Squeezed Angle Fluctuations: Theory

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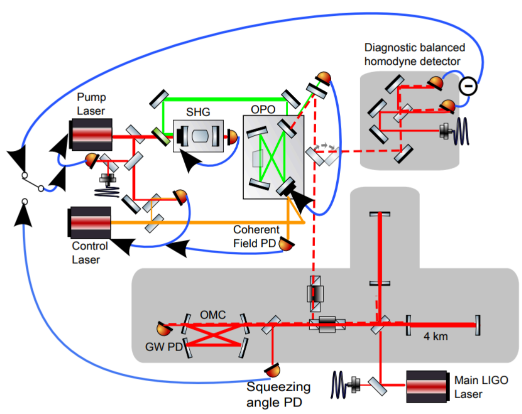
1. Squeezed vacuum source control scheme



Pictured above is the enhanced LIGO squeezer control scheme. It is designed to control several degrees of freedom, namely the frequency and phase of the two lasers, the lengths of the two cavities, and the phase of the squeezed state relative to the local oscillator at the detector.

Laser System:

- Pump laser: locked at 240 kHz bandwidth to interferometer carrier
- Control laser: offset locked +29.5 MHz above pump, 470 kHz bandwidth
- Pump generates 532 nm via SHG, which then pumps the OPO

OPO Control:

- SHG and OPO cavities controlled at 6 kHz and 9 kHz bandwidths
- Control laser injection creates **symmetric sidebands around carrier**

Phase Locking:

- Control laser phase locked to squeezed state phase at 90 kHz bandwidth
- Coherent control sidebands act as **proxy for squeezed angle**

Squeezing Angle Control:

- Squeezing angle photodetector measures phase between coherent sidebands and interferometer carrier
- Feedback loop controls squeezed quadrature with 10 kHz bandwidth using VCO

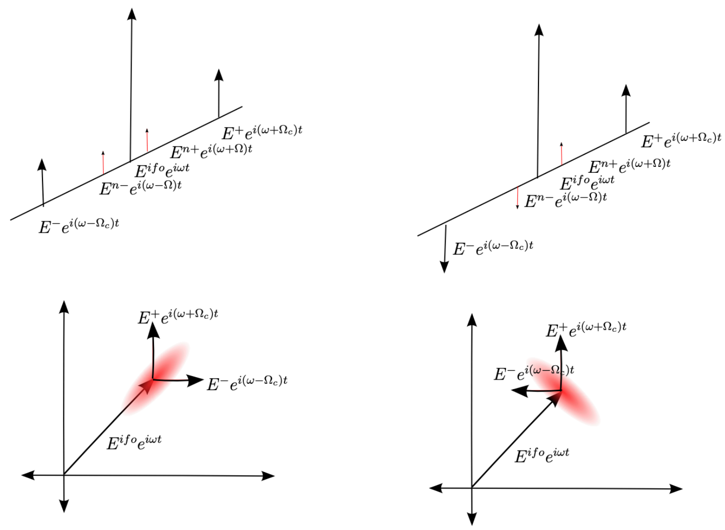
Detection:

- Output mode cleaner filters out coherent sidebands, transmits squeezed field and carrier
- Quantum noise reduction measured via homodyne detection at gravitational wave photodetector
- Alternative: diagnostic homodyne detector can characterize squeezer independently

Key Concepts

- Squeezed light's noise reduction only works in the correct quadrature
- The optimal squeezed angle changes with gravitational wave frequency and can drift over time
- **We can't directly measure the squeezed angle** without destroying the squeezing
- Coherent sidebands serve as a measurable reference for the squeezed angle, enabling active control of the squeezing orientation

2. Squeezing angle control



Pictured here we see the phase relationship between the coherent control sidebands and the local oscillator is related to the phase relationship between the noise sidebands and the local oscillator. This means that the coherent sidebands can be used to sense the squeezing angle.

