Name:______________________________
Period:__________ Date: 11/5/13

Objective for Tuesday 11/5: I can find the determinant of a matrix.
I can solve a system of two equations using Cramer's Rule.

## Notes

### The Determinant

**1 Determinants** Every square matrix has a **determinant**. The determinant of a $2 \times 2$ matrix is called a **second-order determinant**.

↳ specific term for 2×2 matrix

**KeyConcept Second-Order Determinant** (2×2 matrix)

**Words** The value of a second-order determinant is the difference of the products of the two diagonals.

× −  or

**Symbols** $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

**Example** $\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4(6) - (-3)(5) = 39$

$24 - (-15)$

$39$

In our own words:

"cross" multiply in same matrix

↓ subtract products

diagonals

### Our example

$\begin{vmatrix} 7 & 3 \\ 9 & -5 \end{vmatrix}$

$(7)(-5) \quad 9(3)$

$-35 - 27$

$\boxed{-62}$

### Example 1
Find determinant.

$\begin{vmatrix} -2 & 5 \\ 6 & 8 \end{vmatrix}$

$(-2)(8) \quad (6)(5)$

$-16 - 30$

$\boxed{-46}$

### Example 2
Find determinant.

$\begin{vmatrix} -4 & 6 \\ -3 & -2 \end{vmatrix}$

### Example 3
Find determinant.

$\begin{vmatrix} -7 & 3 \\ -9 & 7 \end{vmatrix}$

**Khan Practice**

Show work for

Determinant of a 2x2 matrix

How would you explain finding the determinant to someone who missed class today? Are there any visual cues that you can use to help remember how to find the determinant?

1 Graph
2 Substitution
3 Elimination
4 Matrix
Cramer's Rule

**Cramer's Rule**

**ReadingMath**

Determinants The determinant is used to *determine* whether a system has a unique solution.

**2 Cramer's Rule** You can use determinants to solve systems of equations. If a determinant is nonzero, then the system has a unique solution. If a determinant is 0, then the system either has no solution or infinite solutions. A method called **Cramer's Rule** uses the $\boxed{\text{coefficient matrix}}$. The **coefficient matrix** is a matrix that contains only the coefficients of the system.

number in front of variable

If determinant is 0, no solution (lines don't touch on graph) or infite solutions (same line)

**KeyConcept** Cramer's Rule

Let $C$ be the coefficient matrix of the system $\begin{aligned} ax + by &= m \\ fx + gy &= n \end{aligned}$ $\rightarrow C = \begin{bmatrix} a & b \\ f & g \end{bmatrix}$.

all coefficients

The solution of this system is $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$ and $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$, if $C \neq 0$.

coefficient matrix

In our own words:

**Steps to Cramer's Rule:**

**Test-Taking Tip**

**Cramer's Rule** When the determinant of the coefficient matrix $C$ is 0, the system does not have a unique solution.

no unique solution, no (x, y) point

**Example 4** Solve a System of Two Equations

**Solve the system by using Cramer's Rule.**

$5x - 6y = 15$

$3x + 4y = -29$

① Set up Cramer's Rule

② Define variables

$$x = \frac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|} \qquad \text{Cramer's Rule} \qquad y = \frac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$$

③ Substitute values.

$$= \frac{\begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}} \qquad = \frac{\begin{vmatrix} 5 & 15 \\ 3 & -29 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}}$$

④ Evaluate. by finding determinant (2 times)

$$= \frac{15(4) - (-29)(-6)}{5(4) - (3)(-6)} \qquad = \frac{5(-29) - 3(15)}{5(4) - (3)(-6)}$$

how to find determinant

Multiply.

$$= \frac{60 - 174}{20 + 18} \qquad = \frac{-145 - 45}{20 + 18}$$

Add and subtract.

$$= -\frac{114}{38} \qquad = -\frac{190}{38}$$

Simplify.

$$x = -3 \qquad y = -5$$

The solution of the system is $(-3, -5)$.

