

THE BEST THESIS DEFENSE IS A GOOD THESIS OFFENSE.

Final Review

12.02.21

Multiple regression

Call:

```
lm(formula = mpg ~ hp, data = mtcars)
```

$$\text{MPG} = 30.09 - 0.06(\text{HP})$$

Residuals:

Min	1Q	Median	3Q	Max
-5.7121	-2.1122	-0.8854	1.5819	8.2360

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	30.09886	1.63392	18.421	< 2e-16 ***
hp	-0.06823	0.01012	-6.742	1.79e-07 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.863 on 30 degrees of freedom

Multiple R-squared: 0.6024, Adjusted R-squared: 0.5892

F-statistic: 45.46 on 1 and 30 DF, p-value: 1.788e-07

Multiple regression

Call:

```
lm(formula = mpg ~ trans + hp, data = mtcars)
```

Residuals:

$$\text{MPG} = 26.58 - 0.058(\text{HP}) + 5.27(\text{trans})$$

	Min	1Q	Median	3Q	Max
	-4.3843	-2.2642	0.1366	1.6968	5.8657

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	26.584914	1.425094	18.655	< 2e-16	***
transstick	5.277085	1.079541	4.888	3.46e-05	***
hp	-0.058888	0.007857	-7.495	2.92e-08	***

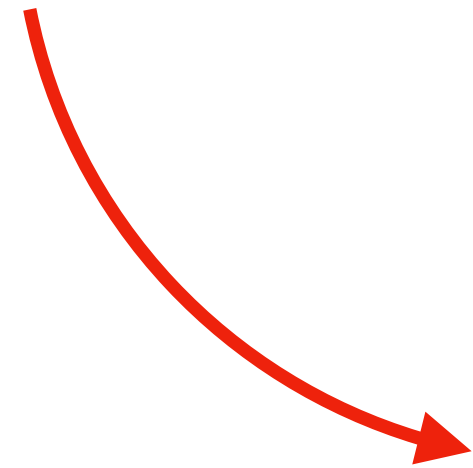
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.909 on 29 degrees of freedom

Multiple R-squared: 0.782, Adjusted R-squared: 0.767

F-statistic: 52.02 on 2 and 29 DF, p-value: 2.55e-10

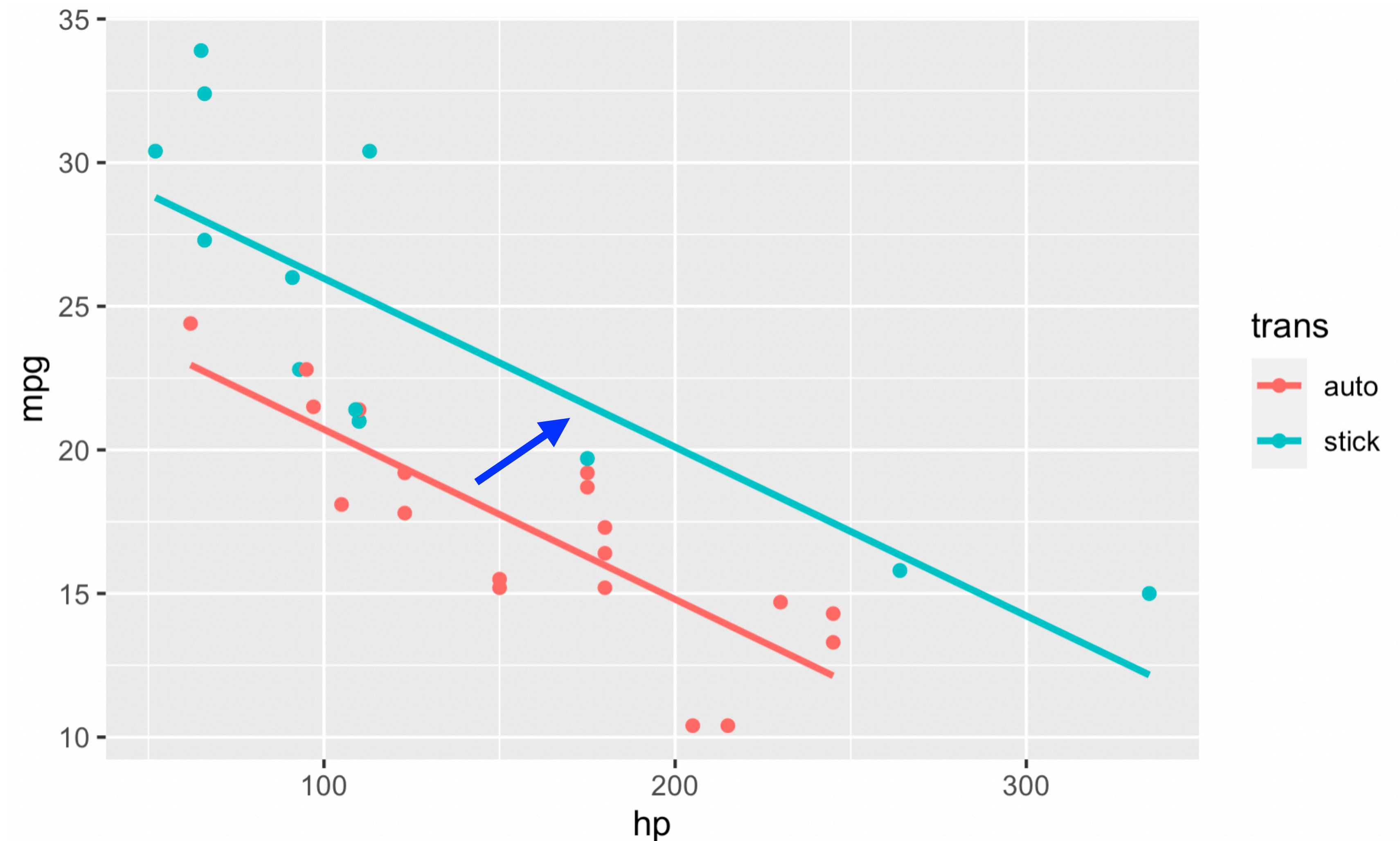
Default is "auto"



