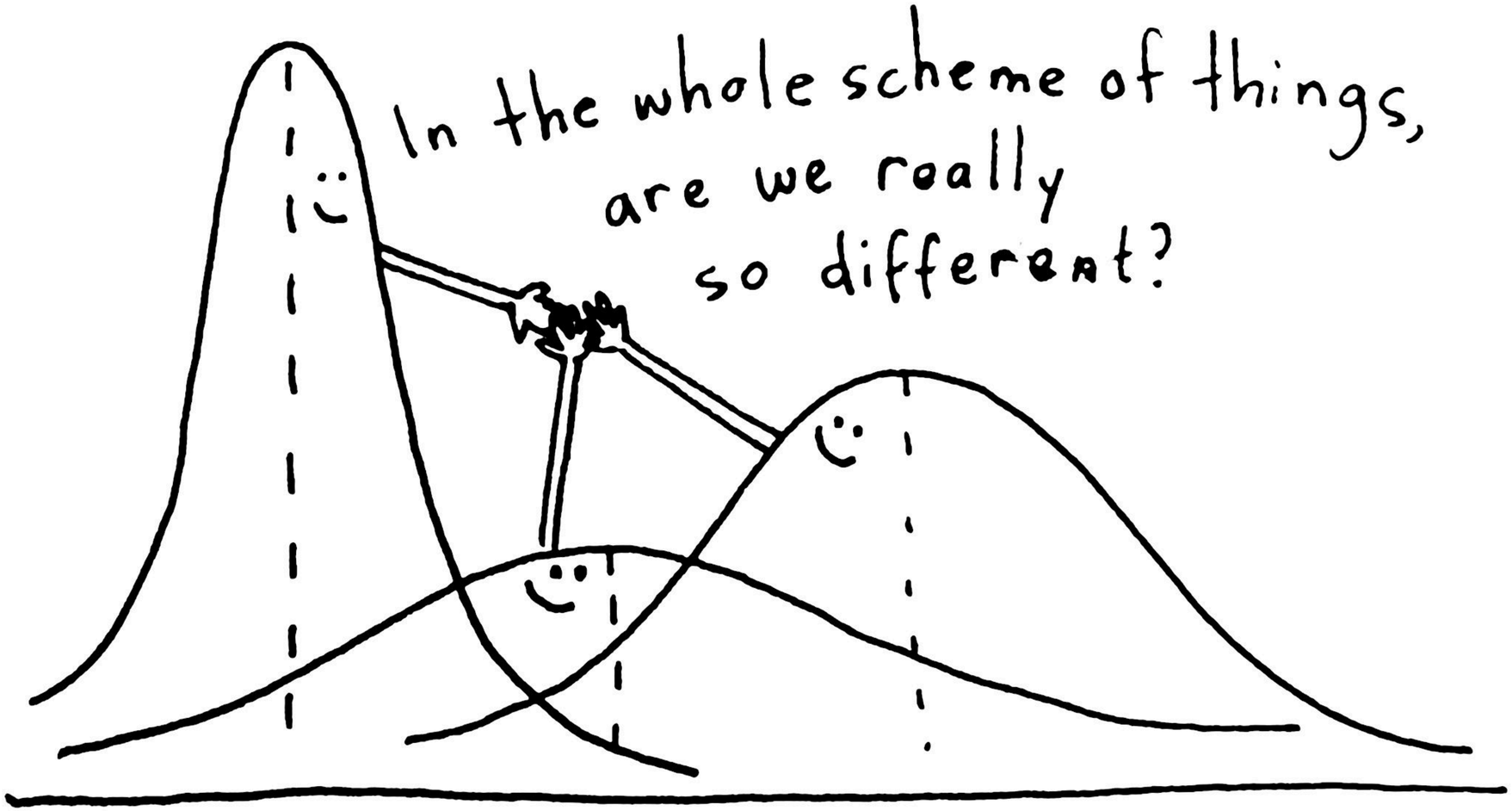
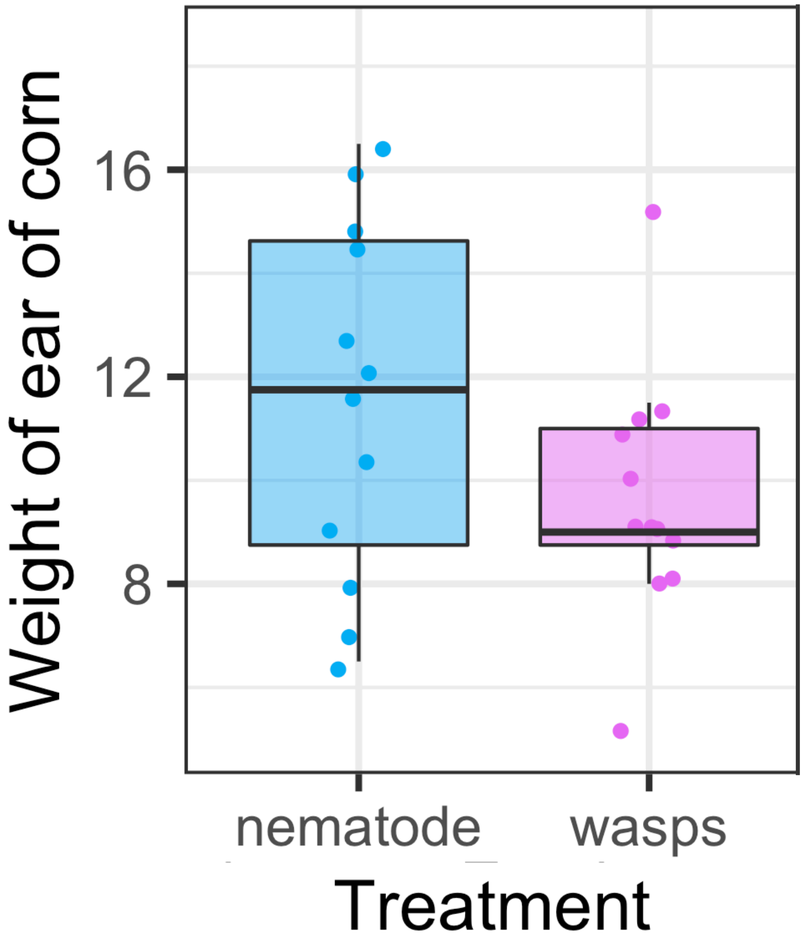


