Gravitational Waves: An Introduction

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Based on Chakrabarty (1999)
https://arxiv.org/pdf/physics/9908041

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Why Study Gravitational Waves?

The Problem

- In Newton's theory: Binary orbital period = constant
- In Einstein's GR: Accelerating masses emit gravitational radiation
- This radiation carries away energy $\Rightarrow$ orbital decay

The Discovery

- Hulse–Taylor binary pulsar (Taylor & Weisberg, 1982)
- Observed orbital period decrease matches GR predictions!
- First indirect evidence for gravitational waves

Key Insight

Gravitational waves are **ripples in spacetime** itself, not waves traveling through space

Gravitational Waves vs Electromagnetic Waves

Electromagnetic Waves:

- Oscillating E and B fields
- Travel through spacetime
- Dipole radiation dominant
- 2 polarization states

Gravitational Waves:

- Oscillating spacetime metric
- Are distortions of spacetime
- Quadrupole radiation dominant
- 2 polarization states

Why Quadrupole?

No gravitational dipole because the center of mass of an isolated system cannot oscillate freely!

The Weak Gravitational Field

Starting Point: Flat Spacetime

In absence of gravity, spacetime is flat with Minkowski metric:

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Weak Field as Perturbation

Add small perturbation $h_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$

Intuition: Like ripples on a pond - the water surface is mostly flat, with small oscillations on top

Raising and Lowering Indices

Key Simplification

In weak field limit, we can raise/lower indices with $\eta^{\mu\nu}$:

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

Why? Corrections would be $\mathcal{O}(h^2)$ - second order in perturbation!

Example

$$\begin{aligned} h_\mu^\nu &= \eta^{\nu\rho} h_{\mu\rho} + \mathcal{O}(h^2) \\ &\approx \eta^{\nu\rho} h_{\mu\rho} \end{aligned}$$

This dramatically simplifies calculations!

Linearized Affine Connection

Full Expression

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\eta^{\lambda\rho}[\partial_{\mu} h_{\rho\nu} + \partial_{\nu} h_{\mu\rho} - \partial_{\rho} h_{\mu\nu}] + \mathcal{O}(h^2)$$

Intuition: The Christoffel symbols tell us how spacetime is curved

- Depend on **derivatives** of the metric
- In flat space: $\Gamma = 0$ everywhere
- With weak field: $\Gamma \sim \partial h$ (first order)

Physical Meaning

$\Gamma^{\lambda}_{\mu\nu}$ tells us how a vector component V^{λ} changes when parallel transported in the μ-ν plane

The Riemann Curvature Tensor

Linearized Form

$$R_{\mu\nu\rho\sigma} = \eta_{\mu\lambda}\partial_\rho\Gamma^\lambda_{\nu\sigma} - \eta_{\mu\lambda}\partial_\sigma\Gamma^\lambda_{\nu\rho}$$

Ricci Tensor (Contract once)

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu} = \frac{1}{2}[\partial_\lambda\partial_\nu h^\lambda_\mu + \partial_\lambda\partial_\mu h^\lambda_\nu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu}]$$

where $\Box = \eta^{\lambda\rho}\partial_\lambda\partial_\rho$ is the flat-space d'Alembertian

Physical meaning: $R_{\mu\nu}$ measures how nearby geodesics converge or diverge

Ricci Scalar and Einstein Tensor

Ricci Scalar (Contract again)

$$R = \eta^{\mu\nu} R_{\mu\nu} = \partial_\lambda \partial_\mu h^{\lambda\mu} - \Box h$$

where $h = \eta^{\mu\nu} h_{\mu\nu}$ is the trace

Einstein Tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} R$$

Why This Combination?

The Einstein tensor $G_{\mu\nu}$ is automatically conserved:

$$\partial_\mu G^{\mu\nu} = 0$$

This matches energy-momentum conservation: $\partial_\mu T^{\mu\nu} = 0$

Linearized Einstein Field Equations

The Equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

where G is Newton's gravitational constant

Expanded Form

$$\frac{1}{2}[\partial_\lambda \partial_\nu h^\lambda_\mu + \partial_\lambda \partial_\mu h^\lambda_\nu - \eta_{\mu\nu}\partial_\mu \partial_\nu h^{\mu\nu} + \eta_{\mu\nu}\Box h - \Box h_{\mu\nu}] = 8\pi G T_{\mu\nu}$$

Problem: These equations don't have unique solutions!
Why? Gauge freedom - we haven't fully specified the coordinate system

What is Gauge Freedom?

