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ARTICLE

Quantifying Partisan Gerrymandering: An Evaluation of the Efficiency Gap Proposal

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Abstract. Electoral districting presents a risk of partisan gerrymandering, the manipulation of electoral boundaries to favor one party over another. For three decades, the Court has failed to settle on a legal test for partisan gerrymandering, and such claims have uniformly failed. Until now. Plaintiffs have prevailed before a three-judge federal panel, leveraging a new measure called the "efficiency gap" that quantifies partisan gerrymandering in terms of two parties' relative efficiency at translating votes for their party into seats in government. The case is now before the Supreme Court, which may embrace the efficiency gap approach and thereby remake the law of electoral districting. Through a synthesis of mathematical and legal analysis, this Article examines the efficiency gap measure, focusing particularly on its underlying methodological choices and electoral assumptions, and its relationship to competitiveness, seats-votes proportionality, and voter turnout.

The efficiency gap is a useful indicative measure of partisan gerrymandering under the circumstances of the instant case, where each party earns about half the votes and a large efficiency gap persists under plausible variations in voter behavior. Relying in part on the efficiency gap measure, the Supreme Court should rule in favor of the plaintiffs. However, a mapmaker can achieve a below-threshold efficiency gap with a skewed bipartisan gerrymander that carves a state up into uncompetitive districts that deny minority parties sufficient representation. For example, a party that earns only 59% of the vote can secure a filibuster- and veto-proof 75% supermajority of the legislature with a below-threshold efficiency gap. For this and other reasons, the Court should not adopt the efficiency gap as the exclusive definitional measure of partisan gerrymandering under all circumstances, such that a plan is invalid if and only if

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it exhibits a large, durable, intended, and unjustified efficiency gap. Instead, the Court should permit some flexibility for scholars, litigants, and courts to refine measurement approaches over time and for varying circumstances. One approach worth future exploration is a variation on the efficiency gap that defines a surplus vote in terms of the full margin of victory and compares wasted vote shares instead of totals. Finally, the Court should be aware that any measure, like the efficiency gap, that compares votes to seats entails the perverse risk that partisan voter suppression may operate to reduce the apparent severity of partisan gerrymanders.

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Introduction

We may be approaching a watershed moment in the Supreme Court's gerrymandering jurisprudence. In three cases over the last three decades,[1] partisan gerrymandering has eluded the Court's grasp. The Court has unanimously recognized that partisan gerrymandering poses a problem of constitutional significance,[2] but repeatedly fractured on whether and how to intervene. A minority of Justices has insisted that partisan gerrymandering presents a nonjusticiable political question susceptible to no judicially discernible and manageable standard,[3] while a majority of Justices has agreed that partisan gerrymandering is justiciable but disagreed among themselves about the proper legal standard.[4] Justice Kennedy, the controlling swing vote, has rejected each proposal, while expressing hope that a suitable standard may one day materialize.[5] In the first thirty years after the Court held partisan gerrymandering justiciable, dozens of plaintiffs raised claims of partisan gerrymandering, but not one survived the motions stage[6]—until now.

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[1] League of United Latin Am. Citizens (LULAC) v. Perry, 548 U.S. 399 (2006); Vieth v. Jubelirer, 541 U.S. 267 (2004); Davis v. Bandemer, 478 U.S. 109 (1986), *abrogated by Vieth*, 541 U.S. 267.

[2] *See* Ariz. State Legislature v. Ariz. Indep. Redistricting Comm'n, 135 S. Ct. 2652, 2658 (2015) ("[T]his Court has recognized, '[that partisan gerrymanders] [are incompatible] with democratic principles.'") (third alteration in original) (quoting *Vieth*, 541 U.S. at 292 (plurality opinion)). Even the *Vieth* plurality, while rejecting judicial intervention for partisan gerrymandering, conceded that "an excessive injection of politics is unlawful" and "setting out to segregate [voters] by political affiliation is . . . lawful" only "so long as one doesn't go too far." 541 U.S. at 293 (plurality opinion) (emphasis omitted).

[3] *LULAC*, 548 U.S. at 511 (Scalia, J., concurring in the judgment in part and dissenting in part) (joined by Justice Thomas); *Vieth*, 541 U.S. at 281 (plurality opinion) (authored by Justice Scalia and joined by Chief Justice Rehnquist and Justices O'Connor and Thomas); *Bandemer*, 478 U.S. at 144 (O'Connor, J., concurring in the judgment) (joined by Chief Justice Burger and Justice Rehnquist).

[4] *LULAC*, 548 U.S. at 413-14 (opinion of Kennedy, J.) ("A plurality of the Court in *Vieth* would have held [partisan gerrymandering] challenges to be nonjusticiable political questions, but a majority declined to do so. We do not revisit the justiciability holding . . . ."); *id.* at 447, 456-57 (Stevens, J., concurring in part and dissenting in part); *id.* at 483 (Souter, J., concurring in part and dissenting in part); *id.* at 491-92 (Breyer, J., concurring in part and dissenting in part); *Vieth*, 541 U.S. at 306 (Kennedy, J., concurring in the judgment); *id.* at 317, 321-23 (Stevens, J., dissenting); *id.* at 344, 346-47 (Souter, J., dissenting); *id.* at 355, 364-67 (Breyer, J., dissenting).

[5] *Vieth*, 541 U.S. at 306 (Kennedy, J., concurring in the judgment). *LULAC*, 548 U.S. at 414 (opinion of Kennedy, J.) (affirming Vieth, in which he expressed hope a standard would ultimately emerge).

[6] *See Vieth*, 541 U.S. at 279-80 (plurality opinion) ("[I]n *all* of the cases we are aware of involving that most common form of political gerrymandering, relief was denied.") (emphasis in original); Nicholas O. Stephanopoulos & Eric M. McGhee, *Partisan Gerrymandering and the Efficiency Gap*, 82 U. CHI. L. REV. 831, 832-833 (2015) [hereinafter Stephanopoulos &

In a case called *Whitford v. Gill*,[7] plaintiffs have successfully challenged Wisconsin's Assembly map as a partisan gerrymander, relying in part on a newly proposed numeric measure and associated legal test called the "efficiency gap." In 2014, political scientist Eric McGhee proposed the numeric measure.[8] In 2015, Eric McGhee and leading election law scholar Nicholas Stephanopoulos developed this numeric measure into a legal test specifically designed to address concerns with prior proposals.[9] In brief, the efficiency gap measure counts the relative number of votes "wasted" by each of two competing political parties; it thereby quantifies the relative efficiency with which each party is able to convert popular support (votes) into governmental power (seats).[10] The legal test classifies as an invalid partisan gerrymander any plan intentionally drawn to produce a large, durable, and unjustified efficiency gap.[11]

Armed with this new measure and associated legal test, the *Whitford* plaintiffs not only survived the motions stage, but won at trial before a panel of three federal judges.[12] The majority opinion does not endorse wholesale the plaintiffs' proposal, but extensively discusses the efficiency gap as strong

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McGhee, *The Efficiency Gap*] ("By our count, claimants' record over this generation-long period [1986-2015] is roughly zero wins and fifty losses."); Easha Anand, Comment, *Finding a Path Through the Political Thicket: In Defense of Partisan Gerrymandering's Justiciability*, 102 CALIF. L. REV. 917, 933 (2014) ("[O]f the thirty-nine decisions surveyed . . . , only one found a gerrymander unconstitutional, and that one decision was subsequently dismissed as moot.").

[7] 218 F. Supp. 3d 837, 854, 910 (W.D. Wis. 2016).

[8] *See* Eric McGhee, *Measuring Partisan Bias in Single-Member District Electoral Systems*, 39 LEGIS. STUD. Q. 55, 68, 77 (2014) [hereinafter McGhee, *Measuring Partisan Bias*] In this article, McGhee called the metric "relative wasted votes."

[9] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 833-34. Throughout the Article, I refer to Eric McGhee and Nicholas Stephanopoulos as the "academic proponents" or simply the "proponents" of the proposed efficiency gap measure and legal test.

[10] *Id*. at 851 (defining the efficiency gap in terms of wasted votes); *id.* at 852 (emphasizing that "[a] gap in a party's favor enables the party to claim more seats, relative to a zero-gap plan, *without claiming more votes*").

[11] Specifically, "large" means the gap exceeds a set numeric threshold, *see id*. at 886-89; "durable" means the gap is robust to sensitivity testing that models plausible shifts in voting patterns, *see id*. at 889-90; and "unjustified" means the gap cannot be explained as the product of legitimate districting criteria consistently applied to the jurisdiction's underlying political geography, *see id*. at 891.

[12] *Whitford*, 218 F. Supp. 3d at 843, 856-57, 903, 930. When a plaintiff sues in federal court to challenge the federal constitutionality of an electoral districting plan for a state's legislature or congressional delegation, federal statutes require the convening of a three-judge federal panel, *see* 28 U.S.C. § 2284(a) (2015), with direct appeal to the Supreme Court, *see id.* § 1253; *see* Cooper v. Harris, 137 S. Ct. 1455, 1464 n.2 (2017); *see also* Shapiro v. McManus, 136 S. Ct. 450, 454-55 (2015) (clarifying that section 2284 requires convening a three-judge panel). *See generally* Joshua A. Douglas, *The Procedure of Election Law in Federal Courts*, 2011 UTAH L. REV. 433 (discussing the operation of relevant statutory provisions).

evidence in support of its conclusion that the map was a partisan gerrymander.[13] Wisconsin appealed directly to the Supreme Court, which stayed the panel's remedial order, ordered full briefing, and heard oral argument on October 3, 2017.[14] The *Whitford* case offers the Court the opportunity to decide whether the efficiency gap provides the legal test it has been waiting for. Were the Court to affirm the panel's finding of partisan gerrymandering, based on the efficiency gap analysis, other evidence, or some combination thereof, it would remake the law of electoral districting in advance of the 2020 redistricting cycle.[15]

As the Court considers *Whitford*, the efficiency gap measure and associated legal test warrant careful and comprehensive examination. Thus far, the reactions in popular media, scholarship, and litigation have been strong and conflicting.[16]

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[13] *Whitford*, 218 F. Supp. 3d at 903 ("[The] evidence is further bolstered by the plaintiffs' use of the 'efficiency gap' . . . to demonstrate that . . . their representational rights have been burdened."); *id.* at 933 (Griesbach, J., dissenting) ("Despite the central role the efficiency gap has played in the case . . . the majority has declined the Plaintiffs' invitation to adopt their standard and uses it only as confirming evidence . . . .").

[14] Defendants' Notice of Appeal at 1, Gill v. Whitford, No. 15CV0421 (W.D. Wis. Feb. 24, 2017); Gill v. Whitford, 137 S. Ct. 2289 (2017) (granting stay of district court judgment pending appeal to Supreme Court); Docket No. 16-1161, *Gill v. Whitford*, SUPREME COURT OF THE UNITED STATES, https://www.supremecourt.gov/search.aspx?filename=/docket/docketfiles/html/public/16-1161.html (last visited Nov. 20, 2017).

[15] Justice Kennedy was the swing vote in *Vieth*, *see* 541 U.S. 267, 306 (2004) (Kennedy, J., concurring in the judgment), and may remain the swing vote in *Whitford* if each of the five present Justices that joined the court since *Vieth* (Roberts, Alito, Sotomayor, Kagan, and Gorsuch) votes like his or her predecessor on the question of partisan gerrymandering. S*ee, e.g.*, Kerr v. Hickenlooper, 759 F.3d 1186, 1193, 1196 (10th Cir. 2014) (Gorsuch, J., dissenting from the denial of rehearing en banc) (approvingly citing Scalia's plurality opinion in *Vieth*). But this possibility is no foregone conclusion. And even if the Court embraces a partisan gerrymandering claim this term in a predicted 5-4 decision, the evolution and refinement of partisan gerrymandering doctrine over time will be determined by the Court as a whole rather than by any single member.

[16] For a small sampling of media coverage of the efficiency gap proposal, see David Daley, *Will Justice Kennedy be the Supreme Court's hero on gerrymandering? Don't count on it*, VOX (Jun. 6, 2017), https://www.salon.com/2017/06/06/will-justice-kennedy-be-the-supreme-courts-hero-on-gerrymandering-dont-count-on-it; Adam Liptak, *When Does Political Gerrymandering Cross a Constitutional Line?*, N.Y. TIMES (May 15, 2017), https://www.nytimes.com/2017/05/15/us/politics/when-does-political-gerrymandering-cross-a-constitutional-line.html.

For scholarly examination of the efficiency gap proposal, see Theodore S. Arrington, *A Practical Procedure for Detecting a Partisan Gerrymander*, 15 ELECTION L.J. 385 (2016); Jowei Chen, *The Impact of Political Geography on Wisconsin Redistricting: An Analysis of Wisconsin's Act 43 Assembly Districting Plan*, 16 ELECTION L.J. (forthcoming Dec. 2017); Edward B. Foley, *Due Process, Fair Play, and Excessive Partisanship: A New Principle for Judicial Review of Election Laws*, 84 U. CHI. L. REV. 655 (2017); Anthony J. McGann et al., *A Discernable and Manageable Standard for Partisan Gerrymandering*, 14 ELECTION L.J. 295

This Article contributes to this evaluative effort by offering a new analysis of the proposed efficiency gap measure, focusing particularly on its underlying methodological choices and electoral assumptions, and its relationship to competitiveness, seats-votes proportionality, and voter turnout.

This analysis bears on the questions before the Court in *Gill v. Whitford*. Is partisan gerrymandering justiciable? If so, what is the governing legal standard? What role, if any, should the efficiency gap measure play in this standard? And under the proper standard, is the Wisconsin Assembly plan a partisan gerrymander? I would suggest the following answers: yes, partisan gerrymandering is justiciable; the principle of partisan symmetry is an appropriate legal standard; and the Wisconsin Assembly plan is a partisan gerrymander.

The efficiency gap is one of multiple useful indicative measures of partisan asymmetry under the circumstances of the instant case, where each party earns about half the votes and a large efficiency gap persists under plausible variations in voter behavior. However, the Court should not adopt the efficiency gap as the exclusive definitional measure of partisan gerrymandering under all circumstances, such that a plan is invalid if and only if it exhibits a large, durable, intended, and unjustified efficiency gap. Instead, the Court should permit some flexibility for scholars, litigants, and courts to refine measurement approaches over time and for varying circumstances. Note that this is precisely the approach suggested by the *amici* of leading academics.[17] Furthermore, the Court should

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(2015); John F. Nagle, *How Competitive Should a Fair Single Member Districting Plan Be?*, 16 ELECTION L.J. 196 (2017); Samuel S.-H. Wang, *Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin*, 15 ELECTION L.J. 367 (2016); Jonathan Krasno et al., *Can Gerrymanders Be Measured? An Examination of Wisconsin's State Assembly* (May 22, 2016), https://ssrn.com/abstract=2783144.

In the *Whitford* panel's post-trial decision on the merits, both the majority and dissent discuss the efficiency gap proposal in depth. *See* 218 F. Supp. 3d at 854-57; *id.* at 933-34, 937-38 (Griesbach, J., dissenting). The panel also discussed the proposal when denying Wisconsin's motion to dismiss and motion for summary judgment. *See* Whitford v. Nichol, 151 F. Supp. 3d 918, 920-22 (W.D. Wis. 2015) (denying motion to dismiss); Whitford v. Nichol, 180 F. Supp. 3d 583, 585. 588-93 (W.D. Wis. 2016) (denying motion for summary judgement).

Meanwhile, another federal lawsuit challenging North Carolina's congressional redistricting plan and also relying on the efficiency gap measure has survived a motion to dismiss. Common Cause v. Rucho, 240 F. Supp. 3d 376, 377-78, 380 (M.D.N.C. 2017) (per curiam). The federal panel in that case consolidated two cases, one brought by a group of plaintiffs led by Common Cause, the other brought by a group of plaintiffs led by the League of Women Voters. *See id.* at 377 & n.1. The two groups of plaintiffs make similar legal arguments, but only the League of Women Voters plaintiffs use the efficiency gap in the discriminatory effect element of the proffered legal test. *See id.* at 380. The *Rucho* panel briefly discussed the efficiency gap proposal when denying defendant's motion to dismiss. *See* 240 F. Supp. 3d at 380-81.

[17] Brief of Heather K. Gerken et al. as Amici Curiae in Support of Appellees at 4, Gill v. Whitford, No. 16-1161 (U.S. Aug. 30, 2017), 2017 WL 3774485 (urging the Court to announce "[p]artisan symmetry" as a "workable principle, one that lends itself to a

acknowledge that partisan gerrymandering is not the only form of political gerrymandering that subverts democratic values, and signal its receptiveness to efforts to define and proscribe other forms of political gerrymandering. Just as excessive departures from partisan symmetry can trigger a partisan gerrymandering claim, perhaps excessive departures from competitiveness should trigger a bipartisan gerrymandering claim, and excessive departures from seats-votes proportionality should trigger a minority protection claim.

Were the Court to embrace an approach of measurement refinement over time, this Article would prove relevant to the process by which "lower courts . . . work out the precise contours of [partisan gerrymandering claim analysis] with time and experience."[18] Additionally, the efficiency gap measure represents a contribution to the election law and political science literature independent from the role it may play in *Whitford*. Political scientists are already producing scholarship that explores the relationship between partisan gerrymandering and other variables of interest, using the efficiency gap measure as the operational definition of partisan gerrymandering.[19] An evaluation of the efficiency gap measure is thus relevant not only to whether and how courts proscribe partisan gerrymandering, but also to how political scientists study it. Finally, in developing the efficiency gap measure, McGhee has discovered significant, surprising relationships between seats-votes curves and properties of wasted vote measures, such as the fact that under traditional definitions parties waste an equal number of votes when a party translates a 1% increase in votes into a 2% increase in seats.[20] This Article identifies other relationships of interest between wasted vote measures, seats-votes proportionality, competitiveness, and voter turnout.

The Article proceeds in five parts. Part I relates the necessary background in a way that frames the subsequent analysis, suggesting the utility and limits of

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manageable test, while allowing the lower courts to work out the precise contours of that test with time and experience."); *id.* at 25 ("Partisan-symmetry tests all answer the same, simple question and rely on a shared standard. But they are flexible enough to accommodate contextual differences, thus allowing courts to choose the test best suited for assessing a particular plan. . . . More importantly, a court can assess a districting plan using more than one symmetry test. Extreme gerrymanders will certainly perform poorly along more than one symmetry measure. Judges can thereby use multiple symmetry tests to assure themselves of the robustness of their assessment and identify extreme outliers."); Brief of Robin Best et al. as Amici Curiae in Support of Appellees at 3, *Whitford*, No. 16-1161 (Sept. 5, 2017), 2017 WL 4311099 (classifying the Wisconsin Assembly plan a partisan gerrymander based on an analysis employing the median-mean measure and 10,000 simulated alternative Assembly maps).

[18] Brief of Gerken et al., *supra* note 17, at 4.

[19] *See, e.g.*, Devin Caughey et al., *Partisan Gerrymandering and the Political Process: Effects on Roll-Call Voting and State Policies*, ELECTION L.J. (forthcoming 2017) (manuscript at 2-3).

[20] *See* McGhee, *Measuring Partisan Bias*, *supra* note 8, at 68-69 eq.5; *id.* at 79-82 app.B (deriving equation 5).

the efficiency gap measure. Political gerrymandering is a multi-normative structural problem the Court has struggled to regulate. The efficiency gap is designed to better measure partisan asymmetry using the ideal of equal wasted votes. But a legal standard for partisan gerrymandering must cohere with both an individual rights framework and a structural account of electoral democracy attentive to the multiple norms at stake. This suggests an inquiry into the efficiency gap's conceptual design and its relationship to competitiveness and seats-votes proportionality.

Part II explores the efficiency gap's conceptual design, examining five choices underlying the measure: the "efficiency principle" McGhee developed as a guide to the measure's design; the equal voter turnout assumption used to reduce the long-form equation to the simplified formula; the method of aggregating wasted votes to produce a single number; the definition and weight of surplus votes; and the two-party assumption.

Part III examines the efficiency gap's relationship to seats-votes proportionality and competitiveness. It shows that the efficiency gap can be understood as a competitiveness gap, expressed in terms of turnout and margin of victory, rather than wasted votes or undeserved seats. The efficiency gap measure may allow or even encourage mapmakers to draw plans that undermine electoral competitiveness and proportionality between votes earned and seats won. This creates a false positive problem, where the measure disfavors normatively desirable plans, and a false negative problem, where the measure favors normatively undesirable plans. The doctrinal tools of intent, justification, and sensitivity analysis only partially address the false positive problem, because mapmakers may fear not just invalidation but litigation. And these tools fail to address the false negative problem, because the efficiency gap proposal offers no mechanism to overcome the presumption of validity triggered by a below-threshold gap. This analysis also suggests unacknowledged measure convergence, where scholars or jurists invoke the competitiveness gap without realizing it is mathematically equivalent to the efficiency gap. Finally, the definition and weight of surplus votes determines the efficiency gap's relationship to the norms of electoral competitiveness and seats-votes proportionality. With a voter-centric definition of surplus votes (using the full margin of victory rather than half thereof) and a party-centric scale (comparing wasted vote totals rather than shares), the efficiency gap would idealize triple-proportionality, exacerbating the extreme vote share problem.

Part IV presents a new wasted vote measure designed to exhibit greater discernibility and structural coherence. This measure defines a surplus vote as the entire (rather than half of the) margin of victory and then compares the parties' wasted vote shares (rather than totals). This conceptual design is more voter-centric in terms of how wasted votes are designed and compared. And the measure bears a relationship to competitiveness and proportionality that better aligns with structural values and electoral reality.

Part V concludes, drawing doctrinal implications from the mathematical and legal analysis preceding it. First, questions of robustness and scope must be

addressed when setting the numeric threshold and computing a challenged plan's efficiency gap. Second, given the measure's normatively fraught relationship with competing democratic norms, courts should only use it as an indicative measure and not as the exclusive definition of partisan gerrymandering.

I. The Efficiency Gap's Power and Limits

A. Gerrymandering as a Multi-Normative Structural Problem

Jurists, scholars, politicians, media, reformers, and ordinary citizens agree that gerrymandering[21] poses a profound threat to democratic values.[22] And for good reason. Electoral districting confers on the mapmaker the power to shape electoral destiny—a power too easily abused.[23] To favor one party over another,

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[21] This Article employs the following terminology throughout: *gerrymandering* refers broadly to any manipulation of electoral boundaries; *malapportionment* refers to manipulation of the population size of electoral districts; *racial gerrymandering* refers to manipulation on the basis of race; *political gerrymandering* refers to manipulation with political intent and effect; *partisan gerrymandering* refers to manipulation intended to benefit one party over another; and *bipartisan gerrymandering* refers to manipulation intended to preserve safe seats for incumbents from both parties. Thus, partisan gerrymandering is one type of political gerrymandering.

[22] *See, e.g.*, Ariz. State Legislature v. Ariz. Indep. Redistricting Comm'n, 135 S. Ct. 2652, 2658 (2015) ("[incompatible] with democratic principles") (alteration in original) (quoting Vieth v. Jubelirer, 541 U.S. 267, 292 (2004) (plurality opinion)); Jack M. Balkin, *The Last Days of Disco: Why the American Political System Is Dysfunctional*, 94 B.U. L. REV. 1159, 1165 (2014) (including "exclusively single-member districts," "first-past-the-post election rules," and "[p]olitical gerrymandering" in a list of causes of "features of our current system that make it dysfunctional"); Robert Draper, *The League of Dangerous Mapmakers*, ATLANTIC (Oct. 2012), https://www.theatlantic.com/magazine/archive/2012/10/the-league-of/309084 ("the most insidious practice in American politics"); *see generally* DAVID DALEY, RATF**KED: THE TRUE STORY BEHIND THE SECRET PLAN TO STEAL AMERICA'S DEMOCRACY (2016).

[23] Samuel Issacharoff, *Gerrymandering and Political Cartels*, 116 HARV. L. REV. 593, 595 (2002) ("There is a core understanding in American politics, going back to the evocative imagery of the gerrymander, that geographically districted elections are subject to ends-oriented manipulation."). The risk of gerrymandering is an inherent feature of the practice of geographic electoral districting, whereby individual representatives for a multimember body are selected through separate elections conducted in geographic subunits (called electoral districts) of the entire jurisdiction represented by the entire body. Many countries eschew districting entirely, opting instead for some system of proportional representation, whereby representation of the entire body is distributed according to the support each party earns in a single election conducted over the entire jurisdiction. *Electoral Systems Around the World*, FAIRVOTE, http://www.fairvote.org/research_electoralsystems_world (last visited Oct. 22, 2017) (surveying thirty-five major democracies, twenty-nine of which use some form of proportional representation (as of 2012)). But from its inception to the present, the American electoral system has relied heavily on geographic electoral districting. Paul L. McKaskle, *Of Wasted Votes and No Influence: An Essay on Voting Systems in the United States*, 35 HOUS. L. REV. 1119, 1124 (1998). Most states have adopted a single-member simple plurality (SMSP) system, under which each electoral district is assigned one seat in the multimember

the mapmaker can simply dilute the influence of the disfavored party's supporters by assigning them to districts where their votes have less impact: by "packing" them into a few districts where their preferred candidates win by overwhelming margins, and/or by "cracking" them into many districts so that their preferred candidates lose each one. Aided by powerful computers—and prevailing patterns of residence and voting—the modern mapmaker can pack and crack with exquisite precision, and thereby distort the way political parties translate popular support (votes) into governmental power (seats).[24] With the stroke of a pen (or keyboard), the mapmaker can confer a legislative majority on a party supported by a minority of voters; or a legislative supermajority on a party supported by a slim majority of voters. As one state legislator put it, the practice of gerrymandering turns the process of electoral districting into "the business of rigging elections."[25] This is why legislatures guard their districting power so jealously,[26] why the districting process is often so partisan and secretive,[27] and why parties expend so many resources drawing and litigating

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body, and each district awards its seat to the candidate who earns the most votes in that district's race. *See* Bruce E. Cain, *Garrett's Temptation*, 85 VA. L. REV. 1589, 1601 (1999); *What Is Redistricting?*, ALL ABOUT REDISTRICTING, http://redistricting.lls.edu/what.php (last visited Mar. 21, 2017) ("Most of our federal legislators, all of our state legislators, and many of our local legislators in towns and counties are elected from districts. These districts divide states and the people who live there into geographical territories."). Electoral districting may in fact offer some advantages over proportional representation systems. *See* Peter H. Schuck, *The Thickest Thicket: Partisan Gerrymandering and Judicial Regulation of Politics*, 87 COLUM. L. REV. 1325, 1350-51 (1987); *see also* Nathaniel Persily, *In Defense of Foxes Guarding Henhouses: The Case for Judicial Acquiescence to Incumbent-Protecting Gerrymanders*, 116 HARV. L. REV. 649, 650 (2002). But it has one profound disadvantage: it is vulnerable to manipulation by political cartographers.

[24] *See Vieth*, 541 U.S. at 345-46 (Souter, J., dissenting) (citing Samuel Issacharoff, *Gerrymandering and Political Cartels*, 116 HARV. L. REV. 593, 624 (2002) [hereinafter Issacharoff, *Political Cartels*]; Pamela S. Karlan, *The Fire Next Time: Reapportionment After the 2000 Census*, 50 STAN. L. REV. 731, 736 (1998); Richard H. Pildes, *Principled Limitations on Racial and Partisan Redistricting*, 106 YALE L.J. 2505, 2553-54 (1997)).

[25] *Vieth*, 541 U.S. at 317 (Kennedy, J., concurring in the judgment) (quoting John Hoeffel, *Six Incumbents Are a Week Away from Easy Election*, WINSTON-SALEM JOURNAL, Jan. 27, 1998, at B1).

[26] For example, when Arizona voters, acting through initiative, transferred districting power from the state legislature to an independent commission, the legislature (unsuccessfully) challenged the constitutionality of this initiative all the way up to the United States Supreme Court. *See Ariz. State Legislature*, 135 S. Ct. at 2658-59.

[27] For example, the Wisconsin Assembly plan challenged in *Whitford* was produced with the use of nondisclosure agreements, expedited legislative procedures, a war room with limited access, and consultation exclusively with members of one party. *See* Emily Bazelon, *The New Front in the Gerrymandering Wars: Democracy vs. Math*, N.Y. TIMES MAG. (Aug. 29, 2017), https://www.nytimes.com/2017/08/29/magazine/the-new-front-in-the-gerrymandering-wars-democracy-vs-math.html (noting that "[n]early all of the 79 Republicans in the Wisconsin Senate and Assembly [visited] the map room, signing the same secrecy pledge to see the new

electoral districting plans.[28] Electoral districting entails districting power; such power invites abuse; we call such abuse gerrymandering. The term—a portmanteau of the surname "Gerry" and the word "salamander"—was coined in 1812 by a critic of the districting plan for the Massachusetts state senate, who likened its serpentine appearance to a salamander, and suggested that Governor Elbridge Gerry was behind it.[29] The term colorfully captures our intuitive sense—and visceral disgust—that manipulation of electoral districts subverts fundamental democratic norms.

But political gerrymandering, like its amphibian namesake, is slippery, repeatedly eluding efforts to curb it, in part because gerrymandering is a slippery concept resistant to precise, consensus-garnering definition and quantification. This is so because gerrymandering is an inherently structural phenomenon, concerning the functioning of a healthy electoral system,[30] and the relevant structural analysis is irreducibly multi-normative. Electoral districting implicates, and gerrymandering threatens, multiple democratic norms—including electoral competition, voter participation, majoritarianism, minority protection, and partisan fairness. There is a high-level consensus that districting power may be abused, but dissensus on the right way to draw electoral districts, and thus disagreement on precisely how to define and measure gerrymandering.[31] Just as doctors disagree on the most salient components of

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shape of their districts" whereas "[n]o Democrat was invited," and that the "Legislature passed the plan a week later, with the support of every Republican . . . and no Democrats").

[28] *See, e.g.*, Lisa Marshall Manheim, *Redistricting Litigation and the Delegation of Democratic Design*, 93 B.U. L. REV. 563, 612-13 (2013); David Daley, *The House the GOP Built: How Republicans Used Soft Money, Big Data, and High-Tech Mapping to Take Control of Congress and Increase Partisanship*, NEW YORK. (Apr. 24, 2016), http://nymag.com/daily/intelligencer/2016/04/gops-house-seats-are-safe-heres-why.html. Former President Obama and former Attorney General Eric Holder are focusing on redistricting reform through a newly formed organization called the National Democratic Redistricting Committee. Edward-Isaac Dovere, *Obama, Holder to Lead Post-Trump Redistricting Campaign*, POLITICO (Oct. 17, 2016), http://www.politico.com/story/2016/10/obama-holder-redistricting-gerrymandering-229868.

[29] *Ariz. State Legislature*, 135 S. Ct. at 2658 n.1 (citing ELMER C. GRIFFITH, THE RISE AND DEVELOPMENT OF THE GERRYMANDER 16-19 (1907)) (describing etymology); *Vieth*, 541 U.S. at 274 (plurality opinion) (citing WEBSTER'S NEW INTERNATIONAL DICTIONARY 1052 (2d ed. 1945)) (describing etymology).

