# Proton Decay

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**Abstract**

Protons are normally considered stable, but this article will look at the possibility of proton decay, the evidence for it, and the experiments aiming to verify its existence.

## 1 Introduction

In the standard model of physics, the proton is considered the only stable baryon, with all other baryons (sigmas, lambdas, neutrons, etc.) expected to eventually decay into it. For example, a free neutron has a half-life of $610.0 \pm 0.76$ seconds[1][2], which is about 10 minutes. However, it is possible that even a free proton will eventually decay, but with a very long half-life (many orders of magnitude longer than the age of the universe), hence it's apparent stability.

## 2 Grand Unified Theory

Whilst no experiments have yet produced results that go against the theoretical predictions of the standard model, it is not considered to be complete, as it does not explain the physical force of gravity, and there are multiple constants that have to be determined from experiment, such as those describing CP violation. This has led to multiple theories being proposed in order to fix or improve upon the standard model. Unification theories, which are one class of such theories, are based on the observation that the strength of the weak and electromagnetic interactions increases with energy, and that the strength of the strong interaction decreases with energy, leading to an expectation that at a certain energy the three forces will be unified. This unification energy is expected to be of the order of $10^{16}$GeV (or $10^{6}$J). The first unification theory was Grand Unified Theory, which predicts the existence of very heavy X and Y bosons (up to $10^{17}$GeV), which allows quarks and leptons to convert between each other via absorption or emission of on of these bosons. A very significant consequence of this is that Baryon number is no longer always conserved, and so the proton is unstable.[3]

## 3 Experimental Detection

No experiment has been successful in observing proton decays as of yet, but they have placed a lower limit on the mean lifetime of a proton of about $10^{34}$ years. This fits the predictions of Grand Unified Theory; with a unification energy of

$10^{16}$GeV, the predicted mean lifetime of a proton is on the order of $10^{37}$years.[3]

Super-Kamiokande is a particle detector in Japan, situated in the Kamioka Mine. It consists of a 33.8m wide, 36.2m high cylindrical tank filled with 32,000 tonnes of ultra pure water[4]. Thousands of photo-multiplier tubes line the tank, and these detect the Cherenkov radiation emitted by particles passing through the water at greater than the speed of light in the medium. It is designed to detect neutrinos, such as those that may be produced by a proton decay.[5] In the detector's 22,500 tonne fiducial volume are $7.5 \times 10^{33}$ protons, so there is a large quantity that could decay, increasing the likelihood that some will decay within a reasonable time frame. The experiment does have background noise in the form of atmospheric neutrinos, but these can be factored out, for example by calculating the total momentum of the particles produced in an interaction; if the observed particles come from a proton decay, by conservation of momentum their overall momentum should be the same as the proton's initial momentum, approximately zero.[4]

Hyper-Kamiokande is the proposed successor to the Super-Kamiokande detector, with two 74m wide, 60m high cylindrical tanks (see Fig 1), each containing almost 10 times the volume of water as Super-Kamiokande: 260,000 tonnes in total, with a fiducial volume of 190,000 tonnes. The detectors will also be lined with 40,000 50cm photomultiplier tubes, and almost 7000 smaller ones. These PMTs have both improved quantum efficiency, allowing them to detect twice as many photons as before (see Fig 2), and greater resolution in their measurements of charge and time. These significant improvements will, among other things, benefit the search for proton decays.[6]

