Test statistic:

$$\chi_s^2 = \sum_{\text{all cells}} \frac{(o_i - e_i)^2}{e_i}$$

Null distribution (approximate):

χ^2 distribution with df $= (r - 1)(k - 1)$

where r is the number of rows and k is the number of columns in the contingency table. This approximation is adequate if $e_i \geq 5$ for every cell. If r and k are large, the condition that $e_i \geq 5$ is less critical and the χ^2 approximation is adequate if the average expected frequency is at least 5, and no expected frequency is less than 1.

The observations must be independent of one another. If paired data are collected for a 2×2 table, then McNemar's test is appropriate (Section 10.8).

Supplementary Exercises 10.S.1–10.S.21

(Note: Exercises preceded by an asterisk refer to optional sections.)

10.S.1 In the Women's Health Initiative Dietary Modification Trial women were randomly assigned to an intervention or a control group. The intervention included counseling sessions designed to reduce fat intake and to increase consumption of fruits and vegetables. Over 6 years data were collected on coronary heart disease (CHD); results are shown in the table.[64] Do the data provide evidence that the intervention makes a difference? The value of the chi-square statistic for this contingency table is $\chi_s^2 = 0.69$.

		Group		
		Intervention	Control	
CHD?	Yes	1,000	1,549	2,549
	No	18,541	27,745	46,286
	Total	19,541	29,294	48,835

(a) State the null and alternative hypotheses in symbols.

(b) The (nondirectional) P-value for the test is 0.407. If $\alpha = 0.10$, what is your conclusion regarding the hypotheses in (a)?

10.S.2 Use the data from Exercise 10.S.1 to construct a 95% confidence interval for (Pr{CHD | intervention} − Pr{CHD | control}).

10.S.3 As part of a study of environmental influences on sex determination in the fish *Menidia,* eggs from a single mating were divided into two groups and raised in either a warm or a cold environment. It was found that 73 of 141 offspring in the warm environment and 107 of 169 offspring in the cold environment were females.[65] In each of the following chi-square tests, use a nondirectional alternative and let $\alpha = 0.05$.

(a) Test the hypothesis that the population sex ratio is 1:1 in the warm environment.

(b) Test the hypothesis that the population sex ratio is 1:1 in the cold environment.

(c) Test the hypothesis that the population sex ratio is the same in the warm as in the cold environment.

(d) Define the population to which the conclusions reached in parts (a)–(c) apply. (Is it the entire genus *Menidia?*)

10.S.4 The cilia are hairlike structures that line the nose and help to protect the respiratory tract from dust and foreign particles. A medical team obtained specimens of nasal tissue from nursery school children who had viral upper respiratory infections, and also from healthy children in the same classroom. The tissue was sectioned, and the cilia were examined with a microscope for specific defects, with the results shown in the accompanying table.[66] The data show that the percentage of defective cilia was much higher in the tissue from infected children (15.7% versus 3.1%). Would it be valid to apply a chi-square test to compare these percentages? If so, do it. If not, explain why not.

	Number of children	Total number of cilia counted	Cilia with defects	
			Number	Percent
Control	7	556	17	3.1
Respiratory infection	22	1,493	235	15.7

10.S.5 A group of mountain climbers participated in a trial to investigate the usefulness of the drug acetazolamide in preventing altitude sickness. The climbers were randomly assigned to receive either drug or placebo during an ascent of Mt. Rainier. The experiment was supposed to be double-blind, but the question arose whether some of the climbers might have received clues (perhaps from the presence or absence of side effects or from a perceived therapeutic effect or lack of it) as to which treatment they were receiving. To investigate this possibility, the climbers were asked (after the trial was over) to guess which treatment they had received.[67] The results can be cast in the following contingency table, for which $\chi_s^2 = 5.07$:

		Treatment received	
		Drug	Placebo
Guess	Correct	20	12
	Incorrect	11	21

Alternatively the same results can be rearranged in the following contingency table, for which $\chi_s^2 = 0.01$:

		Treatment received	
		Drug	Placebo
Guess	Drug	20	21
	Placebo	11	12

Consider the null hypothesis

H_0: The blinding was perfect (the climbers received no clues).

Carry out the chi-square test of H_0 against the alternative that the climbers did receive clues. Let $\alpha = 0.05$. (You must decide which contingency table is relevant to this question.) (*Hint*: To clarify the issue for yourself, try inventing a fictitious data set in which most of the climbers *have* received strong clues, so we would expect a large value of χ_s^2; then arrange your fictitious data in each of the two contingency table formats and note which table would yield a larger value of χ_s^2.)

