STANDARD ERROR OF $\bar{y}_1 - \bar{y}_2$

$$\mathrm{SE}_{(\bar{Y}_1-\bar{Y}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\mathrm{SE}_1^2 + \mathrm{SE}_2^2}$$

CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$

95% confidence interval:

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.025}\mathrm{SE}_{(\bar{Y}_1-\bar{Y}_2)}$$

Critical value $t_{0.025}$ from Student's t distribution with

$$\mathrm{df} = \frac{(\mathrm{SE}_1^2 + \mathrm{SE}_2^2)^2}{\mathrm{SE}_1^4/(n_1-1) + \mathrm{SE}_2^4/(n_2-1)}$$

where $\mathrm{SE}_1 = s_1/\sqrt{n_1}$ and $\mathrm{SE}_2 = s_2/\sqrt{n_2}$.

Confidence intervals with other confidence levels (90%, 99%, etc.) are constructed analogously (using $t_{0.05}$, $t_{0.005}$, etc.).

The confidence interval formula is valid if (1) the data can be regarded as coming from two independently chosen random samples, (2) the observations are independent within each sample, and (3) each of the populations is normally distributed. If n_1 and n_2 are large, condition (3) is less important.

Supplementary Exercises 6.S.1–6.S.23

6.S.1 To study the conversion of nitrite to nitrate in the blood, researchers injected four rabbits with a solution of radioactively labeled nitrite molecules. Ten minutes after injection, they measured for each rabbit the percentage of the nitrite that had been converted to nitrate. The results were as follows[49]:

51.1 55.4 48.0 49.5

(a) For these data, calculate the mean, the standard deviation, and the standard error of the mean.

(b) Assuming the percentage of nitrite converted to nitrate follows a normal distribution, construct a 95% confidence interval for the population mean percentage.

(c) Without doing any calculations, would a 99% confidence interval be wider, narrower, or the same width as the confidence interval you found in part (b)? Why?

6.S.2 The diameter of the stem of a wheat plant is an important trait because of its relationship to breakage of the stem, which interferes with harvesting the crop. An agronomist measured stem diameter in eight plants of the Tetrastichon cultivar of soft red winter wheat. All observations were made 3 weeks after flowering of the plant. The stem diameters (mm) were as follows[50]:

2.3 2.6 2.4 2.2 2.3 2.5 1.9 2.0

The mean of these data is 2.275 and the standard deviation is 0.238.

(a) Calculate the standard error of the mean.

(b) Construct a 95% confidence interval for the population mean.

(c) Define in words the population mean that you estimated in part (b). (See Example 6.1.1.)

6.S.3 Refer to Exercise 6.S.2.

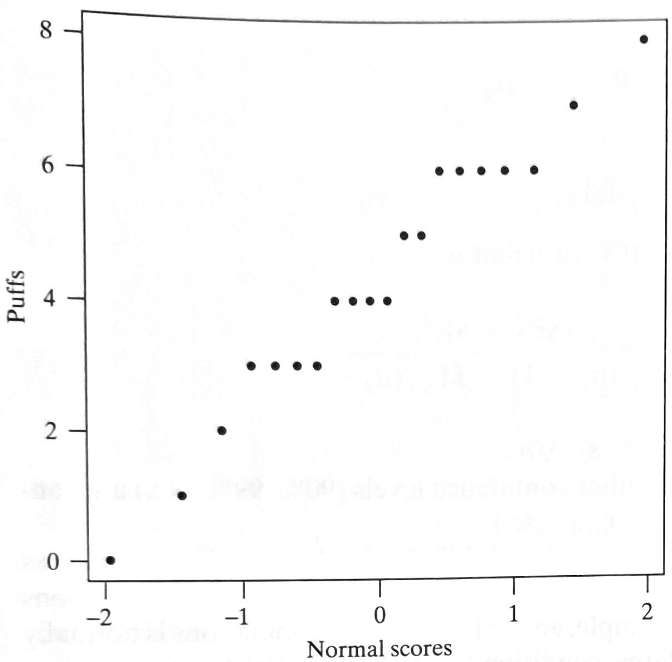
(a) What conditions are needed for the confidence interval to be valid?

