Linear Regression

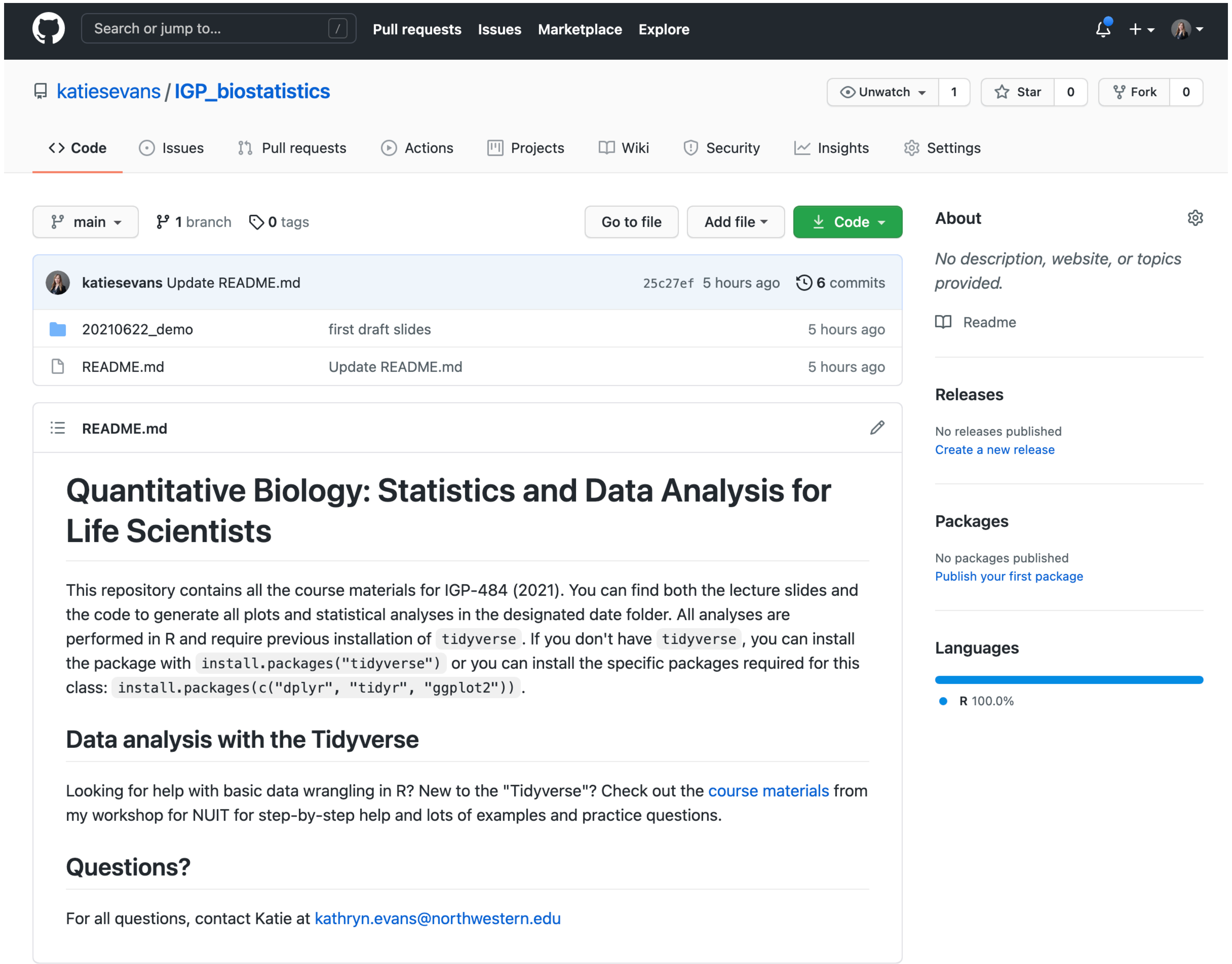
IGP Biostatistics Teaching Demo

Katie Evans

6.22.21

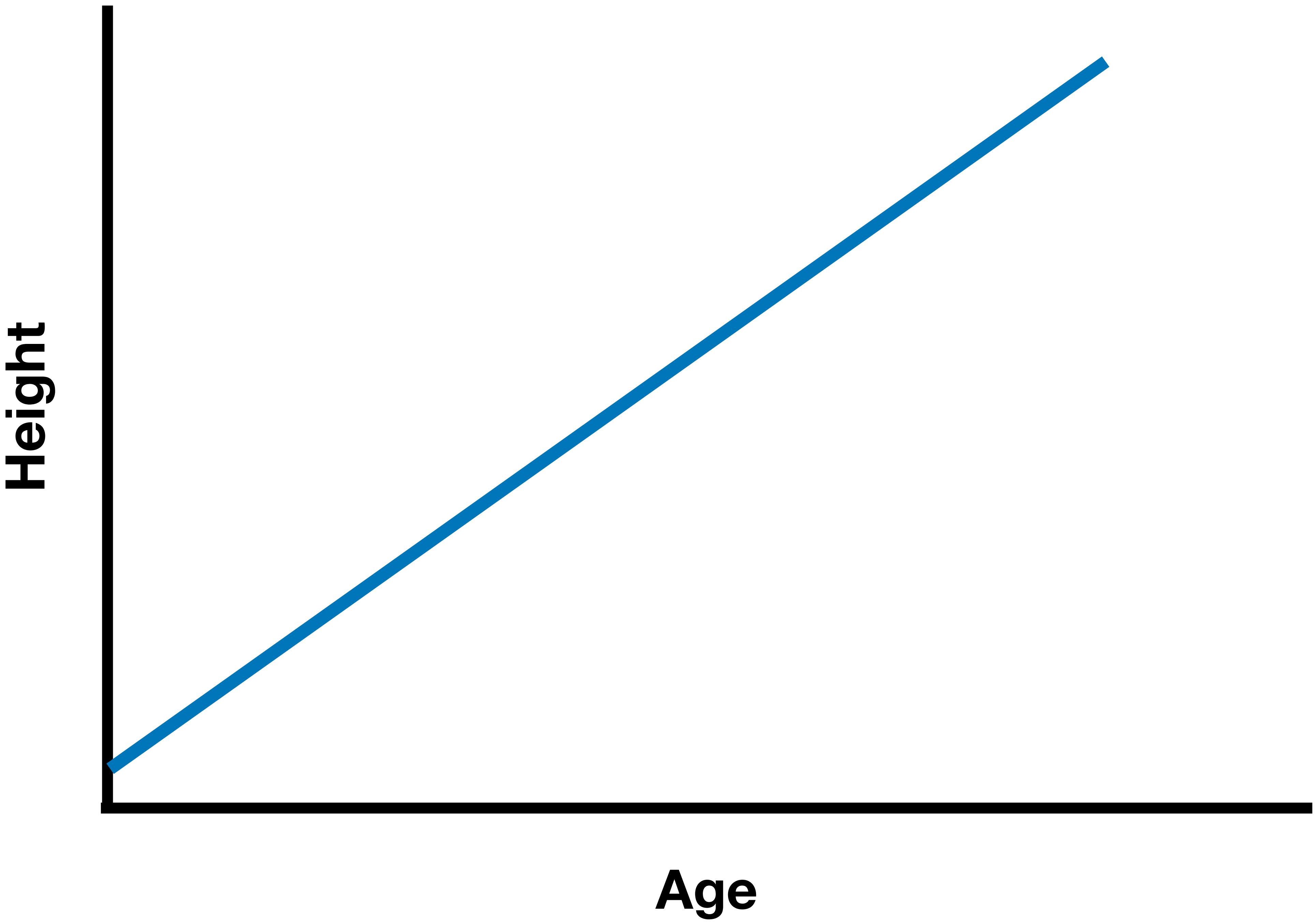
https://github.com/katiesevans/IGP_biostatistics

Course materials: slides and code

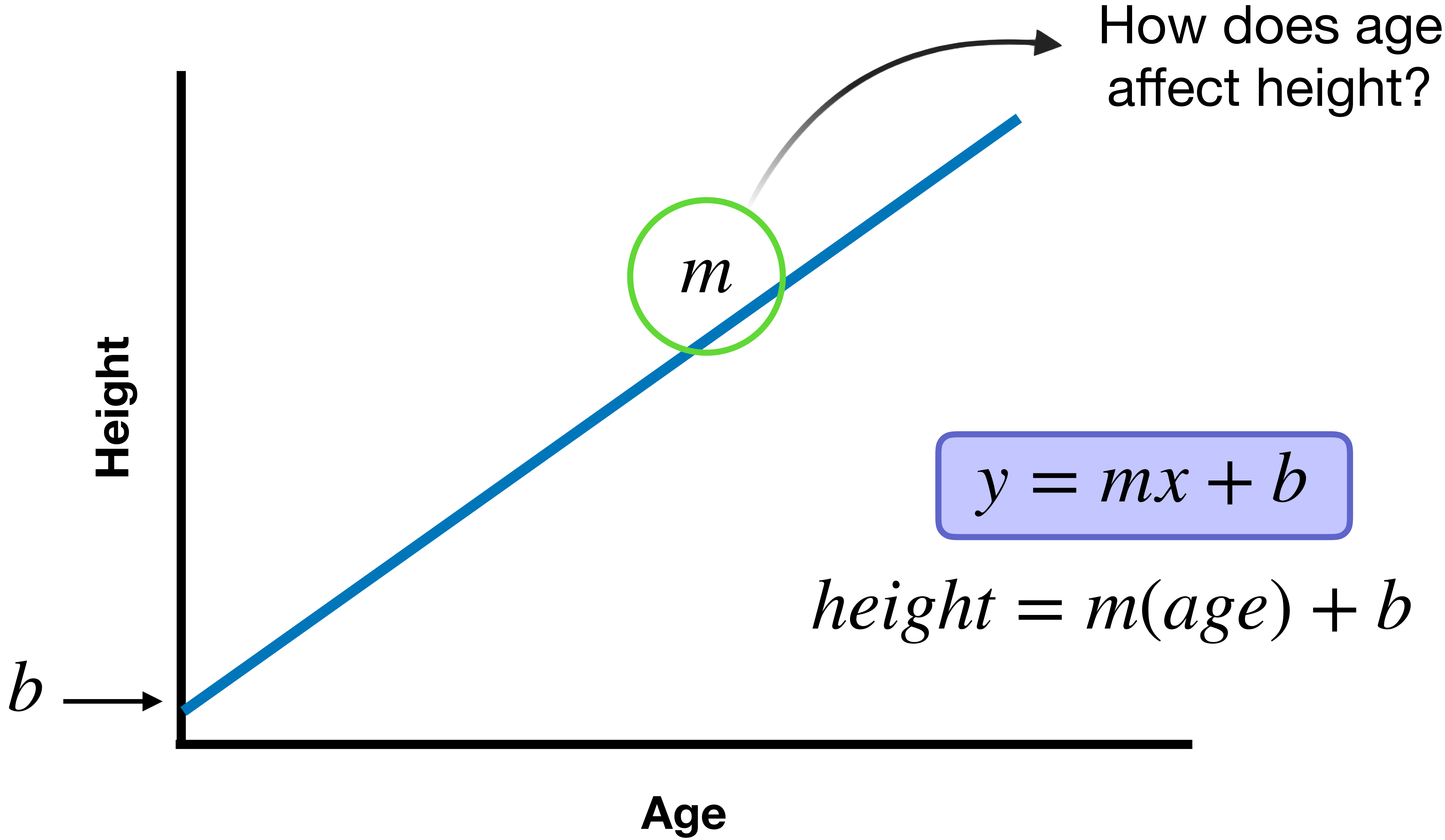


https://github.com/katiesevans/IGP_biostatistics

Linear relationships



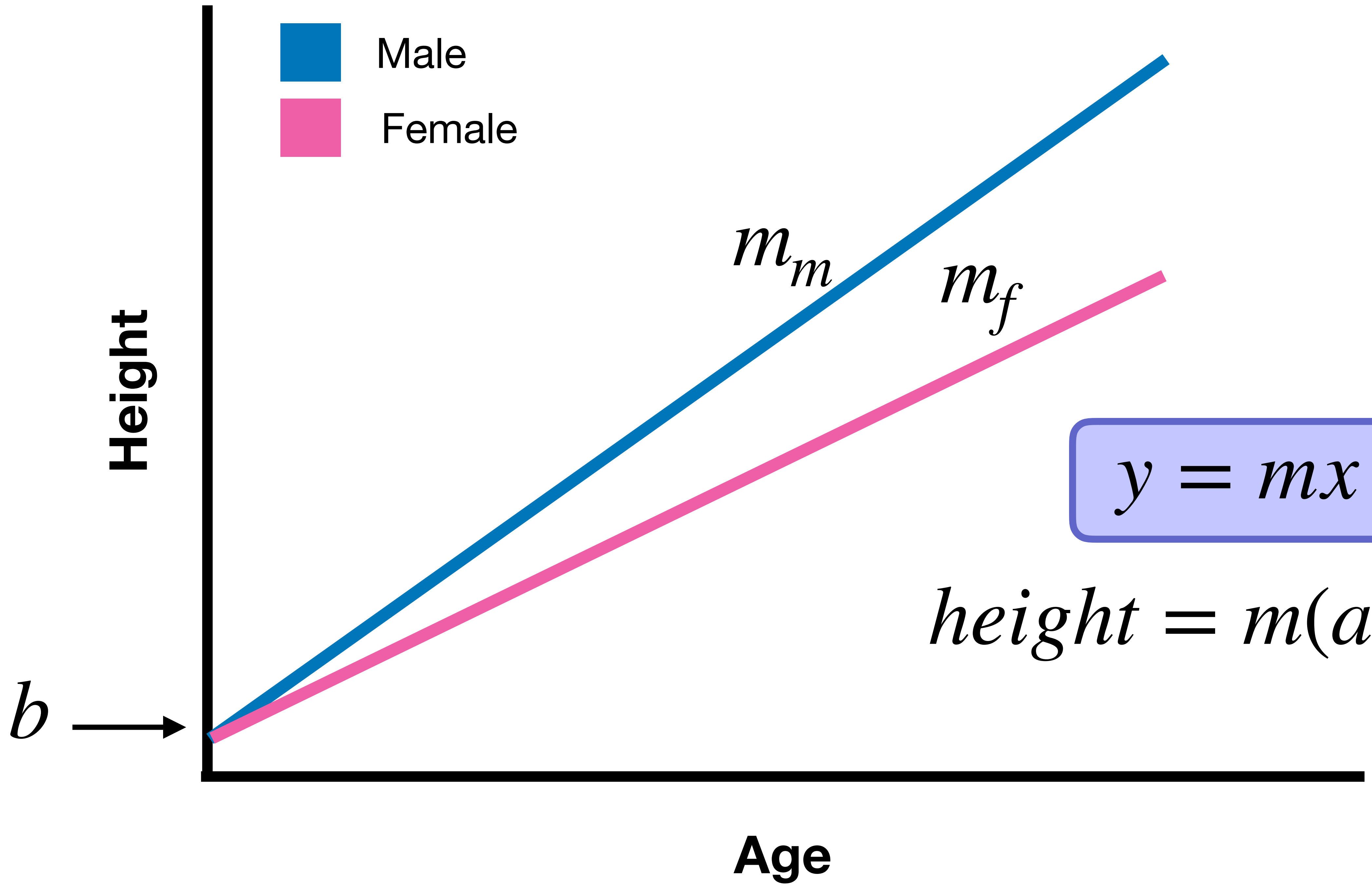
Linear relationships



$$\boxed{y = mx + b}$$

$$height = m(age) + b$$

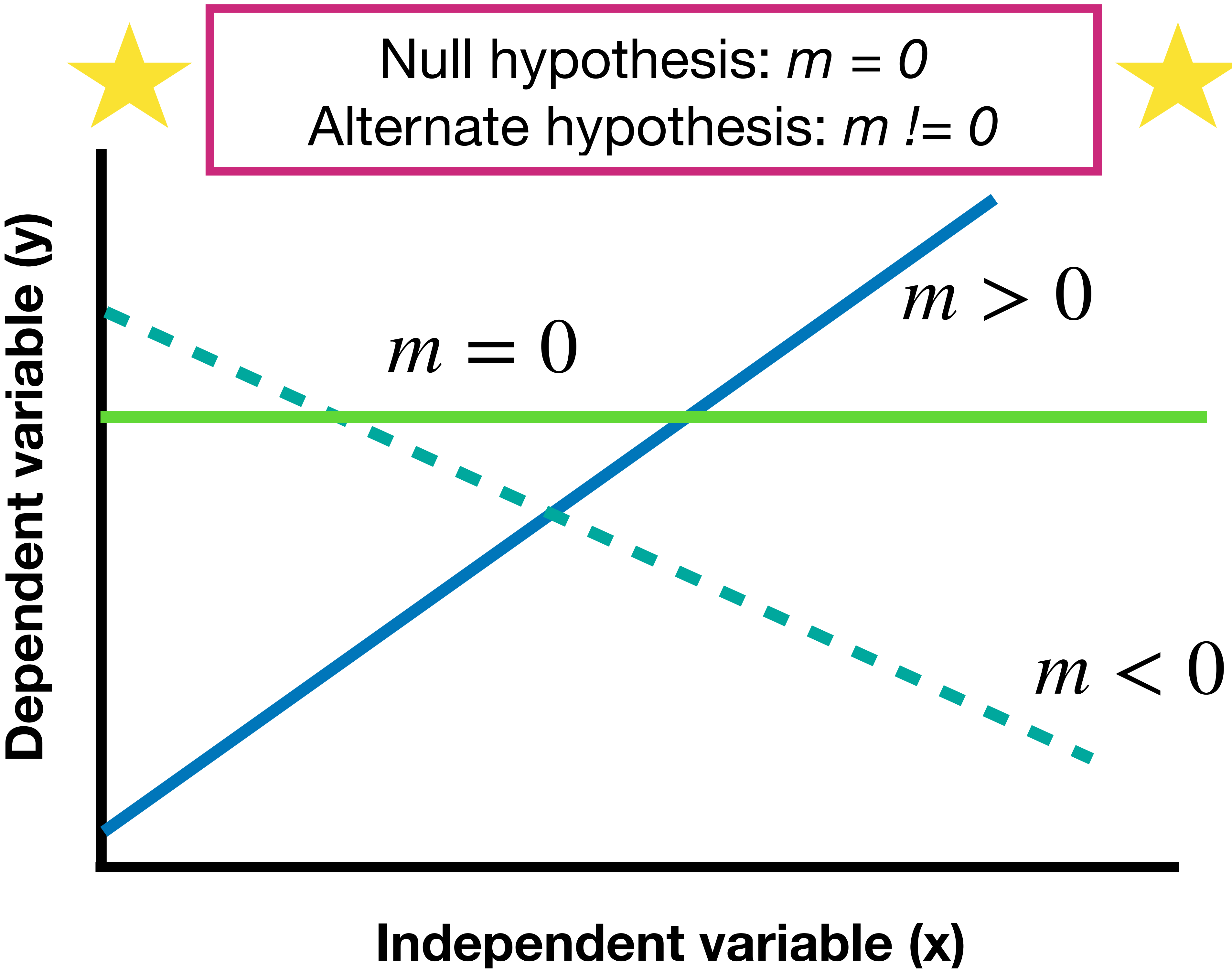
Linear relationships



$$\boxed{y = mx + b}$$

$$height = m(age) + b$$

Linear regression tests for a relationship between variables



Linear regression: an example

$\stackrel{?}{=}$

Independent Variable

Dependent Variable

Linear regression: an example

- Survey 500 people with a range of incomes
- Ask them to rank their happiness on a scale from 1 to 10

