An Experimental Investigation of the Scattering Angle Distribution for α-particles incident on Gold Foil

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Abstract

Alpha particles directed at a thin sheet of material are deflected by the electrostatic repulsion from the nuclei of atoms in the sheet. The distribution of scattering angles from such interactions should be Gaussian around $0°$ for small angles, but larger scattering angles should follow an inverse trigonometric relationship, making large angle deflections much more probable than with a continuation of the normal distribution. Here we show that these predictions hold up well under experiment. We found that there is a high degree of correlation between the scattering angle distribution for small angles and a Gaussian fit curve, with $R^2 = 0.998$. Additionally, we determined that the distribution for larger values fits the expected function of θ very well, with $R^2 = 0.995$, and also obtained the power dependence of count rate on the function of scattering angle as 4.2 ± 0.3, which is within uncertainties of the expected value of 4.

1 Introduction

After discovering the electron in 1897[1], J. J. Thomson proposed the 'plum pudding' model of the atom in his 1904 paper. Thomson's model described the atom as a sphere of uniform positive charge the size of the atom (which was known at the time to be on the order of 10^{-10} m) containing the majority of the atom's mass, and with negatively charged electrons (Thomson called them 'corpuscles') distributed within the positive sphere.[2] Then in 1911, Ernest Rutherford wrote his Nucleus Paper, in which he proposed that in fact the positive charge and most of the atom's mass was concentrated in a very small volume at the centre of the atom. This explained the observations of Geiger and Marsden, who directed alpha particles at thin plates of matter and observed the angles by which the particles were deflected. They found that whilst the distribution for small scattering angles was consistent with the predictions of Thomson's model, the number of alpha particles deflected by larger angles was much more than expected, even observing deflections greater than 90°, which was highly improbable under the Thomson model.[3] This idea of the atomic nucleus is still accepted today.

This experiment aims to verify the scattering angle distributions from Rutherford's paper by confirming that it is roughly Gaussian for small angles, and by determining the power-law dependence for larger angles.

2 Theory

As in Rutherford's original experiment, positively charged alpha(α)-particles will be directed at a thin sheet of gold in an evacuated chamber. These particles will be produced by the radioactive decay of an Americium-241 (Am-241) source into Neptunium-237 (Np-237): [4]

$$ {}^{241}_{95}\mathrm{Am} \ \rightarrow \ {}^{237}_{93}\mathrm{Np} \ + \ {}^{4}_{2}\alpha \tag{1} $$

The beam of particles produced by the source will pass through a collimator in order to be directed at a small point on the gold foil. As the α-particles approach the nucleus of a gold atom on the surface of the foil, they will experience an electrostatic repulsive force F dependent on their distance r from the nucleus

$$ F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \tag{2} $$

where q_1 and q_2 represent the charges of the gold nucleus and alpha particle respectively. [1] This will cause the α-particles to follow a curved path. The scattering angle θ of a detected particle is the angle between the direction of travel of the beam without any deflection, and the line from the incidence point on the gold foil to the detector, as shown in Figure 1.

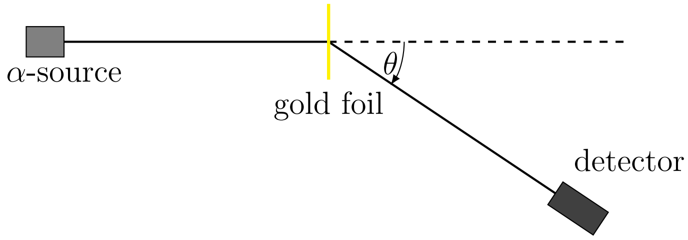

Figure 1: Diagram illustrating how the scattering angle θ is defined relative to the radiation source, gold foil target and detector.

The scattering angle distribution is described by the Rutherford scattering formula [3], which gives the relationship between the count rate $R(\theta)$ and the scattering angle as

$$R(\theta) \propto \csc^4\left(\frac{\theta}{2}\right) \tag{3}$$

or for small values of θ in radians

$$R(\theta) \propto \frac{1}{\theta^4} \tag{4}$$

where $\sin\theta \approx \theta$.

We will determine the count rate for a range of scattering angles, by rotating the detector into position and recording the number of alpha particles detected over a specific time interval using an electronic counter, then dividing the count N by the time period T to obtain the count rate $R(\theta)$ for that scattering angle θ.

$$R(\theta) = \frac{N}{T} \tag{5}$$

Plotting a graph of $\ln R$ against $\ln|\csc(\theta/2)|$ should give a straight line

$$\ln[R(\theta)] = 4\ln\left[\csc\left(\frac{\theta}{2}\right)\right] + \ln k \tag{6}$$

with a gradient of 4 rad s^{-1}, where k is the constant of proportionality.

