Limited English proficiency | 28% | 30% |
| SPED eligible | 10% | 11% |

Note. The sample included 410 students in the ROOTS condition, and 172 in the control condition. Due to district restrictions, free and reduced lunch eligibility and other socioeconomic status indicators were not collected for individual student participants. SPED = special education.

Screening Procedures

A multistep process was utilized to identify students who were at risk for mathematics difficulties. First, all kindergarten students in participating classes with parental consent were screened in late fall of their kindergarten school year. Screening measures included two standardized assessments of early mathematics: Assessing Student Proficiency in Early Number Sense (ASPENS; Clarke, Gersten, Dimino, & Rolhus, 2011) and a research version of the now published Number Sense Screener (NSS; Jordan, Glutting, & Dyson, 2012) titled the Number Sense Brief (NSB; Jordan, Glutting, & Ramineni, 2008). Both measures demonstrated adequate reliability and validity properties based on published materials. Test-retest reliabilities of kindergarten ASPENS measures are in the moderate to high range (.74-.85), and predictive validity of kindergarten ASPENS measures with the TerraNova 3 ranges from .45 to .52 (Clarke et al., 2011). Similarly, NSB authors report an alpha of .84 at the beginning of first grade.

Students were selected for participation if they (a) scored 20 or less (out of a total 33 points) on the NSB, a score that indicates potential long-term mathematics risk for an entering kindergarten student (Jordan et al., 2008), and (b) had a composite score on the ASPENS that placed in the strategic or intensive range, which indicates that a student is “unlikely to reach grade-level performance without receiving some targeted additional instructional support” (Clarke et al., 2011, p. 32). Then, prior to random assignment, students’ ASPENS and NSB scores were converted to standard scores and combined to form a composite standard score. Across

the screened classes, the number of eligible students ranged from 0 to 27. In classes with more than 10 eligible students, the students' composite standard scores were rank ordered and the 10 students with the lowest scores were selected. In cases where there were fewer than 10 eligible students, classes were combined within schools to achieve the necessary samples for random assignment. These procedures resulted in 62 randomly assigned classes across the two cohorts.

Control condition. Core or Tier 1 mathematics instruction delivered in the kindergarten classrooms served as the control condition. All students received core mathematics instruction, and the ROOTS intervention occurred outside of and in addition to core mathematics instruction. To document instructional practices and mathematics content taught in the control condition, research staff administered two surveys and conducted direct observations of core mathematics instruction. The first survey collected a range of teacher demographic information including ethnicity, age, gender, teaching experience, education, and areas of specialization. The second survey examined characteristics of kindergarten classrooms including class size, number of students at risk for mathematics difficulty, and the amount/type of mathematics instruction provided. Information collected in the second survey revealed that classroom teachers used a variety of published (e.g., *Everyday Mathematics*, *Bridges in Mathematics*, *Investigations*) and teacher-developed mathematics programs that varied within and across participating schools.

Core mathematics instruction was delivered approximately 30 min per day, 4 to 5 days per week with instruction occurring through a variety of mediums, including learning centers, small-group activities, and whole-class instruction. During observation periods, some teachers focused on whole number concepts, and others focused on geometry and measurement; however, surveys revealed that Operations and Algebraic Thinking and Geometry were the primary mathematics domains of the Common Core State Standards for Mathematics (CCSS-M) targeted during core instruction. Instructionally, some classroom teachers employed explicit instructional practices, such as teacher modeling, structured practice opportunities for students, and corrective feedback.

