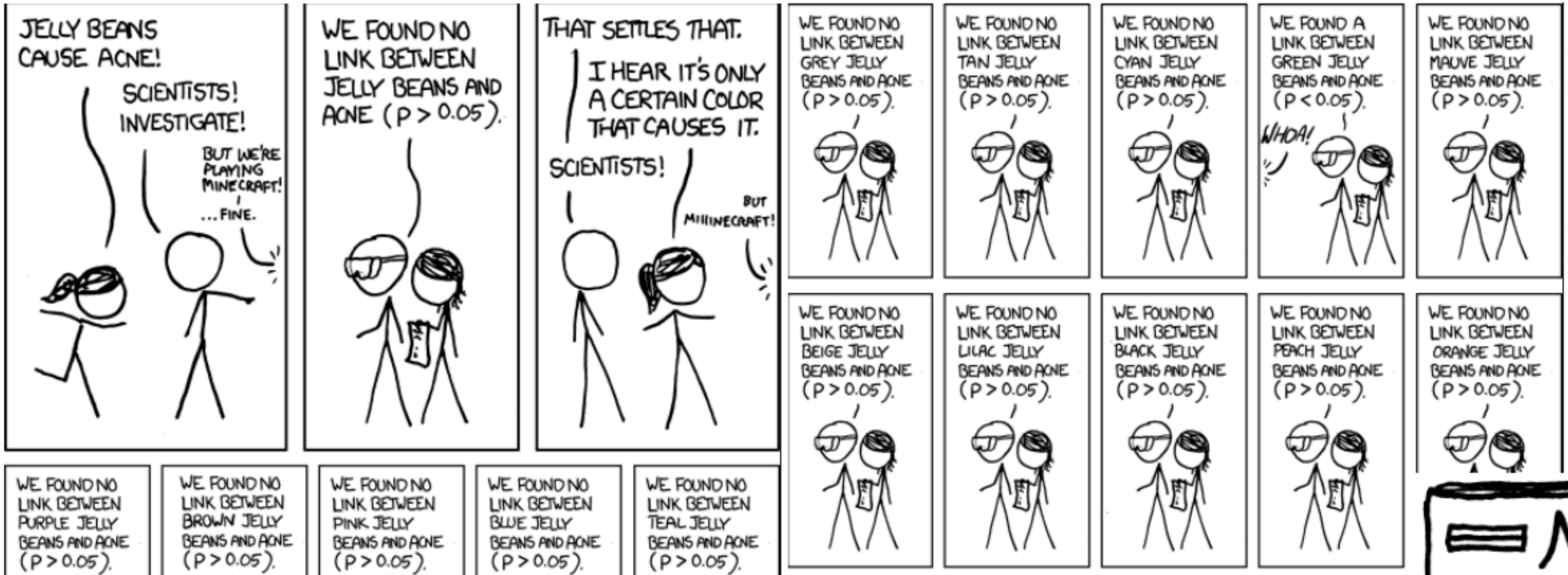
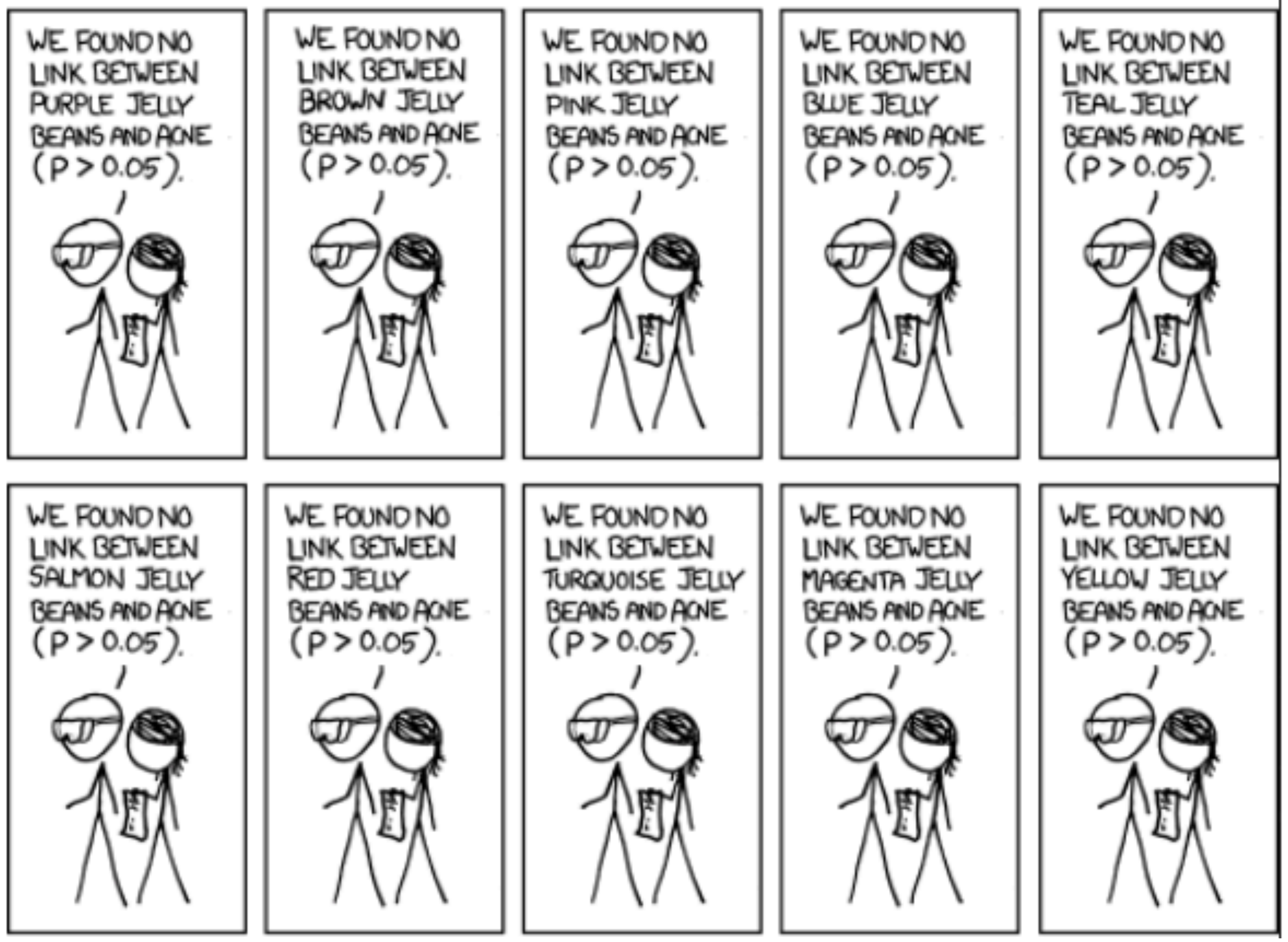
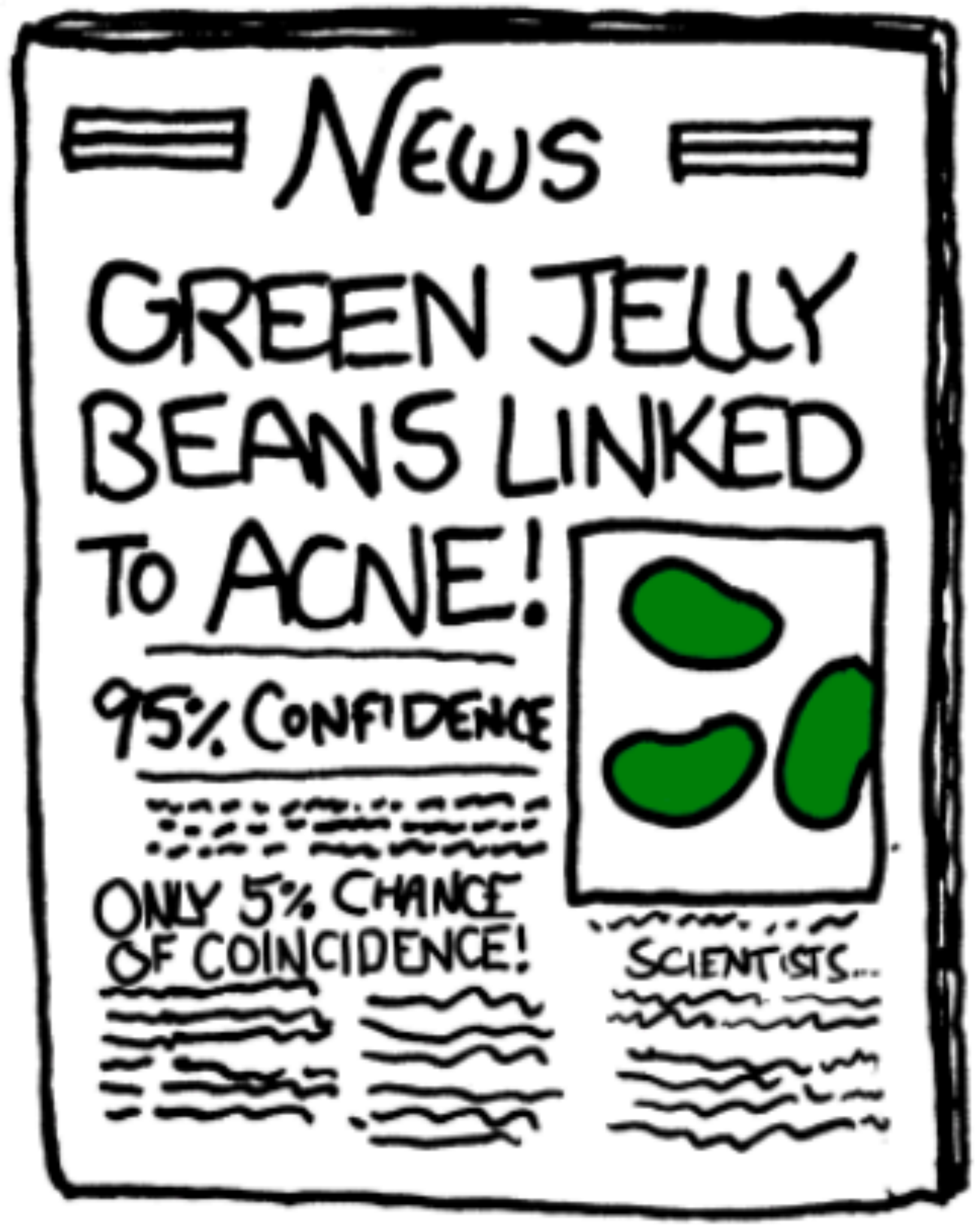




Lecture 08

10.19.21



Refresher Quiz

The blood pressure (average of systolic and diastolic measurements) of each of 25 persons were measured. The average was 94.5 mm Hg with a variance of 225. Use a *t*-test to decide if the underlying population could have a mean of 100 mm Hg.

The blood pressure (average of systolic and diastolic measurements) of each of 25 persons were measured. The average was 94.5 mm Hg with a variance of 225. Use a *t*-test to decide if the underlying population could have a mean of 100 mm Hg.

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
20	1.325	1.725	2.086	2.528	2.845	3.552	3.85
21	1.323	1.721	2.08	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46

The blood pressure (average of systolic and diastolic measurements) of each of 25 persons were measured. The average was 94.5 mm Hg with a variance of 225. Use a *t*-test to decide if the underlying population could have a mean of 100 mm Hg.

1. Generate a hypothesis and choose a significance level

$$H_0 : \mu = 100 \qquad H_A : \mu \neq 100 \qquad \alpha = 0.05$$

2. Calculate test statistic

$$t_s = \frac{\bar{y} - \mu}{s/\sqrt{n}} = \frac{94.5 - 100}{\sqrt{225}/\sqrt{25}} = -1.83$$

3. Calculate the *P*-value

$$H_A : \mu \neq 100$$

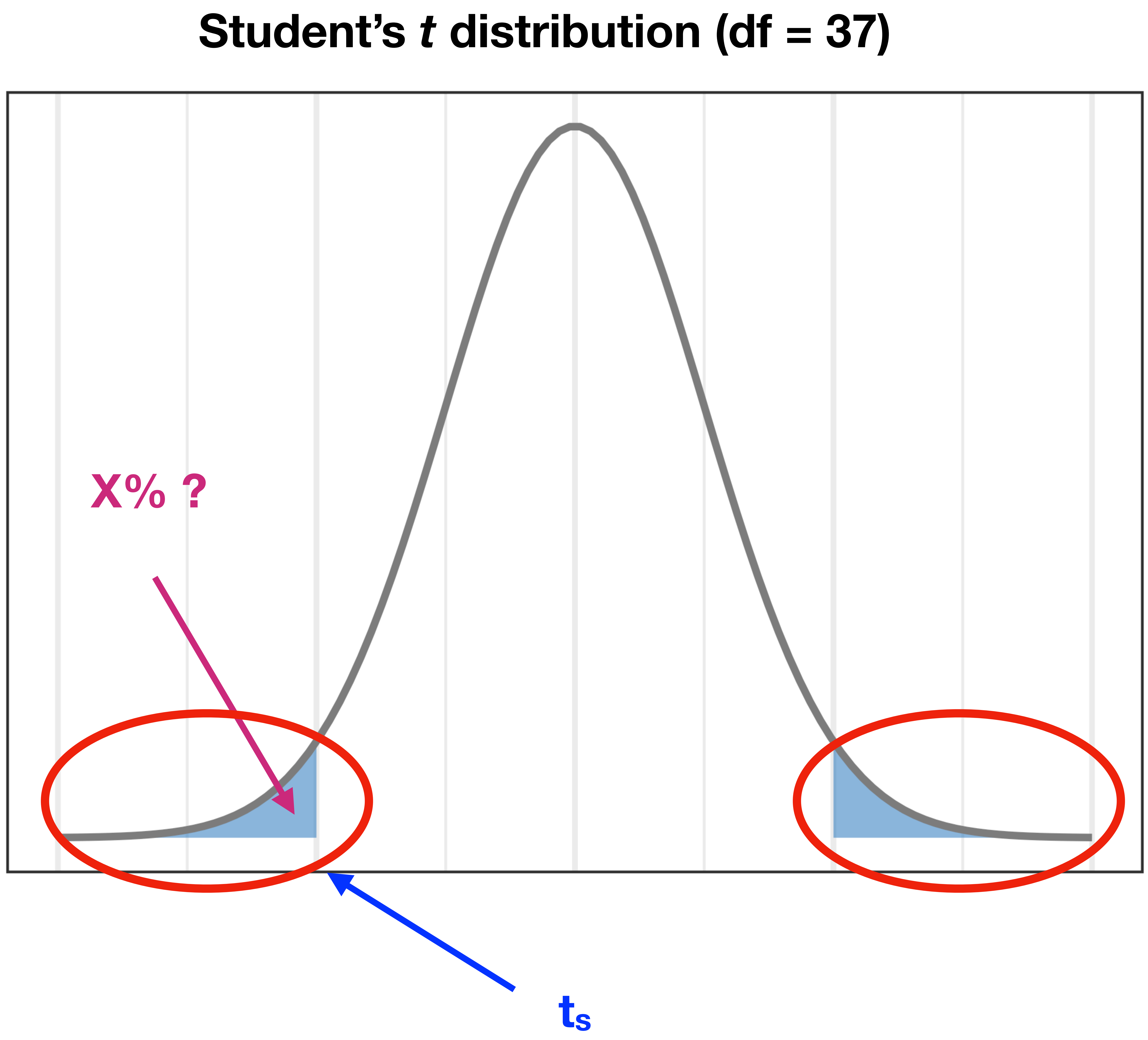
$$\alpha = 0.05$$

$$t_s = -1.83$$

```
> pt(-1.83, 24)*2
[1] 0.07969884
```

P > α

Fail to reject the null!



$$H_A : \mu \neq 100$$

$$\alpha = 0.05$$

$$t_s = -1.83$$

```
> pt(-1.83, 24)*2
[1] 0.07969884
```

P > α

Fail to reject the null!

