Emma Smith
Roger Chen
Leo Carney
Katie Hahm

Bridge Final Report

A. Bridge Design Description

Since none of us had previous experience building bridges, we started out our design process with a little preliminary research and inspiration. We came together with a bunch of findings, as seen in Figure 1 below. We discussed each bridge taking into account our restrictions and knowledge of good truss design. Our favorites included the Camelback, Baltimore, and the Warren. From these we began to sketch our own designs.

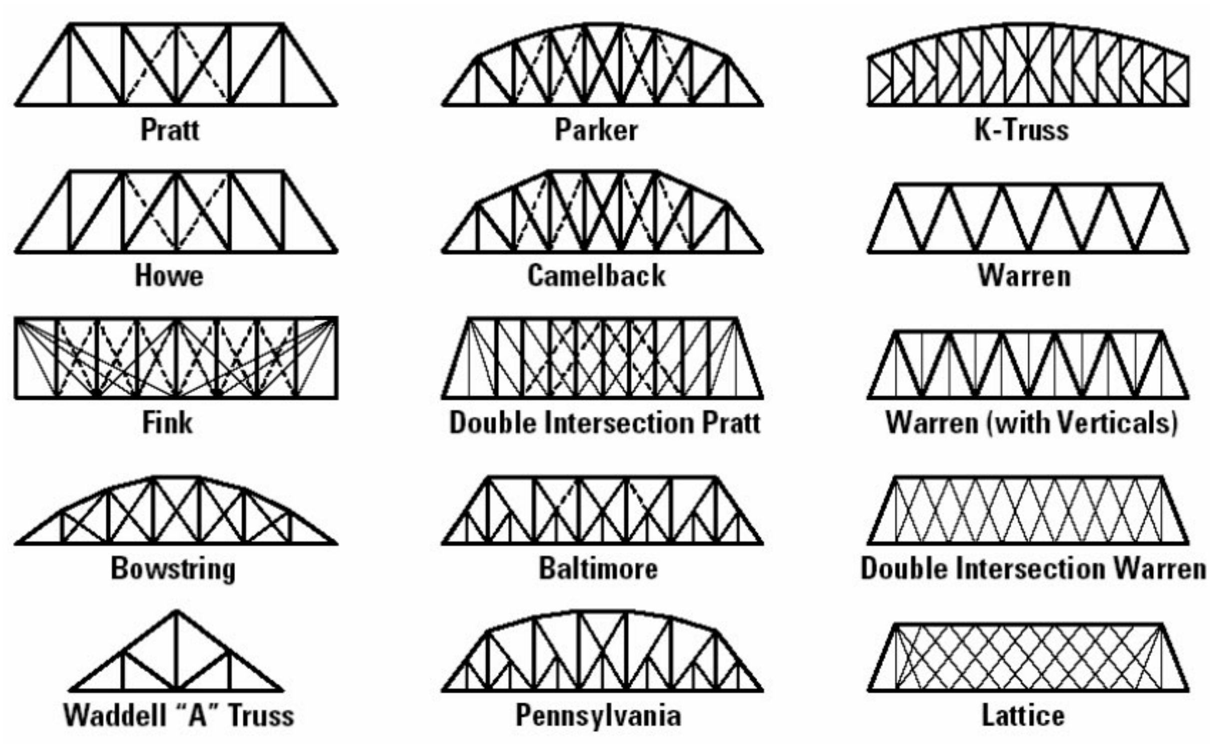


Figure 1: an example of some of the trusses we used for inspiration

In the “flare” phases of our design process, we used sketches to explore our favorite ideas, and more practically, what the connection joints might look like for each. Figure 2 shows some of Leo’s initial sketches.