[30] Heather K. Gerken, *Lost in the Political Thicket: The Court, Election Law, and the Doctrinal Interregnum*, 153 U. PA. L. REV. 503, 508 (2004) ("It is hard to figure out what is 'fair' or 'equal' in districting without speaking in structural terms. Any such conclusion would require a theory of representation, an idea about how a healthy democracy is supposed to function.").

[31] *See Vieth*, 541 U.S. at 307 (Kennedy, J., concurring in the judgment) ("No substantive definition of fairness in districting seems to command general assent."); Krasno et al., *supra* note 16, at 1 ("Partisan gerrymandering shares both of the characteristics of pornography that

health at stake in a treatment decision, legal scholars "are divided as to what [is] the most important structural consideration" that "capture[s] what is truly at stake" in electoral districting.[32] As Stephanopoulos puts it, "[t]wo approaches to redistricting have dominated the academic debate over the last generation: the partisan fairness approach, advocating that district plans treat the major parties symmetrically, and the competitiveness approach, advising that districts be made as competitive as is feasible."[33]

Some of the key democratic norms at stake involve the relationship between the votes a party earns and the seats it wins. I will now introduce some notation and basic properties that help to analyze this relationship, and that will feature prominently throughout my analysis. Let V and S respectively denote the total number of ballots cast and the total number of seats awarded in the election. Assume there is a set of parties (P), and for each party $p \in P$ let V_p, $\bar{V}_p$, V_p^* respectively denote that party's vote total, vote share, and vote margin, and let S_p, $\bar{S}_p$, and S_p^* respectively denote that party's seat total, seat share, and seat margin.[34] A party's vote total (V_p) is simply the number of ballots cast (in all districts) for that party; its vote share ($\bar{V}_p$) is its vote total divided by the total number of ballots cast: $\bar{V}_p = \frac{V_p}{V}$; and its vote margin ($V_p^*$) is the difference between its vote share and 50%: $V_p^* = \bar{V}_p - \frac{1}{2}$. For example, if one hundred ballots are cast ($V = 100$) and a party earns sixty of them ($V_p = 60$), its vote share is sixty percent ($\bar{V}_p = \frac{V_p}{V} = \frac{60}{100} = 0.6$), and its vote margin is ten percent ($V_p^* = \bar{V}_p - \frac{1}{2} = 0.6 - 0.5 = 0.1$). Note that a positive vote margin connotes majority status while a negative vote margin connotes minority status. The seat variables are defined analogously: a party's seat total (S_p) is simply the number of seats won by (candidates of) that party; its seat share ($\bar{S}_p$) is its seat total divided by the total number of seats: $\bar{S}_p = \frac{S_p}{S}$; and its seat margin ($S_p^*$) is the difference between its seat share and 50%: $S_p^* = \bar{S}_p - \frac{1}{2}$. For example, if a congressional plan consists of ten seats ($S = 10$) and a party earns four of them

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Potter Stewart famously wrestled with in his concurring opinion in *Jacobellis*: it is difficult to measure objectively and (therefore) a matter of subjective opinion.").

[32] Nicholas O. Stephanopoulos, *Elections and Alignment*, 114 COLUM. L. REV. 283, 295-96 (2014).

[33] Nicholas O. Stephanopoulos, *The Consequences of Consequentialist Criteria*, 3 U.C. IRVINE L. REV. 669, 673 (2013) [hereinafter Stephanopoulos, *Consequentialist Criteria*].

[34] Under this notation, used consistently throughout the Article, the bar accent indicates a share, while the star superscript indicates a margin.

($V_p = 4$), its seat share is forty percent ($\bar{S}_p = \frac{S_p}{S} = \frac{4}{10} = 0.4$), and its seat margin is negative ten percent ($S_p^* = \bar{S}_p - \frac{1}{2} = 0.4 - 0.5 = -0.1$).

These variables relate in simple ways in the special case where there are only two parties, an assumption generally adopted in the efficiency gap approach and in much of this Article's analysis. In this case, I refer to the two parties as Party *x* and Party *y*, i.e. $P = \{x, y\}$. Since every ballot is cast for, and every seat won by, one party or the other, simple relationships apply. The overall vote (seat) total is the sum of the two parties' vote (seat) totals.

$$V_x + V_y = V$$
$$S_x + S_y = S$$

Respective vote (seat) shares sum to one.[35]

$$\bar{V}_x + \bar{V}_y = 1$$
$$\bar{S}_x + \bar{S}_y = 1$$

Respective vote (seat) margins sum to zero.[36]

$$V_x^* + V_y^* = 0$$
$$S_x^* + S_y^* = 0$$

This means the parties' vote (seat) margins have equal magnitudes but opposite signs: $V_x^* = -V_y^*$ and $S_x^* = -S_y^*$. For this reason, I will sometimes assume, without loss of generality, that Party *x* enjoys a positive vote (seat) margin, and refer to the vote margin, denoted simply as V^*instead of V_x^*, and the seat margin denoted simply as S^*instead of S_x^*. Each party's vote (seat) share can then be expressed in terms of the vote (seat) margin.[37]

$$\bar{V}_x = \frac{1}{2} + V^*;\ \bar{V}_y = \frac{1}{2} - V^*$$
$$\bar{S}_x = \frac{1}{2} + S^*;\ \bar{S}_y = \frac{1}{2} - S^*$$

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[35] Proof: $\bar{V}_x + \bar{V}_y = \frac{V_x}{V} + \frac{V_y}{V} = \frac{V_x + V_y}{V_x + V_y} = 1$; $\bar{S}_x + \bar{S}_y = \frac{S_x}{S} + \frac{S_y}{S} = \frac{S_x + S_y}{S_x + S_y} = 1$.

[36] Proof: $V_x^* + V_y^* = \left(\bar{V}_x - \frac{1}{2}\right) + \left(\bar{V}_y - \frac{1}{2}\right) = \left(\bar{V}_x + \bar{V}_y\right) - \left(\frac{1}{2} + \frac{1}{2}\right) = 1 - 1 = 0$;
$S_x^* + S_y^* = \left(\bar{S}_x - \frac{1}{2}\right) + \left(\bar{S}_y - \frac{1}{2}\right) = \left(\bar{S}_x + \bar{S}_y\right) - \left(\frac{1}{2} + \frac{1}{2}\right) = 1 - 1 = 0$.

[37] Proof: $V^* = V_x^* = \bar{V}_x - \frac{1}{2}$. Thus: $\bar{V}_x = \frac{1}{2} + V^*$; $\bar{V}_y = 1 - \bar{V}_x = 1 - \left(\frac{1}{2} + V^*\right) = \frac{1}{2} - V^*$;
$S^* = S_x^* = \bar{S}_x - \frac{1}{2}$. Thus: $\bar{S}_x = \frac{1}{2} + S^*$; $\bar{S}_y = 1 - \bar{S}_x = 1 - \left(\frac{1}{2} + S^*\right) = \frac{1}{2} - S^*$.

And the difference between vote (seat) shares is twice the vote (seat) margin.[38]

$$\bar{V}_x - \bar{V}_y = 2V^*$$
$$\bar{S}_x - \bar{S}_y = 2S^*$$

Political scientists analyze how a party's seat share ($\bar{S}_p$) does and should vary with its vote share ($\bar{V}_p$) by conceptualizing seat share as a function of vote share: $\bar{S}_p(\bar{V}_p)$. They illustrate this relationship graphically by drawing a "seats-votes curve."[39] Diagram 1 provides an example of a seats-votes curve. A single point on the curve represents an electoral outcome—the seat share a party earns at a given vote share; the curve itself represents a range of outcomes corresponding to different values of the party's vote share. [*Insert Diagram 1*]

Diagram 2 illustrates the concepts of seats-vote proportionality and seats-votes responsiveness. Seats-votes proportionality captures the absolute relationship between vote share and seat share—the ratio between vote share and seat share. Graphically, it is the slope of the line connecting the origin (0,0) to the point of the observed electoral outcome. Seats-votes responsiveness captures the marginal relationship—the ratio between an incremental change in vote share and the corresponding incremental change in seat share. Graphically, it is the slope of the tangent at the point of the observed electoral outcome. In the language of differential calculus, it is the derivate of the seats-votes function at the point of the observed electoral outcome. Seats-votes responsiveness is one measure of electoral competitiveness. The more districts with close races won by small margins of victory, the more seat flips and seat share changes for a given shift in vote share.[40] [*Insert Diagram 2*]

Only when a seats-votes curve is truly curved can proportionality and responsiveness diverge. If a seats-votes curve is a straight line, proportionality and responsiveness are equal. For example, strict proportionality, where a party's seat share is simply its vote share, corresponds to a straight line through the origin with a 45-degree angle. This line runs from the point (0,0) (where a party receives no votes and no seats) to the point (1,1) (where a party receives all the votes and all the seats), passing through the point (0.5,0.5) (where a party receives half the votes and half the seats). Some argue for an ideal of strict proportionality between a party's vote share and its seat share, and on this basis,

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[38] Proof: $\bar{V}_x - \bar{V}_y = \left(\frac{1}{2} + V^*\right) - \left(\frac{1}{2} - V^*\right) = 2V^*$; $\bar{S}_x - \bar{S}_y = \left(\frac{1}{2} + S^*\right) - \left(\frac{1}{2} - S^*\right) = 2S^*$.

[39] A descriptive seat-votes curve estimates the relationship that actually exists in the real world. A prescriptive seats-votes curve indicates the ideal relationship that ought to exist in a healthy, well-functioning democracy.

[40] Stephanopoulos, *Consequentialist Criteria*, *supra* note 33, at 678 ("electoral responsiveness indicate[s] . . . how competitive individual districts are").

propose that we replace our districting-based electoral system with one explicitly based on proportional representation.[41]

In the real world, the actual seats-votes curves estimated by political scientists are not straight lines, but S-shaped curves that exhibit lower responsiveness (flatter slopes) when one party enjoys a large majority and higher responsiveness (steeper slopes) when the electorate is more evenly split between the two parties. Diagrams 3-5 provide examples of linear and non-linear seats-votes curves. *[Insert Diagrams 3-5]* Political scientists have found that average seats-votes responsiveness is usually not one, as strict proportionality would require, but generally closer to two.[42] The result is a "seat bonus," whereby the majority translates a positive vote margin into an even larger seat margin.[43] For example, with 51% vote share (1% vote margin), a party may earn 52% seat share (2% seat margin), instead of the 51% seat share mandated by strict proportionality. Some argue this is normatively undesirable because it departs from strict proportionality;[44] others argue it is normatively desirable, because it incentivizes robust campaigning and promotes a stable, functioning legislative majority.[45]

The S-shaped nature of actual seats-votes curves reflects an accommodation between competing norms, ensuring representation for parties with minority vote share while rewarding parties with majority vote share. As Browning and King aptly put it, this approach "reflects an important principle of the United States two-party, democratic system. It helps majorities form, yet protects the minority party."[46] Specifically, it helps majorities form by exhibiting high seats-votes responsiveness when the two parties earn similar vote share, but protects the minority party by exhibiting low seats-votes responsiveness when one party earns most of the vote share.

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[41] *See generally, e.g.*, DOUGLAS J. AMY, REAL CHOICES/NEW VOICES: HOW PROPORTIONAL REPRESENTATION ELECTIONS COULD REVITALIZE AMERICAN DEMOCRACY (2d ed. 2002).

[42] *See* Nicholas M. Goedert, *Redistricting, Risk, and Representation: How Five State Gerrymanders Weathered the Tides of the 2000s*, 13 ELECTION L.J. 406, 413 (2014); Edward R. Tufte, *The Relationship Between Seats and Votes in Two-Party Systems*, 67 AM. POL. SCI. REV. 540, 542 (1973). The *Whitford* majority emphasized that both parties stipulated that the simplified formula's "implied 2-to-1 votes-to-seats relationship reflects the 'observed average seat/votes curve in historical U.S. congressional and legislative elections.'" Whitford v. Gill, 218 F. Supp. 3d 837, 907 (W.D. Wis. 2016) (quoting Joint Final Pretrial Report at 28 ¶ 105, *Whitford*, 218 F. Supp. 3d 837 (No. 15-00421)).

[43] *See* Mitchell N. Berman, *Managing Gerrymandering*, 83 TEX. L. REV. 781, 806 n.165 (2005); Adam Cox, Commentary, *Partisan Fairness and Redistricting Politics*, 79 N.Y.U. L. REV. 751, 765 (2004).

[44] *See, e.g.*, AMY, *supra* note 41, at 34-41.

[45] *See, e.g.*, Schuck, *supra* note 23, at 1350-51.

[46] Robert X. Browning & Gary King, *Seats, Votes, and Gerrymandering: Estimating Representation and Bias in State Legislative Redistricting*, 9 L. & POL'Y 305, 313 (1987).

The seats-votes framework helps illustrate how electoral districting implicates and involves trade-offs between multiple democratic values. This in turn helps explain why there is no universal agreement on the right way to draw electoral districts. Of course, the Court's task is not to prescribe ideal districting practices as a matter of policy or abstract democratic theory, but to distinguish valid from invalid districting practices as a matter of federal constitutional law. And the question fracturing the Court is not just whether political gerrymandering presents a constitutional problem, but whether the Court can address it. In the language of federal court jurisprudence, the question is whether gerrymandering constitutes a justiciable legal claim the courts can adjudicate or a nonjusticiable political question the courts cannot address, which turns on whether or not the Court can identify a judicially discernible and manageable standard to channel and limit judicial intervention.[47] Thus, gerrymandering presents two distinct but related questions: justiciability (is there an adequate standard to guide judicial intervention?) and identification (what is it?). These questions implicate both the relationship between electoral districting practices and constitutionally significant representational norms and the proper role of the federal judiciary in regulating electoral districting practices pursuant to these norms. Both are democratic problems of profound constitutional significance on which the Constitution provides limited explicit guidance. Although political gerrymanders undoubtedly implicate constitutional values,[48] the Constitution's text offers limited procedural guidance on congressional and state legislative elections.[49] And the precise scope of the federal judicial power to adjudicate

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[47] Specifically, the presence vel non of a judicially discernible and manageable standard is the second of six factors under the political question doctrine; the first is whether the constitutional text provides for resolution of the issue by a coordinate branch; another is whether there is "an unusual need for unquestioning adherence to a political decision already made"; the rest concern whether adjudication would require a policy determination courts cannot make, express disrespect to a coordinate branch, or risk embarrassment from interbranch dissensus. *See Baker v. Carr*, 369 U.S. 186, 217 (1962).

[48] *See supra* note 2. The Constitution provides that members of the House of Representatives are to be "chosen . . . by the People," U.S. CONST. art. I, § 2, cl. 1; guarantees to each state "a Republican Form of Government," *id.* art. IV, § 4, cl. 1; protects freedoms of expression and association, *id.* amend. I; enshrines due process and equal protection, *id*. amend. XIV, § 1; and prohibits race-based electoral discrimination, *id.* amend. XV, § 1. Each of these provisions codifies values threatened by gerrymandering.

[49] Article I vests the federal legislative power in a Congress composed of two multimember legislative bodies: a House, apportioned on a population basis and popularly elected, U.S. CONST. art. I, § 2, cls. 1, 3, and a Senate, apportioned on the basis of equal state suffrage, *id*. art. I, § 3, cl. 1 (two Senators per State); *id.* art. V (constraining amendment of Senate apportionment), with each state's senators originally "chosen by the Legislature thereof," *id.* art. I, § 3, cl. 1 (amended 1913), and now "elected by the people thereof," *id.* amend. XVII. But the Elections Clause does not mandate how to conduct congressional elections; instead, it gives the choice to individual states and Congress. *See id.* art. I, § 4, cl. 1. And while the Constitution assumes that each state has at least one popularly elected legislative body, *see,*

federal constitutional claims is also a question without an explicit textual answer.[50] In this sense, both action and inaction by the Court on political gerrymandering claims present real but ineffable constitutional risks.

B. The Efficiency Gap As an Improved Measure of Partisan Symmetry

Because political gerrymandering is so slippery, it has repeatedly eluded the Court's grasp. The Court has constrained race-based manipulation of district shape[51] and political manipulation of district population size[52]—meaningfully

e.g., *id.* art. I, § 2, cl. 1; *id.* amend. XVII, it says nothing explicitly about how to conduct state legislative elections, implicitly leaving that choice to each state as well.

[50] Article III, Section 2 provides that the "judicial Power shall extend" to an enumerated set of "Cases" and "Controversies," the first of which is "all Cases, in Law and Equity, arising under [federal law]." U.S. CONST. art. III, § 2, cl. 1. While "[i]t is emphatically the province and duty of the judicial department to say what the law is," Marbury v. Madison, 5 U.S. (1 Cranch) 137, 177 (1803), some "subjects are political" and so "can never be examinable by the courts," *id.* at 166. The political question doctrine, like all justiciability doctrines partially discerned from the text of Article III, relates in part to "an idea, which is more than an intuition but less than a rigorous and explicit theory, about the constitutional and prudential limits to the powers of an unelected, unrepresentative judiciary in our kind of government." Elk Grove Unified Sch. Dist. v. Newdow, 542 U.S. 1, 11 (2004) (quoting Allen v. Wright, 468 U.S. 737, 750 (1984)). One may question whether the Court, in developing the political question doctrine, has succeeded in divining, from Article III and its animating structural principles, a discernible and manageable test for justiciability.

[51] When electoral districting implicates race, the Court and Congress have imposed two principal legal constraints. First, pursuant to the Reconstruction Amendments, the Court evaluates a districting plan under strict scrutiny whenever considerations of race predominate, thereby curtailing intentional race-based cracking or packing. *See* Cooper v. Harris, 137 S. Ct. 1455, 1464 (2017); Bethune-Hill v. Va. State Bd. of Elections, 137 S. Ct. 788, 798 (2017); Shaw v. Reno, 509 U.S. 630, 642 (1993). Second, the prohibition on racial vote dilution codified in Section 2 of the Voting Rights Act, as amended in 1982 and construed by the Court, constrains the ability of a mapmaker to crack a sufficiently numerous, politically cohesive, and geographically compact group of minority voters in the presence of racially polarized voting. *See* 52 U.S.C. § 10301 (2015); Bartlett v. Strickland, 556 U.S. 1, 11-12 (2009) (plurality opinion); Johnson v. De Grandy, 512 U.S. 997, 1011 (1994); Growe v. Emison, 507 U.S. 25, 39-40 (1993); Thornburg v. Gingles, 478 U.S. 30, 48-51 (1986). Section 5 of the Voting Rights Act imposed one additional constraint: A covered jurisdiction could only implement a new districting plan, or any other electoral "standard, practice, or procedure", after persuading the Department of Justice or a federal court that the plan would not have a racially retrogressive effect. *See* 52 U.S.C. § 10304 (2015). But since the Court struck down the coverage formula, *see* Shelby County v. Holder, 133 S. Ct. 2612, 2631 (2013), and Congress has yet to adopt a new one, this third constraint on racial gerrymandering is presently inoperative.

[52] For decades, the Court dismissed malapportionment challenges as nonjusticiable political questions, heeding Justice Frankfurter's admonition not to enter the "political thicket," Colegrove v. Green, 328 U.S. 549, 556 (1946), before reversing course, adopting the one person, one vote principle, and thereby launching the reapportionment revolution, *see*

limiting mapmakers' packing and cracking abilities. But the Court has repeatedly fractured on whether and how to intervene when political cartographers manipulate district shape based on party support. In *Davis v. Bandemer*, three Justices argued that partisan gerrymandering was a political question,[53] but six Justices insisted it was justiciable under the Equal Protection Clause[54] and agreed that plaintiffs must demonstrate both discriminatory intent and discriminatory effect.[55] Yet those six Justices disagreed among themselves on the correct legal test for discriminatory effect.[56] For the next eighteen years, the lower courts applied Justice White's standard—and rejected at the motions stage every partisan gerrymandering claim they considered.[57]

In *Vieth v. Jubelirer*, the four conservative Justices argued partisan gerrymandering was a political question.[58] The four liberal Justices insisted it

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Reynolds v. Sims, 377 U.S. 533, 556-58 (1964); Baker v. Carr, 369 U.S. at 197-98 (1962). For examples of scholarship recognizing and exploring the significance of the reapportionment revolution, see GORDON E. BAKER, THE REAPPORTIONMENT REVOLUTION: REPRESENTATION, POLITICAL POWER, AND THE SUPREME COURT 3-6 (1966); GARY W. COX & JONATHAN N. KATZ, ELBRIDGE GERRY'S SALAMANDER: THE ELECTORAL CONSEQUENCES OF THE REAPPORTIONMENT REVOLUTION 12-13 (2002); Gordon E. Baker, *The Unfinished Reapportionment Revolution*, *in* POLITICAL GERRYMANDERING AND THE COURTS 11, 11-16 (Bernard Grofman ed., 1990).

[53] Davis v. Bandemer, 478 U.S. 109, 144 (1986) (O'Connor, J., concurring in the judgment) (joined by Burger, C.J., & Rehnquist, J.).

[54] *Id.* at 113 (plurality opinion) (joined by Brennan, Marshall, & Blackmun, JJ.); *id.* at 161 (Powell, J., concurring in part and dissenting in part) (joined by Stevens, J.).

[55] *Id.* at 127 (plurality opinion); *id.* at 161 (Powell, J., concurring in part and dissenting in part).

[56] Justice White, writing for a four-Justice plurality, proposed a stringent but vague "consistent degradation" test under which a departure from seats-votes proportionality would be insufficient to establish discrimination. *Id.* at 132 (plurality opinion) (joined by Brennan, Marshall, & Blackmun, JJ.) ("[T]he mere lack of proportional representation will not be sufficient . . . . Rather, unconstitutional discrimination occurs only when the [challenged plan] . . . will consistently degrade a voter's or a group of voters' influence on the political process as a whole."). Justice Powell, joined by Justice Stevens, proposed a standard that would have been easier for courts to apply and plaintiffs to meet. *See id.* at 173 & n.13 (Powell, J., concurring in part and dissenting in part) (proposing consideration of multiple factors including "the shapes of voting districts," "adherence to established political subdivision boundaries," "legislative procedures," "population disparities," and "disproportionate election results").

[57] *See* Vieth v. Jubelirer, 541 U.S. 267, 279-80 (2004) (plurality opinion) ("[I]n *all* of the cases we are aware of involving that most common form of political gerrymandering, relief was denied." (emphasis in original)); *id.* at 280 n.6 (citing cases).

[58] *Id.* at 281, 303 (authored by Scalia, J., and joined by Rehnquist, C.J., O'Connor, J., & Thomas, J.).

was justiciable, but offered three different legal tests.[59] Justice Kennedy rejected each standard proposed,[60] but suggested that partisan gerrymandering may be justiciable if a suitable standard could be identified.[61] Justice Kennedy emphasized that judicial intervention required "clear, manageable, and politically neutral standards for measuring the particular burden a given partisan classification imposes on representational rights,"[62] and suggested the First Amendment may offer a better textual basis than the Equal Protection Clause for such standards.[63] Specifically, partisan gerrymandering may infringe "the First Amendment interest of not burdening or penalizing citizens because of their participation in the electoral process, their voting history, their association with a political party, or their expression of political views."[64]

In *League of United Latin American Citizens (LULAC) v. Perry*,[65] the Court fractured along similar lines.[66] Justice Scalia, joined by Justice Thomas, continued to insist that partisan gerrymandering was a nonjusticiable political question.[67] The four liberal Justices continued to favor justiciability and suggest

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[59] Justice Stevens suggested a "predominant motivation standard" based on the *Shaw v. Reno* cause of action for racial gerrymandering. *Id.* at 339, 341 (Stevens, J., dissenting). Justice Souter proposed a burden-shifting framework modelled on Title VII doctrine with a five-factor prima facie case. *Id.* at 346-51 (Souter, J., dissenting) (joined by Ginsburg, J.). Justice Breyer proposed a test based on "unjustified [partisan] entrenchment," whereby a party with a minority of vote share achieves a majority of seat share through "partisan manipulation." *Id.* at 360 (Breyer, J., dissenting). The *Vieth* plaintiffs proposed to demonstrate discriminatory effect by showing that the challenged plan "systematically 'pack[s]' and 'crack[s]' the rival party's voters" and thereby threatens to "thwart the plaintiffs' ability to translate a majority of votes into a majority of seats." *Id.* at 286-87 (plurality opinion) (quoting Brief for Appellants at 19-20, Vieth v. Jubelirer, 541 U.S. 267 (2004) (No. 02-1580), 2003 WL 22070244).

[60] 541 U.S. at 308 (Kennedy, J., concurring in the judgment).

[61] *Id.* at 306.

[62] *Id.* at 307-08.

[63] *Id.* at 314 ("The First Amendment may be the more relevant constitutional provision in future cases that allege unconstitutional partisan gerrymandering."); *id.* at 315 ("Where it is alleged that a gerrymander had the purpose and effect of imposing burdens on a disfavored party and its voters, the First Amendment may offer a sounder and more prudential basis for intervention than does the Equal Protection Clause.").

[64] *Id.* at 314.

[65] 548 U.S. 399 (2006).

[66] *See id.* at 406-07; *supra* note 4 and accompanying text. The case fractured the Court in a particularly severe and complex fashion because it presented both an unsuccessful claim of partisan gerrymandering and a successful claim of racial vote dilution under Section 2 of the Voting Rights Act. Compare *id*. at 423 ("appellants have established no legally impermissible use of political classifications.") with *id*. at 442 ("the totality of the circumstances demonstrates a § 2 violation.")

[67] *LULAC*, 548 U.S. at 511 (Scalia, J., concurring in the judgment in part and dissenting in part).

alternative legal tests.[68] Justice Kennedy again rejected each proffered standard, but left open the possibility that an adequate standard may yet materialize.[69]

But this time, political scientists Gary King, Bernard Grofman, Andrew Gelman, and Jonathan Katz proposed a new partisan "symmetry" standard.[70] Their standard "require[d] that the electoral system treat similarly-situated political parties equally, so that each receives the same fraction of legislative seats for a particular vote percentage as the other party would receive if it had received the same percentage [of the vote]."[71] This principle of partisan symmetry does not require strict seats-votes proportionality.[72] Rather, in the language of the seats-votes framework, this principle requires that the seats-votes curve be symmetric, which entails that it passes through the point (0.5,0.5) so that each of the two major parties gets half the seats when they earn half the votes. Diagram 6 clarifies the difference between partisan symmetry and strict seats-votes proportionality. *[Insert Diagram 6]* If the seats-votes curve accords one party more than half the seats when it earns half the votes, the additional seats constitute a measure of partisan asymmetry that political scientists often refer to as "bias." Diagram 7 provides an example of a curve that violates partisan symmetry, and thereby exhibits bias. *[Insert Diagram 7]*

This notion of tables-turned partisan symmetry is normatively appealing, but it necessarily entails some comparison to a counterfactual hypothetical where the parties' respective vote shares are switched, or each party earns half the votes. To evaluate this hypothetical, an analyst must make some assumptions about the geographic distribution of vote-switchers across electoral districts. The liberal Justices expressed interest in the partisan symmetry concept.[73] But Justice

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[68] *Id.* at 447 (Stevens, J., concurring in part and dissenting in part); *id.* at 483 (Souter, J., concurring in part and dissenting in part) (joined by Ginsburg, J.); *id.* at 491-92 (Breyer, J., concurring in part and dissenting in part).

[69] *Id.* at 414 (plurality opinion) (recounting the *Vieth* majority's refusal to hold partisan gerrymanders nonjusticiable, declining to "revisit the justiciability holding," and "proceed[ing] to examine whether appellants' claims offer the Court a manageable, reliable measure of fairness for determining whether a partisan gerrymander violates the Constitution."); *id.* at 446 (noting that "appellants . . . lack any reliable measure of partisan fairness"); *id.* at 418 (explaining that "a successful claim attempting to identify unconstitutional acts of partisan gerrymandering must . . . show a burden, as measured by a reliable standard, on the complainants' representational rights"). Chief Justice Roberts, joined by Justice Alito, agreed with Justice Kennedy's conclusion that plaintiffs failed to provide an adequate standard, but declined to weigh in on the question of justiciability. *Id.* at 492-93 (Roberts, C. J., concurring in part, concurring in the judgment in part, and dissenting in part).

[70] Brief for Gary King et al. as Amici Curiae in Support of Neither Party at 4-5, Jackson v. Perry, 546 U.S. 1163 (2006) (No. 05-276), 2006 WL 53994.

[71] *Id.*

[72] *Id.* at 7-8.

[73] *See LULAC*, 548 U.S. at 466 (Stevens, J., concurring in part and dissenting in part) ("[T]he symmetry standard, a measure social scientists use to assess partisan bias . . . is undoubtedly

Kennedy identified three concerns with this proposal: (1) it involved "conjecture";[74] (2) it relied on a counterfactual "hypothetical" rather than a directly observed election;[75] and (3) it provided no guidance on how much departure from the ideal is "too much."[76]

Since *LULAC*, partisan gerrymandering has remained in doctrinal limbo. Without a clear federal legal test for partisan gerrymandering, plaintiffs have an incentive to attack political gerrymanders as racial gerrymanders, and defendants have an incentive to justify partisan plans as efforts to comply with the Voting Rights Act. In the context of "conjoined polarization"—the "more consistent alignment of race, party, and ideology since" passage of the original Voting Rights Act[77]—race and party are easy for litigants to conflate and hard for courts to distinguish.[78] And, perversely, by requiring perennial redistricting, the one-person-one-vote standard gave mapmakers new opportunities to manipulate electoral district boundaries, making electoral districting a moving target resistant to judicial oversight.[79]

It is against this backdrop that Eric McGhee and Nicholas Stephanopoulos entered the scene.[80] They designed the efficiency gap to operationalize the same

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'a reliable standard' . . . ." (quoting *id.* at 418 (plurality opinion))); *id.* at 483-84 (Souter, J., concurring in part and dissenting in part) (declining to "rule out the utility of a criterion of symmetry as a test," noting that "[i]nterest in exploring this notion is evident" and suggesting that "further attention could be devoted to [its] administrability").

[74] *Id.* at 420 (plurality opinion) ("The existence or degree of asymmetry may in large part depend on conjecture about where possible vote-switchers will reside.").

[75] *Id.* ("[W]e are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs.").

[76] *Id.* ("[T]he counterfactual plaintiff would face the same problem as the present, actual appellants: providing a standard for deciding how much partisan dominance is too much.").

[77] Bruce E. Cain & Emily R. Zhang, *Blurred Lines: Conjoined Polarization and Voting Rights*, 77 OHIO ST. L.J. 867, 869 (2016) (emphasis omitted).

[78] *See* Cooper v. Harris, 137 S. Ct. 1455, 1472-81 (2017); Richard L. Hasen, *Race or Party, Race as Party, or Party All the Time: Three Uneasy Approaches to Conjoined Polarization in Redistricting and Voting Cases*, WM. & MARY L. REV. (forthcoming 2018) (manuscript at 1-3) (on file with author), https://ssrn.com/abstract=2912403.