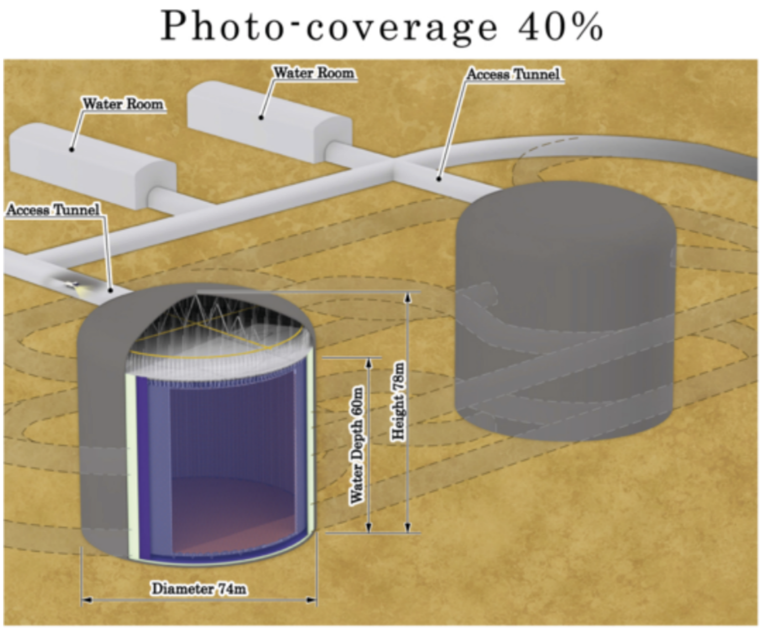


Figure 1: Diagram of the proposed Hyper-Kamiokande Detector [7]

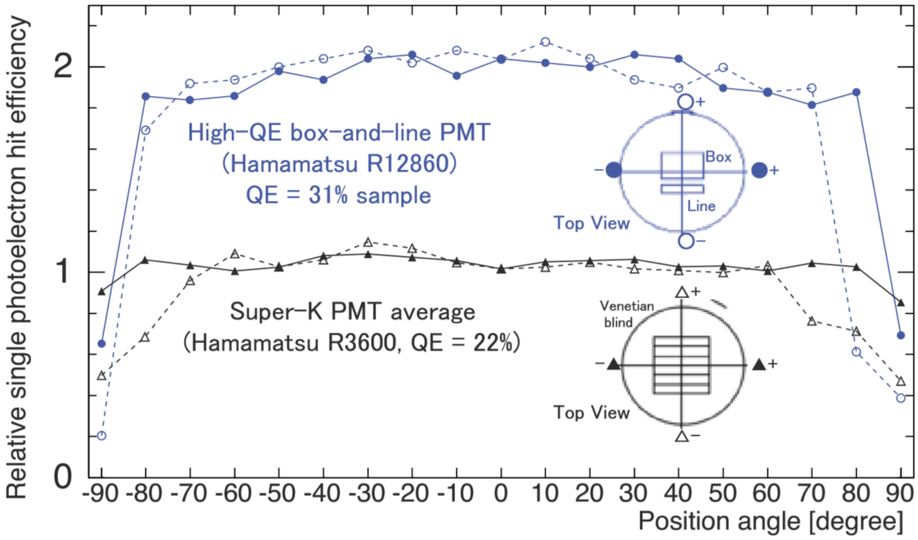


Figure 2: Quantum efficiency comparison of the old and new PMTs [8]

# 4 Mathematical Prediction

An analytic formula for the unification scale ($M_U$) is: [9]

$$\ln \frac{M_U^0}{M_Z} = \frac{2\pi}{187\alpha}\left(7 - \frac{80\alpha}{3\alpha_{3C}} + 8s_W^2\right) + \Delta_U \tag{1}$$

which gives the unification of guage couplings at the unification scale as: [9]

$$M_U^0 = 10^{15.2+0.0312}\text{GeV} \approx 1.7 \times 10^{15}\text{GeV} \tag{2}$$

With the contribution of multiple corrections, the more accurate unification scale is obtained as [10]: [9]

$$M_U = 10^{15.2312 \pm 0.11 \pm 0.221\eta_S \pm 0.655\eta_V}\,\text{GeV} \tag{3}$$

The decay rate for the $p \to e^+\pi^0$ proton decay is calculated by: [11]

$$\Gamma(p \to e^+\pi^0) = \left(\frac{m_p}{64\pi f_\pi^2}\frac{{\alpha_G}^4}{{M_U}^4}\right)|A_L|^2|\overline{\alpha_H}|^2(1 + D' + F)^2 \times R \tag{4}$$

and the inverse decay rate is: [11]

$$\Gamma^{-1}(p \to e^+\pi^0) = \frac{4}{\pi}\frac{f_\pi^2}{m_p}\frac{M_U^4}{\alpha_G^2}\frac{1}{\alpha_H^2 A_R^2}\frac{1}{F_p} \tag{5}$$