ROOTS Intervention

ROOTS is a 50-lesson, Tier 2 kindergarten intervention program designed to build students' proficiency with critical concepts and skills of whole number. In the current study, the intervention was delivered in 20-min, small-group sessions, 5 days per week for approximately 10 weeks. The intervention was scheduled for delivery at a time that did not conflict with students' core mathematics and reading instruction. The

primary aim of ROOTS is to support students who struggle with mathematics in developing conceptual understanding of and procedural fluency with whole number concepts and skills identified in the Common Core State Standards-Mathematics (CCSS-M; Council of Chief State School Officers, 2010). Specifically, the intervention prioritizes topics from two kindergarten domains in the CCSS-M: (a) Counting and Cardinality and (b) Operations and Algebraic Thinking. The intense focus on whole numbers aligns with calls from mathematicians and expert panels to support all students, particularly at-risk learners, in developing robust and lasting number sense (Frye et al., 2013; Gersten et al., 2009; National Mathematics Advisory Panel, 2008).

The ROOTS curriculum aims to strategically link the informal mathematical knowledge students typically acquire prior to school entry with the formal mathematical knowledge developed in their kindergarten year. Thus, explicit instructional design and delivery principles that have been empirically validated to accelerate the mathematics learning of at-risk learners (Baker, Gersten, & Lee, 2002; Clarke et al., 2014; Doabler, Strand Cary et al., 2012; Gersten et al., 2009) are central to the ROOTS intervention. Specifically, ROOTS includes scripted guidelines for interventionists to (a) overtly model and explain what students will learn, (b) provide scaffolded practice opportunities to promote high rates of learning success and eventual learner independence, (c) incorporate visual representations of mathematics to deepen students' conceptual understanding, and (d) deliver timely academic feedback to address student misconceptions and knowledge gaps. Importantly, the ROOTS intervention also emphasizes mathematical discourse or student mathematics verbalizations (Doabler et al., 2015). ROOTS facilitates structured opportunities for struggling learners to verbalize their mathematical thinking and discuss their solution methods for solving whole number problems.

Fidelity of implementation. Research staff were trained to conduct observations of intervention groups to assess fidelity of implementation. Once they established interrater agreement equal to .85 or higher with an expert rater, these trained research staff observed each ROOTS group 3 times across the intervention time period. Observations were used to gauge implementation fidelity or the extent to which interventionists implemented the intervention as intended. The fidelity measure focused on four features of implementation adherence, including the extent to which ROOTS interventionists (a) delivered the prescribed number of activities in the observed lesson, (b) met the observed lesson's instructional objectives, (c) followed the teacher scripting, and (d) used the prescribed mathematics models. The first implementation feature was measured by comparing the number of activities specified in the ROOTS lesson materials with the number of activities taught in the

observed lesson. On average, interventionists taught 4.2 out of 5 ($SD = 0.5$) of the prescribed activities. The final three features were measured using a 4-point scale (4 = *all*, 3 = *most*, 2 = *some*, and 1 = *none*). Interventionists also met target implementation criteria on these three features of implementation adherence with average fidelity measure ratings ranging from 3.3 to 3.6 ($SD = 0.5$ – 0.6 ; Clarke et al., 2016; Doabler et al., 2016).

Intervention effects. Impact analyses conducted using data from the two cohorts included in the present study revealed positive intervention effects for ROOTS participants when compared with their control peers on proximal and distal measures of mathematics achievement. Cohort 1 participants demonstrated gains on the *ROOTS Assessment of Early Numeracy Skills* (RAENS; $g = .75, p < .01$), the Test of Early Mathematics Achievement standard scores ($g = .25, p < .01$), and the ASPENS curriculum-based measures ($g = .55, p < .01$; Clarke et al., 2016). These findings were replicated and extended in Cohort 2 with ROOTS participants demonstrating gains on these three measures ($g = 1.08, p < .01$; $g = .31, p < .01$; and $g = .64, p < .01$, respectively), in addition to the NSB ($g = .40, p < .01$) and standard scores on the kindergarten version of the *Stanford Achievement Test* ($g = .24, p < .05$; Doabler et al., 2016). Measure descriptions are provided below.

Measures

Student-level mathematics achievement data were collected during the kindergarten year at the intervention's pretest (i.e., midwinter) and posttest (i.e., late spring), and at a follow-up approximately 6 months into the students' first-grade year.