[79] *See* Pamela S. Karlan, *John Hart Ely and the Problem of Gerrymandering: The Lion in Winter*, 114 YALE L.J. 1329, 1339 (2005) ("[W]e have moved from entrenchment through inaction to a perhaps even more pathological phenomenon of entrenchment through nonstop action.").

[80] The "efficiency gap" proposal was presented in the academic literature through two related articles: a political science article published in 2014 by Eric McGhee, *see* McGhee, *Measuring Partisan Bias*, *supra* note 8, and a 2015 law review article co-authored by Eric McGhee and Nicholas Stephanopoulos, *see* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6. The 2014 article introduced the numeric measure and demonstrated its key technical properties, *see* McGhee, *Measuring Partisan Bias*, *supra* note 8, at 68-70 app. B, while the 2015 article developed the measure into a proposed legal standard, *see* Stephanopoulos &

principle of partisan symmetry that five Justices found appealing in *LULAC* while addressing each of Justice Kennedy's concerns. The proponents designed the efficiency gap to "aggregate[] all of a district plan's cracking and packing choices into a single, tidy number"[81] and thereby distill "the essence of what critics have in mind when they refer to partisan gerrymandering."[82] The proponents define a partisan gerrymander as "a district plan that results in one party wasting many more votes than its adversary."[83] "Wasted votes include both 'lost' votes (those cast for a losing candidate) and 'surplus' votes (those cast for a winning candidate but in excess of what she needed to prevail)."[84]

To calculate the efficiency gap, "[e]ach party's wasted votes are totaled, one sum is subtracted from the other, and then . . . this difference is divided by the total number of votes cast."[85] Proponents then deploy an assumption to reduce the long-form equation for the efficiency gap (ΔW) into a simplified formula: the difference between the seat margin and twice the vote margin.[86]

$$\Delta W = S^* - 2V^*$$

According to this formula, in an ideal world without partisan gerrymandering (defined as equal wasted votes), each party's seat margin (S^*) would be twice its vote margin (V^*).[87]

$$\Delta W = 0 \text{ if and only if } S^* = 2V^*$$

McGhee, *The Efficiency Gap*, *supra* note 6, at 884-95. Plaintiffs then adopted an efficiency gap approach in litigation challenging Wisconsin's state assembly plan (the *Whitford* litigation) and North Carolina's congressional plan (the *Rucho* litigation). In 2017, McGhee published an article that responds to critiques made here and elsewhere, addressing previously unexamined technical and conceptual aspects of the efficiency gap measure, and offering a more refined and generalized conceptualization. *See* Eric McGhee, *Measuring Efficiency in Redistricting*, 16 ELECTION L.J. 417, 426-31 (2017).

[81] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 834.

[82] *Id.* at 852.

[83] *Id.* at 849-50.

[84] *Id.* at 850-51.

[85] *Id.* at 851-52.

[86] McGhee, *Measuring Partisan Bias*, *supra* note 8, at 68 (equation 5), app.B at 79-82 (deriving equation 5); *see also* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 853 & n.114.

[87] Stephanopoulos & McGhee, *The Efficiency* Gap, *supra* note 6, at 854 ("[E]ach additional percentage point of vote share for a party should result in an extra two percentage points of seat share. This relationship is implied by the efficiency gap formula noted above. If the gap is zero, it can remain at this level only if any shift in seat share is twice the size of any shift in vote share.")

According to this simplified formula, the efficiency gap has a simple interpretation within the seats-votes framework, as illustrated in Diagram 8. *[Insert Diagram 8]* The efficiency gap represents the vertical distance between the observed vote-seat combination and the "ideal" seats-votes curve corresponding to equal wasted votes. On this basis, proponents present the gap as a measure of the "undeserved seat share" attributable to partisan gerrymandering rather than the party's popularity.[88] For a given level of vote share, the ideal curve tells us what seat share a party would achieve under the ideal of partisan symmetry where each party wastes equal votes. Any seat share beyond that level is undeserved in the sense that it is attributable to the partisan asymmetry of the favored party wasting fewer votes than the disfavored party. Like the seats-votes curve for strict proportionality, this one is a straight line, running through the point where each party equally splits votes and seats (0.5,0.5). At this one point, both strict proportionality and equal wasted votes are achieved. But this new seats-votes curve has a slope of *two* instead of *one*. This means that, according to the efficiency gap, the ideal seats-votes relationship is one of double-responsiveness and double-proportionality. As the proponents explain, "[T]he gap offers what scholars to date have been unable to supply: a normative guide as to how large [the seat] bonus should be. To produce partisan fairness—in the sense of equal wasted votes for each party—the bonus should be a precisely twofold increase in seat share for a given increase in vote share."[89] For example, if a party earns 52% vote share, it deserves 54% seat share; if it actually obtains 56% seat share, there is an efficiency gap of 2%. Note that the proponents defined partisan symmetry as equal wasted votes, and then demonstrated that this requirement (under definitions and assumptions I analyze below), is mathematically equivalent to a requirement that the ideal seats-votes curve exhibit double proportionality and responsiveness.[90] Thus, the proponents claim that the seats-votes ratio should be 2:1 because that corresponds to the normative ideal of equal wasted votes.[91]

The 2015 article proposed a legal test for political gerrymandering based on the efficiency gap measure.[92] If the plan's efficiency gap exceeds a numeric

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[88] *Id.* (The efficiency gap "is a measure of undeserved seat share: the proportion of seats a party receives that it would *not* have received under a plan with equal wasted votes." (emphasis in original)).

[89] *Id.* at 854 (footnotes omitted).

[90] *Compare* McGhee, *Measuring Partisan Bias*, *supra* note 8, at 68 eq.2 (defining the measure algebraically), *with id.* eq.5 (presenting simplified formula), *and id.* app.B at 79-80 (deriving the simplified formula). *Compare* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 851 & n.110 (defining the measure linguistically and citing to the algebraic definition in the 2014 article), *with id.* at 853 & n.114 (presenting simplified formula and citing its derivation in the 2014 article).

[91] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 854.

[92] *See id.* at 884-95.

threshold, and sensitivity analysis suggests that the plan will continue to produce an above-threshold gap in future elections, it is presumptively invalid.[93] This presumption can only be overcome if the plan's partisan effect can be justified or explained as the product of legitimate redistricting criteria consistently applied to the jurisdiction's underlying political geography.[94] Such criteria include contiguity, compactness, preservation of local political boundaries, preservation of communities of interest, and compliance with the Voting Rights Act.[95]

The academic proponents originally proposed a numeric threshold of 8% based on their analysis of historical practice—gaps above 8% represent outliers relative to the distribution of gaps produced by modern electoral maps.[96] The *Whitford* plaintiffs' expert, Professor Simon Jackman, proposed a numeric threshold of 7% based on durability—gaps above 7% tend to persist for the life of an electoral map.[97]

The proposed efficiency gap measure and associated legal test were explicitly framed as an effort to improve upon the partisan bias proposal offered by amici and considered by the Court in *LULAC*.[98] It relies on an intuitive and constitutionally discernible concept of symmetric partisan treatment viewed favorably by five Justices in *LULAC*, while addressing the inadequacies Justice Kennedy identified with the symmetry measure proposed by the *LULAC* amici.[99] The measure of partisan symmetry proposed by the *LULAC* amici necessarily relied on assumptions to compare party performance in hypothetical counterfactuals.[100] The efficiency gap compares party performance directly observed in actual election results without necessarily relying on inferential techniques.[101] And the proposed efficiency gap test answers the question of how

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[93] *Id.* at 885, 891.

[94] *Id.* at 884.

[95] *See, e.g.*, Vieth v. Jubelirer, 541 U.S. 267, 284 (2004) (plurality opinion).

[96] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 884, 888-89.

[97] Expert Report of Simon Jackman at 66-69, Whitford v. Gill, 218 F. Supp. 3d 837 (W.D. Wis. 2016) (No. 15-00421), 2015 WL 10091020.

[98] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 895.

[99] *See id.* at 895-99.

[100] League of United Latin Am. Citizens (LULAC) v. Perry, 548 U.S. 399, 420 (2006) (plurality opinion) ("*Amici*'s proposed standard does not compensate for appellants' failure to provide a reliable measure of fairness. The existence or degree of asymmetry may in large part depend on conjecture about where possible vote-switchers will reside. . . . [W]e are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs.").

[101] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 857. However, inferential techniques are used for sensitivity testing and in the case of uncontested districts. *See id.* at 866-67, 889-90 (discussing uncontested districts and sensitivity testing).

much advantage is too much[102] with a numeric threshold of presumptive validity set on the bases of historical practice and durability. An above-threshold efficiency gap is a concrete indication that the electoral map favors one party in a way that is likely to persist for the life of the map and that departs from historical practice. Finally, the measure is distinct from a requirement of strict proportionality, a standard the Court has already rejected. Unlike strict proportionality, this double-proportionality measure permits some seat bonus, but limits the size of this bonus. The limit aligns with electoral reality—political scientists have consistently found approximately two-to-one votes-to-seats ratios in state legislative and congressional elections.[103] And the limit is simple: seats-votes responsiveness can exceed one but not two.

C. Two Potential Concerns with the Efficiency Gap

Faced with the competing constitutional risks presented by claims of partisan gerrymandering, Justices have understandably sought "clear, manageable, and politically neutral standards"[104] and "rules to limit and confine judicial intervention."[105] In an effort to supply such a standard, the *LULAC* amici focused on the principle of partisan symmetry, defined in terms of a tables-turned counterfactual comparison. The efficiency gap similarly focuses on partisan symmetry, but defines it alternatively as symmetric efficiency in the sense of equal wasted votes. To help the Court measure and thereby proscribe partisan gerrymandering, the efficiency gap must simultaneously cohere with an individual rights framework based on a principle of nondiscrimination,[106] and a structural account of electoral democracy attentive to the multiple values implicated by electoral districting. To satisfy this double coherence, the efficiency gap must sensibly define and compare parties' wasted votes in a way that distinguishes normatively desirable from undesirable plans. Notably, there may be tension between individual-rights coherence and structural coherence.

This need for double coherence suggests two potential concerns with the efficiency gap that motivate my subsequent analysis: one regarding its coherence with an individual-rights framework; one regarding its structural implications. First, the efficiency gap addresses concerns with the *LULAC* amici's proposed measure by offering an alternative definition of partisan symmetry based on

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[102] *Cf. LULAC*, 548 U.S. at 420 (plurality opinion) ("[P]laintiff . . . [must] provid[e] a standard for deciding how much partisan dominance is too much.").

[103] See *supra* note 42 and accompanying text.

[104] Vieth v. Jubelirer, 541 U.S. 267, 307-08 (2004) (Kennedy, J., concurring in the judgment).

[105] *Id.* at 307.

[106] This principle of nondiscrimination may be textually grounded in the Equal Protection right to be free from unjustified differential treatment or the First Amendment right to be free from discrimination or punishment on the basis of political affiliation or belief. *See id.* at 313-14 (discussing both textual bases and suggesting the First Amendment may be preferable).

observed election results rather than a hypothetical counterfactual. Instead of requiring that the actual seats-votes curve exhibit (or approximate) symmetry, the efficiency gap requires that the observed election result fall on (or close to) a prescriptive seats-votes curve that corresponds to an ideal of equal wasted votes. Thus, the efficiency gap replaces one equality norm (symmetric hypothetical outcomes) with a new equality norm (equal wasted votes). How intuitive and compelling this new equality norm is depends on how wasted votes are defined and compared. This suggests the importance of examining the efficiency gap's conceptual design. Part II undertakes this examination.

Second, the principle of partisan symmetry the efficiency gap is designed to capture may be a necessary but insufficient condition for a well-functioning democracy. Thus, it is worth considering how the efficiency gap relates to other democratic norms, like seats-votes proportionality and competitiveness, that are relevant to electoral districting but which the efficiency gap was not designed to capture. Part III explores these relationships.

II. The Efficiency Gap's Conceptual Design

The efficiency gap, traditionally defined, reflects a series of interrelated electoral assumptions and methodological choices that warrant more careful examination. First, proponents assume that each district's general election is a two-candidate contest between one candidate from each of the two major parties (which I refer to throughout as party *x* and party *y*). This ensures that every ballot is cast for, and every district race is won by, either party *x* or party *y*. Thus, the set of districts (D) can be split into the set of *x*-won districts (D_x) and the set of *y*-won districts (D_y) and voter turnout (T_i) in district i is simply the sum of the parties' respective vote totals V_{xi} and V_{yi}.

$$\mathrm{D} = \mathrm{D}_x \cup \mathrm{D}_y$$
$$T_i = V_{xi} + V_{yi}$$

When a district has an uncontested general election, the proponents use imputation techniques to estimate what the vote totals would have been had the election been contested.[107]

Second, the proponents deploy a cluster of definitions culminating in the concept of a wasted vote: a lost vote (L_{pi}) is one cast for the losing party: ($L_{pi} = V_{pi}$); an excess[108] vote (E_{pi}) is one cast for the winning party beyond the threshold needed to win (V_{ni}): $E_{pi} = V_{pi} - V_{ni}$; the threshold needed to win (in an assumed two-party race) is half of actual district turnout: $V_{ni} = \frac{T_i}{2}$;[109] and a

[107] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 865-67.

[108] I use the term "excess" instead of "surplus," so that I can use the letter *E* to denote the concept, rather than the letter *S*, which could be confused with "seat" or "share."

[109] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 851.

wasted vote is a lost or an excess vote.[110] Note that the possibility of a tie is ignored, so every district race is won by one party or the other.

The total wasted votes (W_{pi}) for a party p in district i is the sum of lost votes and excess votes cast for that party: $W_{pi} = L_{pi} + E_{pi}$. A party incurs excess votes when it wins a district ($i \in D_p$), and incurs lost votes when it loses a district ($i \notin D_p$). Thus:

$$W_{pi} = \begin{cases} E_{pi} = V_{pi} - V_{ni} = V_{pi} - \frac{T_i}{2} & i \in D_p \\ L_{pi} = V_{pi} & i \notin D_p \end{cases}$$

In short, proponents define a wasted vote by equally weighting lost and surplus votes and defining a surplus vote relative to a threshold of half of turnout. Judge Griesbach, the dissenting voice on the *Whitford* panel, described this definition as "opaque" and "absurd."[111] I will argue that the proponents' definition is plausible, but not the only possible way to define surplus votes. The proponents' threshold of half of turnout represents a party-centric approach. A more voter-centric approach would use a threshold based on the votes cast for the runner-up candidate.

Third, the proponents aggregate values by *party* and *district* to produce a single number for the entire *plan*.[112] For each party, the number of wasted votes over the entire plan is simply the sum over all districts of its wasted votes in each district.

$$W_p = \sum_{i \in D} W_{pi}$$

The plan's efficiency gap (ΔW_{xy}) is "the difference between the parties' respective wasted votes, divided by the total number of votes cast in the election."[113]

$$\Delta W_{xy} = \frac{\sum_{i \in D} W_{yi} - \sum_{i \in D} W_{xi}}{\sum_{i \in D} V_{yi} - \sum_{i \in D} V_{xi}} = \frac{W_y - W_x}{V_y + V_x}$$

This approach compares the parties' relative wasted vote totals: It is zero when each party wastes the same raw number of votes.

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[110] *Id.* ("Wasted votes include both 'lost' votes (those cast for a losing candidate) and 'surplus' votes (those cast for a winning candidate but in excess of what she needed to prevail)."); *id.* ("[A]ny vote for a losing candidate is wasted by definition, but so too is any vote beyond the 50 percent threshold needed (in a two-candidate race) to win a seat.").

[111] Whitford v. Gill, 218 F. Supp. 3d 837, 958 (W.D. Wis. 2016) (Griesbach, J., dissenting).

[112] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 851-52; *Id*. at 852 fig.1 (illustrating the computation of the efficiency gap for a hypothetical plan by aggregating wasted votes by district and party).

[113] *Id.* at 851 (emphasis omitted).

Fourth, proponents adopt an electoral assumption to simplify the long-form equation. Specifically, proponents assume that "each district has exactly the same number of voters."[114] Under this "equal voter turnout" assumption, district-level turnout (T_i) is equal to average turnout (T^*) for every district (i) in the plan (D).

$$T_i = T^* = \frac{\sum_{i \in D} T_i}{S} \quad \text{for every } i \in D$$

Proponents use this assumption to reduce the long-form equation (ΔW) to a simple function of seat margin (S^*) and vote margin (V^*).[115]

$$\Delta W = S^* - 2V^*$$

This simplified formula is much easier to compute, so the proponents use it when analyzing historical election data,[116] as does *Whitford* expert Simon Jackson.[117] Moreover, using this simplified formula, the proponents recast the efficiency gap as a measure of the undeserved seat share attributable to partisan gerrymandering rather than the party's popularity.[118]

In sum, the efficiency gap measure relies on four conceptual moves: (1) the two-party assumption; (2) the definition and weight of surplus votes; (3) the aggregation from district to plan to compare the parties' wasted vote totals; and (4) the equal voter turnout assumption to derive the simplified formula and exploit the associated double-proportionality seats-votes interpretation. In this Part, I examine each move to assess whether the efficiency gap defines and compares wasted votes in a sufficiently discernible and manageable manner. It will prove useful to examine them in reverse order, considering first the equal

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[114] McGhee, *Measuring Partisan Bias*, *supra* note 8, app.B at 79; *see also infra* Part II.B.

[115] *See* McGhee, *Measuring Partisan Bias*, *supra* note 8, app.B at 79-82 (deriving the simplified formula); *see also infra* Part II.B .

[116] The 2015 article never specifies which computation method (long-form equation or simplified formula) was used to analyze historical election data. *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 836. But in a phone conversation with Eric McGhee, I confirmed that this analysis was performed using the simplified formula.

[117] *See* Expert Report of Simon Jackman, *supra* note 97, at 16 ("The assumption of equally-sized districts is especially helpful for the analysis reported below, since the calculation of [the efficiency gap] in a given election then reduces to using the jurisdiction-level quantities [seat share] and [vote share] as in [the simplified formula]. For the analysis of historical election results reported below, it isn't possible to obtain measures of district populations, meaning that we really have no option other than to rely on the jurisdiction-level quantities [seat share] and [vote share] when estimating the [efficiency gap].").

[118] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 853 ("The efficiency gap's second interesting property follows from [the simplified formula]. Simply put, it is a measure of undeserved seat share: the proportion of seats a party receives that it would *not* have received under a plan with equal wasted votes.").

turnout assumption, then the aggregation method from district to plan, then the definition and weight of surplus votes, and finally the assumption that every district's general election is a two-party contest. But first, I briefly examine the principle McGhee has identified as informing these various methodological choices.

A. McGhee's Efficiency Principle

McGhee's design of the efficiency gap is guided by an overarching criterion he calls the "efficiency principle," defined as follows:

> ***The Efficiency Principle***: Any measure of efficiency must indicate a greater advantage for (against) a party when the seat share for that party increases (decreases) without any corresponding increase (decrease) in its vote share.[119]

This principle reflects McGhee's insight that what makes partisan gerrymandering so attractive to partisan mapmakers and so troubling for democracy is that it permits these mapmakers to increase a party's power (seat share) at a constant level of popularity (vote share). According to McGhee, the efficiency principle is "a bedrock condition for a measure of efficiency"; if a measure violates it, "it might be an adequate measure of something else, but it is missing the very essence of an efficient gerrymander."[120] This Part briefly explores and critiques this principle.

1. Implications of the Efficiency Principle

In one sense, the efficiency principle is quite strict. Mathematically, it requires that the measure be a function of seat share with a positive partial derivative with respect to seat share.[121] This means—holding all else (including vote share) constant—an increase in seat share increases the measure, and a decrease in seat share decreases the measure. Assuming that seat share is itself a function of vote share with a non-negative derivate,[122] then any measure satisfying the efficiency principle must itself be a function of vote share with a

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[119] McGhee, *Measuring Efficiency in Redistricting*, *supra* note 80, at 418.

[120] *Id.*

[121] *See id.* at 23 ("This can also be phrased more formally: to satisfy the [efficiency principle], a measure's partial derivative with respect to seat share must be positive"). The partial derivative of a function with respect to a given variable captures the incremental rate of change of the function with respect to that variable when all other variables are held constant.

[122] This is a modest assumption, violated only in the odd event that a party's vote share goes up but its seat share goes down (or vice versa). McGann calls this the assumption of "nonnegative responsiveness" and describes it as "a minimal requirement that is met by every reasonable single vote electoral system, including the first-past-the post system used in the United States." McGann et al., *supra* note 16, at 302-03.

non-positive derivate with respect to vote share. Thus, any measure satisfying the efficiency principle must be a function of both seat share and vote share with a positive partial derivative with respect to seat share and a non-positive partial derivative with respect to vote share. But the measure cannot depend on any variable independent of seat share and vote share, such as overall competitiveness or voter turnout. If it did, one could hold vote share constant, increase seat share and thereby increase the measure, but then modify the independent variable so as to return the measure to its original value. The result would be an increase in seat share at constant vote share without any corresponding increase in the measure—a violation of the efficiency principle.

Thus, any measure satisfying the efficiency principle must be a function only of seat share and vote share, with a positive partial derivative with respect to seat share, a non-positive partial derivative with respect to vote share, and no dependence whatsoever on any variable independent of vote share and seat share. This means we can set the measure equal to zero, and then solve for seat share as a function of vote share. The result is a seats-votes curve corresponding to symmetric partisan efficiency as defined by this measure.[123] The measure must then represent the extent to which a given electoral outcome departs from this ideal symmetric seats-votes curve. In sum, the efficiency principle implicitly requires that the analyst specify a single ideal seats-votes curve, and then measure the extent to which an electoral outcome departs from it. Each efficiency-principle-compliant measure corresponds to one such ideal seats-votes curve.

In another sense, though, the efficiency principle is quite flexible, because it imposes minimal constraints on the associated ideal seats-votes curve. Any function satisfies the efficiency principle so long as it is a function of seat share ($\bar{S}$) and vote share ($\bar{V}$) with properly-signed partial derivatives. If we impose the further requirement that the measure is linear in both $\bar{S}$ and $\bar{V}$, the unsurprising result is a linear seats-votes curve, and the only remaining task is to determine its slope, i.e., its degree of proportionality and responsiveness. As we shall see, when we define the efficiency gap in terms of parties' relative wasted vote totals, the result is a linear seats-votes curve, with the degree of proportionality and responsiveness determined by the definition and weight of surplus votes.

Any measure that satisfies the efficiency principle will encounter several challenges. First, the measure may be better suited to plans with high district numerosity, like state legislative plans and congressional plans in populous states, than to plans with low district numerosity like congressional plans in less populous states or plans for local bodies like city councils and school boards.

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[123] For example, the simplified formula satisfies the efficiency principle because it depends only on seat margin and vote margin. If one sets the simplified formula to zero, and then solves for seat margin as a function of vote margin, the result is the seats-votes curve illustrated in Diagram 8 *supra*.

The reason is that when plans have a small number of districts, seat share ($\bar{S}$) is constrained to a few values, so the observed electoral outcome will necessarily depart significantly from the ideal seats-votes curve unless the system happens to exhibit the right vote share ($\bar{V}$). For example, if a state, like Idaho, has only two congressional districts, there are only two possible values for seat margin: zero (each party wins one district) or fifty percent (one party wins both districts). Unless this state has the right vote share, it will necessarily have a large efficiency gap.[124] This suggests that that the efficiency gap measure may prove more useful when analyzing state legislative plans like the one at issue in *Whitford* than congressional plans like the one at issue in *Rucho*.

Second, any measure that satisfies the efficiency principle is vulnerable to the perverse risk that partisan voter suppression will reduce the apparent severity of a partisan gerrymander as quantified by the measure. If a party's supporter is unable to cast a ballot, assuming this ballot does not determine the outcome of the district election, the result will be that the party achieved the same seat share with less vote share, indicating greater efficiency for that party. This suggests that partisan voter suppression could reduce the apparent severity of partisan gerrymander. I explore this risk in greater detail in Part II.E.2.

2. A Modified Efficiency Principle

McGhee assumes that a partisan mapmaker's goal is to maximize her party's seat share at a specified level of vote share.[125] The efficiency principle aligns the measure with this objective. But this may be a simplification of the strategic calculus motivating real-world partisan mapmakers. It may be more accurate to assume the mapmaker seeks to maximize her party's expected seat share over a plausible range of vote shares.[126] If the mapmaker is too confident in her prediction of vote share, and too greedy in her desire to maximize seat share at predicted vote share, the result may be a "dummymander"[127] that inures to the benefit of the other party when actual vote share departs from the mapmaker's

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[124] For a detailed treatment of the efficiency gap's low district numerosity challenge, see Wendy K. Tam Cho, *Measuring Partisan Fairness: How Well Does the Efficiency Gap Guard Against Sophisticated as well as Simple-Minded Modes of Partisan Discrimination?*, 166 U. PA. L. REV. ONLINE 17, 21-27 (2017).

[125] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 850 ("Our analysis begins with the premise that the goal of a partisan gerrymander is to win as many seats as possible given a certain number of votes.")

[126] *See* Guillermo Owen & Bernard Grofman, *Optimal Partisan Gerrymandering*, 7 POL. GEO. Q. 5, 7 (1988) (arguing that efforts to maximize expected seat share "lead to strategies for sophisticated optimal partisan gerrymandering which differ from the classic 'recipe' of seeking to control as many districts as possible by paper-thin margins.").

[127] Bernard Grofman & Thomas L. Brunell, *The Art of the Dummymander: The Impact of Recent Redistrictings on the Partisan Makeup of Southern House Seats*, *in* REDISTRICTING IN THE NEW MILLENNIUM 183, 184 (Peter F. Galderisi ed., 2005).

prediction.[128] To account for this strategic calculus under conditions of electoral uncertainty, I propose the following modified efficiency principle:

> ***The Modified Efficiency Principle***: Any measure of efficiency must indicate a greater advantage for (against) a party when the seat share for that party increases (decreases) without any corresponding increase (decrease) in its vote share *unless its expected seat share decreases (increases) under plausible variation in that vote share*.

To see the difference between the original principle and this modified one, consider a mapmaker who alters a plan so that the majority party wins more seats by smaller margins with the same number of votes. This change allows the majority party to increase its seat share at constant vote share, but only by making the plan more competitive and thereby increasing the risk that seats will flip to the other party under plausible variation in vote share. In this scenario, a measure that satisfies the original efficiency principle must indicate greater advantage for the majority party, because it won more seat share with the same vote share. In contrast, a measure that satisfies the modified efficiency principle may indicate less advantage for the majority party because its expected seat share over plausible variation in vote share may decrease. Unlike the original efficiency principle, the modified one permits a measure to take into account the plan's increased competitiveness.

B. The Equal Turnout Assumption

Recall that McGhee derives the simplified formula ($\Delta W = S^* - 2V^*$) by explicitly assuming that "each district has exactly the same number of voters."[129] Under this equal turnout assumption, for every district (i) in the plan (D), district-level turnout (T_i) is equal to average turnout (T^*).

$$T_i = T^* \text{ for each } i \in D$$

[128] This is why Justice O'Connor, arguing for non-justiciability in *Bandemer*, suggested that "political gerrymandering is a self-limiting enterprise." Davis v. Bandemer, 478 U.S. 109, 152 (1986) (O'Connor, J., concurring in the judgment).

[129] McGhee, *Measuring Partisan Bias*, *supra* note 8, app.B at 79 (deriving the simplified formula). The first line of derivation explicitly assumes that "each district has exactly the same number of voters" so that "proportions can be substituted for raw votes in all of the formulas [and so] the total vote in each district becomes equal to 1.0," and the sum of district vote totals "is simply the total number of districts in the electoral system." *Id.*; *see also* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 853 n.114 (citing McGhee, *Measuring Partisan Bias*, *supra* note 8, app.B at 79-80).

Thus, the simplified formula, by design, does not account for inter-district variation in voter turnout.[130] If a real-world election satisfies this assumption, the long-form formula for the efficiency gap and the simplified seats-votes formula, will both compute precisely the same number.

$$\text{If } T_i = T^* \text{ for each } i \in D \text{ then } \frac{W_y - W_x}{V_y + V_x} = S^* - 2V^*$$

However, questions of plausibility and robustness arise whenever an assumption (like equal voter turnout) underlies a shorthand equation (like the simplified formula) for a measure (like the efficiency gap).[131] How plausible is the assumption of equal voter turnout in each district? How sensitive is the efficiency gap to departures from this condition? To address these questions, I relax the equal voter turnout assumption and derive a more generalized simplified formula that expresses the efficiency gap in terms of statewide seat and vote margin but makes no *ex ante* assumption about inter-district variation in voter turnout.

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[130] *See* McGhee, *Measuring Partisan Bias*, *supra* note 8, at 83 n.6 ("This necessarily assumes away differences in efficiency due to turnout . . . . [B]ut turnout variation is still a worthy topic of study. In fact, future research could use [the long-form equation] instead of [the simplified formula] to explore the subject.").

[131] Last term the Court clarified that a state may—but declined to address whether it must—comply with the one-person-one-vote principle by equalizing the number of people—as opposed to the number of voters—in each district. *See* Evenwel v. Abbott, 136 S. Ct. 1120, 1126-27 (2016). Thus, equal total population might be constitutionally required, but neither equal *voter* population nor equal voter *turnout* is constitutionally required. For example, it is "plainly permissible for jurisdictions to measure equalization by the *total* population of state and local legislative districts," *id.* at 1126-27 (emphasis added), even if this produces an electoral map that "measured by a *voter*-population baseline . . . [exhibits a] maximum population deviation exceed[ing] 40%," *id.* at 1125 (emphasis added). The scholarship presenting the efficiency gap proposal, published pre-*Evenwel*, characterized the relevant equality conditions in a way that may generate confusion. *Compare* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 853 (stating that the simplified formula assumes that "all districts are equal in *population* (which is constitutionally *required*)" (emphasis added)), *with* McGhee, *Measuring Partisan Bias*, *supra* note 8, at 68 ("In the special case where . . . districts are equal in *population* . . . [the long-form efficiency gap] reduces to [the simplified form]." (emphasis added)), *and id.* at 83 n.6 ("Ignoring *turnout* differences in this way is legally *mandated* for redistricting in the United States." (emphasis added)). If the assumption used to derive the simplified formula were constitutionally required, it would necessarily be satisfied in real elections, and so questions of plausibility and robustness would be moot. Yet the relevant assumption is not equal total population, but equal voter turnout. This assumption is not constitutionally required; a constitutionally valid electoral map may exhibit small inter-district variation in total population, but large variation in the population of eligible voters, and even larger variation in actual voter turnout. Thus, there is no guarantee that real elections will exhibit (or even approximate) equal voter turnout, and so questions of plausibility and robustness warrant attention.