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
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23	1.319	1.714	2.069	2.5	2.807	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.06	2.485	2.787	3.45	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.66
30	1.31	1.697	2.042	2.457	2.75	3.385	3.646
60	1.296	1.671	2	2.39	2.66	3.232	3.46

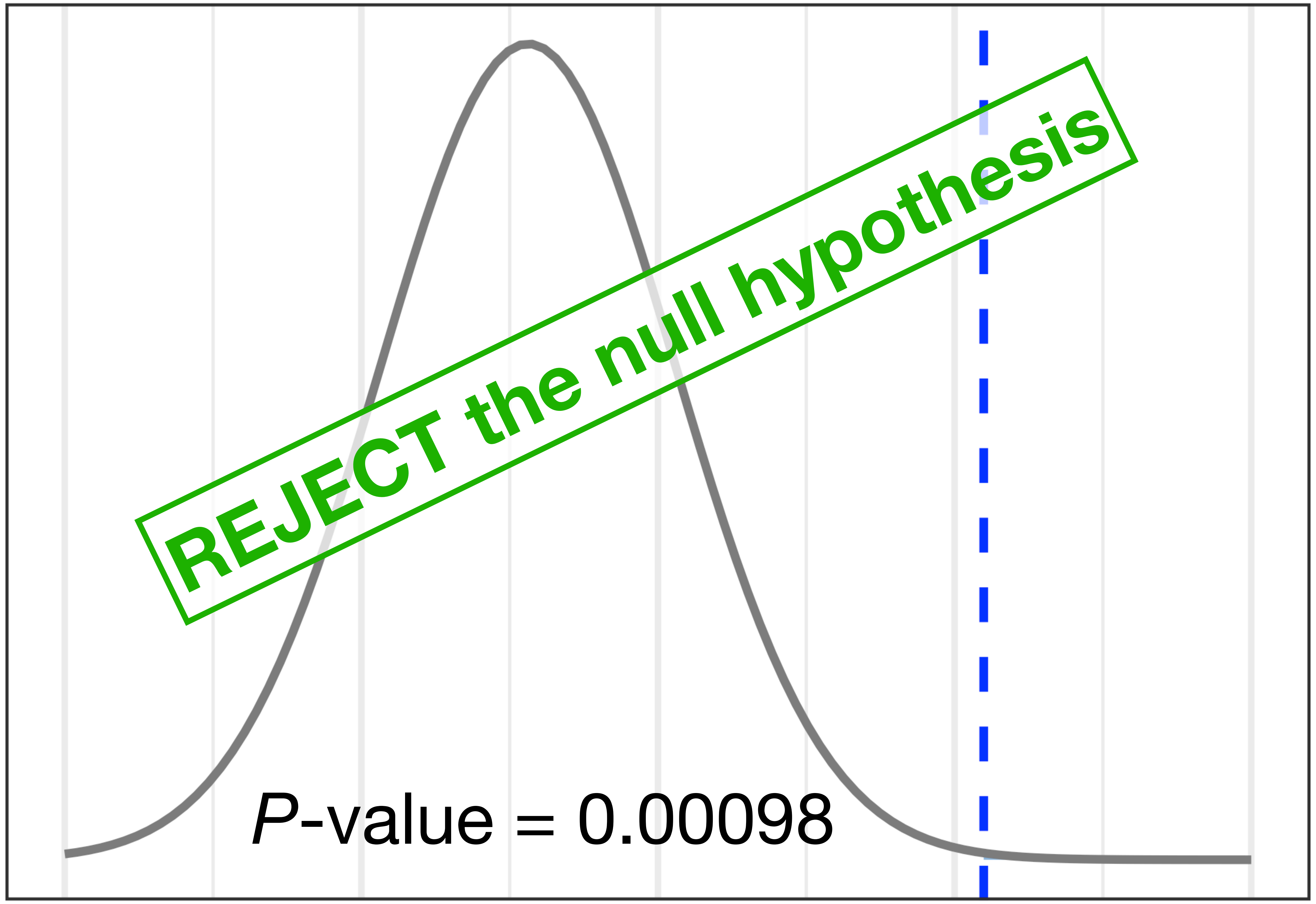
The *t* **statistic** for hypothesis testing

$\bar{y} = 32.81cm^2$

$s = 2.48cm^2$

$\mu = 30cm^2$

$\boxed{\alpha = 0.05}$



$t_s = 4.23$

$$H_0 : \bar{y} = \mu$$

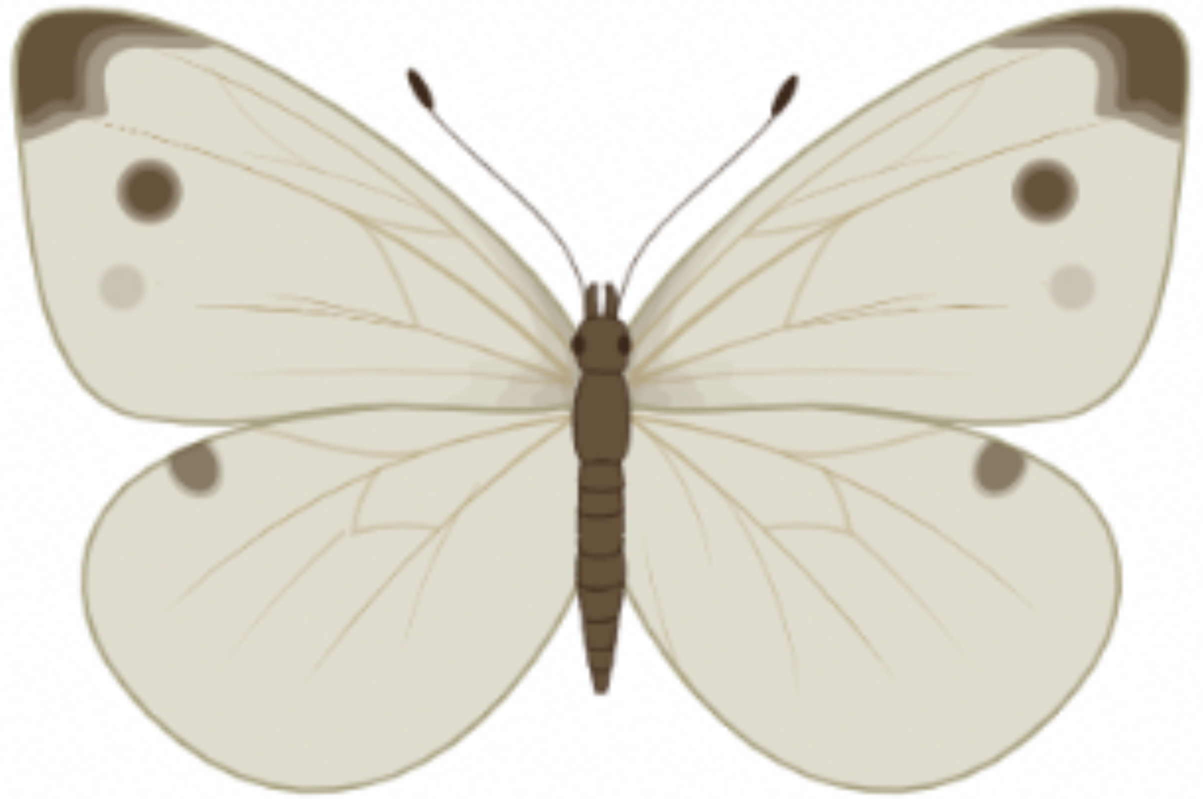
$$H_A : \bar{y} \neq \mu$$

```
> pt(4.23, 13, lower.tail = F)*2
```

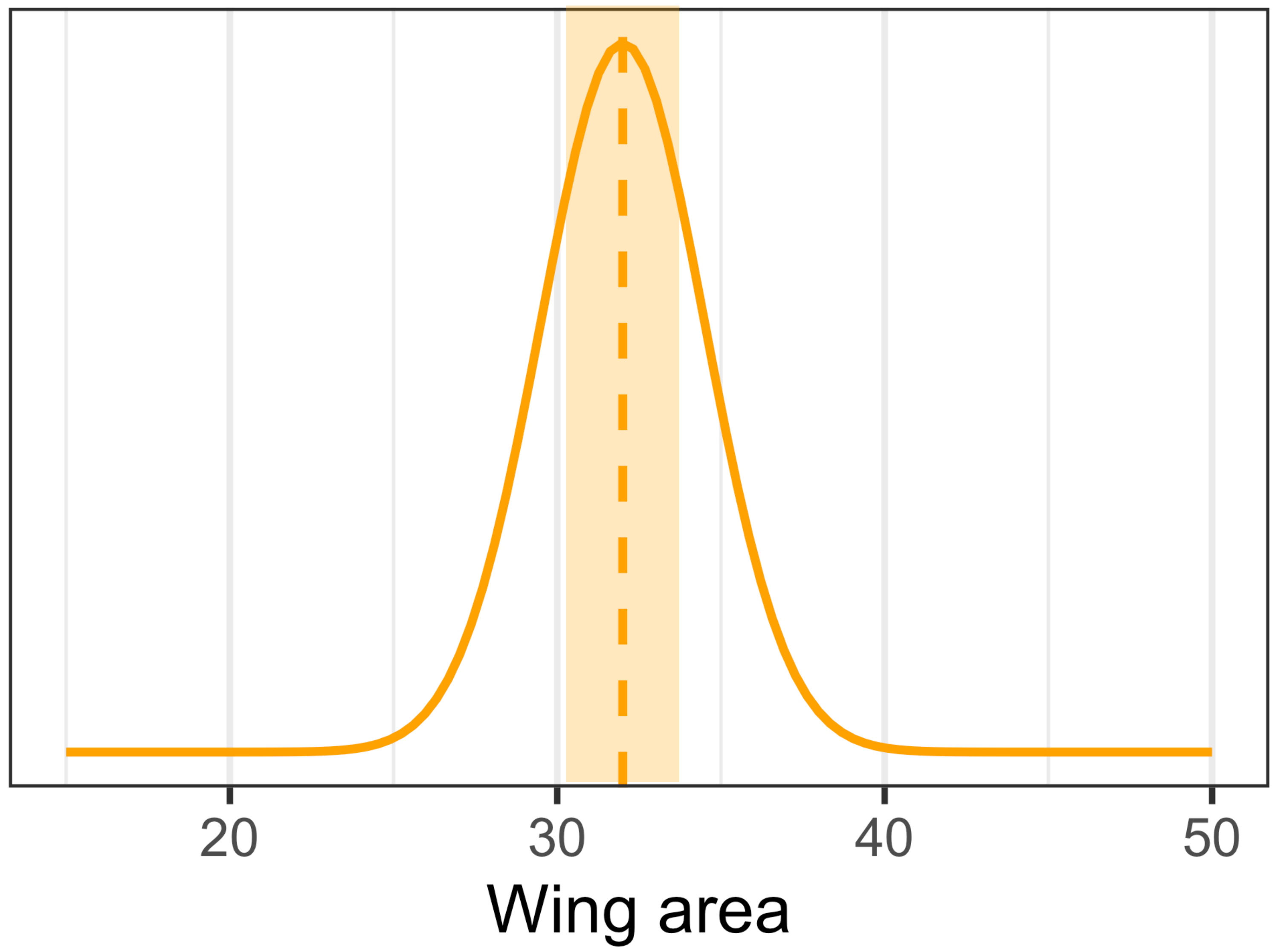
Comparing populations: difference between means

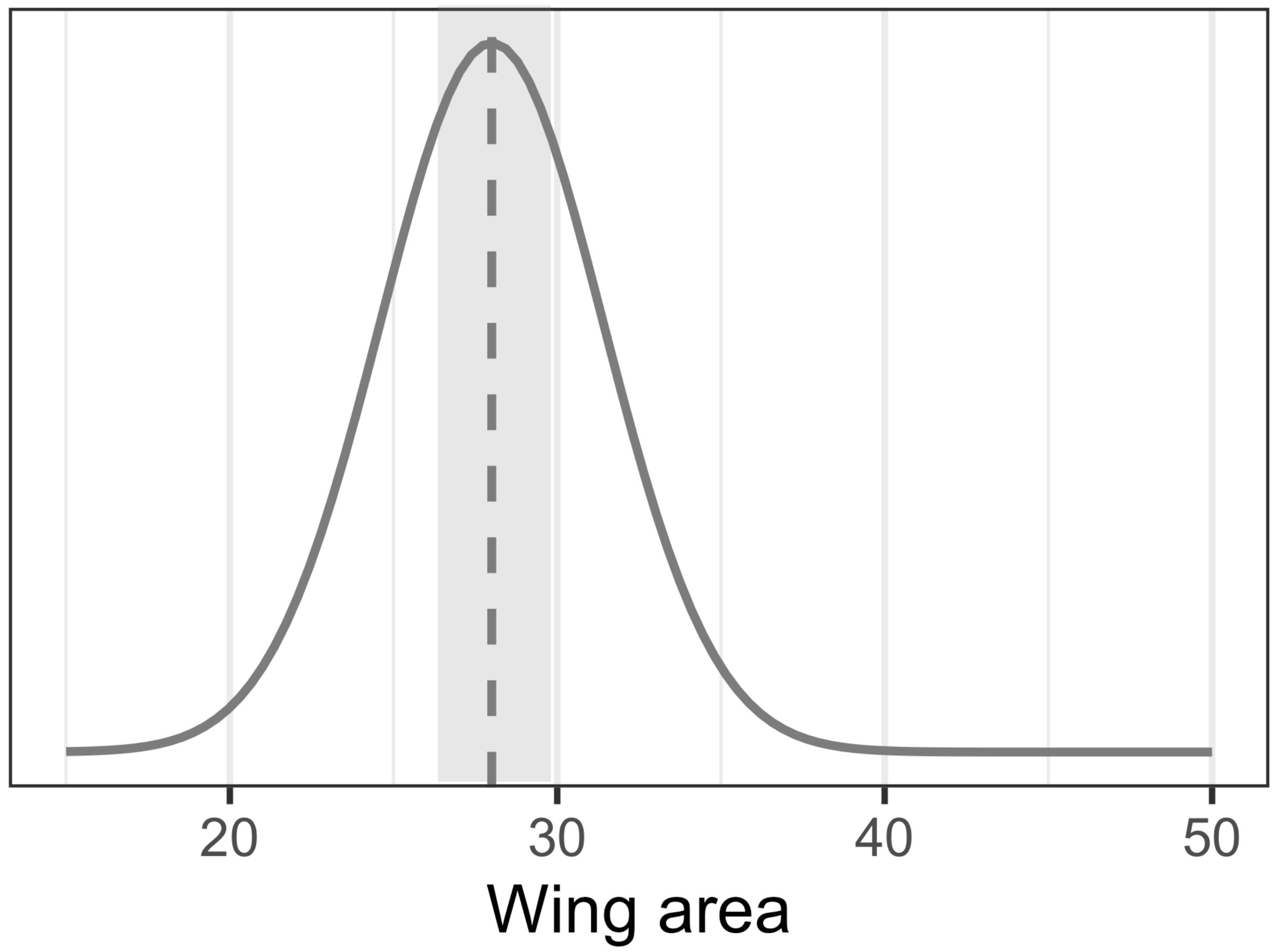
$$\bar{y}_1 = 32$$
$$s_1 = 2.5$$

$$\bar{y}_1 - \bar{y}_2 = 4$$



$$\bar{y}_2 = 28$$
$$s_2 = 3.4$$





Comparing populations: hypothesis testing

The null hypothesis:

$$H_0 : \mu_1 = \mu_2$$

The alternative hypothesis:

$$H_A : \mu_1 \neq \mu_2$$

An introduction to hypothesis testing

The city of Chicago received 48.8 inches of snow last winter (2020-2021) and 34.8 inches of snow the previous winter (2019-2020).

The null hypothesis:

$$H_0 : \mu_1 = \mu_2$$

Chicago received the <u>same</u> amount of snow in 2019 and 2020

The alternative hypothesis:

$$H_A : \mu_1 \neq \mu_2$$

Chicago received <u>a different</u> amount of snow in 2019 and 2020

Source: https://www.weather.gov/lot/Chicago_seasonal_snow

Comparing populations: hypothesis testing

In a certain clinical trial, 30 patients received the drug treatment and 30 patients received the placebo. After two months of treatment, disease progress was measured.

The null hypothesis:

$$H_0 : \mu_1 = \mu_2$$

The patients in the control group have the <u>same</u> disease progression as those in the treatment group.

The alternative hypothesis:

$$H_A : \mu_1 \neq \mu_2$$

The patients in the control group have <u>a different</u> disease progression as those in the treatment group.

Quick note: association vs. causation

In a certain clinical trial, 30 patients received the drug treatment and 30 patients received the placebo. After two months of treatment, disease progress was measured.

- **Association** is not **causation**
- Harder to determine cause and effect relationship from **observational study**
    - *Could be confounding facts*

The alternative hypothesis:

$$H_A : \mu_1 \neq \mu_2$$

The patients in the control group have <u>a different</u> disease progression as those in the treatment group.

Comparing populations: hypothesis testing

The null hypothesis:

$$H_0 : \mu_1 = \mu_2 \longrightarrow H_0 : \underline{\mu_1 - \mu_2} = 0$$

The alternative hypothesis:

$$H_A : \mu_1 \neq \mu_2 \longrightarrow H_A : \underline{\mu_1 \neq \mu_2} = 0$$

How do we choose between these two hypotheses?

Comparing populations: the *t* statistic

The *t* test is a standard method of choosing between these hypotheses

$$H_0 : \mu_1 = \mu_2 \qquad H_A : \mu_1 \neq \mu_2$$

Test statistic:

t is in units of SE

$$t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{Y}_1 - \bar{Y}_2}}$$

Variation in differences of means from random samples

How far the difference between the two means are from 0 (null hypothesis)

Adding and subtracting random variables

$\mu_1 = 300; \sigma_1 = 22$

$\mu_2 = 368; \sigma_2 = 26$

What is the overall difference in means between the populations (black and white)?

What is the variance in the difference of means between these two populations?

$$\mu_{1-2} = \mu_1 - \mu_2$$
$$= -68$$

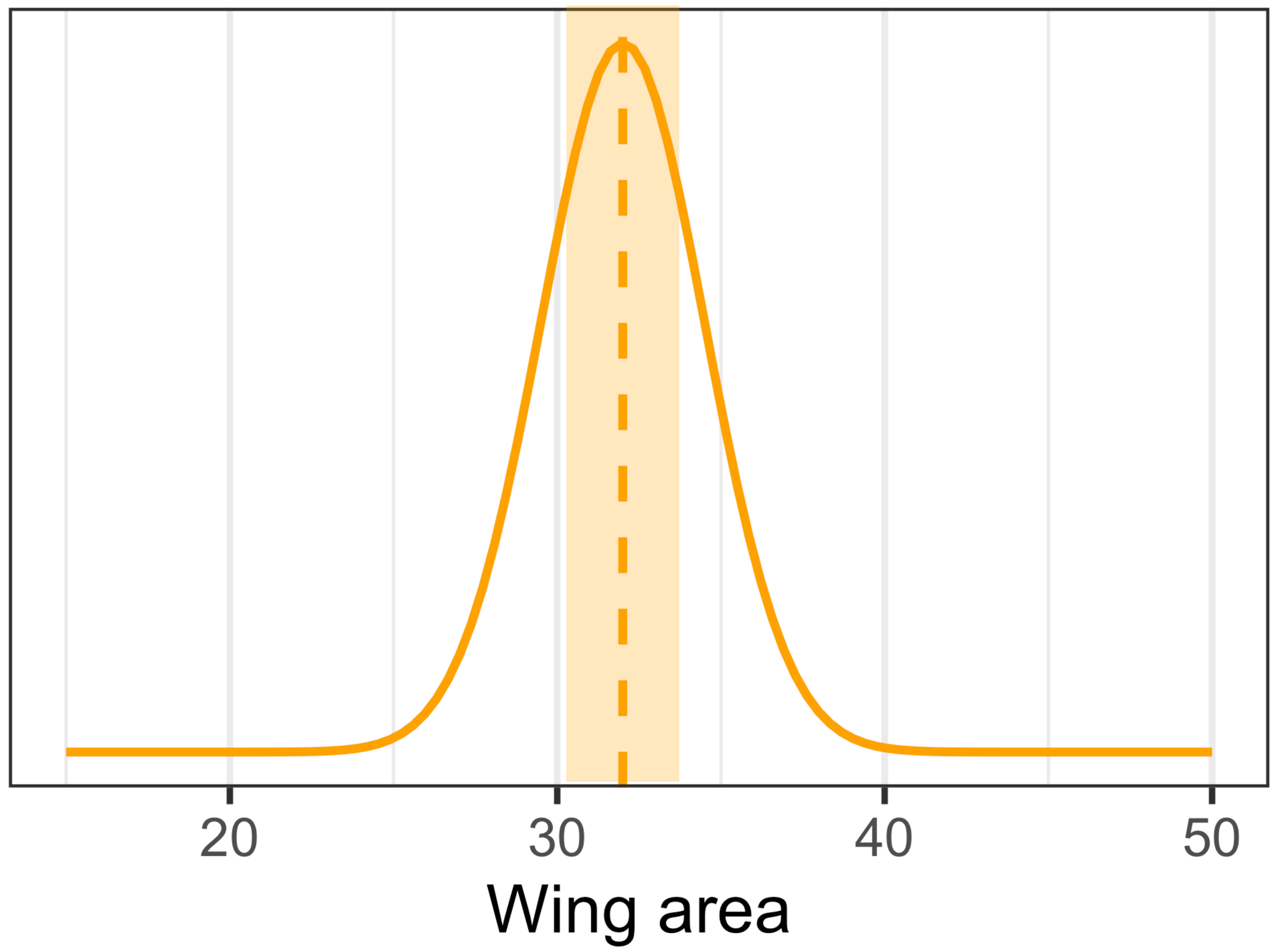
$$\sigma^2_{1-2} = \sigma^2_1 + \sigma^2_2$$
$$\sigma^2_{1-2} = 22^2 + 26^2$$
$$= 1160$$