RAENS. RAENS (Doabler, Clarke, & Fien, 2012) is an individually administered assessment consisting of 32 items. Items assess aspects of counting and cardinality, number operations, and the base-10 system. In an untimed setting, students are asked to count and compare groups of objects; write, order, and compare numbers and quantities; label visual number models (e.g., 10-frames); and write and solve single-digit addition expressions and equations. RAENS's predictive validity ranges from .68 to .83 with measures of mathematics achievement including the *Test of Early Mathematics Ability—Third Edition* (TEMA-3) and the NSB. Interrater scoring agreement was reported at 100% (Clarke et al., 2014), and internal consistency was high (Cronbach's $\alpha = .91$; Clarke et al., 2016). Informal number skills items ($n = 7$) were devoid of written numerals, but included number words or relational terms, such as *make a group of items larger than a given group* or *use colored counters to make a group of five*. Formal number skills ($n = 25$) items were those involving numerals, such as *circle the number that is*

less or *identify the missing number*. The RAENS was administered at pretest and posttest time periods to assess key features of early numeracy.

TEMA-3. The TEMA-3 (Ginsburg & Baroody, 2003) was administered at pretest and posttest. TEMA-3 is a standardized, norm-referenced, individually administered measure of beginning mathematical ability. The TEMA-3 assesses mathematical understanding for children ranging in age from 3 to 8 years 11 months. The TEMA-3 addresses children's conceptual and procedural understanding of math, including counting and basic calculations. The TEMA-3 manual reports alternate-form and test-retest reliabilities of .97 and .82 to .93, respectively. For concurrent validity with other math outcome measures, the TEMA-3 manual reports coefficients ranging from .54 to .91. Due to publisher permissions, only total raw scores were collected for the current study sample; thus, sample-specific reliability analyses were not conducted.

Stanford Achievement Test—10th Edition (SAT-10). The SAT-10 (Pearson Education, 2008) measure is a group-administered, standardized, norm-referenced test with two mathematics subtests, Problem Solving and Procedures. The SAT-10 is a standardized achievement test with adequate and well-reported validity ($r = .67$) and reliability (Cronbach's $\alpha = .93$). Reliability for the current study sample was consistent with publisher results (Cronbach's $\alpha = .92$). The SAT-10 was administered as a follow-up assessment midway through the first grade. SAT-10 data were collected for 457 of the participating students. SAT-10 total raw scores were used in these analyses.

Data Analysis

Preliminary descriptive analyses were conducted using SPSS 20.0 for Mac OS (IBM, 2011), and all subsequent models were investigated using the weighted least squares means and variance (WLSMV) estimator for categorical variables in Mplus 7.3 (Muthén & Muthén, 2013). Model building processes and assessments of model fit were conducted using best practice recommendations and common rules of thumb (Kline, 2011).

First, using a two-factor model of pre- and posttest RAENS items, latent gain score factors were modeled to capture the change in informal and formal number skills from pretest to posttest. Then, the informal and formal number skills gain factors were regressed on ROOTS condition to examine the extent to which latent gains differed for intervention participants, as specified in the first research question. Next, measures of kindergarten and first-grade mathematics achievement (i.e., TEMA-3 and SAT-10) were regressed on the informal and formal gain score factors to investigate how gains in early number skills were related to

Table 2. Descriptive Statistics and Correlations for Mathematics Measures ($n = 583$).

| Variable | 1 | 2 | 3 | 4 | <i>M</i> (<i>SD</i>) |
|-------------------------|-----|-----|-----|-----|------------------------|
| 1. RAENS pretest total | | | | | 12.98 (6.31) |
| 2. RAENS posttest total | .59 | | | | 22.09 (6.69) |
| 3. TEMA-3 pretest | .77 | .61 | | | 17.36 (6.84) |
| 4. TEMA-3 posttest | .67 | .75 | .75 | | 25.68 (7.85) |
| 5. SAT-10 raw score | .51 | .45 | .49 | .51 | 32.34 (10.99) |