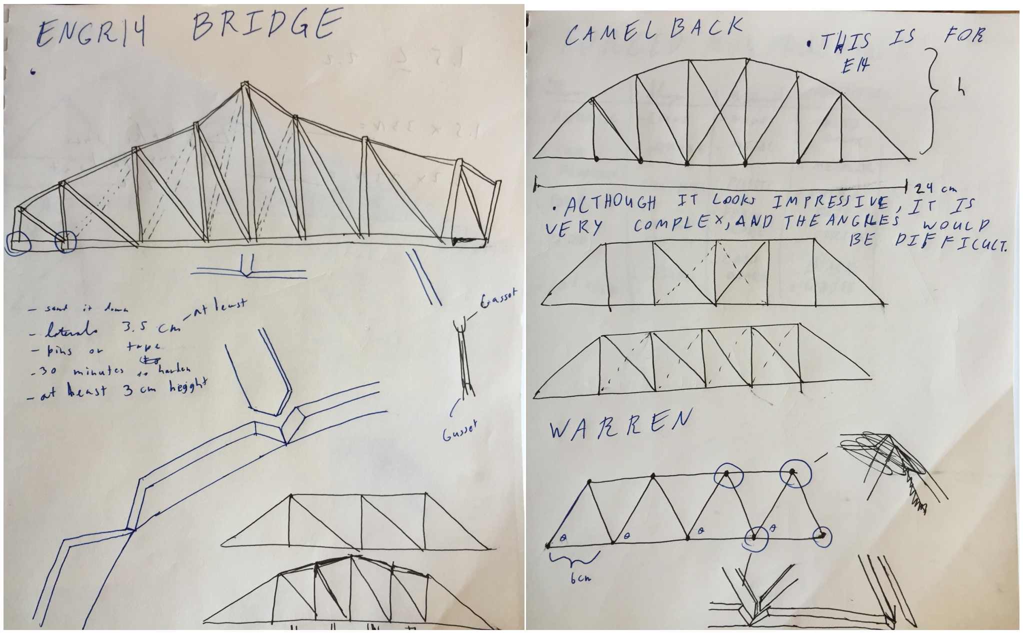


Figure 2: Some explorations of a “sloped roof” concept from Leo’s notebook

Knowing that our team’s collective schedules were all quite busy, and that building time would be pretty tight, we established some clear design priorities in our first meeting:

- Meet all design requirements established by the E14 teaching team
- Have a design that isn’t the traditional Howe or Pratt
- Find a simple design that would reduce unnecessary assembly and algebra time for us, while still giving us the opportunity to learn about building with balsa wood and properly analyzing a bridge design
- While we’re going for simplicity, attempt to have one of the lightest designs in the class. This means having the fewest members.

We decided in the end to move toward a repeating triangle form, distinct looking from previous trusses we analyzed, but also light and sturdy.

We were pleased with our final design, as it met all of the criteria we established. While non-traditional, our design is simple, light, and requires relatively little detailed cutting. Each member is either 5 or 6 cm in length. Not only does this help our bridge become lighter, it also protects our members from buckling since the possibility of buckling increases with length of members.

As noted in our Design Specification Table (Table 1), we have 9 joints and 15 members. This ensures that our design is statically determinate because 2*9 - 3 = 15. In addition, our bridge is composed of a series of congruent triangles. This helped reduce human error during the building process, because we were repeating the same small process of making a triangle over and over, so we become more familiar with the sequence of necessary steps. This also made the bridge easy to assemble in the “assembly line” fashion, where one person could repeatedly cut identical pieces and pass them on to the builder.

As indicated in Table 1, our bridge is relatively short since our height is 3.98 cm. We have four members making up the bottom of the bridge, so each member on the top and bottom of the bridge is 6cm long to make it span 24 cm. Each member that makes up the inside of the bridge (to complete the triangle) is about 5 cm long. Consequently, our bridge consists of short members, so we do not have to worry about buckling, and we only have to meet the maximum load capacity requirements.