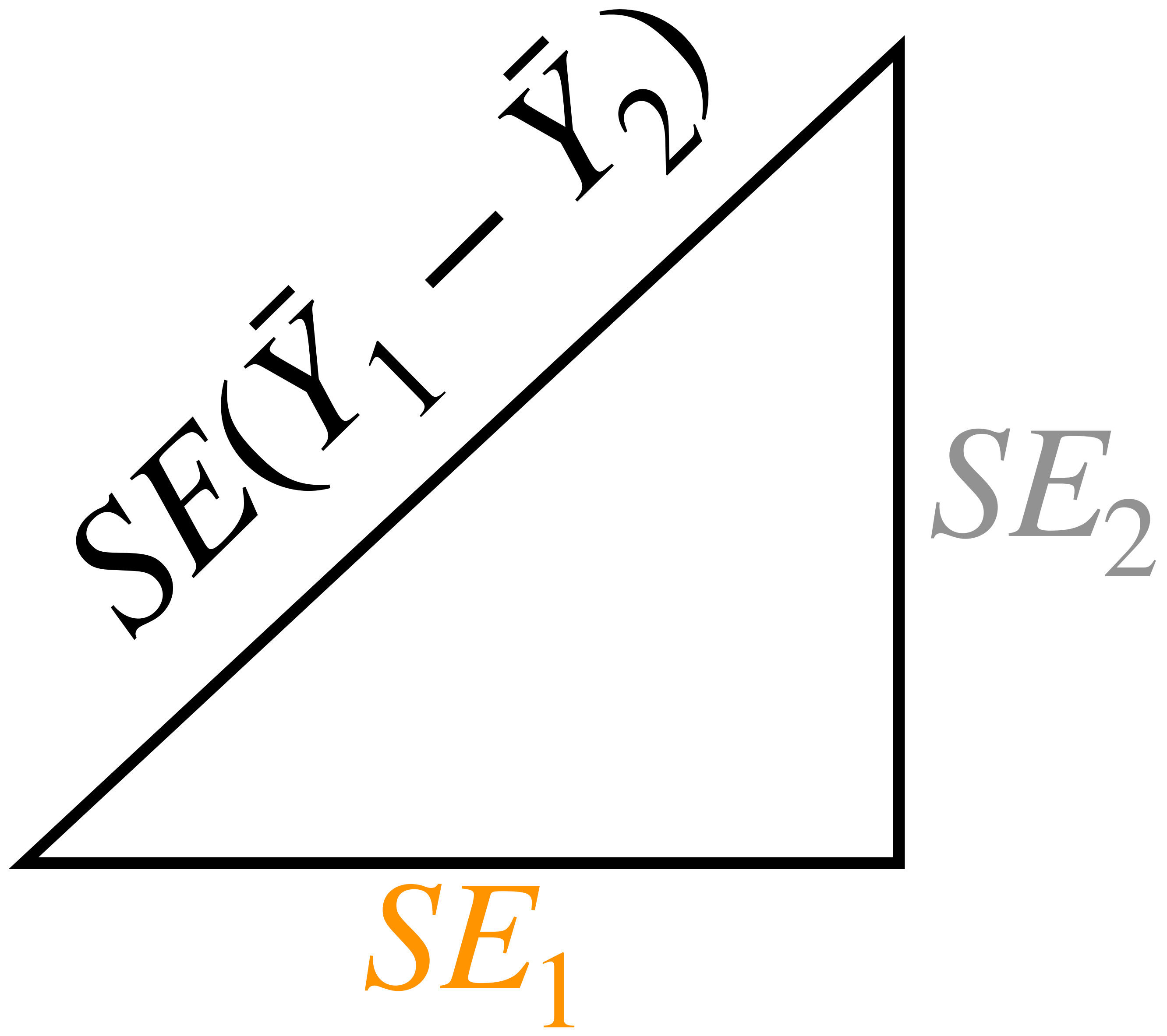
$$\sigma_{1-2} = 34.06$$

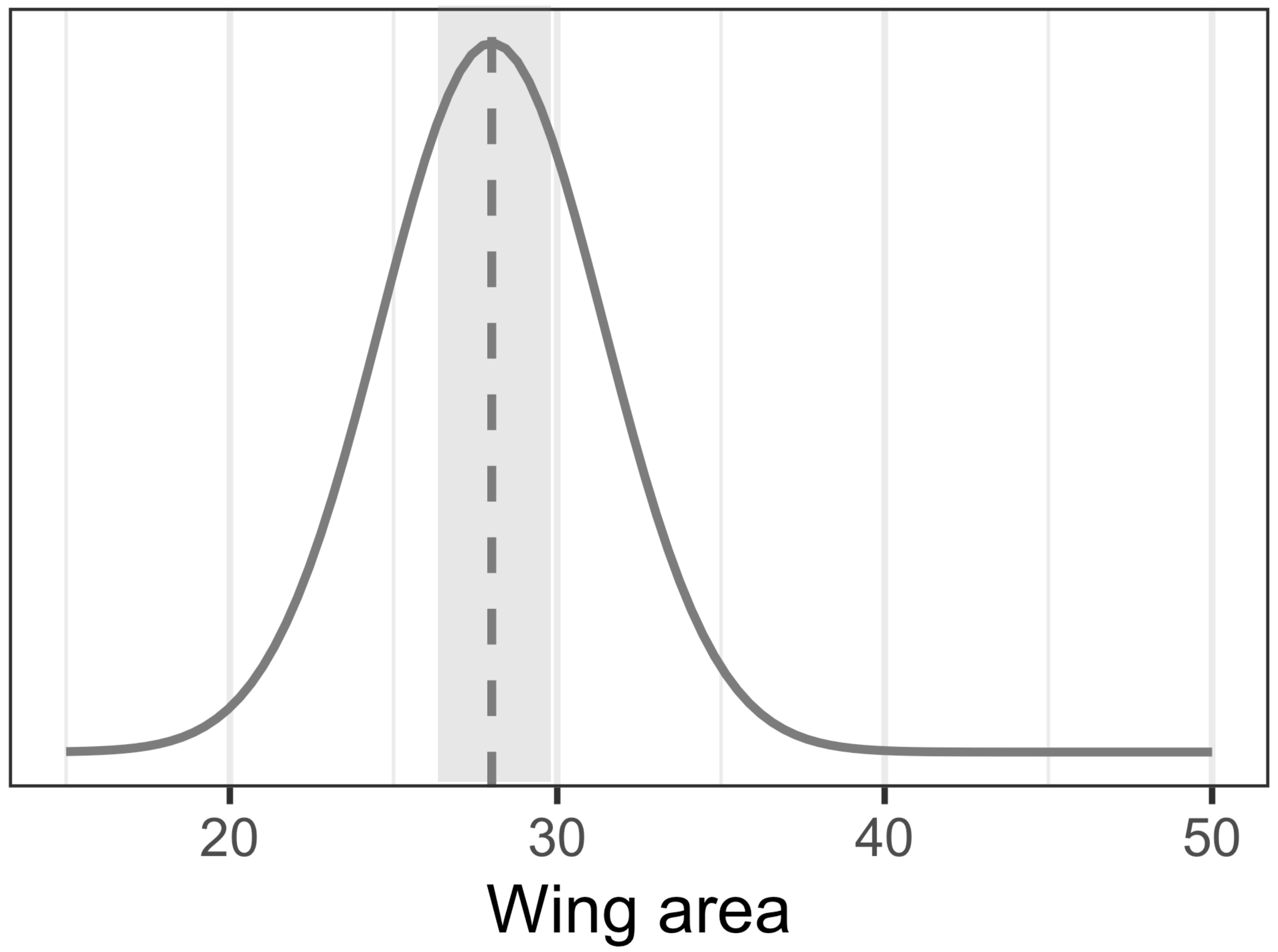
If you want to see the math: link here

Standard error of $\bar{Y}_1 - \bar{Y}_2$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}}$$



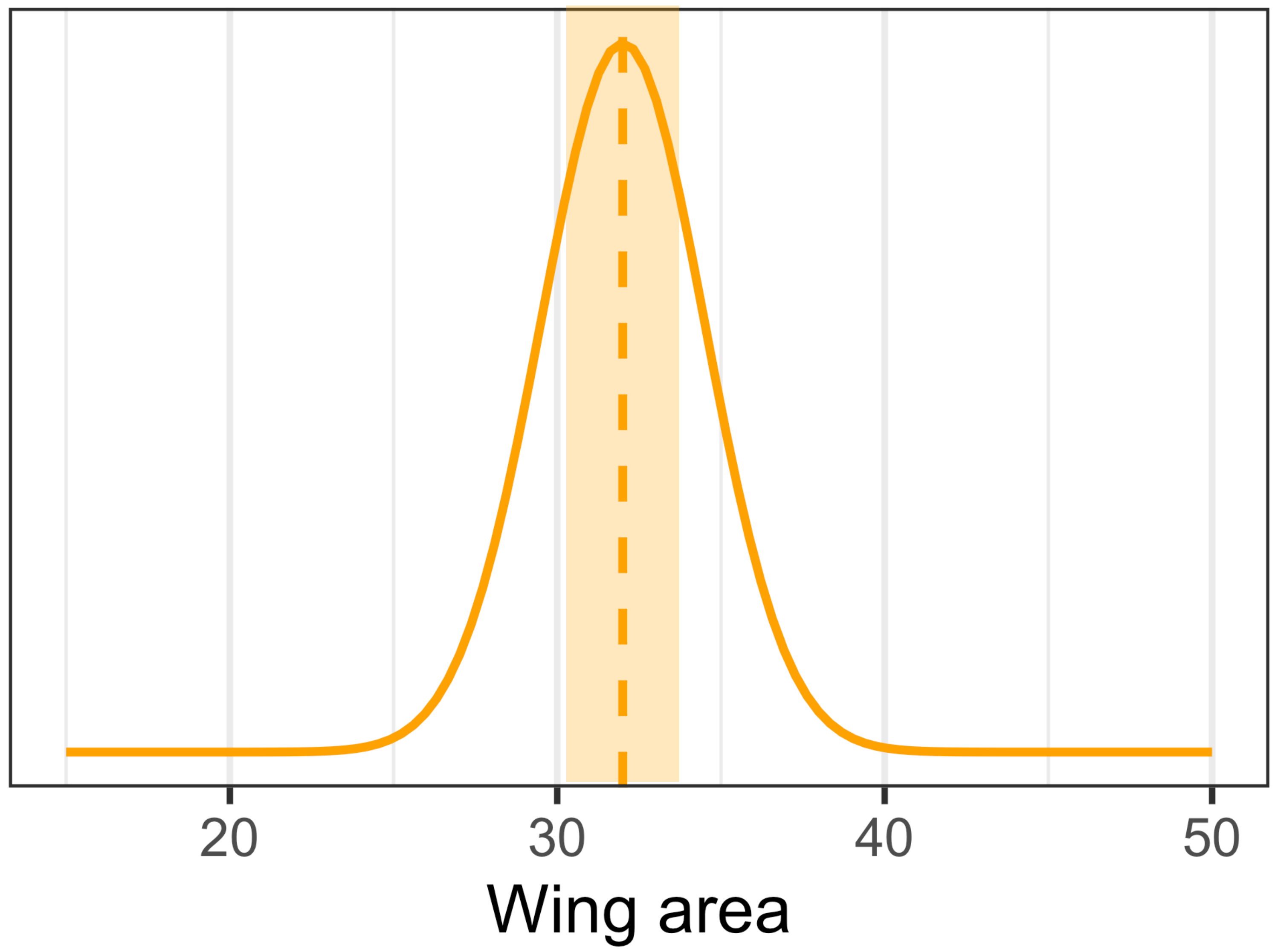


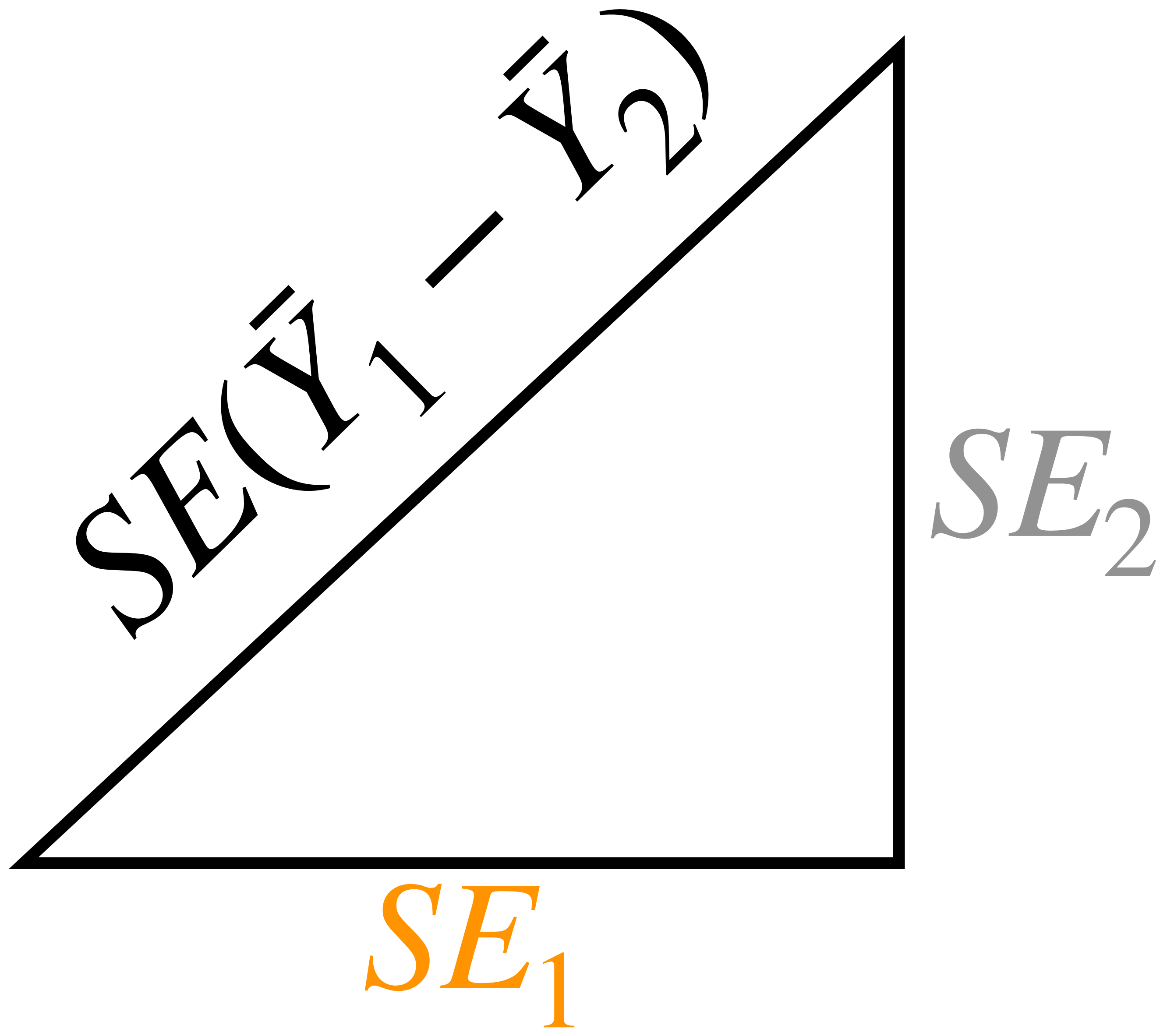


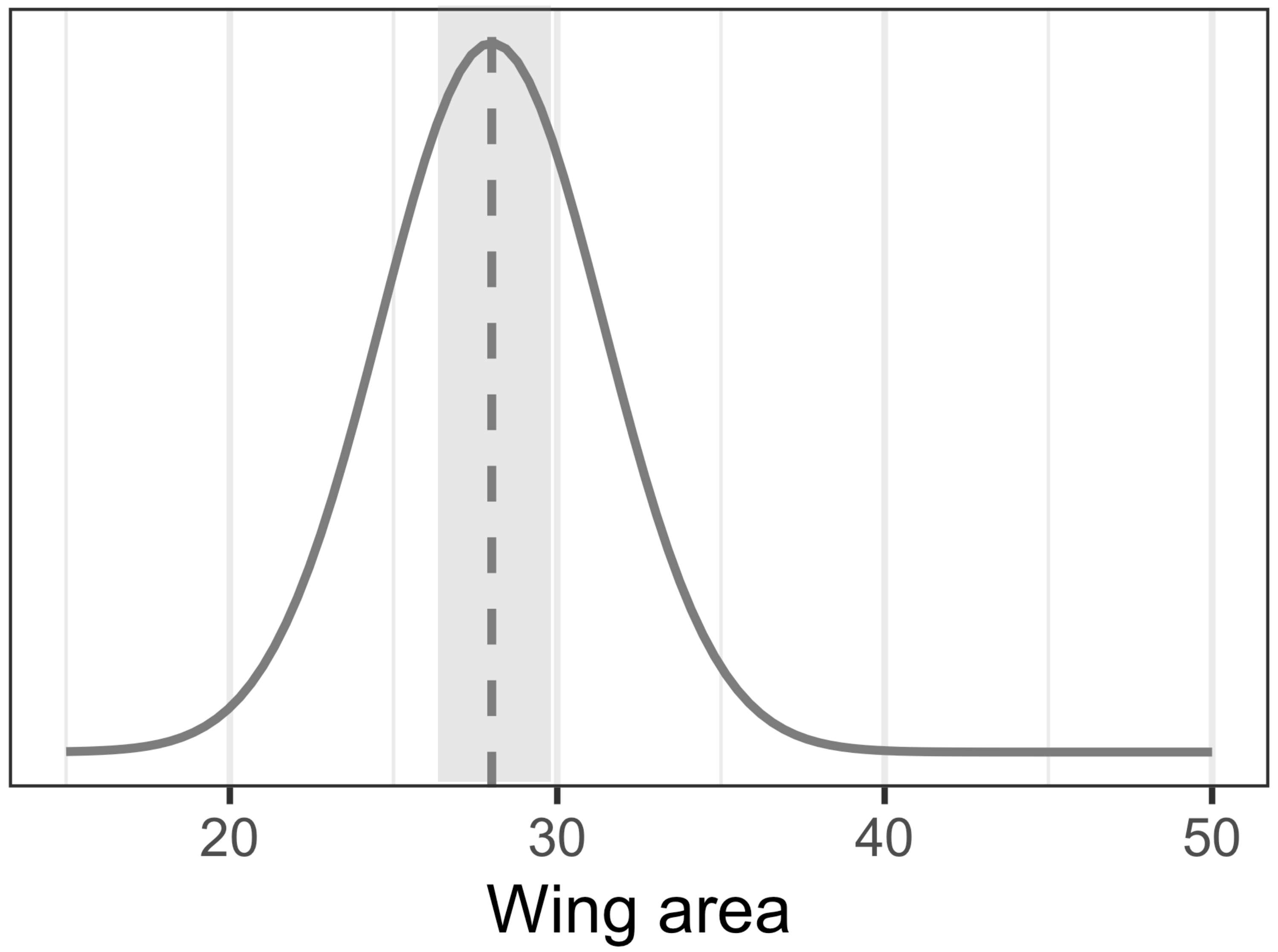
Standard error of $\bar{Y}_1 - \bar{Y}_2$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} \qquad SE^2_{\bar{Y}} = \frac{s^2}{n}$$