$\stackrel{?}{=}$

Independent Variable **Dependent Variable**

Linear regression: an example

```
income_data <- read.csv("income.data.csv")
```

income	happiness
3.862647	2.3144890
4.979381	3.4334898
4.923957	4.5993734
3.214372	2.7911138
7.196409	5.5963983
3.729643	2.4585559
4.674517	3.1929918
4.498104	1.9071368
3.121631	2.9424499
4.639914	3.7379416
4.632840	3.1754061
2.773179	2.0090465
7.119479	5.9518141

```
summary(income_data$income)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  1.506   3.006   4.424   4.467   5.992   7.482
```

(data is in $10k)

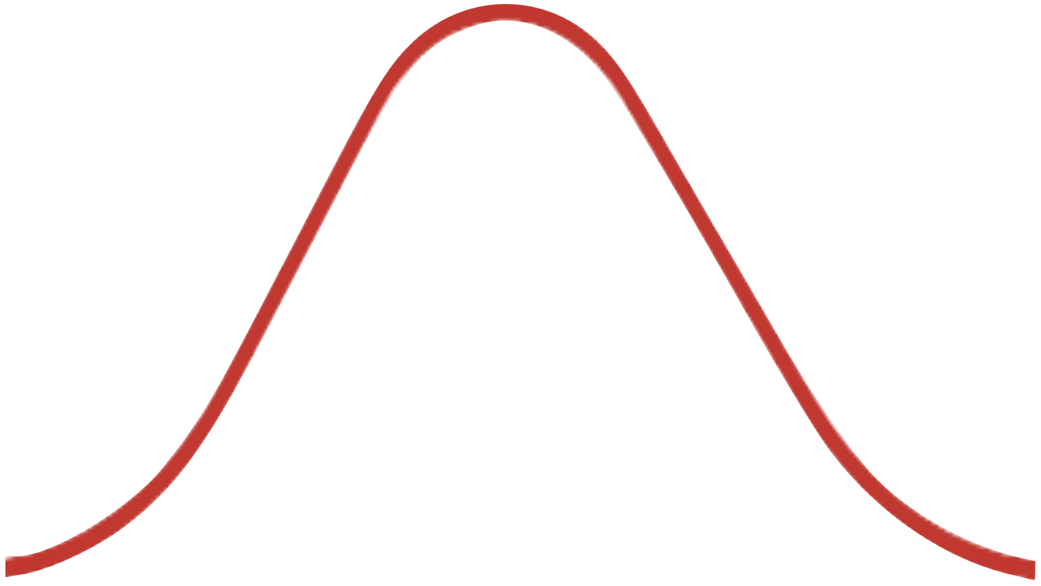
```
summary(income_data$happiness)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  0.266   2.266   3.473   3.393   4.503   6.863
```

(no very happy people…)

https://www.scribbr.com/statistics/simple-linear-regression/

Assumptions for linear regression

- Homogeneity of variance
  - *The error is similar across all samples*
- Independence of observations
  - *No hidden groups or relationships*
- Normality
  - *Data follows normal distribution*



The experiment setup

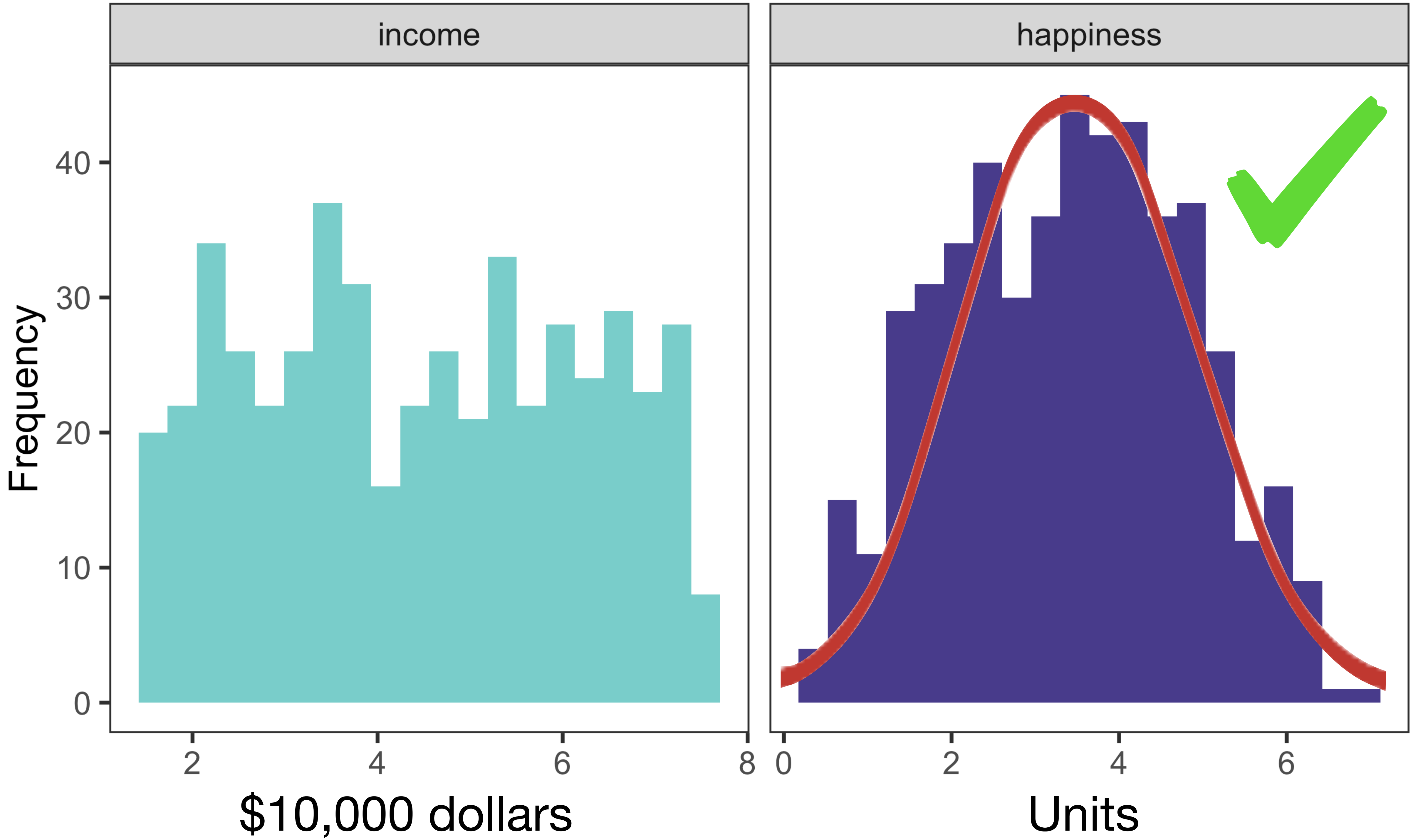
Base R

```
hist(income_data$income)
hist(income_data$happiness)
```

ggplot2

```
income_data %>%
    tidyr::gather(var, value) %>%
    ggplot2::ggplot(.) +
    ggplot2::aes(x = value) +
    ggplot2::geom_histogram() +
    ggplot2::facet_grid(var)
```

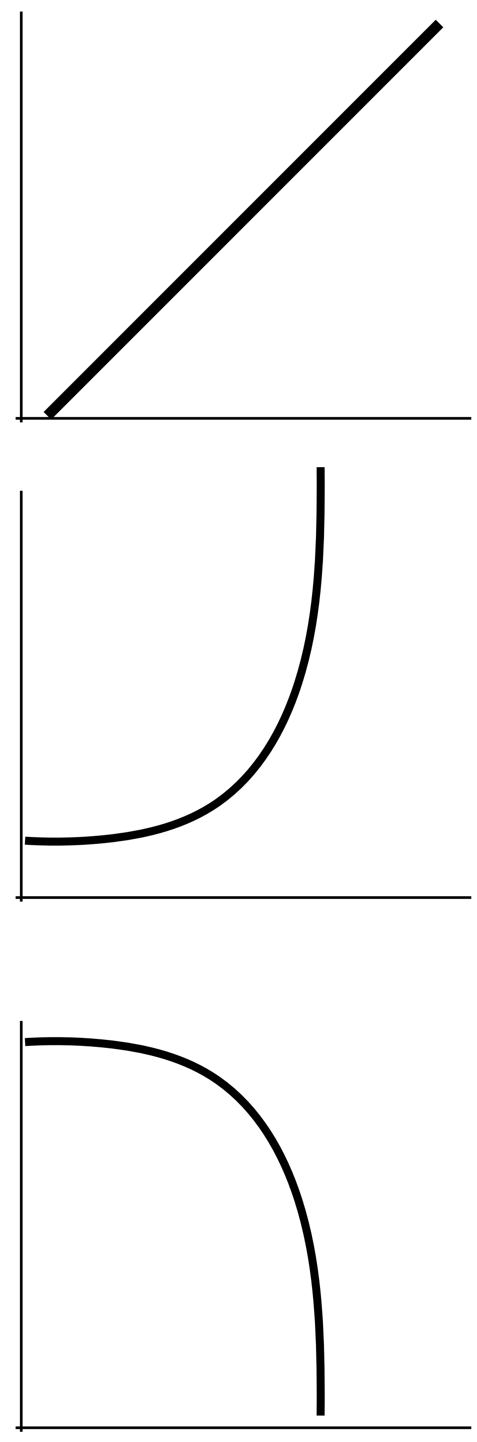
The experiment setup



Independent Variable **Dependent Variable**

Assumptions for linear regression

- Homogeneity of variance
    - *The error is similar across all samples*
- Independence of observations
    - *No hidden groups or relationships*
- Normality
    - *Data follows normal distribution*
- Linear relationship
    - *As opposed to exponential or other*



Plotting data in R

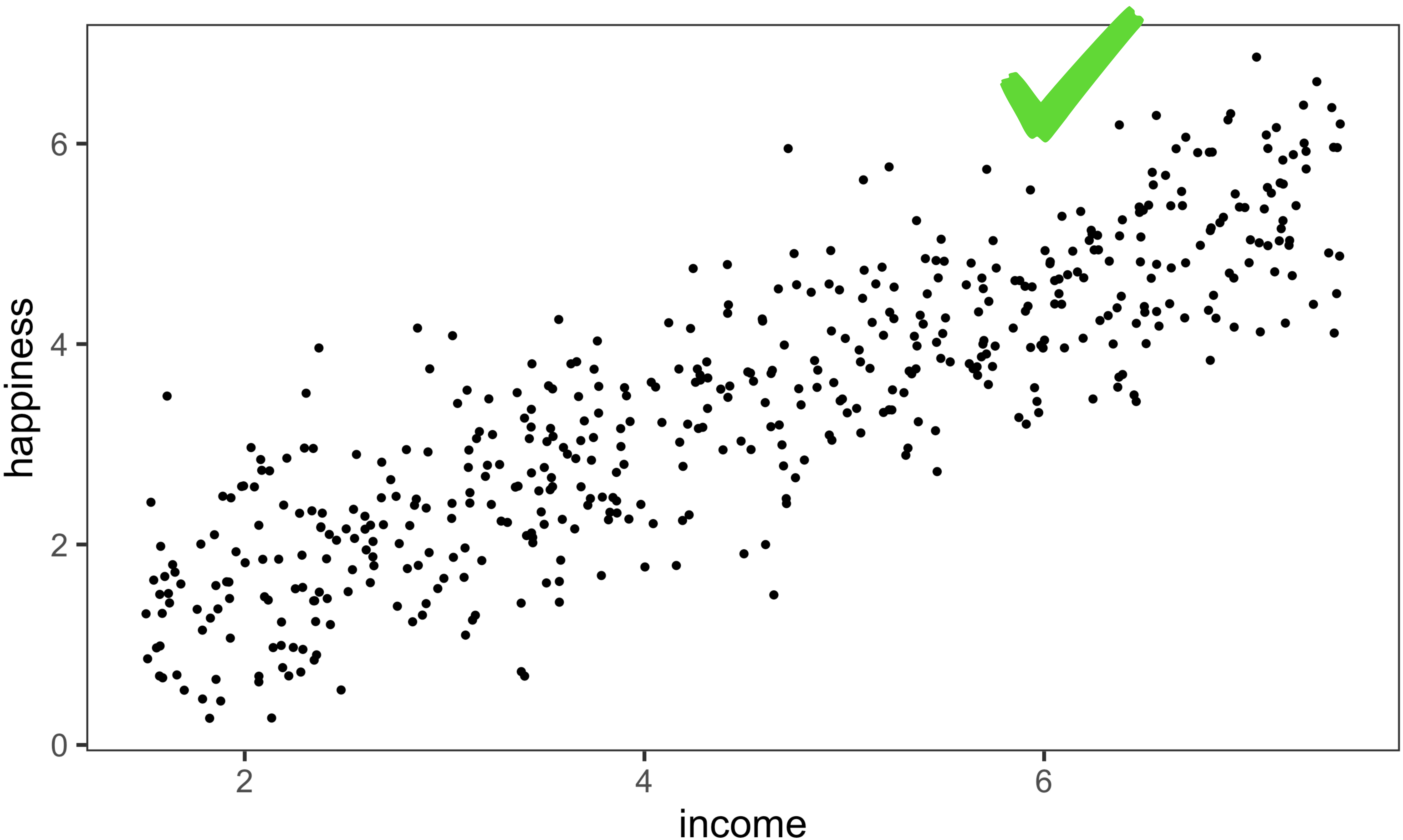
Base R

```
plot(input_data$income, input_data$happiness)
```

ggplot2

```
ggplot2::ggplot(income_data) +
    ggplot2::aes(x = income, y = happiness) +
    ggplot2::geom_point()
```

The experiment setup



The nuts and bolts

$(y = b + mx + e)$

$$\boxed{y = \beta_0 + \beta_1 X + \epsilon}$$

Predicted value of
dependent variable
(i.e. happiness)

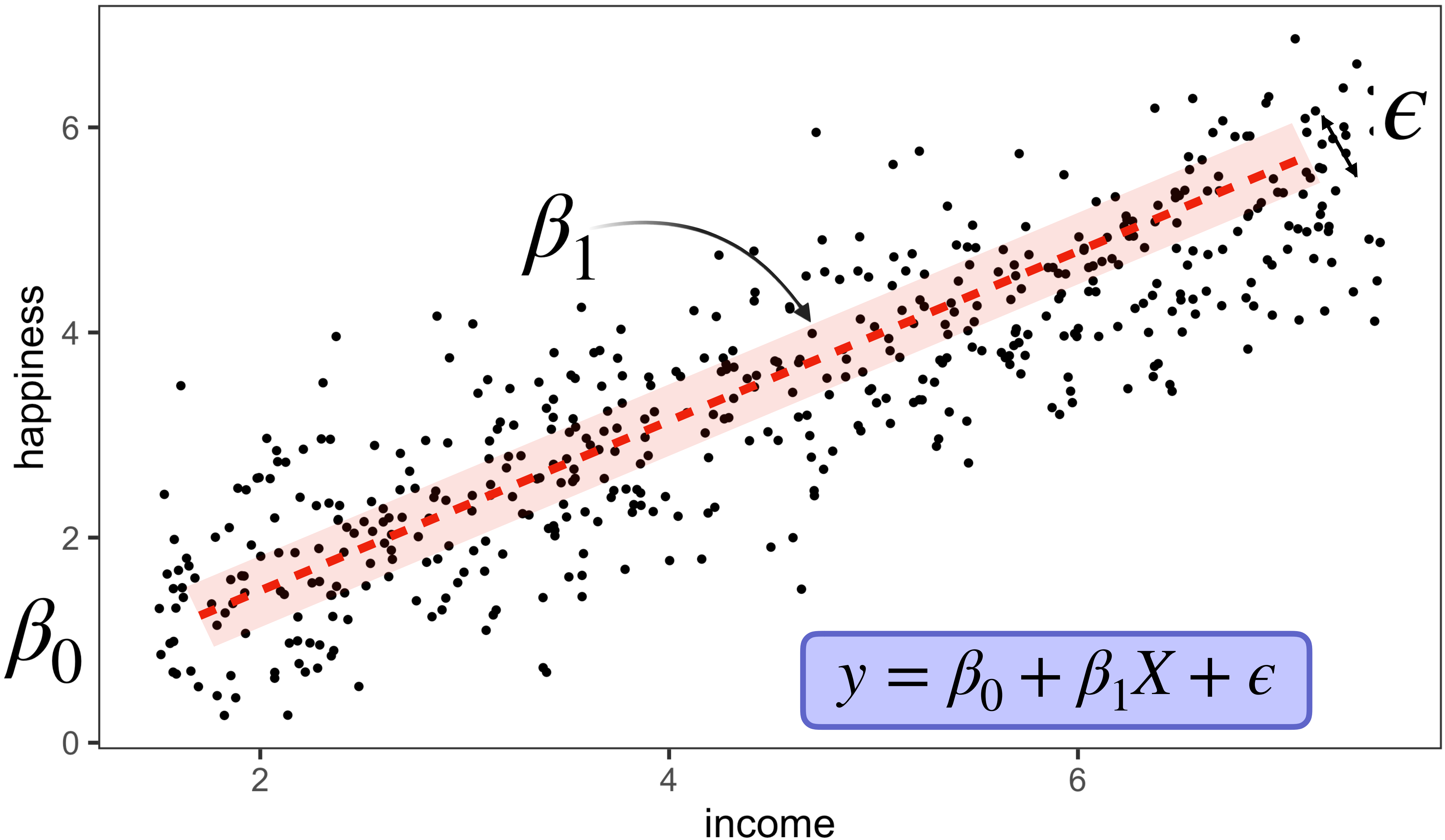
Intercept
(i.e. predicted happiness
at $0 income)

Regression coefficient
(i.e. How much we expect
y to change with x)