**CHECK** by substitution

$5(-3) - 6(-5) \stackrel{?}{=} 15$   $x = -3, y = -5$

$-15 + 30 \stackrel{?}{=} 15$   Simplify.

$15 = 15$ ✓

$3(-3) + 4(-5) \stackrel{?}{=} -29$   $x = -3, y = -5$

$-9 - 20 \stackrel{?}{=} -29$   Simplify.

$-29 = -29$ ✓

**Example 4**

Solve using Cramer's Rule.

$4x - 2y = -2$

$-x + 3y = 13$

$m = -2$

$n = 13$

$a = 4$

$b = -2$

$f = -1$

$g = 3$

$$4x - 2y = -2$$
$$-x + 3y = 13$$

$$x = \frac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{\cancel{|C|}} \quad \frac{20}{10} \qquad y = \frac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{\cancel{|C|}} \quad \frac{50}{10}$$

$$|C| = \begin{vmatrix} 4 & -2 \\ -1 & 3 \end{vmatrix}$$

$(4)(3) \quad (-1)(-2)$

$12 - 2$

$10$

$$\begin{vmatrix} -2 & -2 \\ 13 & 3 \end{vmatrix} \qquad \begin{vmatrix} 4 & -2 \\ -1 & 13 \end{vmatrix}$$

$(-2)(3) \quad (-2)(13) \qquad (4)(13) \quad (-1)(-2)$

$-6 - -26 \qquad 52 - 2$

$20 \qquad 50$

$(2, \; 5)$

**Example 5**
Solve using Cramer's Rule.
2x – 3y = 12
-6x + y = -20

**Example 6**
Solve using Cramer's Rule.
5x + 2y = 8
2x – 3y = 7

Do you find using Cramer's rule to solve for systems of equations to be easy or challenging? Why do you feel that way?

Name:________________________________
Period:___________ Date: 11/5/13

**Class Work – Cramer's Rule & Determinants**
**DUE: At the end of class**

Directions: Use Cramer's Rule to determine the solution to the system of equations.

1) $x - 5y = -5$
$-4x - 2y = 20$

2) $-x + 5y = 2$
$x - 2y = -2$

3) $2x + 2y = 0$
$4x - y = -20$

4) $3x - 4y = 1$
$-5x + 2y = 3$

**Class Work - Determinants (for more practice)**

**Evaluate the determinant of each matrix.**

1) $\begin{bmatrix} 0 & -4 \\ -6 & -2 \end{bmatrix}$

2) $\begin{bmatrix} -6 & 0 \\ 6 & -6 \end{bmatrix}$

3) $\begin{bmatrix} -1 & 1 \\ -1 & 4 \end{bmatrix}$

4) $\begin{bmatrix} 0 & 4 \\ 6 & 5 \end{bmatrix}$

**Find the answer to the system using Cramer's Rule.**

7) $-x + 4y = -2$
$-2x + 5y = -4$

8) $-5x - 5y = 25$
$-2x - 4y = 16$

Name:______________________________
Period:___________ Date: 11/5/13

Objective for Tuesday 11/5: I can find the determinant of a matrix.
I can solve a system of two equations using Cramer's Rule.

**Notes**

**The Determinant**

**1 Determinants** Every square matrix has a **determinant**. The determinant of a $2 \times 2$ matrix is called a **second-order determinant**.

→ 2×2 matrix determinant

**KeyConcept Second-Order Determinant** (2×2 matrix)

Words: The value of a second-order determinant is the difference (−) of the products (×) of the two diagonals.