***10.S.6** Desert lizards *(Dipsosaurus dorsalis)* regulate their body temperature by basking in the sun or moving into the shade, as required. Normally the lizards will maintain a daytime temperature of about 38°C. When they are sick, however, they maintain a temperature about 2° to 4° higher—that is, a "fever." In an experiment to see whether this fever might be beneficial, lizards were given a bacterial infection; then 36 of the animals were prevented from developing a fever by keeping them in a 38° enclosure, while 12 animals were kept at a temperature of 40°. The following table describes the mortality after 24 hours.[68] How strongly do these results support the hypothesis that fever has survival value? Use Fisher's exact test against a directional alternative. Let $\alpha = 0.05$.

	38°	40°
Died	18	2
Survived	18	10
Total	36	12

10.S.7 Consider the data from Exercise 10.S.6. Analyze these data with a chi-square test. Let $\alpha = 0.05$.

10.S.8 In a randomized clinical trial, 154 women with breast cancer were assigned to receive chemotherapy. Another 164 women were assigned to receive chemotherapy combined with radiation therapy. Survival data after 15 years are given in the following table.[69] Use these data to conduct a test of the null hypothesis that type of treatment does not affect survival rate.

	Chemotherapy only	Chemotherapy and radiation therapy
Died	78	66
Survived	76	98
Total	154	164

Here is computer output for a chi-square test.

```
X-squared = 3.47, df = 1,  p-value = 0.0625
```

(a) Compute the sample proportions.

(b) Compute the expected frequencies.

(c) If $\alpha = 0.05$, what is your conclusion regarding H_0?

(d) Suppose a directional alternative that the addition of radiation therapy enhances survival had been used. In this case what would be your conclusion regarding H_0?

***10.S.9** Refer to the data in Exercise 10.S.8.

(a) Calculate the sample odds ratio.

(b) Find a 95% confidence interval for the population value of the odds ratio.

10.S.10 Two drugs, zidovudine and didanosine, were tested for their effectiveness in preventing progression of HIV disease in children. In a double-blind clinical trial, 276 children with HIV were given zidovudine, 281 were given didanosine, and 274 were given zidovudine plus didanosine. The following table shows the survival data for the three groups.[70] Use these data to conduct a test of the null hypothesis that survival and treatment are independent. Let $\alpha = 0.10$.

	Zidovudine	Didanosine	Zidovudine and didanosine
Died	17	7	10
Survived	259	274	264
Total	276	281	274

10.S.11 The blood types of malaria patients at a clinic in India were compared with those obtained in a sample of

visitors to a nearby hospital. The data are shown in the following table.[71] Use these data to conduct a test of the null hypothesis that blood type is independent of contracting malaria.

	A	B	O	AB	Total
Malaria cases	138	199	106	33	476
Controls	229	535	428	96	1,288

Here is computer output for a chi-square test.

```
X-squared = 34.929,  p-value = 1.26e-07
```

(a) If the null hypothesis were true, how many malaria cases would we expect to see in individuals with type A blood in similar studies of this size?

(b) How many degrees of freedom are there?

(c) If $\alpha = 0.05$, what is your conclusion regarding H_0?

10.S.12 The habitat selection behavior of the fruitfly *Drosophila subobscura* was studied by capturing flies from two different habitat sites. The flies were marked with colored fluorescent dust to indicate the site of capture and then released at a point midway between the original sites. On the following 2 days, flies were recaptured at the two sites. The results are summarized in the table.[72] The value of the chi-square statistic for this contingency table is $\chi^2_s = 10.44$. Test the null hypothesis of independence against the alternative that the flies preferentially tend to return to their site of capture. Let $\alpha = 0.01$.

		Site of recapture	
		I	II
Site of original capture	I	78	56
	II	33	58

10.S.13 In the garden pea *Pisum sativum,* seed color can be yellow (Y) or green (G), and seed shape can be round (R) or wrinkled (W). Consider the following three hypotheses describing a population of plants:

$$H_0^{(1)}: \Pr\{\text{Y}\} = \frac{3}{4}$$

$$H_0^{(2)}: \Pr\{\text{R}\} = \frac{3}{4}$$

$$H_0^{(3)}: \Pr\{\text{R}|\text{Y}\} = \Pr\{\text{R}|\text{G}\}$$

The first hypothesis asserts that yellow and green plants occur in a 3:1 ratio; the second hypothesis asserts that round and wrinkled plants occur in a 3:1 ratio, and the third hypothesis asserts that color and shape are independent. (In fact, for a population of plants produced by a certain cross—the dihybrid cross—all three hypotheses are known to be true.)

Suppose a random sample of 1,600 plants is to be observed, with the data to be arranged in the following contingency table:

		Color	
		Y	G
Shape	R		
	W		
			1,600

Invent fictitious data sets as specified, and verify each answer by calculating the estimated conditional probabilities. (*Hint*: In each case, begin with the marginal frequencies.)

(a) A data set that agrees perfectly with $H_0^{(1)}$, $H_0^{(2)}$, and $H_0^{(3)}$.

(b) A data set that agrees perfectly with $H_0^{(1)}$ and $H_0^{(2)}$ but not with $H_0^{(3)}$.