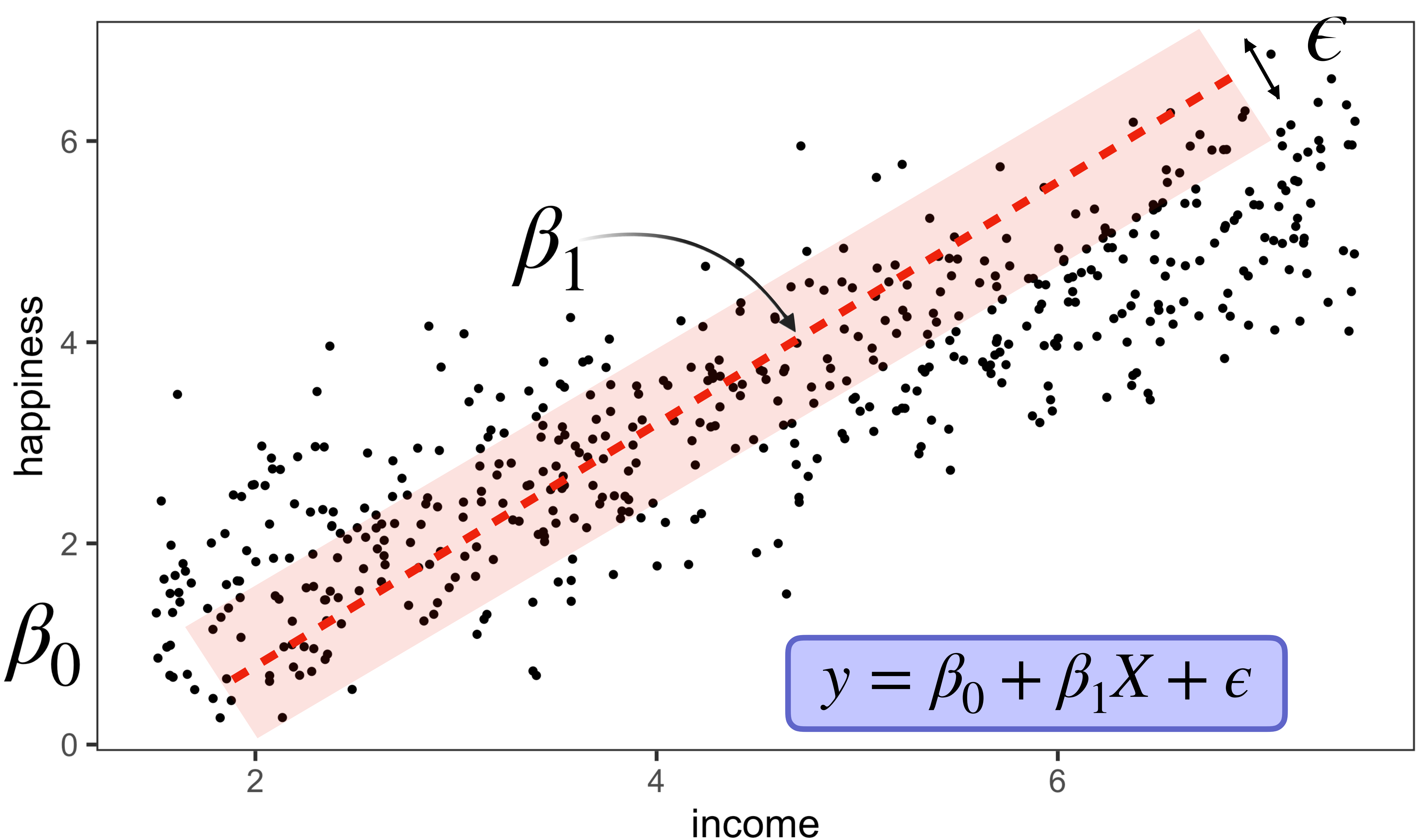
Independent variable
(i.e. income)

Error
(i.e. variation
in the estimate)

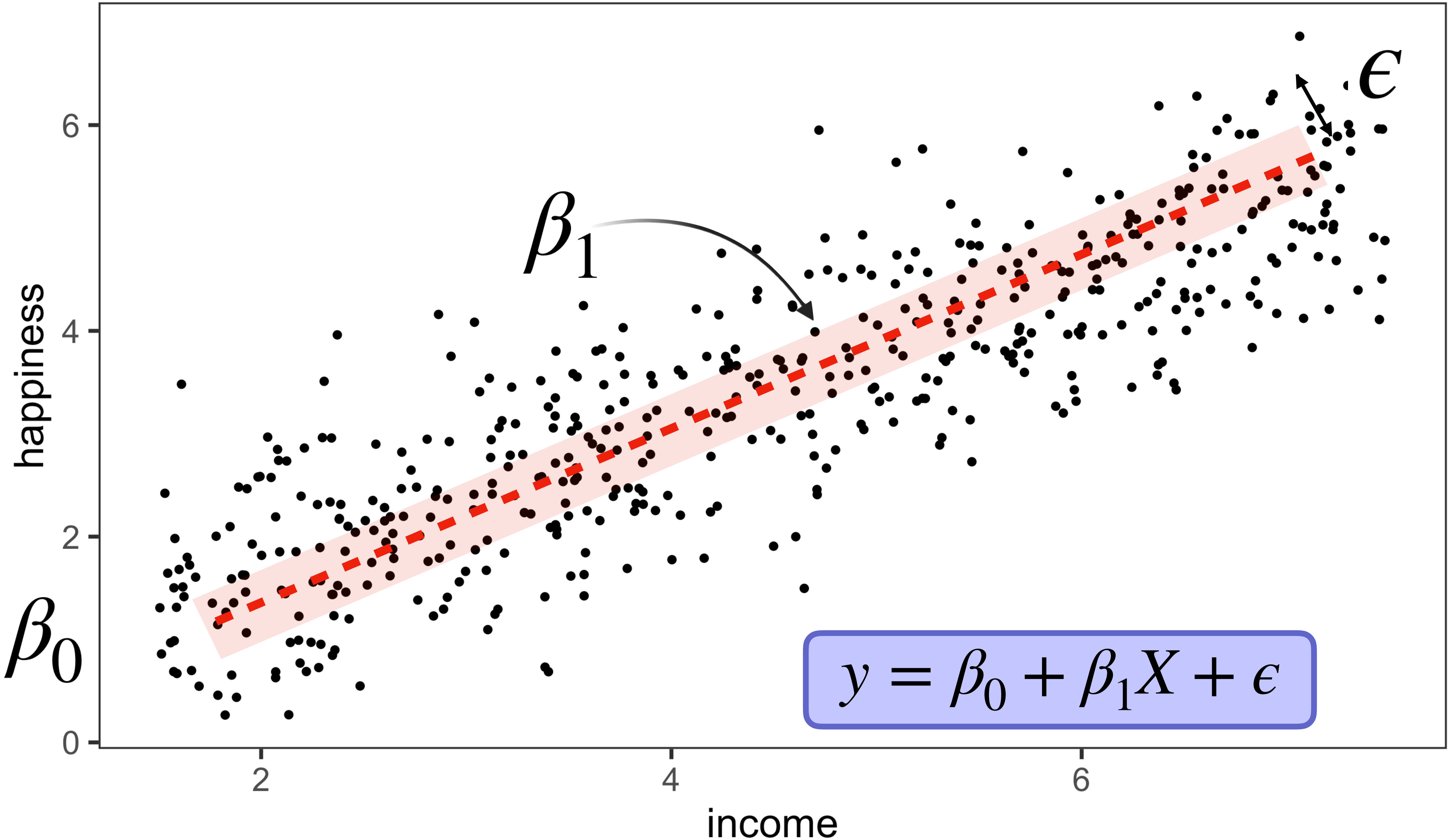
The regression coefficient



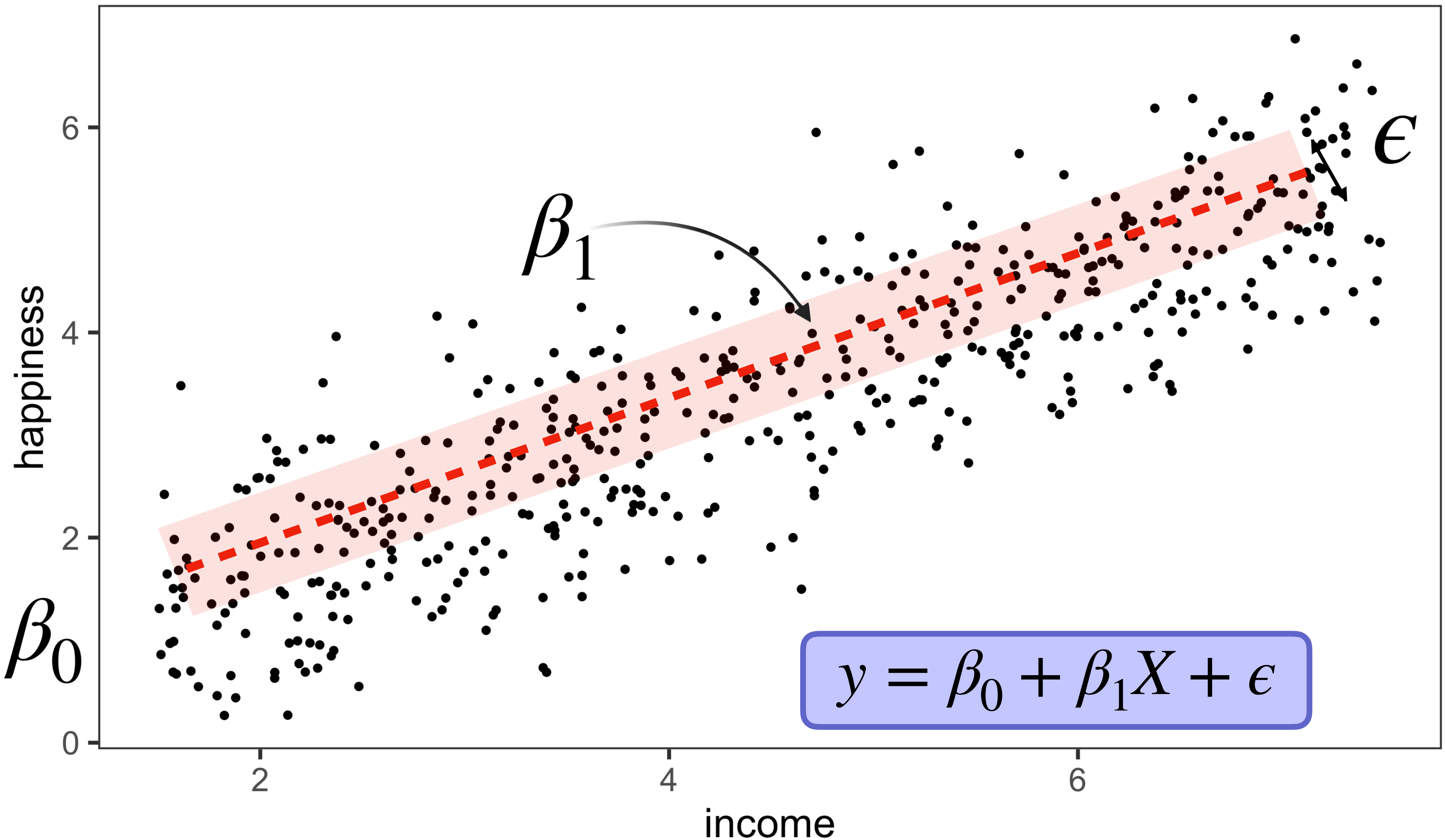
The regression coefficient



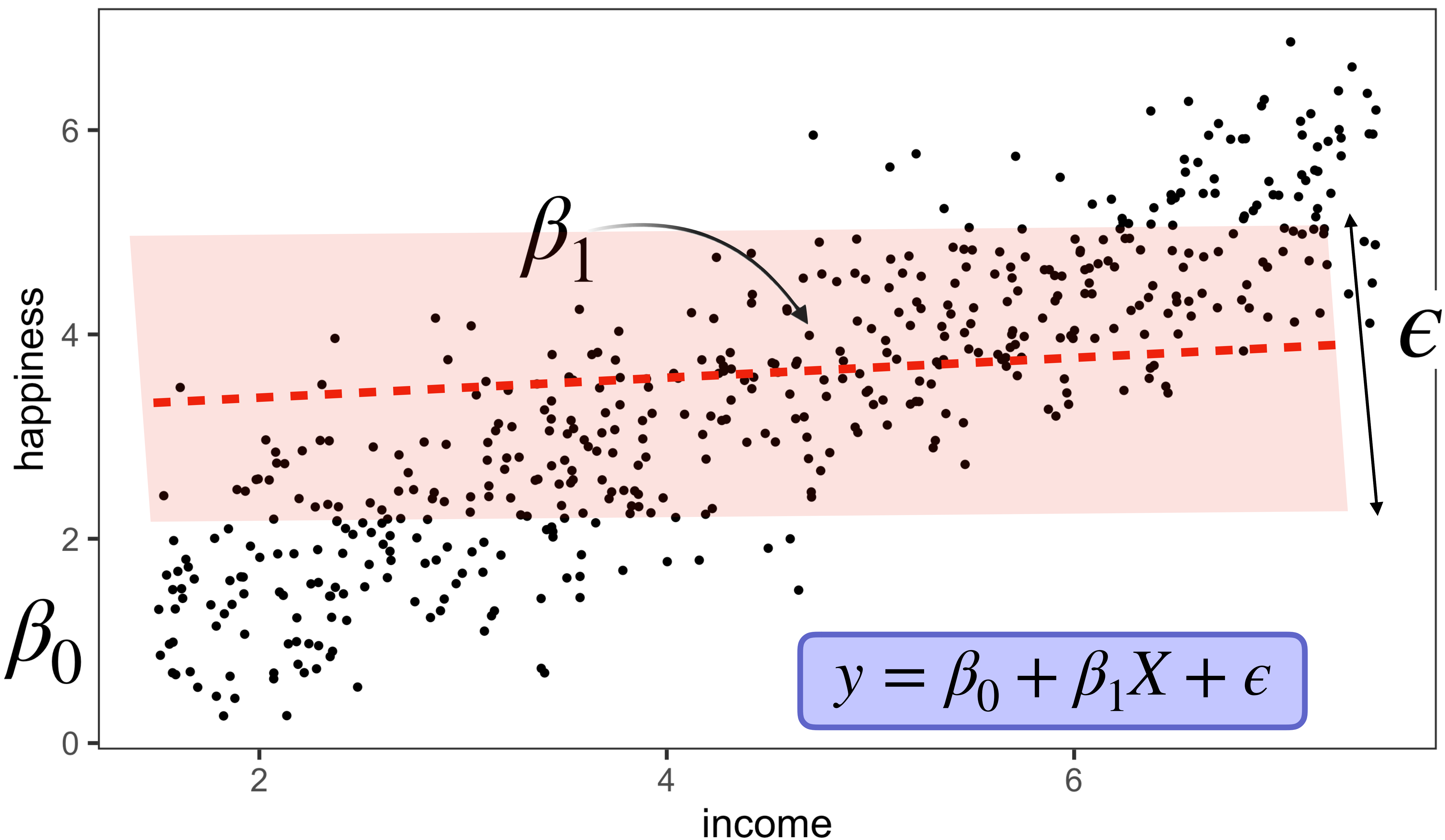
The regression coefficient



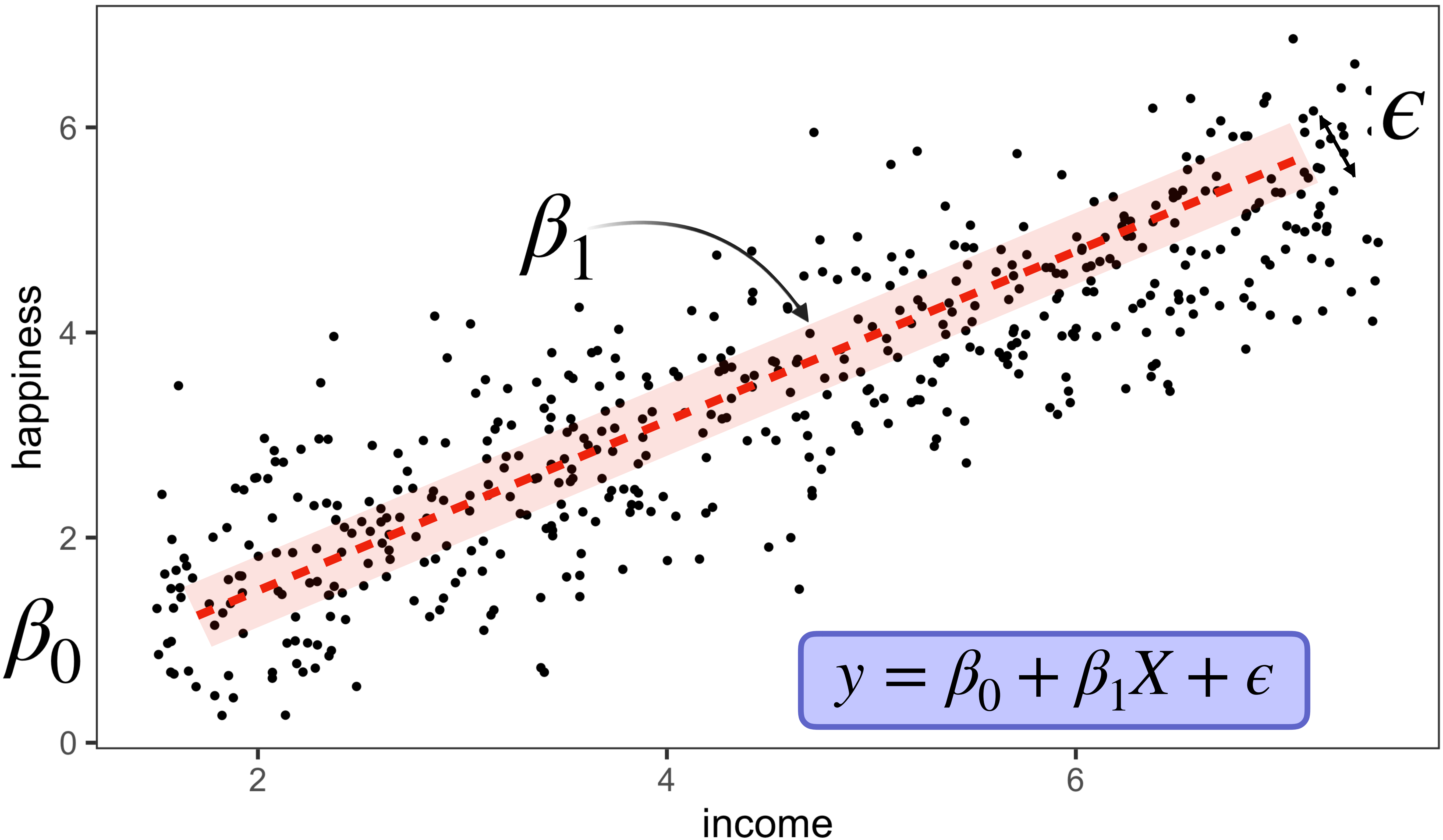
The regression coefficient



The regression coefficient



The regression coefficient



Linear regression in R

```
lm(happiness ~ income, data = income_data)
```

$$y = \beta_0 + \beta_1 X + \epsilon$$

```
Call:
lm(formula = happiness ~ income, data = income_data)

Coefficients:
(Intercept)       income
     0.2043       0.7138
```