Coordinate Transformations

Under small coordinate transformation: $x'^{\mu} = x^{\mu} + \xi^{\mu}(x)$

$$h'_{\mu\nu} = h_{\mu\nu} - \partial_{\mu}\xi_{\nu} - \partial_{\nu}\xi_{\mu}$$

Intuition: Like choosing different coordinate grids on a map

- Physical content doesn't change
- But mathematical description does
- Need to "fix gauge" to get unique solutions

Example

Analogy: In EM, the vector potential A^{μ} has gauge freedom:

$$A'^{\mu} = A^{\mu} + \partial^{\mu}\chi$$

We fix gauge with Lorenz condition: $\partial_{\mu}A^{\mu} = 0$

The Lorenz/Harmonic Gauge

Gauge Condition

$$\partial_\lambda h^\lambda_\mu = \frac{1}{2}\partial_\mu h$$

Why "Harmonic"? In full GR, this is:

$$g^{\mu\nu}\Gamma^\lambda_{\mu\nu} = 0$$

The coordinates x^λ satisfy $\Box x^\lambda = 0$ (harmonic functions!)

Simplified Einstein Equations

With this gauge:

$$\Box h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\Box h = -16\pi G T_{\mu\nu}$$

Trace-Reversed Perturbation

Definition

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

Why do this? The equations become even simpler!

Inverse Relation

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h}$$

where $\bar{h} = \eta^{\mu\nu}\bar{h}_{\mu\nu} = -h$

Simplified Gauge Condition

$$\partial_\mu \bar{h}^{\mu\lambda} = 0$$

Beautiful Einstein Equations

$$\Box\bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

Gravitational Waves in Vacuum

Vacuum Equations

When $T_{\mu\nu} = 0$ (no matter):

$$\Box \bar{h}_{\mu\nu} = 0$$

This is the wave equation!

Plane Wave Solution

$$\bar{h}_{\mu\nu} = A_{\mu\nu} e^{ik_\alpha x^\alpha}$$

where:

- $A_{\mu\nu}$ = constant amplitude tensor
- k_α = wave vector

Intuition: Just like EM plane waves $E = E_0 e^{i(k \cdot x - \omega t)}$

The Dispersion Relation

Plugging into Wave Equation

$$\Box \bar{h}_{\mu\nu} = 0 \quad \Rightarrow \quad k_\alpha k^\alpha = 0$$

Physical Meaning

k^α is a **null vector** - tangent to a light ray!

$$k_\alpha k^\alpha = -\omega^2 + |\mathbf{k}|^2 = 0$$

$$\boxed{\omega^2 = |\mathbf{k}|^2}$$

Conclusion: Gravitational waves travel at the speed of light!

Written as $k_\mu = (\omega, \mathbf{k})$ with $\omega = |\mathbf{k}|$

Constraints from Lorenz Gauge

Gauge Condition

$\partial_\mu \bar{h}^{\mu\lambda} = 0$ for plane waves gives:

$$k_\alpha A^{\alpha\beta} = 0$$

Physical Interpretation: The amplitude is orthogonal (transverse) to the wave vector

- Started with 10 independent components in $A_{\mu\nu}$ (symmetric 4×4)
- This condition removes 4 components
- Left with 6 independent components

Example

For wave traveling in z-direction: $k_\mu = (\omega, 0, 0, \omega)$

$$k_\alpha A^{\alpha\beta} = 0 \quad \Rightarrow \quad A^{0\beta} = A^{3\beta}$$

Further Gauge Freedom

Still Have Freedom!