(c) A data set that agrees perfectly with $H_0^{(3)}$ but not with $H_0^{(1)}$ or $H_0^{(2)}$.

***10.S.14** A study of 36,080 persons who had heart attacks found that men were more likely to survive than were women. The following table shows some of the data collected in the study.[73]

		Men	Women
Survived at least 24 hours?	Yes	25,339	8,914
	No	1,141	686
	Total	26,480	9,600

(a) Calculate the odds ratio for comparing survival of men to survival of women.

(b) Calculate a 95% confidence interval for the population value of the odds ratio.

(c) Does the odds ratio give a good approximation to the relative risk for these data? Why or why not?

***10.S.15** In the study described in Exercise 10.9.6, one of the variables measured was whether the subjects had used any products containing phenylpropanolamine. The odds ratio was calculated to be 1.49, with stroke victims more likely than the control subjects to have used a product containing phenylpropanolamine.[61] A 95% confidence interval for the population value of the odds ratio is (0.84, 2.64). Interpret this confidence interval in the context of this setting.

10.S.16 (Computer exercise) In a study of the effects of smoking cigarettes during pregnancy, researchers examined the placenta from each of 58 women after childbirth. They noted the presence or absence (P or A) of a particular placental abnormality—atrophied villi. In addition, each woman was categorized as a nonsmoker (N), moderate smoker (M), or heavy smoker (H). The following table shows, for each woman, an ID number (#) and the results for smoking (S) and atrophied villi (V).[74]

#	S	V	#	S	V	#	S	V	#	S	V
1	N	A	16	H	P	31	M	A	46	M	A
2	M	A	17	H	P	32	M	A	47	H	P
3	N	A	18	N	A	33	N	A	48	H	P
4	M	A	19	M	P	34	N	A	49	H	A
5	M	A	20	N	P	35	N	A	50	N	P
6	M	P	21	M	A	36	H	P	51	N	A
7	H	P	22	H	A	37	N	A	52	M	P
8	N	A	23	M	P	38	H	P	53	M	A
9	N	A	24	N	A	39	H	P	54	H	P
10	M	P	25	N	P	40	N	A	55	H	A
11	N	A	26	N	A	41	M	A	56	M	P
12	N	P	27	N	A	42	N	A	57	H	P
13	H	P	28	M	P	43	H	A	58	H	P
14	M	A	29	N	A	44	M	A			
15	M	P	30	N	A	45	M	P			

(a) Test for a relationship between smoking status and atrophied villi. Use a chi-square test at $\alpha = 0.05$.

(b) Prepare a table that shows the total number of women in each smoking category, and the number and percentage in each category who had atrophied villi.

(c) What pattern appears in the table of part (b) that is not used by the test of part (a)?

***10.S.17** Researchers studied the cellular telephone records of 699 persons who had automobile accidents. They determined that 170 of the 699 had made a cellular telephone call during the 10-minute period prior to their accident; this period is called the hazard interval. There were 37 persons who had made a call during a corresponding 10-minute period on the day before their accident; this period is called the control interval. Finally, there were 13 who made calls both during the hazard interval and the control interval.[75] Do these data indicate that use of a cellular telephone is associated with an increase in accident rate? Analyze these data using McNemar's test. Use a directional alternative and let $\alpha = 0.01$.

		Call during control interval?	
		Yes	No
Call during hazard interval?	Yes	13	157
	No	24	505

10.S.18 Prior to an influenza season subjects were randomly assigned to receive either a flu vaccine or a placebo. During that season there were 28 cases of the flu among 813 vaccine recipients and 35 cases of the flu among the 325 subjects who were given the placebo.[76] Do these data indicate that the vaccine was effective? Conduct an appropriate test using a directional alternative with $\alpha = 0.05$.

***10.S.19** Refer to the data in Exercise 10.S.18.

(a) Calculate the sample odds ratio.

(b) Find a 95% confidence interval for the population value of the odds ratio.

10.S.20 Consider Exercise 9.S.18. The romantic partners of the 36 men discussed in Exercise 9.S.18 were also tested, in the same manner as the men (i.e., they were blindfolded and asked to identify their partner by touching the backs of the hands of three men, one of whom was their partner). Among the women, 25 were successful and 11 were not. Are these data significant evidence for the hypothesis that men and women differ in their ability to identify their partners?

Consider an appropriate test that uses a nondirectional alternative.

Here is computer output for a chi-square test.

```
X-squared = 4.59, df = 1, p-value = 0.03219
```

(a) State the null and appropriate directional alternative hypotheses in context.

(b) Compute the sample proportions and the expected frequencies.

(c) If $\alpha = 0.05$, what is your conclusion regarding H_0?

10.S.21 A researcher found that 54 out of 66 randomly selected trees in an arboretum in northern Ohio were native species.[77] This compared to 42 out of 72 randomly chosen trees in "managed" land in the same general area. Is there significant evidence of a difference in proportions? If so, how strong is that evidence?