Note. Correlations calculated using pairwise deletion. All correlations are statistically significant at $p < .001$. RAENS = *ROOTS Assessment of Early Numeracy Skills*; TEMA-3 = *Test of Early Mathematics Ability—Third Edition*; SAT-10 = *Stanford Achievement Test—10th Edition*.

mathematics achievement and gains on summative mathematics measures. Finally, group difference tests were conducted to examine how relationships between the early number skills latent gain factors and mathematics achievement differed based on intervention status. Because standard chi-square difference tests cannot be conducted based on the nonnormal distribution—adjusted chi-square values and degrees of freedom generated by the WLSMV estimator in Mplus 7.3, the DIFFTEST procedure was used to examine group differences (Muthén & Muthén, 2013).

Missing data. As noted above, 4% of the sample ($n = 27$) left the study before posttesting was complete. Because prior analyses of study attrition (Clarke et al., 2016; Doabler et al., 2016) revealed no statistically significant relationships between study completion and any other analytic variables (i.e., pretest and posttest measures), cases without at least one posttest measure were omitted from these analyses. All other randomly missing data (i.e., missing data resulting from a participant being unavailable at the time of testing) were accounted for by the maximum likelihood estimation methods employed in the analyses, which employ the expected information matrix by default (Muthén & Muthén, 2013).

Results

All measures met the assumptions for the analyses conducted, and there were moderate to strong correlations among all mathematics measures as indicated in Table 2. After establishing the adequacy and fit of the pre-post two-factor latent gain model representing gain in informal and formal early number skills from pretest to posttest where RAENS Items 1 to 7 loaded on the informal factor and Items 8 to 32 loaded on the formal factor— $\chi^2(1482) = 3,207.96, p < .001$; root mean square error of approximation (RMSEA) = .04; comparative fit index (CFI) = .90—intervention condition (i.e., ROOTS participation) was entered as a predictor of informal and formal number skills gains. The model demonstrated adequate fit— $\chi^2(1536) = 2,817.01,$

$p < .001$; RMSEA = .04; CFI = .90—and indicated that ROOTS participation was associated with statistically significantly larger gains on both informal and formal early number skills. See Figure 1 for an illustration of this structural model with key parameter values.

Next, associations between gains in informal and formal early number skills and mathematics achievement at the end of kindergarten were examined by regressing TEMA-3 posttest scores on the informal and formal early number skills gain scores and TEMA-3 pretest scores to evaluate the relationship between early number skills gains and concurrent mathematics achievement gains in kindergarten. This model demonstrated adequate fit, $\chi^2(1592) = 4,243.15, p < .001$; RMSEA = .05; CFI = .55, with the predictors explaining nearly 72% of the variance in TEMA-3 posttest scores. Controlling from pretest scores, informal skill latent gains were not statistically significantly associated with TEMA-3 posttest scores; however, formal early number skill gains were uniquely related to TEMA-3 posttest scores, $\beta = .37, SE = 0.04, p < .001$. Finally, SAT-10 total raw scores were regressed on the early number skills latent gain factors and TEMA-3 pretest scores to evaluate associations between early number skill gains and first-grade math achievement controlling for kindergarten mathematics achievement. This model also demonstrated adequate fit— $\chi^2(1592) = 4,253.64, p < .001$; RMSEA = .05; CFI = .51—and explained approximately 32% of the variance in SAT-10 scores. Again, only formal early number skills gains were associated with SAT-10 scores, $\beta = .17, SE = 0.07, p < .05$.