(Variance of the mean)



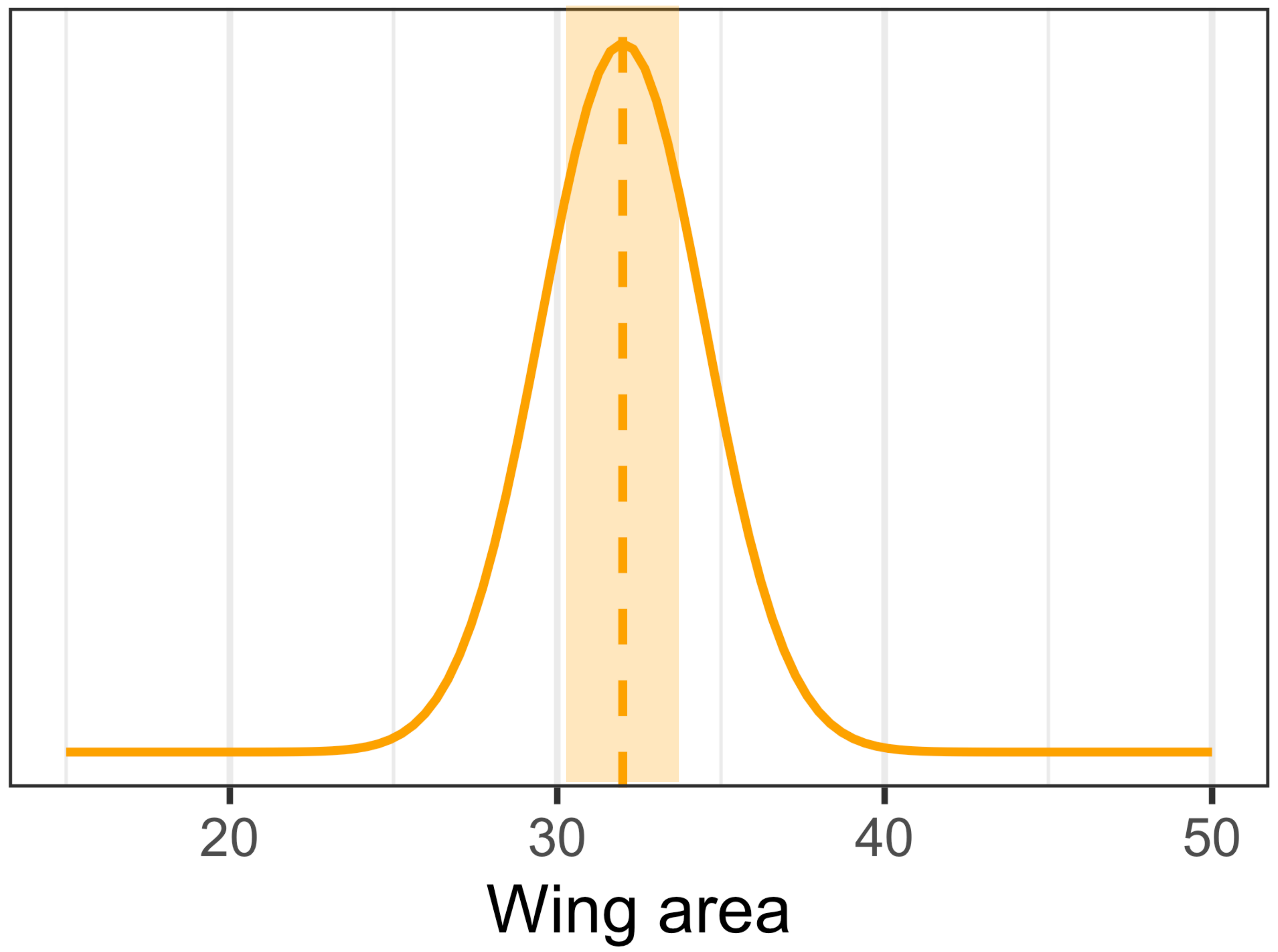


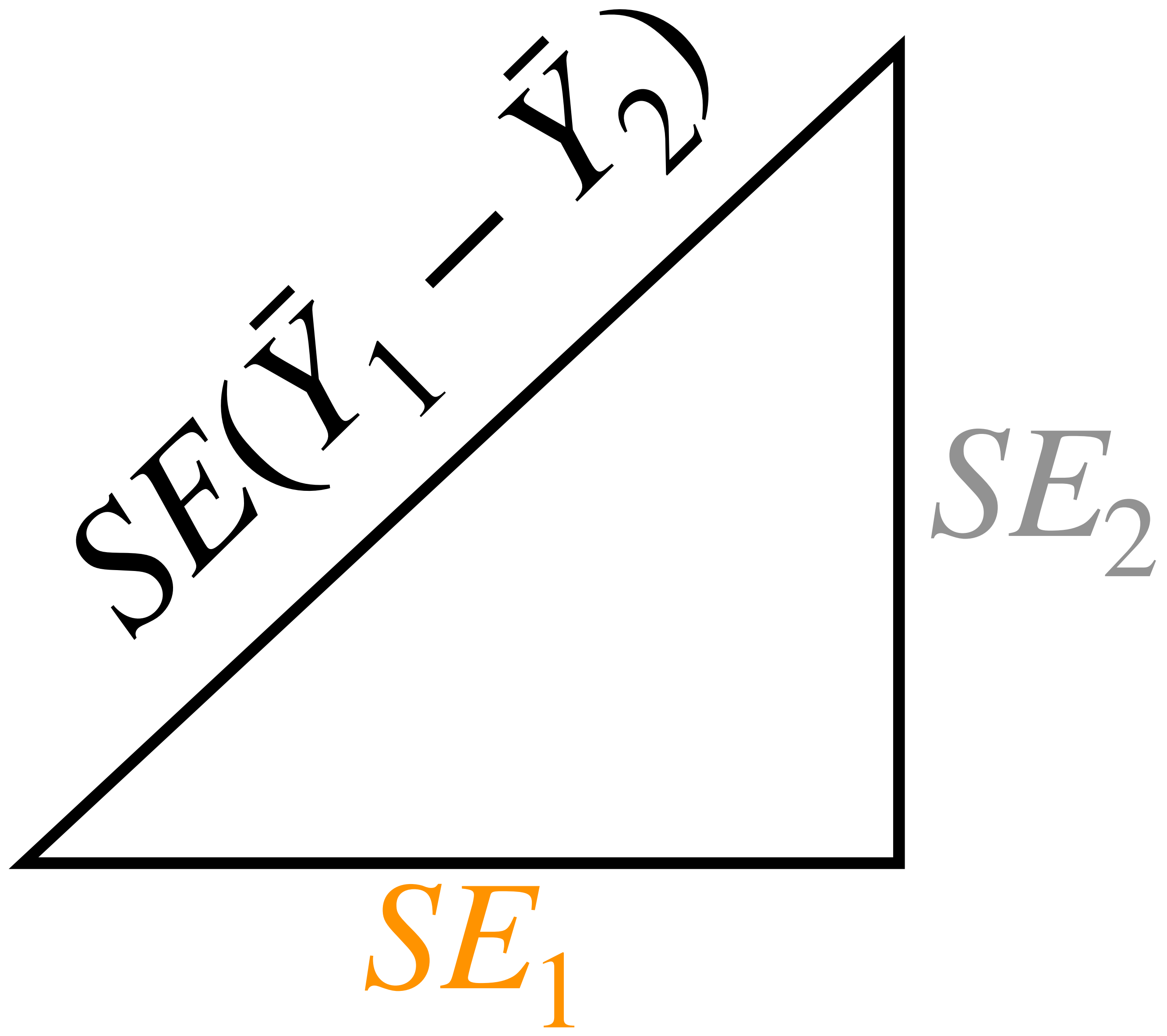


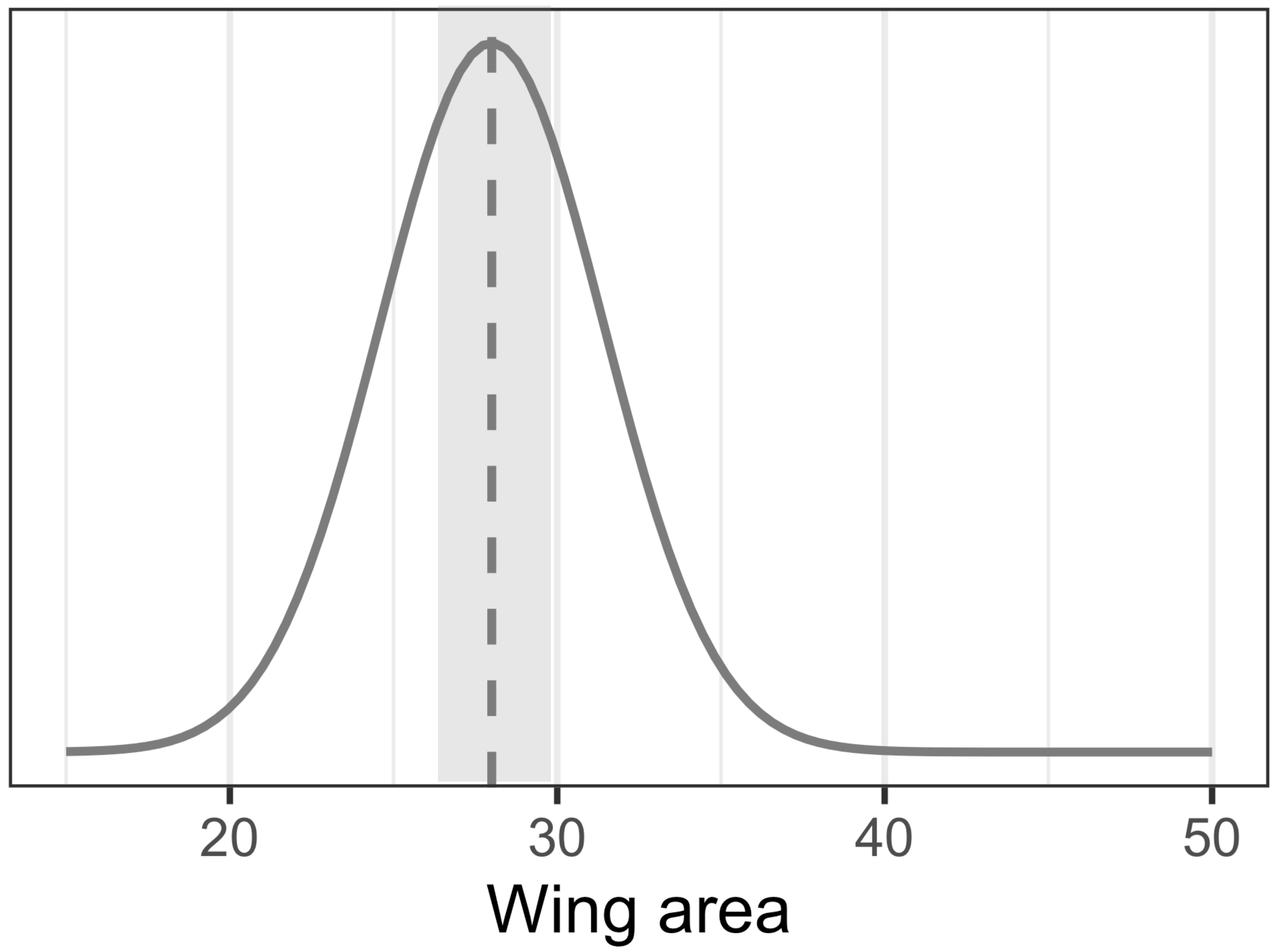
Standard error of $\bar{Y}_1 - \bar{Y}_2$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} \qquad SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{SE_1^2 + SE_2^2}$$