(b) Are these conditions met? How do you know?

(c) Which of these conditions is most important?

6.S.4 Refer to Exercise 6.S.2. Suppose that the data on the eight plants are regarded as a pilot study, and the agronomist now wishes to design a new study for which he wants the standard error of the mean to be only 0.03 mm. How many plants should be measured in the new study?

6.S.5 A sample of 20 fruitfly (*Drosophila melanogaster*) larva were incubated at 37 °C for 30 minutes. It is theorized that such exposure to heat causes polytene chromosomes located in the salivary glands of the fly to unwind, creating puffs on the chromosome arm that are visible under a microscope. The following normal quantile plot supports the use of a normal curve to model the distribution of puffs.[51]



The average number of puffs for the 20 observations was 4.30, with a standard deviation of 2.03.

(a) Construct a 95% confidence interval for μ.

(b) In the context of this problem, describe what μ represents. That is, the confidence interval from part (a) is a confidence interval for what quantity?

(c) The normal probability plot shows the dots lining up on horizontal bands. Is this sort of behavior surprising for this type of data? Explain.

6.S.6 Over a period of about 9 months, 1,353 women reported the timing of each of their menstrual cycles. For the first cycle reported by each woman, the mean cycle time was 28.86 days, and the standard deviation of the 1,353 times was 4.24 days.[52]

(a) Construct a 99% confidence interval for the population mean cycle time.

(b) Because environmental rhythms can influence biological rhythms, one might hypothesize that the population mean menstrual cycle time is 29.5 days, the length of the lunar month. Is the confidence interval of part (a) consistent with this hypothesis?

6.S.7 Refer to the menstrual cycle data of Exercise 6.S.6.

(a) Over the entire time period of the study, the women reported a total of 12,247 cycles. When all of these cycles are included, the mean cycle time is 28.22 days. Explain why one would expect that this mean would be smaller than the value 28.86 given in Exercise 6.S.6. (*Hint*: If each woman reported for a fixed time period, which women contributed more cycles to the total of 12,247 observations?)

(b) Instead of using only the first reported cycle as in Exercise 6.S.6, one could use the first four cycles for each woman, thus obtaining $1{,}353 \times 4 = 5{,}412$ observations. One could then calculate the mean and standard deviation of the 5,412 observations and divide the SD by $\sqrt{5412}$ to obtain the SE; this would yield a much smaller value than the SE found in Exercise 6.S.6. Why would this approach not be valid?

6.S.8 For the 28 lamb birthweights of Example 6.2.2, the mean is 5.1679 kg, the SD is 0.6544 kg, and the SE is 0.1237 kg.

(a) Construct a 95% confidence interval for the population mean.

(b) Construct a 99% confidence interval for the population mean.

(c) Interpret the confidence interval you found in part (a). That is, explain what the numbers in the interval mean. (*Hint*: See Examples 6.3.4 and 6.3.5.)

(d) Often researchers will summarize their data in reports and articles by writing $\bar{y} \pm$ SD (5.17 ± 0.65) or $\bar{y} \pm$ SE (5.17 ± 0.12). If the researcher of this study is planning to compare the mean birthweight of these Rambouillet lambs to another breed, Booroolas, which style of presentation should she use?

6.S.9 Refer to Exercise 6.S.8.

(a) What conditions are required for the validity of the confidence intervals?

(b) Which of the conditions of part (a) can be checked (roughly) from the histogram of Figure 6.2.1?

(c) Twin births were excluded from the lamb birthweight data. If twin births had been included, would the confidence intervals be valid? Why or why not?