Finally, model invariance group difference tests were conducted to examine differences in relationships between early number skills gains and overall mathematics achievement. For each mathematics achievement indicator, a fully constrained model where the predictive paths between the latent gain factors and the mathematics achievement measure were fixed to be the same for both ROOTS participants and control students was compared with a model where the same paths were freely estimated for each sample. Whereas DIFFTEST results revealed no statistically significant differences between the fixed and free models for the TEMA-3, $\chi^2(2) = 1.60, p = .45$, there were statistically significant differences in the SAT-10 model, $\chi^2(2) = 6.34, p < .05$. Based on this statistically significant finding, follow-up DIFFTEST analyses were conducted to examine the extent to which each path (i.e., formal and informal) differed. These analyses revealed statistically significant differences in the relationships between first-grade mathematics achievement and gains on both informal early number skills, $\chi^2(1) = 4.69, p < .05$, and formal early number skills, $\chi^2(1) = 4.30, p < .05$, based on intervention participation. See Figure 2 for an illustration of these models and parameter estimates for each group.

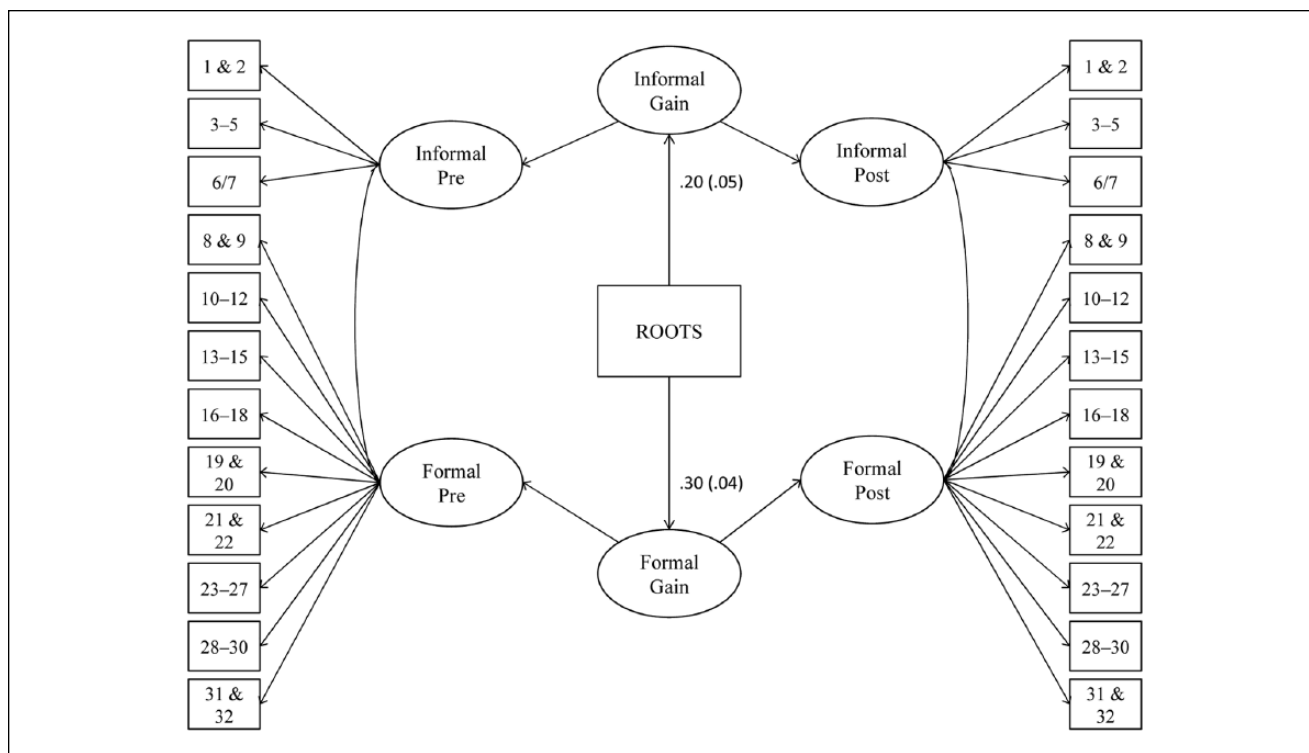


Figure 1. ROOTS intervention participation and gains in early number skills structural model with key standardized parameter estimates.