Variability in each estimate ADDS to the total variability in the difference



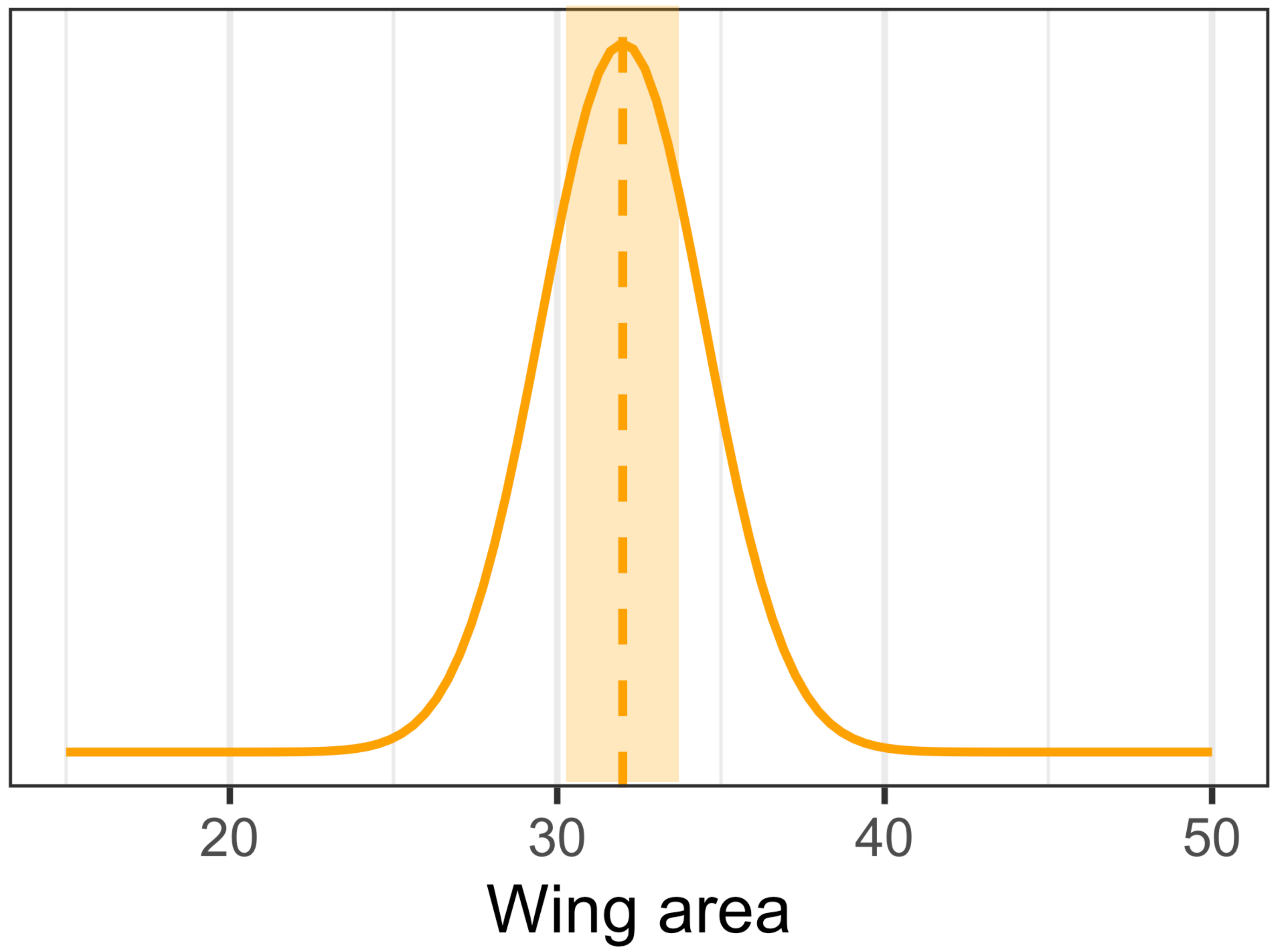


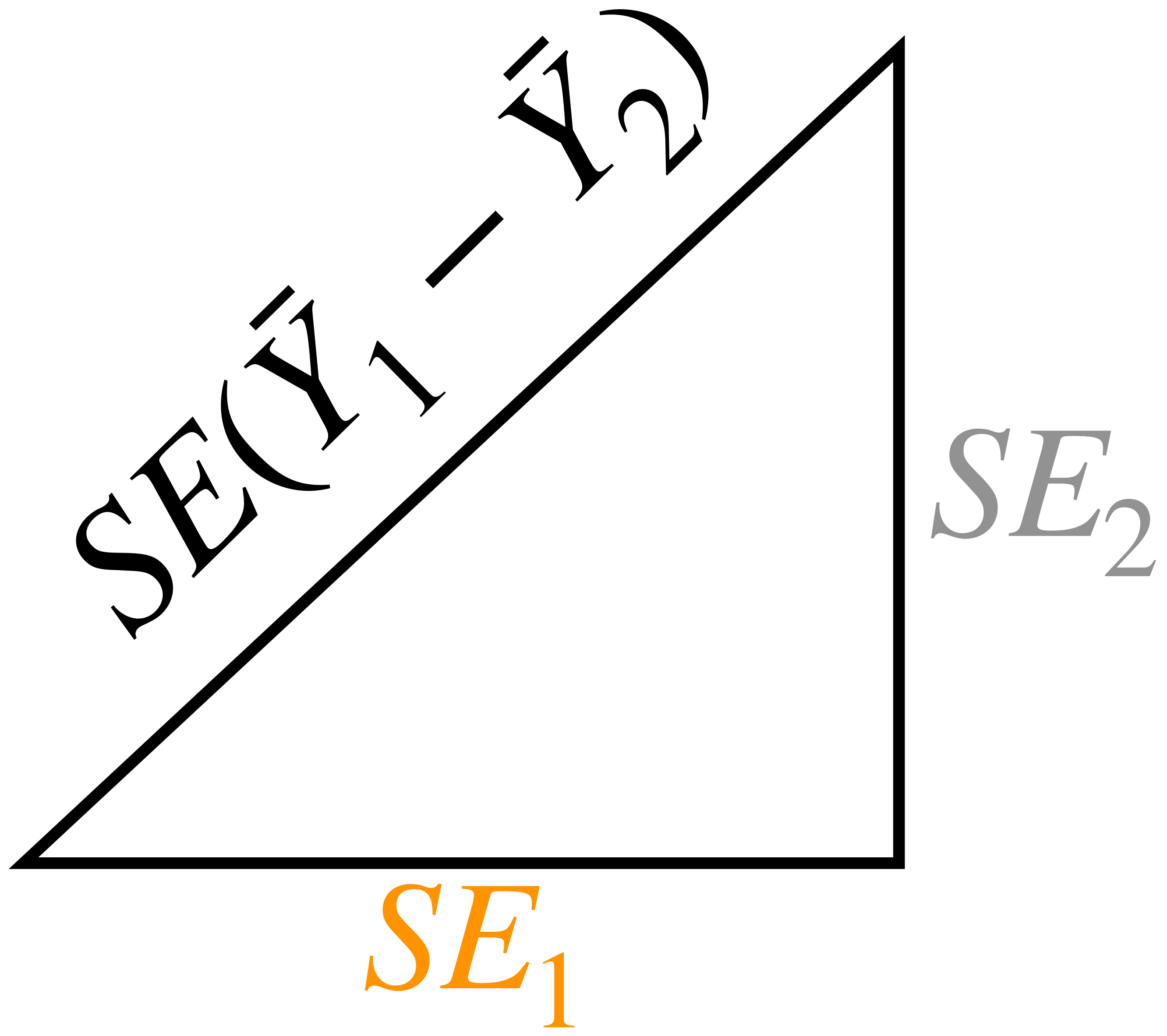


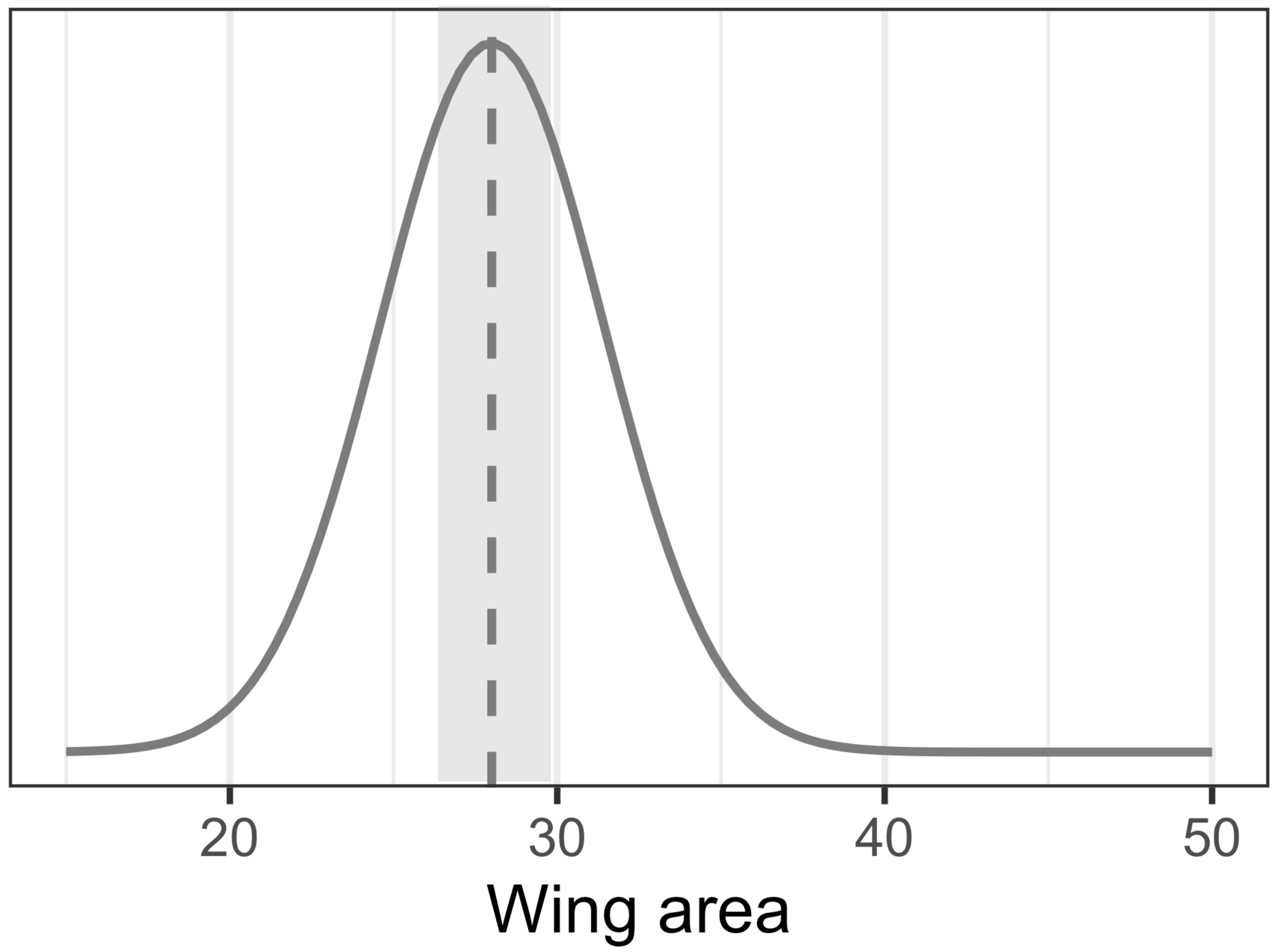
Standard error of $\bar{Y}_1 - \bar{Y}_2$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} \qquad SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}$$

Variability in each estimate ADDS to the total variability in the difference







Standard error of $\bar{Y}_1 - \bar{Y}_2$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} \qquad SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}$$

Variability in each estimate ADDS to the total variability in the difference

- Whether we add $\bar{Y}_2$ to $\bar{Y}_1$ or subtract, we **add their variances**
- The "noise" associated with $\bar{Y}_2$ (i.e. SE_2) adds to the overall uncertainty
- The greater the variability in $\bar{Y}_2$, the greater the variability in $\bar{Y}_1 - \bar{Y}_2$

Standard error of $\bar{Y}_1 - \bar{Y}_2$

$$SE_{\bar{Y}} = \frac{s}{\sqrt{n}} \qquad SE_{\bar{Y}_1 - \bar{Y}_2} = \sqrt{\frac{2.5^2}{14} + \frac{3.4^2}{12}} = 1.18$$

Variability in each estimate ADDS to the total variability in the difference

- Whether we add $\bar{Y}_2$ to $\bar{Y}_1$ or subtract, we **add their variances**
- The "noise" associated with $\bar{Y}_2$ (i.e. SE_2) adds to the overall uncertainty
- The greater the variability in $\bar{Y}_2$, the greater the variability in $\bar{Y}_1 - \bar{Y}_2$

Standard error of $\bar{Y}_1 - \bar{Y}_2$

Calculate the standard error of the difference between Sample 1 and Sample 2

$$SE_1 = \frac{s_1}{\sqrt{n_1}}$$

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_2 = \frac{s_2}{\sqrt{n_2}}$$

Standard error of $\bar{Y}_1 - \bar{Y}_2$

Calculate the standard error of the difference between Sample 1 and Sample 2

$$SE_1 = \frac{4.3}{\sqrt{6}}$$

$$SE_1 = 1.75$$

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_2 = \frac{5.7}{\sqrt{12}}$$

$$SE_2 = 1.65$$

$$SE_{1-2} = \sqrt{SE_1^2 + SE_2^2}$$

Standard error of $\bar{Y}_1 - \bar{Y}_2$

Calculate the standard error of the difference between Sample 1 and Sample 2

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_1 = \frac{4.3}{\sqrt{6}}$$

$$SE_1 = 1.75$$

$$SE_2 = \frac{5.7}{\sqrt{12}}$$

$$SE_2 = 1.65$$

$$SE_{1-2} = \sqrt{1.75^2 + 1.65^2}$$

$$SE_{1-2} = 2.41$$

Standard error of $\bar{Y}_1 - \bar{Y}_2$

Calculate the standard error of the difference between Sample 1 and Sample 2

	Sample 1	**Sample 2**
n	6	12
y	40	50
s	4.3	5.7

$$SE_{1-2} = \sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}$$

Standard error of $\bar{Y}_1 - \bar{Y}_2$

Calculate the standard error of the difference between Sample 1 and Sample 2

	Sample 1	**Sample 2**
n	6	12
y	40	50
s	4.3	5.7

$$SE_{1-2} = \sqrt{\frac{4.3^2}{6} + \frac{5.7^2}{12}}$$

$$SE_{1-2} = 2.41$$

Comparing populations: the *t* statistic

The *t* test is a standard method of choosing between these hypotheses

$$H_0 : \mu_1 = \mu_2 \qquad H_A : \mu_1 \neq \mu_2$$

Test statistic:

t is in units of SE

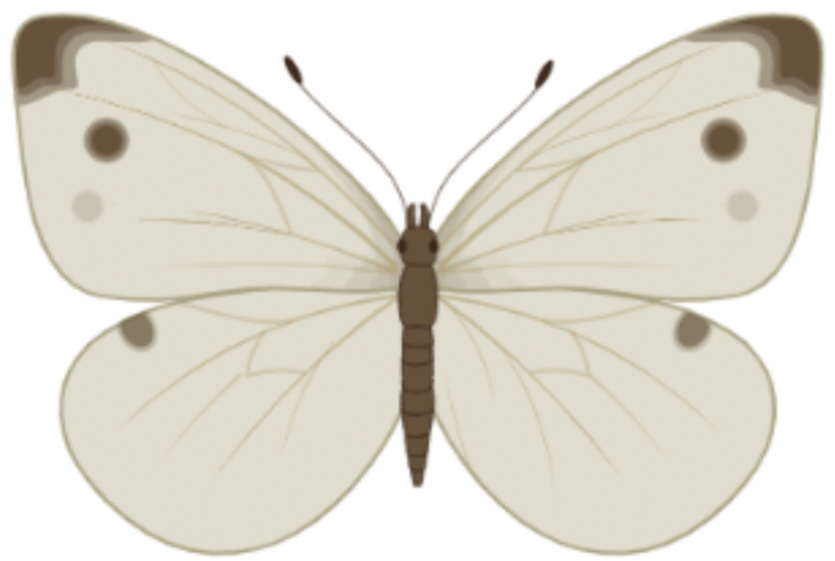
$$t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{Y}_1 - \bar{Y}_2}}$$

Variation in differences of means from random samples

How far the difference between the two means are from 0 (null hypothesis)

Comparing populations: the t statistic

$\bar{y}_1 = 32$
$s_1 = 2.5$



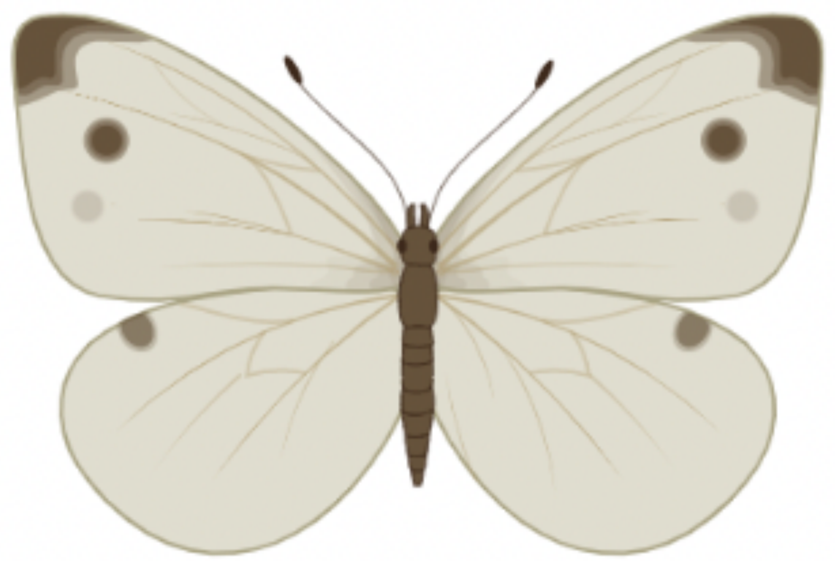
$\bar{y}_2 = 28$
$s_2 = 3.4$

$$t_s = \frac{\bar{y}_1 - \bar{y}_2}{SE_{\bar{y}_1 - \bar{y}_2}} \qquad (SE_{\bar{Y}_1 - \bar{Y}_2} = 1.18)$$

$$t_s = \frac{32 - 28}{1.18} = 3.39$$

Comparing populations: the *t* statistic

$\bar{y}_1 = 32$
$s_1 = 2.5$



$\bar{y}_2 = 28$
$s_2 = 3.4$

1. Generate a hypothesis and choose a significance level

$$H_0 : \bar{y}_1 = \bar{y}_2 \qquad H_A : \bar{y}_1 \neq \bar{y}_2 \qquad \alpha = 0.05$$

2. Calculate test statistic

$$t_s = \frac{32 - 28}{1.18} = 3.39$$

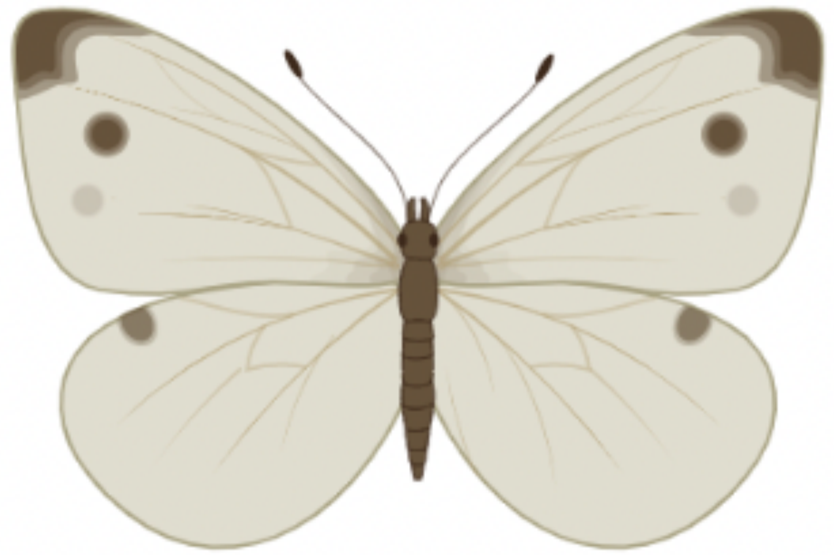
3. Calculate the *P*-value

Conservative estimate

Use smaller of n_1-1 and n_2-1 for degrees of freedom*

Comparing populations: the *t* statistic

$\bar{y}_1 = 32$
$s_1 = 2.5$



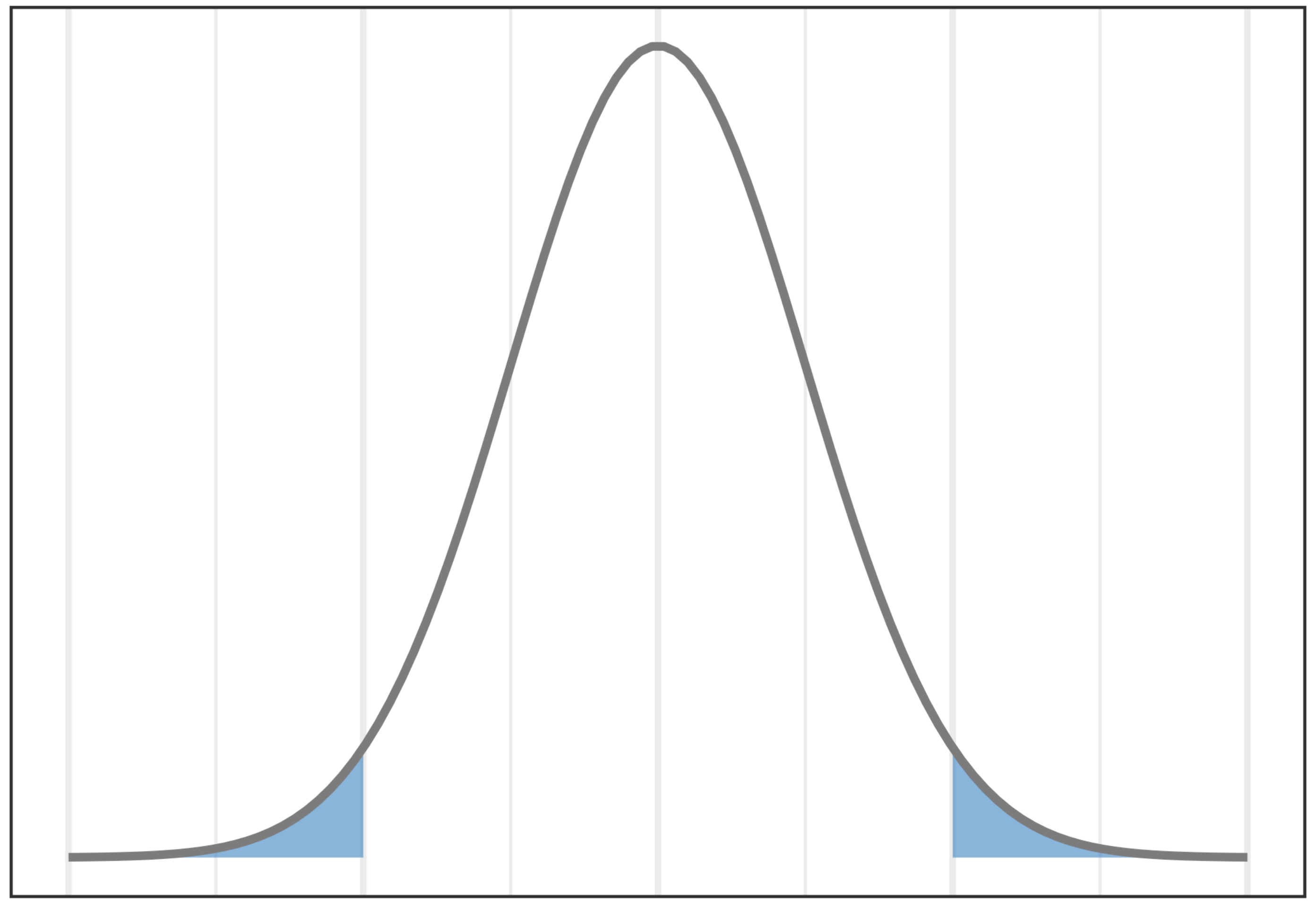
$\bar{y}_2 = 28$
$s_2 = 3.4$

1. Generate a hypothesis and choose a significance level

$$H_0 : \bar{y}_1 = \bar{y}_2 \qquad H_A : \bar{y}_1 \neq \bar{y}_2 \qquad \alpha = 0.05$$