1. The Turnout Gap

To explain the impact of turnout variation on the efficiency gap, I first introduce the concept of a turnout gap. When we relax the equal turnout assumption, we can still denote by T^* the average turnout across all districts: $T^* = \frac{\sum_{i \in D} T_i}{S}$. But now each district may have a turnout above or below (or equal to) average turnout. Let ΔT_i denote the proportional difference between actual turnout in district i and average turnout over all districts: $\Delta T_i = \frac{T_i - T^*}{T^*}$.[132] Let ΔT_p denote the average value of ΔT_i over districts won by party $p \in \{x, y\}$: $\Delta T_p = \frac{\sum_{i \in D_p} \Delta T_i}{S_p}$.[133] Define the turnout gap[134] as the product of party x seat share ($\bar{S}_x$) and the average proportional difference in x-won districts (ΔT_x).

$$\Delta T_{xy} = \bar{S}_x \Delta T_x$$

The turnout-generalized seats-votes formula is:[135]

$$\Delta W_{xy} = S^* - 2V^* + \Delta T_{xy}$$

This more generalized simplified formula is just like the prior one, except it now contains an additional term—variable ΔT_{xy}—that precisely quantifies the effect of voter turnout on the efficiency gap. The long-form equation reduces to the original simplified formula if—and only if—the turnout gap is zero. Note that $\bar{S}_x$

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[132] ΔT_i is positive when district i has higher than average turnout; negative when district i has lower than average turnout; and zero when district i has average turnout. For example, ΔT_i = 0.05 when that district's turnout is 5% higher than average turnout.

[133] By design, with only two parties, the seat-share-weighted sum of ΔT_x and ΔT_y is zero. $0 = \sum_D \Delta T_i = \sum_{D_x} \Delta T_i + \sum_{D_y} \Delta T_i = S_x \Delta T_x + S_y \Delta T_y = \bar{S}_x \Delta T_x + \bar{S}_y \Delta T_y$. Equivalently, $\bar{S}_x \Delta T_x = -\bar{S}_y \Delta T_y$. This makes intuitive sense because *x*-won districts will have above-average turnout only if *y*-won districts have correspondingly below-average turnout (and vice versa).

[134] Relying on the fact that $\bar{S}_x \Delta T_x = -\bar{S}_y \Delta T_y$ (as derived in note 133 *supra*), the turnout gap can also be expressed as half the seat-share-weighted difference of ΔT_x and ΔT_y:

$$\Delta T_{xy} = \bar{S}_x \Delta T_x = \frac{\bar{S}_x \Delta T_x + \bar{S}_x \Delta T_x}{2} = \frac{\bar{S}_x \Delta T_x - \bar{S}_y \Delta T_y}{2}$$

Alternatively, the turnout gap can be expressed as the sum of proportional turnout differences over *x*-won districts divided by the number of districts.

$$\Delta T_{xy} = \bar{S}_x \Delta T_x = \frac{S_x}{S} \frac{\sum_{i \in D_x} \Delta T_i}{S_x} = \frac{\sum_{i \in D_x} \Delta T_i}{S}$$

This is essentially how McGhee defines the turnout gap. *See* McGhee, *Measuring Efficiency in Redistricting*, *supra* note 80, at 427 (equations [4] and [5] and associated text) and 438-39 (Appendix – Derivation of Equation 4). Thus, McGhee and I derive equivalent turnout-generalized seats-votes formulae. Note that I discussed turnout effects with McGhee while preparing this Article, a draft of which McGhee cites in his 2017 piece. *See id*. at 428.

[135] *See* Technical Appendix, Part A for the proof of this result.

is never negative, so the sign of the turnout gap depends on the sign of ΔT_x, which captures whether turnout in x-won districts is above or below average turnout. When x-won districts exhibit above-average turnout, ΔT_x and thus the turnout gap are positive, and the long-form equation exceeds the simplified formula, registering greater advantage for party x. When x-won districts exhibit below-average turnout, ΔT_x and thus the turnout gap are negative, and the long-form equation is less than the simplified formula, registering greater advantage for party y.

2. The Size of the Turnout Gap

In some cases, the turnout gap may be so small it has no meaningful effect on the efficiency gap analysis and can thus be safely ignored. The 2012 Wisconsin Assembly election under the plan challenged in *Whitford* appears to be one such case. Consider the analysis of Dr. Kenneth Mayer, plaintiffs' expert in the *Whitford* litigation. Under the actual plan, of 2,844,676 votes cast, Democrats wasted 877,445 votes, while Republicans wasted only 544,893 votes, leading to an efficiency gap of 11.69% according to the long-form calculation.[136] In this plan, Republicans received only 1,389,958 out of the total 2,844,676 votes cast, but won 57 out of 99 districts, for an efficiency gap of 9.85% according to the simplified formula.[137] The long-form value exceeds the short-form value because there is a turnout gap of 1.84% in favor of Republicans.[138] But both values indicate above-threshold gaps in favor of Republicans.[139] For this reason,

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[136] Expert Report of Kenneth R. Mayer at 46 tbl.10, Whitford v. Gill, 218 F. Supp. 3d 837 (W.D. Wis. 2016) (No. 15-00421), 2015 WL 10091018.

$$\frac{W_y - W_x}{V_y + V_x} = \frac{877{,}445 - 544{,}893}{2{,}844{,}676} = 0.1169.$$

See id. at 45-46, 46 tbl.10.

[137] *Id.* at 46 tbl.10.

$$S^* - 2V^* = \left(\frac{57}{99} - 0.5\right) - 2\left(\frac{1{,}389{,}958}{2{,}844{,}676} - 0.5\right) = 0.0985.$$

[138] Proof: $= \Delta T_{xy} = \left(\frac{W_y - W_x}{V_y + V_x}\right) - (S^* - 2V^*) = 0.0184.$

[139] Interestingly, Professor Simon Jackman, plaintiffs' other expert, computed a gap of 13% for the 2012 election using the simplified formula. Expert Report of Simon Jackman, *supra* note 97, at 16, 36. However, in addition to employing different computation methods (long-form versus simplified formula), Professors Mayer and Jackman may have also employed different imputation methods to account for uncontested assembly races. Expert Report of Simon Jackman, *supra* note 97, at 24-30; Expert Report of Kenneth R. Mayer, *supra* note 136, at 44-45. Of 99 assembly races, 27 were uncontested. Expert Report of Kenneth R. Mayer, *supra* note 136, at 44. *See infra* Part II.E.1 for a discussion of the efficiency gap's sensitivity to alternative imputation strategies. Note that the *Whitford* panel majority compared the value Mayer computed with the long-form equation to the value Jackman computed with the

even though the *Whitford* majority regarded the long-form equation as "preferable" to the simplified formula, it was "not troubled" by the choice of computational technique, given that "both methods yield an historically large, pro-Republican [gap]."[140]

But the turnout gap is not always insignificant. Consider Indiana's 2014 congressional election, the results of which are provided below in Table 1.

simplified formula, rather than the respective values Mayer computed with the respective formulas. *See* Whitford v. Gill, 218 F. Supp. 3d 837, 904-05 and 907 (W.D. Wis. 2016).

[140] *Whitford*, 218 F. Supp. 3d at 907-08. This argument neglects the fact that both the academic proponents and Professor Jackman analyzed historical election data using only the simplified formula, not the long-form equation. *See supra* notes 116, 117. In other words, the majority is relying on the fact that the gap of 11.69% that Mayer calculated with the long-form equation is large compared to gaps calculated for historical election data with the simplified formula. Essentially, the majority assumed a high correlation between the results produced by the two computation methods. Such a high correlation may very well exist, but it would ideally be computed rather than assumed. The *Whitford* majority also emphasized that defendant's expert Goedert "described the simplified method as 'an appropriate and useful summary measure,'" and both parties stipulated that the simplified formula's "implied 2-to-1 votes-to-seats relationship reflects the 'observed average seat/votes curve in historical U.S. congressional and legislative elections.'" *Id.* at 907 (quoting Expert Report of Nicholas Goedert at 5-6, *Whitford*, 218 F. Supp. 3d 837 (No. 15-00421), 2015 WL 10091017). Finally, the *Whitford* majority cautioned that "[w]ere there record evidence indicating that [the simplified formula] did not correlate highly with both the [long-form] and electoral reality, we would have reason to doubt its validity." *Id.* at 907-08.

Table 1

DISTRICT	TOTAL VOTES BY PARTY		TOTAL TURNOUT
	R	D	
1	51,000	86,579	137,579
2	85,583	55,590	141,173
3	102,889	39,771	142,660
4	94,998	47,056	142,054
5	105,277	49,756	155,033
6	102,187	45,509	147,696
7	46,887	61,443	108,330
8	103,344	61,384	164,728
9	101,594	55,016	156,610

Election results as reported by the Indiana Secretary of State. *See* Official Election Results, Indiana General Election, November 4, 2014, http://www.in.gov/apps/sos/election/general/general2014?page=office&countyID=-1&officeID=5&districtID=-1&candidate=

Out of 1,295,863 ballots cast, the Democrats wasted 379,150 votes, but the Republicans wasted only 268,782 votes, for an efficiency gap of 8.5% in Republicans' favor according to the long form equation.[141] With only 61% of the statewide vote share, the Republicans won seven of nine districts, for an efficiency gap of 5.3% according to the simplified formula.[142] The discrepancy between the long-form equation (8.5%) and the simplified formula (5.3%) is a function of the turnout gap. District turnout is 143,985 on average, but it ranges from a low of 108,330 in Democrat-won District 7 to a high of 164,728 in Republican-won District 8, and turnout in Republican-won districts is 4.2%

[141] Plugging the values into the relevant equation: $\Delta W_{xy} = \frac{W_y - W_x}{V_y + V_x} = \frac{379{,}150 - 268{,}782}{1{,}295{,}863} = 0.085$.

[142] Plugging the values into the relevant equation: $S^* - 2V^* = (\frac{7}{9} - 0.50) - 2(0.61 - 0.50) = 0.053$.

above average, for a turnout gap of 3.2%.[143] The turnout gap is the difference between the long-form and simplified computations (3.2% = 8.5% - 5.3%). Note that the long-form value (the departure from equal wasted votes) is greater than the short-form value (the departure from double proportionality) because Republican-won districts exhibit above-average turnout, producing a positive turnout gap. If the "real" efficiency gap is defined by the long-form equation, the simplified formula underestimates it. If an 8% threshold were used for congressional plans, contrary to the original recommendation of the academic proponents, whether this election produced an above-threshold or below-threshold gap would depend on the choice of computation method. In the 2014 congressional elections, Texas had a turnout gap of 8.9%[144]—a number that is itself larger than the proposed numeric threshold for state legislative plans.

Data analysis beyond the scope of this Article can compute the turnout gaps (and thus the efficiency gaps under the long-form equation) for historical elections, but historical patterns do not guarantee future trends. Note that any factor that disproportionately decreases Democratic turnout will generally tend to generate a turnout gap in Republicans' favor. The reason is that Democrat-won districts have more Democrats, so a uniform decrease in Democratic turnout will have a larger impact on the turnout in Democrat-won districts than in Republican-won districts. This suggests that the turnout gap has a systematic tendency to increase whenever electoral rules have this differential partisan impact on turnout. Many believe, with good reason, that this is precisely the impact and intent of many recent electoral reforms.[145]

However common or rare the occurrence, when the turnout gap is large enough, the choice of computation method matters, presenting questions of robustness and correspondence.[146]

Partly in response to a draft of this Article,[147] McGhee recently published a new piece that addresses several features of the efficiency gap measure, including its relationship to turnout, that were previously unexamined.[148] In that piece, McGhee derives the same turnout gap I present *supra*.[149] McGhee views the efficiency gap's dependence on turnout as a violation of his efficiency

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[143] Plugging the values into the relevant equation: $\Delta T_{xy} = \bar{S}_x \Delta T_x = \frac{7}{9}(0.042) = 0.032$.

[144] *See* Technical Appendix (computing these turnout gaps from officially reported election results.)

[145] *See infra* Part II.E.2.

[146] In Part V below, I discuss in detail the questions of robustness and correspondence raised by this and other issues.

[147] McGhee, *Measuring Efficiency in Redistricting*, *supra* note 80, at 428.

[148] *Id*. at 426-32.

[149] *Id*. at 427 (equations [4] and [5] and associated text) and 438-39 (Appendix – Derivation of Equation 4).

principle.[150] To see why, consider a district where party x wins but wastes fewer votes. Keep constant the parties' respective vote shares in that district, but increase district turnout. This change will increase the wasted vote disparity in that district and thus the efficiency gap as originally defined, indicating greater advantage for party x. But party x has increased its vote share with no change in seat share, suggesting less advantage for party x. To solve this problem, McGhee essentially proposes a new definition of wasted votes where the threshold needed to win (V_{ni}) is not half of district turnout ($\frac{T_i}{2}$) but half of average turnout ($\frac{T^*}{2}$).[151] This definition eliminates the turnout gap from the equation, so that the simplified formula expresses the efficiency gap, not approximately based on electoral assumptions, but exactly based on the new definition.[152] Full consideration of this new approach lies beyond the scope of this paper, which focuses on the efficiency gap as originally presented in scholarship and litigation. But note that this new definition reflects the view of a mapmaker assessing an entire plan, rather than a voter participating in an individual district.[153]

C. The Aggregation Method from District to Plan

That the efficiency gap, traditionally defined, compares the total number of wasted votes is noteworthy for two reasons. First, it facilitates an analytical technique that defines a plan's gap as an (turnout-weighted) average of district level wasted vote disparities. Second, there is an alternative aggregation method that compares wasted vote shares rather than wasted vote totals.

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[150] *Id*. at 427.

[151] McGhee does not say he is changing the definition of a surplus vote from $E_{pi} = V_{pi} - \frac{T_i}{2}$ to $E_{pi}^{new} = V_{pi} - \frac{T^*}{2}$. Instead, he says that a party's total wasted votes (W_p) must be adjusted by adding an "effective vote deviation" defined as $\sum_{i \in D_p}(\frac{T_i}{2} - \frac{T^*}{2})$. *Id*. at 427 (equation [6]). But adding this effective vote deviation term is equivalent to changing the definition of a surplus vote.

Let $W_p = \sum_{i \in D_p} E_{pi} + \sum_{i \notin D_p} L_{pi}$ and let $W_p^{new} = \sum_{i \in D_p} E_{pi}^{new} + \sum_{i \notin D_p} L_{pi}$. Then:

$W_p + \sum_{i \in D_p}(\frac{T_i}{2} - \frac{T^*}{2}) = \sum_{i \in D_p} E_{pi} + \sum_{i \notin D_p} L_{pi} + \sum_{i \in D_p}(\frac{T_i}{2} - \frac{T^*}{2}) = \sum_{i \in D_p}(V_{pi} - \frac{T_i}{2} + \frac{T_i}{2} - \frac{T^*}{2}) + \sum_{i \notin D_p} L_{pi} = \sum_{i \in D_p}(V_{pi} - \frac{T^*}{2}) + \sum_{i \notin D_p} L_{pi} = \sum_{i \in D_p} E_{pi}^{new} + \sum_{i \notin D_p} L_{pi} = W_p^{new}$

[152] *Id*. at 428.

[153] Consider a plan with average turnout of 100, a high-turnout district that party x wins $105 - 95$ and a low-turnout district that party x wins 40-10. If surplus votes are defined relative to a baseline of half of average turnout ($100/2 = 50$), then party x incurs 55 surplus votes in the high-turnout district ($105 - 50 = 50$) and *negative* ten surplus votes in the low-turnout district ($40 - 50 = -10$). These surplus vote totals make more sense from the perspective of a mapmaker assessing an entire plan than a voter participating in a specific district.

1. The Analytical Technique: From District-Level Disparity to Plan-Level Gap

Proponents' approach involves a two-step aggregation process: first, sum wasted votes over districts; second, compute the difference.[154] One can switch the order of aggregation by defining a district-level wasted vote disparity and then expressing a plan's efficiency gap as the weighted average of district-level disparities. Specifically, define the district-level wasted vote disparity (Δw_{xyi}) between Party *x* and Party *y* in district i as the parties' relative wasted votes in that district, expressed as a proportion of district-level voter turnout.

$$\Delta w_{xyi} = \frac{W_{yi} - W_{xi}}{T_i}$$

A plan's efficiency gap ($\Delta W_{xy}(P)$) is the weighted average of district-level wasted vote disparities, where each district's wasted vote disparity ($\Delta w_{xyi}(P_i)$) is weighted by its turnout (T_i).[155]

$$\Delta W_{xy} = \frac{\sum_D T_i \Delta w_{xyi}}{\sum_D T_i}$$

Under the equal turnout assumption, the plan's efficiency gap is simply the unweighted average district disparity.[156]

$$\Delta W_{xy} = \frac{\sum_D \Delta w_{xyi}}{\mathrm{S}}$$

Since a plan's gap is an average of its districts' disparities, one simple way to achieve a zero gap is to maintain zero disparity in each district. Formally, define the set of zero-disparity districts (Π_i^0) as the set of districts that produce a wasted vote disparity of zero, and define the set of zero-gap plans (Π^0) as the set of plans that produce an efficiency gap of zero.

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[154] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 851-52.

[155] By definition, $\Delta W_{xy} = \frac{W_y - W_x}{V_x + V_y}$, $\Delta w_{xyi} = \frac{W_{yi} - W_{xi}}{T_i}$, $W_p = \sum_D W_{pi}$, $V_p = \sum_D V_{pi}$, and $V_{xi} + V_{yi} = T_i$. Thus: $\Delta W_{xy} = \frac{W_y - W_x}{V_x + V_y} = \frac{\sum_D W_{yi} - W_{xi}}{\sum_D V_{xi} + V_{yi}} = \frac{\sum_D T_i (\frac{W_{yi} - W_{xi}}{T_i})}{\sum_D T_i} = \frac{\sum_D T_i \Delta w_{xyi}}{\sum_D T_i}$.

[156] $S = \sum_D 1$, and under the equal turnout assumption, $T_i = T^*$ for each $i \in D$ .Thus:

$$\Delta W_{xy} = \frac{\sum_D T_i \Delta w_{xyi}}{\sum_D T_i} = \frac{\sum_D T^* \Delta w_{xyi}}{\sum_D T^*} = \frac{\sum_D \Delta w_{xyi}}{\sum_D 1} = \frac{\sum_D \Delta w_{xyi}}{\mathrm{S}}.$$

$$\Pi_i^0 = \{D_i \mid \Delta w_{xyi} = 0\}$$
$$\Pi^0 = \{D \mid \Delta W_{xy} = 0\}$$

One way to construct a zero-gap plan is to create a plan composed entirely of zero-disparity districts.

$$D^0 = \bigcup D_i \text{ where for each } i\ D_i \in \Pi_i^0$$

I call such a plan a simple zero-gap plan. By design, it maintains a zero efficiency gap.[157] The simple zero-gap plan is not the only zero-gap plan. More generally, a plan can achieve a zero plan-level gap even if it exhibits non-zero district-level wasted vote disparities, so long as these disparities average out so that both parties waste the same number of votes overall. But so long as we restrict our attention to plans with equal voter turnout in each district, each zero-gap plan can be converted to (and from) a simple zero-gap plan by performing the appropriate series of voter swaps, whereby two districts swap two voters—one party *x* supporter for one party *y* supporter—without altering any district election outcome.[158] A voter swap corresponds to a marginal change in the electoral boundary between two adjacent districts. A significant change to a district boundary can be understood as a series of incremental voter swaps. By design, a swap changes the wasted vote disparities in the participating districts, but maintains the same number of wasted votes for each party overall. Starting with a simple zero-gap plan, such a swap produces a plan that is still zero-gap but no longer simple. The appropriate series of voter swaps can convert any zero-gap plan to or from a simple zero-gap plan.

This provides a useful way to investigate the efficiency gap: Construct a simple zero-gap plan, examine its properties, and determine which properties vary under gap-preserving voter swaps. I use this technique in Part III.

2. The Alternative Approach: Comparing Wasted Vote Shares

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[157] Since both parties waste the same number of votes in each district, the parties must waste the same number of votes overall.

[158] Formally, if we start with an equal-turnout plan ($D = \{V_{xk}, V_{yk}\}$), and take two districts D_i and D_j, a single voter swap between the two districts produces a new plan $D' = \{V'_{xk}, V'_{yk}\}$ with the following properties:

$$V'_{xk} = \begin{cases} V_{xk} + 1 & k = i \\ V_{xk} - 1 & k = j \\ V_{xk} & k \neq i, j \end{cases} \qquad V'_{yk} = \begin{cases} V_{yk} - 1 & k = i \\ V_{yk} + 1 & k = j \\ V_{yk} & k \neq i, j \end{cases}$$

In this swap, district D_i swaps an *x* supporter for a *y* supporter, district D_j swaps a *y* supporter for an *x* supporter, and all other districts remain unchanged.

The proponents compare the respective parties' total number of wasted votes.

$$\Delta W_{xy} = \frac{W_y - W_x}{V_y + V_x}$$

Anthony J. McGann has criticized this approach, arguing that "it is not obvious that each party having an equal absolute number of wasted votes is uniquely fair,"[159] and noting that one could alternatively require that that each party should waste the same *share* of votes, rather than the same *number* of votes.[160] Following this suggestion, John F. Nagle has developed what he calls a "voter-centric" measure that compares relative wasted vote shares rather than relative wasted votes as a proportion of all ballots cast.[161]

$$\Delta W_{xy}^{V} = \frac{W_y}{V_y} - \frac{W_x}{V_x}$$

The two aggregation methods superficially appear nearly identical, but they can produce markedly different results. Suppose 6,000,000 votes are cast, with 4,000,000 for Party *x* and 2,000,000 for Party *y*, and suppose each party wastes 1,500,000 votes. This means that 75% of ballots cast by Party *y* supporters are wasted, while only 37.5% of ballots cast by Party *x* supporters are wasted. Each party wastes the same number of raw votes, but a Party *y* supporter is twice as likely as a Party *x* supporter to waste her vote. While a party may focus more on the total number of votes it wastes, an individual voter may care more about the likelihood that her vote will be wasted, which depends on the share rather the total of votes wasted by her preferred party. This is why Nagle describes relative wasted vote totals as "party-centric" and relative waste vote shares as "voter-centric."[162] One could argue that the "voter-centric" approach based on wasted vote shares better coheres with an individual-rights framework based on a particular voter's Equal Protection or First Amendment interest in participating free from discrimination based on political affiliation, belief, or expressive conduct.

Note that the choice between a party-centric (wasted vote totals) and a voter-centric (wasted vote shares) scale for the asymmetry comparison may be particularly consequential for minor parties that earn a small number of total votes, and usually waste all of them (because they win no seats). Such a minor party will have a low wasted vote total, but a high wasted vote share—possibly

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[159] McGann et al., *supra* note 16, at 296.

[160] *Id.*

[161] Nagle, *supra* note 16, at 201.

[162] *Id.* at 201.

100%. Thus, a voter-centric scale may facilitate greater receptivity to a claim that an electoral map is gerrymandered to disadvantage a minor party.

Nagle explored the mathematical properties of a wasted vote measure using a voter-centric scale of wasted vote shares. However, Nagle defined a surplus vote as half the victory margin rather the full victory margin, noting that the alternative was mathematically equivalent to unequal weighting of lost and surplus votes.[163] Nagle then rejected unequal weighting of lost and surplus votes on the ground that it would violate McGhee's efficiency principle.[164] Part IV proposes a new measure that compares the share of wasted votes with surplus votes defined as the full victory margin. This new measure is voter-centric in terms of both the scale of comparison and the definition of wasted votes.

D. The Definition and Weight of Surplus Votes

Both the definition and the weight of surplus votes are methodological choices—not self-defining concepts. They are susceptible to competing interpretations, and selecting among them requires deliberation and transparency. The proponents and *Whitford* litigants define a surplus vote using a threshold of half of total votes, and they equally weight lost and surplus votes.[165] But these two distinct yet related methodological choices have not been adequately explained.

The proponents simply define a wasted vote as a lost *or* surplus vote,[166] assuming implicitly and without explanation that lost and surplus votes should be equally weighted. When considered from the perspective of the party, equal weighting makes sense: Whether lost or surplus, a vote is equally wasted in the sense that it could be more effective if cast in another district. But when considered from the perspective of an individual voter, lost and surplus votes are not obviously equivalent. True enough, both the voter who casts the lost vote and the voter who casts the surplus vote may regret that her vote could have been more effective in another district. But the voter who casts a surplus vote gets to be represented by the candidate of her choice. Not so with the voter who casts a lost vote. Faced with a choice between casting a lost vote and casting a surplus vote, I would prefer the latter option, and I suspect most other voters would, too.

The definition of surplus votes is similarly unexplained and even less intuitive. The proponents define a surplus vote as one "cast for a winning

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[163] *Id.* at 199 & n.16, 203 & n.24.

[164] *Id.* at 203 ("However, as Eric McGhee has kindly pointed out, the possibility that different values of S for the same vote V may give the same value of bias violates a fundamental principle for bias measures, namely, gerrymandering might be able to increase S for the same V and not be detected by the measure of bias." (citation omitted)).

[165] *See supra* Part II.

[166] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 850-51.

candidate but *in excess of what she needed to prevail*."[167] Thus, $E_{pi} = V_{pi} - V_{ni}$ where V_{ni} denotes the number of votes the "winning candidate . . . needed to prevail."[168] But what is V_{ni}? Let V_{i1} and V_{i2} respectively denote the number of ballots cast for the first and second place candidates. Recall that, under the two-party assumption, the sum of the party's respective vote totals is the district's total voter turnout: $T_i = V_{yi} + V_{xi}$. The difference is the victory margin (M_i)—a primary measure of electoral competitiveness.[169]

$$M_i = V_{1i} - V_{2i} = |V_{xi} - V_{yi}| = \begin{cases} V_{xi} - V_{yi} & i \in D_x \\ V_{yi} - V_{xi} & i \in D_y \end{cases}$$

Under a plurality voting system, the candidate with the most votes wins. This suggests that the number of votes the "winning candidate . . . needed to prevail" is the number of votes earned by her most popular opponent—i.e., $V_{ni} = V_{2i}$.[170] Under this interpretation, the number of surplus votes is simply the victory margin.

$$E'_{pi} = V_{pi} - V_{ni} = V_{1i} - V_{2i} = M_i$$

For example, suppose 100 ballots are cast, and the victor prevails with a vote tally of 65 to 35, then $V_{ni} = V_{2i} = 35$ and $E'_{pi} = V_{pi} - V_{ni} = 65 - 35 = 30$.

But this is not the proponents' interpretation. Instead, they define necessary votes (V_{ni}) as half of actual voter turnout ($\frac{T_i}{2}$).[171] Under this interpretation, the number of surplus votes is half the victory margin.

$$E_{pi} = V_{pi} - V_{ni} = V_{1i} - \frac{T_i}{2} = V_{1i} - \frac{V_{1i} + V_{2i}}{2} = \frac{V_{1i} - V_{2i}}{2} = \frac{M_i}{2}$$

For example, if the victor prevails 65-35, $V_{ni} = \frac{T_i}{2} = 50$ and $E_{pi} = V_{pi} - V_{ni} = 65 - 50 = 15$.[172]

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[167] *Id.* at 834, 851 (emphasis added).

[168] *Id.*

[169] *See* Stephanopoulos, *Consequentialist Criteria*, *supra* note 33, at 678.

[170] Technically, the number of necessary votes is one more than the runner-up's vote total. Like other scholars, litigants, and jurists engaged in efficiency gap analysis, I ignore this "plus one" technicality.

[171] *See supra* note 109, 110.

[172] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 851 n.107 ("Assume, for example, that Candidate A receives 65 percent of the vote and Candidate B receives 35 percent. Then 15 percent of Candidate A's votes . . . are wasted . . . .").

The proponents justify this approach by invoking "the 50 percent threshold needed (in a two-candidate race) to win a seat."[173] It is true that a candidate must earn more than half the votes to win a two-way race. But the proponents' invocation of the 50% threshold fails to clarify the basis for their definition, because both definitional approaches can be explained with reference to a 50% threshold. The distinction between surplus and necessary votes logically entails a hypothetical counterfactual where a candidate earns fewer votes but still prevails. If the victor earned the same vote total as the runner-up ($V_{1i} = V_{2i}$), half of turnout would be the runner-up's vote total: $\frac{T_i}{2} = \frac{V_{1i}+V_{2i}}{2} = \frac{2V_{2i}}{2} = V_{2i}$. But the proponents, without adequate explanation, apply the 50% threshold to the actual vote total rather than the vote total in the hypothetical counterfactual.

This is the definition of surplus votes that Judge Griesbach, the dissenting judge on the *Whitford* panel, attacked as "opaque" and "absurd,"[174] Using a baseball analogy, Judge Griesbach suggested that surplus votes must be defined as the entire vote margin rather than half the vote margin:

> Just as a baseball game is not decided by reference to total runs, an election is not decided by a fraction of total votes. Instead, the number of votes needed to win is simply the number one more than the losing candidate won, and therefore anything beyond that should be counted as a "wasted" vote.[175]

The proponents' definition of surplus votes is not absurd, though it could be explained more clearly. It simply defines surplus votes as the number of voter swaps possible without altering the outcome. Because each swap exchanges one *x*-supporter for one *y*-supporter, it decreases the victory margin by two votes, so the total number of possible outcome-preserving swaps is half the victory margin. This reflects the perspective of the mapmaker under the equal voter turnout assumption. Each voter swap represents a marginal adjustment to the district boundaries—one that changes the district of only the two homes where the respective swapped voters reside. This definition of surplus votes is not absurd, but it does privilege the party-centric perspective of the mapmaker under the stylized assumption of equal voter turnout. From the perspective of an individual voter who supports a particular party, her choice is either to cast her ballot or to stay home. Her vote is wasted if she could have stayed home without altering the outcome. From the perspective of the mapmaker focused on a partisan voter, his choice is which district to assign her. The mapmaker could leave the voter in her current district, or swap her with a voter from another

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[173] *Id.* at 851; *see also id.* at 834 n.14 ("[W]e also assume that 50 [out of 100] votes are needed to win a district"); *id.* at 851 n.107 ("[V]ictory in a two-candidate race is achieved with 50 percent of the vote (plus one).").

[174] Whitford v. Gill, 218 F. Supp. 3d 837, 958 (W.D. Wis. 2016) (Griesbach, J., dissenting).

[175] *Id.* (emphasis omitted).

district supporting the other party. Her vote is wasted if he could swap her without altering the outcome in the original district.

Judge Griesbach's baseball analogy implicitly adopts a voter-centric approach.[176] Just as a voter is assigned to a single district, Judge Griesbach considers a single baseball game. But in a real election, there are multiple districts, and a partisan mapmaker is concerned with the total number of districts won. To put this in baseball terms would require an odd sort of athletic competition. Suppose multiple baseball games are played simultaneously, each game between one team from each of two parties. Further suppose that a run for team *x* in game one could be swapped for a run for team *y* in game two. And the party's ultimately goal was to win as many games as possible with a fixed number of runs. This competition has the salient features at play in partisan districting. Under these conditions, it is not absurd to define a team's surplus runs as those more than half of a game's total runs. That is simply the number of run-swaps possible without altering a game's outcome.