Lecture 15

11.23.21

Comparing means of many samples

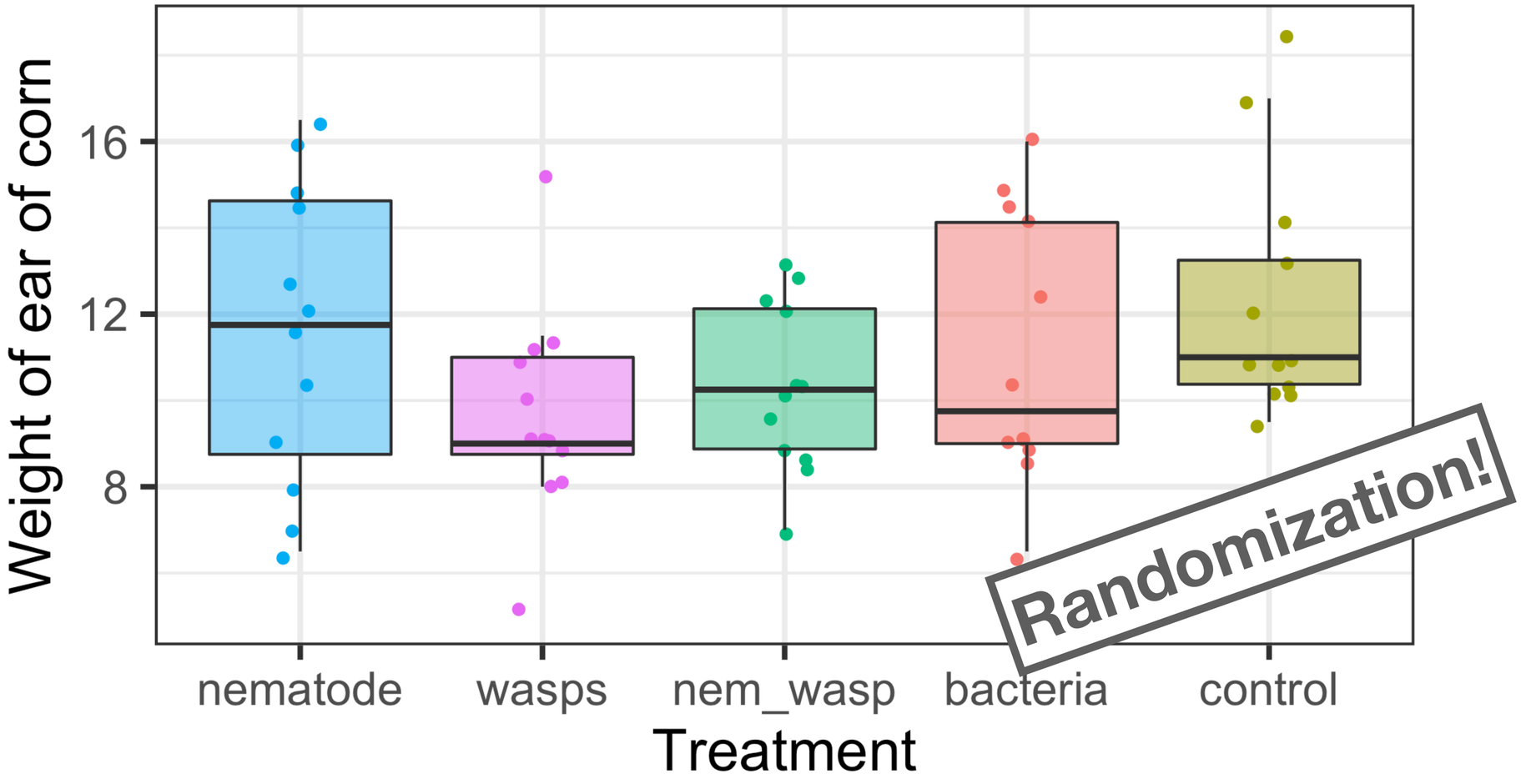
Q: Is there a difference between these groups?



