Chapter 1

Gaussian Beams

In optics, a Gaussian beam is an idealized beam of electromagnetic radiation whose amplitude envelope in the transverse plane is given by a Gaussian function. The electric and magnetic field amplitude profiles along a circular Gaussian beam of a given wavelength and polarization are determined by two parameters: the waist w_0, which is a measure of the width of the beam at its narrowest point, and the position z relative to the waist.

1.1 Modes

The Gaussian beam is a transverse electromagnetic (TEM) mode. Modes in waveguides can be classified as follows:

- **Transverse electromagnetic (TEM) modes:** Neither electric nor magnetic field in the direction of propagation
- **Transverse electric (TE) modes:** No electric field in the direction of propagation. These are sometimes called H modes because there is only a magnetic field along the direction of propagation (H is the conventional symbol for magnetic field)
- **Transverse magnetic (TM) modes:** No magnetic field in the direction of propagation. These are sometimes called E modes because there is only an electric field along the direction of propagation.

In a laser with cylindrical symmetry, the transverse mode patterns are described by a combination of a Gaussian beam profile with a Laguerre polynomial. The modes are denoted TEM_{pl} where p and l are integers labeling the radial and angular mode orders, respectively. With $p = l = 0$, the TEM_{00} mode is the lowest order. It is the fundamental transverse mode of the laser resonator and has the same form as a Gaussian beam.

These higher order modes pop up in cavities, if the beam isn't perfectly aligned (too wide, or lopsided), then we can use these harmonics to describe what we see.

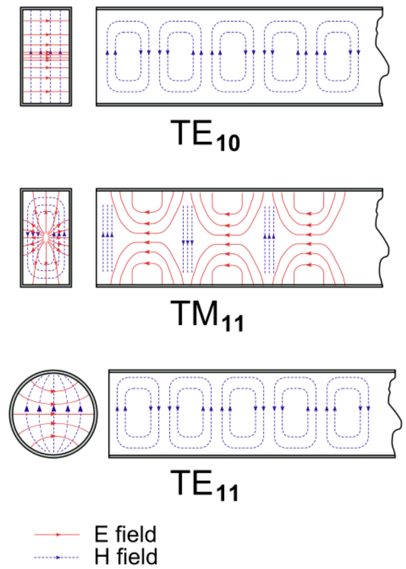

Figure 1.1: Different Mode Examples

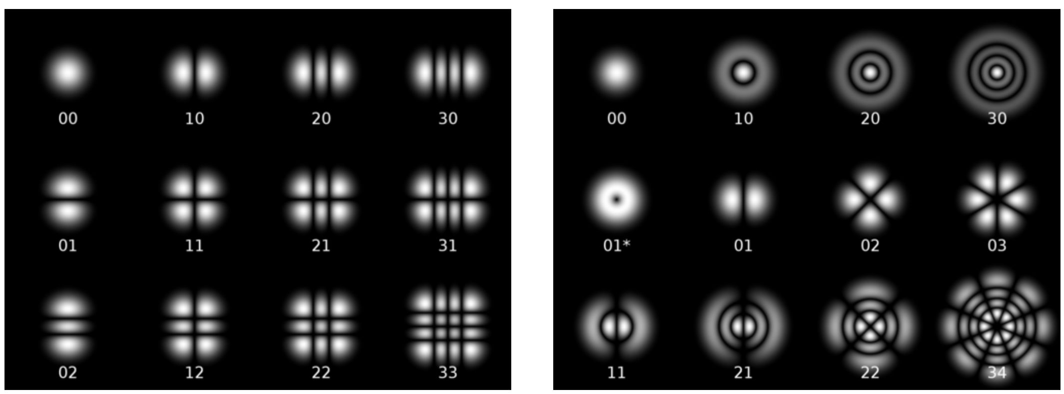

Figure 1.2: Left: Cylindrical transverse mode patterns TEM(pl)
Right: Rectangular transverse mode patterns TEM(mn)

1.2 Beam Characteristics

1.2.1 Waist and Curvature

At a position z along the beam (measured from the focus), the spot size parameter w is given by a hyperbolic relation:

$$w(z) = w_o\sqrt{1 + (\frac{z}{z_R})^2}$$

where $z_R = \frac{\pi w_0 n}{\lambda}$ is called the Rayleigh range, and n is the refractive index of the medium. The radius of the beam w(z), at any position z along the beam, is related to the full width at half maximum (FWHM) of the intensity distribution at that position according to: $w(z) = \frac{FWHM(z)}{\sqrt{2ln2}}$.

The wavefronts have zero curvature (radius $= \infty$) at the waist. Wavefront curvature increases away from the waist, with the maximum rate of change occurring at the Rayleigh distance, $z = \pm z_R$. Beyond the Rayleigh distance, $|z| > z_R$, the curvature again decreases in magnitude, approaching zero as z $\rightarrow \pm\infty$. The distance between the two points $z = \pm z_R$ is called the confocal parameter or depth of focus of the beam. The curvature is often expressed in terms of its reciprocal, R, the radius of curvature:

$$R(z) = z(1 + (\frac{z}{z_R})^2)$$

The radius of curvature reverses sign and is infinite at the beam waist where the curvature goes through zero.

Many laser beams have an elliptical cross-section. Also common are beams with waist positions which are different for the two transverse dimensions, called astigmatic beams. These beams can be dealt with using the above two evolution equations, but with distinct values of each parameter for x and y and distinct definitions of the z = 0 point.

