Question 6, Homework 1, Katie Hutchinson, ehe9vz

(1) $m(a+bX) = a + b \times m(x)$

$$m(a+bx) = \frac{1}{N}\sum_{i=1}^{N} a + bx_i$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i\right)$$

a is a constant

$$= \frac{1}{N}\left(Na + b\sum_{i=1}^{N} x_i\right)$$

b is a constant

$$= a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

m(x)

$$m(a+bx) = a + b \times m(x)$$

(2) $\text{cov}(X,X) = s^2$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(x_i - m(x))$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2$$

$= s^2$ ← by definition

(3) $\text{cov}(X, a+bY) = b \times \text{cov}(X,Y)$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(a+by - m(a+by))$$

$\hookrightarrow z_i = a+by$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(z_i - m(a+by))$$

$\hookrightarrow m(a+by) = a+bm(Y)$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\underbrace{((a+by_i)-(a+bm(Y)))}$$

$(a+by_i - a - bm(Y))$

$(by_i - bm(Y))$

$(b(y_i - m(Y)))$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(b(y_i - m(Y)))$$

$$b \times \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$$

$= b \times \text{cov}(X,Y)$

(4) $\text{cov}(a+bX, a+bY) = b^2\,\text{cov}(X,Y)$

$$\frac{1}{N}\sum_{i=1}^{N}(a+bX - m(a+bX))(a+by - m(a+by))$$

$(a+bx) - m(a+bx)$

$(a+bx) - a + b(mx)$

$(a+bx - a + bmx)$

$(bx - b(mx))$

$[b(x - m(x))][b(y - m(y))]$

$$\frac{b^2}{N}\sum_{i=1}^{N}(x_i - mx)(y_i - my)$$

$= b^2\,\text{cov}(X,Y)$

(5) $\text{med}(a+bX) = a + b \times \text{med}(x)$

Yes, the median only depends on the order of the data. Adding a simply shifts all the points by the same amount, while multiplying it by b stretches out the points. In both transformations, the order is still the same.

$IQR(a+bx) = a + b \times IQR(x)$

$Q1(a+bX) = a + bQ1(x)$

$Q3(a+bX) = a + bQ3(x)$

⇓

$IQR(a+bx) = (a+bQ3) - (a+bQ1)$

$= b(Q3 - Q1)$

$= b\,IQR(x)$

This is not true since $IQR(a+bx)$ is equal to $b\,IQR(x)$ instead of $a + b\,IQR(X)$.

(6) Ex: $X = \{1, 4\}$ $\quad X^2 = \{1, 16\}$

$m(x) = \frac{1+4}{2} = 2.5$ $\quad m(x^2) = \frac{1+16}{2} = 8.5$

$(m(x))^2 = 6.25$ $\quad 8.5 \neq 6.25$

$\sqrt{X} = \{1, 2\}$ $\quad 1.5 \neq 1.58$

$m(\sqrt{X}) = \frac{1+2}{2} = 1.5$

$\sqrt{m(x)} = \sqrt{2.5} \approx 1.58$