Symbols: $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ← straight brackets means determinant

Example: $\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4(6) - (-3)(5) = 39$

In our own words:

$24 - (-15)$

$24 + 15 = \boxed{39}$

**Our example**

$\det \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \to \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}$ $2(8) - 6(4)$

$16 - 24$

$\boxed{-8}$

**Example 1**
Find determinant.

$\begin{vmatrix} -2 & 5 \\ 6 & 8 \end{vmatrix}$

$(-2)(8) - (5)(6)$

$-16 - 30$

$\boxed{-46}$

**Example 2**
Find determinant.

$\begin{vmatrix} -4 & 6 \\ -3 & -2 \end{vmatrix}$

$8 - (-18)$

$\boxed{26}$

**Example 3**
Find determinant.

$\begin{vmatrix} -7 & 3 \\ -9 & 7 \end{vmatrix}$

$(-7)(7) - (3)(-9)$

$-49 - (-27)$

$-49 + 27 = \boxed{-22}$

**Khan Practice**
Show work for
Determinant of a 2x2 matrix

How would you explain finding the determinant to someone who missed class today? Are there any visual cues that you can use to help remember how to find the determinant?

1. Graph (intersection)
2. Substitute
3. Eliminate
4. Cramer's Rule

**Cramer's Rule**

**Reading Math**
**Determinants** The determinant is used to *determine* whether a system has a unique solution.

**2 Cramer's Rule** You can use determinants to solve systems of equations. If a determinant is nonzero, then the system has a unique solution. If a determinant is 0, then the system either has no solution or infinite solutions. A method called **Cramer's Rule** uses the coefficient matrix. The **coefficient matrix** is a matrix that contains only the coefficients of the system.

# in front of variable

If 0, then no solution or infinite solutions

If det ≠ 0, then there is a solution

**Key Concept** Cramer's Rule

Let $C$ be the coefficient matrix of the system $\begin{aligned} ax + by &= m \\ fx + gy &= n \end{aligned} \rightarrow \begin{bmatrix} a & b \\ f & g \end{bmatrix} = C$

The solution of this system is $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$ and $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$, if $C \neq 0$.

determinant of coefficient matrix

In our own words:

**Steps to Cramer's Rule:**

**Test-Taking Tip**

Cramer's Rule When the determinant of the coefficient matrix $C$ is 0, the system does not have a unique solution.

**Example 4** Solve a System of Two Equations

Solve the system by using Cramer's Rule. $|C| =$ coefficient matrix

$5x - 6y = 15$

$3x + 4y = -29$

① Write formulas

Cramer's Rule

$$x = \frac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|} \qquad y = \frac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$$

② Define $a=$ $b=$ $m=$ $f=$ $g=$ $n=$

Substitute values. ③ into formula

$$x = \frac{\begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 5 & 15 \\ 3 & -29 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}}$$

Evaluate. ④ by finding determinant

$$= \frac{15(4) - (-29)(-6)}{5(4) - (3)(-6)} \qquad = \frac{5(-29) - 3(15)}{5(4) - (3)(-6)}$$

Multiply.

$$= \frac{60 - 174}{20 + 18} \qquad = \frac{-145 - 45}{20 + 18}$$

Add and subtract. finding det.

$$= -\frac{114}{38} \qquad = -\frac{190}{38}$$

⑤ Simplify.

$x = -3 \qquad y = -5$

The solution of the system is $(-3, -5)$.

**CHECK** by substitution

$5(-3) - 6(-5) \stackrel{?}{=} 15$ $\quad x = -3, y = -5$

$-15 + 30 \stackrel{?}{=} 15$ $\quad$ Simplify.

$15 = 15$ ✓

$3(-3) + 4(-5) \stackrel{?}{=} -29$ $\quad x = -3, y = -5$

$-9 - 20 \stackrel{?}{=} -29$ $\quad$ Simplify.

$-29 = -29$ ✓

**Example 4**

Solve using Cramer's Rule.

$4x - 2y = -2$

$-x + 3y = 13$

$a = 4$

$b = -2$

$f = -1$

$g = 3$

$m = -2$

$n = 13$

$$|C| = \begin{vmatrix} 4 & -2 \\ -1 & 3 \end{vmatrix} = \boxed{10}$$

$$\boxed{x} = \frac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|} = \frac{20}{10} \qquad \boxed{y} = \frac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|} = \frac{50}{10}$$

$$\begin{vmatrix} -2 & -2 \\ 13 & 3 \end{vmatrix} = \boxed{20} \qquad \begin{vmatrix} 4 & -2 \\ -1 & 13 \end{vmatrix} = \boxed{50}$$

$x = 2 \qquad y = 5$

(2, 5)

**Example 5**
Solve using Cramer's Rule.
2x – 3y = 12
-6x + y = -20

**Example 6**
Solve using Cramer's Rule.
5x + 2y = 8
2x – 3y = 7

Do you find using Cramer's rule to solve for systems of equations to be easy or challenging? Why do you feel that way?