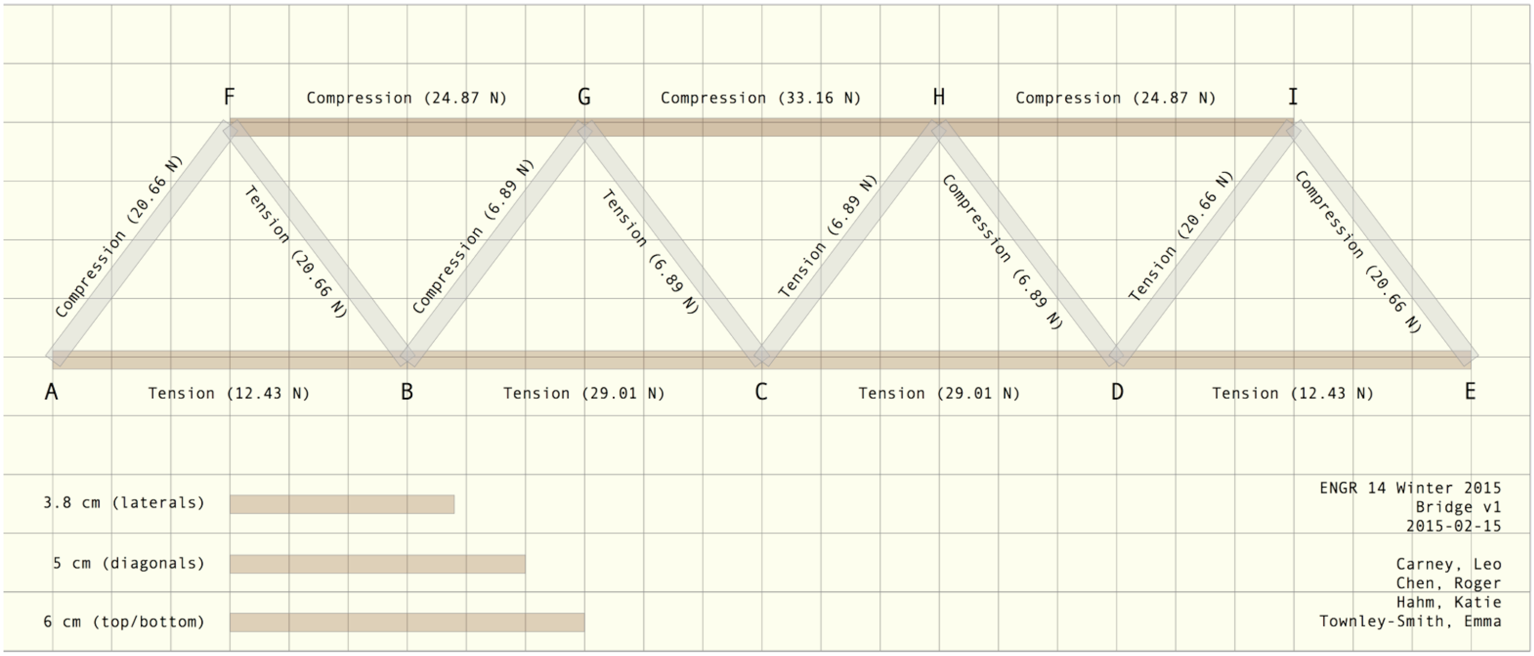
The members at the bottom corners of our bridge meet at the angle we determined our height from, 53 degrees -- see section B for more details on how we chose this angle that ultimately determined our dimensions. The joints in on the top corners of our bridge are formed from three members, so we had to sand the corners down to make them form all 360 degrees of the joint, so that we make sure that all lines of action point to the same point. The joints throughout the middle of the bridge mostly meet from four members. Each member at this joint had to be cut in a 45 degree angle such that all four make up a straight line on the bottom or top.

To make our design into a three dimensional bridge, we had to connect two of our trusses with lateral members. These lateral members did not have to be cut at an angle since all 9 joints were flat in the y direction. We joined the two trusses together with lateral members of 3.6 cm in length, just wide enough to clear the roadbed requirements.