Even in Lorenz gauge, can still make transformations:

$$x'^{\alpha} = x^{\alpha} + \xi^{\alpha}(x^{\beta})$$

as long as $\Box \xi^{\alpha} = 0$

Choose Special Form

$$\xi^{\alpha} = C^{\alpha} e^{i k_{\beta} x^{\beta}}$$

where C^{α} are constants

Goal: Use this freedom to impose additional conditions and isolate the physical degrees of freedom

The TT Gauge Conditions

Three Conditions

1. **Transverse:** $k_\alpha A^{\alpha\beta} = 0$ (from Lorenz gauge)
2. **Traceless:** $A^\mu_\mu = 0$
3. **Orthogonal to reference 4-velocity:** $A_{\mu\nu} U^\beta = 0$

Consequences:

- Used up all gauge freedom
- Remaining components are physically meaningful
- $\bar{h}^{TT}_{\alpha\beta} = h^{TT}_{\alpha\beta}$ (trace-reversal trivial)

Degrees of Freedom

Started with 10 $\rightarrow$ 6 (Lorenz) $\rightarrow$ 2 (TT gauge)
Only 2 physical polarization states!

Explicit TT Gauge

Choose Reference Frame

Let $U^{\alpha} = (1, 0, 0, 0)$ (rest frame) Wave propagates in z-direction:
$k^{\mu} = (\omega, 0, 0, \omega)$

Conditions Give

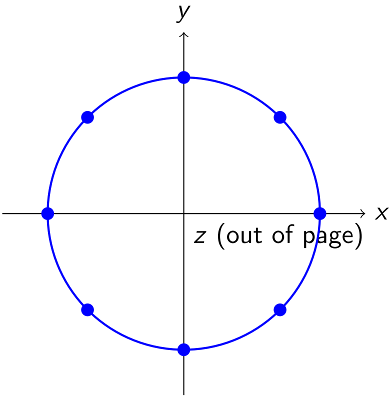
- $A_{\mu\nu} U^{\beta} = 0 \Rightarrow A_{\mu 0} = 0$ for all μ
- $k_{\alpha} A^{\alpha\beta} = 0 \Rightarrow A_{\alpha 3} = 0$ for all α
- $A^{\mu}_{\mu} = 0 \Rightarrow A_{11} + A_{22} = 0$

TT Amplitude Matrix

$$A^{TT}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{+} & A_{\times} & 0 \\ 0 & A_{\times} & -A_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Only 2 free parameters: A_{+} and $A_{\times}$ (the two polarizations!)

Visualizing the TT Gauge



Initial configuration

Setup: Ring of test particles, wave propagating perpendicular to page
Key Point: Only x and y positions change; z and t unaffected in TT gauge

Effect on Test Particles

Geodesic Deviation Equation

For nearby particles with separation ζ^α:

$$\frac{d^2\zeta^\alpha}{d\tau^2} = R^\alpha_{\beta\gamma\delta} U^\beta U^\gamma \zeta^\delta$$

Intuition: Curvature causes geodesics to converge/diverge

In TT Gauge (slow motion limit)

For $U^\nu = (1, 0, 0, 0)$ and particle at rest initially:

$$\frac{\partial^2\zeta^\alpha}{\partial t^2} = -\zeta^\mu R^\alpha_{0\mu 0}$$

Computing the Riemann Tensor

In TT Gauge

$$R^x_{0x0} = -\frac{1}{2} h^{TT}_{xx,00}$$
$$R^y_{0x0} = -\frac{1}{2} h^{TT}_{xy,00}$$
$$R^y_{0y0} = -\frac{1}{2} h^{TT}_{yy,00} = +\frac{1}{2} h^{TT}_{xx,00}$$

All other independent components vanish

Key observation: $R^y_{0y0} = -R^x_{0x0}$ due to tracelessness!
This leads to:

- When x-separation increases, y-separation decreases
- When x-separation decreases, y-separation increases

Equations of Motion

For particle initially at $(\epsilon, 0, 0)$ (along x-axis)

$$\frac{\partial^2 \zeta^x}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2}{\partial t^2} h_{xx}^{TT}, \quad \frac{\partial^2 \zeta^y}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2}{\partial t^2} h_{xy}^{TT}$$

For particle initially at $(0, \epsilon, 0)$ (along y-axis)

$$\frac{\partial^2 \zeta^y}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2}{\partial t^2} h_{yy}^{TT} = -\frac{1}{2}\epsilon \frac{\partial^2}{\partial t^2} h_{xx}^{TT}$$

$$\frac{\partial^2 \zeta^x}{\partial t^2} = \frac{1}{2}\epsilon \frac{\partial^2}{\partial t^2} h_{xy}^{TT}$$

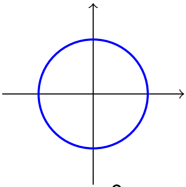
Notice: Both h_{xx} and h_{xy} contribute, but in different ways!