Using estimated values, this gives the proton lifetime: [11]

$$\tau_p^{SU(5)} \simeq 10^{33.110 \pm 0.440 \pm 0.884|\eta_S| \pm 2.62|\eta_V|}\text{yrs} \tag{6}$$

We can use the lifetime $\tau$ to calculate the number $N$ out of the initial $N_0$ protons that remain undecayed after an amount of time $t$ [2]:

$$N = N_0 e^{-t/\tau} \tag{7}$$

Using this equation, and an approximate value for $\tau$ of $10^{33}$ yrs, we can plot a decay graph (Figure 3). Note that the unit of the time scale in Figure 3 is $10^{33}$ (1 Billion Trillion Trillion or 1 Decillion) years, which for scale is about 70 billion trillion times the current age of the universe ($\sim$ 13.8 billion years).

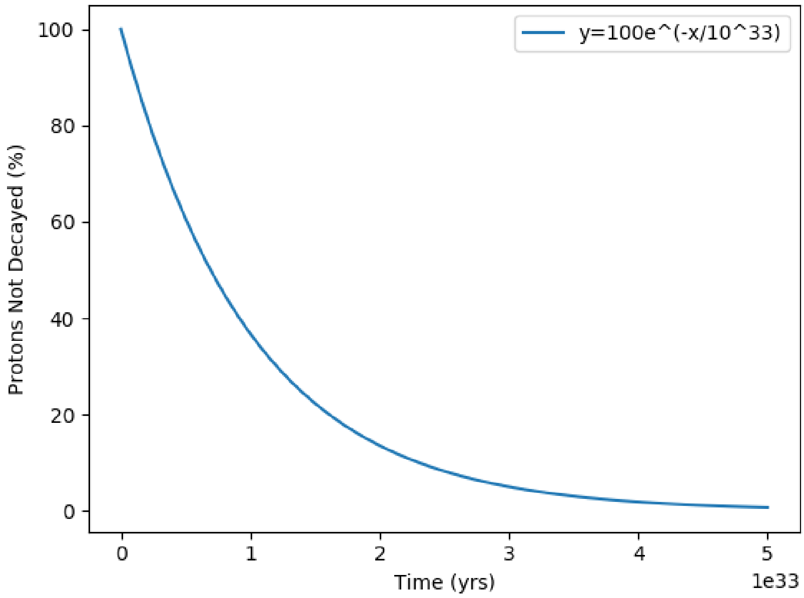


Figure 3: Percentage of protons remaining as a function of time, plotted using matplotlib in python.

Table 1 contains some more predictions of upper limits on proton lifetimes for different superheavy scalar and guage boson (S and V respectively) mass splittings [12].

| $\frac{M_S}{M_U}$ | $\frac{M_V}{M_U}$ | $\tau_p$(yrs) |
|---|---|---|
| 10 | 1 | $9.77 \times 10^{33\pm0.44}$ |
| 10 | 2 | $6.00 \times 10^{34\pm0.44}$ |
| 8 | 3 | $1.42 \times 10^{35\pm0.44}$ |
| 6 | 4 | $2.35 \times 10^{35\pm0.44}$ |
| 5 | 5 | $3.59 \times 10^{35\pm0.44}$ |
| 3 | 6 | $3.68 \times 10^{35\pm0.44}$ |
| 1 | 10 | $5.32 \times 10^{35\pm0.44}$ |
| 20 | 1 | $1.80 \times 10^{34\pm0.44}$ |

Table 1: Proton lifetime estimations for different splitting factors [13]

## 5 Summary

Proton decays, if observed, would provide evidence for Grand Unified Theory, as well as suggest the existence of two new particles, the X and Y bosons. We have not yet detected such decays, telling us that if they do occur, they must have a lifetime of at least $10^{34}$ years, as if it were any shorter we should've detected them already. This fits with the theory's prediction of a lifetime on the order of $10^{37}$ years.[3] However, a new detector has been proposed in Japan which is capable of detecting $p \to e^{+} + \pi^{0}$ and $p \to \bar{\nu} + K^{+}$ decays, and is large and sensitive enough to detect longer lifetimes, so it may not be long until we either observe proton decays, or increase our lower bound on possible proton lifetimes even further.[6]

## References


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