For every 1 unit increase in income, there is a 0.7 unit increase in happiness

Linear regression in R

```
lm(happiness ~ income, data = income_data)
```

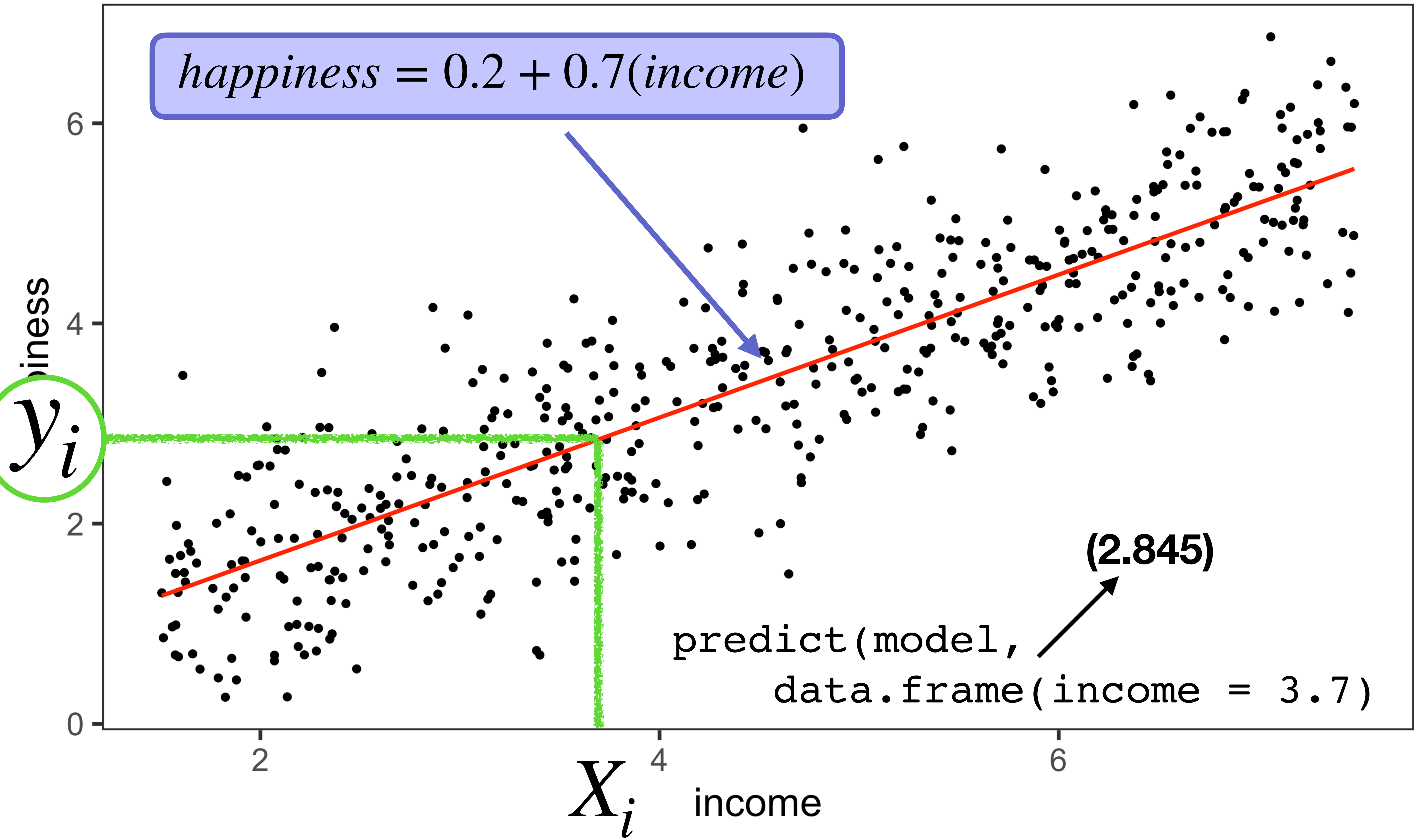
$$happiness = 0.2 + 0.7(income)$$

```
Call:
lm(formula = happiness ~ income, data = income_data)

Coefficients:
(Intercept)       income
     0.2043       0.7138
```

For every 1 unit increase in income, there is a 0.7 unit increase in happiness

Linear regression in R



Plotting linear regression in R

Base R

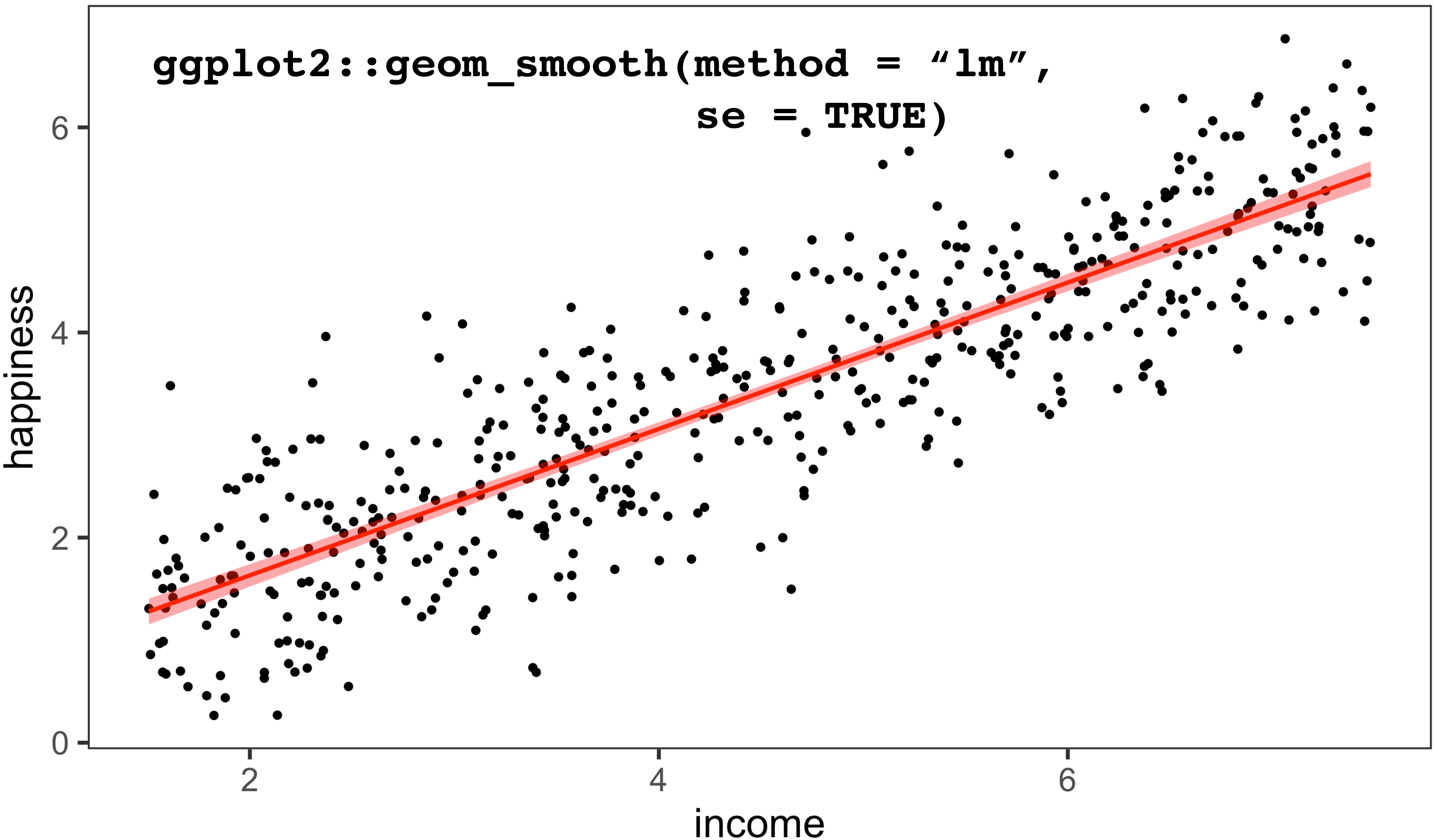
```r
plot(input_data$income, input_data$happiness)

abline(lm(happiness ~ income, data = input_data))
```

ggplot2

```r
ggplot2::ggplot(income_data) +
    ggplot2::aes(x = income, y = happiness) +
    ggplot2::geom_point() +
    ggplot2::geom_smooth(method = "lm")
```

Linear regression in R



Linear regression in R

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

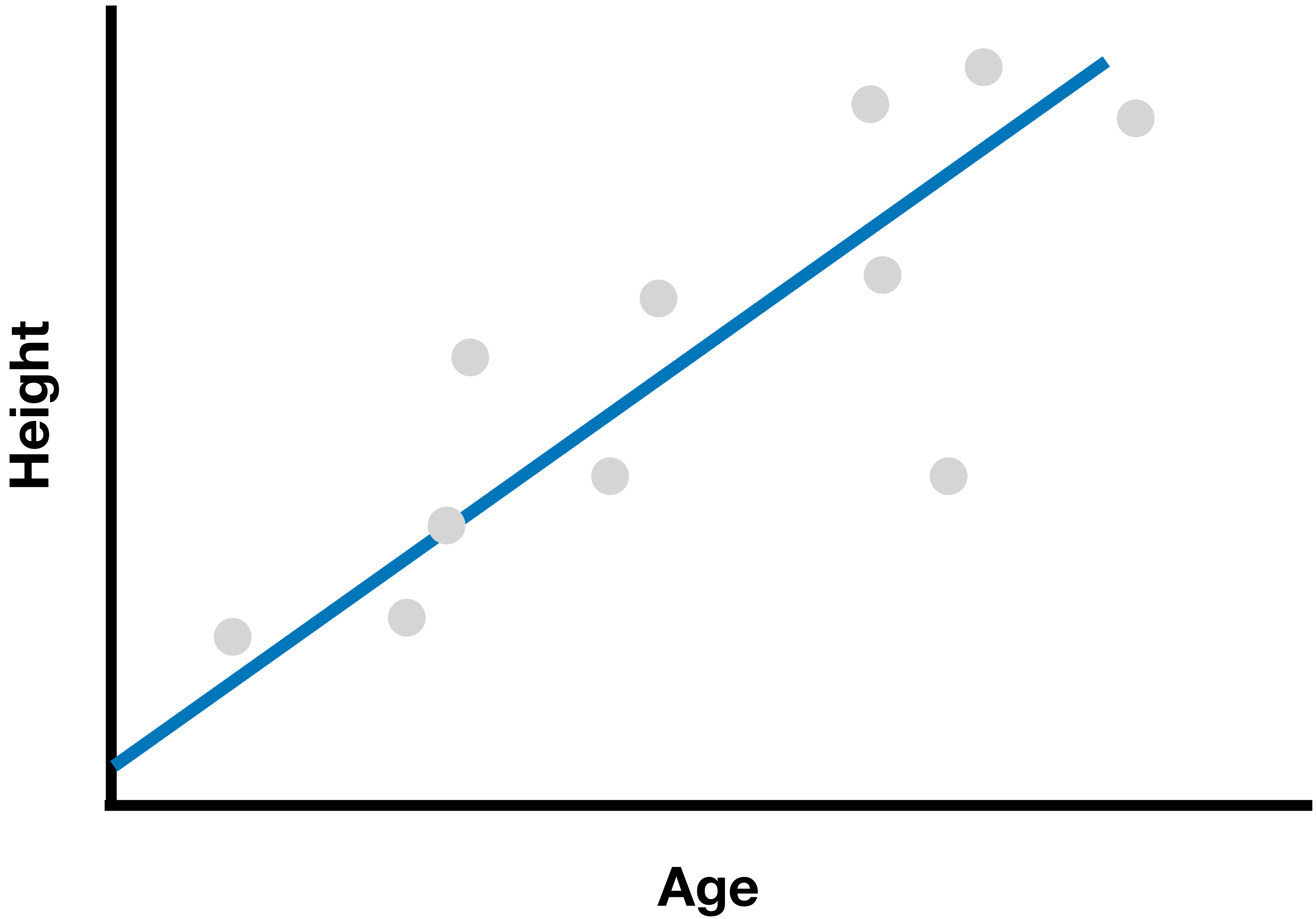
Residuals:
     Min       1Q   Median       3Q      Max
-2.02479 -0.48526  0.04078  0.45898  2.37805

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.20427    0.08884   2.299   0.0219 *
income       0.71383    0.01854  38.505   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7181 on 496 degrees of freedom
Multiple R-squared:  0.7493,	Adjusted R-squared:  0.7488
F-statistic:  1483 on 1 and 496 DF,  p-value: < 2.2e-16
```

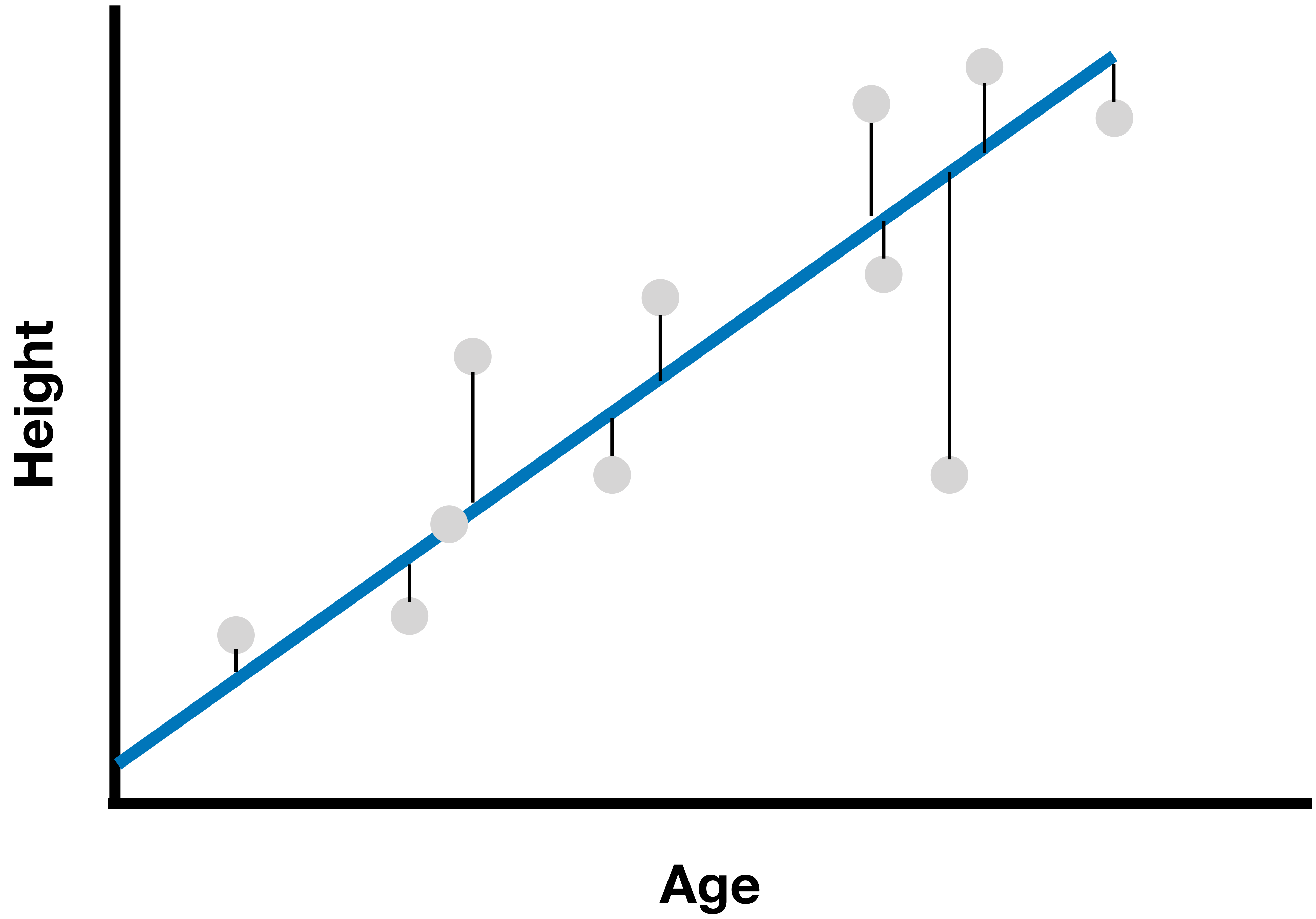
How well the model fits the data

Using residual values



Using residual values

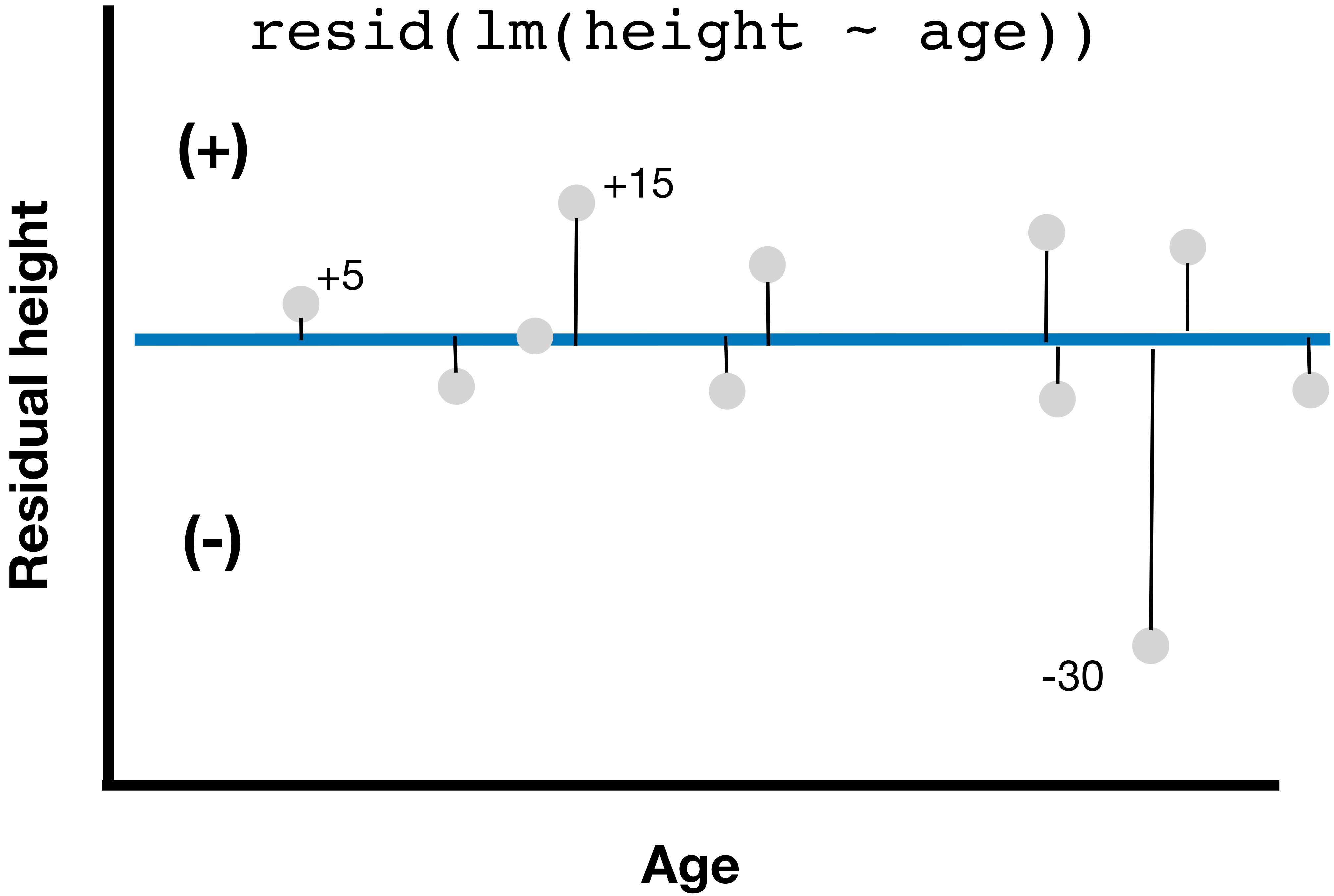
residual = observed - fitted



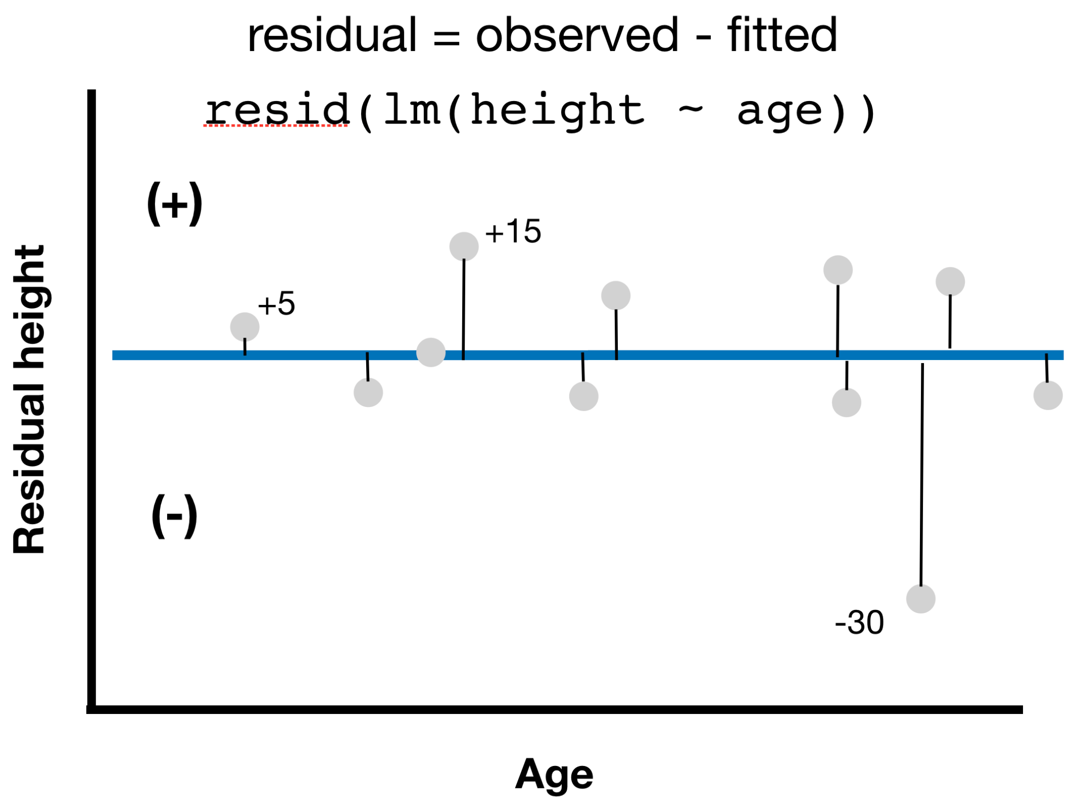
Using residual values

residual = observed - fitted