Plus Polarization $(+)$

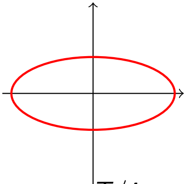
Wave Form

$h^{TT}_{xx} \neq 0$, $h^{TT}_{xy} = 0$

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_+ \cos(\omega t) & 0 & 0 \\ 0 & 0 & -A_+ \cos(\omega t) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$t = 0$



$t = T/4$

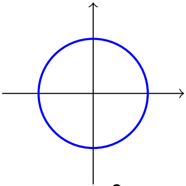
Effect: Stretches along x, compresses along y (and vice versa)

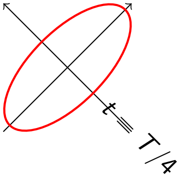
Cross Polarization ($\times$)

Wave Form

$h_{xx}^{TT} = 0, \; h_{xy}^{TT} \neq 0$

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & A_\times \cos(\omega t) & 0 \\ 0 & A_\times \cos(\omega t) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$





Effect: Rotated 45 relative to + polarization

Polarization States Summary

Key Properties

- Both are **transverse** (perpendicular to propagation)
- Both are **quadrupolar** (stretch-squeeze pattern)
- Rotated by 45 with respect to each other
- **Spin-2**: Rotating by 180 gives same state!

Compare to EM Waves

EM (Spin-1)	**GW (Spin-2)**
2 polarizations	2 polarizations
Rotate 180 = flip	Rotate 180 = same
Dipole pattern	Quadrupole pattern
90 between polarizations	45 between polarizations

The Source Problem

Einstein Equations with Matter

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

Goal: Find $\bar{h}_{\mu\nu}$ at position $\mathbf{x}$ and time t given source $T_{\mu\nu}$
Strategy: Use Green's function method

- Same technique as in electromagnetism
- Analogous to retarded potentials
- Accounts for finite speed of propagation

Green's Function Method

Green's Function Definition

$$\Box G(x^\mu - y^\mu) = \delta^{(4)}(x^\mu - y^\mu)$$

where $\delta^{(4)}$ is the 4D Dirac delta function

General Solution

$$\bar{h}_{\mu\nu}(x^\alpha) = -16\pi G \int d^4y \, G(x^\alpha - y^\alpha) T_{\mu\nu}(y^\alpha)$$

Intuition:

- Each point of the source contributes to $\bar{h}$
- G tells us how much each point contributes
- Integrate over all source points

Retarded Green's Function

Explicit Form

$$G(x^\mu - y^\mu) = -\frac{1}{4\pi|\mathbf{x}-\mathbf{y}|}\delta[|\mathbf{x}-\mathbf{y}| - (t-t')]\theta(t-t')$$

where:

- $\mathbf{x} = (x^1, x^2, x^3)$ and $\mathbf{y} = (y^1, y^2, y^3)$
- $t = x^0$ and $t' = y^0$
- θ is the Heaviside step function

Physical meaning:

- Only sources in the **past light cone** contribute
- Delta function: only at $|\mathbf{x}-\mathbf{y}| = t - t'$ (light travel time)
- $1/|\mathbf{x}-\mathbf{y}|$: strength decreases with distance

Integrated Solution

After Integrating over t'

$$\bar{h}_{\mu\nu}(t, \mathbf{x}) = 4G \int d^3y \frac{1}{|\mathbf{x} - \mathbf{y}|} T_{\mu\nu}(t_R, \mathbf{y})$$

where the **retarded time** is:

$$t_R = t - |\mathbf{x} - \mathbf{y}|$$

Crucial Point

The field at $(\mathbf{x}, t)$ depends on the source at earlier time t_R!
Time for signal to travel from $\mathbf{y}$ to $\mathbf{x}$ is $|\mathbf{x} - \mathbf{y}|$

Example: If source is 1 light-year away, we see what it was doing 1 year ago!