Table 1. Bridge design specification table

	Design Requirements	**Your Design Specs**	**Your Built and Tested Bridge**
Number of joints	n/a	9	9
Number of members	n/a	15	15
Span	24 cm	24 cm	24.8 cm
Clear width	3.5 cm	3.5 cm	3.6 cm
Clear height	>3.5 cm	3.98 cm	4.1 cm
Total height	<=10 cm	3.98 cm	4.1 cm
Cost (team expenditure)	<=$10	$0	$0
Weight	Minimized	4.8 grams	4.8 grams
Design approach	Non-standard preferred	Non-standard	Non-standard
Safety Factor (SF) requirement	Min: 1.5 Max: 2.2	2.1	

Figure 3. Scale drawing of bridge

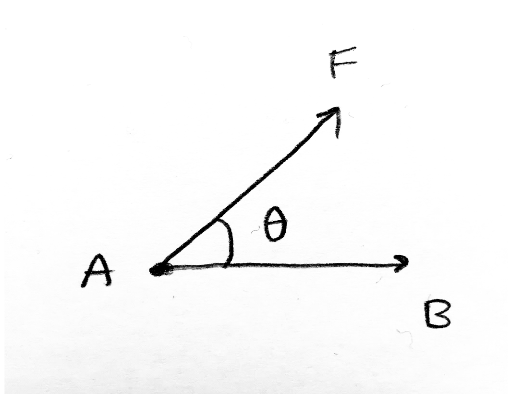


B. Analysis and Construction of Bridge

We first performed an equilibrium analysis of the entire bridge, finding the external forces at joints A and E with a design load of 33N on the truss.

From there, we used the method of joints to make an expression for each of the forces at joints A, F, G and B in terms of θ, the optimum internal angle as shown in Figure 4. Because of the symmetry of the truss design, analyzing these four joints allowed us to determine all the forces in the rest of the truss.

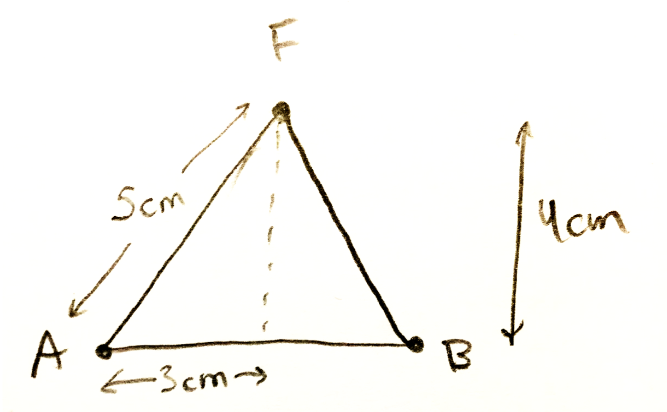
Figure 4. Angle θ indicates the optimum internal angle as discussed in the text.



We used a spreadsheet (see Appendix A) to look at all of the possible angle measurements for θ -- 0 to 90 degrees, inclusive -- and calculate the resulting truss height and safety factor at each angle. The truss height had to exceed 3.5cm in order to allow the roadbed to pass through, and the safety factor for the compression member with the highest load -- member GH -- had to fall between 1.5 and 2.2. We knew to look at the members in compression rather than the members in tension, because the maximum internal tensile load was about 735N, much greater than the maximum internal compressive load of about 70N. We determined that member GH was the member with the highest load by creating an expression for the force in each member in terms of θ, and choosing the expression with the greatest value.

Of the angles that fulfilled both of the above conditions, we decided to choose 53° for our θ, because this made each of our "half" triangles a 3-4-5 triangle, as shown in Figure 5. Though this choice created difficult angles for us to measure precisely between the members, it gave us easy member lengths to cut from our stock material. We were able to create the angles with reasonable precision by sanding the balsa wood and comparing it to our planar model of the truss, created in Sketch.