Note. Boxes on the left and right sides of the figure represent RAENS items administered at pretest and posttest. RAENS = ROOTS Assessment of Early Numeracy Skills.

Discussion

The current study applied structural equation modeling to examine mathematics achievement in a large-scale multi-year, multisite efficacy trial of a kindergarten mathematics intervention. Analyses explored how gains in informal and formal early number skills differed based on intervention condition, were associated with mathematics achievement, and were differentially associated with mathematics achievement for intervention participants. In whole, findings from this study lend support for the continued provision of intensive, instructionally sound mathematics intervention that emphasizes both informal and formal early number skills development to establish strong mathematical foundations and foster continued mathematics achievement for all young learners. Specific findings and implications for special education, early mathematics instruction, and intervention are discussed in the sections that follow.

Intervention and Early Number Skills

Consistent with previous intervention research findings (see Bryant et al., 2008; Clements & Sarama, 2007; Dyson et al., 2013), participation in a whole number, small-group intervention was associated with gains in both informal and

formal early number skills tasks. Small-group instruction that emphasizes student participation and prioritizes the provision of numerous opportunities for students to respond to teacher prompts and verbalize mathematical thinking appears to be linked with improved informal number skills. Similarly, given the explicit focus on building whole number skills within the ROOTS curriculum, it stands to reason that intervention participation was associated with greater gains in formal early number skills, as well. In whole, these findings lend support to the hypothesis that teacher instruction on and student work with early mathematics concepts go hand in hand with early number skills development, and the assertion that intensive, targeted instruction on early numeracy is critically linked to mathematics achievement.

Early Number Skill Gains and Mathematics Achievement

Formal early number skills gains were statistically significantly associated with mathematics achievement gains assessed at the end of kindergarten and at first-grade follow-up. That is, students who demonstrated improved formal early number skills in kindergarten were predicted to perform better on measures of mathematics achievement at