Unit 2: Linear and Absolute Value Functions – 2.17
Algebra II – Mrs. Cutrona

Name:________________________________
Period:___________ Date: 11/5/13

**Class Work – Cramer's Rule & Determinants**
**DUE: At the end of class**

Directions: Use Cramer's Rule to determine the solution to the system of equations.

1) $x - 5y = -5$
$-4x - 2y = 20$

2) $-x + 5y = 2$
$x - 2y = -2$

3) $2x + 2y = 0$
$4x - y = -20$

4) $3x - 4y = 1$
$-5x + 2y = 3$

**Class Work - Determinants (for more practice)**

**Evaluate the determinant of each matrix.**

1) $\begin{bmatrix} 0 & -4 \\ -6 & -2 \end{bmatrix}$

2) $\begin{bmatrix} -6 & 0 \\ 6 & -6 \end{bmatrix}$

3) $\begin{bmatrix} -1 & 1 \\ -1 & 4 \end{bmatrix}$

4) $\begin{bmatrix} 0 & 4 \\ 6 & 5 \end{bmatrix}$

**Find the answer to the system using Cramer's Rule.**

7) $-x + 4y = -2$
$-2x + 5y = -4$

8) $-5x - 5y = 25$
$-2x - 4y = 16$

Name:________________________________
Period:____________ Date: 11/5/13

Objective for Tuesday 11/5: I can find the determinant of a matrix.
I can solve a system of two equations using Cramer's Rule.

## Notes

### The Determinant

**1 Determinants** Every square matrix has a **determinant**. The determinant of a $2 \times 2$ matrix is called a **second-order determinant**. → 2×2 matrix

**KeyConcept** Second-Order Determinant

2×2 matrix

| | |
|---|---|
| Words | The value of a second-order determinant is the difference ⊖ of the products ⊗ of the two diagonals. (like cross multiply) |
| Symbols | $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ ← showing determinant of a matrix |
| Example | $\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4(6) - (-3)(5) = 39$ |

~~In our own words:~~

$24 - (-15)$

$= \boxed{39}$

### Our example

### Example 1

Find determinant.

$\begin{vmatrix} -2 & 5 \\ 6 & 8 \end{vmatrix}$

$(-2)(8) \quad (6)(5)$

$-16 - 30$

$\boxed{-46}$

### Example 2

Find determinant.

$\begin{vmatrix} -4 & 6 \\ -3 & -2 \end{vmatrix}$

$(-4)(-2) - (-3)(6)$

$8 - (-18)$

$\boxed{26}$

### Example 3

Find determinant.

$\begin{vmatrix} -7 & 3 \\ -9 & 7 \end{vmatrix}$

$(-7)(7) - (-9)(3)$

$-49 - (-27)$

$-49 + 27 = \boxed{-22}$

| | |
|---|---|
| **Khan Practice**<br>Show work for<br>Determinant of a 2x2 matrix | |

How would you explain finding the determinant to someone who missed class today? Are there any visual cues that you can use to help remember how to find the determinant?

1 Graph
2 Substitute
3 Eliminate
4 Cramer's Rule

**Cramer's Rule**

**ReadingMath**

**Determinants** The determinant is used to *determine* whether a system has a unique solution.

**2 Cramer's Rule** You can use determinants to solve systems of equations. If a determinant is nonzero, then the system has a unique solution. If a determinant is 0, then the system either has no solution or infinite solutions. A method called **Cramer's Rule** uses the coefficient matrix. The **coefficient matrix** is a matrix that contains only the coefficients of the system.