t-test!

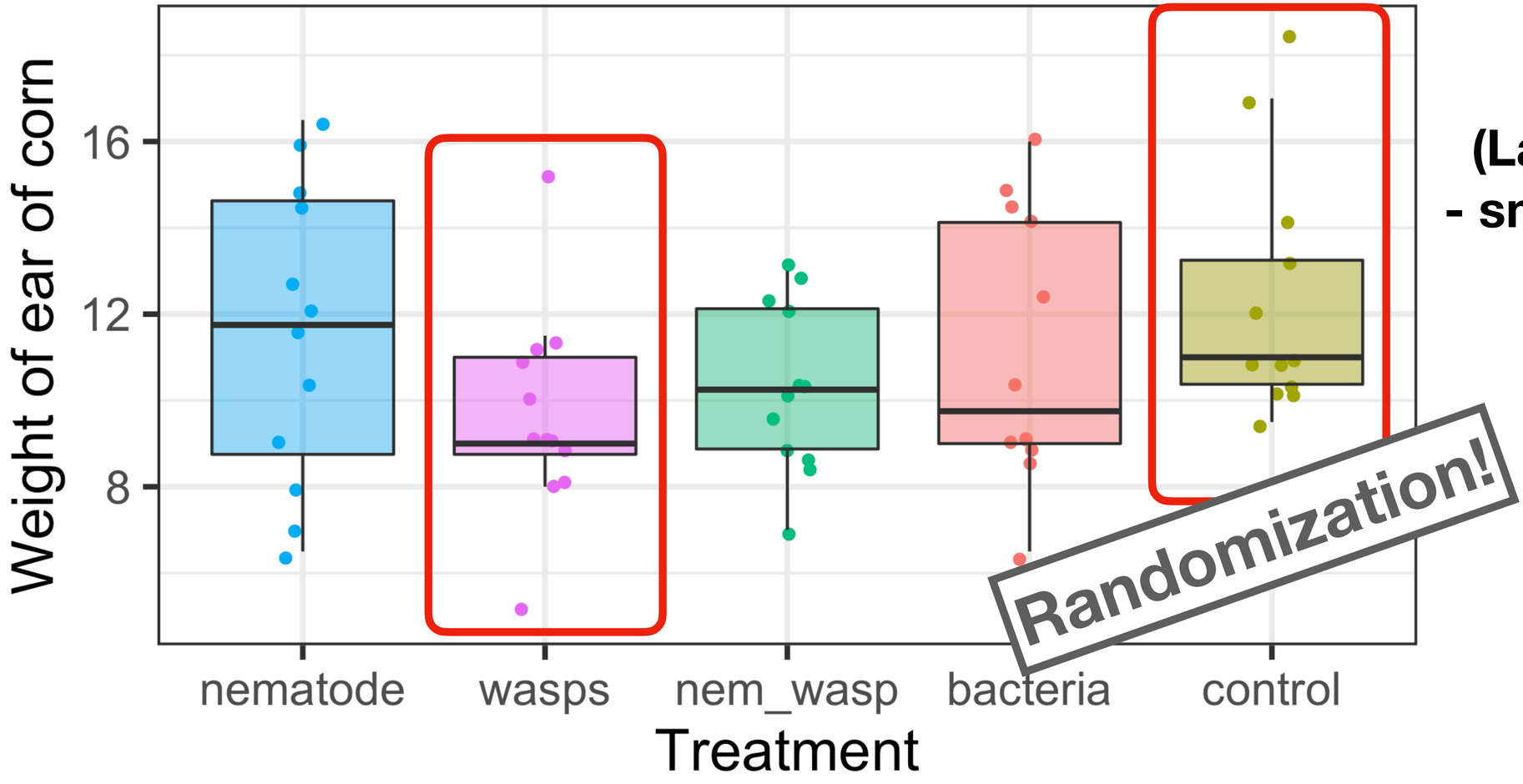
Comparing means of many samples

Q: Is there a difference between these groups?