6.S.10 Researchers measured the number of tree species in each of 69 vegetational plots in the Lama Forest of Benin, West Africa.[53] The number of species ranged from a low of 1 to a high of 12. The sample mean was 6.8 and the sample SD was 2.4, which results in a 95% confidence interval of (6.2, 7.4). However, the number of tree species in a plot takes on only integer values. Does this mean that the confidence interval should be (7, 7)? Or does it mean that we should round off the endpoints of the confidence interval and report it as (6, 7)? Or should the confidence interval really be (6.2, 7.4)? Explain.

6.S.11 As part of a study of natural variation in blood chemistry, serum potassium concentrations were measured in 84 healthy women. The mean concentration was 4.36 mEq/l, and the standard deviation was 0.42 mEq/l. The table presents a frequency distribution of the data.[54]

Serum potassium (mEq/l)	Number of women
[3.1, 3.4)	1
[3.4, 3.7)	2
[3.7, 4.0)	7
[4.0, 4.3)	22
[4.3, 4.6)	28
[4.6, 4.9)	16
[4.9, 5.2)	4
[5.2, 5.5)	3
[5.5, 5.8)	1
Total	84

(a) Calculate the standard error of the mean.

(b) Construct a histogram of the data and indicate the intervals $\bar{y} \pm$ SD and $\bar{y} \pm$ SE on the histogram. (See Figure 6.2.1.)

(c) Construct a 95% confidence interval for the population mean.

(d) Interpret the confidence interval you found in part (c). That is, explain what the numbers in the interval mean. (*Hint*: See Examples 6.3.4 and 6.3.5.)

6.S.12 Refer to Exercise 6.S.11. In medical diagnosis, physicians often use "reference limits" for judging blood chemistry values; these are the limits within which we would expect to find 95% of healthy people. Would a 95% confidence interval for the mean be a reasonable choice of "reference limits" for serum potassium in women? Why or why not?

6.S.13 Refer to Exercise 6.S.11. Suppose a similar study is to be conducted next year, to include serum potassium measurements on 200 healthy women. Based on the data in Exercise 6.S.11, what would you predict would be

(a) the SD of the new measurements?

(b) the SE of the new measurements?

6.S.14 An agronomist selected six wheat plants at random from a plot, and then, for each plant, selected 12 seeds from the main portion of the wheat head; by weighing, drying, and reweighing, she determined the percentage moisture in each batch of seeds. The results were as follows[55]:

62.7	63.6	60.9	63.0	62.7	63.7

(a) Calculate the mean, the standard deviation, and the standard error of the mean.

(b) Assuming the percent moisture in each batch follows a normal distribution, construct a 90% confidence interval for the population mean.

6.S.15 As part of the National Health and Nutrition Examination Survey (NHANES), hemoglobin levels were checked for a sample of 1139 men age 70 and over.[56] The sample mean was 145.3 g/l and the standard deviation was 12.87 g/l.

(a) Use these data to construct a 95% confidence interval for μ.

(b) Does the confidence interval from part (a) give limits in which we expect 95% of the sample data to lie? Why or why not?

(c) Does the confidence interval from part (a) give limits in which we expect 95% of the population to lie? Why or why not?

6.S.16 The following data are 16 weeks of weekly fecal coliform counts (MPN/100 ml) at Dairy Creek in San Luis Obispo County, California.[57]

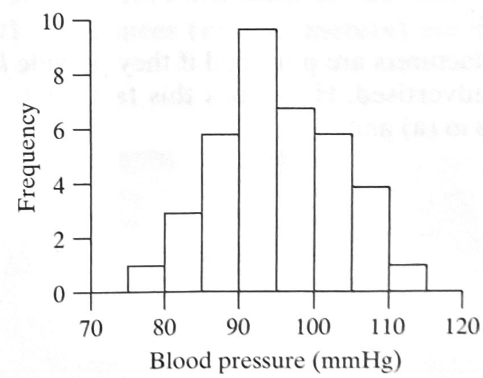
203	215	240	236	217	296	301	190
197	203	210	215	270	290	310	287

(a) Counts above 225 MPN/100ml are considered unsafe. What type of one-sided interval (upper- or lower-bound) would be appropriate to assess the safety of this creek? Explain your reasoning.