→ # in front of variable

→ matrix w/ coefficients

**KeyConcept Cramer's Rule**

Let $C$ be the coefficient matrix of the system $ax + by = m$, $fx + gy = n$ → $\begin{bmatrix} a & b \\ f & g \end{bmatrix} = |C|$

The solution of this system is $\boxed{x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|} \text{ and } y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}}$, if $C \neq 0$.

formula

Det

0 → no solution

≠0, → one unique solution

In our own words:

**Steps to Cramer's Rule:**

**Test-Taking Tip**

**Cramer's Rule** When the determinant of the coefficient matrix $C$ is 0, the system does not have a unique solution.

**Example 4** Solve a System of Two Equations

Solve the system by using Cramer's Rule.

$5x - 6y = 15$
$3x + 4y = -29$

$x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$   Cramer's Rule   $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$

① write formula

$= \dfrac{\begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}}$   Substitute values.   $= \dfrac{\begin{vmatrix} 5 & 15 \\ 3 & -29 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}}$

② Find values: a= b= f= g= m= n=
③ into formula

$= \dfrac{15(4) - (-29)(-6)}{5(4) - (3)(-6)}$   Evaluate.   $= \dfrac{5(-29) - 3(15)}{5(4) - (3)(-6)}$

④ by finding determinant

$= \dfrac{60 - 174}{20 + 18}$   Multiply.   $= \dfrac{-145 - 45}{20 + 18}$

$= -\dfrac{114}{38}$   Add and subtract.   $= -\dfrac{190}{38}$

$x = -3$   Simplify.   $y = -5$

⑤ by dividing

The solution of the system is $(-3, -5)$.

⑥ by sub.

**CHECK** $5(-3) - 6(-5) \stackrel{?}{=} 15$   $x = -3, y = -5$
$-15 + 30 \stackrel{?}{=} 15$   Simplify.
$15 = 15$ ✓

$3(-3) + 4(-5) \stackrel{?}{=} -29$   $x = -3, y = -5$
$-9 - 20 \stackrel{?}{=} -29$   Simplify.
$-29 = -29$ ✓

$(-3, -5)$

**Example 4**
Solve using Cramer's Rule.
$4x - 2y = -2$
$-x + 3y = 13$

$a = 4$
$b = -2$
$m = -2$
$f = -1$
$g = 3$
$n = 13$

$ax + by = m$
$fx + gy = n$

$x = \begin{vmatrix} m & b \\ n & g \end{vmatrix}$   $y = \begin{vmatrix} a & m \\ f & n \end{vmatrix}$

$|c| = \begin{vmatrix} 4 & -2 \\ -1 & 3 \end{vmatrix}$

10

$\begin{vmatrix} -2 & -2 \\ 13 & 3 \end{vmatrix}$   $\begin{vmatrix} 4 & -2 \\ -1 & 13 \end{vmatrix}$

$(-2)(3) - (13)(-2)$
$-6 - (-26)$
$-6 + 26$
$x = 20$

$4(13) - (-1)(-2)$
$52 - 2$
$50$
$y = 50$

$x = \frac{20}{10}$ $y = \frac{50}{10}$

$x = 2$ $y = 5$

$(2, 5)$

**Example 5**
Solve using Cramer's Rule.
2x – 3y = 12
-6x + y = -20

**Example 6**
Solve using Cramer's Rule.
5x + 2y = 8
2x – 3y = 7

Do you find using Cramer's rule to solve for systems of equations to be easy or challenging? Why do you feel that way?

Unit 2: Linear and Absolute Value Functions – 2.17 Name:________________________________
Algebra II – Mrs. Cutrona Period:___________ Date: 11/5/13

**Class Work – Cramer's Rule & Determinants**
**DUE: At the end of class**

Directions: Use Cramer's Rule to determine the solution to the system of equations.