Multiple regression

$$\text{MPG} = 26.58 - 0.058(\text{HP}) + 5.27(\text{trans})$$

Default is “auto”



Multiple regression

Call:

```
lm(formula = mpg ~ hp + factor(gear), data = mtcars)
```

Residuals:

Min	1Q	Median	3Q	Max
-4.4937	-2.3586	-0.8277	2.2753	7.7287

```
> unique(mtcars$gear)
```

```
[1] 3 4 5
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	27.88193	2.10908	13.220	1.47e-13	***
hp	-0.06685	0.01105	-6.052	1.59e-06	***
factor(gear)4	2.63486	1.55164	1.698	0.100575	
factor(gear)5	6.57476	1.64268	4.002	0.000417	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.154 on 28 degrees of freedom

Multiple R-squared: 0.7527, Adjusted R-squared: 0.7262

F-statistic: 28.41 on 3 and 28 DF, p-value: 1.217e-08

Multiple regression

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**MPG = 27.88 - 0.066(HP) +
2.63(gear4) + 6.57(gear5)**

(Default is gear3)

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Gear4 and gear5 = binary (0 or 1)

Multiple regression

Call:

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lm(formula = mpg ~ hp + factor(gear), data = mtcars)
```

Residuals:

Min	1Q	Median	3Q	Max
-4.4937	-2.3586	-0.8277	2.2753	7.7287

$$\text{MPG} = 27.88 - 0.066(\text{HP}) + 2.63(\text{gear4}) + 6.57(\text{gear5})$$

$$\text{HP} = 10, \text{gear} = 3$$

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	27.88193	2.10908	13.220	1.47e-15
hp	-0.06685	0.01105	-6.052	1.59e-06 ***
factor(gear)4	2.63486	1.55164	1.698	0.100575
factor(gear)5	6.57476	1.64268	4.002	0.000417 ***

$$\text{MPG} = 27.88 - 0.066(10) + 2.63(0) + 6.57(0)$$

`predict(model, data.frame(hp = 10, gear = 3))`

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 $\text{HP} = 10, \text{gear} = 4$

$$\text{MPG} = 27.88 - 0.066(10) + 2.63(1) + 6.57(0)$$

Residual standard error: 3.154 on 28 degrees of freedom

Multiple R-squared: 0.7527, Adjusted R-squared: `predict(model, data.frame(hp = 10, gear = 4))`

F-statistic: 28.41 on 3 and 28 DF, p-value: 1.217e-08

Multiple regression

```
> mtcars$gear <- factor(mtcars$gear, levels = c(4,3,5)))
```

```
lm(formula = mpg ~ hp + gear, data = mtcars)
```

Residuals:

Min	1Q	Median	3Q	Max
-4.4937	-2.3586	-0.8277	2.2753	7.7287

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	30.51679	1.34393	22.707	< 2e-16	***
hp	-0.06685	0.01105	-6.052	1.59e-06	***
gear3	-2.63486	1.55164	-1.698	0.1006	
gear5	3.93990	2.04730	1.924	0.0645	.