Far-Field Approximation

Assumptions

1. Source is **isolated** (localized in space)
2. Source is **far away**: $|\mathbf{x}| \gg |\mathbf{y}|$ for all $\mathbf{y}$ in source
3. Source is **slow** (non-relativistic): $v \ll c$

Simplification

$$|\mathbf{x} - \mathbf{y}| \approx R = |\mathbf{x}|$$

in both denominator and retarded time

Far-Field Solution

$$\bar{h}_{\mu\nu}(\omega, \mathbf{x}) = 4G\,\frac{e^{i\omega R}}{R} \int d^3y\ \tilde{T}_{\mu\nu}(\omega, \mathbf{y})$$

Why Focus on Spatial Components?

Gauge Condition in Fourier Space

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \quad \Rightarrow \quad i\omega \bar{h}^{0\nu} = \partial_i \bar{h}^{i\nu}$$

Implication: If we know spatial components $\bar{h}^{ij}$, we can determine time components $\bar{h}^{0\nu}$

Strategy

1. Calculate $\bar{h}^{ij}$ from T^{ij}
2. Use energy-momentum conservation to relate T^{ij} to T^{00}
3. Express everything in terms of energy density

Why? Energy density T^{00} is easier to work with physically!

Energy-Momentum Conservation

Conservation Law

$$\partial_\mu T^{\mu\nu} = 0$$

Fourier Space

$$\partial_i \tilde{T}^{i\nu} = i\omega \tilde{T}^{0\nu}$$

Key manipulation:

$$\begin{aligned}\int d^3y \, \tilde{T}^{ij} &= -\int d^3y \, y^i (\partial_k \tilde{T}^{kj}) \\ &= i\omega \int d^3y \, y^i \, \tilde{T}^{0j}\end{aligned}$$

Used integration by parts (surface term vanishes for localized source)

From Momentum to Energy Density

Symmetrize

$$\int d^3y\, \tilde{T}^{ij} = \frac{i\omega}{2} \int d^3y (y^i\, \tilde{T}^{0j} + y^j\, \tilde{T}^{0i})$$

Apply Conservation Again

$$\int d^3y (y^i\, \tilde{T}^{0j} + y^j\, \tilde{T}^{0i}) = - \int d^3y\, y^i y^j (\partial_l\, \tilde{T}^{0l})$$

Final Result

$$\int d^3y\, \tilde{T}^{ij} = -\frac{\omega^2}{2} \int d^3y\, y^i y^j\, \tilde{T}^{00}$$

Success! Expressed stress components in terms of energy density!

The Quadrupole Moment

Definition

$$\tilde{q}^{ij}(\omega) = 3 \int d^3y \, y^i y^j \, \tilde{T}^{00}(\omega, \mathbf{y})$$

Why factor of 3? Convention to match multipole expansion

In Terms of Quadrupole Moment

$$\int d^3y \, \tilde{T}^{ij} = -\frac{\omega^2}{6} \tilde{q}^{ij}$$

Physical Interpretation

q^{ij} is the quadrupole moment of the mass distribution

- Measures deviation from spherical symmetry
- Second moment of mass distribution
- Zero for spherically symmetric sources!

Final Result: Quadrupole Formula

Fourier Space

$$\tilde{\bar{h}}_{ij}(\omega, \mathbf{x}) = -\frac{2G\omega^2}{3R} e^{i\omega R} \tilde{q}_{ij}(\omega)$$

Time Domain

$$\bar{h}_{ij}(t, \mathbf{x}) = \frac{2G}{3R} \frac{d^2}{dt^2} q_{ij}(t_R)$$

where $t_R = t - R$ is the retarded time

The Quadrupole Formula

Gravitational wave $\propto$ Second time derivative of quadrupole moment

This is why:

- Static sources don't radiate ($\ddot{q} = 0$)
- Spherical sources don't radiate ($q = 0$)

Why Quadrupole and Not Dipole?

For EM: Dipole Radiation

Dipole moment: $\mathbf{d} = \int \mathbf{r}\rho(\mathbf{r})d^3r$
Can oscillate freely: $\ddot{\mathbf{d}} \neq 0$

For GW: No Dipole!