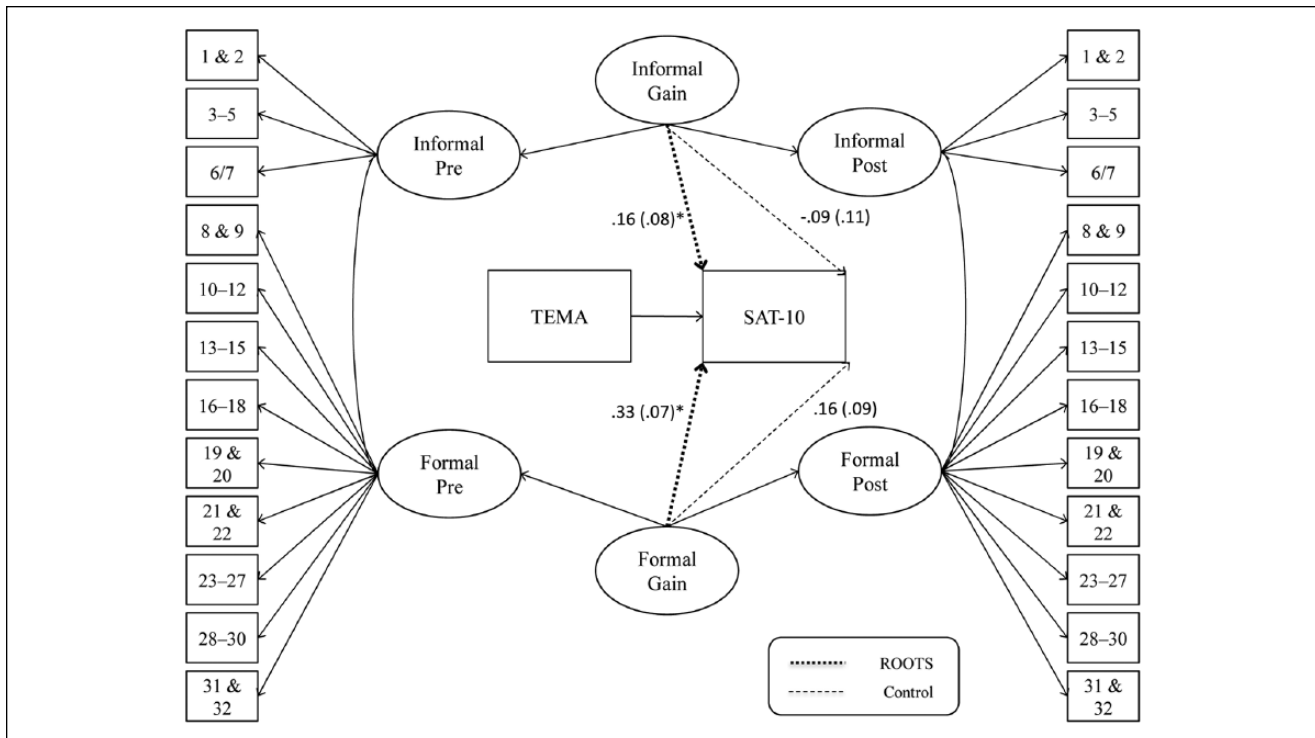


Figure 2. Structural model with key standardized parameter estimates illustrating the moderation of the relations between gains in early number skills and first-grade mathematics by ROOTS intervention participation.

Note. Boxes on the left and right sides of the figure represent RAENS items administered at pretest and posttest. RAENS = *ROOTS Assessment of Early Numeracy Skills*.

the end of kindergarten and again in the first grade. These findings are consistent with prior study findings indicating a consistent relationship between formal early number skills and global mathematics achievement such that students who demonstrate robust formal early number skills also tend to exhibit strong general mathematics skills (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Schneider et al., 2017). The relations present in these analyses reiterate the importance of formal numeracy development in supporting general mathematics achievement (De Smedt, Noël, Gilmore, & Ansari, 2013; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013), and provide support for the continued focus on whole number and formal early number skills intervention in the early grades.

Differential Skill and Achievement Relations

Moderation analyses conducted in the present study revealed differential relations between gains in early number skills and mathematics achievement for intervention participants when compared with their at-risk, control peers. Specifically, both formal and informal early number skills were positively and statistically significantly associated with first-grade mathematics achievement for intervention participants, whereas these relationships were not statistically significant for the at-risk, control sample.

The emergence of an association between informal number skills gains and first-grade mathematics achievement for intervention participants is perhaps most intriguing and bears further exploration. For example, the significant relationship between informal early number skills and summative achievement at first-grade follow-up suggests that at-risk students who are taught to analyze and generate visual- or quantity-based number depictions and gain facility with the utilization of verbal number representations in early mathematics operations may show greater gains in general mathematics development. Explaining mathematical thinking and reasoning and developing nonnumerical representations are important aspects of demonstrating mastery of early mathematics concepts. Thus, at-risk students could benefit from efforts to boost informal early number skills, especially when combined with formal number skills activities, and this may provide a potential focal area for intervention development and/or intensification.

Limitations and Future Research

The early number skills investigated here were restricted to just some variations of informal and formal early number skills, and all items incorporated some aspect of symbolic (i.e., number names or representations) and relational (i.e., more or less) understanding. Although the research is

somewhat inconclusive, nonsymbolic skills are also potentially important to the development of global mathematics skills (Chen & Li, 2014; Schneider et al., 2017), and gains in nonsymbolic number skills, especially in the context of early mathematics intervention, bear exploration. Whereas previous research indicates that differences in nonsymbolic skills become more pronounced in the latter grades (Noël & Rousselle, 2011), nonsymbolic number models provide the foundation for whole number understandings (Benoit et al., 2013) and may be critical to at-risk kindergarten students who have limited formal mathematics exposure. Adding nonsymbolic assessments to early number skills batteries is recommended to parse out the extent to which nonsymbolic number skills gains, in addition to informal and formal early number skills, may make unique contributions to mathematics achievement.