2. Calculate test statistic

$$t_s = \frac{32 - 28}{1.18} = 3.39$$



3. Calculate the *P*-value

$$df_1 = n_1 - 1 = 13 \qquad df_2 = n_2 - 1 = 11$$

$$t_s = 3.39$$

$$df = 11$$

$$\alpha = 0.05$$

$$\boxed{0.002 < P < 0.01}$$

```
> pt(3.39, 11, lower.tail = F)*2
```

```
[1] 0.00603
```

REJECT the null hypothesis

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
[illegible]	[illegible]	1.782	2.179	2.681	3.055	3.93	4.318
[illegible]	[illegible]	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373
1000	1.282	1.646	1.962	2.33	2.581	3.098	3.3
Inf	1.282	1.645	1.96	2.326	2.576	3.091	3.291

Another example

Use a *t*-test to compare the difference of means between Sample 1 and Sample 2

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_{1-2} = \sqrt{\frac{4.3^2}{6} + \frac{5.7^2}{12}}$$

$$SE_{1-2} = 2.41$$

Use a *t*-test to compare the difference of means between Sample 1 and Sample 2

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_{1-2} = 2.41$$

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.14
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
120	1.289	1.658	1.98	2.358	2.617	3.16	3.373

Use a *t*-test to compare the difference of means between Sample 1 and Sample 2

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_{1-2} = 2.41$$

1. **Generate a hypothesis and choose a significance level**

$$H_0 : \bar{y}_1 = \bar{y}_2 \qquad \alpha = 0.05$$

$$H_A : \bar{y}_1 \neq \bar{y}_2$$

2. Calculate test statistic

$$t_s = \frac{y_1 - y_2}{SE_{1-2}}$$

3. Calculate the *P*-value

Use a *t*-test to compare the difference of means between Sample 1 and Sample 2

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_{1-2} = 2.41$$

$$df_1 = 5$$

$$df_2 = 11$$

1. **Generate a hypothesis and choose a significance level**

$$H_0 : \bar{y}_1 = \bar{y}_2 \qquad \alpha = 0.05$$

$$H_A : \bar{y}_1 \neq \bar{y}_2$$

2. **Calculate test statistic**

$$t_s = \frac{40 - 50}{2.41} = -4.14$$

3. **Calculate the *P*-value**

Use a *t*-test to compare the difference of means between Sample 1 and Sample 2

	Sample 1	Sample 2
n	6	12
y	40	50
s	4.3	5.7

$$SE_{1-2} = 2.41$$

$$t_s = 4.14$$

$$df = 5$$

$$\alpha = 0.05$$

$$\boxed{0.002 < P < 0.01}$$

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
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6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781

```
> pt(4.14, 5, lower.tail = F)*2
```

```
[1] 0.0089
```

REJECT the null hypothesis

Confidence interval for $\bar{Y}_1 - \bar{Y}_2$

95% confidence interval for one sample

$$\bar{y} \pm t_{0.025} SE_{\bar{Y}}$$

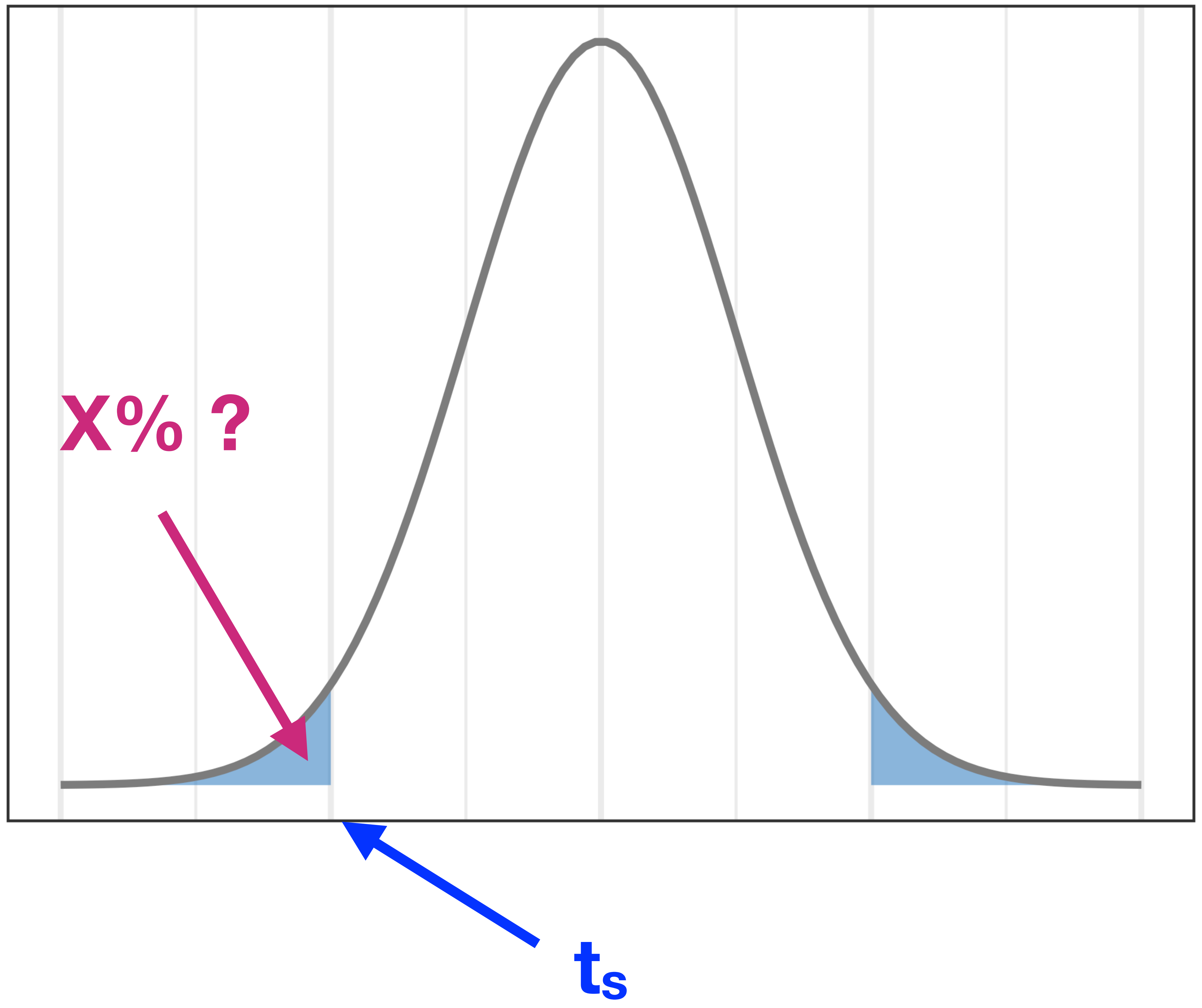
95% confidence interval for difference of means (two samples)

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.025} SE_{(\bar{Y}_1 - \bar{Y}_2)}$$

Use smaller of n_1-1 and n_2-1 for degrees of freedom*

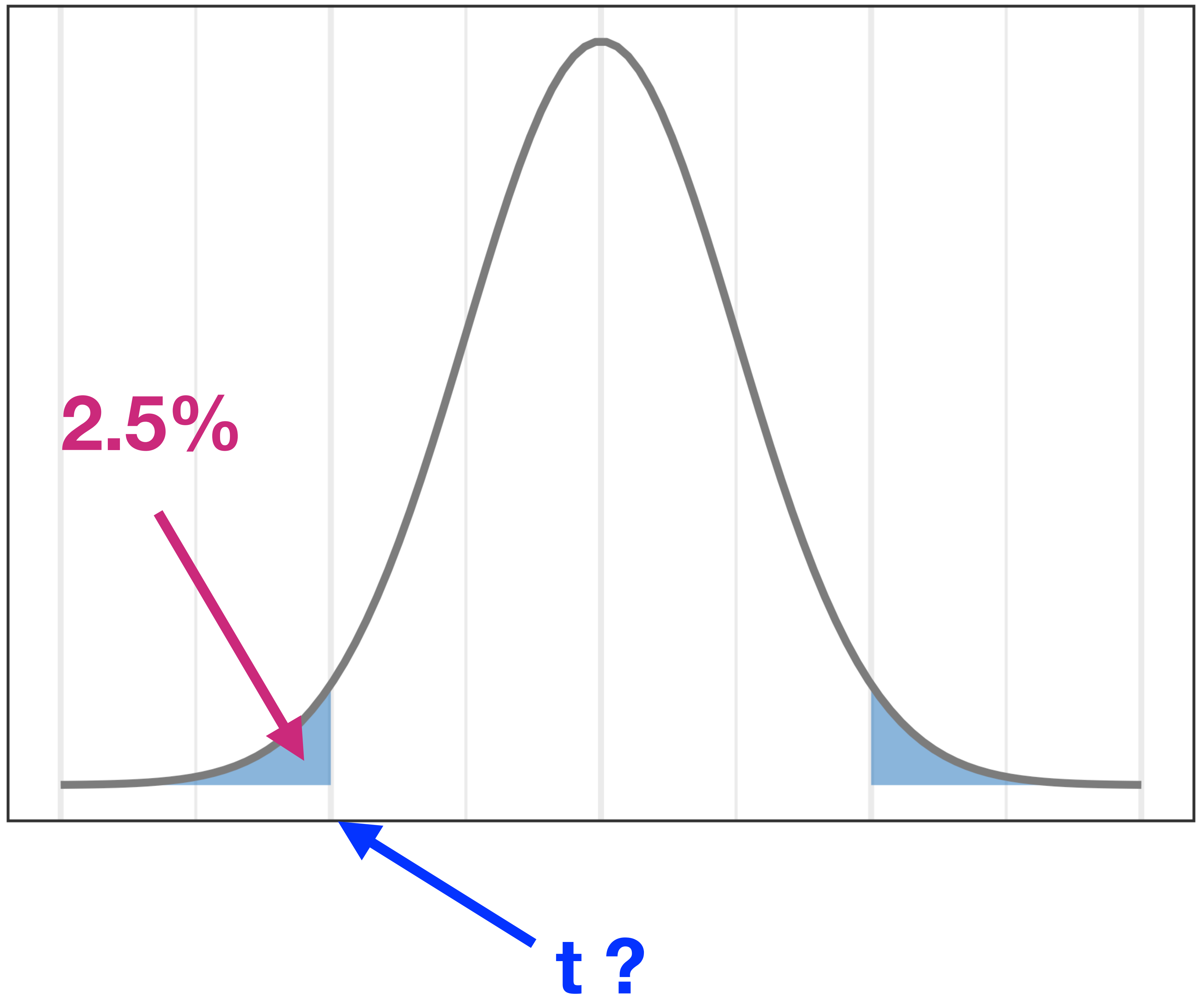
Comparing *t*-test and confidence intervals

***t*-test**




```
> pt(t, df)*2
```

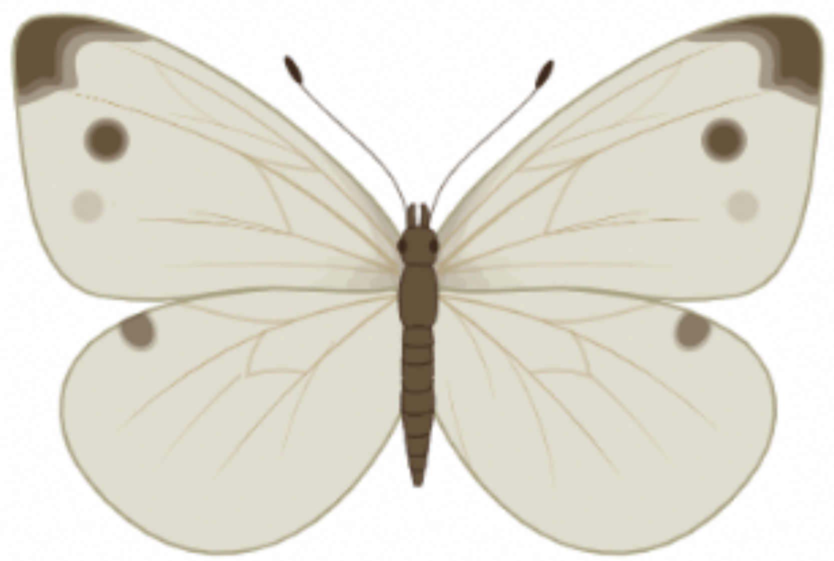
Confidence interval




```
> qt(p, df)
```

Confidence interval for $\bar{Y}_1 - \bar{Y}_2$

$\bar{y}_1 = 32$
$s_1 = 2.5$



$\bar{y}_2 = 28$
$s_2 = 3.4$

95% confidence interval for difference of means (two samples)

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.025} SE_{(\bar{Y}_1 - \bar{Y}_2)}$$

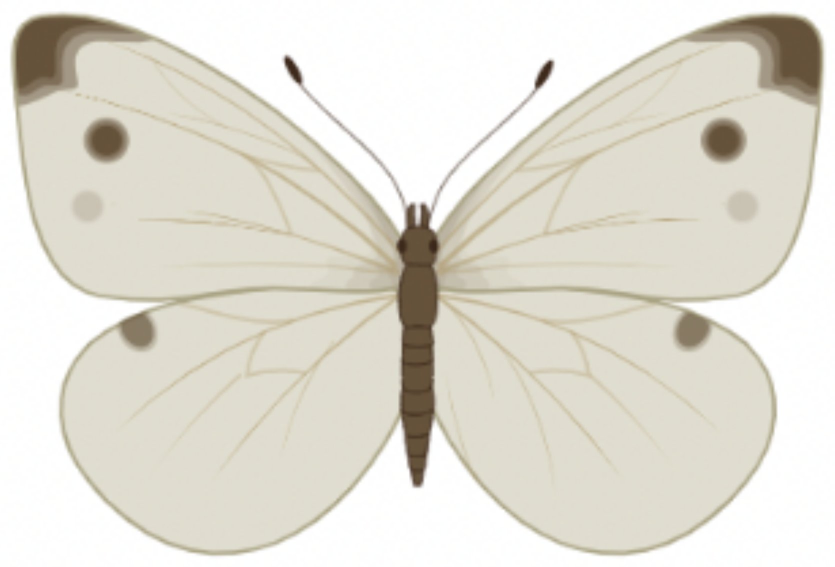
Use smaller of n_1-1 and n_2-1 for degrees of freedom*

$$(32 - 28) \pm t_{0.025}(1.18)$$

$$df = 11$$

Confidence interval for $\bar{Y}_1 - \bar{Y}_2$

$\bar{y}_1 = 32$
$s_1 = 2.5$



$\bar{y}_2 = 28$
$s_2 = 3.4$

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
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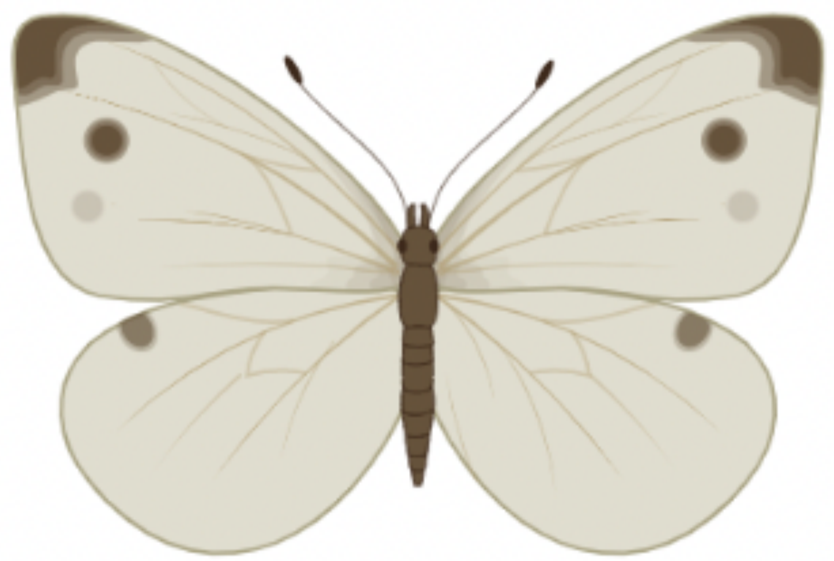
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.93	4.318
13	1.35	1.771	2.16	2.65	3.012	3.852	4.221
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Confidence interval for $\bar{Y}_1 - \bar{Y}_2$

$\bar{y}_1 = 32$
$s_1 = 2.5$



$\bar{y}_2 = 28$
$s_2 = 3.4$

95% confidence interval for difference of means (two samples)

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.025} SE_{(\bar{Y}_1 - \bar{Y}_2)}$$

Use smaller of n_1-1 and n_2-1 for degrees of freedom*

$$(32 - 28) \pm t_{0.025}(1.18)$$

$df = 11$

```
> qt(0.975, 11)
```

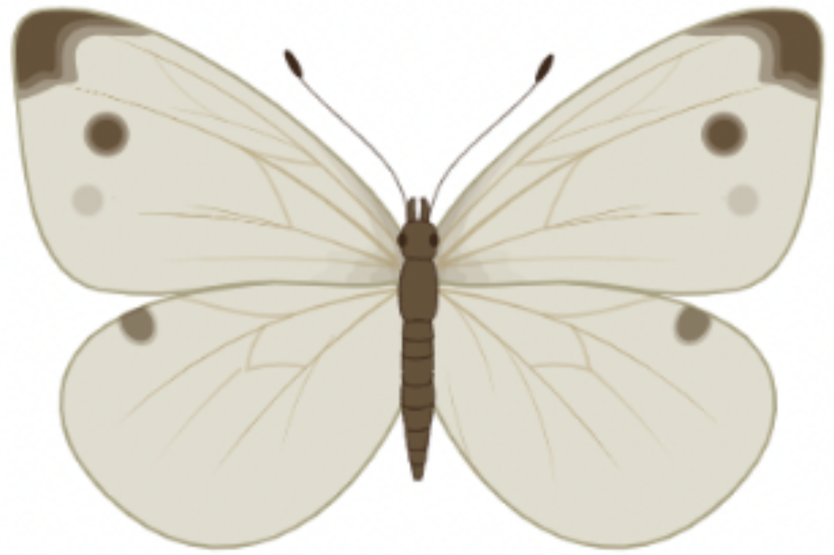
$t_s = 2.2$

Does NOT include 0!!

