

### In-Class Assignment 10

1. Define an exponential random variable with parameter  $\lambda = 1/3$ . Generate 2 sets of simulations of the random variable (call them `exp_1` and `exp_2`) of 10,000 simulations each. Create a third variable `su` which is the sum of `exp_1` and `exp_2`. Verify that
    - i. `val = 1/3`
    - ii. `exp_1 <- rexp(10000, val)`
    - iii. `exp_2 <- rexp(10000, val)`
    - iv. `su <- exp_1 + exp_2`
  - b.  $E(su) = E(exp\_1) + E(exp\_2)$ 
    - i. `print(mean(exp_1) + mean(exp_2)) = 6.075177`
    - ii. `print(mean(su)) = 6.075177`
  - c.  $Var(su) = Var(exp\_1) + Var(exp\_2)$ 
    - i. `print(var(exp_1) + var(exp_2)) = 18.3664`
    - ii. `print(var(su)) = 18.65352`
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2. Create 1,000,000 simulations of a multivariate normal random variable similar to the one you worked with in ICA 9 (set  $var(X) = var(Y) = 1.5$ , and  $cor(X,Y) = -0.8$ ), and call it `mvn`. `x` will be the first column of `mvn` and `y` will be the second column (for example, `x = mvn[,1]`). Create a new variable `z` which is the sum of the two outputs from the simulation. Verify from the experimental data that
  - a. `min <- -4`
  - b. `max <- 5`
  - c. `var_x <- 1.5`
  - d. `var_y <- 1.5`
  - e. `cor_xy <- -0.8`
  - f. `mvn <-  
 cbind(c(var_x, cor_xy*sqrt(var_x)*sqrt(var_y)), c(cor_xy*sqrt(var_x)*sqrt(var_y), var_y))`
  - g. `x <- mvn[,1]`
  - h. `y <- mvn[,2]`
  - i. `z <- sum(x,y)`

- j.  $\text{Cov}(x, y) = -1.2$ 
    - i. `cov(x,y)`
    - ii. `[1] -1.2`
  - k.  $E(z) = E(x) + E(y)$ 
    - i. `print(mean(z)) = 0.3`
    - ii. `print(mean(x) + mean(y)) = 0.3`
  - l.  $\text{Var}(z) = \text{Var}(x) + \text{Var}(y) + 2\text{Cov}(x, y)$ 
    - i. `print(var(x) + var(y) + 2*cov(x,y)) = 1.87653`
    - ii. `print(var(z)) = 1.81346`
3. For this problem we will look at sample means of an exponential distribution with mean value 3 (parameter is  $1/3$ ). Create 20,000 simulation sets of 10,000 simulations each for this distribution. Calculate the means of the 20,000 sets and plot a histogram. Does the histogram look approximately like a normal distribution?
- a. `param <- 1/3`
  - b. `um<-rexp(10000, param)`
  - c. `round(mean(um))`
  - d. `largev<- replicate(20000,um)`
  - e. `mean(largev)`
  - f. `hist(largev)`

3

`[1] 3.038718`