Comparing means of many samples

Q: Is there a difference between these groups?



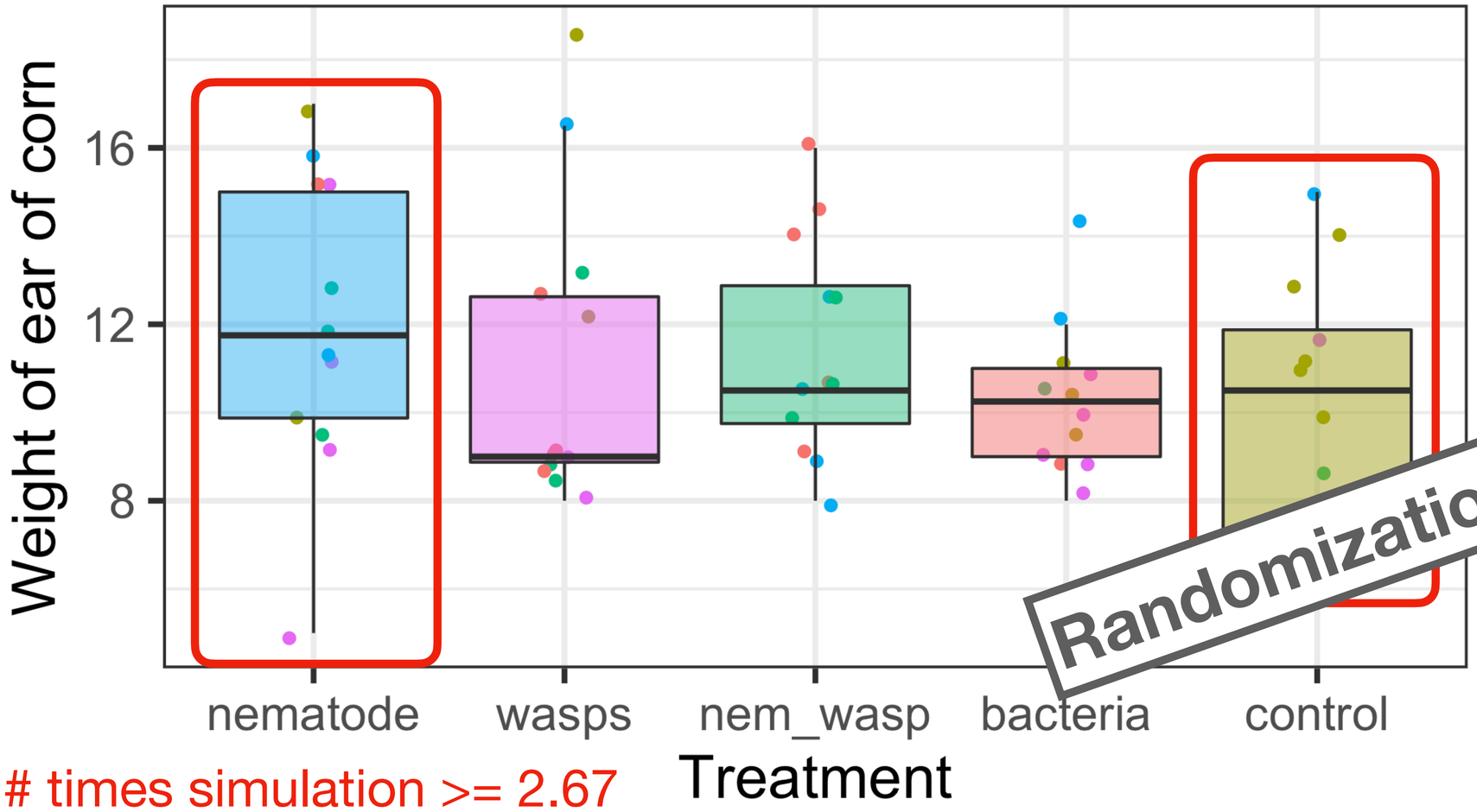
(Largest mean - smallest mean)

12.29-9.62

2.67

Comparing means of many samples

Q: Is there a difference between these groups?