4 ± 2.596 → **(1.404, 6.596)**

Comparing populations: the t statistic

$\bar{y}_1 = 32$
$s_1 = 2.5$



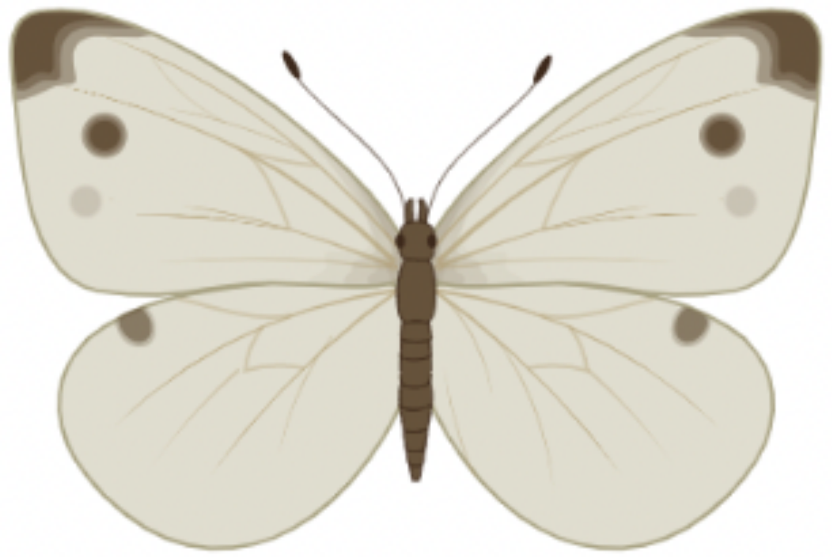
$\bar{y}_2 = 28$
$s_2 = 3.4$

```r
# set random seed
set.seed(76)

# make populations for butterfly
y1 <- rnorm(14, 32, 2.5)
y2 <- rnorm(12, 28, 3.4)

# calculate t test with t.test
t.test(y1, y2)
```

$\bar{y}_1 = 32$
$s_1 = 2.5$



$\bar{y}_2 = 28$
$s_2 = 3.4$

$t_s = 3.39$

$df = 11$

```
# set random seed
set.seed(76)

# make populations for butterfly
y1 <- rnorm(14, 32, 2.5)
y2 <- rnorm(12, 28, 3.4)

# calculate t test with t.test
t.test(y1, y2)
```

```
	Welch Two Sample t-test

data:  y1 and y2
t = 4.2051, df = 23.351, p-value = 0.0003288
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 1.646552 4.829972
sample estimates:
mean of x mean of y
 31.37680  28.13854
```

P = 0.00603

(1.404, 6.596)

Degrees of freedom: two samples

Just for example, no need to memorize this equation!!!

$$df = \frac{(SE_1^2 + SE_2^2)^2}{SE_1^4/(n_1 - 1) + SE_2^4(n_2 - 1)}$$

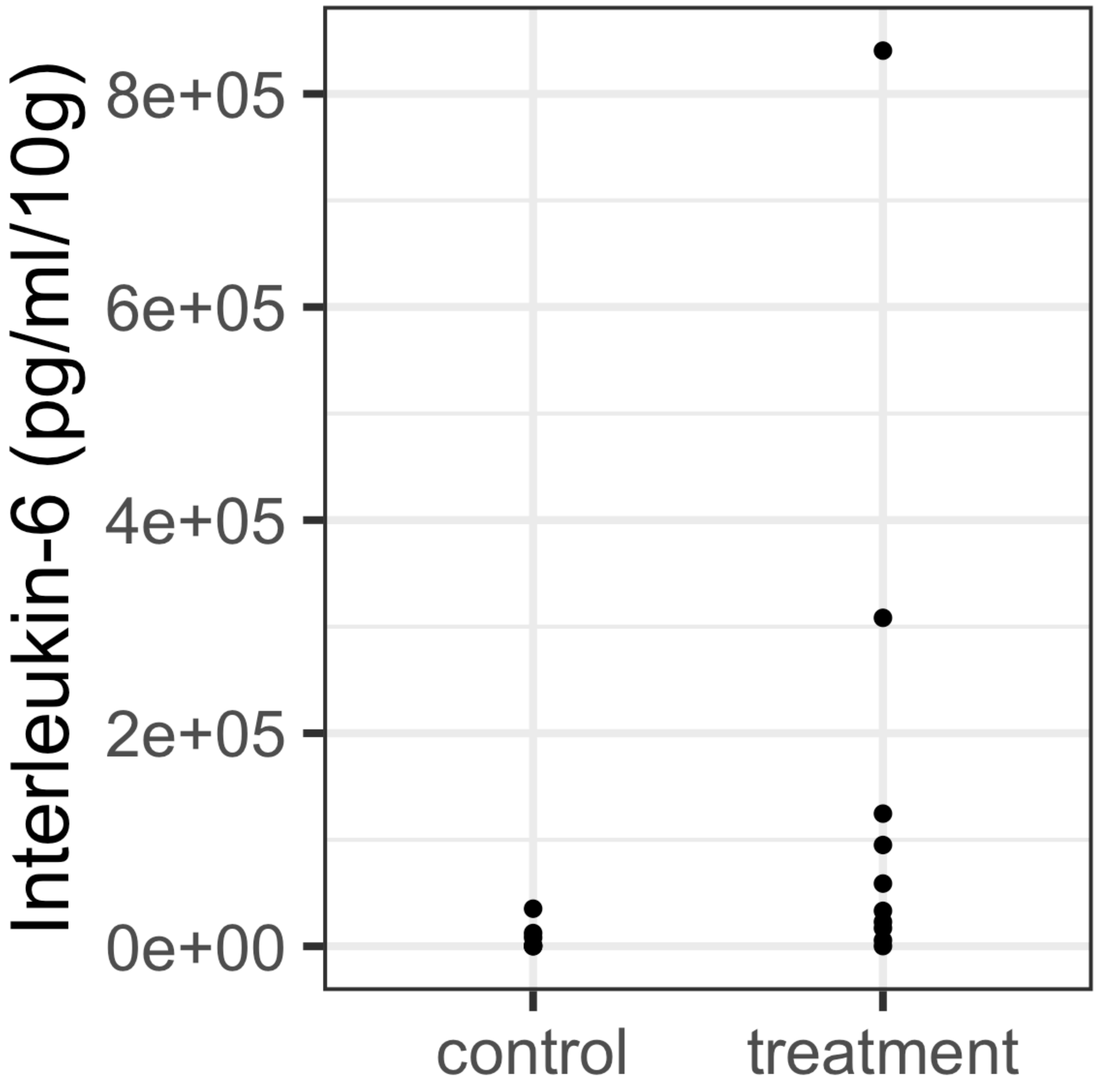
$$df = \frac{(0.668^2 + 0.982^2)^2}{0.668^4/(14 - 1) + 0.982^4(12 - 1)}$$

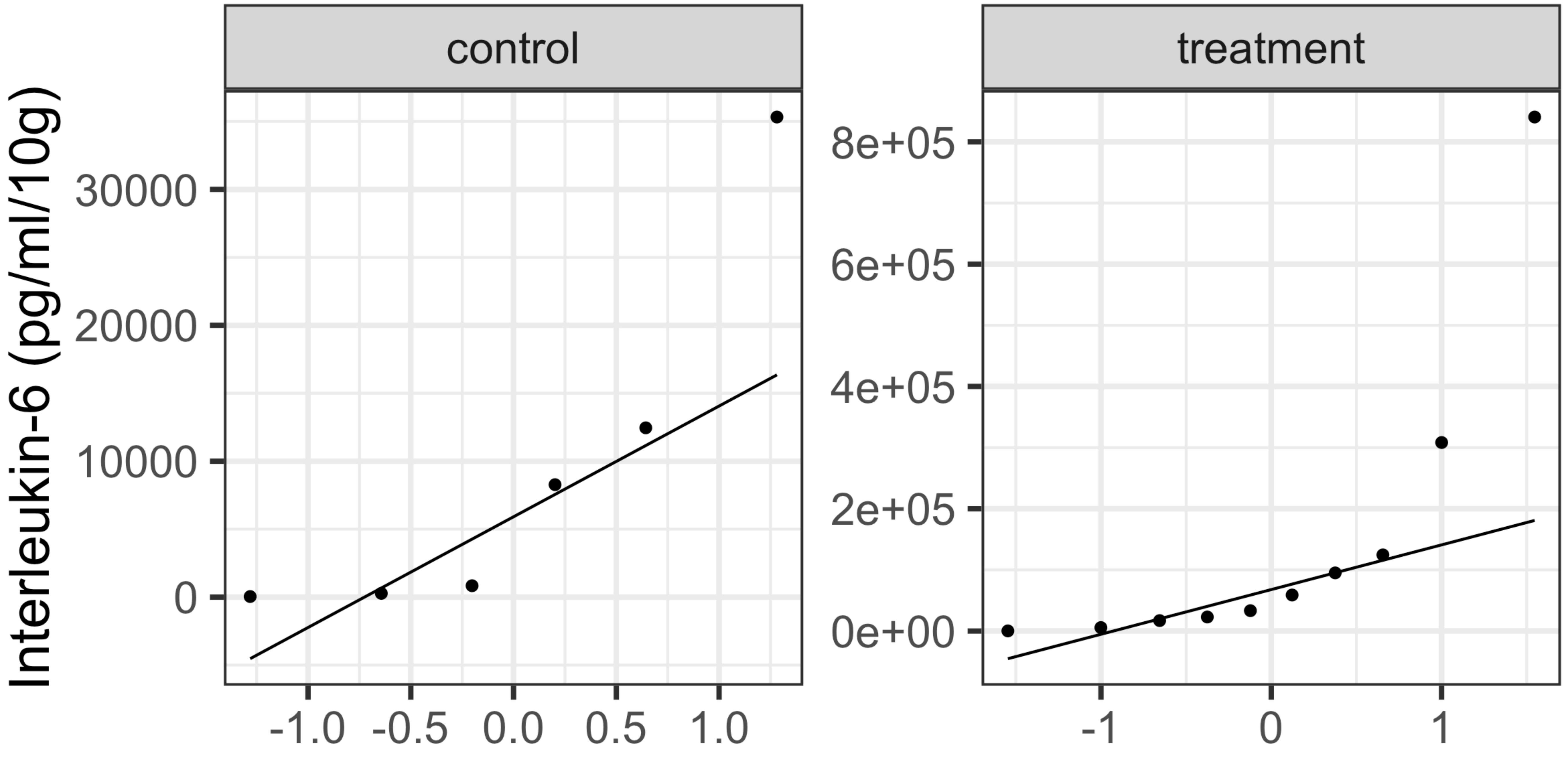
$df = 19.2$ **(Compared to 23)**

Assumptions for comparing populations

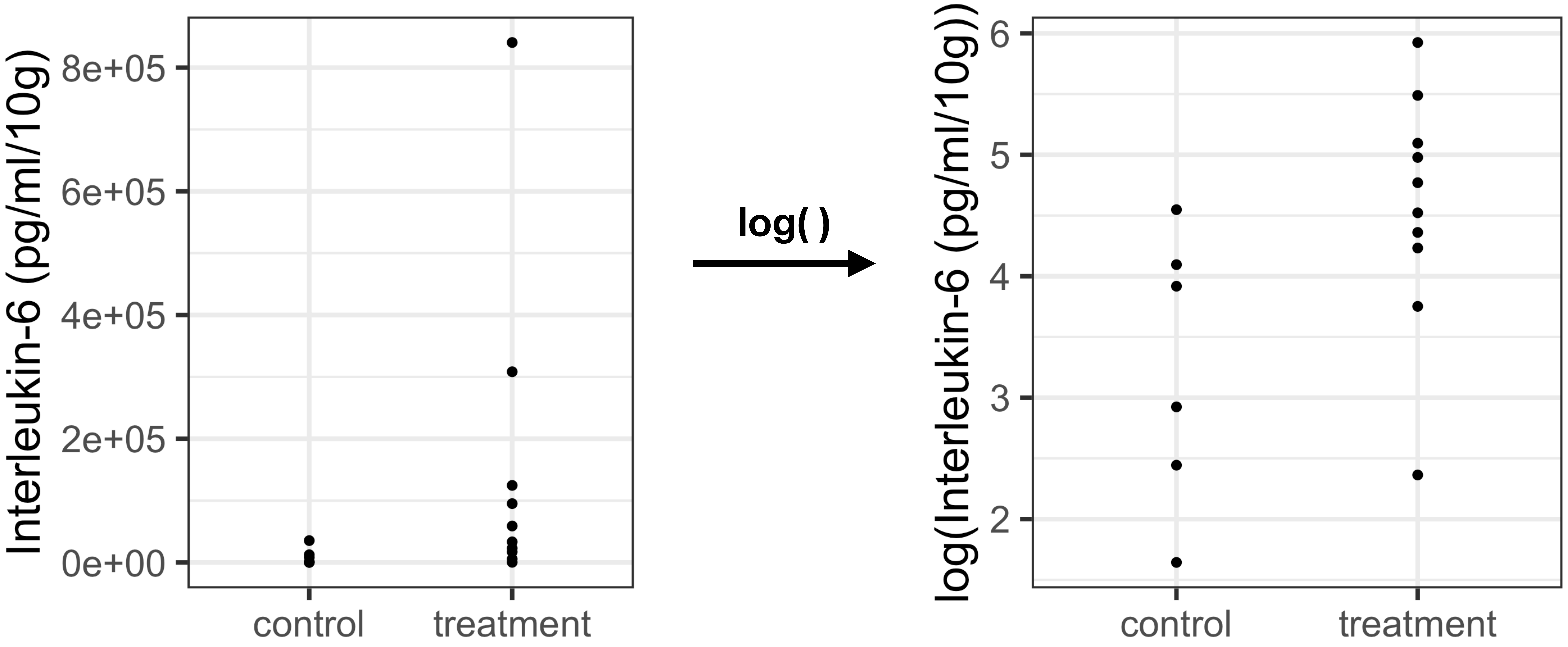
- **Conditions on the design of the study:**
  - (1) Data is a <u>random sample</u> from respective <u>large populations</u>
  - (2) The two samples must be independent of each other
- **Conditions on the form of the population distribution**
  - (3) If n is small, the population distribution must be ~normal
  - (4) If n is large, the population distribution doesn't have to be normal