In short, the proponents' party-centric approach is one plausible way to define and weight surplus votes. But it is not the only way, nor is it necessarily the most intuitive way. Just as Nagle has recently considered McGann's suggestion of comparing wasted vote shares, he has also noted that surplus votes can be alternatively defined as the entire vote margin,[177] and that wasted votes can be generalized as a weighted sum of lost and surplus votes.[178] Part III.F similarly relaxes the definition and weight of surplus votes, and it derives a more generalized formula that quantifies the precise impact of these methodological choices on the efficiency gap measure. My results accord with Nagle's, but my approach demonstrates the critical role these methodological choices play in calibrating the measure's relationship between competing norms of electoral competitiveness and seats-votes proportionality.

E. The Two-Party Assumption

The proponents make the electoral assumption that "there are only two parties."[179] The efficiency gap measure is by definition a bilateral comparison—it takes two parties and compares their relative efficiency by calculating the difference in their respective wasted vote totals. I use the term "focal parties" to refer to the two parties that are the focus of the efficiency-gap measure's bilateral comparison, and the term "peripheral candidates" to refer to candidates unaffiliated with the two focal parties. I call the two focal parties party *x* and

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[176] *See id*.

[177] Nagle, *supra* note 16, at 199, 199 n.16, 203 n.24.

[178] *Id.* at 200.

[179] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 853.

party *y*.[180] When proponents assume "there are only two parties," they not only concentrate on the two focal parties subject to the bilateral comparison, but also ignore any ballot cast for a peripheral candidate. Because the proponents emphasize partisan fairness between the two major political parties,[181] they make the simplifying assumption that every district race is a contest between one party *x* candidate and one party *y* candidate. This "two-party" assumption actually consists of three related assumptions: in each district's general election (1) no ballots are cast for peripheral candidates; (2) no more than one candidate runs from each focal party; and (3) no race is uncontested, so that a candidate runs from each focal party.

Section 1 examines the consequences when the third assumption fails, and analysts must impute results for uncontested races. Implicit in the efficiency gap approach is one more assumption: that the vote shares earned by the two parties reflect their relative popular support among the electorate. Section 2 examines the consequences when this final assumption fails because electoral administration differentially prevents or discourages supporters of one party from casting ballots.

1. Uncontested Races

When a district race is uncontested, the proponents suggest a strategy of imputation to estimate what would have occurred had the race been contested:

> Going forward, we encourage other scholars to explore a range of imputation techniques to ensure that the direction of a gerrymander (if not its size) is robust to any particular strategy. But this catholic philosophy has its limits. We strongly discourage analysts from either dropping uncontested races from the computation or treating them as if they produced unanimous support for a party. The former approach eliminates important information about a plan, while the latter assumes that coerced votes accurately reflect political support. Neither correctly represents how the gerrymandering party itself would view its plan.[182]

Uncontested district races present problems of correspondence, robustness, and scope for the efficiency gap. If no imputation is permitted, we could either

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[180] The efficiency gap is a signed measure. Its absolute value indicates the extent of the gerrymander, while its sign indicates which party the gerrymander favors. I define all relevant concepts so that a positive gap favors party *x* and a negative gap favors party *y*. Obviously, the two primary parties of interest are the major political parties. Whenever I discuss the efficiency gap between Republicans and Democrats, I adopt the convention that party *x* connotes Republicans and party *y* connotes Democrats.

[181] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 838-39 (characterizing the efficiency gap as a "new measure of *partisan* symmetry" designed to capture "the idea that a plan should treat the *major parties* symmetrically" (emphasis added)).

[182] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 867.

omit the uncontested election from the dataset (presenting a problem of scope) or count the uncontested district and register it as indicating unanimous support for the winning party (presenting a problem of normative correspondence). If imputation is permitted, we must choose a fair and accurate method of imputation (presenting a challenge of robustness).

The need to impute election results in uncontested districts prompts two analytical points related to our overall assessment of the efficiency gap measure. First, the imputation method chosen may, under the right circumstances, have a significant, and even outcome-determinative, impact on the measure of a plan's efficiency gap. Second, uncontested races are a sign of uncompetitive districts, so there may be an unfortunate association between a plan's uncompetitiveness and the sensitivity of the efficiency gap calculation to imputation method.

My first point is practical. Proponents recognize that employing different imputation methods may present robustness problems but do not quantify how significantly the choice of imputation method might impact the resulting efficiency gap. It is possible, however, to mathematically estimate how sensitive the gap is to different imputation approaches using the simplified formula: $\Delta W_{xy} = S^* - 2V^*$. When an analyst imputes vote share for an uncontested district, she changes only the vote totals, not the winning party. The seat margin stays the same, but the vote margin changes. Imagine two analysts employing different imputation methods that produce different imputed vote margins in uncontested races. Consider the following notation:

Notation	Description
$\bar{S}_U$	the share of uncontested districts
$\bar{V}_C$	the average district vote share in contested districts
$\bar{V}_{1U}$	the average district vote share in uncontested districts as estimated by Analyst 1
$\bar{V}_{2U}$	the average district vote share in uncontested districts as estimated by Analyst 2
$\bar{V}_1$	the vote share as estimated by Analyst 1
$\bar{V}_2$	the vote share as estimated by Analyst 2
$\Delta W_{1,2}$	the difference between the efficiency gap estimated by Analyst 1 and the efficiency gap estimated by Analyst 2.

Note that subscripts 1 and 2 respectively denote Analyst 1 and 2, while subscript U denotes uncontested districts. We can quantify the impact of imputation under the equal voter turnout assumption as follows.[183]

[183] The share of contested districts is $1 - \bar{S}_U$, $\bar{V}_1 = \bar{S}_U\bar{V}_{1U} + (1 - \bar{S}_U)\bar{V}_C$, and $\bar{V}_2 = \bar{S}_U\bar{V}_{2U} + (1 - \bar{S}_U)\bar{V}_C$. Thus:

$$\bar{V}_2 - \bar{V}_1 = [\bar{S}_U\bar{V}_{2U} + (1 - \bar{S}_U)\bar{V}_C] - [\bar{S}_U\bar{V}_{1U} + (1 - \bar{S}_U)\bar{V}_C] = \bar{S}_U(\bar{V}_{2U} - \bar{V}_{1U}).$$

And:

$$\Delta W_{12} = 2\bar{S}_U(\bar{V}_{2U} - \bar{V}_{1U})$$

The greater the difference between imputed average vote shares, the greater the difference in computed gaps. Note that the analyst who imputes the higher average vote share will compute the lower gap. By increasing the estimate of Party *x* support, the analyst increases the estimate of deserved seat share, and thereby decreases the estimate of undeserved seat share, thus decreasing the estimate of the extent to which the plan favors Party *x*. The greater the share of districts with uncontested races, the greater the difference in computed gaps. The more uncontested races, the more the imputation technique matters. For example, assume that one third of districts in a state hold uncontested elections, Analyst 1 concludes that Party *x* would have, on average, earned 70% of the vote in uncontested races, but Analyst 2 concludes that Party *x* would have, on average, won 73% of the vote in uncontested races. Analyst 1 would compute an efficiency gap 2% higher than Analyst 2.[184]

This effect of the imputation method upon the statewide efficiency gap is not merely a theoretical concern. The choice of imputation method could have significant practical consequences for the overall validity of the plan. In the 2012 Wisconsin assembly election, 27 of 99 assembly races were uncontested and therefore imputed.[185] Applying the simplified formula to this election, but using different imputation methods, Professor Mayer computed an efficiency gap of 9.85% while Professor Jackman computed an efficiency gap of 13%.[186] The difference between these two estimates is greater than the difference between Professor Mayer's estimate and the numeric threshold,[187] suggesting that imputation method may have a non-trivial impact on efficiency gap analysis.

There may be real-world cases where the existence *vel non* of an above-threshold gap depends on the imputation method used for uncontested races.[188] If the efficiency gap proposal were adopted as a legal test, plaintiffs alleging a political gerrymander may strategically employ imputation methods that would

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$$\Delta W_{12} = \left[S^* - 2\left(\bar{V}_1 - \tfrac{1}{2}\right)\right] - \left[S^* - 2\left(\bar{V}_2 - \tfrac{1}{2}\right)\right] = 2(\bar{V}_2 - \bar{V}_1) = 2\bar{S}_U(\bar{V}_{2U} - \bar{V}_{1U}).$$

[184] In this example, $\bar{S}_U = \frac{1}{3}$, $\bar{V}_{1U} = 0.70$, and $\bar{V}_{2U} = 0.73$. Thus:

$$\Delta W_{12} = 2\bar{S}_U(\bar{V}_{2U} - \bar{V}_{1U}) = 2\tfrac{1}{3}(0.73 - 0.70) = 0.02.$$

[185] Expert Report of Kenneth R. Mayer, *supra* note 136, at 39 ("Twenty three [sic] Democratic candidates were uncontested…By contrast, only four Republicans were uncontested.")

[186] *See supra* notes 136, 139.

[187] The academic proponents suggested a threshold of 8% based on historical trends. *See supra* note 96. Professor Jackman suggested a threshold of 7% based on durability analysis. *See supra* note 97.

[188] *Cf.* Expert Report of Simon Jackman, *supra* note 97, at 22 ("Uncontested races are common in state legislative elections, and are even the norm in some states.").

produce larger gaps; defendants may strategically employ competing imputation methods that produce lower gaps; and the battle of the experts could be outcome-determinative.

With the imputation of hypothetical election results, one of the advantages of the efficiency gap over other partisan symmetry measures is limited. The efficiency gap may be particularly appealing—especially to Justice Kennedy—because it relies upon directly observed election data, not hypothetical results.[189] But if calculating the gap requires imputing hypothetical election results, and if the size of the gap depends on which imputation method the analyst selects, the gap is a less straightforward measure of real-world data. The Court could restrict use of efficiency gap analysis to circumstances when a plan produces a (durable) above-threshold gap under any plausible imputation method. This would address the concern with hypotheticals and avoid the battle of the experts, but at the cost of limiting the circumstances when plaintiffs can deploy the efficiency gap, and potentially, when plaintiffs can win partisan gerrymandering claims.

It is particularly troubling that the efficiency gap calculation is more sensitive to imputation method when a plan has more uncontested races because an uncontested race is a signal that a district is highly uncompetitive.[190] Thus, a highly uncompetitive plan may produce a large proportion of uncontested races, rendering efficiency gap analysis more sensitive to imputation method. This dynamic may make efficiency gap analysis harder to use for uncompetitive plans, thereby perversely creating a (further) incentive for mapmakers to draw uncompetitive plans. In Part III, I argue that adoption of the efficiency gap as the definition of partisan gerrymandering may also unintentionally incentivize uncompetitive plans because the sensitivity of the efficiency gap to vote swings is a function of a plan's responsiveness. Combined, these considerations suggest that overreliance on the efficiency gap may present the risk of a vicious circle: Adoption of the efficiency gap as the definition of partisan gerrymandering may unintentionally encourage mapmakers to draw uncompetitive plans that produce a high proportion of uncontested races; and courts may struggle to evaluate these plans because the high proportion of uncontested races renders efficiency gap analysis more sensitive to imputation methods.

2. Voter Suppression

Because the efficiency gap is a measure of the relative number of wasted votes for each party, the only data points that it registers in assessing partisan

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[189] *See* League of United Latin Am. Citizens v. Perry, 548 U.S. 399, 419-20 (2006) (opinion of Kennedy, J.) ("[W]e are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs.")

[190] Expert Report of Kenneth R. Mayer, *supra* note 136, at 39 ("[U]ncontested races occur largely when one party sees zero probability of winning because the majority party has such overwhelming majorities in the district.").

fairness are ballots cast. In its most direct application, an analyst computes the efficiency gap produced by a given plan in a given election by inputting into the long-form equation (or simplified formula) the actual votes cast in that election by district and party (or the vote margin and seat margin produced by that election) after imputations for uncontested races. In a more advanced application, an analyst estimates the results of a hypothetical election using regression techniques on historical and contemporaneous data correlated with election outcomes, and then inputs those results into the relevant equation to compute the efficiency gap a given plan would likely produce in that hypothetical election.

The efficiency gap's singular focus on ballots cast means that it cannot detect any obstacles voters face in casting ballots, and cannot perceive electoral administration rules that disproportionately impact supporters of one party. If one such rule—for example, a stringent photo identification requirement for in-person voting—thwarts a voter's effort to cast a ballot, the efficiency gap detects no problem; the measure simply assumes the relevant party has one ballot less support from the electorate. Unintentionally, this dynamic may reward, and thereby further incentivize, voter suppression efforts, because the suppression may make a gerrymander seem less like a gerrymander—it can hide gerrymanders from the efficiency gap. Suppressing one party's statewide vote total can have the effect of reducing the overall gap. If the Democratic Party enacts a partisan gerrymander, and then adopts electoral reforms that differentially burden Republican voters, any resulting decrease in Republican turnout would operate to reduce the apparent pro-Democratic advantage conferred by the districting plan as measured by the efficiency gap. Conversely, if the Republican Party enacts a partisan gerrymander, and then adopts electoral reforms that differentially burden Democratic voters, any resulting decrease in Democratic turnout would operate to reduce the apparent pro-Republican advantage conferred by the districting plan as measured by the efficiency gap.[191]

Unfortunately, the Court considers the efficiency gap at a moment in American politics when electoral administration is a partisan issue, with Democrats more likely to support "ballot access" measures ostensibly designed to reduce the perceived risk that an eligible voter will encounter difficulty casting a ballot, and Republicans more likely to support "ballot integrity" measures ostensibly designed to reduce the perceived risk that an ineligible voter will cast

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[191] To be clear, I do not claim that this lack of sensitivity to voter suppression is a problem unique to the efficiency gap. It is a feature of any measure of partisan gerrymandering that quantifies an ideal or acceptable relationship between votes won and seats won. Specifically, it is a feature of any measure that satisfies the efficiency principle. Any definition that uses popular support as demonstrated by ballots cast to justify the number of seats won may incentivize voter suppression, because political actors seeking to demonstrate greater relative support can inflate the appearance of that support by making it harder for their competitor's supporters to vote.

a ballot.[192] In the four years since the Supreme Court invalidated the Voting Rights Act's coverage formula and thereby rendered inoperative its preclearance regime,[193] twenty states have adopted such electoral reforms,[194] including notably the two states currently defending against partisan gerrymandering claims based on efficiency gap approaches: Wisconsin[195] and North Carolina.[196] It is difficult to confidently estimate the impact of electoral reforms on voter turnout,[197] and it is not the objective of this Article to advance the debate about

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[192] *See*, *e.g.*, Stephen Ansolabehere, *Access Versus Integrity in Voter Identification Requirements*, 63 N.Y.U. ANN. SURV. AM. L. 613, 613-16 (2008).

[193] *See* Shelby Cty. v. Holder, 133 S. Ct. 2612, 2631 (2013).

[194] New Voting Restrictions in America, Brennan Ctr. for Justice 2-11 (2017), https://www.brennancenter.org/sites/default/files/analysis/New_Voting_Restrictions.pdf (Alabama, Arizona, Arkansas, Georgia, Indiana, Iowa, Kansas, Mississippi, Missouri, Nebraska, New Hampshire, North Carolina, North Dakota, Ohio, Rhode Island, South Carolina, Tennessee, Texas, Virginia, and Wisconsin).

[195] *See id.* at 8. Wisconsin adopted a stringent voter identification law, which was softened by the Wisconsin Supreme Court after a federal district court concluded it violated the Fourteenth Amendment and the Voting Rights Act, a conclusion the Seventh Circuit reversed after the law was softened. *See* Frank v. Walker, 17 F. Supp. 3d 837, 863, 878-80 (E.D. Wis. 2014), *rev'd*, 768 F.3d 744 (7th Cir.), *cert. denied*, 135 S. Ct. 1551 (2015); Milwaukee Branch of NAACP v. Walker, 851 N.W.2d 262, 281 (Wis. 2014) (employing saving construction of administrative regulation such that voter would not be required to pay fees for any document required for issuance of photo ID); *see also* Frank v. Walker, 819 F.3d 384, 388 (7th Cir. 2016) (remanding to district court to consider claims of voters unable to obtain a qualifying photo ID with reasonable effort); Richard L. Hasen, *Softening Voter ID Laws Through Litigation: Is It Enough?*, 2016 WIS. L. REV. FORWARD 100, 110-11 (explaining litigation challenging Wisconsin's voter identification law in federal and state courts).

[196] *See* Brennan Ctr. for Justice, *supra* note 194, at 11. North Carolina was subject to preclearance prior to *Shelby County*, because some of its counties fell under the coverage formula. *See Jurisdictions Previously Covered by Section 5*, U.S. DEP'T OF JUSTICE (Aug. 6, 2015), https://www.justice.gov/crt/jurisdictions-previously-covered-section-5. Once the Court struck down the coverage formula, the North Carolina legislature quickly enacted an omnibus electoral reform bill with five key provisions: (1) a voter identification requirement (for in person but not mail-in voting) limited to forms of identification that white people tend to have and black people tend to lack; (2) a reduction in early voting days; (3) elimination of same-day registration; (4) elimination of out-of-precinct voting; and (5) elimination of preregistration. N.C. State Conference of NAACP v. McCrory, 831 F.3d 204, 216-18 (4th Cir. 2016), *cert. denied*, 137 S. Ct. 1399 (2017). The Fourth Circuit concluded that this omnibus bill constituted intentional racial discrimination, "target[ing] African Americans with almost surgical precision," *id.* at 214-15, and thereby "[u]sing race as a proxy for party . . . to win an election," *id.* at 222. Note that North Carolina is a state where "African-American race is a better predictor for voting Democratic than party registration." *Id*. at 225.

[197] Turnout may go down after an electoral reform, but this correlation does not tell us how much of the turnout effect was caused by the reform as opposed to other factors like reduced

whether, and to what extent, "ballot integrity" measures like those adopted in Wisconsin and North Carolina differentially burden and thereby reduce participation among Democratic-leaning voters.[198] My only claim is that *if* electoral administration reform had the differential turnout effect some fear, that in turn would impact the efficiency gap analysis.

Consider the following. In the *Whitford* litigation, plaintiffs proposed a numeric threshold of 7% and plaintiff's expert, Mayer, computed an efficiency gap for the actual plan with the simplified formula of about 9.85%.[199] According to the simplified formula $\Delta W_{xy} = S^* - 2V^*$, holding seat share constant, every 0.5% decrease in Democratic vote share decreases by 1% the Republican advantage conferred by an electoral map as estimated by the efficiency gap.

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enthusiasm for races and candidates. We may be able to estimate the number of people potentially affected by an electoral reform—like the number of registered voters who presently lack required voter identification. But this does not tell us the number of voters actually affected—like the number of registered voters turned away on Election Day for lack of proper identification. Another complicating factor is that voter suppression efforts may affect turnout through information or misinformation. For example, an individual may actually be permitted by law to cast a ballot, but decline to try to vote based on the mistaken assumption that she is not.

[198] For a sampling of this debate, see Shelley de Alth, *ID at the Polls: Assessing the Impact of Recent State Voter ID Laws on Voter Turnout*, 3 HARV. L. & POL'Y REV. 185, 186 (2009) ("[P]hoto and non-photo ID laws decreased turnout by between 1.6 and 2.2 percentage points" but "states that amended their ID laws more recently experienced increased voter turnout, whereas states that changed their voting laws prior to 2004 showed a decline in turnout."); Robert S. Erikson & Lorraine C. Minnite, *Modeling Problems in the Voter Identification-Voter Turnout Debate*, 8 ELECTION L.J. 85, 98 (2009) (concluding that "the existing science regarding vote suppression [is] incomplete and inconclusive . . . . not because of any reason to doubt the suppression effect but rather because the data that have been analyzed to date do not allow a conclusive test."); Jack Citrin et al., *The Effects of Voter ID Notification on Voter Turnout: Results from a Large-Scale Field Experiment*, 13 ELECTION L.J. 228, 235 (2014) (suggesting that, with voter notification, laws requiring voter identification may increase rather than suppress turnout); Samuel Issacharoff, *Ballot Bedlam*, 64 DUKE L.J. 1363, 1381 (2015) ("To date, empirical studies . . . have been unable to find any substantial decline either in overall turnout or in the turnout of racial minorities as a result of these laws."); *id.* at 1382 n. 67 (collecting studies); Pamela S. Karlan, *Turnout, Tenuousness, and Getting Results in Section 2 Vote Denial Claims*, 77 OHIO ST. L.J. 763, 774 (2016) ("[A]s a practical matter, determining whether a challenged practice has depressed minority turnout can be extraordinarily complex."); *id.* at 774 n.63 (citing studies); Spencer Overton, *Voter Identification*, 105 MICH. L. REV. 631, 659-61, 659 nn.136-41, 660 nn.143-44, 661 nn.150-53 (2007) (citing studies suggesting that senior citizens, young people, people of color, people with disabilities, low-income people, and transient persons differentially lack driver's licenses); Michael J. Pitts, *Empirically Assessing the Impact of Photo Identification at the Polls Through an Examination of Provisional Balloting*, 24 J.L. & POL. 475, 480 (2008) (concluding that a study of provisional ballots cast in Indiana's 2008 primary election "likely provide[s] a little something for both proponents and opponents of photo identification.").

[199] Expert Report of Kenneth R. Mayer at 46 tbl.10, Whitford v. Gill, 218 F. Supp. 3d 837 (W.D. Wis. 2016) (No. 15-00421), 2015 WL 10091018.

About 2.8 million ballots were cast in the 2012 Wisconsin assembly election;[200] 0.5% of this number is 14,000 ballots. Thus, if 14,000 fewer Democrats cast ballots, the pro-Republican efficiency gap would go down about 1%. If about 40,000 fewer Democrats cast ballots, the pro-Republican efficiency gap would fall below the numeric threshold. If 140,000 fewer Democrats cast ballots, the pro-Republican efficiency gap would be eliminated entirely. A federal judge in another case concluded that over 300,000 registered voters in Wisconsin—9% of registered voters—lack the required photo identification,[201] a "substantial number of the 300,000 plus eligible voters who lack a photo ID are low-income,"[202] and "it is likely that a substantial number of the 300,000 plus voters who lack a qualifying ID will be deterred from voting."[203] Of course, these considerations do not establish the intent or effect of Wisconsin's photo ID law. But they do suggest the risk inherent in the efficiency gap's focus on ballots cast. A party eager to enact a partisan gerrymander that would withstand legal challenge based on the efficiency gap would have a strong incentive to engage in voter suppression efforts that reduced the turnout of its opponents' supporters.

The only way to avoid this perverse relationship between the turnout effects of partisan voter suppression and the measure of partisan gerrymandering is to explicitly account for partisan turnout effects in the measure. This can be accomplished by applying the efficiency gap measure to a hypothetical election result estimated in a way that controls for turnout-reducing electoral reforms or practices. In essence, this technique would ask what efficiency gap a plan would produce without voter suppression, rather than what efficiency gap a plan would produce with voter suppression. The drawback is this makes the measure further reliant on hypotheticals and more sensitive to modeling assumptions. But it would avoid the risk of rewarding and further incentivizing partisan voter suppression. At the very least, this approach warrants consideration under the circumstances present in Wisconsin and North Carolina, where the same lawmakers that enacted the challenged maps enacted electoral reforms—subsequently called into question by federal courts[204]—that risk partisan voter suppression and a corresponding underestimation of the extent of advantage those maps conferred on the Republican Party. More generally, if efficiency gap measures are to play a role in partisan gerrymandering claims, this problem warrants further consideration.

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[200] *Id.*

[201] *Frank*, 17 F. Supp. 3d at 854.

[202] *Id.*

[203] *Id.* at 862.

[204] *See* N.C. State Conference of NAACP v. McCrory, 831 F.3d 204, 215 (4th Cir. 2016), *cert. denied*, 137 S. Ct. 1399 (2017) (North Carolina); *Frank*, 17 F. Supp. 3d at 863, 878-80 (Wisconsin).

III. The Efficiency Gap's Relationship to Proportionality and Competitiveness

The last Part focused on the efficiency gap's conceptual design and its underlying methodological choices. This Part focuses on the efficiency gap's relationship to seats-votes proportionality and competitiveness. The efficiency gap was explicitly designed to capture asymmetrical partisan efficiency, not proportionality or competitiveness. Yet the efficiency gap bears a relationship to proportionality and competitiveness that warrants better understanding and consideration.

As vote margin increases, the efficiency gap approves increasing departures from strict seats-votes proportionality. A party with 75% vote share can win every seat and achieve a zero efficiency gap. In fact, the efficiency gap will report a disadvantage for a party with more than 75% vote share even if that party wins every seat. More realistically, a party with 59% vote share can win a 75% supermajority in the legislature with a below-threshold efficiency gap.

The efficiency gap bears a more nuanced relationship to competitiveness. The efficiency gap is a measure of differential, not overall competitiveness, but the efficiency gap's sensitivity to vote swings is a function of seats-votes responsiveness, which depends on the proportion of relatively competitive districts. This relationship reveals that scholars and jurists may be referring to the efficiency gap without realizing it. And it suggests that adoption of the efficiency gap as the exclusive definition of a partisan gerrymander would permit and perhaps encourage mapmakers to draw uncompetitive plans.

The analysis proceeds as follows. Subpart A explains why proportionality and competitiveness matter. Partisan symmetry may be a necessary but insufficient condition for a well-functioning democracy. We should be concerned by extreme departures from proportionality and competitiveness, especially when they occur simultaneously.

Subpart B adopts the analytic technique introduced in Part II.C.1 to offer a more intuitive understanding of the efficiency gap's operation. At the level of an individual district, the wasted vote disparity is a function of the district's competitiveness; it is zero when one party wins 75-25, such that the victory margin is precisely half of turnout; and it jumps discontinuously when seat flips from one party to the other. From this district-level analysis, we can derive key features of the measure's plan-level operation.

Subpart C examines the efficiency gap's relationship to proportionality. The proponents recognize an extreme vote share problem, where a majority with 75% vote share can win all the seats and achieve a zero efficiency gap. I argue that the problem is not confined to this point. For example, a majority with 59% vote share can win a veto- and filibuster-proof 75% supermajority in the legislature with a below-threshold gap.[205] Subpart D examines the efficiency gap's

[205] Most prominently, in the U.S. Senate, sixty votes (three-fifths or 60% of the senators) are needed to overcome a filibuster by voting that the debate "shall be brought to a close[.]" Senate

relationship to competitiveness. I define the competitiveness gap as the seat-share weighted difference in average competitiveness between *x*-won and *y*-won districts, and show that under the equal voter turnout assumption the efficiency gap is equal to the competitiveness gap. This means a mapmaker can reduce the efficiency gap by unpacking, and thereby increasing overall competitiveness, or by decracking, and thereby decreasing overall competitiveness. While a zero gap can be achieved at any level of competitiveness, a zero gap can only be maintained at a relatively low level of competitiveness, because the sensitivity of the gap to vote swings is a function of seats-votes responsiveness.

Subpart E offers a normative assessment of the efficiency gap's relationship to proportionality and competitiveness. I suggest this relationship presents two problems: a false positive problem, where the measure flags as suspect normatively desirable plans that reflect efforts to promote proportionality or competitiveness; and a false negative problem, where the measure fails to detect normatively problematic plans, like skewed bipartisan gerrymanders with excessive seat bonuses and insufficient representation for minority parties. I suggest that the doctrinal tools of intent, justification, and sensitivity analysis only partially address the false positive problem, and fail to address the false negative problem for want of a mechanism to overcome the presumption of validity triggered by a below-threshold gap.

Subpart F demonstrates how the efficiency gap's relationship to proportionality and competitiveness depends on the definition and weight of surplus votes.

A. The Relevance of Proportionality and Competitiveness

The efficiency gap's proponents, like the *LULAC* amici, have good reason to focus on partisan symmetry rather than other democratic norms like seats-votes proportionality and competitiveness. For one thing, proportionality and competitiveness are in tension with each other. A maximally competitive system, where each district race is a razor-thin nail-biter, may depart radically from proportionality, because a small uniform swing across all districts in favor of one party would result in that party winning every seat with just over half the votes. Conversely, a strictly proportional system exhibits relatively low competitiveness, because any incremental change in vote share must translate into an equal change in seat share, and so the number of competitive districts is

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Comm. on Rules & Admin., Standing Rules of the Senate, S. Doc. No. 113-18, at 15-16 (2013). Parallel practices are less common in state legislatures. Page Scobee, *Ahoy! The Future of the Filibuster*, National Conference of State Legislatures Blog (Jun. 26, 2016), http://www.ncsl.org/blog/2016/06/29/ahoy-the-future-of-the-filibuster.aspx ("Approximately 10 states have a cloture rule that requires more than a simple majority.") (citing a report by a legislative analyst for the Connecticut General Assembly, Meghan Reilly, States Limiting Legislative Debate (2009), https://www.cga.ct.gov/2009/rpt/2009-R-0249.htm).

necessarily limited. Moreover, the Court has demonstrated deep skepticism about the constitutional significance of proportionality or competitiveness. The Court has repeatedly insisted that the Constitution does not require strict seats-votes proportionality.[206] And in *Gaffney v. Cummings*,[207] the Court was untroubled by the uncompetitiveness of a bipartisan gerrymander that carved the state up into safe Democratic districts and safe Republican districts so as to achieve seats-votes proportionality.[208] Thus, the Court may reject as foreclosed by precedent any legal test that essentially requires strict proportionality or maximal competitiveness. In contrast, five Justices have expressed interest in partisan symmetry, a normatively appealing standard that closely tracks nondiscrimination principles familiar to both Equal Protection and First Amendment law.[209]

But this does not render proportionality and competitiveness irrelevant. Strict proportionality may not be required, but a significant departure from proportionality may be relevant to our assessment of a districting plan. So too with competitiveness. A plan need not maximize competitiveness, but we may be rightly concerned if a plan needlessly and intentionally minimizes it. And we ought to be particularly concerned by a plan that simultaneously departs significantly from both proportionality and competitiveness. Note that *Gaffney* approved a bipartisan gerrymander that reduced competitiveness to achieve seats-votes proportionality, not a plan that departed from both competitiveness and proportionality.[210]

If proportionality and competitiveness matter, the principle of partisan symmetry proposed by the *LULAC* amici and invoked by the efficiency gap proponents may constitute a necessary but insufficient condition of a well-functioning electoral system. Take some extreme examples. Consider a winner-takes-all system, where whichever party earns more votes gets all the seats (and the two parties split the seats if they precisely split the votes). This system satisfies the principle of partisan symmetry, and it maximizes competitiveness,

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[206] *See, e.g.*, Vieth v. Jubelirer, 541 U.S. 267, 288 (2004) (plurality opinion) ("[Appellants'] standard rests upon the principle that groups . . . have a right to proportional representation. But the Constitution contains no such principle. It guarantees equal protection of the law to persons, not equal representation in government to equivalently sized groups.").

[207] 412 U.S. 735 (1973).

[208] *Id.* at 752-54.

[209] LULAC v. Perry, 548 U.S. 399, 483–84 (2006) (Souter, J., joined by Ginsburg, J., concurring in part and dissenting in part) ("[N]or do I rule out the utility of a criterion of symmetry as a test . . . . Interest in exploring this notion is evident, see *ante*, at 2610–2611 principal opinion); *ante*, at 2636 – 2638 (STEVENS, J., concurring in part and dissenting in part); *post*, at 2651–2652 (BREYER, J., concurring in part and dissenting in part). Perhaps further attention could be devoted to the administrability of such a criterion at all levels of redistricting and its review.").