Figure 5. Our 3-4-5 triangle design element



Our assumptions were as follows:

- We're on Earth -- gravity is in the -y direction
- Our truss is rigid and will not deform
- We are treating the truss as a planar system in equilibrium
- System is massless
- The design load is evenly distributed over joints B, C, and D

Once we chose our angle θ, we were able to determine the force in each member based on our assumed design load of 33N per truss, or 66N in total. Since none of our members were over 6 cm in length, we assumed that the failure mode of all compressive members would be yielding rather than buckling. See Table 2 for a list of member loads, and Appendix B for the corresponding hand calculations.

Table 2. Member force table of member loads for applied load of 66 N (SF 2.1). Shaded row indicates member predicted to fail at applied load.

Member	Load (N)	Tension or Compression?	Member Length (cm)	Failure Mode	Load Capacity (N)	Safety Factor
AB	12.43	Tension	6	Yield	735	59.1
BC	29.01	Tension	6	Yield	735	25.3
CD	29.01	Tension	6	Yield	735	25.3
DE	12.43	Tension	6	Yield	735	59.1
AF	20.66	Compression	5	Yield	70	3.4
FB	20.66	Tension	5	Yield	735	35.6
BG	6.89	Compression	5	Yield	70	10.2
GC	6.89	Tension	5	Yield	735	106.7
CH	6.89	Tension	5	Yield	735	106.7
HD	6.89	Compression	5	Yield	70	10.2
DI	20.66	Tension	5	Yield	735	35.6
IE	20.66	Compression	5	Yield	70	3.4
FG	24.87	Compression	6	Yield	70	2.8
GH	33.16	Compression	6	Yield	70	2.1
HI	24.87	Compression	6	Yield	70	2.8

Once we were confident in our calculations, we proceeded to construction. Before beginning, we assembled a high level plan, as shown below.

Construction day plan:

- Re-sync up on dimensions and bridge template
- Cut all members to rough length (for two trusses, parallel building)
- Begin gluing and pinning
    - While two people glue and pin, two people work on the report in the Google doc

We began our construction at 10:30am on Sunday, sitting in a circle in the lounge of Toyon with supplies stacked up in the middle. After syncing up about our plan, Katie, Leo, and Roger set to cutting the members to rough lengths while Emma worked on documenting the process and writing the report. Leo and Roger took charge of sanding, pinning, and gluing the members of each of the trusses simultaneously, while Emma and Katie continued work on the report and triaged requests for supplies as necessary.

This ‘assembly line’ sort of process worked well for us, as shown in Figure 6.

Figure 6. Our construction environment, featuring E14 students Roger Chen and Leo Carney

One problem we ran into with sanding the balsa wood members down to length was splintering and cracking, as shown in Figure 7. If the constructor pressed down too hard while sanding the member, it would crack in the center. If the constructor sanded against the grain of the balsa wood, it would splinter from the rough edges. We think our rough x-acto knife cutting work probably contributed to this problem, since it was difficult to cleanly and completely cut through the wood.

Figure 7. A cracked piece of balsa wood that didn’t make it into our final truss

We used q-tips to apply the wood glue, and to spread it out from the joint if extra beads appeared. The wax paper provided by the teaching team successfully stopped our trusses from becoming glued to the foamcore.

We ran into the most trouble in our construction process when we tried to connect the two trusses with lateral members. While our trusses came together nicely on their own, they weren't perfectly symmetrical, which made them difficult to connect precisely. The lateral members needed to be slightly different lengths to account for differences in sanding and glue. Any slight bends or non-perpendicular lateral members caused the bridge to lean. We initially tried to glue the bottom lateral members to one truss first, and the top lateral members to the other, and then connect all of the laterals at once. This approach failed, because there were too many members to simultaneously align. We ended up breaking off and sanding down some of the laterals from this first attempt, and instead attaching all bottom laterals to both trusses, and then all top laterals to both trusses.