1) $x - 5y = -5$
$-4x - 2y = 20$

2) $-x + 5y = 2$
$x - 2y = -2$

3) $2x + 2y = 0$
$4x - y = -20$

4) $3x - 4y = 1$
$-5x + 2y = 3$

**Class Work - Determinants (for more practice)**

**Evaluate the determinant of each matrix.**

1) $\begin{bmatrix} 0 & -4 \\ -6 & -2 \end{bmatrix}$

2) $\begin{bmatrix} -6 & 0 \\ 6 & -6 \end{bmatrix}$

3) $\begin{bmatrix} -1 & 1 \\ -1 & 4 \end{bmatrix}$

4) $\begin{bmatrix} 0 & 4 \\ 6 & 5 \end{bmatrix}$

**Find the answer to the system using Cramer's Rule.**

7) $-x + 4y = -2$
$-2x + 5y = -4$

8) $-5x - 5y = 25$
$-2x - 4y = 16$

Unit 2: Linear and Absolute Value Functions – 2.17
Algebra II – Mrs. Cutrona

Name:________________________________
Period:____________ Date: 11/5/13

Objective for Tuesday 11/5: I can find the determinant of a matrix.
I can solve a system of two equations using Cramer's Rule.

**Notes**

**The Determinant**

**1 Determinants** Every square matrix has a determinant. The determinant of a $2 \times 2$ matrix is called a second-order determinant.

→ 2×2 matrix

**KeyConcept Second-Order Determinant**

| | |
|---|---|
| Words | The value of a second-order determinant is the difference of the products of the two diagonals. |
| Symbols | $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ |
| Example | $\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4(6) - (-3)(5) = 39$ |

−  ×

← straight up/down lines w/ matrix is "determinant"

$24 - (-15)$

In our own words:

$24 + 15$

$\boxed{39}$

**Our example**

$\det \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} \rightarrow \begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix}$

$(4)(7) - (5)(6)$

$28 - 30$

$\boxed{-2}$

**Example 1**
Find determinant.

$\begin{vmatrix} -2 & 5 \\ 6 & 8 \end{vmatrix}$

$-46$

**Example 2**
Find determinant.

$\begin{vmatrix} -4 & 6 \\ -3 & -2 \end{vmatrix}$

$26$

**Example 3**
Find determinant.

$\begin{vmatrix} -7 & 3 \\ -9 & 7 \end{vmatrix}$

**Khan Practice**
Show work for
Determinant of a 2x2 matrix

How would you explain finding the determinant to someone who missed class today? Are there any visual cues that you can use to help remember how to find the determinant?

① graphing (intersection)
② eliminate
③ substitute
④ Cramer's Rule

**Cramer's Rule**

**ReadingMath**
**Determinants** The determinant is used to *determine* whether a system has a unique solution.

**2 Cramer's Rule** You can use determinants to solve systems of equations. If a determinant is nonzero, then the system has a unique solution. If a determinant is 0, then the system either has no solution or infinite solutions. A method called **Cramer's Rule** uses the coefficient matrix. The **coefficient matrix** is a matrix that contains only the coefficients of the system.

**KeyConcept** Cramer's Rule

Let $C$ be the coefficient matrix of the system $ax + by = m$, $fx + gy = n$ → $\begin{bmatrix} a & b \\ f & g \end{bmatrix}$

matrix of coefficients

The solution of this system is $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$ and $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$, if $C \neq 0$.

coefficient matrix

det.

0
no solution
or infinite solutions

non zero
$\neq 0$
unique solution

In our own words:

**Steps to Cramer's Rule:**

$ax + by = m$

$fx + gy = n$

**Test-Taking Tip**

**Cramer's Rule** When the determinant of the coefficient matrix $C$ is 0, the system does not have a unique solution.

**Example 4 Solve a System of Two Equations**

Solve the system by using Cramer's Rule.