[210] *See id.* at 752.

but it permits an extreme departure from seats-votes proportionality and thus denies any representation to the minority party. In contrast, consider a system where each party gets half the seats no matter how many votes they earn. This system also satisfies the principle of partisan symmetry, and it ensures minority representation, but it eliminates competitiveness. Whereas competitive races promote accountability,[211] safe districts shift the action from the general election to the primary, pushing legislators to ideological extremes, promoting polarization and gridlock,[212] and reducing the responsiveness of legislators to the general electorate.[213] The proliferation of safe districts[214] may also discourage high-quality challengers, reduce party mobilization, and depress voter participation,[215] giving incumbents an advantage unrelated to their prior

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[211] *See* Stephanopoulos, *Consequentialist Criteria*, *supra* note 33, at 676-77 & nn.21, 30 (citing scholarship).

[212] *See* Lillian V. Smith, Note, *Recreating the "Ritual Carving": Why Congress Should Fund Independent Redistricting Commissions and End Partisan Gerrymandering*, 80 BROOK. L. REV. 1641, 1644 (2015) (Gerrymandering "reduc[es] the competitiveness of elections, contribut[es] to the systemic entrenchment of partisan interests, and perpetuat[es] congressional deadlock."). Recent Congresses have been notoriously unproductive. *See* Drew Desilver, *In late spurt of activity, Congress avoids 'least productive' title*, PEW RESEARCH CENTER (Dec. 29, 2014), http://www.pewresearch.org/fact-tank/2014/12/29/in-late-spurt-of-activity-congress-avoids-least-productive-title/. *But see* Drew Desilver, *Congressional productivity is up – but many new laws overturn Obama-era rules*, PEW RESEARCH CENTER (Aug. 29, 2017), http://www.pewresearch.org/fact-tank/2017/08/29/115th-congress-productivity/. When legislators fear primary challengers but take general elections for granted, the predictable result is increased partisanship and gridlock. MARK TUSHNET, THE NEW CONSTITUTIONAL ORDER 19-24 (2003).

[213] *See* Josh Chafetz, *The Phenomenology of Gridlock*, 88 NOTRE DAME L. REV. 2065, 2086 (2013) (considering the possibility that "the combination of partisan primaries and bipartisan gerrymandering are resulting in a legislature that cannot be said to be broadly responsive to the American people"); Samuel Issacharoff & Pamela S. Karlan, *Where to Draw the Line?: Judicial Review of Political Gerrymanders*, 153 U. PA. L. REV. 541, 574 (2004) ("The perverse consequence of the incumbent gerrymander is that it skews the distribution politically by driving the center out of elected office at the legislative level.").

[214] By one count, the number of swing congressional districts plunged from 103 in 1992 to 35 in 2012. Nate Silver, *As Swing Districts Dwindle, Can a Divided House Stand?*, N.Y. TIMES (Dec. 27, 2012), http://fivethirtyeight.blogs.nytimes.com/2012/12/27/as-swing-districts-dwindle-can-a-divided-house-stand; *see also* John Nichols, *Why Redistricting Threatens Democracy*, NATION (Feb. 1, 2012), https://www.thenation.com/article/why-redistricting-threatens-democracy (arguing that redistricting "explains why the vast majority of races are not competitive").

[215] *See, e.g.*, League of United Latin Am. Citizens (LULAC) v. Perry, 548 U.S. 399, 471 n.10 (2006) (Stevens, J., concurring in part and dissenting in part) (explaining multiple ways in which "[s]afe seats may harm the democratic process"); Richard H. Pildes, *The Constitution and Political Competition*, 30 NOVA L. REV. 253, 260 (2006) ("[I]t is well documented that competitive elections encourage the appearance of strong challengers to incumbents and increase voter turnout and party mobilization."). Note that some scholars and jurists emphasize

performance or present popularity. For these reasons, it may be problematic if the Court proscribes partisan gerrymandering in a way that neglects entirely norms of competitiveness and seats-votes proportionality.

One approach is to account for different norms with different causes of action: a partisan gerrymandering claim for excessive departures from partisan symmetry, a bipartisan gerrymandering claim for excessive departures from competitiveness, and a minority protection claim for excessive departures from seats-votes proportionality. This approach warrants consideration going forward. But at this moment, when no other political gerrymandering claim exists, it is prudent to consider the implications for other democratic norms of a partisan gerrymandering test designed to vindicate only the principle of partisan symmetry. If mapmakers can avoid excessive partisan asymmetry by sacrificing competitiveness, minority representation, or both, and if the Court prohibits excessive partisan asymmetry without offering any legal protection or incentive for competitiveness or minority representation, the result may be skewed incentives to produce problematic electoral plans. This is not a criticism of the efficiency gap measure itself, which was sensibly designed with the exclusive goal of quantifying partisan asymmetry. Rather, it is a call for careful study and use of the measure in a context where partisan symmetry is a necessary but insufficient condition of well-functioning elections. No legal doctrine presently exists to protect or incentivize other important democratic norms like proportionality and competitiveness.

B. The Simple Zero-Gap Plan

To develop a deeper and more intuitive understanding of the efficiency gap's relationship to proportionality and competitiveness, this Subpart examines the characteristics of districts and plans that waste the same number of votes.

1. Measuring Competitiveness

I begin the analysis by defining and deriving some properties of a district election's victory margin. Recall that V_{1i} and V_{2i} respectively denote the number of ballots cast for the first- and second-place candidates in a given district. The sum is the district's total voter turnout: $T_i = V_{1i} + V_{2i} = V_{yi} + V_{xi}$. The difference is the victory margin (M_i)—a primary measure of electoral competitiveness.[216]

$$M_i = V_{1i} - V_{2i} = |V_{xi} - V_{yi}| = \begin{cases} V_{xi} - V_{yi} & i \in D_x \\ V_{yi} - V_{xi} & i \in D_y \end{cases}$$

voter participation directly as a primary democratic value. *See* Stephanopoulos, *Elections and Alignment*, *supra* note 32, at 297 (describing Justice Breyer's, Christopher Elmendorf's, and Spencer Overton's views on the primacy of voter participation).

[216] *See* Stephanopoulos, *Consequentialist Criteria*, *supra* note 33, at 678.

The proportional margin of victory (m_i) is the district-level margin of victory (M_i) expressed as a proportion of district-level voter turnout (T_i): $m_i = \frac{M_i}{T_i}$. Rearranging, we can express V_{1i} and V_{2i} in terms of victory margin (M_i) and turnout (T_i).[217]

$$\frac{V_{xi} - V_{yi}}{T_i}$$

$$V_{1i} = \frac{T_i + M_i}{2} = \frac{T_i}{2}(1 + m_i)$$

$$V_{2i} = \frac{T_i - M_i}{2} = \frac{T_i}{2}(1 - m_i)$$

Let $m_p = \frac{\sum_{i \in D_p} m_i}{S_p}$ denote the average proportional victory margin in *p*-won districts and let $m = \frac{\sum_{i \in D} m_i}{S}$ denote the average proportional victory margin over all districts. Note that the seat-share-weighted sum of m_x and m_y is m and the seat-share-weighted difference of m_x and m_y is twice the vote margin (V^*).[218]

$$m = \bar{S}_x m_x + \bar{S}_y m_y$$
$$2V^* = \bar{S}_x m_x - \bar{S}_y m_y$$

This means we can express m_x and m_y in terms of m, V^*, and S^*.[219]

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[217] These expressions can be derived as follows:

$$V_{1i} = \frac{(2V_{1i} + 0)}{2} = \frac{(V_{1i} + V_{1i}) + (V_{2i} - V_{2i})}{2} = \frac{(V_{1i} + V_{2i}) + (V_{1i} - V_{2i})}{2} = \frac{T_i + M_i}{2} = \frac{T_i}{2}(1 + m_i)$$

$$V_{2i} = \frac{(2V_{2i} + 0)}{2} = \frac{(V_{2i} + V_{2i}) + (V_{1i} - V_{1i})}{2} = \frac{(V_{1i} + V_{2i}) - (V_{1i} - V_{2i})}{2} = \frac{T_i - M_i}{2} = \frac{T_i}{2}(1 - m_i)$$

[218] This expression can be derived as follows:

$$2V^* = \bar{V}_x - \bar{V}_y = \frac{V_x - V_y}{V} = \frac{\sum_D V_{xi} - \sum_D V_{yi}}{ST^*} = \frac{\sum_D (V_{xi} - V_{yi})}{ST^*}$$

$$2V^* = \frac{\sum_{D_x}(V_{xi} - V_{yi}) - \sum_{D_y}(V_{yi} - V_{xi})}{ST^*} = \frac{\sum_{D_x} T_i m_i - \sum_{D_y} T_i m_i}{ST^*}$$

$$2V^* = \frac{\sum_{D_x} T^* m_i - \sum_{D_y} T^* m_i}{ST^*} = \frac{\sum_{D_x} m_i - \sum_{D_y} m_i}{S} = \frac{S_x m_x - S_y m_y}{S}$$

$$2V^* = \bar{S}_x m_x - \bar{S}_y m_y.$$

[219] Relying on the expressions $\bar{S}_x = \frac{1}{2} + S^*, \bar{S}_y = \frac{1}{2} - S^*, m = \bar{S}_x m_x + \bar{S}_y m_y$, and $2V^* = \bar{S}_x m_x - \bar{S}_y m_y$, these results can be derived as follows:

$$m_x = \frac{m + 2V^*}{1 + 2S^*}$$
$$m_y = \frac{m - 2V^*}{1 - 2S^*}$$

These equations will prove critical in Part IV.

2. A District's Wasted Vote Disparity Is a Discontinuous Linear Function of Its Margin of Victory

At the level of an individual district, the wasted vote disparity (Δw_{xyi}) is a discontinuous linear function of the victory margin m_i.

$$\Delta w_{xyi} = \frac{W_{yi} - W_{xi}}{T_i} = \begin{cases} \frac{1}{2} - m_i & i \in D_x \\ m_i - \frac{1}{2} & i \in D_y \end{cases}$$

With ties excluded, the disparity (Δw_{xyi}) is zero if and only if the victor prevails by half of turnout[220]—regardless of which party wins. With this victory margin, the winner and the loser earn votes at a 3:1 ratio, so three quarters of ballots are cast for the winner, one quarter for the loser. Thus, the winner's minimizing vote share (v^*) is 75%. For example, suppose 100 ballots are cast, 75 for party *x* and 25 for party *y*. Party *x* wins 75% vote share, and prevails by a margin of victory of 50 votes, which is precisely half of turnout. Under the proponent's formulation, party *y* wastes 25 lost votes, while party *x* wastes 25 surplus votes ($75 - 50$), so each party wastes the same number of votes, and the district's wasted vote disparity is zero.

Let D_i^0 denote a minimizing district, D_i^{x0} an *x*-won minimizing district, and D_i^{y0} a *y*-won minimizing district.

$$m_x = \frac{2\bar{S}_x m_x + 0}{2\bar{S}_x} = \frac{(\bar{S}_x m_x + \bar{S}_y m_y) + (\bar{S}_x m_x - \bar{S}_y m_y)}{2(\frac{1}{2} + S^*)} = \frac{m + 2V^*}{1 + 2S^*}$$
$$m_x = \frac{2\bar{S}_x m_x + 0}{2\bar{S}_x} = \frac{(\bar{S}_x m_x + \bar{S}_y m_y) + (\bar{S}_x m_x - \bar{S}_y m_y)}{2(\frac{1}{2} + S^*)} = \frac{m + 2V^*}{1 + 2S^*}$$

[220] This makes intuitive sense. The proponents note that precisely half of all votes are wasted. *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 851 & n.107. Thus, equal wasted votes occur when each party wastes one quarter of ballots cast. Mira Bernstein & Moon Duchin, *A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap*, 64 NOTICES AM. MATHEMATICAL SOC'Y 1020, 1022 (2017).

$$D_i^{x0} = \left(\tfrac{3}{4}T_i, \tfrac{1}{4}T_i\right), D_i^{y0} = (\tfrac{1}{4}T_i, \tfrac{3}{4}T_i)$$

Call one half the minimizing victory margin (m^0) and define the competitiveness score (c_i) as the difference between the minimizing (m^0) and the actual (m_i) victory margin. Then the wasted vote disparity is:

$$\Delta w_{xyi} = \begin{cases} c_i & D_x \\ -c_i & D_y \end{cases}, c_i = m^0 - m_i, \ m^0 = 0.5$$
$$\Delta w_{xyi} = c_i = 0 \textit{ if and only if } m_i = m^0 = 0.5$$

A district's disparity is simply its competitiveness score, the difference between the minimizing and the actual victory margin, with a sign convention such that a relatively competitive district favors the winning party while a relatively uncompetitive district favors the losing party. When m_i exceeds m^0, c_i is negative because the district is less competitive than the minimizing level. This favors the losing party, because surplus votes exceed lost votes. When m_i is less than m^0, c_i is positive because the district is more competitive than the minimizing level. This favors the winning party because lost votes exceed surplus votes. The district-level disparity is positive (favors party *x*) when *x* wins a relatively competitive district or *y* wins a relatively uncompetitive district; the disparity is negative (favors party *y*) when *x* wins a relatively uncompetitive district or *y* wins a relatively competitive district.

The simple zero-gap plan has a zero-competitiveness gap, because each district has a zero-competitiveness score. Since each district is won by half of turnout ($m_i = \ m^0 = \frac{1}{2}$), party *x* earns $\frac{T^*}{2}$ more votes in *x*-won districts, and party *x* earns $\frac{T^*}{2}$ fewer votes in *y*-won districts. Since the plan must assign each voter to one district, the seat margin must be double the vote margin.[221]

$$2V^* = \frac{V_x - V_y}{V} = \frac{T^*(\sum_{D_x} m^0 - \sum_{D_y} m^0)}{ST^*} = \frac{\frac{1}{2}(S_x - S_y)}{S} = S^*$$

Recall that the simple zero-gap plan can be transformed to or from any zero-gap plan with the appropriate series of voter swaps. By design, a voter swap preserves equal voter turnout[222] and statewide vote share.[223] Since no swap alters a

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[221] The proof relies on the equalities $\bar{V}_x - \bar{V}_y = 2V^*$and $\bar{S}_x - \bar{S}_y = 2S^*$ derived *supra* at note 38.

[222] Each participating district exchanges one voter for another and thus maintains the same voter turnout.

[223] Proof: for all districts $i, j, k \in D$

district's election outcome, the series preserves seat share. Since the series preserves vote share, seat share, and equal turnout, it must preserve the efficiency gap. Thus, the equality $S^* = 2V^*$ is a feature of any zero-gap plan, not just the simple zero-gap plan. This offers another way to understand the double proportionality and double responsiveness that emerges from the proponents' approach:[224] It is the seats-votes relationship exhibited by a simple minimizing plan composed exclusively of minimizing districts. When each party earns half the votes, the simple minimizing plan accords each party half the seats. When one party earns 75% of the votes, the simple minimizing plan accords that party all the seats, because each district is won by that party 75-25.

Diagram 9 illustrates the relationship between a district's disparity and the vote share difference between parties *x* and *y*. The diagram looks like a "double back slash"—two downward sloping lines with a discontinuous jump at the 50% mark when the seat flips from one party to the other. Much of the measure's operation at the plan-level can be intuited from this district-level relationship. Since a party can win a single seat with a zero-plan disparity and 75% of the vote, a party can win all the seats with a zero-plan gap and 75% of the votes. Since a district's disparity is its competitiveness score, the efficiency gap is the seat-share-weighted difference in average competitiveness. And since a district's disparity jumps discontinuously when a seat flips, the sensitivity of a plan's efficiency gap to vote swings is a function of how many of its districts are competitive.

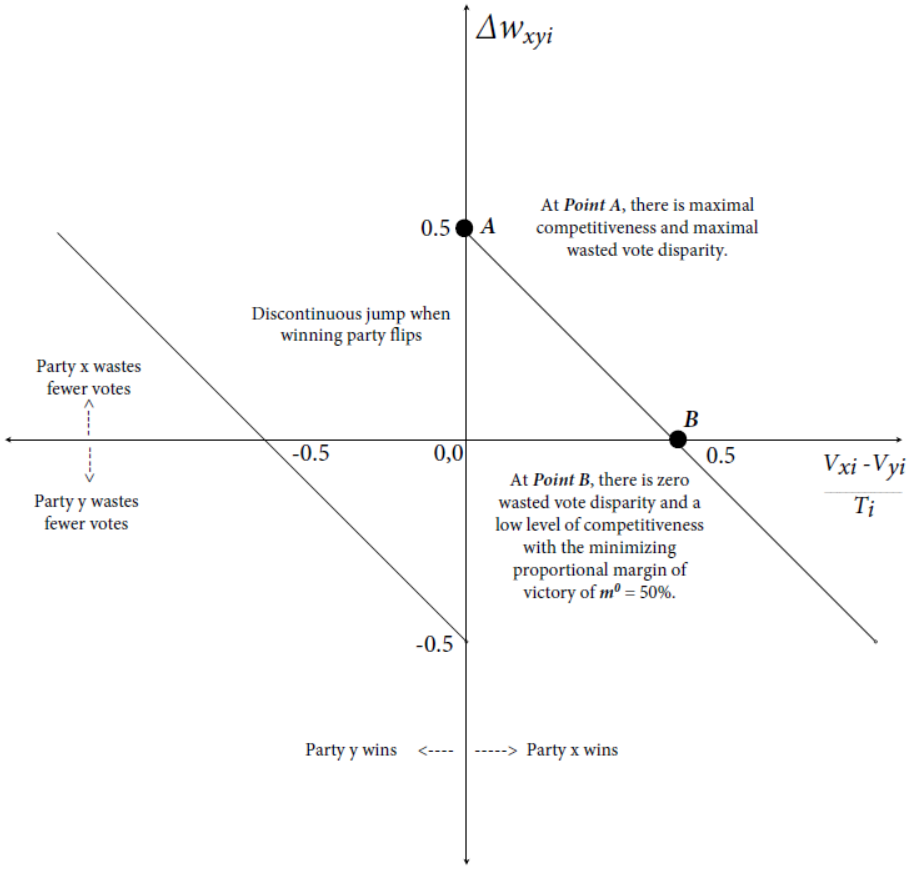
Diagram 9.

$$V_x' = \sum_k V_{xk}' = V_{xi} + 1 + V_{xj} - 1 + \sum_{k \neq i,j} V_{xk} = \sum_k V_{xk} = V_x$$

$$V_y' = \sum_k V_{yk}' = V_{yi} - 1 + V_{yj} + 1 + \sum_{k \neq i,j} V_{yk} = \sum_k V_{yk} = V_y$$

Since $V_x' = V_x$ and $V_y' = V_y$ the swap preserves the vote margin.

[224] *See* McGhee, *Measuring Partisan Bias*, *supra* note 8, at 68-69 eq.5; *id.* at 79-82 app.B (deriving equation 5); Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 853 and n. 114.



C. The Efficiency Gap and Proportionality

The efficiency gap measures undeserved seat share relative to a baseline of double proportionality, not strict proportionality. That the efficiency gap is distinct from a requirement of strict proportionality may be a doctrinal virtue, as it avoids the argument that its use is foreclosed by precedent holding that the Constitution does not require strict proportionality.[225] But the efficiency gap's departure from strict proportionality is normatively problematic, particularly as one party's vote share increases. A party with 75% vote share can win every seat and achieve a zero efficiency gap. In fact, the efficiency gap will report a

[225] *See supra* Part III.A, particularly n. 206 and accompanying text.

disadvantage for a party with more than 75% vote share even if that party wins every seat.[226]

The academic proponents recognize that the efficiency gap measure "fails to capture the idea of fairness" in this case, but they conclude that "this is not a problem that is especially relevant to real-world redistricting" because "results this lopsided are extremely rare."[227] In other words, the proponents concede the normative correspondence problem, but address that problem by limiting the scope of the measure's operation: "All an analyst must do is flag elections in which a party received at least 75 percent of the statewide vote and 100 percent of the seats."[228] The proponents, writing in 2015, noted that "[n]o party has received more than 75 percent of the aggregate vote in state legislative elections since 1982, and there are only 18 such cases out of 800 in congressional elections (all of them either in the South or in states with fewer than four House districts)."[229]

The proponents provide strong evidence that it is historically rare for a majority party to enjoy a vote share above 75%. But this fact does not eliminate the concerns highlighted by this scenario. Eighteen cases out of eight hundred (about 2%) is a tiny proportion, but it is a significant absolute number of cases. Five of the eighteen cases involved a state exhibiting high vote share for a single congressional election—Wyoming in 1984, Mississippi in 1990, South Dakota and West Virginia in 1998, and Louisiana in 2000.[230] The other thirteen cases involved a state exhibiting high vote share in multiple congressional elections—twice for North Dakota (1984 and 1986); thrice for Alaska (2000, 2002, and 2004) and Hawaii (1984, 1992, and 2008); and five times for Vermont (1982, 1984, 1990, 1992, 1996).[231] For these four states, the efficiency gap would repeatedly "fail[] to capture the idea of fairness at stake" in the election results.[232]

Moreover, these are only the cases where majority vote share exceeds 75%, the vote share needed to win every seat with a gap of zero. The problem may

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[226] For example, if party *x* wins every seat ($S^* = 0.5$) with 80% vote share ($V^* = 0.3$), the simplified formula computes a gap of negative 10%:

$$\Delta W = S^* - 2V^* = (0.5) - 2(0.3) = -0.10$$

The negative sign indicates that the gap disadvantages party *x* and favors party *y*, even though party *x* won every seat with only 80% of the vote.

[227] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 863-64.

[228] *Id.* at 863.

[229] *Id*.

[230] *Id.* at 863-64 n. 148.

[231] *Id.* at 863-64 n. 148.

[232] *Id.* at 863.

reach its apex at this point, but the problem is not confined to this point.[233] The proposed standard does not require a zero gap; it requires only a gap below the numeric threshold triggering a presumption of invalidity. Let ΔW^* denote this threshold. The efficiency gap will fall below this threshold whenever the following inequality obtains:[234]

$$\bar{V} > \frac{(\bar{S} - \Delta W^* + \frac{1}{2})}{2}$$

With proponent's suggested threshold of $\Delta W^* = 0.08$, this reduces to:

$$\bar{V} > \frac{\bar{S}}{2} + 0.21$$

Thus, a party with vote share exceeding 71% can win every seat with a below-threshold gap ($\bar{S} = 1$ if the party wins every seat, so $\bar{V} > \frac{1}{2} + 0.21 = 0.71$). If the presumption of validity is irrebuttable, a majority with this vote share can capture every seat with a plan impervious to judicial scrutiny.

Thus far, we have assumed the majority insists on every last seat. But if the majority is willing to throw the minority a bone, it can enjoy less-than-total domination and a below-threshold gap at lower vote share. For example, with a vote share above 59%, the majority can achieve 75% seat share with a below-threshold gap ($0.59 > \frac{0.75}{2} + 0.21 = 0.585$). Presumably, 71% vote share is more common than 75% vote share, and 59% vote share is more common still.

Finally, historical patterns do not necessarily predict future trends. In the coming decades, more states may consistently exhibit high vote share. And if the Court were to adopt a legal standard for partisan gerrymandering that gave free rein to majorities with sufficiently high vote share, majorities would have an ever-stronger incentive to achieve it. This could encourage desirable behavior—like voter persuasion—or undesirable behavior—like voter suppression. Even if a majority party earned a sufficiently high vote share through legitimate means, courts might be unable to regulate that majority's subsequent redistricting decisions.

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[233] *Compare id.* ("this scenario is easily identified"), *with id.* ("the unexpected results that begin to emerge when one party receives an extraordinarily high vote share").

[234] Since $S^* = \bar{S} - \frac{1}{2}$, $V^* = \bar{V} - \frac{1}{2}$, and $\Delta W = S^* - 2V^*$(under the simplified formula), this inequality ensures a below-threshold gap:

If $\bar{V} > \frac{\bar{S} - \Delta W^* + \frac{1}{2}}{2}$ then $\bar{V} > \frac{\left(\bar{S} - \frac{1}{2}\right) - \Delta W^*}{2} + \frac{1}{2}$ and so $2\left(\bar{V} - \frac{1}{2}\right) > \left(\bar{S} - \frac{1}{2}\right) - \Delta W^*$ and so $S^* - 2V^* < \Delta W^*$ and so $\Delta W < \Delta W^*$.

For these reasons, the academic proponents may be too quick to conclude that the operation of the measure in cases of high vote share "is not a problem that is especially relevant to real-world redistricting"[235] because vote shares exceeding 75% are "extremely rare."[236]

D. The Efficiency Gap and Competitiveness

The efficiency gap bears a more nuanced relationship to competitiveness. The value of the efficiency gap at a specified level of vote share is a function of differential average competitiveness between x-won and y-won districts – a function I call the "competitiveness gap." The sensitivity of the efficiency gap to changes in vote share is a function of responsiveness, which is closely related to a plan's overall competitiveness. Section 1 explains the competitiveness gap. Section 2 explains the relationship between responsiveness and the efficiency gap's sensitivity to changes in vote share.

1. The Competitiveness Gap

This section explains the relationship between the efficiency gap and the competitiveness gap. I define the competitiveness gap and show that it equals the efficiency gap under equal turnout. I then explore two implications of this equality. First, scholars and jurists may invoke the competitiveness gap (or something close to it) without recognizing its relationship to the efficiency gap. Second, at a given level of vote share, a mapmaker can achieve a zero gap with either high or low overall competitiveness. Specifically, a mapmaker can eliminate or reduce an efficiency gap in one of two ways: unpacking, which increases a plan's overall competitiveness; or decracking, which decreases a plan's overall competitiveness.

a. Definition

Let $c_p = \frac{\sum_{i \in D_p} c_i}{S_p}$ denote the average competitiveness score in p-won districts and let $c = \frac{\sum_{i \in D} c_i}{S}$ denote the average competitiveness score over all districts. Note that average competitiveness is the difference between the minimizing and the average victory margin; $c = m^0 - m, c_p = m^0 - m_p$;[237] and c is the seat-

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[235] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 864.

[236] *Id.* at 863.

[237] Proof: Relying on $c = \frac{\sum_{i \in D} c_i}{S}$, $c_p = \frac{\sum_{i \in D_p} c_i}{S_p}$, $m = \frac{\sum_{i \in D} m_i}{S}$, $m_p = \frac{\sum_{i \in D_p} m_i}{S}$

share-weighted sum of c_x and c_y: $c = \bar{S}_x c_x + \bar{S}_y c_y$. Define the competitiveness gap (ΔC_{xy}) as the seat-share-weighted difference of c_x and c_y.

$$\Delta C_{xy} = \bar{S}_x c_x - \bar{S}_y c_y$$

While c measures a plan's overall average competitiveness, the competitiveness gap ΔC_{xy} measures a plan's differential average competitiveness, comparing the average competitiveness of x-won and y-won districts.

b. Equality

Under equal voter turnout, the efficiency gap is the competitiveness gap.

$$\Delta W_{xy} = \frac{\sum_D \Delta w_{xyi}}{\text{S}} = \frac{\sum_{D_x} c_i + \sum_{D_y} -c_i}{\text{S}} = \frac{S_x c_x - S_y c_y}{\text{S}} = \bar{S}_x c_x - \bar{S}_y c_y = \Delta C_{xy}$$

And the competitiveness gap reduces to the simplified seats-votes formula.

$$\begin{aligned}\Delta C_{xy} = \ \bar{S}_x c_x - \bar{S}_y c_y = \ \bar{S}_x (m^0 - m_x) - \bar{S}_y \left(m^0 - m_y\right) \\ = \frac{1}{2}\left(\bar{S}_x - \bar{S}_y\right) - (\bar{S}_x m_x - \bar{S}_y m_y) = \ S^* - 2V^*\end{aligned}$$

Thus, under equal turnout, one can frame the measure as relative wasted votes, undeserved seat share, or differential average competitiveness.

$$\Delta W_{xy} = S^* - 2V^* = \Delta C_{xy}$$

Note that a zero efficiency gap does not necessarily entail equal average competitiveness in *x*-won and *y*-won districts. This equality obtains only in the special case where each party wins half the seats. In the more general case, the relative competitiveness of *x*-won and *y*-won districts needed to achieve a zero efficiency gap will depend on the relative seat shares. Formally, let $\tilde{c} = \frac{c_x}{c_y}$ denote the competitiveness ratio, and $\tilde{s} = \frac{S_x}{S_y}$ denote the seat ratio. Then the sign of the

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$$c = \frac{\sum_{i \in D} c_i}{S} = \frac{\sum_{i \in D} (m^0 - m_i)}{S} = \frac{\sum_{i \in D} m^0 - \sum_{i \in D} m_i}{S} = \frac{Sm^0 - Sm}{S} = m^0 - m$$

$$c_p = \frac{\sum_{i \in D_p} c_i}{S_p} = \frac{\sum_{i \in D_p} (m^0 - m_i)}{S_p} = \frac{\sum_{i \in D_p} m^0 - \sum_{i \in D_p} m_i}{S_p} = \frac{S_p m^0 - S_p m_p}{S_p} = m^0 - m_p$$

efficiency gap (ΔW) depends on whether the product of the seat ratio ($\tilde{s}$) and the competitiveness ratio ($\tilde{c}$) is greater than, equal to, or less than one.[238]

$$\begin{cases} \Delta W > 0 & \tilde{s}\tilde{c} > 1 \\ \Delta W = 0 & \tilde{s}\tilde{c} = 1 \\ \Delta W < 0 & \tilde{s}\tilde{c} < 1 \end{cases}$$

When the product is greater than one, the competitiveness gap and thus the efficiency gap is positive, indicating an advantage for party x; when the product is less than one, the competitiveness gap and thus the efficiency gap is negative, indicating an advantage for party y; when the product is equal to one, the competitiveness gap and thus the efficiency gap is zero, indicating partisan fairness in the sense of equal wasted votes. Thus, when each party wins half the seats ($\tilde{s} = 1$), a zero efficiency gap requires that x-won and y-won districts exhibit equal average competitiveness ($\tilde{c} = 1$). When party x wins a majority of seats ($\tilde{s} > 1$), a zero efficiency gap requires that y-won districts are more competitive than x-won districts on average ($\tilde{c} < 1$). Specifically, if party x wins two-thirds of the seats ($\tilde{s} = 2$), a zero efficiency gap requires that y-won districts are twice as competitive as x-won districts on average ($\tilde{c} = 1/2$).

c. Unacknowledged Measure Convergence

The equivalence of the efficiency gap and the competitiveness gap suggests that scholars and jurists may be referring to the efficiency gap measure (or something quite like it) without realizing it. For example, Samuel S.-H. Wang recently proposed three tests for partisan gerrymandering, including the following "lopsided outcomes test":

> *Test 2 (the lopsided outcomes test)*: Compare the difference between the share of Democratic votes in the districts that Democrats win, and the share of Republican votes in the districts that Republicans win. This test works because in a partisan gerrymander, the targeted party wins lopsided victories in a small number of districts, while the gerrymandering party's wins are engineered to be relatively narrow. To compare the winning vote shares for the two parties, I use a grouped *t*-test, an extremely common statistical test.[239]

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[238] Using $\Delta W = \Delta C = \bar{S}_x c_x - \bar{S}_y c_y$, $c = \frac{c_x}{c_y}$, and $s = \frac{S_x}{S_y} = \frac{\bar{S}_x}{\bar{S}_y}$, the equality can be derived as follows: $\Delta W = 0$ if and only if $\bar{S}_x c_x - \bar{S}_y c_y = 0$ if and only if $\bar{S}_x c_x = \bar{S}_y c_y$ if and only if $\frac{\bar{S}_x}{\bar{S}_y}\frac{c_x}{c_y} = 1$ if and only if $sc = 1$. The inequalities can be derived analogously.