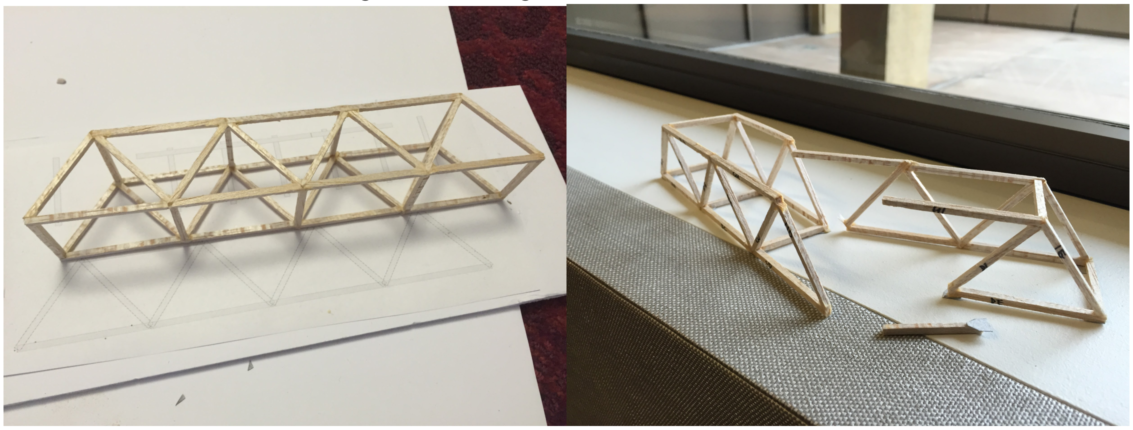
C. Testing Results & Failure Analysis

We tested our bridge through the traditional E14 method: hanging weight from a roadbed that slid into the bridge, resting on the laterals. One of our team members continued to add weight that hung from the roadbed until our bridge failed. See the test sheet, in Appendix C, for further details of the results.

Our prediction was that our bridge should fail at member GH as a result of compressive yielding. Our bridge met the safety factor requirement, coming in at 2.1 -- so member GH, with a load of 33.16 N, should fail at a load of 69.64 N (just over 7 lbs). Since the design load for the whole bridge was 66 N, this means that our entire bridge should have failed at a load of 138.6 N (just over 30 lbs).

On the day of testing, loading did not go exactly as intended. Rather than our ideal envisioned gradual load, the loads for the bridge were only available in chunky water bottle units that had to be stacked into a round bucket suspended from the roadbed. Since we had to add the load in chunks, there was no good way to gradually load our balsa wood bridge to the full intended load. As a result, the sand was unevenly distributed in the bucket, and it caused a disproportionately heavy load to be distributed to one side of our bridge. This caused our bridge to break at joint D, as visible in Figure 8.

Figure 8. Our bridge before and after the test



We believe that had we been able to add the load to the bucket slowly -- perhaps by pouring in the sand, or by loading it in sandwich bags instead of in water bottles -- we would have seen the predicted behavior with our bridge. As it happened, our bridge broke just at 99N, or at a safety factor of 1.5.

Other possible causes of or contributions toward this unusual failure include:

- Variation in the stiffness and strength of the balsa wood. Some of the pieces that we used seemed strong and stiff, and others were more pliable -- these qualities could affect how easily a member would bend and break.
- Variation in the quality of our gluing. The two trusses were constructed by two different team members, and then glued together by a third team member, so it is possible that there was variation in how much glue was applied and how long it was allowed to soak into the wood before the pieces set. This would affect the strength of the bridge especially at the joints, where we saw unusual breakage.