Gravitational "dipole": $\mathbf{d}_g = \int \mathbf{r}T^{00}(\mathbf{r})d^3r$
But this is just the **center of mass**!
For isolated system: $\frac{d}{dt}\mathbf{d}_g =$ constant (momentum conservation)
Therefore: $\ddot{\mathbf{d}}_g = 0$ always!

Key Insight

The center of mass of an isolated system can't accelerate freely, but the center of charge can!

Physical Examples

Non-radiating Systems

1. **Spherically symmetric collapse:** $q_{ij} \propto \delta_{ij}$ (Birkhoff's theorem)
2. **Rotating sphere:** Axial symmetry $\Rightarrow \ddot{q}_{ij} = 0$
3. **Single particle in orbit:** Center of mass stationary $\Rightarrow \ddot{q}_{ij} = 0$

Radiating Systems

1. **Binary system:** Both position and orientation change $\Rightarrow \ddot{q}_{ij} \neq 0$
2. **Supernova (asymmetric):** Rapid asymmetric collapse $\Rightarrow$ large $\ddot{q}_{ij}$
3. **Rotating neutron star (deformed):** "Mountain" on surface $\Rightarrow \ddot{q}_{ij} \neq 0$

Example: Binary System

Setup

Two masses m_1, m_2 in circular orbit, separation a, angular frequency Ω
Positions: $\mathbf{r}_1 = (a_1 \cos \Omega t, a_1 \sin \Omega t, 0)$ and similar for m_2

Quadrupole Moment Component

$$q_{11} = m_1 r_1^2 \cos^2(\Omega t) + m_2 r_2^2 \cos^2(\Omega t) + \dots$$

Second Derivative

$$\ddot{q}_{11} \propto -\Omega^2 \cos(2\Omega t)$$

Key result: Gravitational wave frequency $= 2\times$ orbital frequency!
Physical reason: Quadrupole pattern repeats twice per orbit

The Complete Picture

What We've Learned

1. Gravitational waves are ripples in spacetime itself
2. They propagate at the speed of light
3. Two polarization states: $+$ and $\times$ (spin-2)
4. Generated by time-varying quadrupole moments
5. Amplitude decreases as $1/R$ from source

Central Equations

Wave equation: $\Box h_{\mu\nu} = -16\pi G T_{\mu\nu}$
Dispersion: $\omega^2 = |\mathbf{k}|^2$ (speed of light)
Quadrupole formula: $h_{ij} \sim \frac{G}{R}\ddot{q}_{ij}(t_R)$

Energy and Power

Energy Loss Rate

For a radiating system:

$$\frac{dE}{dt} = -\frac{G}{5} \left\langle \dddot{q}_{ij} \, \dddot{q}^{ij} \right\rangle$$

where $\langle \cdot \rangle$ denotes time average

Intuition:

- Energy carried away by gravitational waves
- Proportional to third derivative of quadrupole moment!
- Explains orbital decay in binary systems

Example

For PSR 1913+16:

- Predicted power: $\sim 7 \times 10^{24}$ Watts
- Observed orbital decay matches prediction to 0.2%!

Detection Principles

How Detectors Work

Measure change in distance between test masses:

$$\Delta L / L \sim h \sim 10^{-21}$$

for typical astrophysical sources

Detector Types

- **Laser interferometers:** LIGO, Virgo, KAGRA (ground-based)
- **Pulsar timing arrays:** Use pulsars as galactic-scale detector
- **Space-based:** LISA (future) - longer wavelengths

Challenge

Detecting $\Delta L \sim 10^{-18}$ meters (1/1000 proton diameter!) requires:

- Exquisite vibration isolation
- Ultra-stable lasers
- Sophisticated signal processing

Physical Intuition Summary

Key Conceptual Points

1. **Spacetime is dynamical:** Not a fixed stage, but can oscillate
2. **Transverse nature:** Only perpendicular components affected
3. **Quadrupole pattern:** Stretch one way, squeeze perpendicular
4. **Retardation:** Effects propagate at finite speed (light speed)
5. **Weakness:** Gravity is weakest force $\Rightarrow$ hard to detect

The Big Picture

Gravitational waves provide a new way to observe the universe:

- Complementary to electromagnetic observations
- See through opaque matter
- Direct probe of strong-field gravity
- Test general relativity in extreme conditions