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Multiple regression

These are different because they are compared to a different starting point...

Coefficients:

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(Intercept)	30.51679	1.34393	22.707	< 2e-16 ***
hp	-0.06685	0.01105	-6.052	1.59e-06 ***
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These are the same because it is still the same model...

Hypothesis testing + p -values

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Residuals:

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Coefficients:

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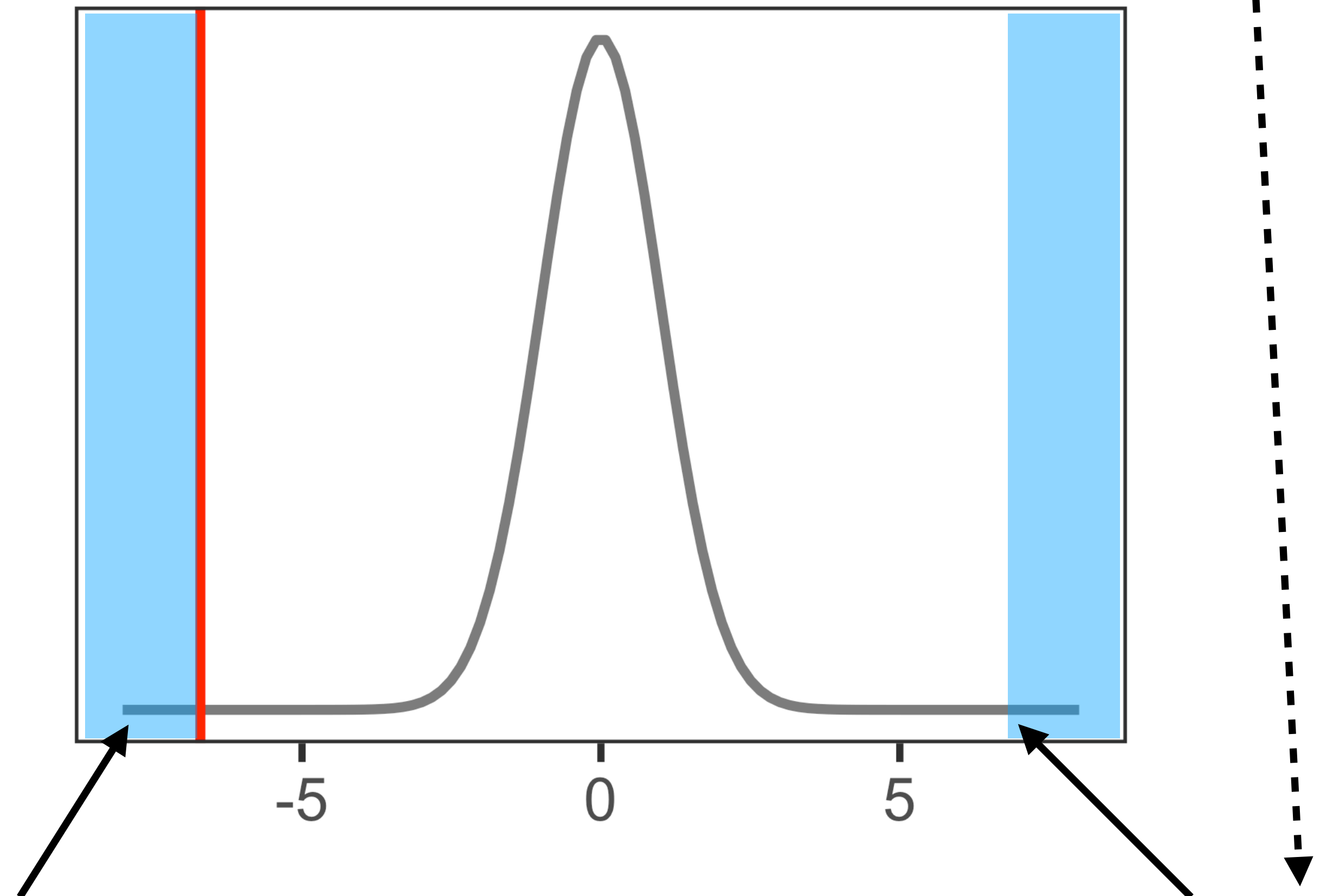
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Residual standard error: 3.863 on 30 degrees of freedom