(b) Using 95% confidence, construct the interval chosen in part (a).

(c) Based on your interval in part (b), what conclusions can you make regarding the safety of the water?

6.S.17 The blood pressure (average of systolic and diastolic measurements) of each of 38 persons were measured.[58] The average was 94.5 (mm Hg). A histogram of the data is shown.



Which of the following is an approximate 95% confidence interval for the population mean blood pressure? Explain.

(i) 94.5 ± 16

(ii) 94.5 ± 8

(iii) 94.5 ± 2.6

(iv) 94.5 ± 1.3

6.S.18 Suppose you wished to estimate the mean blood pressure of students at your school to within 2 mmHg with 95% confidence.

(a) Using the data displayed in Exercise 6.S.17 as pilot data for your study, determine the (approximate) sample size necessary to achieve your goals. (*Hint*: You will need to use the graph to make some visual estimates.)

(b) Suppose your school is a small private college that only has 500 students. Would the interval based on your sample size be valid? Explain. Do you think it would be too wide or too narrow?

6.S.19 It is known that alcohol consumption during pregnancy can harm the fetus. To study this phenomenon, 10 pregnant mice will receive a low dose of alcohol. When each mouse gives birth, the birthweight of each pup will be measured. Suppose the mice give birth to a total of 85 pups, so the experimenter has 85 observations of Y = birthweight. To calculate the standard error of the mean of these 85 observations, the experimenter could calculate the standard deviation of the 85 observations and divide by $\sqrt{85}$. On what grounds might an objection be raised to this method of calculating the SE?

6.S.20 Is the nutrition information on commercially produced food accurate? In one study, researchers sampled 13 packages of a certain frozen reduced-calorie chicken entrée with a reported calorie content of 252 calories per package. The mean calorie count of the sampled entrées was 306 with a sample standard deviation of 51 calories.[59]

(a) Compute a 95% confidence interval for the population mean calorie content of the frozen entrée.

(b) Based on this interval computed in part (a), what do you think about the reported calorie content for this entrée?

(c) Manufacturers are punished if they provide *less* food than advertised. How does this fact relate to your results in (a) and (b)?

6.S.21 Nitric oxide is sometimes given to newborns who experience respiratory failure. In one experiment, nitric oxide was given to 114 infants with respiratory failure. This group was compared to a control group of 121 infants with respiratory failure. The length of hospitalization (in days) was recorded for each of the 235 infants. The mean in the nitric oxide sample was $\bar{Y}_1 = 36.4$; the mean in the control sample was $\bar{Y}_2 = 29.5$. A 95% confidence interval for $\mu_1 - \mu_2$ is $(-2.3, 16.1)$, where μ_1 is the population mean length of hospitalization for infants who get nitric oxide and μ_2 is the mean length of hospitalization for infants in the control population.[60]

(a) True or false (and say why): We are 95% confident that μ_1 is greater than μ_2, since most of the confidence interval is greater than zero.

(b) True or false (and say why): We are 95% confident that the difference between μ_1 and μ_2 is between −2.3 days and 16.1 days.

(c) True or false (and say why): 95% of the nitric oxide infants were hospitalized longer than the mean of the control infants.

6.S.22 Consider the data from Exercise 6.S.20(a) which led to a 95% confidence interval for $\mu_1 - \mu_2$ of $(-2.3, 16.1)$.

(a) True or false (and say why): We are 95% confident that the difference between $\bar{Y}_1$ and $\bar{Y}_2$ is between −2.3 days and 16.1 days.

(b) True or false (and say why): If this experiment were repeated many times (with the same sample sizes), then 95% of the time $\bar{y}_1 - \bar{y}_2$ would be between −2.3 and 16.1.

(c) True or false (and say why): We are 95% confident that nitric oxide has no effect on the mean length of hospitalization because zero is in the confidence interval.

6.S.23 Suppose you are discussing confidence intervals with someone who has a limited statistics background. Briefly explain, without doing any calculations, why a 100% confidence interval for a mean is infinitely wide.