```
resid(lm(height ~ age))
```



Using residual values



- Mean of zero
- Equally positive as negative
- No remaining shape
- Low residual values

Linear regression in R

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
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-2.02479 -0.48526  0.04078  0.45898  2.37805 

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```

How well the model fits the data

Using residual values

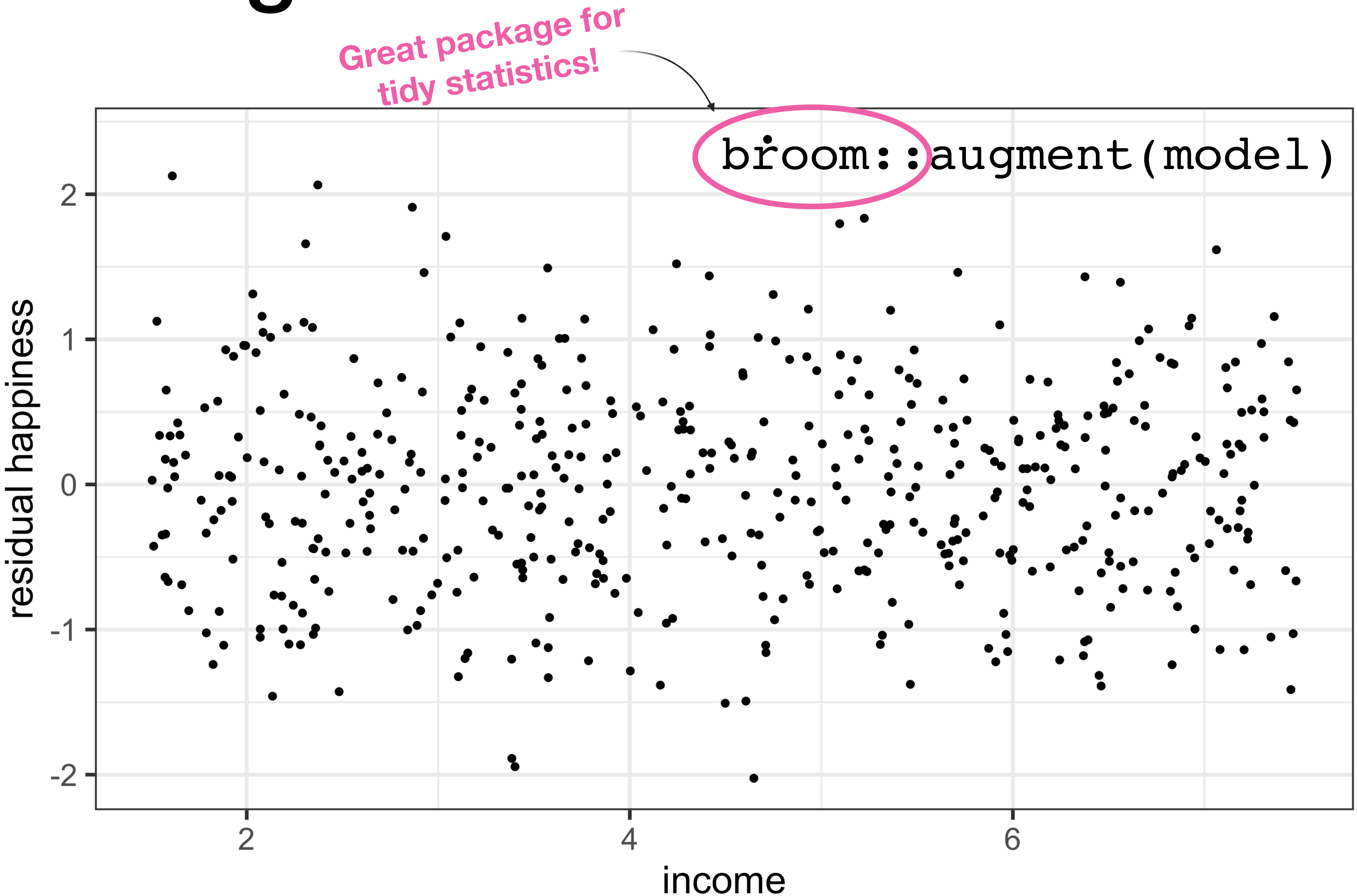
```
# assign linear model to variable
model <- lm(happiness ~ income, data = income_data)


# calculate residuals
residuals <- income_data %>%
    dplyr::mutate(resid = resid(model))


# plot predicted data
ggplot2::ggplot(residuals) +
    ggplot2::aes(x = income, y = resid) +
    ggplot2::geom_point()

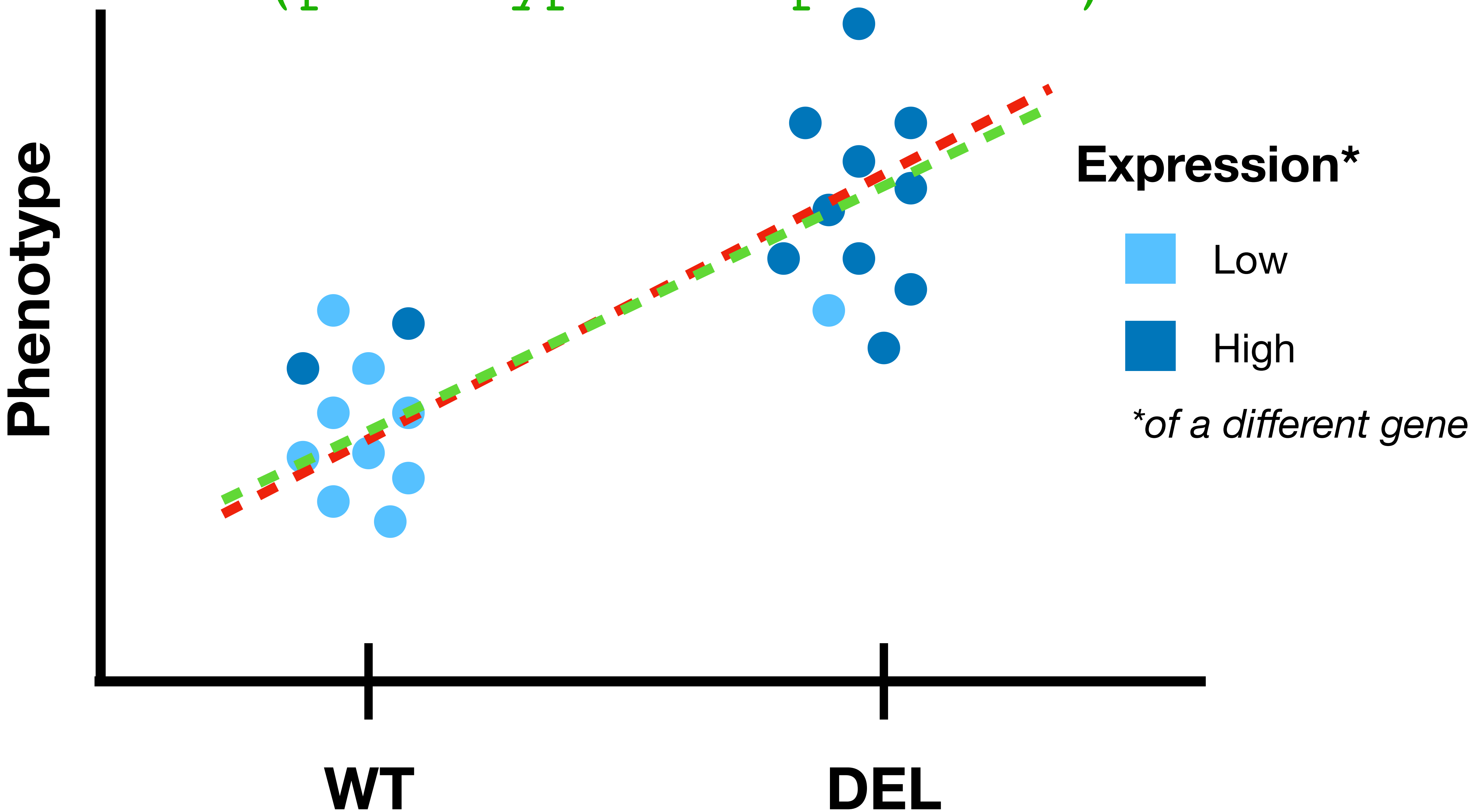

# plot in base R
plot(residuals$income, residuals$resid)
```

Using residual values



Residual values (teaser!)

```
lm(phenotype ~ strain)
lm(phenotype ~ expression)
```



(Evans and Andersen 2020)

Linear regression in R

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
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F-statistic:  1483 on 1 and 496 DF,  p-value: < 2.2e-16
```

Slope and intercept

Linear regression in R

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
     Min       1Q   Median       3Q      Max
-2.02479 -0.48526  0.04078  0.45898  2.37805

Coefficients:
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```

How much variation

Linear regression in R

```
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Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
     Min       1Q   Median       3Q      Max 
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Coefficients:
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F-statistic:  1483 on 1 and 496 DF,  p-value: < 2.2e-16
```

Test statistic

Linear regression in R

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
     Min       1Q   Median       3Q      Max
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```

p-value

Linear regression in R

```
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lm(formula = happiness ~ income, data = income_data)

Residuals:
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Multiple R-squared:  0.7493,	Adjusted R-squared:  0.7488
F-statistic:  1483 on 1 and 496 DF,  p-value: < 2.2e-16
```

Is intercept significantly different from zero?