Assumptions for comparing populations

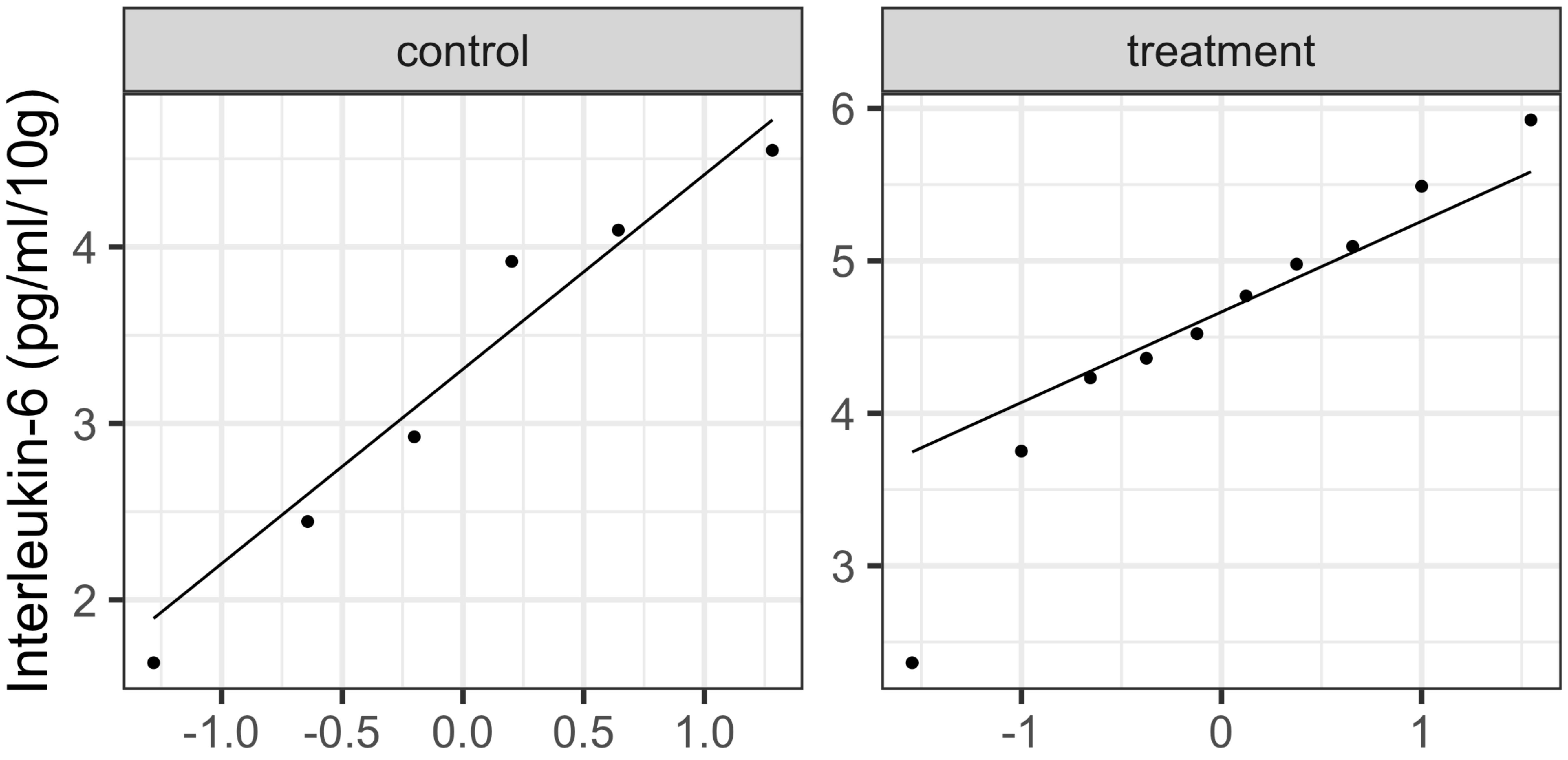


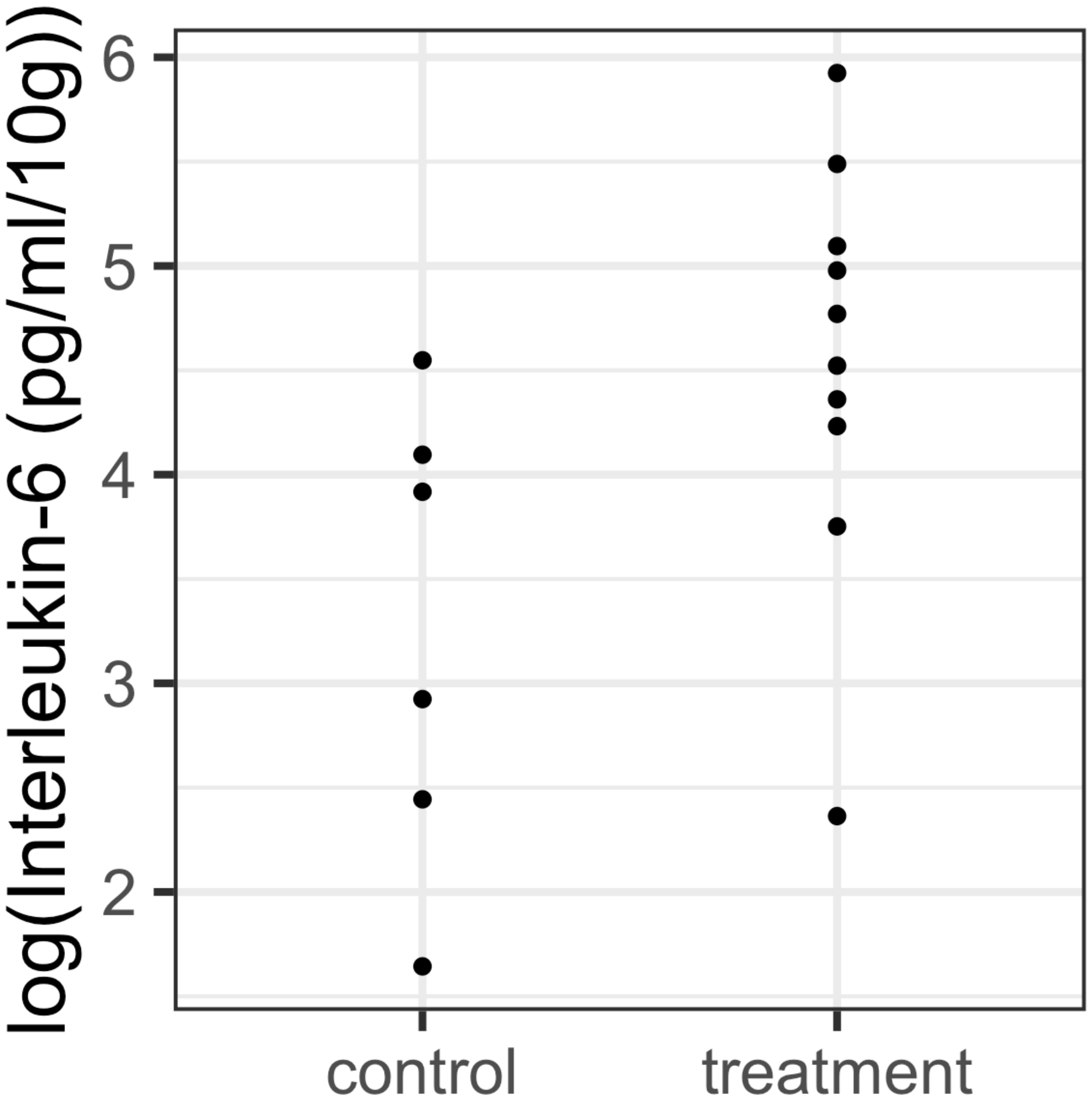


Assumptions for comparing populations



Assumptions for comparing populations





Assumptions for comparing populations

Non-transformed

```
        Welch Two Sample t-test

data:  il$control and il$treatment
t = -1.7179, df = 9.0826, p-value = 0.1196
alternative hypothesis: true difference in means
 is not equal to 0
95 percent confidence interval:
 -326709.82   44453.02
sample estimates:
mean of x mean of y
   9536.5  150664.9
```

Log-transformed

```
        Welch Two Sample t-test

data:  il$log_control and il$log_treatment
t = -2.3319, df = 9.6879, p-value = 0.04269
alternative hypothesis: true difference in means
 is not equal to 0
95 percent confidence interval:
 -2.52145436 -0.05188761
sample estimates:
mean of x mean of y
 3.262180  4.548851
```

The paired-sample design

You are performing a study to see how heart rate changes during exercise. You measure 20 individual's heart rates before exercise and directly after five minutes of vigorous cardiovascular exercise.

Independent samples?

Individual	**Pre**	**Post**
1	69	143
2	72	150
…	…	…
20	57	170

Observations = 40

Sample size, n = 20

The **paired-sample** design

You are performing a study to see how heart rate changes during exercise. You measure 20 individual's heart rates before exercise and directly after five minutes of vigorous cardiovascular exercise.

Independent samples?

Individual	Pre	Post	Difference (D)
1	69	143	143-69
2	72	150	150-72
...	...	...	...
20	57	170	170-57

Observations = 40

Sample size, n = 20

The **paired-sample** design

You are performing a study to see how heart rate changes during exercise. You measure 20 individual's heart rates before exercise and directly after five minutes of vigorous cardiovascular exercise.

Independent samples?

Use difference (D) as a single sample now

Observations = 40

Sample size, n = 20

Individual	Pre	Post	Difference (D)
1	69	143	74
2	72	150	78
…	…	…	…
20	57	170	113

Examples of paired-sample designs

- **Pre-test & post-test samples**: factor is measured before and after
- **Cross-over trials**: patients switch treatments half-way through a trial
- **Matched samples**: individuals are matched based on characteristics
- **Duplicate measurements**: technical replicates
- **Pairing by time**: observations made at the same time, month, etc.

Paired-sample designs aim to ***reduce bias*** *and* ***increase precision***

Paired-sample *t* test and confidence

Hypothesis testing for paired samples is done exactly the same as hypothesis testing for one sample as long as you use the difference sample distribution

$\bar{d} = 88.3$

$s_D = 5.4$

$SE_{\bar{D}} = 1.20$

$$SE_{\bar{D}} = \frac{s_D}{\sqrt{n_D}}$$

$$= \frac{5.4}{\sqrt{20}}$$

Paired-sample t test and confidence

Hypothesis testing for paired samples is done exactly the same as hypothesis testing for one sample as long as you use the difference sample distribution

$$\bar{d} = 88.3$$

$$s_D = 5.4$$

$$SE_{\bar{D}} = 1.20$$

$$\bar{d} \pm t_{0.025} SE_{\bar{D}}$$

$$(df = n_D - 1)$$

```
qt(0.975, df = 19)
```

$$88.3 \pm 2.09(1.20)$$

Paired-sample *t* test and confidence

Hypothesis testing for paired samples is done exactly the same as hypothesis testing for one sample as long as you use the difference sample distribution

$$\bar{d} = 88.3 \pm 2.5$$

$$s_D = 5.4$$

$$SE_{\bar{D}} = 1.20$$

$$\bar{d} \pm t_{0.025} SE_{\bar{D}}$$

$$(df = n_D - 1)$$

`qt(0.975, df = 19)`

$$88.3 \pm 2.09(1.20)$$

Paired-sample t test and confidence

Hypothesis testing for paired samples is done exactly the same as hypothesis testing for one sample as long as you use the difference sample distribution

$$\bar{d} = 88.3 \pm 2.5$$

$$s_D = 5.4$$

$$SE_{\bar{D}} = 1.20$$

$$t_s = 73.5$$

$$H_0 : \mu_{\bar{D}} = 0 \qquad H_A : \mu_{\bar{D}} \neq 0$$

$$t_s = \frac{\bar{d} - 0}{SE_{\bar{D}}}$$

$$t_s = \frac{88.3}{1.20}$$

Paired-sample *t* test and confidence

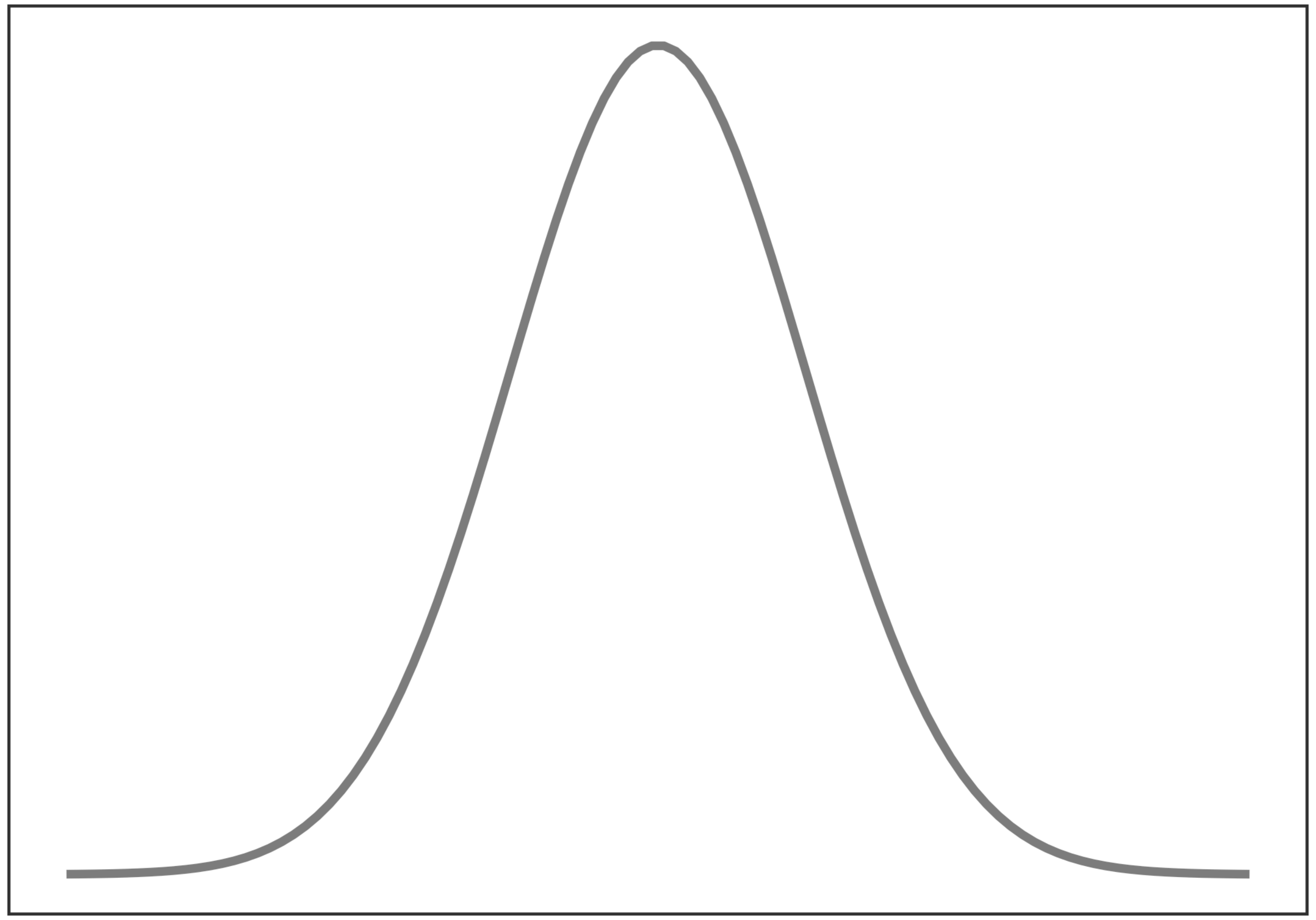
Hypothesis testing for paired samples is done exactly the same as hypothesis testing for one sample as long as you use the difference sample distribution

$\bar{d} = 88.3 \pm 2.5$

$s_D = 5.4$

$SE_{\bar{D}} = 1.20$

$t_s = 73.5$



`pt(73.5, df = 19) = 0`

$t_s = 73.5$

cAMP is a substance that can mediate cellular response to hormones. In a certain study, oocytes from four *Xenopus* females were divided into two batches: one batch was exposed to progesterone and the other was not. After 2 minutes, each batch was assayed for its cAMP content. Use a *t* test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

Frog	Control	Progesterone
1	6.01	5.23
2	2.28	1.21
3	1.51	1.40
4	2.12	1.38

cAMP is a substance that can mediate cellular response to hormones. In a certain study, oocytes from four *Xenopus* females were divided into two batches: one batch was exposed to progesterone and the other was not. After 2 minutes, each batch was assayed for its cAMP content. Use a *t* test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

Frog	Control	Progesterone
1	6.01	5.23
2	2.28	1.21
3	1.51	1.40
4	2.12	1.38

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
3	1.638	2.353	3.182	4.541	5.841	10.214	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.61
5	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.25	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587

cAMP is a substance that can mediate cellular response to hormones. In a certain study, oocytes from four *Xenopus* females were divided into two batches: one batch was exposed to progesterone and the other was not. After 2 minutes, each batch was assayed for its cAMP content. Use a *t* test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

Frog	Control	Prog.	Diff.
1	6.01	5.23	0.78
2	2.28	1.21	1.07
3	1.51	1.40	0.11
4	2.12	1.38	0.74
Mean	2.98	2.31	0.68
SD	2.05	1.95	0.40

Independent samples?

PAIRED *t* test!

1. **Generate a hypothesis and choose a significance level**

2. **Calculate the differences**

3. **Calculate test statistic**

4. **Calculate the *P*-value**

Use a *t* test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

$$\bar{d} = 0.68$$
$$s_d = 0.40$$
$$n_d = 4$$

1. Generate a hypothesis and choose a significance level

$$H_0 : \bar{d} = 0 \qquad H_A : \bar{d} \neq 0 \qquad \alpha = 0.10$$

2. Calculate the differences

3. Calculate test statistic

$$t_s = \frac{\bar{d}}{SE_{\bar{D}}} = \frac{0.68}{0.40/\sqrt{4}} = 3.4$$

4. Calculate the *P*-value

$$df = n_d - 1 = 3$$

Use a t test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

$$\bar{d} = 0.68$$

$$s_d = 0.40$$

$$n_d = 4$$

$$t_s = 3.4$$

$$df = 3$$

0.02 < P < 0.05

	P						
one-tail	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	0.2	0.1	0.05	0.02	0.01	0.002	0.001
DF							
1	3.078	6.314	12.706	31.821	63.656	318.289	636.578
2	1.886	2.92	4.303	6.965	9.925	22.328	31.6
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6	1.44	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.86	2.306	2.896	3.355	4.501	5.041

```
> pt(3.4, 3, lower.tail = F)*2
```

```
[1] 0.0424
```

REJECT the null hypothesis

Use a *t* test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

```r
# create data table
df <- data.frame(control = c(6.01, 2.28, 1.51, 2.12),
                 progesterone = c(5.23, 1.21, 1.40, 1.38)) %>%
    dplyr::mutate(diff = control - progesterone)

# calculate t test with t.test
t.test(df$control, df$progesterone, paired = T)
```

```
        Paired t-test

data:  df$control and df$progesterone
t = 3.3387, df = 3, p-value = 0.04443
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.0315868 1.3184132
sample estimates:
mean of the differences
                  0.675
```

$t = 3.4$

$P = 0.0424$

Use a *t* test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

```r
# create data table
df <- data.frame(control = c(6.01, 2.28, 1.51, 2.12),
                 progesterone = c(5.23, 1.21, 1.40, 1.38)) %>%
    dplyr::mutate(diff = control - progesterone)

# calculate t test with t.test
t.test(df$diff)
```

```
        One Sample t-test

data:  df$diff
t = 3.3387, df = 3, p-value = 0.04443
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.0315868 1.3184132
sample estimates:
mean of x
    0.675
```

t = 3.4

P = 0.0424

Use a *t* test to investigate the effect of progesterone on cAMP. Let H_A be nondirectional and let $\alpha = 0.10$

```r
# create data table
df <- data.frame(control = c(6.01, 2.28, 1.51, 2.12),
                 progesterone = c(5.23, 1.21, 1.40, 1.38)) %>%
    dplyr::mutate(diff = control - progesterone)

# calculate t test with t.test
t.test(df$control, df$progesterone, mu = 1, paired = T)
```

$$t_s = \frac{\bar{d} - 1}{SE_{\bar{D}}}$$

$$= -1.6 \checkmark$$

```
        Paired t-test

data:  df$control and df$progesterone
t = -1.6075, df = 3, p-value = 0.2063
alternative hypothesis: true difference in means is not equal to 1
95 percent confidence interval:
 0.0315868 1.3184132
sample estimates:
mean of the differences
                  0.675
```

Assumptions for paired-sample analysis

- **Conditions on the design of the study:**
  - The *differences* (D's) must be regarded as a random sample from some large population
- **Conditions on the form of the population distribution**
  - The population distribution of the D's must be normal (or sample size must be large ~ approx. normal)