(Largest mean - smallest mean)

12.29-9.62

2.67

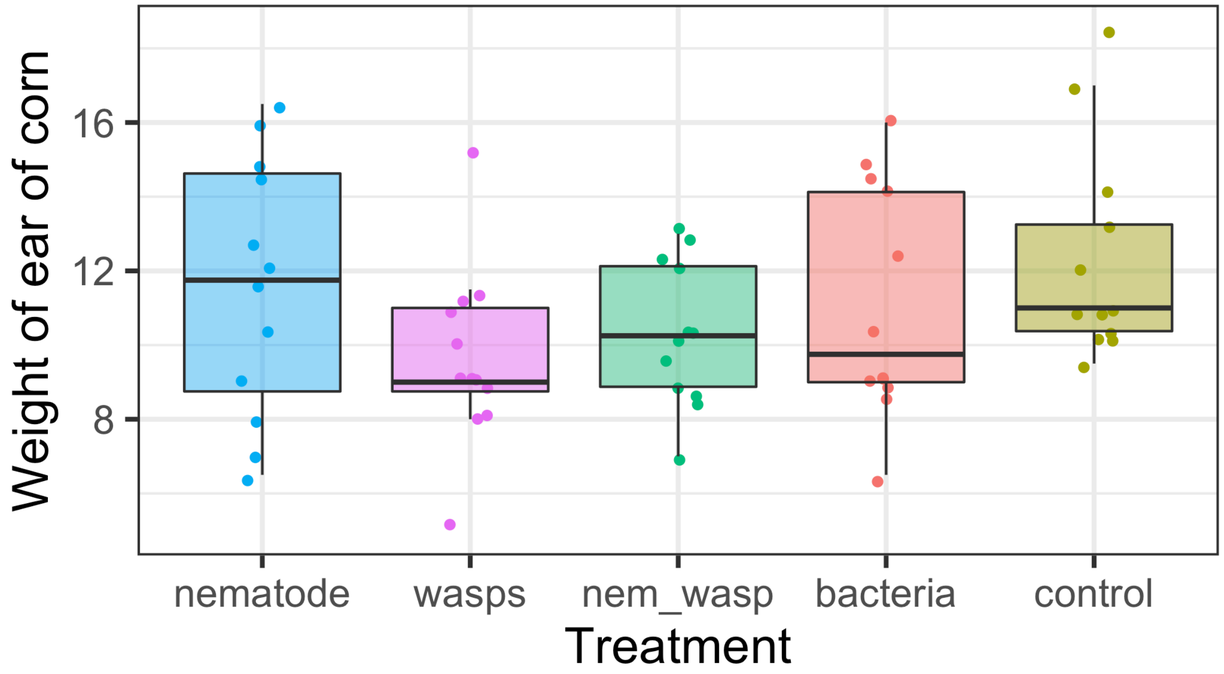
(1) 12-10.1

1.9

P-value = $\frac{\text{\# times simulation >= 2.67}}{\text{\# simulations}}$

Comparing means of many samples

ANOVA (analysis of variance)