Although the RAENS factor structure utilized in these analyses demonstrated adequate model fit, the two-factor model is just one of many possible factor structures for this measure. Previous research has suggested the presence of various factor structures for early mathematics including a trifactor model (Purpura & Logan, 2015) and a factor structure that increased from four to six unique early number skills factors across kindergarten (Ryoo et al., 2015). Although outside of the scope of the current work, the types of early number skills tasks included in the RAENS could also be delineated by math task type (i.e., numeral knowledge, number combinations, quantity discrimination) or by the depth of the task (i.e., conceptual vs. procedural or application vs. recall). Future research is warranted to explore potential alternate factor structures, and verify the psychometric properties of RAENS assessment.

Finally, although important learner characteristics and demographics were not included in the analyses conducted here, recent research findings have revealed links between various learner characteristics and the retention of intervention effects in the years that follow (Bailey, Duncan, Odgers, & Yu, 2017). For example, findings from studies of mathematics development among low-income children suggest that students from low-income backgrounds may be less likely to experience adequate growth in early number skills (Jordan et al., 2007), and may benefit from targeted support in certain early number skills domains (Rittle-Johnson, Fyfe, Hofer, & Farran, 2016). Similarly, investigations examining relations between a range of domain general skills such as attention, executive functioning, self-regulation, persistence, and mind-sets have revealed important associations and mediated relations among early mathematics achievement, later mathematics achievement, and demographic characteristics (Blair, Ursache, Greenberg, & Vernon-Feagans, 2015; Claro, Paunesku, & Dweck, 2016; Hassinger-Das, Jordan, Glutting, Irwin, & Dyson, 2014; Verdine, Irwin, Golinkoff, & Hirsh-Paske, 2014). Future investigations are needed to explore potential differences in

early number skills gains and the moderating effects of important malleable and fixed learner characteristics.

Conclusion

As emphasized in a recently articulated research agenda (Alcock et al., 2016), the continued exploration of links between gains in early number skills and overall mathematics achievement is critical to understanding how best to support the development of early whole number understandings and ensure that all students are set on successful learning trajectories. The current study examined how early number skills gains are related to mathematics achievement and found that intervention participants demonstrated positive, significant relations between early number skills gains and later mathematics achievement. These findings are significant because they provide some evidence of the critical nature of both formal and informal number skills development in supporting later mathematics achievement for at-risk students, and extend intervention effect findings to examine how skills gained via intervention participation are positively associated with mathematics achievement. Knowledge of relations between early number skills and mathematics achievement can improve efforts to provide intervention instruction that is targeted to specific learner needs and support screening efforts to bolster intervention response and promote long-term gains in mathematics achievement.

Possessing a strong foundation of formal early number skills allows students to apply number knowledge to solve mathematical problems, learn basic principles of measurement and geometry, and perform operations in base-10 systems. In turn, strong quantity-based competencies and informal early number skills provide critical contextual information for formal early number skills. If at-risk students are able to visualize and verbalize basic number properties and extrapolate verbal number relational understandings of quantities of 10 or less to larger multi-digit numbers, they may be more likely to experience continued growth and achievement in mathematics. Creating interventions that emphasize the development and monitoring of growth in informal number skills through the explicit incorporation of informal number skills content in early mathematics curricula has the potential to help at-risk students and potential nonresponders bridge gaps in their numerical understandings. Such inclusion may in turn lead to sustained learning gains in mathematics.

Understanding how gains in early number skills parallel mathematics achievement gains within the context of a Tier 2 intervention program can also support efforts to become more sophisticated with screening approaches in the early grades. Measuring these early number skills at kindergarten entry has utility for instructional planning and identifying students who may be at risk for mathematics difficulties and

who may benefit from early, targeted mathematics intervention that is aligned to their learner profile. Given the relation between informal number skills gains and mathematics achievement for intervention participants, monitoring and continuing to support the development of informal number skills as part of formal learning activities may be a key aspect of effective, intensive mathematics intervention in the early grades.

Declaration of Conflicting Interests

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