3 Experimental Details

The experimental apparatus was set up as shown in Figure 2, with an evacuated chamber containing a detector fixed to the chamber wall, and a pivoting α-particle source and gold foil connected together, so that they rotate with each other. The detector output was connected via a uniaxial cable to a pulse discriminator, in order to filter out signal noise pulses, which was then connected to an electronic counter able to count the number of input pulses for various time intervals.

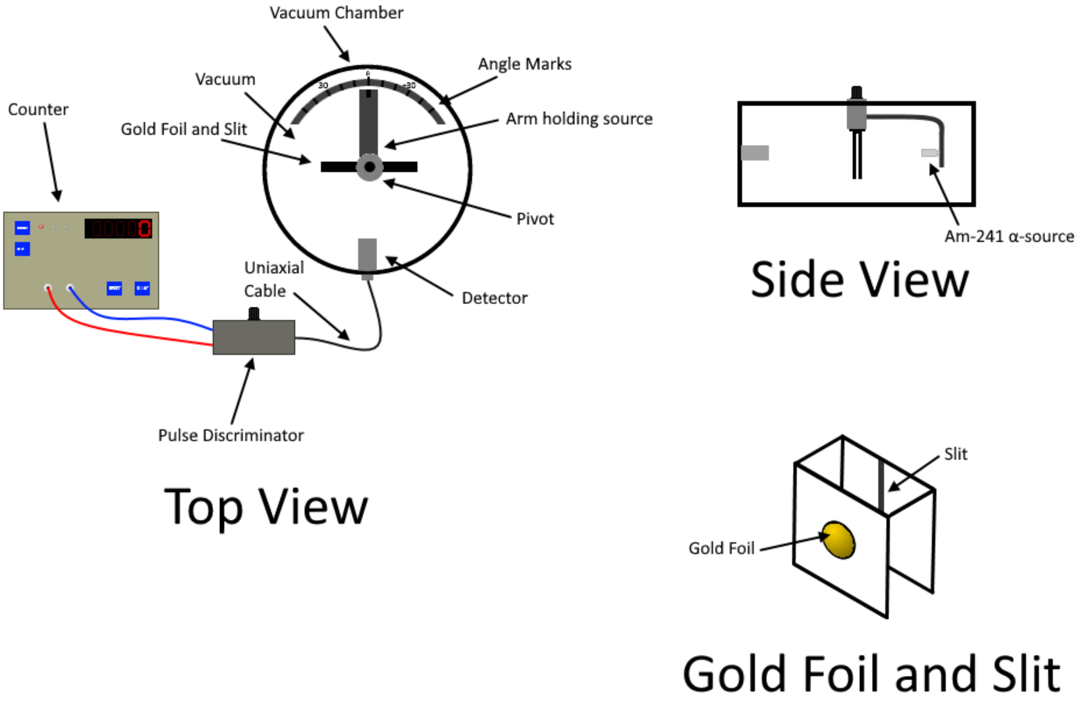

Figure 2: Diagram showing the experimental setup used, and showing multiple views of the evacuated chamber containing the α-particle source, gold foil, and detector.

For a given scattering angle, the gold foil and α-particle source were rotated to align the marker with the corresponding angle mark on the chamber lid (see Figure 3). Then the electronic counter was configured to count for an appropriate time period, before being started. After the time period had elapsed and the counter had stopped counting, the count displayed was recorded. A range of angles from -7.5 to 15 degrees in 2.5 degree intervals were recorded with 120 second counting periods, in addition to a few larger angles up to 30 degrees, recorded with up to 400 second periods.

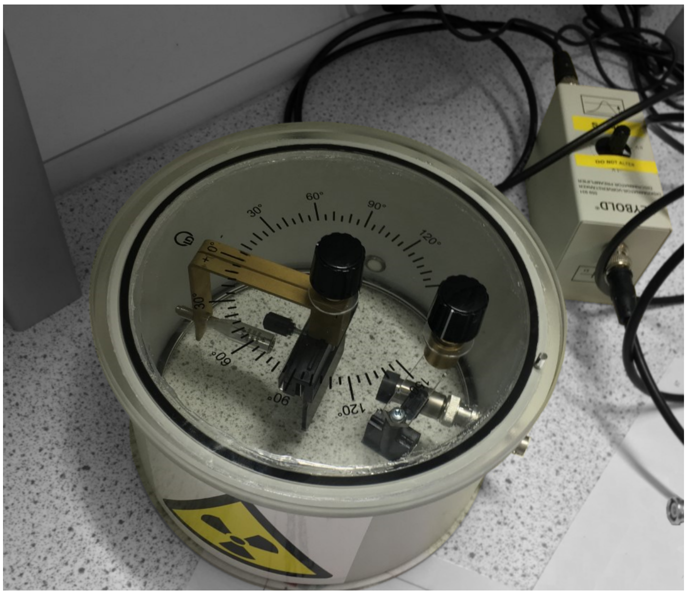

Figure 3: An edited photo of the evacuated chamber and pulse discriminator, with the background desaturated.