How Coherent Sidebands Work:

- Control beam injected at ω + Ωc generates second sideband at ω − Ωc via nonlinear interaction in OPO
- Phase relationship between these sidebands **mirrors** the phase relationship of the squeezed audio sidebands
- Both experience identical disturbances (cavity length, crystal temperature, pump field fluctuations)
- Sidebands travel same path as squeezed field, acquiring same phase shifts

What They Measure:

- Phase between sidebands and local oscillator reveals **which quadrature is squeezed**

Two Control Loops Needed:

1. **Coherent locking field phase (ψ = φ+ − φ−):**
    - Controls relative phase between injected and generated sidebands

- Sensed in reflection off OPO rear coupler

2. **Squeezing angle ($\varphi = \varphi_+ - \varphi_{ifo}$):**
    - Controls phase of control field relative to local oscillator
    - Sensed at the main readout detector
    - This is the actual error signal that maintains optimal squeezing orientation

We need the control sidebands to measure the squeezing angle, but we don't want them at the final detector because they add noise.

The Solution:

- Measure the squeezing angle error signal **before** the output mode cleaner (at the anti-symmetric port)
- The output mode cleaner then **filters out** the control sidebands before the readout detector
- Only the squeezed field + interferometer carrier reach the gravitational wave detector

This Creates a Problem:

- The output mode cleaner is also a **spatial filter** - it only transmits the fundamental TEM_{00} mode
- Since we measure the error signal before this spatial filtering happens, the measurement includes higher order spatial modes

3. Squeezing angle control with higher-order modes

"The squeezing angle control error signal is generated by the beat between the interferometer field and the coherent squeezing angle control fields, both of which can have misalignments"

What this means:

- When the two light fields at slightly different frequencies mix on a photodetector, they create an oscillating signal at the difference frequency (the beat note)

In this system:

- **Interferometer field**: at frequency ω (the carrier/local oscillator)
- **Control sidebands**: at frequencies $\omega + \Omega c$ and $\omega - \Omega c$
- When these hit the photodetector together, they interfere
- Creates oscillating photocurrent at frequency Ωc (the offset frequency, ~29.5 MHz)
- The **amplitude and phase** of this Ωc signal depends on the relative phase between the interferometer field and the control sidebands
- The phase of the beat signal tells you the squeezing angle
- By demodulating at Ωc, you extract this phase information as your error signal
- So we are using one field as a phase reference to measure another field

The misalignment issue:

- If either beam is spatially misaligned, they don't overlap perfectly on the detector
- This changes the beat signal because you're mixing different spatial mode components
- Higher-order modes contaminate the measurement

Assuming that at the photo-detector both fields are mostly in the TEM_00 mode with a small component in the TEM_ij higher-order mode we can write the total field at the photo-detector as:

$$
\begin{aligned}
E(x,y,z,t) &= E^{ifo} + E^{+} + E^{-} \\
&= \sum_{ij}\{(a_{00}^{ifo}u_{00}(x,y,z) + a_{ij}^{ifo}u_{ij}(x,y,z))e^{i\phi_{ifo}} \\
&+ (a_{00}^{+}u_{00}(x,y,z) + a_{ij}^{+}u_{ij}(x,y,z))e^{i(\Omega_c t+\phi_{+})} \\
&+ (a_{00}^{-}u_{00}(x,y,z) + a_{ij}^{-}u_{ij}(x,y,z))e^{i(-\Omega_c t+\phi_{-})}\}e^{i\omega t} + c.c.
\end{aligned}
$$

Higher-order modes will different phases than TEM_{00} modes, with the phase difference between modes **at the photodetector** is defined as the difference between the Gouy phase shifts in the interferometer field versus the coherent locking field. Because of the Gouy phase shift there is a difference between the phase of the higher order modes and the 00 modes:

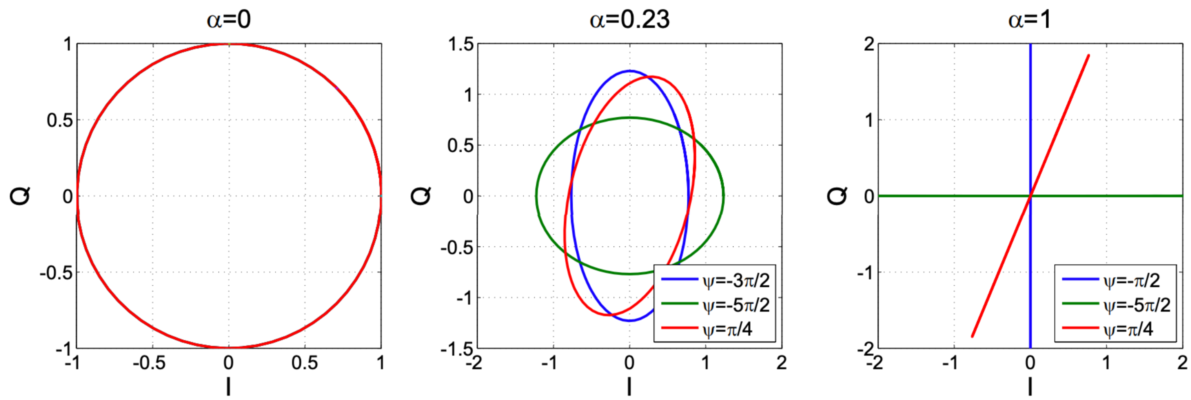
$$\gamma_{ij}^{ifo} = \left|\frac{a_{ij}^{ifo}}{a_{00}^{ifo}}\right| \qquad \gamma_{ij}^{clf} = \left|\frac{a_{ij}^{+}}{a_{00}^{+}}\right| = \left|\frac{a_{ij}^{-}}{a_{00}^{-}}\right|$$

$$a_{ij}^{ifo} = \gamma_{ij}^{ifo} a_{00}^{ifo} e^{i\phi_{ij}^{ifo}} \qquad a_{ij}^{clf} = \gamma_{ij}^{clf} a_{00}^{clf} e^{i\phi_{ij}^{clf}}$$

A common misalignment will cause the same Gouy phase shift in the two beams, but there may also be relative misalignments that cause a relative phase shift:

$$\phi_{ij} = \phi_{ij}^{ifo} - \phi_{ij}^{clf}$$

4. Error signals without higher order modes



Error signals with I and Q demodulation phases without higher order modes as the phase rotates and is held constant. Without any misalignments the demodulated signals are simply:

$$I_{err} \propto \cos\left(-\phi + \theta_{dm}\right) + \alpha \cos\left(\phi - \psi + \theta_{dm}\right)$$

$$Q_{err} \propto \sin\left(-\phi + \theta_{dm}\right) + \alpha \sin\left(\phi - \psi + \theta_{dm}\right)$$

Demodulation Signals in more Detail

- **I** tells you the projection onto the horizontal axis
- **Q** tells you the projection onto the vertical axis

Beating the coherent control sidebands (at frequencies $\omega \pm \Omega_c$) against the interferometer carrier field (at frequency ω) at a photodetector produces a signal oscillating at the offset frequency Ω_c. This oscillating photocurrent is then mixed with reference signals $\cos(\Omega_c t - \theta_{dm})$ and $\sin(\Omega_c t - \theta_{dm})$ and low-pass filtered to extract these I (in-phase) and Q (quadrature) components.

Error Signals

The behavior depends on α (the sideband amplitude ratio):

- $\alpha = 0$ (no generated sideband): Error signal traces a circle in I-Q space as φ changes
- $\alpha = 1$ (equal sidebands): Error signal becomes a line, similar to Pound-Drever-Hall, can be zeroed in either quadrature
    - As φ varies, the I and Q signals maintain a fixed linear relationship: $Q/I = \tan((\psi/2) - \varphi + \theta_{dm})$. In the I-Q plane, this traces out a **straight line through the origin** rather than a closed curve like a circle or ellipse. The line's angle depends on ψ (the phase between the two control sidebands, controlled by the coherent locking field loop) and θ_{dm} (the demodulation phase you choose).
    - If you want to lock using the I quadrature ($I_{err} = 0$), adjust θ_{dm} until the line lies entirely along the Q axis; then I remains zero as φ fluctuates slightly, while Q provides a strong restoring signal that indicates which direction φ has drifted.
- $\alpha \approx 0.23$ (actual experimental value): Error signal traces an ellipse in I-Q space
    - This elliptical shape is important because it means there's only one specific orientation—determined by the demodulation phase θ_{dm}—where locking on the I quadrature (setting $I_{err} = 0$) corresponds to the correct squeezing angle.
    - When perfectly aligned, one can optimize squeezing by adjusting θ_{dm} until the ellipse's semi-major axis aligns with the locking axis, which minimizes the signal in the unlocked Q quadrature. However, **misalignments distort this picture**: they add higher-order mode contributions with their own Gouy