The Gouy phase is a phase shift gradually acquired by a beam around the focal region. At position z the Gouy phase of a fundamental Gaussian beam is given by:

$$\Psi(z) = arctan(\frac{z}{z_R})$$

The Gouy phase results in an increase in the apparent wavelength near the waist $z = 0$.

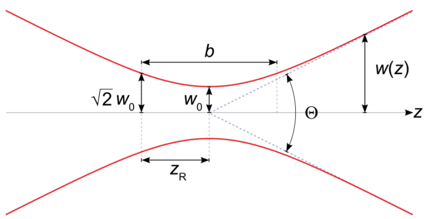

Figure 1.3: Beam Characteristics

1.2.2 Beam Divergence

Although the tails of a Gaussian function never actually reach zero, for the purposes of the following discussion the "edge" of a beam is considered to be the radius where r = w(z). That is where the intensity has dropped to $\frac{1}{e^2}$ of its on-axis value. Now, for $z >> z_R$ the parameter w(z) increases linearly with z. This means that far from the waist, the beam "edge" is cone-shaped.

The angle between that cone (whose r = w(z)) and the beam axis (r = 0) defines the divergence of the beam:

$$\theta = \lim_{z\rightarrow\infty} arctan(\frac{w(z)}{z})$$

.

We use the paraxial approximation. The divergence is inversely proportional to the spot size, for a given wavelength, a Gaussian beam that is focused to a small spot diverges rapidly as it propagates away from the focus. Conversely, to minimize the divergence of a laser beam in the far field (and increase its peak intensity at large distances) it must have a large cross-section at the waist.

One last important quantity is the complex beam parameter: it is a complex number that specifies the properties of a Gaussian beam at a particular point z along the axis of the beam:

$$q(z) = z + \frac{i\pi n w_0^2}{\lambda_0} = z + z_R i$$

where where z is the location, relative to the location of the beam waist, at which q is calculated, and z_R is the Rayleigh range.

Chapter 2

Beam Profiling

2.1 Definitions

Beam profiling means measuring and describing the shape and quality of a laser beam. Lasers aren't perfect pencil-thin lines, they have a spatial distribution of intensity (usually a Gaussian spot), and that spot can change size, symmetry, or alignment as the beam travels. We want to align the beam so we can do things like fiber coupling.

One important parameter is the "waist". A beam waist is the location where a Gaussian laser beam has its minimum radius. At this point the beam is at its narrowest, the wavefront is perfectly flat, and the beam diverges symmetrically before and after this point.

2.1.1 A La Mode

This is a Matlab package for beam profiling. For my purposes I want to understand where to put my first lens after the laser. In this case, the module says $L_1 = 4$ inches.

One note about A la Mode is that the online version of MatLab lacks some functionality that I manually replaced by creating a script in the alm-master file "createCellArrayOfFunctions.m" since the command apparently doesn't work otherwise. Here is the link to the MatLab drive: `https://drive.mathworks.com/sharing/c169d8a7-6cc5-40e0-bfb9-224285ae47ce`.

2.2 Components

There are a lot of acronyms in AMO, here are some of the components and what they mean + what they do:

- **NPRO Laser (Non-Planar Ring Oscillator):** Provides single-frequency, narrow-linewidth laser output
- **Optical Isolator:** Uses Faraday rotation to allow light through in only one direction. In other words it breaks time reversal symmetry since magnetic fields rotate light polarization in the same direction regardless of propagation direction, so we sort of block backwards propagating light.

- **Steering Mirrors:** Provide angular and position control of beam, can be used to decouple components.

- **Mode-Matching Lenses:** Transform beam size and divergence to match fiber acceptance

- **Iris:** Adjustable aperture to mark alignment, so you can check if the beam is centered.

- **Beam Dump:** A little block that absorbs the energy so we don't have any stray beams (oh no).

2.3 Fiber Coupling

Again, the laser has a waist size that expands/diverges as it travels. Meanwhile, the fiber core is tiny, and it only accepts a beam with the right size and divergence. To get high coupling efficiency, we need to mode-match: make the laser beam's spatial mode (waist size and location) match the fiber's mode. We need to align the beam position and angle, so there are many degrees of freedom to be considered.

Thor Labs has a nice guide to this: https://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=3812, where under this page's 'Fiber Coupling" tab, they have an equation that I will paraphrase here.

$$\phi_{spot} = \frac{4\lambda f}{\pi D}$$

Aspheric lenses are commonly chosen to couple incident light with a diameter of 1 - 5 mm into a single mode fiber. We currently have lenses with f 4, 5, 6 and 11 mm. Our laser is 1064 nm, and we are using an FC/PC single mode patch cable. Thus, it has a mode field diameter (MFD) of 6.6 ± 0.5 µm. We can solve the equation for the desired focal length:

$$f = \frac{\pi D \cdot MFD}{4\lambda} = \frac{\pi \cdot D \cdot 6.6 \times 10^{-6} m}{4 \cdot 1064 \times 10^{-9} m}$$

Ok, we still need to know the focal length but we are closer. All we need to figure out is D, the diameter of collimated beam incident on the lens.

Thor labs says "For maximum coupling efficiency into single mode fibers, the light should be an on-axis Gaussian beam with its waist located at the fiber's end face, and the waist diameter should equal the MFD" Thus, $2 \times waist = MFD$.