4 Experimental Results and Discussion

The count rate for each angle was calculated by dividing each count N by the corresponding time period T. The uncertainty in each count was equal to $\sqrt{N}$, and the uncertainty in T due to the electronic counter is assumed to be negligible, so the uncertainty in each count rate was calculated as $\frac{\sqrt{N}}{T}$. The count rates for smaller angles ($|\theta| \leq 10°$) and their uncertainties were used to plot the calibration curve shown in Figure 4. A Gaussian fit for the small angle data had its centre at -1.19 ± 0.19 degrees (with $R^2 = 0.998$ to 3sf), so this value is assumed to be the systematic error in the angle measurements, and so all angle values were adjusted by adding 1.19° to correct this.

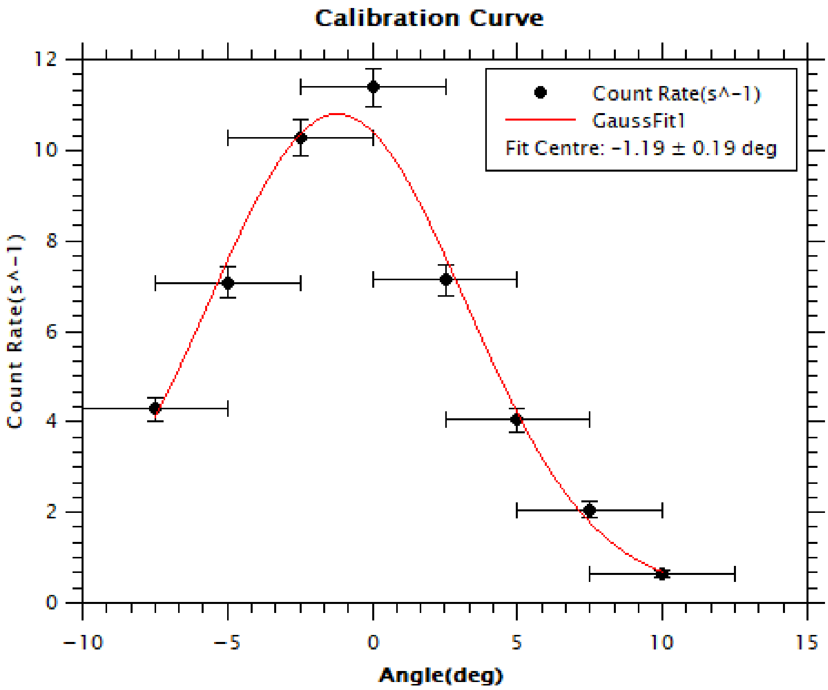

Figure 4: Plot of count rate against angle with a Gaussian curve fitted to determine the systematic error in angle measurements.

With the corrected angle values, the natural logarithm of $|\csc\frac{\theta}{2}|$ was calculated for each angle, and the uncertainties propagated through:

$$\alpha_{\ln|\csc\frac{\theta}{2}|} = \frac{\alpha_\theta}{|\tan\frac{\theta}{2}|} \tag{7}$$

The natural logarithm of all count rates R was also calculated, and the uncertainties obtained from:

$$\alpha_{\ln R} = \frac{\alpha_R}{R} = \frac{1}{\sqrt{N}} \tag{8}$$

With these calculated values the graph of $\ln R$ against $\ln|\csc\frac{\theta}{2}|$ (Figure 5) was plotted. For angles greater in magnitude than 10 degrees (i.e. $\ln|\csc\frac{\theta}{2}| < 2.44$), the plot is consistent with a straight line, and a linear fit of those points gives a line with y-intercept -10.1 ± 0.06 s^{-1} and gradient 4.2 ± 0.3 rad s^{-1}. The fit also gives a linear correlation coefficient of $R^2 = 0.995$ to 3sf, indicating that 99.5% of the variance in $\ln R$ values is justified by the $\ln|\csc\frac{\theta}{2}|$ values for a linear relationship, so this is a good fit.