D. Summary of Project Processes

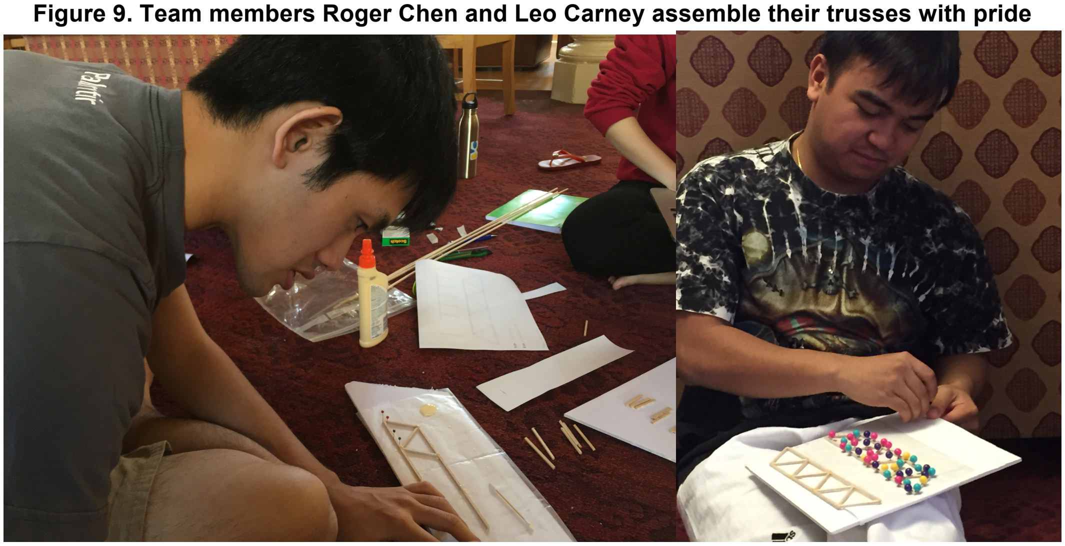
As a team, we first decided to convene on February 10th for brainstorming, sketching, and calculations before we met with our Bridge Project Coach, Kai Jun Chew. Some of these sketches and ideas are included in part A of this report. We also briefly discussed our schedules and established design priorities based on the reasonable time that everyone could commit to the project. We planned the rest of our meeting times and rough deadlines for the different components of the project so that we could have a strong plan to present for the Thursday check-in.

After that initial meeting, we communicated largely by email amongst ourselves and between ourselves and KJ until our marathon building session on Sunday, February 15th. Due to some calculation errors on our part -- and a graceful rescue by KJ's emails -- we were able to finalize our bridge dimensions and member load calculations on Saturday. We finished up the to-scale building template on Sunday morning, and constructed the bridge and worked on the report together during Sunday afternoon. We refined the report on Monday afternoon, and felt good about the bridge and draft we delivered on Tuesday.

We divided up the work based on preferences and strengths. In the beginning, we were all capable brainstormers and truss analyzers, so we met together to make design decisions and align around a general vision of the project. Up until the first class check-in, we agreed that we should all understand the analysis of the bridge. We emailed back and forth several times to check and confirm the dimensions and angles of the bridge so that we wouldn't have any unfortunate surprises during our building time.

Leo and Roger were eager to build as shown in Figure 9, applying the tips that Jose had demonstrated during class, so they took most of the responsibility for assembling the trusses and sanding them once the glue set. Emma and Katie provided supplemental help by cutting balsa wood to rough lengths, measuring and cutting the cardstock for the gussets, and retrieving materials. Katie and Roger put the trusses together using lateral members.