[239] Samuel S.-H. Wang, *Three Tests for Practical Evaluation of Partisan Gerrymandering*, 68 STAN. L. REV. 1263, 1306 (2016) (footnotes omitted).

Just like the efficiency gap, this "lopsided outcomes test" (LOT) measures differential average competitiveness. There are three differences, but only one is substantive. First, the efficiency gap focuses on the competitiveness score (c_i), whereas the LOT focuses on the victor's vote share (v_i^*)—"the share of Democratic votes in the districts that Democrats win, and the share of Republican votes in the districts that Republicans win."[240] These two variables are linearly related—$v_i^* = \frac{3-2c_i}{4}$.[241] Second, the LOT uses a statistical test—"a grouped *t*-test"[242]—to estimate the likelihood that the average competitiveness in *x*-won districts is different from the average competitiveness in *y*-won districts, whereas the efficiency gap simply computes the difference.

The third difference is substantive: the LOT does not account for seat share. Note that the block quote above makes no mention of seat share. This means the LOT measures unweighted differential average competitiveness, assuming that the competitiveness ratio should be one, whereas the efficiency gap measures seat-share-weighted differential average competitiveness, assuming the competitiveness ratio should be equal to the seat ratio. As the seat ratio approaches one, this last difference drops out. But when the seat ratio departs significantly from one, the two tests will diverge. When one party wins most seats, the efficiency gap—but not the LOT—permits what I call a "competitiveness bonus." This means that a party with a positive seat margin can win relatively more competitive districts while maintaining a zero competitiveness gap, so long as the competitiveness ratio equals the seat ratio.

Similarly, the *Whitford* majority separated its discriminatory effect analysis into two parts. First, the majority discussed evidence of discriminatory effect other than the efficiency gap.[243] Only after concluding that "the evidence we have just described certainly makes a firm case on the question of discriminatory effect" did the majority proceed to discuss how "that evidence is further bolstered by the plaintiffs' use of the 'efficiency gap.'"[244] But as part of its initial discussion, the majority focused on the competitiveness gap, noting that Democratic-won districts were far less competitive on average than Republican-won districts—that Democrats were packed into "safe" districts.[245] The majority did not realize that in talking about differential competitiveness, it was actually talking about the efficiency gap.

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[240] *Id.*

[241] This relationship can be derived as follows:

$$v_i^* = \frac{V_{i1}}{T_i} = \frac{T_i+M_i}{2T_i} = \frac{1}{2} + \frac{m_i}{2} = \frac{1}{2} + \frac{\frac{1}{2}-c_i}{2} = \frac{1}{2} + \frac{1}{4} - \frac{c_i}{2} = \frac{3-2c_i}{4}.$$

[242] Wang, *supra* note 239, at 1306.

[243] *See* Whitford v. Gill, 218 F. Supp. 3d 837, 898-903 (W.D. Wis. 2016).

[244] *Id.* at 903.

[245] *Id.* at 898-99.

d. Unpacking Versus Decracking

The equivalence of the efficiency gap and the competitiveness gap also clarifies that plans with low efficiency gaps can, but need not, exhibit competitiveness. The efficiency gap is a function of differential competitiveness, not overall competitiveness. To achieve a zero efficiency gap, the competitiveness ratio must equal the seat ratio, but overall competitiveness can take on any value. A zero-gap plan can be highly competitive or highly uncompetitive, so long as it is not differentially competitive in the sense that the competitiveness ratio departs from the seat ratio. The mapmaker can design minimizing plans with districts as competitive or uncompetitive as she pleases while still achieving a zero efficiency gap. She can even make some districts more competitive than others, so long as the relative competitiveness of the average *x*-won district and the average *y*-won district is proportional to the relative number of seats won by parties *x* and *y*. What she cannot do (if she wants a zero or low efficiency gap) is systematically vary competitiveness by party so that the competitiveness ratio departs from the seat ratio. For example, the mapmaker cannot achieve a zero or low efficiency gap by drawing a plan that awards each party half the seats but with party *x* generally winning competitive races and party *y* generally winning by landslides. That competitiveness gap would entail an efficiency gap in favor of party *x*. It would systematically pack and crack party *y* supporters, which would produce an efficiency gap (Party *y* would waste more votes), and equivalently, a competitiveness gap (*x*-won districts would be excessively competitive), and equivalently, undeserved seat share relative to the ideal seats-votes ratio of two.

To eliminate (or limit) this undeserved seat share, the mapmaker must flip one or more seats from the favored party to the disfavored party. But just as there are two fundamental gerrymandering strategies—packing and cracking—so there are two analogous strategies to flip the requisite seat(s) and thereby eliminate (or reduce) a large efficiency/competitiveness gap: *unpacking* and *decracking*. Unpacking flips the requisite seats by transferring supporters of the disfavored party from relatively uncompetitive districts won by the disfavored party. This unpacking strategy makes the districts won by the disfavored party, and thus the plan over all, more competitive. Decracking flips the requisite seats by transferring supporters of the disfavored party from relatively competitive districts won by the favored party. This decracking strategy makes the districts won by the favored party, and thus the plan over all, less competitive. Table 2 demonstrates these two strategies.

Table 2

DISTRICT	**ORIGINAL PLAN**		**UNPACKING PLAN**		**DECRACKING PLAN**	
	x	***y***	***X***	***Y***	***X***	***y***

1	**700**	300	**700**	300	**800**	200
2	**700**	300	**700**	300	**800**	200
3	**700**	300	**700**	300	**700**	300
4	**540**	460	440	**560**	440	**560**
5	**540**	460	440	**560**	440	**560**
6	**540**	460	**540**	460	**540**	460
7	**540**	460	**540**	460	**540**	460
8	**540**	460	**540**	460	**540**	460
9	350	**650**	450	**550**	350	**650**
10	350	**650**	450	**550**	350	**650**
Total Won	**8**	**2**	**6**	**4**	**6**	**4**

In both the unpacking plan and the decracking plan, districts 4 and 5 swap enough *x*-voters for *y*-voters to flip those seats from *x*-won to *y*-won, thereby producing a seat margin twice the vote margin and a zero efficiency gap. But the unpacking plan swaps voters out of *y*-won districts 9 and 10, making those districts and the plan overall more competitive. In contrast, the decracking plan swaps voters out of *x*-won districts 1 and 2, making those districts and the plan overall less competitive.

2. Sensitivity and Responsiveness

The preceding section illustrated how both competitive and uncompetitive plans can produce a zero efficiency gap. But while a plan of any competitiveness level can achieve a zero (or low) gap at one level of vote share, only a plan with a specified level of competitiveness can maintain a zero (or low) gap over a range of vote share. This is so because the sensitivity of the efficiency gap is a function of overall competitiveness.

a. Sensitivity as a Function of Responsiveness

Ignoring turnout effects, the efficiency gap (ΔW) is a function of two variables—the statewide vote margin (V^*) and the statewide seat margin (S^*): $\Delta W = S^* - 2V^*$. But the statewide seat margin (S^*) is itself a function of the statewide vote margin (V^*): $S^* = S^*(V^*)$. This latter function is the real-world—not the ideal—seats-votes curve. The responsiveness (r) of this curve tells us how much the seat margin changes for a given incremental change in vote margin. Mathematically, responsiveness (r) is the derivative of the function $S^*(V^*)$ with respect to V^*: $r = \frac{dS^*(V^*)}{V^*}$; graphically, it is the slope of the tangent to the seats-votes curve at a specified point. Responsiveness is a measure of

competitiveness because the more districts with small victory margins, the greater the change in seat share for a given change in vote share.

The sensitivity of the efficiency gap to a change in statewide vote margin is captured by the derivative with respect to V^* of $\Delta W = S^*(V^*) - 2V^*$.

$$\frac{d\Delta W}{dV^*} = r - 2$$

According to this equation, the sensitivity of the efficiency gap to a change in vote margin depends on the responsiveness of the actual seats-votes curve. If responsiveness is precisely equal to two, the increase in seat margin perfectly offsets the increase in vote margin and the efficiency gap remains constant. If responsiveness is greater than two, the increase in vote margin triggers an overcompensatory increase in seat margin, and the efficiency gap increases. If responsiveness is less than two, the increase in vote margin triggers an undercompensatory increase in seat margin, and the efficiency gap decreases.

This relationship means that both highly competitive (r>2) and highly uncompetitive (r<2) plans entail a risk: they may produce a zero gap at expected vote share but an above-threshold gap if vote share departs sufficiently from the expectation. However, the highly competitive plan may present the greater risk because it requires a smaller change in vote share to produce an above-threshold gap. With a highly competitive plan, a small change in vote share can flip many seats, and that large change in seat share can produce an above threshold gap. With a highly uncompetitive plan, a small change in vote share will flip no seats, so a 4% vote swing is needed to produce an 8% gap.[246]

b. The Robust, Minimizing Plan

Now consider a mapmaker who knows (or can estimate well) the current statewide vote margin, but who recognizes the vote margin may vary over time with changing electoral circumstances. Suppose this mapmaker wishes to design a plan that is both *minimizing* in the sense that it produces a zero efficiency gap under the current statewide vote margin, and *robust* in the sense that it maintains a zero (or low) efficiency gap if vote margin varies. How can the mapmaker design such a robust, minimizing plan? The trick is to find a plan that produces a particular seat-vote relationship—one that exhibits the *right ratio* at the current

[246] Assume the plan produces a zero gap ($S^* - 2V^* = 0$) at expected vote share (V^*) and a 4% uniform vote swing fails to flip a single seat. Thus, the seat margin is still S^*, but the vote margin is now ($V^* + 0.04$), so the efficiency gap is $S^* - 2(V^* + 0.04) = S^* - 2V^* - 0.08 = -0.08$. For both the highly competitive and highly uncompetitive plans, the above-threshold gap may not be durable under sensitivity analysis. However, there is uncertainty about whether and how sensitivity analysis will play a role doctrinally. Will mapmakers prefer to avoid generating unstable above-threshold gaps? If so, they may prefer less competitive plans.

vote margin (double proportionality), and one that flips seats at the *right rate* as vote margin varies (double responsiveness).

Return to our numeric example. Presently, the statewide vote margin is five percent (*x* wins 5,500 votes, or 55%, and *y* wins 4,500 votes, or 45%). To produce a zero efficiency gap, the statewide seat margin must be 10% ($0 = S^* - 2(0.05)$ so $S^* = 0.1$). Thus, party *x* must win 6 of the 10 seats ($S^* = \bar{S}_x - \frac{1}{2}$ so $0.1 = \bar{S}_x - \frac{1}{2}$ and thus $\bar{S}_x = 0.6$). And for every 5% change in vote margin, one seat must flip (i.e., $0 = S^* - 2(0.1)$ so $S^* = 0.2 = \bar{S}_x - \frac{1}{2}$ and thus $\bar{S}_x = 0.7$, or 7 out of 10 seats). Assuming uniform swing, this means that the proportional vote margin in each district must be a distinct multiple of 5%. The result is to spread out the competitiveness of each district so that seats flip at the right rate.

Table 3

DISTRICT	TOTAL VOTES BY PARTY			
	X	***Y***	**V*i**	**Winner**
1	775	225	.275	X
2	725	275	.225	X
3	675	325	.175	X
4	625	375	.125	X
5	575	425	.075	X
6	525	475	.025	X
7	475	525	-.025	Y
8	425	575	-.075	Y
9	375	625	-.125	Y
10	325	675	-.175	Y

Note that only 2 of 10 districts are relatively competitive. District 6 currently favors party *x* but would flip if party *y* earned a 5% uniform vote swing; District 7 currently favors party *y* but would flip if party *x* earned a 5% uniform vote swing; all other districts would have the same winning party. As this example illustrates, a mapmaker eager to produce and maintain a zero efficiency gap must necessarily limit the competitiveness of the plan she draws. Most districts must be safe seats.

E. False Negatives and False Positives

The preceding examination reveals how the efficiency gap measure privileges a form of symmetric partisan efficiency over electoral competitiveness and strict seats-votes proportionality. If proportionality and competitiveness

matter, the efficiency gap's relationship to them presents two problems of normative correspondence: the measure may favor normatively undesirable plans (the false negative problem); and the measure may disfavor normatively desirable plans (the false positive problem). Consider each problem in turn.

First, a plan may be gerrymandered in a way the measure cannot detect. A plan may achieve the ideal of equal wasted votes at the expense of both competitiveness and seats-votes proportionality. In this sense, the efficiency gap measure has a significant false negative problem: it approves plans that exhibit one democratic ideal (equal wasted votes) even if those plans subvert competitiveness, seats-votes proportionality, or both.

Second, the measure may flag as suspect a normatively desirable plan. A plan that produces rough proportionality even when one party enjoys a significant popular majority will necessary produce a large efficiency gap in favor of the minority party. While both competitive and uncompetitive plans can produce low efficiency gaps, a more competitive plan may present a greater risk of an above-threshold gap in the face of vote swings. In this sense, the efficiency gap measure has a significant false positive problem: it flags as suspect plans that depart from the ideal of equal wasted votes, even if that departure reflects an effort to promote democratic norms like competitiveness and seats-votes proportionality.

By privileging the ideal of equal wasted votes, the efficiency gap measure not only favors some plans that subvert competitiveness and proportionality (the false negative problem), but also disfavors some plans that promote competitiveness or proportionality (the false positive problem). For these reasons, the efficiency gap measure may promote fairness in the sense of symmetric partisan efficiency but unintentionally encourage uncompetitive elections that accord one party a legislative majority disproportional to its popular support.

The efficiency gap proposal consists not only of this measure, but also sensitivity analysis, intent, and justification elements.[247] These other doctrinal tools mitigate but do not eliminate the false positive problem. If a highly competitive plan produces a large efficiency gap in one election, that gap will likely be unstable under sensitivity analysis. And a plan that promotes competitiveness or proportionality is more likely to be justified by legitimate districting principles and less likely to be branded the result of discriminatory intent. But these tools do not eliminate the problem. Mapmakers may fear not only liability, but also litigation. An above-threshold gap may be unstable, unintended, or justified, but it is likely to invite legal challenge, and may only

[247] The academic proponents suggested sensitivity analysis, Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 889-90, and a justification element, *id*. at 891, but not an intent element, *id*. at 832 n.2. The *Whitford* plaintiffs also offered evidence of partisan intent. *Whitford*, 218 F. Supp. 3d at 890-96.

prevail after the considerable delay, expense, and risk of a trial before a three-judge federal panel.

Perhaps more significantly, these doctrinal tools do not address the false negative problem, because proponents offer no mechanism to overcome the presumption of validity triggered by a below-threshold gap.[248] If the Court embraced the efficiency gap as the definitional measure of partisan gerrymandering, plans that consistently produced low efficiency gaps would be judicially bulletproof. This would be problematic in jurisdictions where one party enjoyed a large majority. This party could achieve a low gap by carving the state up into safe districts that accorded it most or all of the seats.

F. Definition and Weight of Surplus Votes

The preceding Subparts examined the efficiency gap's relationship to proportionality and competitiveness under its traditional definition. This Subpart demonstrates how those relationships vary when we modify the definition of wasted votes. Specifically, I relax the definition and weight of surplus votes using parameters alpha (α) and beta (β), and derive a more generalized simplified formula as a function of these parameters.[249]

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[248] The proponents repeatedly refer to both the presumption of invalidity triggered by an above-threshold gap and the presumption of validity triggered by a below-threshold gap as presumptive rather than irrebuttable. Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 884 ("courts would need to choose an efficiency gap threshold above which district plans would be presumptively unlawful and below which they would be presumptively valid."); *id*. at 886 ("specifying an efficiency gap level above which plans would be presumptively unlawful and below which they would be presumptively legitimate."); *id*. at 890 ("courts may be reluctant in early cases to set particular levels above which plans are presumptively unlawful and below which they are presumptively legitimate."); *id*. at 891 ("throughout our discussion to this point, we have spoken of presumptive rather than irrebuttable validity and invalidity.") However, while the proponents explain in detail how a state could rebut the presumption of invalidity, *see id*. 891-95, they say nothing about how a plaintiff could rebut the presumption of validity. Moreover, proponents suggest that their "approach would neatly slice *Vieth*'s Gordian knot, informing lower courts and political actors, in clear quantitative terms, exactly '[h]ow much political . . . effect is too much.'" *Id*. at 886 (quoting *Vieth*, 541 US at 297 (Scalia) (plurality)). This characterization suggests that a plan that produces a below-threshold gap must be valid because its political effect is not too much. Finally, the *Whitford* panel majority seems to have understood the plaintiff's proposal to include an irrebuttable presumption of validity for low-gap plans. *See* Whitford v. Gill, 218 F. Supp. 3d 837, 908 (W.D. Wis. 2016) ("a challenge to a map enacted with egregious partisan intent but demonstrating a low EG also will fail because the plaintiffs cannot demonstrate the required discriminatory effect.")

[249] I use the term "parameter" to denote a variable that an analyst selects, rather than a variable that an analyst observes. Each parameter—alpha and beta—represents a real number the analyst selects.

Alpha (α) captures weighting of surplus votes. The efficiency gap, as proposed, measures the number of wasted votes through the equation:

$$W_{pi} = L_{pi} + E_{pi}$$

In order to weight lost and surplus votes differently, we can simply multiply the number of excess votes by the parameter alpha (α) which we can adjust to capture how heavily we wish to weight surplus votes as compared to lost votes. When we add alpha (α), the wasted votes equation becomes:

$$W_{pi} = L_{pi} + \alpha E_{pi}$$

Under the proponents' definition, alpha (α) is set to one. If, alternatively, we set alpha (α) to zero, then we would ignore surplus votes entirely. If we set alpha (α) to 0.5, then we would weight surplus votes half as heavily as lost votes. If we set alpha (α) greater than one, we would weight surplus votes more heavily than lost votes.

Beta (β) captures the definition of surplus votes. The efficiency gap as proposed defines the number of surplus votes as one half of the margin of victory:

$$E_{pi} = \frac{1}{2} M_i$$

But surplus votes could also be plausibly defined as the entire margin of victory. Thus, we can replace the number 0.5 with the parameter beta (β) to allow an analyst to vary the definition of a surplus vote. If we do so, we get the following generalized equation defining surplus votes:

$$E_{pi} = \beta M_i$$

We can set beta (β) to one half (using proponents' definition) or to one (using the alternative definition)—or at some intermediate value.[250]

Gamma (γ) captures the combined impact of weight and definition. Gamma (γ) is simply the product of alpha and beta.

$$\gamma = \alpha\beta$$

We can now repeat all the relevant steps of our prior anlaysis, but this time in terms of these parameters to quantify the precise impact of the way we define

[250] An analyst might use an intermediate value if she thought that both definitions had some merit and some flaws, and that the best approach was somewhere in between.

and weight surplus votes. The long-form definition-generalized efficiency gap is:

$$\Delta W_{xy}(\gamma) = \frac{W_y - W_x}{V_y + V_x}$$

where

$$W_p = \sum_D W_{pi}$$

and

$$W_{pi} = \begin{cases} E_{pi} = \gamma M_i & i \in D_p \\ L_{pi} = V_{pi} & i \notin D_p \end{cases}$$

As we shall soon see, it will prove useful to define the measure's generalized ideal responsiveness as the following function of parameter γ: $r(\gamma) = 1 + 2\gamma$. Note that when $\alpha = 1$ and $\beta = \frac{1}{2}$, as under the proponents' traditional definition, $\gamma = \alpha\beta = \frac{1}{2}$ and $r = 1 + 2\gamma = 2$. This is the responsiveness of the ideal seats-votes curve associated with the simplified formula.

At the level of an individual district, the wasted vote disparity (Δw_{xyi}) remains a simple linear discontinuous function of the proportional margin of victory (m_i), but that function now depends on ideal responsiveness (r).[251]

$$\Delta w_{xyi} = \begin{cases} \frac{1}{2}(1 - rm_i) & i \in D_x \\ -\frac{1}{2}(1 - rm_i) & i \in D_y \end{cases}$$

Excluding ties, this function is zero if and only if $m_i = \frac{1}{r}$. Call this the generalized minimizing proportional margin of victory, denoted $m^0(\gamma)$. For example, when $\alpha = 1$ and $\beta = \frac{1}{2}$, as under the proponents' traditional definition,

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[251] Since $V_{2i} = \frac{T_i}{2}(1 - m_i)$, both excess and lost votes can be expressed in terms of m_i:

$E_{pi} = \gamma M_i = \gamma m_i T_i$ and so $\frac{E_{pi}}{T_i} = \gamma m_i$

$L_{pi} = V_{pi} = V_{2i} = \frac{T_i}{2}(1 - m_i)$ and so $\frac{L_{pi}}{T_i} = \frac{1}{2}(1 - m_i)$

By definition $\Delta w_{xyi} = \frac{W_{yi} - W_{xi}}{T_i}$ and $r = 1 + 2\gamma$, so we can express Δw_{xyi} in terms of m_i:

For $i \in D_x$, $\Delta w_{xyi} = \frac{W_{yi} - W_{xi}}{T_i} = \frac{L_{yi}}{T_i} - \frac{E_{xi}}{T_i} = \frac{1}{2}(1 - m_i) - \gamma m_i = \frac{1}{2}(1 - rm_i)$

For $i \in D_y$, $\Delta w_{xyi} = \frac{W_{yi} - W_{xi}}{T_i} = \frac{E_{yi}}{T_i} - \frac{L_{xi}}{T_i} = -\left(\frac{L_{yi}}{T_i} - \frac{E_{xi}}{T_i}\right) = -\frac{1}{2}(1 - rm_i)$

$\gamma = \alpha\beta = \frac{1}{2}$, $r = 1 + 2\gamma = 2$, and $m^0 = \frac{1}{r} = \frac{1}{2}$. This is the minimizing victory margin derived *supra*, where the victor earns 75% vote share.

Define the generalized competitiveness score $c_i(\gamma)$ as the difference between this new minimizing margin of victory $m^0(\gamma)$ and the actual margin of victory m_i: $c_i(\gamma) = m^0(\gamma) - m_i$. At the level of an individual district, the wasted vote disparity Δw_{xyi} is a simple function of $c_i(\gamma)$ and $r(\gamma)$:[252]

$$\Delta w_{xyi} = \begin{cases} \frac{r}{2} c_i & i \in D_x \\ -\frac{r}{2} c_i & i \in D_y \end{cases}$$

With this new competiveness score, define average competitiveness in p-won districts as $c_p(\gamma) = \frac{\sum_{D_p} c_i(\gamma)}{S_p}$. Note that, just as before, $c_p(\gamma) = m^0(\gamma) - m_p$ where m_p is the average victory margin in p-won districts. Finally, define the competitiveness gap as the seat-share-weighted difference between average competitiveness in x-won and y-won districts.

$$\Delta C_{xy}(\gamma) = \bar{S}_x c_x(\gamma) - \bar{S}_y c_y(\gamma)$$

With these new definitions, the efficiency gap and the competitiveness gap are related by the factor $\frac{r}{2}$:[253]

$$\Delta W_{xy}(\gamma) = \frac{r}{2} \Delta C_{xy}(\gamma).$$

The definition-generalized simplified formula is:

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[252] By definition, $c_i = m^0 - m_i$ and $m^0 = \frac{1}{r}$. Rearranging: $m_i = \frac{1}{r} - c_i$. Thus:

$1 - r m_i = 1 - r\left(\frac{1}{r} - c_i\right) = 1 - 1 + r c_i = r c_i$ And so:

$$\Delta w_{xyi} = \begin{cases} \frac{1}{2}(1 - r m_i) & i \in D_x \\ -\frac{1}{2}(1 - r m_i) & i \in D_y \end{cases} = \begin{cases} \frac{r}{2} c_i & i \in D_x \\ -\frac{r}{2} c_i & i \in D_y \end{cases}$$

[253] This result is readily derived by expressing the efficiency gap as the average wasted voted disparity.

$$\Delta W_{xy} = \frac{\sum_D \Delta w_{xyi}}{S} = \frac{\frac{r}{2}[\sum_{D_x} c_i + \sum_{D_y} -c_i]}{S} = \frac{\frac{r}{2}[S_x c_x - S_y c_y]}{S} = \frac{r}{2}[\bar{S}_x c_x - \bar{S}_y c_y] = \frac{r}{2} \Delta C_{xy}$$

$$S^* - r(\gamma)V^*$$

Note that this is just like the traditional simplified formula, with the constant 2 replaced by the variable r, which is a function of the parameter γ. Of course, under proponents' traditional definition, r is 2 and the generalized simplified formula reduces to the traditional one. More generally, the definition-generalized simplified formula and the definition-generalized competitiveness gap are also related by the same factor $\frac{r}{2}$:[254]

$$\frac{r}{2}\Delta C_{xy}(\gamma) = S^* - rV^*$$

Combining these two results, with these new, generalized, definitions, under equal voter turnout, the efficiency gap, competitiveness gap, and simplified formula are related as follows.

$$\Delta W_{xy}(\gamma) \ = \frac{r}{2}\Delta C_{xy}(\gamma) = S^* - rV^*$$

Note that, under proponents' traditional definition, r is 2, and so the long-form equation and simplified formula equal the competitiveness gap. In the general case, the long-form equation and simplified formula equal the competitiveness gap multiplied by the factor $\frac{r}{2}$.

As the preceding examination reveals, even when we relax the definition and weight of surplus votes, the measure still captures relative wasted votes, undeserved vote share, and differential competitiveness. The measure still generally privileges equal wasted votes over competitiveness and seats-votes proportionality. But the precise trade-offs between these competing norms are determined by the precise values the analyst selects for parameters alpha and beta. The following table summarizes the effects of varying these two parameters.

Table 4

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[254] Relying on the following definitions and equalities: $\Delta C_{xy} = \bar{S}_x c_x - \bar{S}_y c_y$, $c_p = m^0 - m_p$, $m^0 = \frac{1}{r}$, $S^* = \frac{\bar{S}_x - \bar{S}_y}{2}$, and $\bar{S}_x m_x - \bar{S}_y m_y = 2V^*$. Substituting and rearranging:

$$\frac{r}{2}\Delta C_{xy} = \frac{r}{2}[\bar{S}_x c_x - \bar{S}_y c_y] = \frac{r}{2}[\bar{S}_x(m^0 - m_x) - \bar{S}_y(m^0 - m_y)]$$
$$= \frac{r}{2}\frac{1}{r}(\bar{S}_x - \bar{S}_y) - \frac{r}{2}(\bar{S}_x m_x - \bar{S}_y m_y) = S^* - \frac{r}{2}2V^* = S^* - rV^*$$

	Weight of surplus vote relative to lost vote	Relationship between surplus votes and vote margin	Product of alpha and beta	Ideal responsiveness	Generalized simplified formula	Minimizing victory margin	Minimizing winner's vote share	Statewide vote share that enables total capture with zero gap (expressed as percentage)	Statewide vote share that enables ($\bar{S}$) supermajority with sub-threshold gap (expressed as percentage)
Parameter	α	β	γ	r	ΔW	m^0	v^*	$V_{100\%,0\%}$	$V_{100\%,8\%}, V_{75\%,8\%}$
Equation			$\alpha\beta$	$1+2\gamma$	$S^* - rV^*$	$\frac{1}{r}$	$\frac{1+m^0}{2}$	v^*	$\frac{\left(\bar{S}-\frac{1}{2}\right)-\Delta W^*}{r}+\frac{1}{2}$
OPTION 1 (Alternative Definition)	1	1	1	3	$S^* - 3V^*$	1/3	2/3	67%	64%, 56%
OPTION 2 ***Traditional Interpretation***	1	½	½	2	$S^* - 2V^*$	½	¾	75%	71%, 59%
OPTION 3 ***Alternative Interpretation***	½	1	½	2	$S^* - 2V^*$	½	¾	75%	71%, 59%
OPTION 4 ***Alternative Definition & Weighting***	½	½	¼	3/2	$S^* - \frac{3}{2}V^*$	2/3	5/6	83%	78%, 61%
OPTION 5 ***Alternative Weighting***	0	1	0	1	$S^* - V^*$	1	1/1	100%	92%, 67%

The table's columns correspond to values of interest. The first two – α and β – are the parameters selected by the analyst to capture the weight and definition of surplus votes. γ is the product of α and β. $r = 1 + 2\gamma$, a simple function of γ, determines the responsiveness of the ideal seats-votes curve. The generalized simplified formula is just like the traditional one, except vote margin (V^*) is multiplied by responsiveness (r), which is a function of γ. The minimizing victory margin (m^0) is the multiplicative inverse of responsiveness (r): $m^0 = \frac{1}{r}$. The winner's minimizing vote share (v^*) is a simple function of m^0:[255] $v^* = \frac{1+m^0}{2}$. And the vote share a party must exceed to maintain a gap below threshold ΔW^* with a seat share of $\bar{S}$ is $\bar{V}_{\bar{S},\Delta W^*} = \frac{\left(\bar{S}-\frac{1}{2}\right)-\Delta W^*}{r} + \frac{1}{2}$.[256] Note that the winner's minimizing vote share is the vote share a party needs to win every seat ($\bar{S} = 1$) with a zero gap ($\Delta W^* = 0$):[257] $\bar{V}_{1,0} = v^*$. This makes sense, because a simple

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[255] Since $V_{1i} = \frac{T_i}{2}(1 + m_i)$, *see supra* note 217, $v^* = \frac{V_{1i}}{T_i} = \frac{1}{2}(1 + m_i)$.

[256] Since $S^* = \bar{S} - \frac{1}{2}$, $V^* = \bar{V} - \frac{1}{2}$, and $\Delta W = S^* - rV^*$(under the generalized simplified formula), this inequality ensures a below-threshold gap:

If $\bar{V} > \frac{\left(\bar{S}-\frac{1}{2}\right)-\Delta W^*}{r} + \frac{1}{2}$ then $r\left(\bar{V} - \frac{1}{2}\right) > \left(\bar{S} - \frac{1}{2}\right) - \Delta W^*$ and so $S^* - rV^* < \Delta W^*$ and so $\Delta W < \Delta W^*$.

[257] Proof: $\bar{V}_{1,0} = \frac{\left(1-\frac{1}{2}\right)-0}{r} + \frac{1}{2} = \frac{1}{2}(1 + m^0) = v^*$.

plan composed exclusively of x-won districts will produce a zero gap, seat share of one, and statewide vote share equal to the winner's minimizing vote share.