$5x - 6y = 15$

$3x + 4y = -29$

① Write FORMULA

$$x = \frac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|} \qquad \text{Cramer's Rule} \qquad y = \frac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$$

← det. of coefficient matrix

$$= \frac{\begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}} \qquad \text{Substitute values.} \qquad = \frac{\begin{vmatrix} 5 & 15 \\ 3 & -29 \end{vmatrix}}{\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix}}$$

② Write values ③ into formula

a=
b=
f=
g=
m=
n=

$$= \frac{15(4) - (-29)(-6)}{5(4) - (3)(-6)} \qquad \text{Evaluate.} \qquad = \frac{5(-29) - 3(15)}{5(4) - (3)(-6)}$$

④ find det.

$$= \frac{60 - 174}{20 + 18} \qquad \text{Multiply.} \qquad = \frac{-145 - 45}{20 + 18}$$

det.

$$= -\frac{114}{38} \qquad \text{Add and subtract.} \qquad = -\frac{190}{38}$$

$$= -3 \qquad \text{⑤ Simplify.} \qquad = -5$$

→ divided

The solution of the system is $(-3, -5)$.

⑥ **CHECK** by substitution

$5(-3) - 6(-5) \stackrel{?}{=} 15$ $\quad x = -3, y = -5$

$-15 + 30 \stackrel{?}{=} 15$ $\quad$ Simplify.

$15 = 15$ ✓

$3(-3) + 4(-5) \stackrel{?}{=} -29$ $\quad x = -3, y = -5$

$-9 - 20 \stackrel{?}{=} -29$ $\quad$ Simplify.

$-29 = -29$ ✓

**Example 4**
Solve using Cramer's Rule.
$4x - 2y = -2$
$-x + 3y = 13$

$a = 4$
$b = -2$
$m = -2$
$f = -1$
$g = 3$
$n = 13$

$ax + by = m$

$fx + gy = n$

$$|C| = \begin{vmatrix} 4 & -2 \\ -1 & 3 \end{vmatrix} \qquad (4)(3) - (-1)(-2)$$

$12 - 2$

$10$

$$x = \begin{vmatrix} m & b \\ n & g \end{vmatrix} \qquad y = \begin{vmatrix} a & m \\ f & n \end{vmatrix}$$

$$\begin{vmatrix} -2 & -2 \\ 13 & 3 \end{vmatrix} \qquad \begin{vmatrix} 4 & -2 \\ -1 & 13 \end{vmatrix}$$

$(-2)(3) - (-2)(13)$ $\qquad$ $(4)(13) - (-2)(-1)$

$-6 - (-26)$ $\qquad$ $52 - 2$

$-6 + 26$ $\qquad$ $50$

$20$

10 (2, 5) 10

**Example 5**
Solve using Cramer's Rule.
$2x - 3y = 12$
$-6x + y = -20$

**Example 6**
Solve using Cramer's Rule.
$5x + 2y = 8$
$2x - 3y = 7$

Do you find using Cramer's rule to solve for systems of equations to be easy or challenging? Why do you feel that way?

Unit 2: Linear and Absolute Value Functions – 2.17
Algebra II – Mrs. Cutrona

Name:______________________________
Period:___________ Date: 11/5/13

**Class Work – Cramer's Rule & Determinants**
**<u>DUE: At the end of class</u>**

<u>Directions</u>: Use Cramer's Rule to determine the solution to the system of equations.

1) $x - 5y = -5$
$-4x - 2y = 20$

2) $-x + 5y = 2$
$x - 2y = -2$

3) $2x + 2y = 0$
$4x - y = -20$

4) $3x - 4y = 1$
$-5x + 2y = 3$

**Class Work - Determinants (for more practice)**

**Evaluate the determinant of each matrix.**

1) $\begin{bmatrix} 0 & -4 \\ -6 & -2 \end{bmatrix}$

2) $\begin{bmatrix} -6 & 0 \\ 6 & -6 \end{bmatrix}$

3) $\begin{bmatrix} -1 & 1 \\ -1 & 4 \end{bmatrix}$

4) $\begin{bmatrix} 0 & 4 \\ 6 & 5 \end{bmatrix}$

**Find the answer to the system using Cramer's Rule.**

7) $-x + 4y = -2$
$-2x + 5y = -4$

8) $-5x - 5y = 25$
$-2x - 4y = 16$