Figure 9. Team members Roger Chen and Leo Carney assemble their trusses with pride



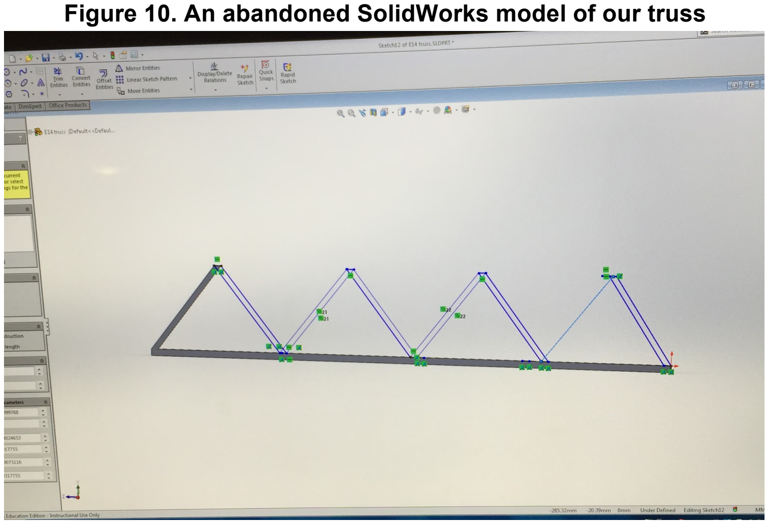
Emma and Katie took the lead on the report. Emma put together a Google Doc with all of the necessary components and descriptions, and continued to write up and document the construction process throughout the building cycle. She also wrote the hand-calculations that created the load report. Katie helped detail the bridge design process and the decisions that went into the final design.

Roger created the bridge diagram from Emma's calculations, which came from Leo's initial sketches. Katie also performed a set of load calculations and verified that the team was on the same page. After the bridge was tested, Emma wrote the failure analysis and integrated the comments and critiques from the teaching team. Overall, the pieces of the project came together quite cohesively without any last-minute rushing or fumbling.

Reflecting on the project, we were all quite content with the outcome. No one team member took on a disproportionate amount of work, and everyone had the opportunity to work on the components of the project that interested them most. We carefully planned our work time together, and no one felt like we had meetings that were unnecessary or disrespectful of team member schedules.

One element of the project that could have been planned better was the to-scale diagram of the truss used for building on construction day. Emma had originally planned to do an engineering drawing in SolidWorks, and ended up making about half of the truss as a model before giving up, as seen in Figure 10. From her SolidWorks experience in ME 203, she thought that the SolidWorks model might be easier than using some other software.

Figure 10. An abandoned SolidWorks model of our truss



While SolidWorks is quick for simple extruded shapes, Emma found that the geometries her team had selected were not simple to create with the software. Since the initial sketches for each triangle shared endpoints, any time the dimensions of the truss were adjusted, it would create multiple errors. For what the team actually needed from the end product, SolidWorks had an excess amount of functionality and specificity.

The team briefly experimented with Photoshop, but Photoshop's tools for exact dimensioning and scale work are not easy to use. Roger ended up using a simpler software called Sketch, usually used by UI/UX designers to create mockups, to create the final scale model.

In the end, our bridge performed as expected and our report was largely complete by the test day, Tuesday, reducing workload for the team members for the rest of the week.

Appendix

A. Spreadsheet used to calculate the optimum internal angle

1					
2					
3	**theta**	**theta in rad**	**height**	**Fgh**	**safety factor**
4	1	0.01745328	0.05236515	-839.48532	0.08338442
5	2	0.03490656	0.10476222	-418.4611	0.16727959
6	3	0.05235983	0.1572232	-277.54716	0.25220939
7	4	0.06981311	0.20978026	-206.65744	0.3387248
8	5	0.08726639	0.26246577	-163.77362	0.42741927
53	50	0.87266389	3.57525543	-95.01974	0.73668903
54	51.5	0.89884381	3.77151102	-60.357505	1.15975635
55	51	0.89011717	3.70468577	-69.701111	1.00428816
56	52	0.90757044	3.83981883	-52.427098	1.33518738
57	53	0.92502372	3.98112799	-39.476403	1.7732112
58	54	0.942477	4.12913885	-29.032239	2.41111269
59	55	0.95993028	4.28443663	-20.07038	3.48772661
94	90	1.570795	2261087.99	24.0617956	2.90917607
95					

B. Hand calculations to support member force table

C. Test sheet