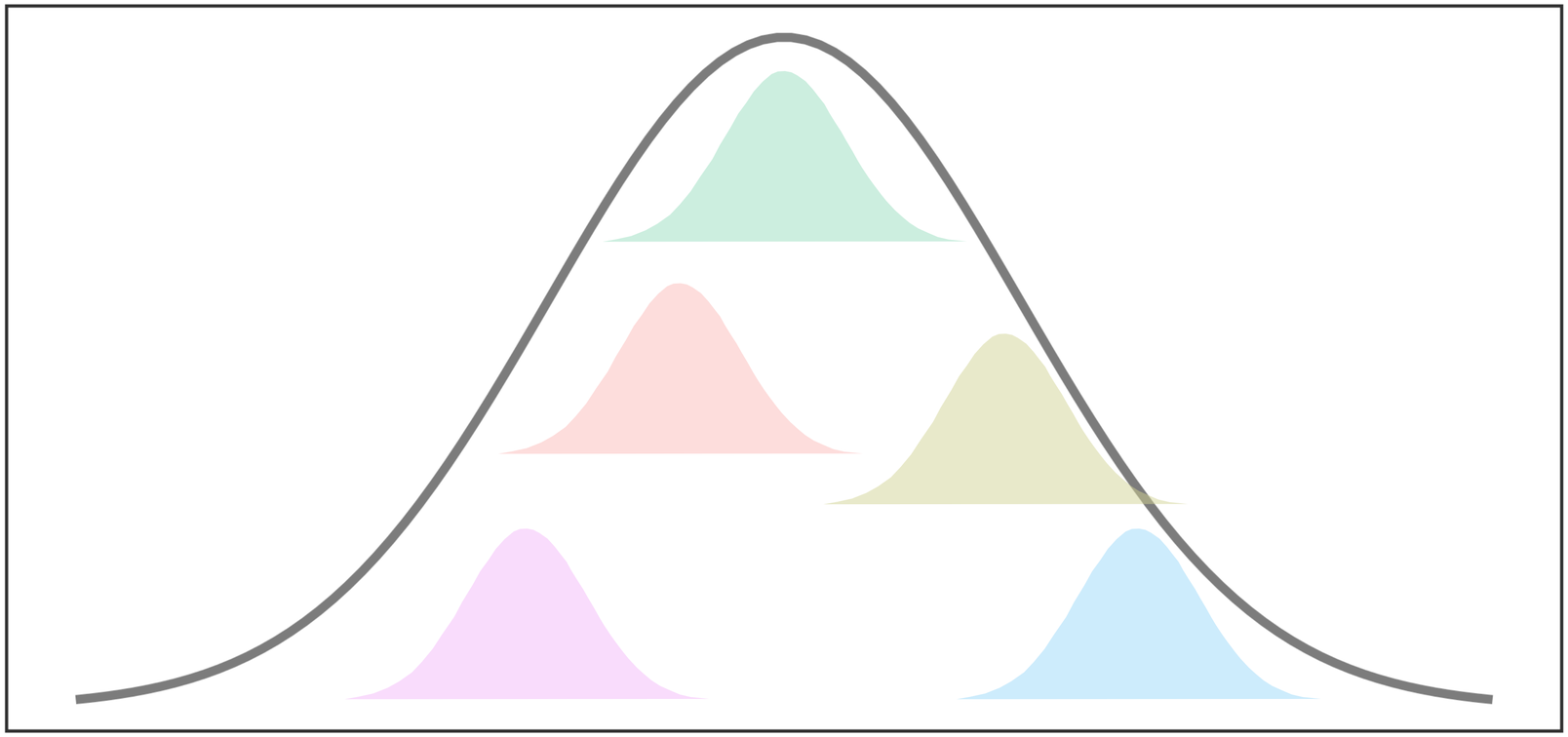


$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_A : Null is false (i.e. at least some of the means are different)

Comparing means of many samples

ANOVA (analysis of variance)