p-value

Linear regression in R

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
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Coefficients:
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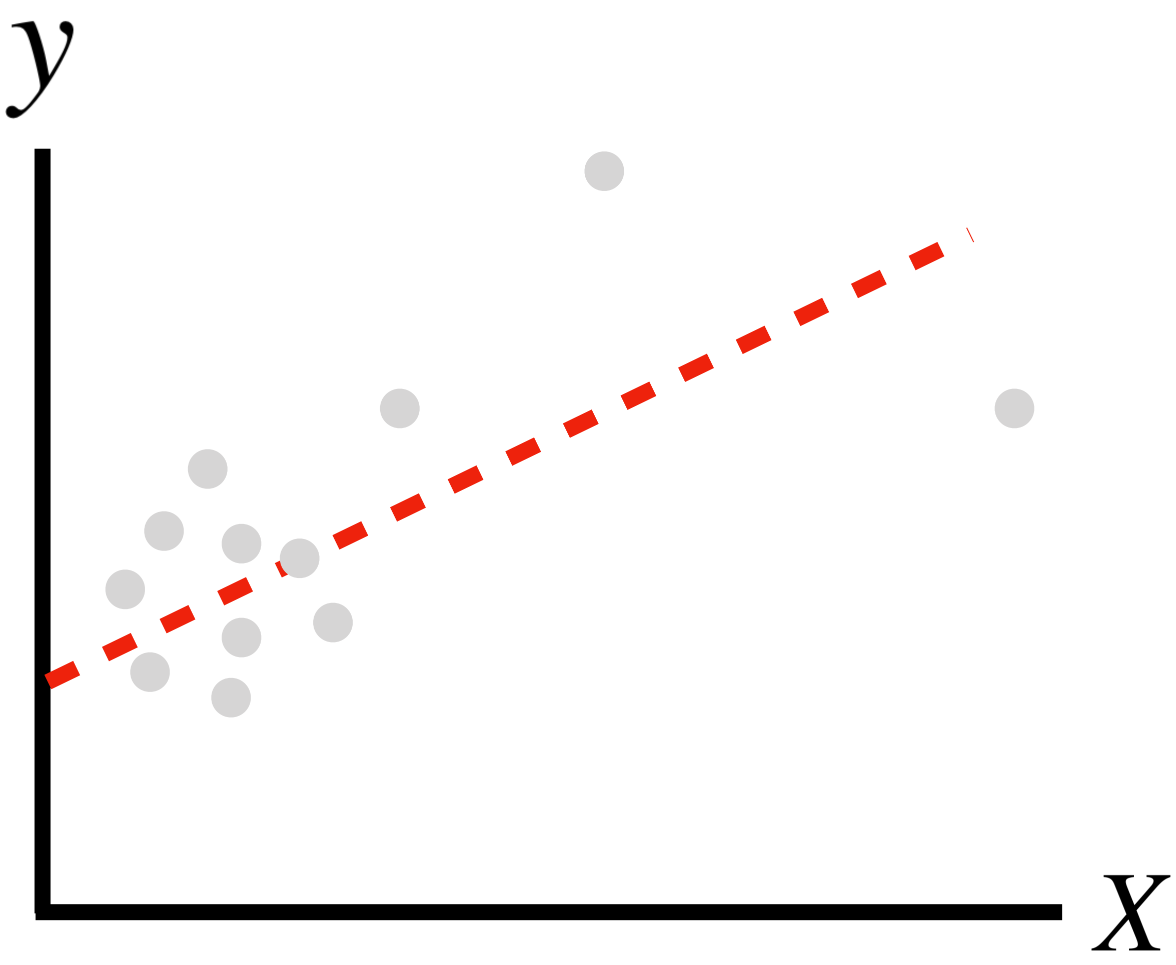
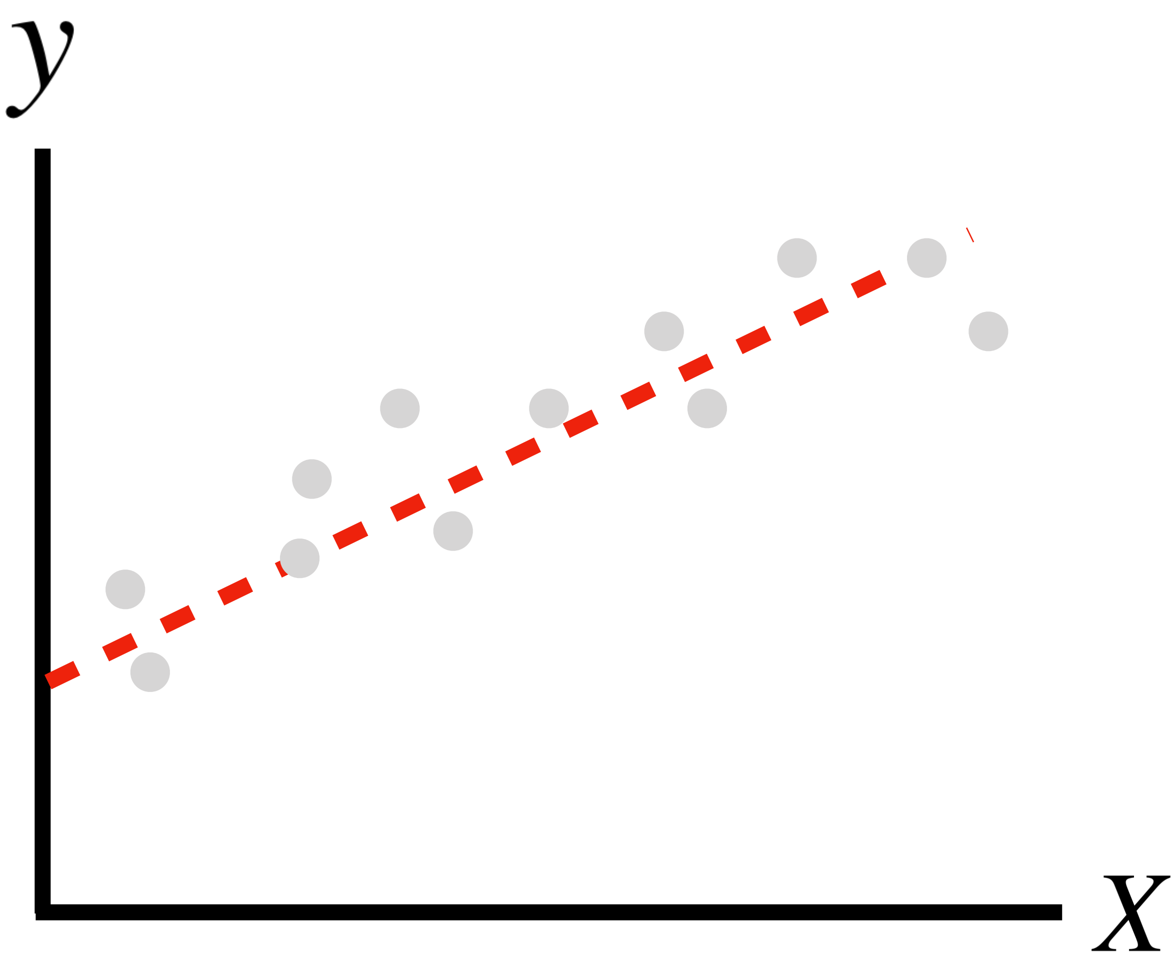
F-statistic:  1483 on 1 and 496 DF,  p-value: < 2.2e-16
```

More fit values

Linear regression in R

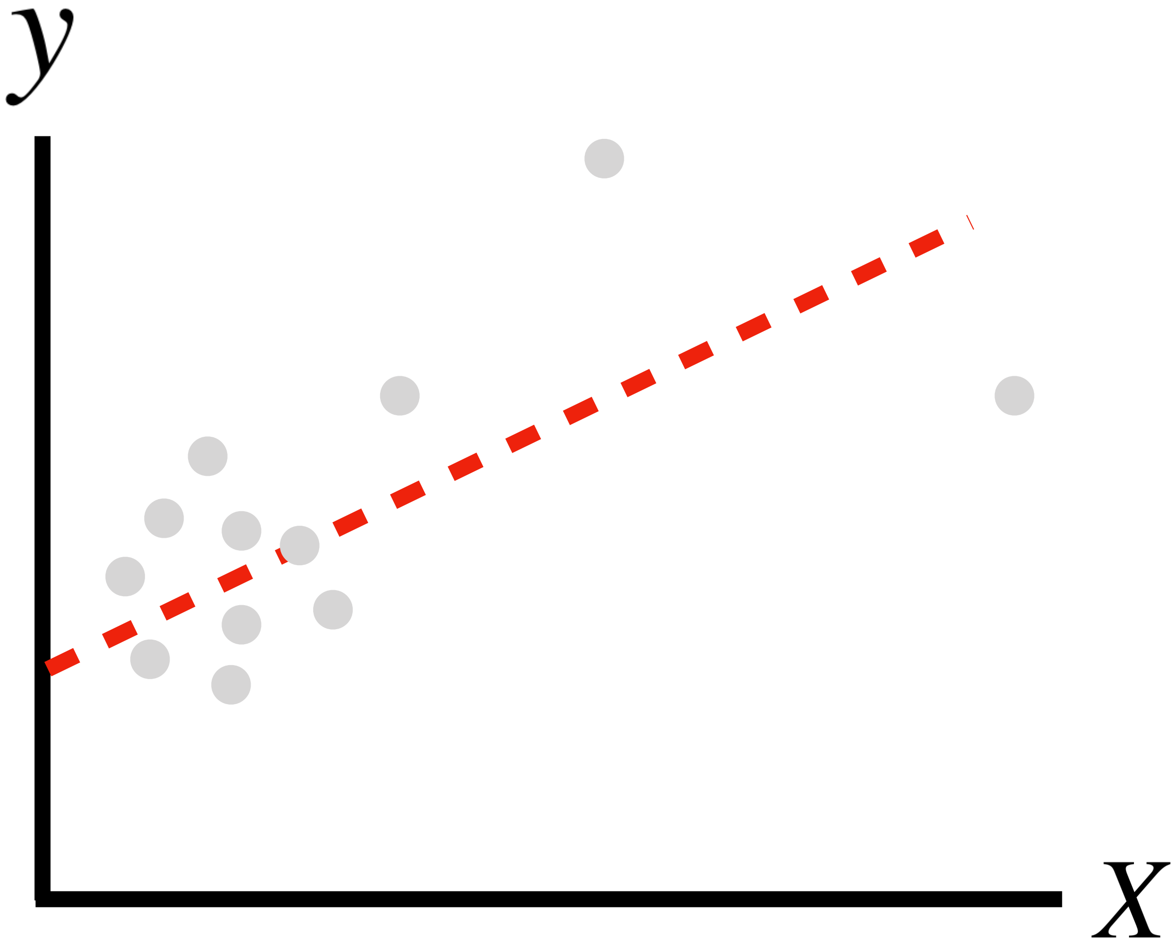
Which model has the higher R^2 value?

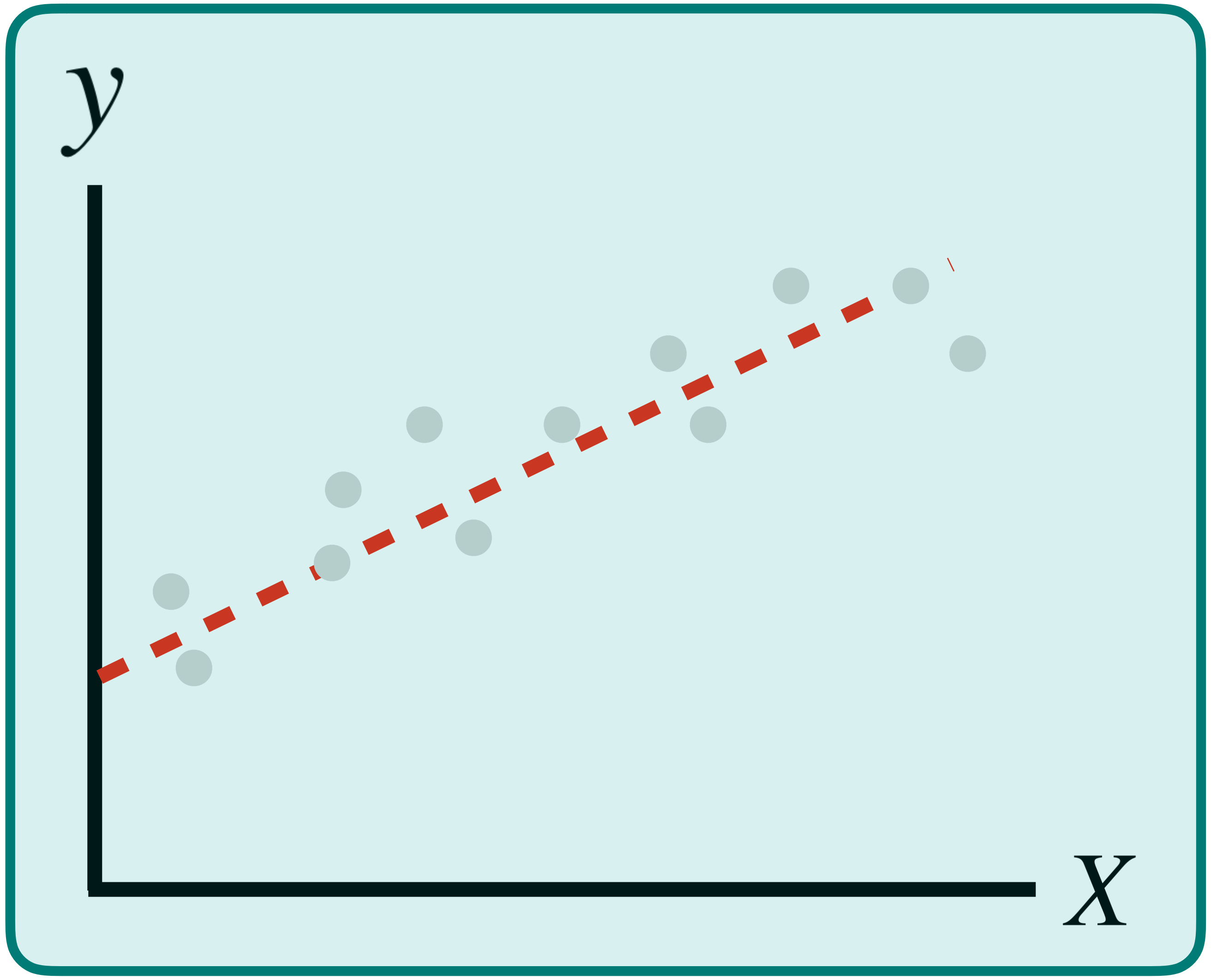




Linear regression in R

Which model has the higher R^2 value?

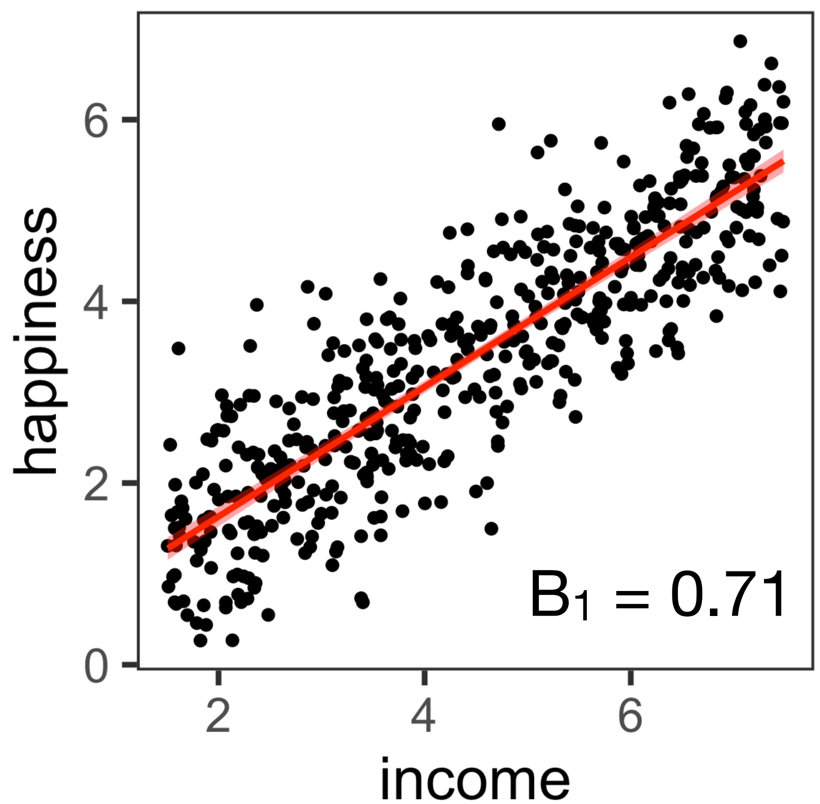




Regression vs. correlation

Regression: Describes how one variable (x) affects another variable (y)

Correlation: Quantifies the direction and strength of the relationship between two variables (x and y)




```
cor(happiness, income)
```

0.865

(0.865^2 = R^2 = 0.75)

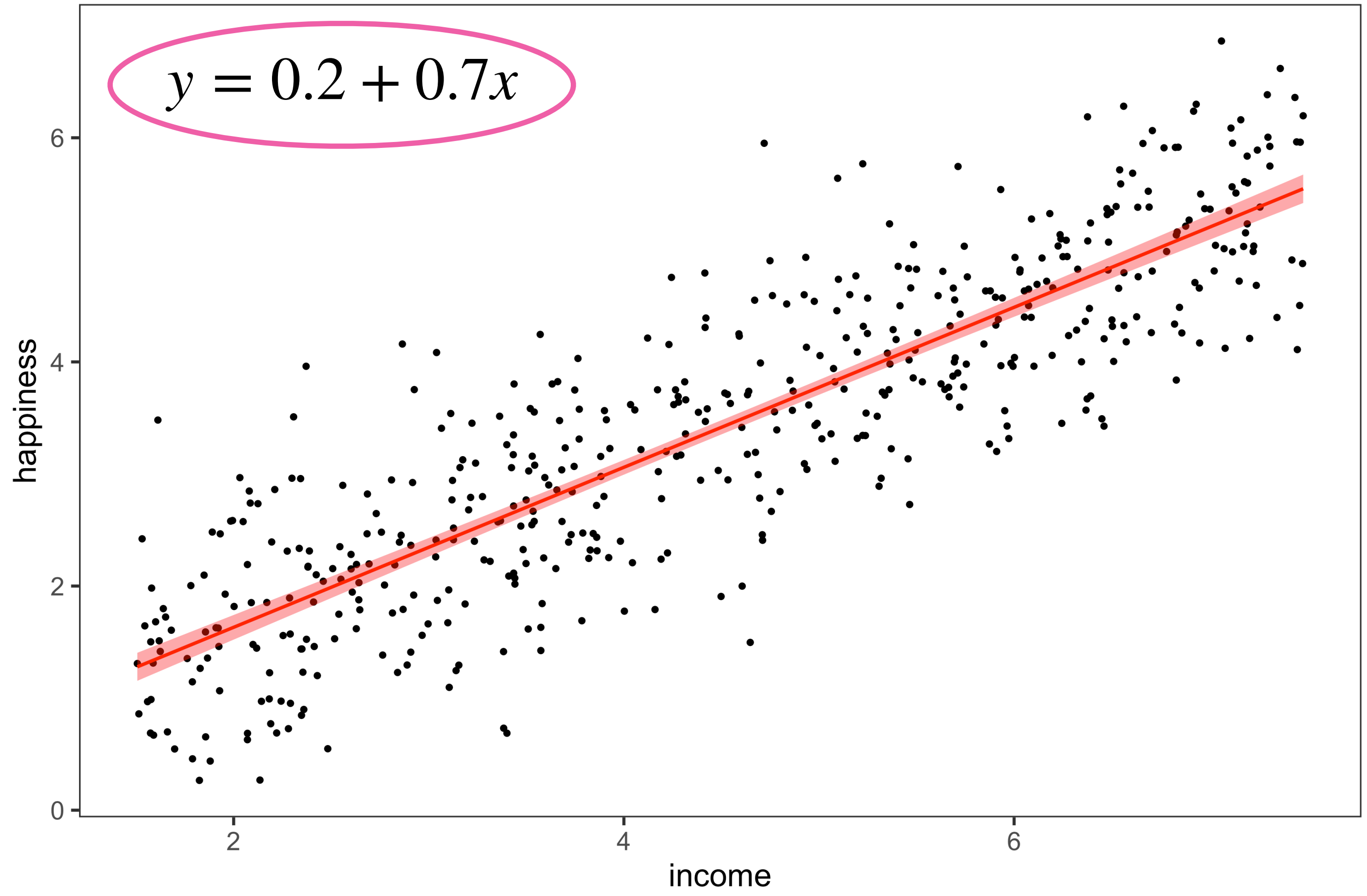
Regression vs. correlation

Regression: Describes how one variable (x) affects another variable (y)

Correlation: Quantifies the direction and strength of the relationship between two variables (x and y)

	Correlation	Regression
When to use	When summarizing direct relationship between two variables	To predict or explain responses
Able to quantify direction of relationship?	Yes	Yes
Able to quantify strength of relationship?	Yes	Yes
Able to show cause and effect?	No	Yes
Able to predict and optimize?	No	Yes
X and Y are interchangeable?	Yes	No

Reporting results



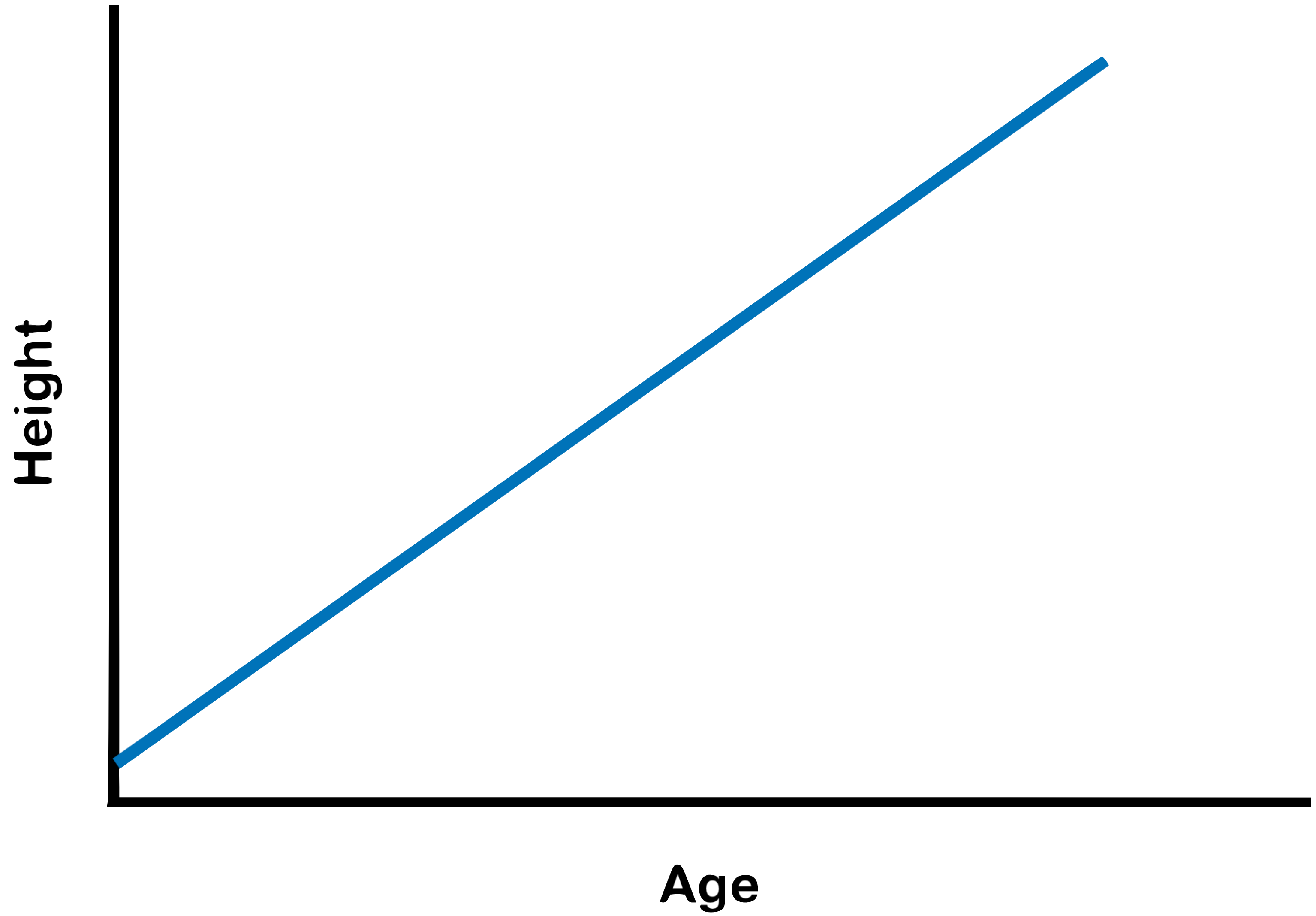
B_1

R^2

A simple linear regression was performed to test if income significantly predicted happiness. *(Describe experiment here)*. The analysis indicated that the model *(equation here)* explained 75% of the variation in happiness and that income did significantly predict happiness with a 0.71 unit increase in happiness for every $10,000 increase in income (B_1 = 0.71 +/- 0.018, *p*-value < 2e-16).

Summary: linear regression

