

Objective for Monday 11.4: I can use the idea of dot-product to multiply two matrices together.

Matrix Multiplication

Dot product

When you multiply matching rows/columns and then add them up.

Dimensions for multiplication

The inner dimensions of both matrices have to be same. If they are not equal, then the product is undefined.

To multiply two matrices together

In order to be able to multiply matrices,

- 1.) Determine if you can multiply → are inner dimensions the same?
- 2.) Create the answer matrix using the inner dimensions
- 3.) Multiply the first row in the first matrix by the first column in the second matrix
- 4.) Add the products together
- 5.) Record result in the first row, first column of answer matrix
- 6.) Repeat steps 3-4 with the first row in first matrix and second column in second matrix
- 7.) Record result in the first row, second column of answer matrix
- 8.) Repeat with same pattern

Examples

Example 1

$$A = \begin{bmatrix} 4 & -6 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 7 \\ 5 & -9 \end{bmatrix}$$

- a. What are the dimensions of A? 2×2 of B? 2×2
 b. Can you multiply them together?

Yes b/c inner dimensions are same

- c. Find AB

$$A = \begin{bmatrix} 4 & -6 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 7 \\ 5 & -9 \end{bmatrix}$$

$$\begin{bmatrix} -42 & 82 \\ 9 & -13 \end{bmatrix}$$

$$\begin{array}{l} [4(-3) + (-6)(5)] \quad [4(7) + (-6)(-9)] \\ -12 + -30 \qquad \qquad \qquad 28 + 54 \\ \hline -42 \end{array}$$

$$\begin{array}{l} [2(-3) + 3(5)] \quad [2(7) + 3(-9)] \\ -6 + 15 \qquad \qquad \qquad 14 + (-27) \\ \hline 9 \end{array}$$

Example 2

$$K = \begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix} \text{ and } L = \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$$

- a. What are the dimensions of K? 2×3 of L? 3×2
 b. Can you multiply them together?

Yes --- inner dimensions are same

- c. Find KL

$$K = \begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$$

$$\begin{array}{cc} -40 & 64 \\ 22 & 1 \end{array}$$

$$\begin{aligned} 2(4) + 9(-6) + -3(2) \\ 8 + (-54) + 6 \\ -40 \\ 4(2) + 9(7) + (-3)(1) \\ 8 + 63 + (-3) \\ 64 \\ 9(4) + 1(-6) + 0(-2) \\ 36 + 6 \\ 0 \\ 4(2) + (-1)(1) + 0 \\ 8 - 1 + 0 \\ 7 \end{aligned}$$

Example 3

At a particular swim meet, 7 points were awarded for each first-place finish, 4 points for each second, and 2 points for each third. Which school won the meet? Hint: Make 2 matrices.

School	First place	Second place	Third place
Central	4	7	3
Franklin	8	9	1
Hayes	10	5	3
Lincoln	3	3	6

$$\begin{array}{c|ccc} & 1^{\text{st}} & 2^{\text{nd}} & 3^{\text{rd}} \\ \hline C & 4 & 7 & 3 \\ F & 8 & 9 & 1 \\ H & 10 & 5 & 3 \\ L & 3 & 3 & 6 \end{array}$$

$$\begin{aligned} 7 \text{ pt} &= 1^{\text{st}} \\ 4 \text{ pt} &= 2^{\text{nd}} \\ 2 \text{ pt} &= 3^{\text{rd}} \end{aligned}$$

$$\begin{bmatrix} 7 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 62 \\ 94 \\ \hline \end{bmatrix}$$

$$\begin{array}{c} C \\ F \\ H \end{array} \quad \begin{array}{c} 4 \times 3 \\ 3 \times 1 \end{array} \quad \begin{array}{c} 28 + 28 + 6 \\ 56 + 36 + 2 \\ 94 \end{array}$$