phase shifts (the φij terms) that effectively tilt, shift, or distort the ellipse in I-Q space.

For the squeezing angle in terms of control sideband phases:

- θ = π/2 for amplitude squeezing → φ = (ψ+π)/2
- θ = 0 for phase squeezing → φ = ψ/2

Squeezing	Antisqueezing
$\theta_{sqz} = \frac{\pi}{2}$	$\theta_{sqz} = 0$
$\phi = \frac{\psi+\pi}{2}$	$\phi = \frac{\psi}{2}$
$I_{err} \propto (1-\alpha)\cos(\phi - \theta_{dm})$	$I \propto (1+\alpha)\cos(\phi - \theta_{dm})$
$Q_{err} \propto (1-\alpha)\sin(\theta_{dm} - \phi)$	$Q \propto (1+\alpha)\sin(\theta_{dm} - \phi)$
Squeezing angle locked using I_{err}	
$I = 0$	$I = 0$
$\phi = \pi/2 + \theta_{dm}$	$\phi = \pi/2 + \theta_{dm}$
$\psi = 2\theta_{dm}$	$\psi = 2\theta_{dm} + \pi$
$Q = -(1-\alpha)$	$Q = -(1+\alpha)$

5. Effect of a misalignment on locking point

$$
\begin{aligned}
I &\propto (1+\alpha)\left[\cos(\phi - \theta_{dm}) + \Sigma_{ij}\gamma_{ij}^{ifo}\gamma_{ij}^{clf}\cos(\phi - \theta_{dm} - \phi_{ij})\right] \\
Q &\propto (1-\alpha)\left[\sin(\theta_{dm} - \phi) + \Sigma_{ij}\gamma_{ij}^{ifo}\gamma_{ij}^{clf}\sin(\theta_{dm} - \phi + \phi_{ij})\right]
\end{aligned}
$$

The coherent locking field control loop maintains ψ constant, but the misalignment shifts where φ locks. Since the locking condition is I = 0 and the small angle approximation to first order for the gamma terms as they are <<1, an equation for the locking point error:

$$
\begin{aligned}
\Delta\phi &= \frac{\sum_{ij}\gamma_{ij}^{ifo}\gamma_{ij}^{clf}\sin\phi_{ij}}{(1+\sum_{ij}\gamma_{ij}^{ifo}\gamma_{ij}^{clf}\cos\phi_{ij}})\\
&\approx \sum_{ij}\gamma_{ij}^{ifo}\gamma_{ij}^{clf}\sin\phi_{ij}
\end{aligned}
$$

where γ_{ij} represents the amplitude fraction in each higher-order mode and φ_{ij} is the relative Gouy phase difference.

Static misalignment of one beam increases the phase noise contribution from fluctuations in the other beam. Since beam jitter likely involves multiple optics at different Gouy phases, the total contribution is difficult to predict. The error contribution is first order from **combined static misalignment + beam jitter.**

$$
\begin{aligned}
\Delta\theta_{sqz}(t) &= (\sin(\bar{\phi}_{ij}+\delta\phi_{ij}(t))\delta\gamma_{ij}^{ifo}(t)(\bar{\gamma}_{ij}^{clf}+\delta\gamma_{ij}^{clf}(t))\\
&\approx \bar{\gamma}_{ij}^{clf}\delta\gamma_{ij}^{ifo}(1+\bar{\phi}_{ij})\delta\phi_{ij}(t)
\end{aligned}
$$