```
lm(height ~ age)
```

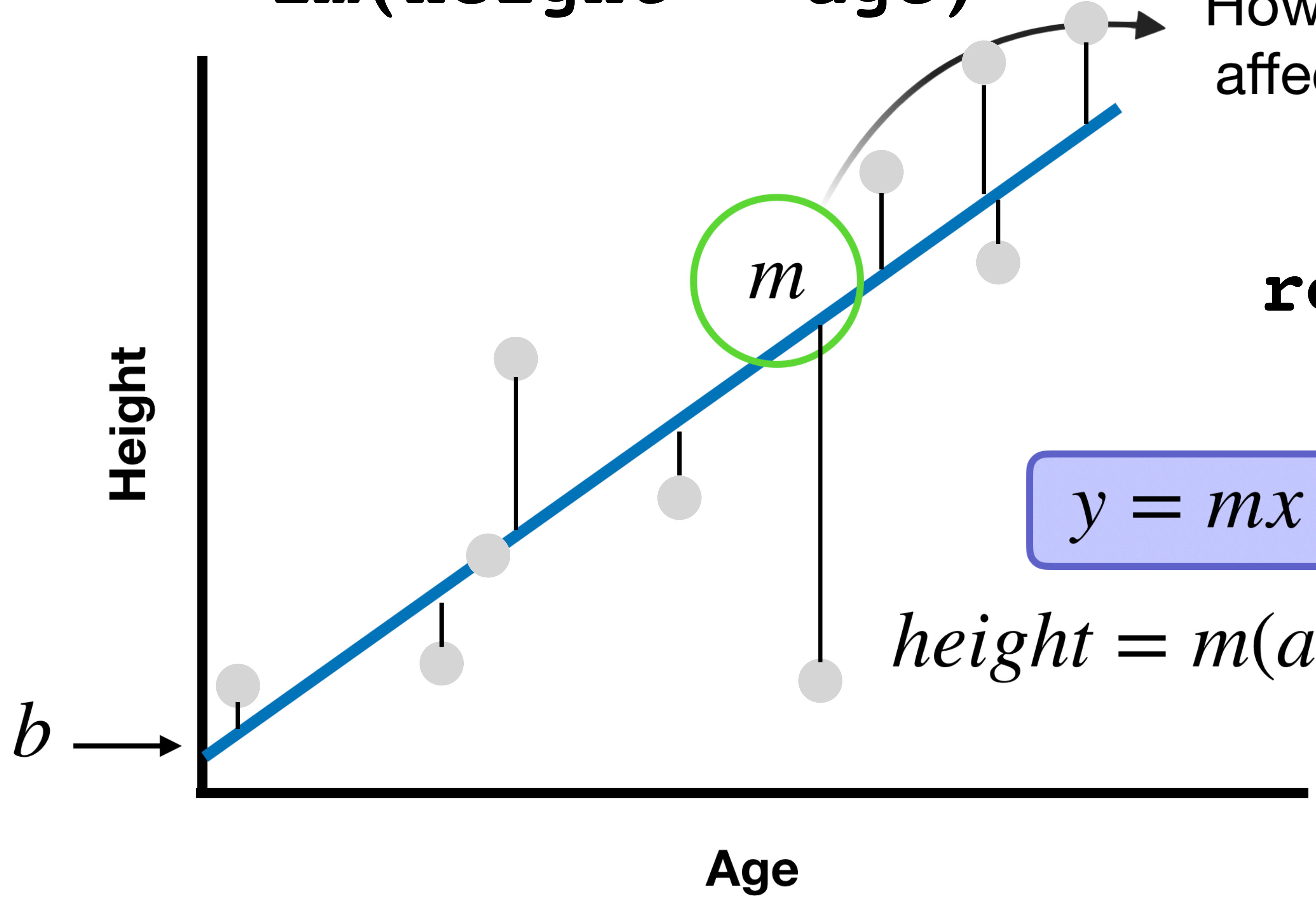


https://github.com/katiesevans/IGP_biostatistics

Summary: linear regression

`lm(height ~ age)`

How does age affect height?



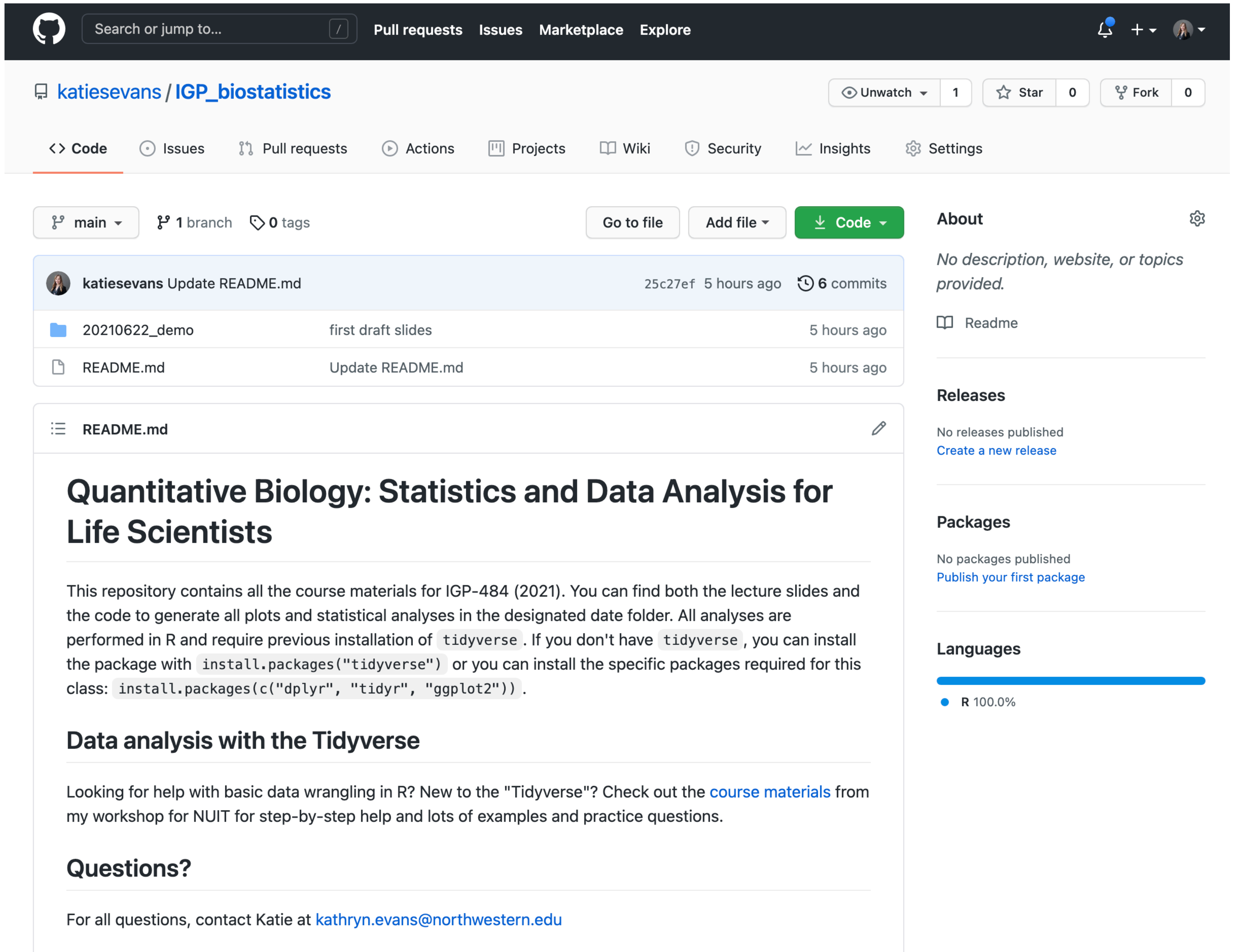
`resid(model)`

$$\boxed{y = mx + b}$$

$$height = m(age) + b$$

https://github.com/katiesevans/IGP_biostatistics

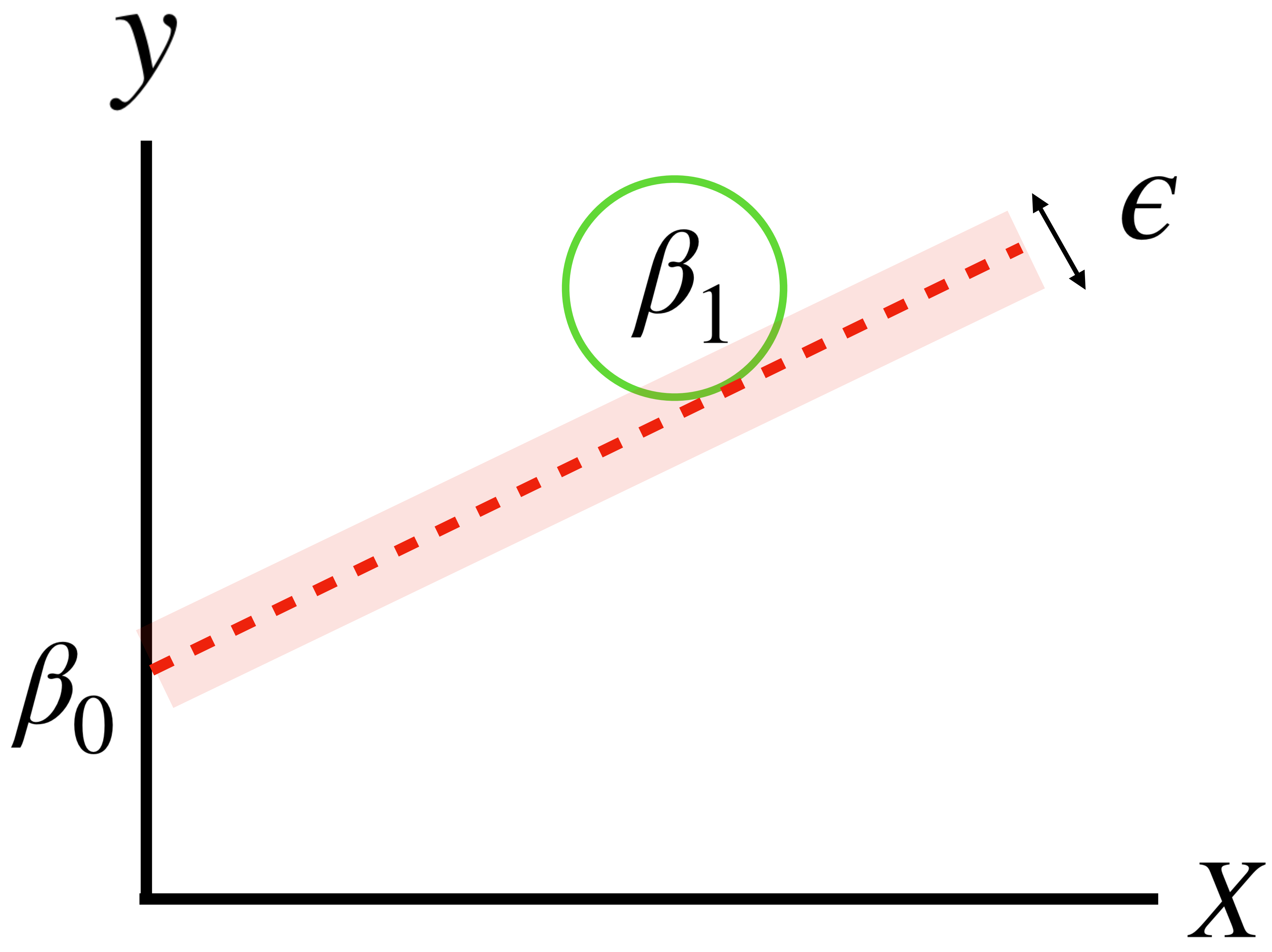
Course materials: slides and code



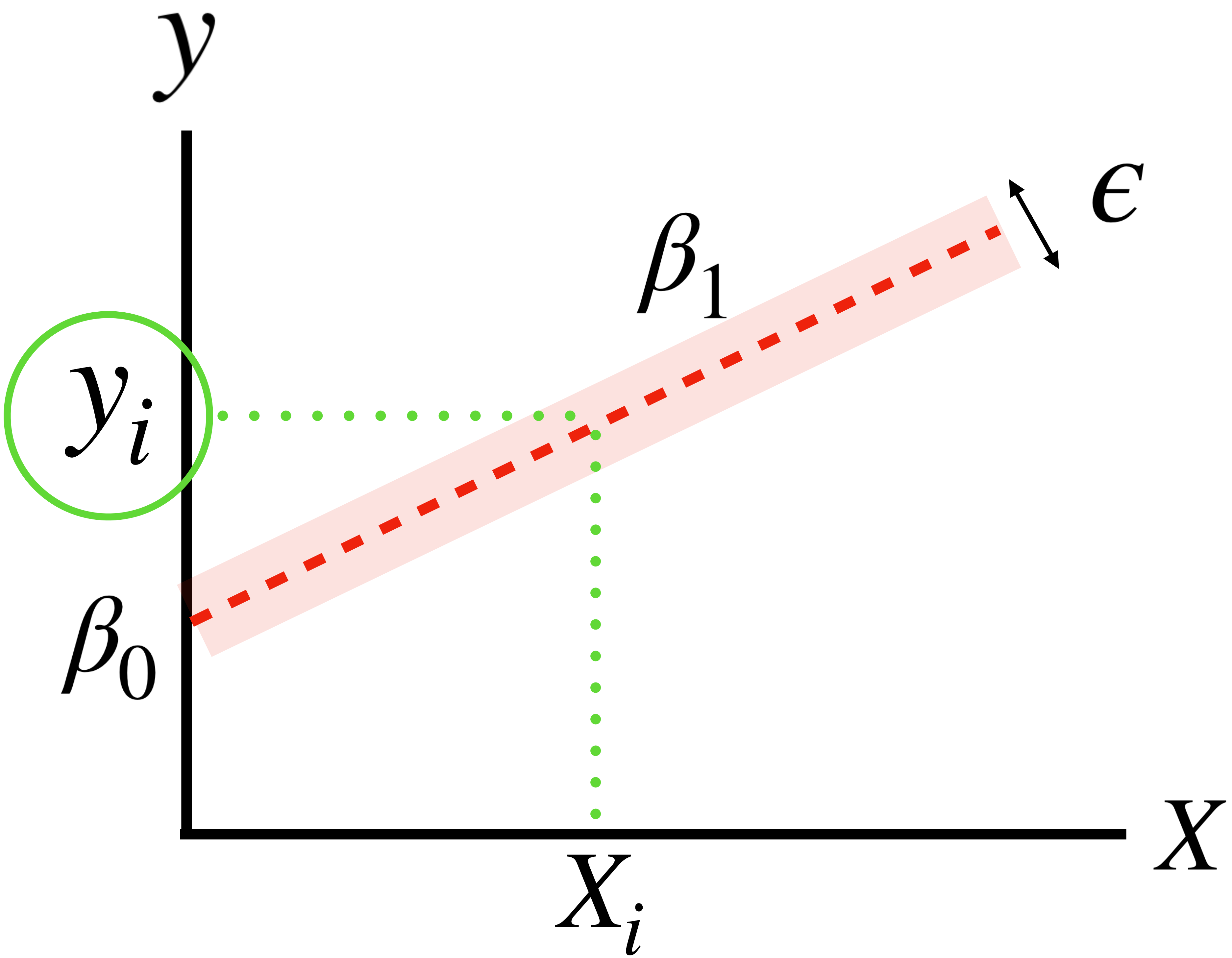
https://github.com/katiesevans/IGP_biostatistics

Extra Slides

Predicting values



Predicting values



Predicting values

```
# assign linear model to variable
model <- lm(happiness ~ income, data = income_data)
```

Income from $15k to $75k at intervals of $1,000

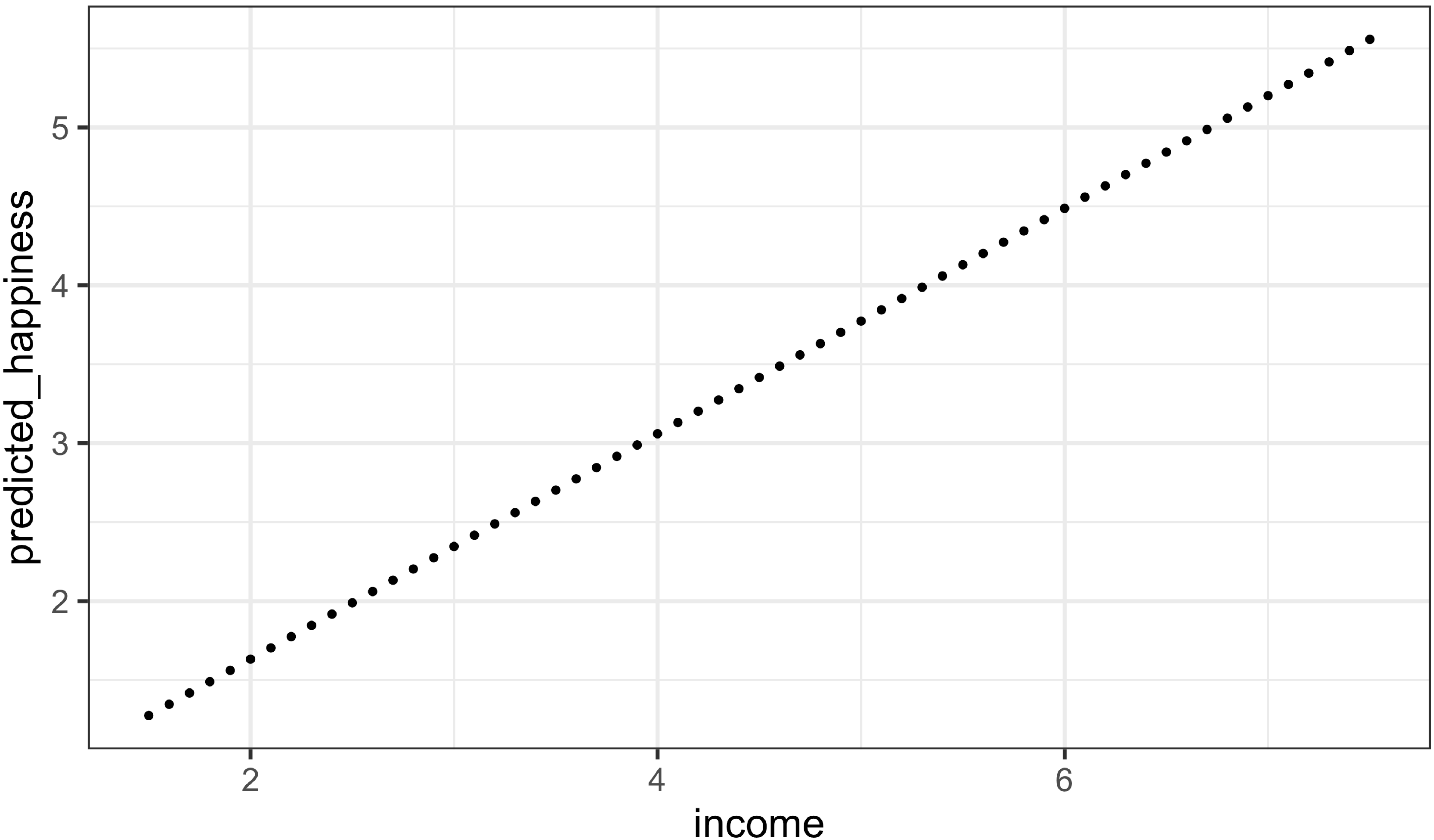
```
# generate predicted data for set of values
predicted_data <- data.frame(income = seq(1.5, 7.5, 0.1)) %>%
    dplyr::mutate(predicted_happiness = predict(model, newdata = .))