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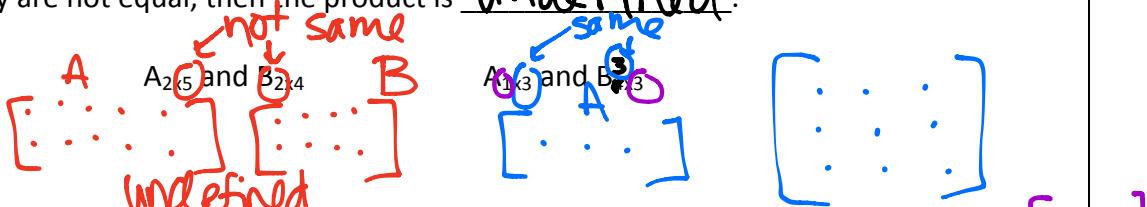
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**To multiply two matrices together**

In order to be able to multiply matrices,

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- 8.) Repeat with same pattern

$$\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

1x3 answer [- - -]

Examples

Example 1

$$A = \begin{bmatrix} 4 & -6 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 7 \\ 5 & -9 \end{bmatrix}$$

- a. What are the dimensions of A? 2 x 2 of B? 2 x 2

b. Can you multiply them together?

Yes b/c the inner dimensions are the same.

c. Find AB

$$A = \begin{bmatrix} 4 & -6 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 7 \\ 5 & -9 \end{bmatrix}$$

$$\begin{aligned} 4(-3) + -6(5) \\ -12 + -30 \\ -42 \end{aligned}$$

$$\begin{aligned} 4(7) + (-6)(-9) \\ 28 + 54 \\ 82 \end{aligned}$$

$$\begin{bmatrix} -42 & 82 \\ 9 & -13 \end{bmatrix}$$

$$\begin{aligned} 2(-3) + (3)(5) \\ -6 + 15 \\ 9 \end{aligned}$$

$$\begin{aligned} 2(7) + 3(-9) \\ 14 + (-27) \\ -13 \end{aligned}$$

answer matrix

Example 2

$K = \begin{bmatrix} 2 & 9 & -3 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ and $L = \begin{bmatrix} 4 & 2 & 7 \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{bmatrix}$

answer

- a. What are the dimensions of K? 3×3 of L? 3×2
- b. Can you multiply them together?

Yes b/c inner dimensions same

- c. Find KL

$$\begin{bmatrix} -40 \\ 22 \\ 64 \end{bmatrix} \quad \begin{bmatrix} 1 \end{bmatrix}$$

$$\begin{array}{c}
 \begin{array}{l}
 2(4) + 9(-6) + (-3)(-2) \\
 8 - 54 + 6 \\
 -40
 \end{array}
 \quad
 \begin{array}{l}
 2(2) + 7(9) + (-3)(1) \\
 4 + 63 - 3 \\
 64
 \end{array}
 \\
 \hline
 \begin{array}{l}
 4(4) + (-1)(-6) + (-2)(0) \\
 16 + 6 + 0 \\
 22
 \end{array}
 \quad
 \begin{array}{l}
 4(2) + (-1)(7) + 1(0) \\
 8 - 7 + 0 \\
 1
 \end{array}
 \end{array}$$

Example 3

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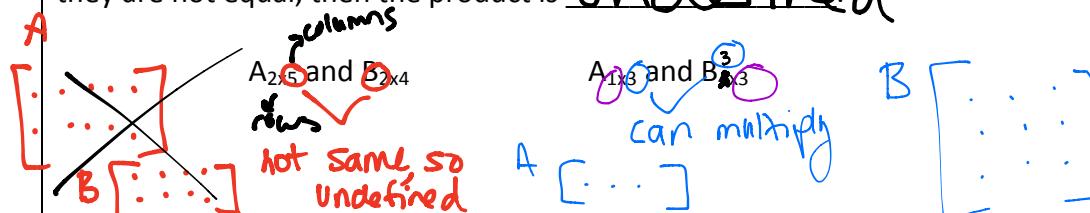
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To multiply two matrices together