Multiple R-squared: 0.6024, Adjusted R-squared: 0.5892

F-statistic: 45.46 on 1 and 30 DF, p-value: 1.788e-07

$\text{pt}(6.742, 30, \text{lower.tail} = \text{F})$



$\text{pt}(-6.742, 30, \text{lower.tail} = \text{T})$ -----► $8.95\text{e-}08$

+

$8.95\text{e-}08$

Directional hypothesis

If I only care if the HP has a NEGATIVE regression coefficient...

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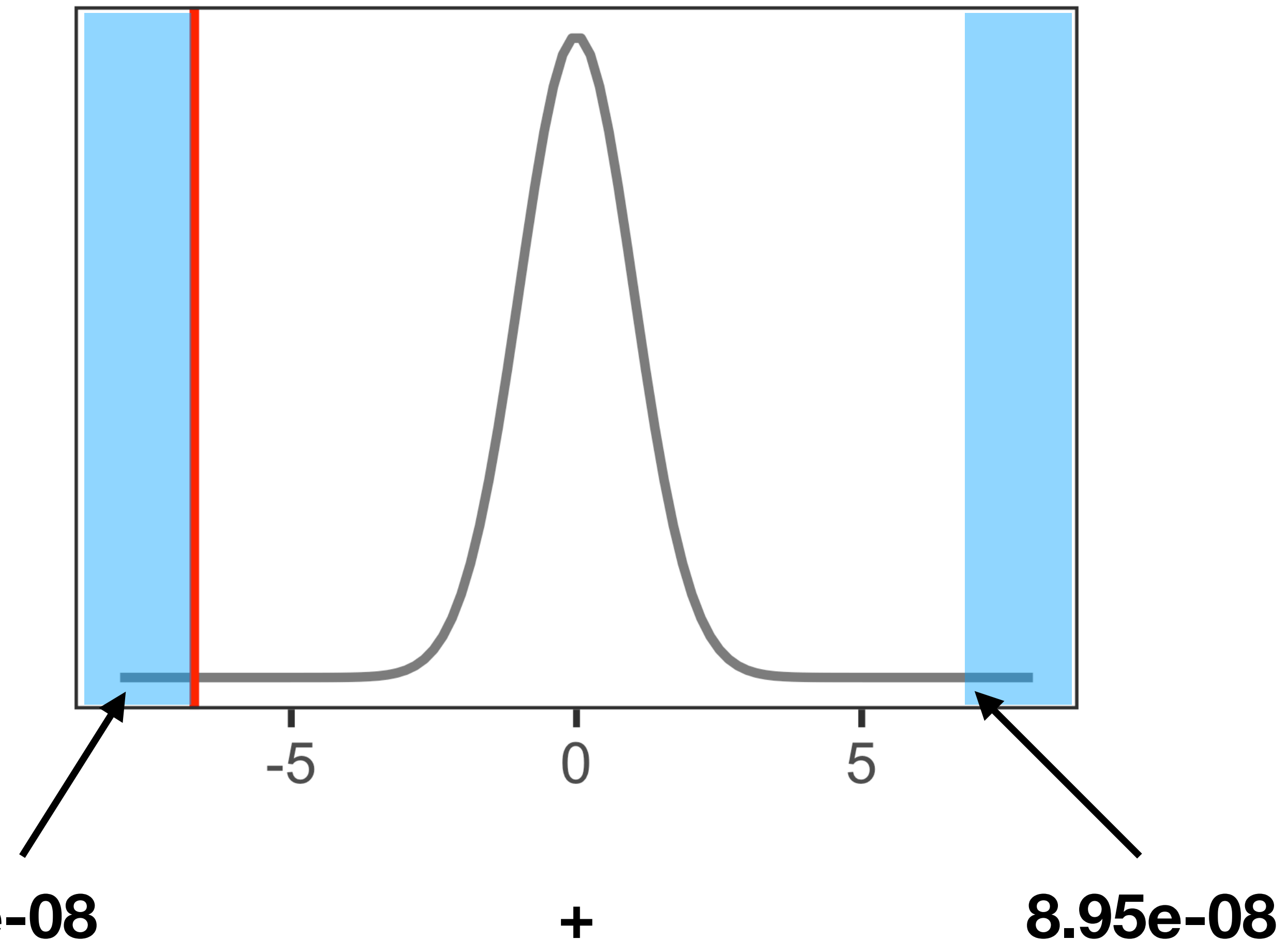
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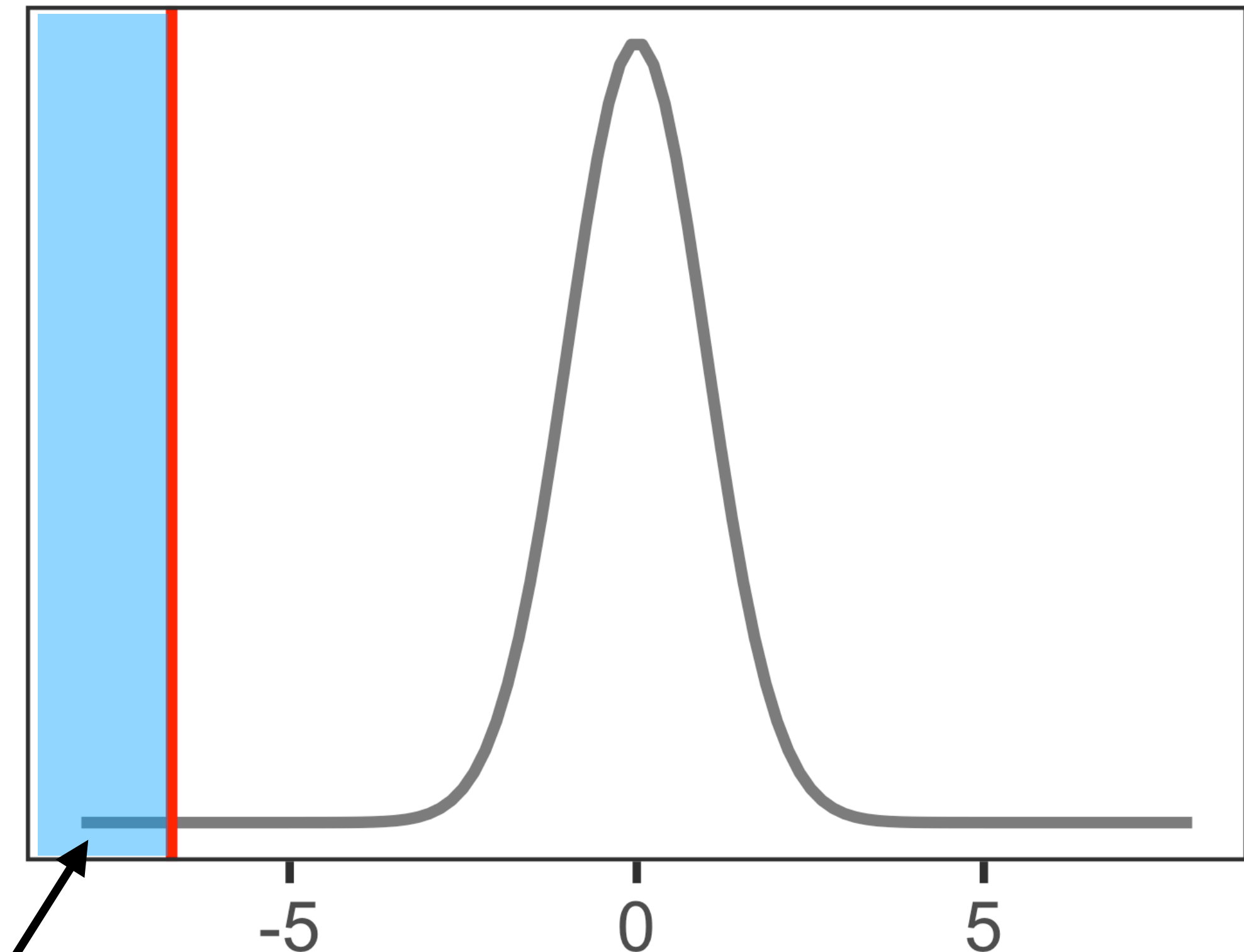
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`pt(-6.742, 30, lower.tail = T)` -----> **8.95e-08** = $1.79e-07 / 2$

Directional hypothesis

If I only care if the HP has a POSITIVE regression coefficient...