# plot predicted data
ggplot2::ggplot(predicted_data) +
    ggplot2::aes(x = income, y = predicted_happiness) +
    ggplot2::geom_point()


# plot in base R
plot(predicted_data$income, predicted_data$predicted_happiness)
```

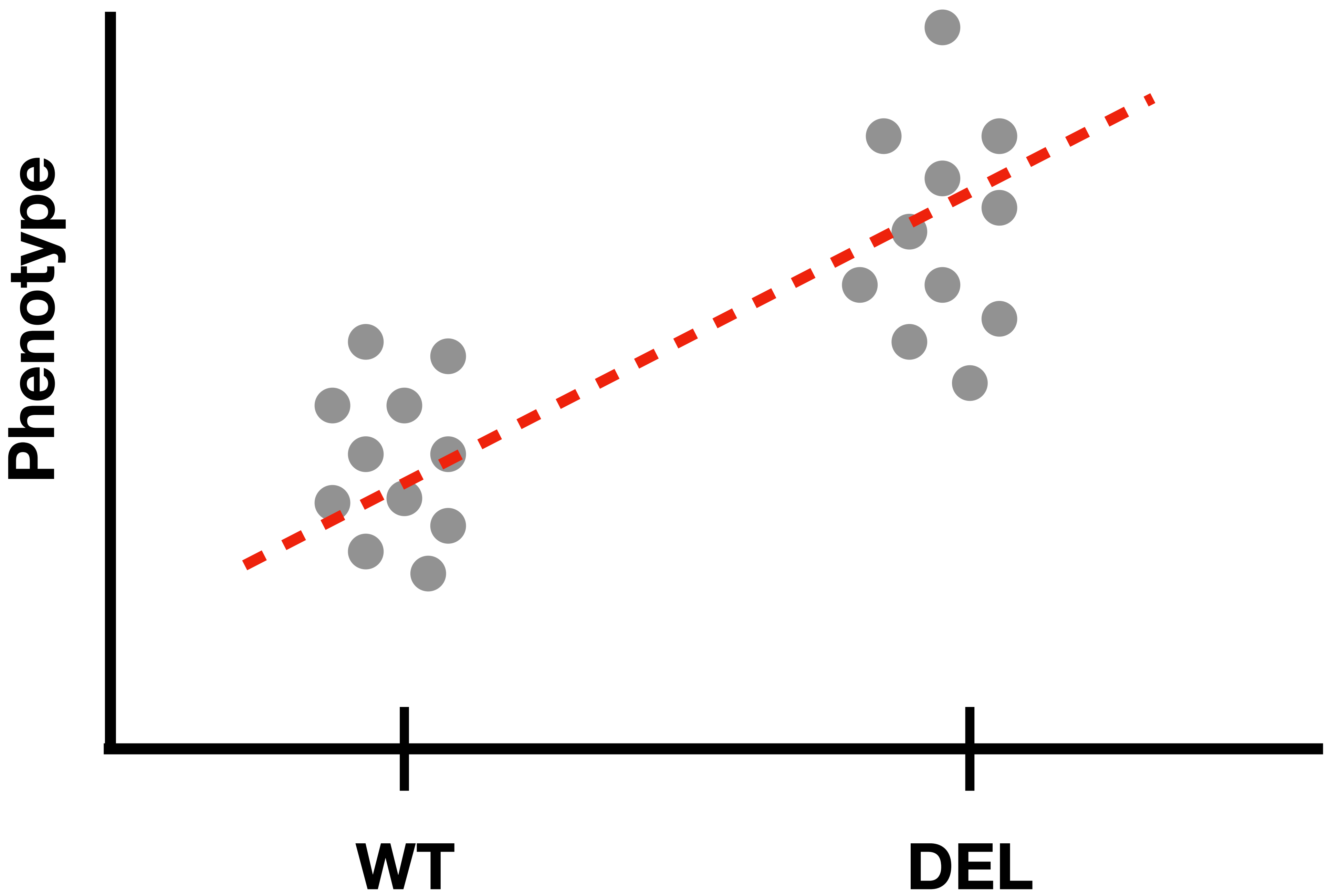
Important to not predict values outside of the data range!

Predicting values



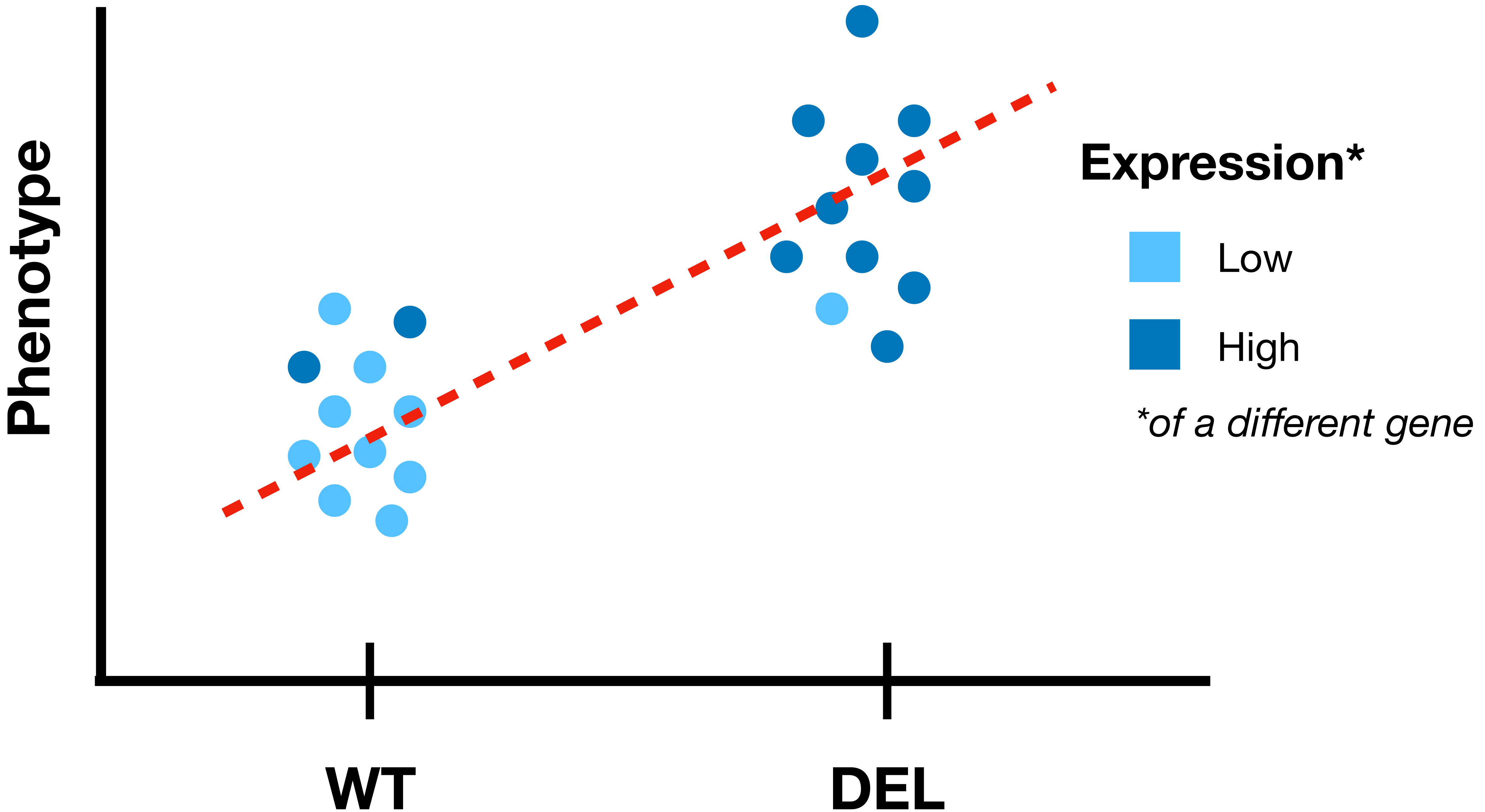
Using residual values

`lm(phenotype ~ strain)`



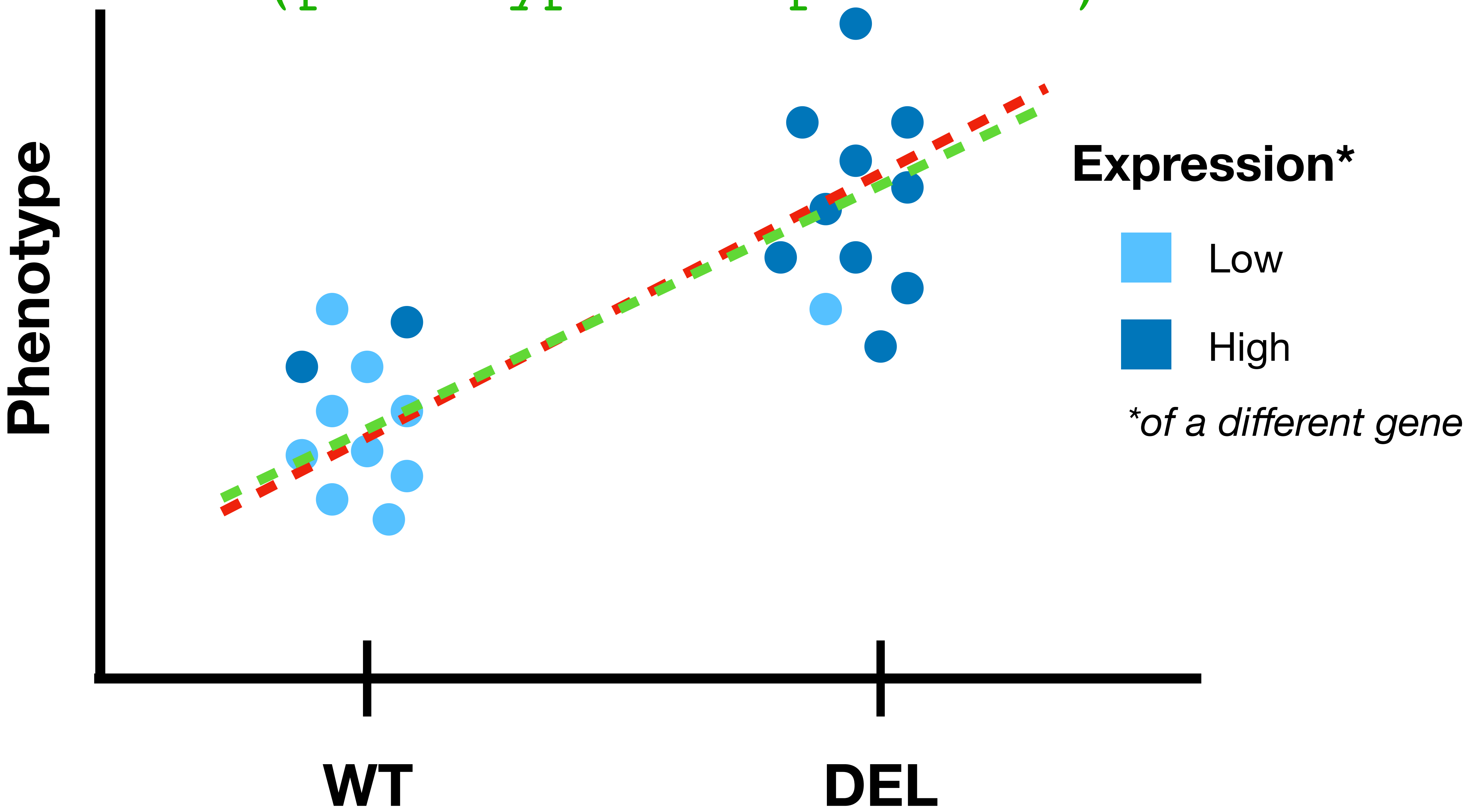
Using residual values

`lm(phenotype ~ strain)`



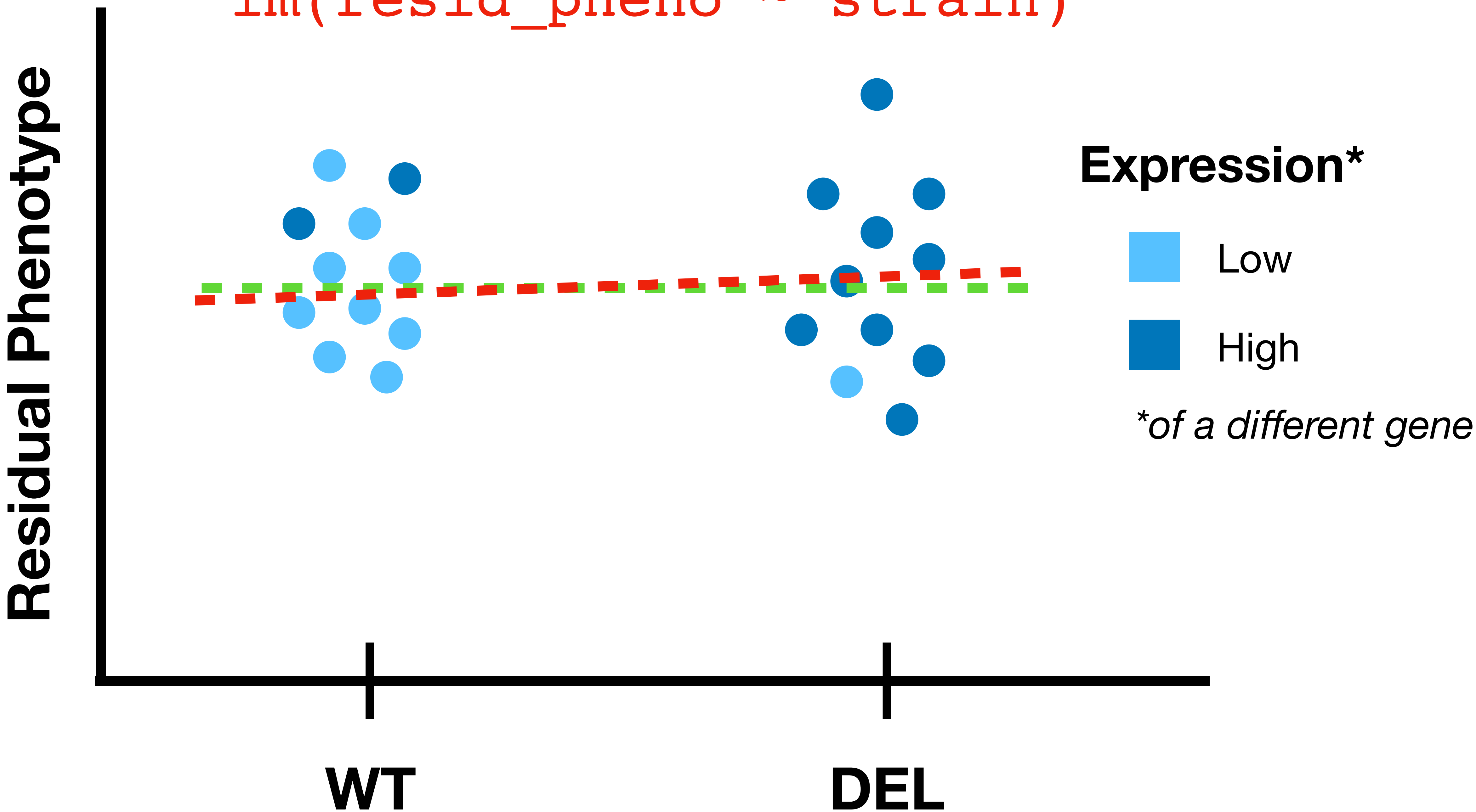
Using residual values

```
lm(phenotype ~ strain)
lm(phenotype ~ expression)
```



Using residual values

```
resid(lm(phenotype ~ expression))
lm(resid_pheno ~ strain)
```



Using residual values

Great package for tidy statistics!

```
broom::augment(model)
```

happiness	income	.fitted	.resid	.std.resid	.hat	.sigma	.cooksd
2.3144890	3.862647	2.961527	-0.64703769	-0.90205658	0.002251376	0.7182356	9.180461e-04
3.4334898	4.979381	3.758680	-0.32519010	-0.45334273	0.002183070	0.7186765	2.248227e-04
4.5993734	4.923957	3.719116	0.88025693	1.22713119	0.002147257	0.7177335	1.620203e-03
2.7911138	3.214372	2.498771	0.29234235	0.40772807	0.003053610	0.7187050	2.545969e-04
5.5963983	7.196409	5.341251	0.25514736	0.35655407	0.006973369	0.7187334	4.463777e-04
2.4585559	3.729643	2.866585	-0.40802919	-0.56888086	0.002370292	0.7185909	3.844547e-04
3.1929918	4.674517	3.541060	-0.34806836	-0.48520140	0.002036760	0.7186549	2.402367e-04
1.9071368	4.498104	3.415132	-1.50799483	-2.10208970	0.002008681	0.7156164	4.446893e-03
2.9424499	3.121631	2.432570	0.50987997	0.71118370	0.003214180	0.7184589	8.154585e-04
3.7379416	4.639914	3.516360	0.22158191	0.30888007	0.002027982	0.7187563	9.693831e-05
3.1754061	4.632840	3.511309	-0.33590329	-0.46824107	0.002026383	0.7186666	2.225930e-04
2.0090465	2.773179	2.183836	-0.17478978	-0.24388421	0.003919933	0.7187824	1.170366e-04
5.9518141	7.119479	5.286336	0.66547825	0.92983921	0.006697419	0.7181987	2.914819e-03
5.9605473	7.466653	5.534158	0.42638937	0.59616497	0.008005262	0.7185679	1.434066e-03

Using residual values

Great package for tidy statistics!

```
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