**STT 380**

**In-Class Activity 20**

The Markov Chain we will look at handles the transition of 4 states, gives as follows:

$s_1 \rightarrow 0.2\, s_1 + 0.3\, s_2 + 0.1\, s_3 + 0.4\, s_4$

$s_2 \rightarrow \quad 0.1\, s_2 + 0.7\, s_3 + 0.2\, s_4$

$s_3 \rightarrow \quad 0.2\, s_2 + 0.1\, s_3 + 0.7\, s_4$

$s_4 \rightarrow \quad 0.8\, s_2 + 0.1\, s_3 + 0.1\, s_4$

1. What is the transition matrix for this Markov Chain?
   a. P <- cbind(c(.2,0.3,0.1,0.4),c(0,0.1,0.7,0.2), c(0,0.2,0.1,0.7), c(0,0.8,0.1,0.1))
   b. P

```
     [,1] [,2] [,3] [,4]
[1,]  0.2  0.0  0.0  0.0
[2,]  0.3  0.1  0.2  0.8
[3,]  0.1  0.7  0.1  0.1
[4,]  0.4  0.2  0.7  0.1
```

2. Take an initial state (1, 0, 0, 0).
   a. What is the next state after 1 transition?
      i. x0 <- as.matrix(c(1,0,0,0))
      ii. x1 <- P %*% x0
      iii. x1 =     [,1]
      iv. [1,]  0.2
      v. [2,]  0.3
      vi. [3,]  0.1
      vii. [4,]  0.4
      viii.
      ix. What is the state after 10 transitions?
         1. x0 <- as.matrix(c(1,0,0,0))
         2. x <- x0
         3. for(i in 1:10){

4. x <- P %*% x
5. print(x)
6. }
7. [1,] 0.0000001024
8. [2,] 0.3602174411
9. [3,] 0.3140708890
10. [4,] 0.3257115675

b. Is there a stable final state?
   i. eigen(P)
   ii. eigen() decomposition
   iii. $values
   iv. [1]  1.00+0.0000000i -0.35+0.4873397i -0.35-0.4873397i  0.20+0.0000000i
   v. eigenvector corresponding to eigenvalue 1 is final state, one of the values (first one) is one so yes

c.

d. We will simulate an arbitrary initial state with the Dirichlet distribution. Define an initial state with the command t(as.matrix(rdirichlet(1,rep(1,4)))) Find the end state after burn-in (50 for burn-in)
   i. library(DirichletReg)
   ii. state <- t(as.matrix(rdirichlet(1,rep(1,4))))
   iii. x <- state
   iv. #initial burn-in
   v. for(i in 1:50){
   vi. x <- P %*% x
   vii. }
   viii. #collect end-states
   ix. y <- matrix(nrow=4, ncol = 50)
   x. for(i in 1:50){
   xi. x <- P%*%x
   xii. y[,i]<- x }
   xiii. mean(y[1,])

xiv.mean(y[2,])

xv. mean(y[3,])

xvi.1.1577e-38

xvii.[1] 0.3592233

xviii.[1] 0.315534