Call:

```
lm(formula = mpg ~ hp, data = mtcars)
```

Residuals:

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-5.7121	-2.1122	-0.8854	1.5819	8.2360

Coefficients:

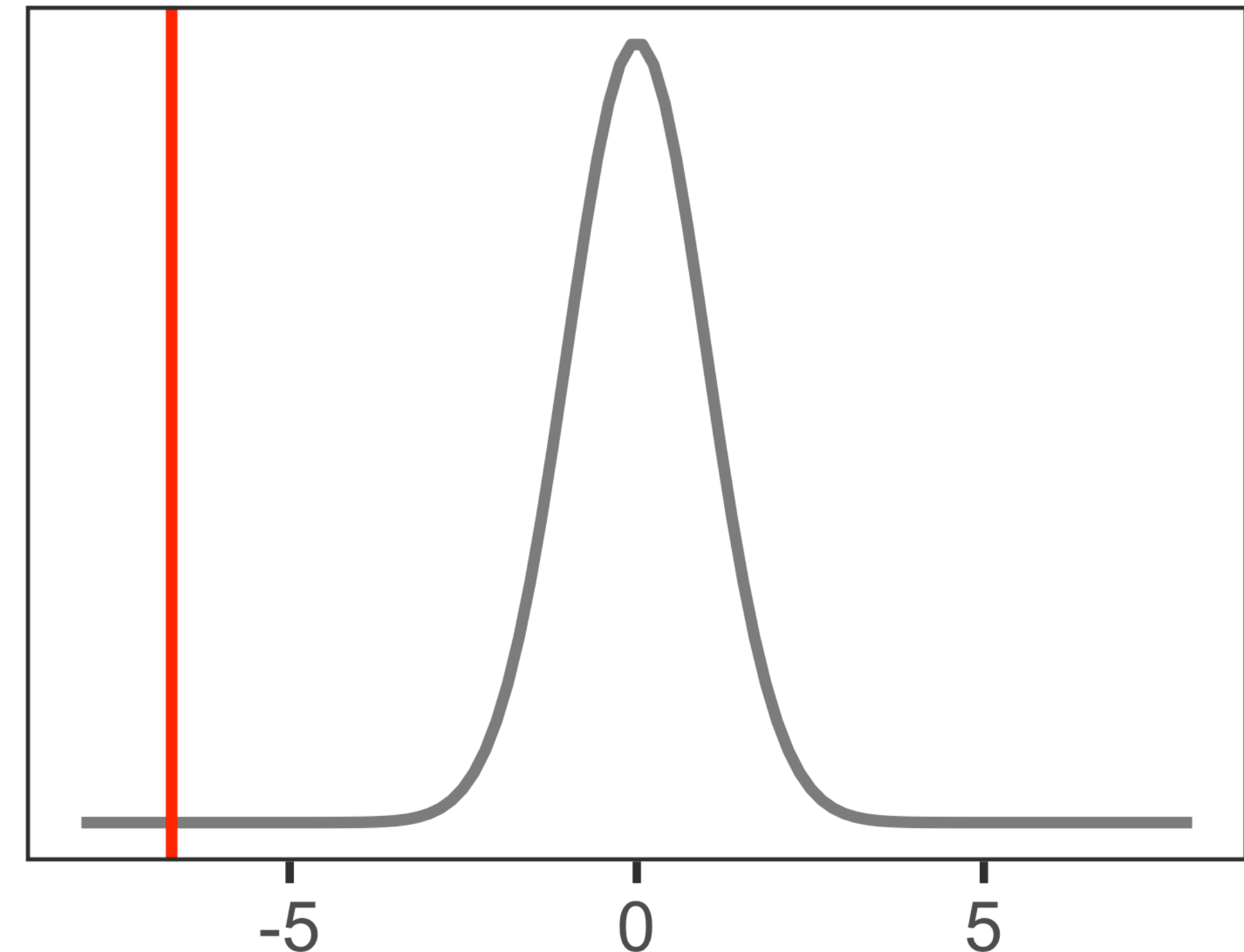
	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	30.09886	1.63392	18.421	< 2e-16 ***
hp	-0.06823	0.01012	-6.742	1.79e-07 ***

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It goes in the wrong direction, so p-value = 0.5 (i.e. not significant)