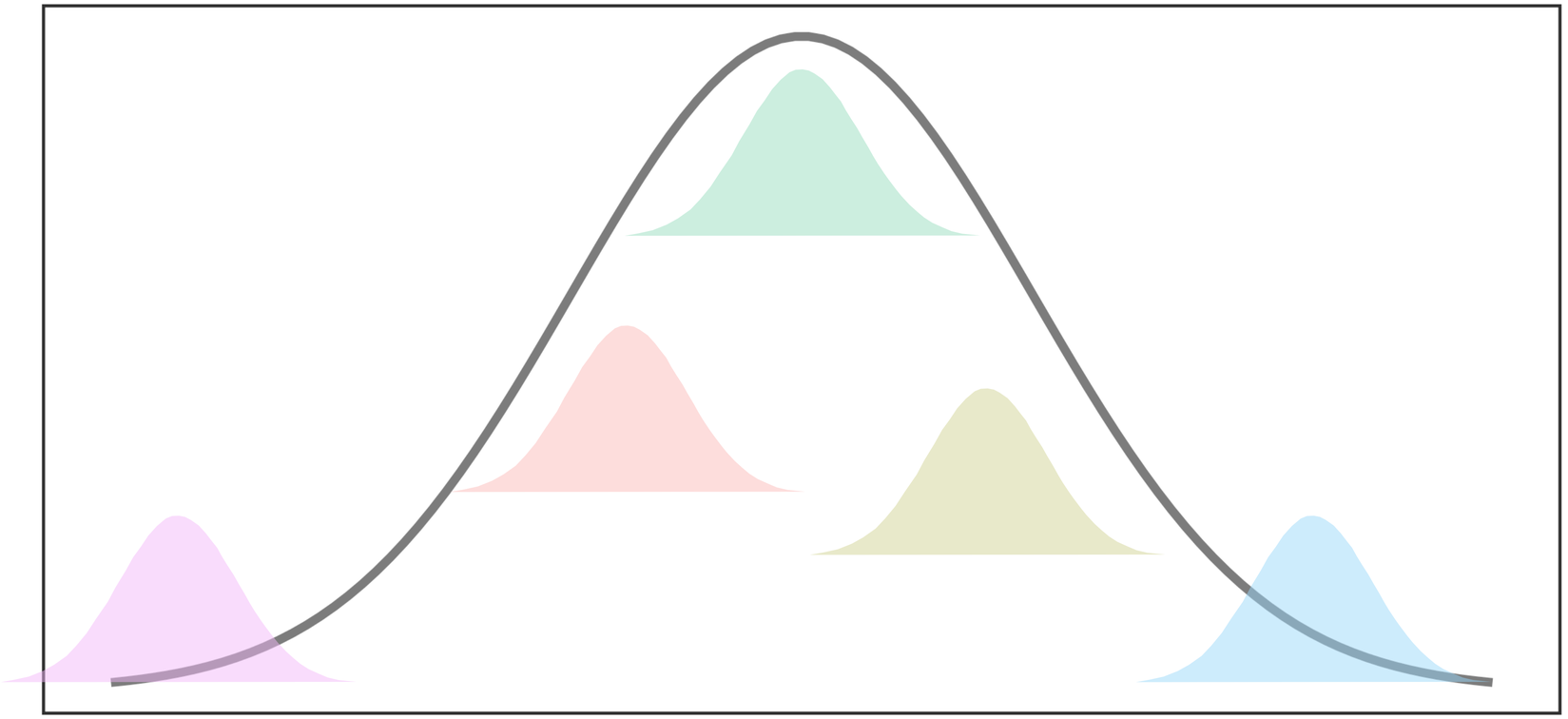


★ H_0 : All groups came from same population distribution

H_A : Null is false (i.e. at least some of the means are different)

Comparing means of many samples

ANOVA (analysis of variance)



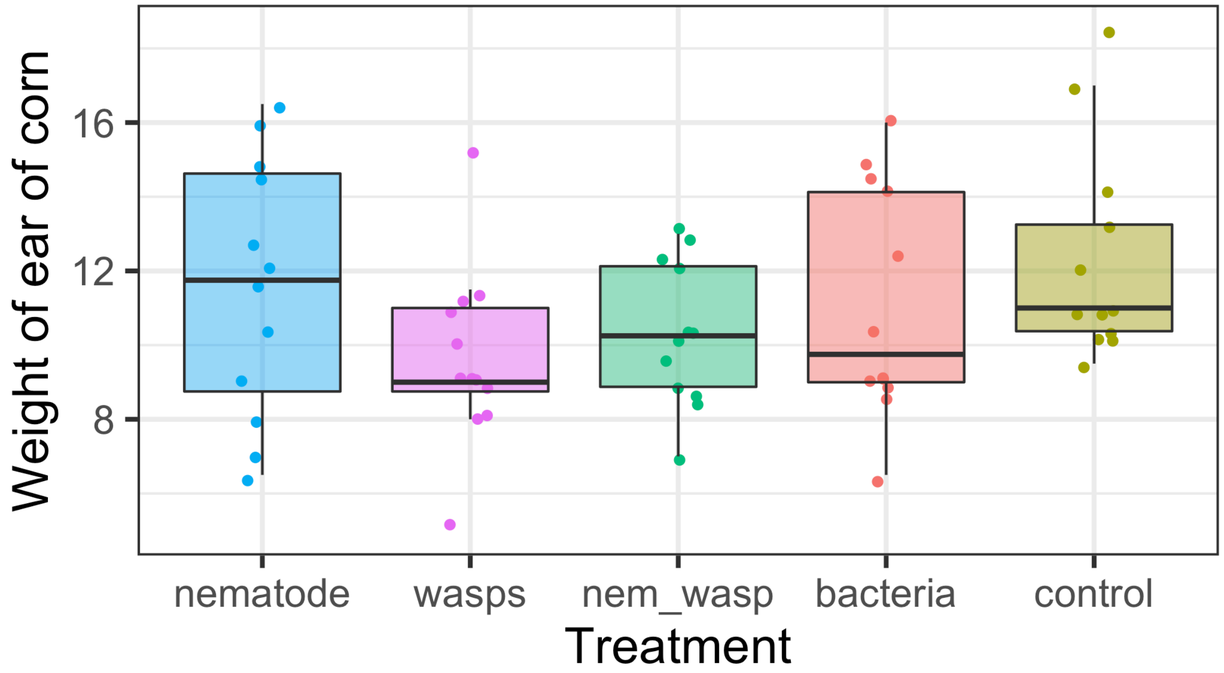
H_0 : All groups came from same population distribution

H_A : Null is false (i.e. at least some of the means are different)

Comparing means of many samples

✗ Pairwise *t*-tests?

Problem of multiple comparisons!