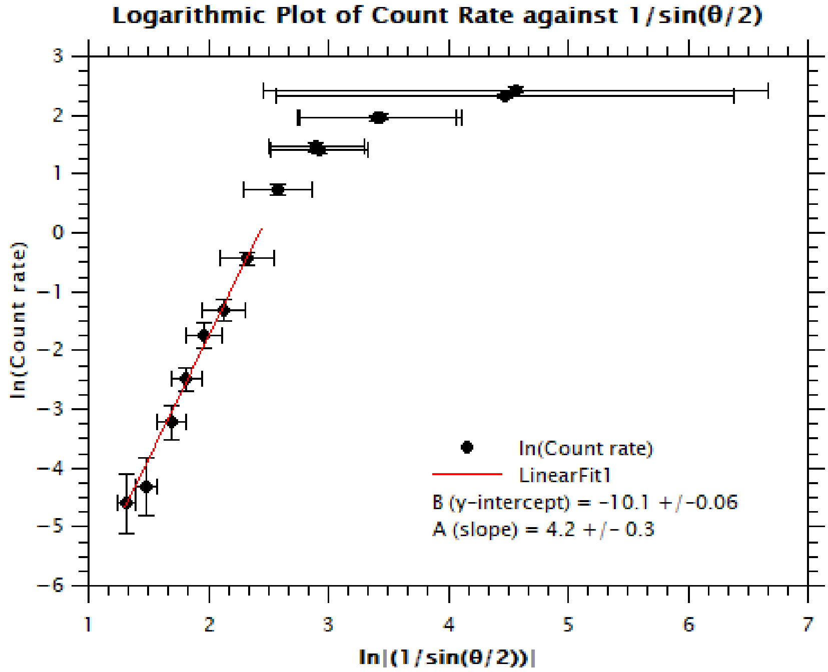

Figure 5: Plot of $\ln R$ against $\ln|\csc\frac{\theta}{2}|$, with a linear fit applied for $\theta > 10°$ to determine the power law dependence of count rate on scattering angle, given by the fit line's slope.

The value found as the gradient for the linear fit, 4.2 ± 0.3 rad s^{-1}, is in very good agreement with the expected gradient of 4 rad s^{-1} from Equation 6, which lies within the range of the value's uncertainty. This supports the power law dependence on the scattering angle in Equation 3 and thus also supports the Rutherford scattering formula, showing that the observations match the predictions of Rutherford's model of the atom quite well. Additionally, the correlation coefficient for the Gaussian fit in Figure 4 was $R^2 = 0.998$, demonstrating that the scattering angle distribution is approximately Gaussian for small angles.

5 Conclusions

The results of this experiment match the predictions of the Rutherford model of the atom, supporting Rutherford's idea of a very dense, positively charged nucleus taking up a very small volume at the centre of the atom and containing most of its mass, an idea which is still agreed upon in modern physics, although the exact nature of the nucleus has been refined since. The Gaussian curve fit for smaller scattering angles gave a correlation of $R^2 = 0.998$, which is very close to 1, and supports the prediction that the scattering is normally distributed for small angles. Importantly, for larger angles the results fit the relation proposed by Rutherford (in Equation 3) very well with $R^2 = 0.995$, and a gradient of 4.2 ± 0.3 rad s^{-1} was given for the fit line found, implying that the result is in agreement with the accepted power law dependence of count rate on scattering angle of 4, within uncertainties.

The main errors in the experiment were in the angle measurements, as the scale used was graduated in five degree intervals, giving each measurement an uncertainty of $2.5°$, however the actual uncertainties were probably much smaller than this. Therefore repeating the experiment with a much more precise scale or other angle measuring method would improve the precision of our results.

Also, the point on the calibration curve in Figure 4 for $0°$ appears to be much further away from the Gaussian fit than all the other points, and is the only point for which the fit curve does not go within the range of its error bars. This suggests that the value of the count rate for that point may be slightly anomalous, so repeating the measurement for that point may yield a better fit, and thus a more accurate value for the systematic error, allowing the angle measurements to be corrected better.

References

[1] Young H D, Freedman R A. *University Physics*. 14th ed. Harlow: Pearson; 2016.

[2] Thomson J J. On the structure of the atom: an investigation of the stability and periods of oscillation of a number of corpuscles arranged at equal intervals around the circumference of a circle; with application of the results to the theory of atomic structure. *Philosophical Magazine*. 1904; 39(1): 237-265.

[3] Rutherford E. The Scattering of α and β Particles by Matter and the Structure of the Atom. *Philosophical Magazine*. 1911; 21(1): 669-688.

[4] Skorobogatov G A, Bondarevskiib S I, Eremina V V, Timofeev S A. Consequences of ^{241}Am α-Decay in Some Solid Matrices. *Radiochemistry*. 2011; 53(5): 523-528.