As this table shows, the parameter γ is like a dial the analyst can turn to calibrate the relationship between the efficiency gap, competitiveness, and seats-votes proportionality. When γ is one, the minimizing district is won 67-33, mitigating the competitiveness problem, but the minimizing plan exhibits triple seats-votes proportionality, exacerbating the proportionality problem. When γ is zero, the minimizing district is won 100-0, exacerbating the competitiveness problem, but the minimizing plan exhibits strict seats-votes proportionality, eliminating the proportionality problem.

The proponents eschew either of these "pure" approaches.[258] Rather than turning the dial all the way in one direction or the other, they adjust the dial to an intermediate position, setting γ equal to one half. This intermediate calibration avoids the most extreme tension with either norm, opting instead for more moderate tension with both norms.

There are two mathematically equivalent ways to set γ equal to one half: (1) defining a surplus vote as half the vote margin while equally weighting lost and surplus votes; or (2) defining a surplus vote as the entire vote margin while weighting a surplus vote half as heavily as a lost vote. In the latter approach, the definition is more intuitive, but the weight is more arbitrary and therefore harder to discern and justify. Proponents opt instead for the former approach, adopting a more intuitive weight but a less intuitive definition. In this sense, defining and weighting surplus votes involves a trade-off not only between competitiveness and seats-votes proportionality, but also between structural coherence and individual-rights coherence.

IV. A New Wasted Vote Measure

In this section I suggest the plausibility and superiority of a measure that compares shares rather than absolute numbers of wasted votes, with a surplus vote defined as the full vote margin, and lost and surplus votes weighted equally. The measure is more discernible within the prevailing individual rights framework because it is voter- rather than party-centric in both its scale and its definition of surplus vote. And the measure is more structurally resonant because it coheres more closely with electoral reality and with the democratic values of seats-votes proportionality and competitiveness.

Consider a modified efficiency gap measure with a voter-centric scale and definition of surplus vote.

$$\Delta W_{xy}^{V} = \frac{W_y}{V_y} - \frac{W_x}{V_x}$$

[258] *See supra* note 109, 110.

where

$$W_p = \sum_D W_{pi}$$

and

$$W_{pi} = \begin{cases} \gamma(V_{pi} - V_{2i}) & i \in D_p \\ V_{pi} & i \notin D_p \end{cases}$$

Recall that we can express vote totals, and thus wasted votes, in terms of victory margin and turnout.[259]

$$W_{pi} = \begin{cases} \gamma T_i m_i & i \in D_p \\ \frac{T_i}{2}(1 - m_i) & i \notin D_p \end{cases}$$

$$W_p = \sum_D W_{pi} = \sum_{D_p'} \frac{T_i}{2}(1 - m_i) + \sum_{D_p} \gamma T_i m_i$$

By applying equal voter turnout ($T_i = T^*$), $\frac{S_x}{S} = \bar{S}$, and $\frac{S_y}{S} = 1 - \bar{S}$, $V_x = ST^*\bar{V}$, and $V_y = ST^*(1 - \bar{V})$, we can express each party's wasted vote share as a function of these variables, the parameter (γ), and the average victory margin in *x*-won and *y*-won districts, m_x and m_y:[260]

$$\frac{W_y}{V_y} = \frac{2\bar{S} - 2\bar{S}m_x + 4\gamma(1-\bar{S})m_y}{4(1-\bar{V})} \qquad \text{(EQ1)}$$

$$\frac{W_x}{V_x} = \frac{4\gamma\bar{S}m_x + 2(1-\bar{S}) - 2(1-\bar{S})m_y}{4\bar{V}} \qquad \text{(EQ2)}$$

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[259] *See supra* Part III(B)(1), particularly n. 217 and accompanying text.

[260] By definition, the following equalities hold:
$\bar{S} = \bar{S}_x = \frac{S_x}{S}$, so $1 - \bar{S} = \bar{S}_y = \frac{S_y}{S}$. $\bar{V} = \bar{V}_x = \frac{V_x}{V}$, so $1 - \bar{V} = \bar{V}_y = \frac{V_y}{V}$.
$T^* = \frac{V}{S}$, so $V = ST^*$, $V_x = V\bar{V}_x = ST^*\bar{V}$, and $V_y = V\bar{V}_y = ST^*(1 - \bar{V})$.
$\sum_{D_x} 1 = S_x = S\bar{S}$ and $\sum_{D_y} 1 = S_y = S(1 - \bar{S})$.
$m_x = \frac{\sum_{D_x} m_i}{S_x}$, so $\sum_{D_x} m_i = S_x m_x = S\bar{S}m_x$. $m_y = \frac{\sum_{D_y} m_i}{S_y}$, so $\sum_{D_y} m_i = S_y m_y = S(1 - \bar{S})m_y$.
Using these equalities:
$W_y = \sum_{D_x} \frac{T^*}{2}(1 - m_i) + \sum_{D_y} \gamma T^* m_i = \frac{T^*}{2}\sum_{D_x} 1 - \frac{T^*}{2}\sum_{D_x} m_i + \gamma T^* \sum_{D_y} m_i$
$W_y = \frac{T^*}{2}S\bar{S} - \frac{T^*}{2}S\bar{S}m_x + \gamma T^* S(1 - \bar{S})m_y = \frac{ST^*}{4}(2\bar{S} - 2\bar{S}m_x + 2\gamma 2(1 - \bar{S})m_y)$
$\frac{W_y}{V_y} = \frac{\frac{ST^*}{4}(2\bar{S} - 2\bar{S}m_x + 2\gamma 2(1-\bar{S})m_y}{ST^*(1-\bar{V})} = \frac{2\bar{S} - 2\bar{S}m_x + 2\gamma 2(1-\bar{S})m_y}{4(1-\bar{V})}$ (EQ1)
$W_x = \sum_{D_x} \gamma T^* m_i + \sum_{D_y} \frac{T^*}{2}(1 - m_i) = \gamma T^* \sum_{D_x} m_i + \frac{T^*}{2}\sum_{D_y} 1 - \frac{T^*}{2}\sum_{D_y} m_i$
$W_x = \gamma T^* S\bar{S}m_x + \frac{T^*}{2}S(1 - \bar{S}) - \frac{T^*}{2}S(1 - \bar{S})m_y = \frac{ST^*}{4}(2\gamma 2\bar{S}m_x + 2(1 - \bar{S}) - 2(1 - \bar{S})m_y)$
$\frac{W_x}{V_x} = \frac{\frac{ST^*}{4}(2\gamma 2\bar{S}m_x + 2(1-\bar{S}) - 2(1-\bar{S})m_y)}{ST^*\bar{V}} = \frac{2\gamma 2\bar{S}m_x + 2(1-\bar{S}) - 2(1-\bar{S})m_y}{4\bar{V}}$ (EQ2)

Nagle proceeded as though m_x and m_y were independent variables.[261] But recall that the seat-share weighted sum of m_x and m_y is the average victory margin over all districts (m) and the seat-share weighted difference of m_x and m_y is twice the vote margin.[262]

$$m = \bar{S}_x m_x + \bar{S}_y m_y$$
$$2V^* = \bar{S}_x m_x - \bar{S}_y m_y$$

Let $\bar{S}$ and $\bar{V}$ denote seat share and vote share for party x. Then $\bar{S}_x = \bar{S}$, $\bar{S}_y = 1 - \bar{S}$, and $V^* = \bar{V} - \frac{1}{2}$. Thus, we can express these relationships in terms of $\bar{S}$ and $\bar{V}$ as follows:

$$m = \bar{S}m_x + (1 - \bar{S})m_y \quad \text{(EQ3)}$$
$$2\bar{V} - 1 = \bar{S}m_x - (1 - \bar{S})m_y \quad \text{(EQ4)}$$

Rearranging, we can express m_x and m_y in terms of m, $\bar{V}$, and $\bar{S}$.

$$2\bar{S}m_x = m + 2\bar{V} - 1 \quad \text{(EQ5)}$$
$$2(1 - \bar{S})m_y = m - 2\bar{V} + 1 \quad \text{(EQ6)}$$

Substituting (EQ5) and (EQ6) into (EQ1) and (EQ2) and simplifying, we can express each party's wasted vote share as a function of $\bar{S}$, $\bar{V}$, γ, and m.

$$\frac{W_y}{V_y} = \frac{2\bar{S} - m(1-2\gamma) - 2\bar{V}(1+2\gamma) + 1(1+2\gamma)}{4(1-\bar{V})} \quad \text{(EQ7)}$$
$$\frac{W_x}{V_x} = \frac{-2\bar{S} - m(1-2\gamma) + 2\bar{V}(1+2\gamma) + 1(1-2\gamma)}{4\bar{V}} \quad \text{(EQ8)}$$

The new measure is the difference between each party's wasted vote share. Subtracting (EQ8) from (EQ7) and simplifying:

$$\Delta W^V{}_{xy} = \frac{2\bar{S} - 2\bar{V}(1+2\gamma) - (1 - m(1-2\gamma))(1-2\bar{V}) + 2\gamma}{4\bar{V}(1-\bar{V})} \quad \text{(EQ9)}$$

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[261] Nagle called his parties *A* and *B* instead of *x* and *y* and focused on "the average A vote for those districts won by A and the average A vote for those districts won by B." Nagle, *supra* note 16, app. B at 208. These two values are related to the parties' respective average victory margins. Only after setting equal the parties' respective wasted vote shares did Nagle conclude that these two values "are not independent; [one] can be determined from [the other] through a quadratic formula." *Id.* at 201, app. B at 209. However, even when respective wasted vote shares are not equal, the two values are related to one another through equations (EQ3) and (EQ4).

[262] *See supra* note 218 and accompanying text.

Setting $\Delta {W^V}_{xy}$ to zero yields the following ideal seats-votes formula:

$$\bar{S} = (1 + 2\gamma)\bar{V} + \frac{1}{2}\big(1 - m(1 - 2\gamma)\big)(1 - 2\bar{V}) - \gamma$$

Expressed in terms of seat margin ($S^* = \bar{S} - \frac{1}{2}$) and vote margin ($V^* = \bar{V} - \frac{1}{2}$), this ideal seats-votes formula is:

$$S^* = (2\gamma - m(2\gamma - 1))V^*$$

With equal weighting of lost and surplus votes ($\alpha = 1$) and the proponents' definition of surplus votes based on half the vote margin ($\beta = \frac{1}{2}$), $\gamma = \frac{1}{2}$, so the margin of victory term (m) drops out and the ideal seats-votes relationship reduces to strict proportionality.

$$S^*\left(\gamma = \frac{1}{2}\right) = \left(2\frac{1}{2} - m\left(2\frac{1}{2} - 1\right)\right)V^* = \big(1 - m(0)\big)V^* = V^*$$

This result accords with Nagle.[263] However, if we maintain equal weighting of lost and surplus votes ($\alpha = 1$) but alternatively define surplus votes based on the entire vote margin ($\beta = 1$), so that $= 1$, then the margin of victory term m remains and the ideal seats-votes relationship reduces to the following:

$$S^*(\gamma = 1) = \big((2)(1) - m((2)(1) - 1)\big)V^* = (2 - m)V^*$$

Under maximal uncompetitiveness, when each prevailing candidate earns all the votes ($m \rightarrow 1$), this reduces to strict proportionality. Under maximal competitiveness, when each prevailing candidate wins by a single vote ($m \rightarrow 0$), this reduces to the same double proportionality that emerges from the proponents' party-centric approach based on equal wasted vote totals (not shares) and surplus votes defined based on half (not all) of the victory margin. But unlike the proponents' approach, this seats-votes curve depends on the margin of victory term m. As m varies from 0 to 1, the slope varies from 2 to 1. The seat bonus is still capped at two, but now the majority party can only achieve that maximal seat bonus if it maximizes competitiveness. If the system is less than maximally competitive, the seat bonus must be less than two.

Nagle recognized that his voter-centric approach under an alternative definition of surplus votes[264] would produce an ideal seats-votes curve that

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[263] *See id.* at 202 eq.6.

[264] Nagle used the proponents' definition of surplus votes as half the victory margin, but noted that the alternative definition was equivalent to doubling the relative weight of surplus votes. *See id.* at 199 & n.16, 203 & n.24.

depended on competitiveness.[265] But Nagle, following McGhee, views this dependence on competitiveness as an undesirable—indeed fatal—feature of the measure under the alternative definition of surplus votes ($\gamma \neq \frac{1}{2}$):

> However, as Eric McGhee has kindly pointed out, the possibility that different values of [seat share] for the same vote [vote share] may give the same value of bias violates a fundamental principle for bias measures, namely, gerrymandering might be able to increase [seat share] for the same [vote share] and not be detected by the measure of bias. The mechanism to do this is to draw the lines to change the average district votes. Making districts more competitive allows a gerrymandering party that has [majority vote share] to increase its [seat share] with no change in this measure of bias when [$\gamma > \frac{1}{2}$].[266]

Nagle and McGhee are right that when $\gamma = 1$ or more generally $\gamma > \frac{1}{2}$, the majority party can increase its seat share while maintaining constant vote share and equal wasted vote shares by increasing average district competitiveness. And this property violates McGhee's strict efficiency principle, which defines efficiency as increasing seat share at constant vote share.[267] But this property does not necessarily violate a modified efficiency principle, which defines efficiency as increasing expected seat share at constant vote share. This is because the majority party can only increase its seat share at current vote share by increasing the competitiveness of the system, which makes the outcome less robust to vote swings.

But there is another way to address the measure's dependence on competitiveness when $\gamma > \frac{1}{2}$. Rather than using the system's actual average margin of victory (m), the analyst can compare the difference in wasted vote shares that would have obtained if the system exhibited an ideal baseline level of average margin of victory ($\hat{m}$). Political scientists estimate that real-world seats-votes curves tend to exhibit competitiveness that varies with vote share, exhibiting high competitiveness when the majority enjoys a modest vote margin and low competitiveness when the majority enjoys a significant vote margin.[268] This produces an S-shaped curve that is relatively flat far away from the (0.5,0.5) point where each party earns half the votes but relatively steep near the (0.5,0.5) point. This accommodation between competitiveness and proportionality favors the majority when the minority is large but protects the minority when it is small. Because the seat bonus decreases with majority vote share, the majority must earn almost all the votes in order to win all the seats.

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[265] *See id.* at 203 ("Making districts more competitive allows a [majority] party . . . to increase its [seat share] with no change in this measure . . . .").

[266] *Id.* at 203 (citation omitted).

[267] McGhee, *Measuring Efficiency in Redistricting*, *supra* note 80, at 418.

[268] Robert X. Browning & Gary King, *Seats, Votes, and Gerrymandering: Estimating Representation and Bias in State Legislative Redistricting*, 9 L. & POL'Y 305, 313 (1987).

This suggests an alternative wasted vote measure. First, construct an average margin of victory measure $\widehat{m}(V^*)$ that aligns with these sensible features of electoral reality. Second, compute the difference in the parties' wasted vote shares (not totals), using the full (not half) definition of surplus votes ($\gamma = 1$), that would obtain under $\widehat{m}(V^*)$.[269] This measure may be more discernible, as it adopts more intuitive, voter-centric approaches to scale and definition. And it may avoid the extreme vote share problem, where a majority can completely (or effectively) shut out a minority and capture all (or most) of the seats while maintaining a zero gap. Moreover, this measure satisfies McGhee's strict efficiency principle. The drawback, of course, is that it relies on a baseline competitiveness measure the analyst selects, which introduces conjecture. However, the analyst could derive this relationship from real election data.[270] Thus the baseline would be electoral reality of what the relationship generally is, not the analyst's subjective judgment about what the relationship ought to be. A large gap would indicate that the majority has earned significantly more seats than the majority would have if the plan equalized the parties' respective wasted vote shares under a level of competitiveness consistent with the generally prevailing electoral relationship between competitiveness and vote share.

V. Doctrinal Implications and Conclusions

This Article has analyzed the methodological choices underlying the efficiency gap measure, as well as the tensions between the measure and democratic norms of proportionality, competitiveness, and voter participation. This analysis has implications for the Court as it rules in *Gill v. Whitford*. I suggest the Court should reaffirm the justiciability of partisan gerrymandering

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[269] To make this idea more concrete, I offer a simple example purely for illustrative purposes. Suppose the analyst selected the following competitiveness measure: $\widehat{m}(V^*) = 4V^{*2}$. This is the simplest function that satisfies three sensible properties: maximal competitiveness at minimal vote margin, $\widehat{m}(0) = 4 * 0 = 0$; minimal competitiveness at maximal vote margin, $\widehat{m}\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 = 4\left(\frac{1}{4}\right) = 1$; and symmetric treatment of parties, $\widehat{m}(-V^*) = 4(-V^*)^2 = 4V^{*2} = \widehat{m}(V^*)$. The analyst would then substitute this competitiveness measure into the seats-votes formula $S^*(\gamma = 1) = (2 - m)V^*$.

$$S^*(\gamma = 1) = \left(2 - \widehat{m}(V^*)\right)V^* = \left(2 - \left(4V^{*2}\right)\right)V^* = 2V^* - 4V^{*3}$$

Expressed in terms of seat share and vote share, the relationship is:

$$\bar{S} = 2\left(\bar{V} - \frac{1}{2}\right) - 4\left(\bar{V} - \frac{1}{2}\right)^3 + \frac{1}{2}$$

This equation represents a nonlinear seats-votes curve which exhibits higher responsiveness when vote margin is small (steeper near the middle) and lower responsiveness when vote margin is large (flatter near the ends).

[270] Such a statistical exercise lies beyond the scope of this Article but warrants future consideration.

claims, adopt the conclusion of the three-judge panel that the Wisconsin Assembly plan constitutes an unlawful partisan gerrymander, and acknowledge that efficiency gap analysis provides evidence of that plan's discriminatory effect. As its architects intended, that plan operated to pack and crack Democratic voters such that many more Democratic voters than Republican voters cast votes that had no effect on electoral outcomes. The efficiency gap analyses performed by plaintiffs' experts Professors Mayer and Jackman capture this effect by computing efficiency gap values that indicate a significant advantage for the Republican party that constitutes an historical outlier compared to modern elections and is likely to persist in future elections.

While the Court should view efficiency gap analysis as helpful evidence supporting the panel's conclusion, it should not adopt the efficiency gap as the exclusive definitional measure of partisan gerrymandering, such that a plan constitutes a partisan gerrymander if and only if it produces a sufficiently large, durable, intended, and unjustified gap. Instead, the Court should recognize the efficiency gap as one indicative measure of partisan gerrymandering while affording some flexibility for courts, litigants, and scholars to refine measurement approaches over time. Both methodological and normative considerations support this approach.

A. Methodological Considerations

Were the Court to proclaim the efficiency gap the definitional measure of partisan gerrymandering, it would place a heavy burden on the efficiency gap to assess the validity of all future plans in a way that yields one correct answer. Yet the answer efficiency gap analysis yields may depend on a series of methodological choices that analysts are still exploring: what imputation methods to use for uncontested races, how to account for variation in voter turnout, how to define and weight surplus votes, and whether to compare wasted vote totals or shares.[271] Each choice has the potential to change the gap calculated; all combined may have a cumulative effect. If, as proposed, a challenged plan's efficiency gap is compared to some numeric threshold, both the challenged plan's gap and the numeric threshold itself may depend on methodological approach.

At the level of a single plan, methodological choices may (1) change the sign of the gap and thereby toggle the assessment of which party is favored; (2) drive the gap above or below the numeric threshold; or (3) change the magnitude of the gap enough to influence the justification analysis (for example, a proffered justification may suffice to explain a gap of 10% but not a gap of 20%). And while used to assess an individual plan's efficiency gap, the numeric threshold

[271] Another question is whether to look to statewide or district-level races when calculating the gap. *See, e.g.*, Krasno et al., *supra* note 16, at 5 & n. 11 (justifying use of statewide rather than district-level races).

is itself based on a claim about the distribution of gaps associated with modern elections—and therefore dependent on methodological choices.

Consider how the proponents arrived at a numeric threshold of 8% for state legislative maps. The 2015 article used only the simplified formula to compute the efficiency gaps for a large number of congressional and state legislative elections between 1972 and 2012.[272] Based on the distributions, the authors proposed an 8% numeric threshold for state legislative maps.[273] They reasoned that a "gap of at least eight points placed a [state legislative] plan in the worst 12 percent of all plans in this period, also about 1.5 standard deviations from the mean."[274]

However, this 8% threshold depends on all of the methodological choices discussed to this point. We know that variation in imputation method, as described in Part II.E.1, can sometimes produce significant changes in the gap. Because the gaps were calculated using the simplified formula rather than the long-form calculation, all differential turnout effects were ignored. And, of course, all gaps were calculated with proponents' definition and weighting of surplus votes and by comparing wasted vote totals rather than shares. The precise impact of all of these methodological choices can only be quantified by recalculating all of the historical gaps with alternative methodological approaches—an intensive project that is certainly worth undertaking, but which lies beyond the scope of this Article. Thus, there is substantial uncertainty as to whether and to what degree the threshold of 8% would change if the underlying choices were modified.

It should be noted at this point that the proponents set a different threshold for congressional plans—two undeserved seats—than the 8% threshold for state legislative plans.[275] In arriving at this threshold, proponents added a new

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[272] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 867-68. *See supra* note 248.

[273] *See id.* at 887. In the *Whitford* litigation, plaintiffs' expert, Professor Jackman, used only the simplified formula to compute the efficiency gaps for a similarly large number of elections. *See supra* note 117. Based on these distributions, Jackman proposed a 7% numeric threshold. Whitford v. Gill, 218 F. Supp. 3d 837, 860 (W.D. Wis. 2016)

[274] Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 888-89.

[275] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 837. Note that the League of Women Voters in the *Rucho* litigation have not focused on the two-seat threshold but rather on a numeric efficiency gap threshold. It appears from plaintiff's complaint that, in calculating historical gaps across congressional plans, and evaluating the extent of North Carolina's deviation from historical norms, states with low district-numerosity were also excluded from that historical baseline. *See* Common Cause v. Rucho, 240 F. Supp. 3d 376, 380-81 (M.D.N.C. 2017) (per curiam) ("According to the League Amended Complaint, the Plan produced an efficiency gap of 19 percent in the 2016 election, which is 'in approximately the worst *4 percent* of the historical distribution, and the single worst score of all relevant congressional plans in the country in 2016.'" (quoting Amended Complaint ¶¶ 61-62, League

methodological choice to the mix: When calculating the historical undeserved seat share in congressional races, they omitted all election results from states with fewer than eight districts.[276] This choice was justified on the ground that "redistricting in smaller states has only a minor influence on the national balance of power."[277] Yet twenty-two states (comprising more than one-fifth of all representatives) have more than one but fewer than eight congressional districts.[278] Removing these states from the full data set eliminates those data points from consideration and may significantly change the historical analysis.[279]

Special problems do arise in trying to apply the efficiency gap measure to plans with low district numerosity. If the threshold is set at two undeserved congressional seats then voters in many small states could never make political gerrymandering claims because there could never be two undeserved seats. Conversely, if the threshold is set at 8%, maps with low district numerosity may be particularly likely to exhibit above-threshold gaps. For example, if a state, like Idaho, has only two congressional districts, there are only two possible values for seat margin: zero (each party wins one district) or fifty percent (one party wins both districts). Unless this state has the right vote share, it will necessarily have a large efficiency gap. We see, then, a problem of scope. The efficiency gap either works poorly or does not work at all in capturing gerrymandering dynamics in plans with low district numerosity, such as congressional maps for small states and electoral maps for local governing bodies like city councils and school boards.

In short, efficiency gap analysis is not robust to methodological choice and electoral circumstances. And those methodological choices are currently the subject of dynamic debate. For example, the *Whitford* panel majority viewed the simplified formula as a "shortcut,"[280] and the long-form equation as "preferable."[281] McGhee now suggests the long-form equation violates his efficiency principle and the simplified formula represents the proper definition

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of Women Voters of N.C. v. Rucho, No. 1:16-cv-1164 (M.D.N.C. Feb. 10, 2017), Doc. No. 41)).

[276] *See* Stephanopoulos & McGhee, *The Efficiency Gap*, *supra* note 6, at 868.

[277] *Id*.

[278] *See Apportionment Population and Number of Representatives, by State: 2010 Census*, U.S. CENSUS BUREAU, https://www.census.gov/population/apportionment/data/2010_apportionment_results.html (last visited Feb. XX, 2018). In total, these states were allocated ninety-one representatives—21% of the total number of representatives nationwide. *Id.*

[279] If one calculates the efficiency gap for all 2014 congressional maps, eight of the ten states whose congressional plans produce the highest gaps are small states that would have been excluded from the analysis by the proponents.

[280] Whitford v. Gill, 218 F. Supp. 3d 837, 907-08 (W.D. Wis. 2016).

[281] *Id*. at 907.

of the efficiency gap.[282] Nagle has suggested comparison of wasted vote shares rather than totals.[283] I have proposed a new wasted vote measure that compares wasted vote shares rather than totals as Nagle suggests while defining surplus votes as the full victory margin.[284] This new measure allows responsiveness to vary with vote share in a way that may better align with democratic values and electoral reality. This proposal, like all the methodological choices implicated by efficiency gap analysis, warrants further exploration. If the efficiency gap is *the* measure of partisan gerrymandering, courts will struggle when a plan's validity depends on methodological approach, and litigants will have a powerful incentive to advance the methodological approach that best supports their cause. If the efficiency gap is only an indicative measure of partisan gerrymandering, courts will have more flexibility to give it more or less weight depending on whether its assessment of the plan at issue is more or less robust to methodological choice.

B. Normative Considerations

The other reason the Court should recognize the efficiency gap as an indicative but non-definitional measure is that it exhibits tensions with democratic norms of electoral competitiveness, seats-votes proportionality, and voter participation.

The efficiency gap measure is at best agnostic and at worst antagonistic toward the goal of electoral competitiveness. Because the measure privileges the perspective of mapmakers serving party interests, it does not recognize harm to voters when elections are uncompetitive. The measure fails to recognize even extreme bipartisan gerrymanders, so long as the efficiency gap remains low. And a more competitive plan poses a greater risk of an above-threshold gap in the face of vote swings.

With a simplifying assumption of equal voter turnout across districts, the efficiency gap is equivalent to double seats-votes proportionality. It idealizes districting plans in which a party supported by 75% of the electorate wins all of the seats. To the extent that normative intuitions support a system in which vote share and seat share should be roughly equal, the efficiency gap undermines that norm—and not only in extreme scenarios.

Finally, the efficiency gap measure, like other measures justifying seats won in terms of ballots cast, registers voter suppression simply as reduced support for the targeted party. In some circumstances, suppressing the disfavored party's

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[282] *See supra* notes 150-152 and accompanying text.

[283] *See supra* notes 161-164 and accompanying text.

[284] *See supra* Part IV.

voters will actually serve to lower the gap. Thus, the efficiency gap does not condemn—and may in fact encourage—voter suppression.

The other doctrinal elements of the proposed legal test—sensitivity analysis, intent, and justification—only partially address the false positive problem. In pursuit of competitiveness or proportionality, a mapmaker may devise a districting plan that produces an above-threshold gap, but still avoid invalidation unless that gap is durable, unjustified, and intentional. But these doctrinal tools are imperfect; they may not always work, and even when they do, they avoid invalidation, not litigation. In contrast, a below-threshold efficiency gap triggers a presumption of validity, and the other doctrinal elements do nothing to address this false negative problem. Were the Court to embrace the efficiency gap as the definitional measure, any plan that wastes (roughly) equal votes would enjoy absolute immunity from judicial scrutiny, no matter how severely it subverted other democratic norms like competitiveness or seats-votes proportionality. In this way, the efficiency gap proposal would provide mapmakers a powerful incentive to draw uncompetitive plans that accord the majority a "double-proportionality" seat bonus. Such plans can sensibly be called gerrymanders if that term implicates norms of competitiveness and seats-votes proportionality. Yet those are the plans the efficiency gap would be most likely to approve, and thus the plans mapmakers would be encouraged to design.

Since electoral districting implicates—and gerrymandering threatens—multiple democratic norms, it is unsurprising that a single measure fails to capture them all. Indeed, it may be that no one measure can satisfactorily reduce to a single number the multiple democratic norms at stake. I do not fault the efficiency gap measure for failing to perform an impossible task. And I recognize that the efficiency gap does measure one significant democratic norm, which may powerfully capture, and thereby help curb, certain partisan gerrymanders. For this reason, the efficiency gap is a helpful indicative measure that courts can and should consider when analyzing claims of partisan gerrymandering—so long as that consideration recognizes its technical and normative limits.

APPENDIX

Part A.

The turnout-generalized seats-votes formula is:

$$\Delta W_{xy} = S^* - 2V^* + \Delta T_{xy}$$

where $\Delta T_{xy} = \bar{S}_x \Delta T_x$, $\Delta T_p = \frac{\sum_{D_p} \Delta T_i}{S_p}$, $\Delta T_i = \frac{T_i - T^*}{T^*}$, and $T^* = \frac{\sum_D T_i}{S}$

Proof:

$$W_y - W_x = \sum_D W_{yi} - \sum_D W_{xi} = \sum_D (W_{yi} - W_{xi}) = \sum_{D_x} (L_{yi} - E_{xi}) + \sum_{D_y} (E_{yi} - L_{xi})$$

$$W_y - W_x = \sum_{D_x} (V_{yi} - (V_{xi} - \frac{T_i}{2})) + \sum_{D_y} ((V_{yi} - \frac{T_i}{2}) - V_{xi}) = \sum_D (V_{yi} - V_{xi}) + \sum_{D_x} \frac{T_i}{2} - \sum_{D_y} \frac{T_i}{2}$$

$$W_y - W_x = V_y - V_x + \sum_{D_x} \frac{T^*}{2} - \sum_{D_y} \frac{T^*}{2} + \sum_{D_x} \frac{(T_i - T^*)}{2} - \sum_{D_y} \frac{(T_i - T^*)}{2}$$

$$W_y - W_x = \tfrac{T^*}{2}(S_x - S_y) - (V_x - V_y) + [\tfrac{T^*}{2} \textstyle\sum_{D_x} \frac{(T_i - T^*)}{T^*} - \frac{T^*}{2} \sum_{D_y} \frac{(T_i - T^*)}{T^*}]$$

$$\Delta W = \frac{W_y - W_x}{V_y + V_x} = \frac{\frac{T^*}{2}(S_x - S_y) - (V_x - V_y) + \frac{T^*}{2}[S_x \Delta T_x - S_y \Delta T_y]}{V}$$

$$\Delta W = \frac{T^*(S_x - S_y)}{2ST^*} - \frac{V_x - V_y}{V} + \frac{T^*}{2T^*}[\frac{S_x \Delta T_x - S_y \Delta T_y}{S}]$$

$$\Delta W = \frac{S_x - S_y}{2S} - \frac{V_x - V_y}{V} + \frac{\bar{S}_x \Delta T_x - \bar{S}_y \Delta T_y}{2}$$

$$\Delta W = S^* - 2V^* + \bar{S}_x \Delta T_x$$

$$\Delta W = S^* - 2V^* + \Delta T_{xy}$$