In order to be able to multiply matrices,

- 1.) Determine if you can multiply → are inner dimensions the same?
- 2.) Create the answer matrix using the ~~inner~~ outer dimensions
- 3.) Multiply the first row in the first matrix by the first column in the second matrix
- 4.) Add the products together
- 5.) Record result in the first row, first column of answer matrix
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Examples

Example 1

$$A = \begin{bmatrix} 4 & -6 \\ 12 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 7 \\ 5 & -9 \end{bmatrix}$$

- a. What are the dimensions of A? 2×2 b. Can you multiply them together? 2×2

- c. Find AB

Yes, we can multiply b/c inner numbers are same

$$\begin{bmatrix} -42 & 82 \\ 9 & -13 \end{bmatrix}$$

$$\begin{aligned} 4(-3) + (-6)(5) \\ -12 + (-30) \\ -42 \end{aligned}$$

$$\begin{aligned} 2(-3) + 3(5) \\ -6 + 15 \end{aligned}$$

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Example 2

$$K = \begin{bmatrix} 2 & 9 & -3 \\ 1 & 4 & -1 \end{bmatrix}$$

$$L = \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$$

- a. What are the dimensions of K?
b. Can you multiply them together?

2×3

of L?

3×2

Same

- c. Find KL

$$2(4) + 9(-6) + (-3)(2) \quad 2(2) + 9(7) + (-3)(1)$$

$$4(4) + (-1)(-6) + 0(-2) \quad 4(2) + (-1)(7) + (0)(1)$$

Example 3

At a particular swim meet, 7 points were awarded for each first-place finish, 4 points for each second, and 2 points for each third. Which school won the meet? **Hint: Make 2 matrices.**

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$A_{2 \times 5}$ and $B_{2 \times 4}$

$A_{1 \times 3}$ and $B_{3 \times 3}$

inner dimension not same
undefined

same! can multiply

To multiply two matrices together

dot product

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Example 1

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- a. What are the dimensions of A?
b. Can you multiply them together?

SAME So can multiply

$$2 \times 2 \text{ of } B: 2 \times 2$$

- c. Find AB

$$\begin{bmatrix} -12 & 82 \\ 9 & -13 \end{bmatrix}$$

$$\begin{aligned} 4(-3) + -6(5) \\ -12 + (-30) \\ -42 \end{aligned}$$

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Example 2

$$K = \begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix} \quad \text{and} \quad L = \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$$

ANSWER

- a. What are the dimensions of K?
b. Can you multiply them together?

2 \times 3

of L?

3 \times 2

- c. Find KL

$$\begin{array}{c} 2(4) + 9(-6) + (-3)(-2) \\ (4)(4) + (-1)(-6) + (0)(-2) \\ (4)(2) + (-1)(7) + (0)(1) \end{array}$$

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$$\begin{aligned} 1^{\text{st}} &= 7 \text{ pt} \\ 2^{\text{nd}} &= 4 \text{ pt} \\ 3^{\text{rd}} &= 2 \text{ pt} \end{aligned}$$

$$\begin{matrix} & 1^{\text{st}} & 2^{\text{nd}} & 3^{\text{rd}} \\ C & 4 & 7 & 3 \\ F & 8 & 9 & 1 \\ H & 10 & 5 & 3 \\ L & 3 & 3 & 6 \end{matrix}$$

$$\begin{matrix} & 1^{\text{st}} & 2^{\text{nd}} & 3^{\text{rd}} \\ & 7 & 4 & 2 \end{matrix} = \begin{matrix} & 1^{\text{st}} & 2^{\text{nd}} & 3^{\text{rd}} \\ & 7 & 4 & 2 \end{matrix}$$

$(4 \times 7) + (7 \times 4) + (3 \times 2)$
 $28 + 28 + 6 = 62$

4 \times 3 3 \times 1