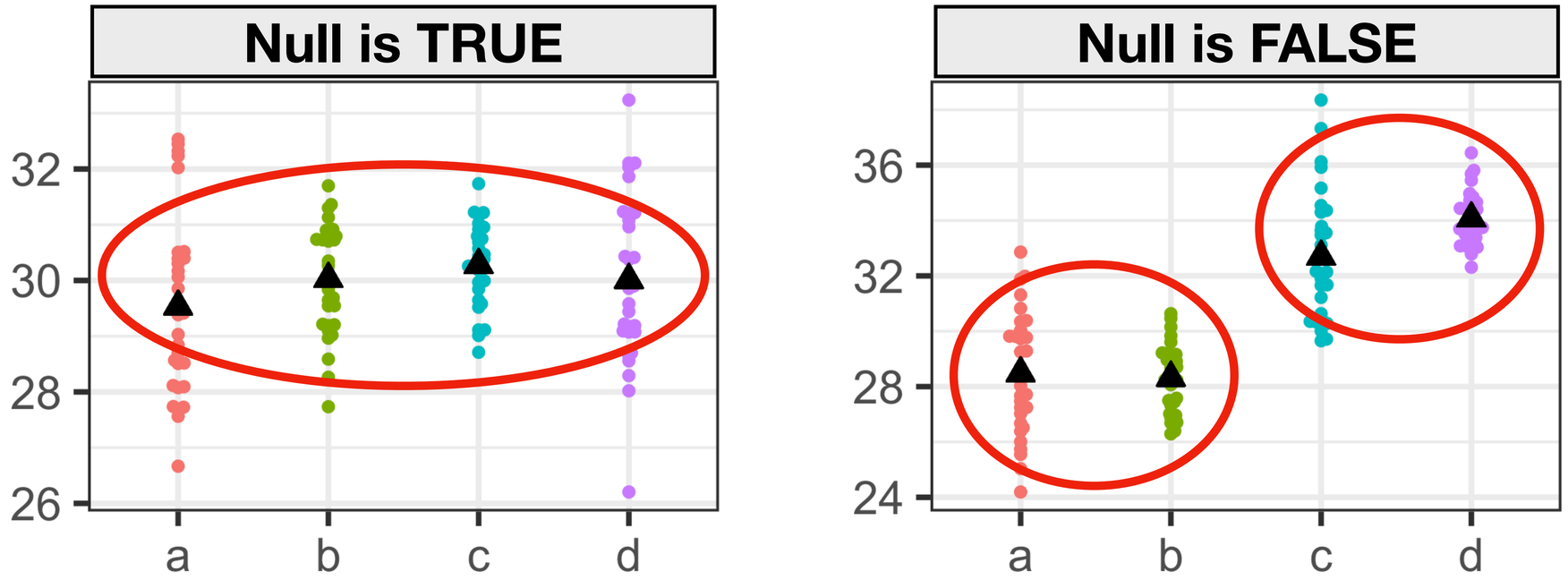
$$P(+) = 1 - (1 - 0.05)^n$$

# Groups	# Tests	P(+)
2	1	0.05
3	3	0.142
4	6	0.264
5	10	0.401

$$H_0 : \mu_1 = \mu_2 \quad H_0 : \mu_1 = \mu_4 \quad H_0 : \mu_2 = \mu_3$$

$$H_0 : \mu_1 = \mu_3 \quad H_0 : \mu_1 = \mu_5 \quad H_0 : \mu_2 = \mu_4 \quad \ldots$$

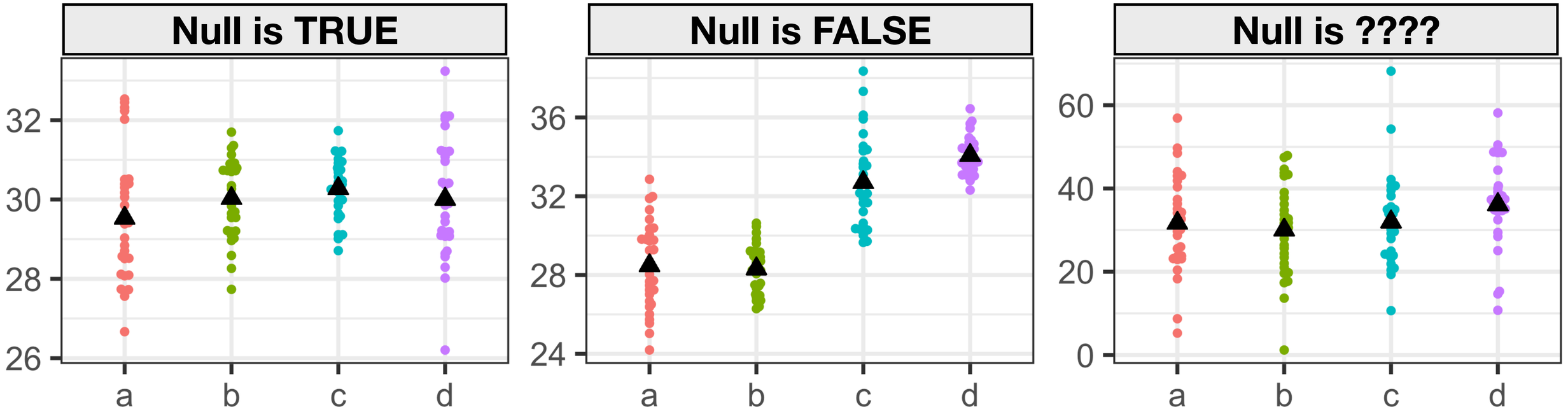
A graphical view of ANOVA



$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_A : Null is false (i.e. at least some of the means are different)

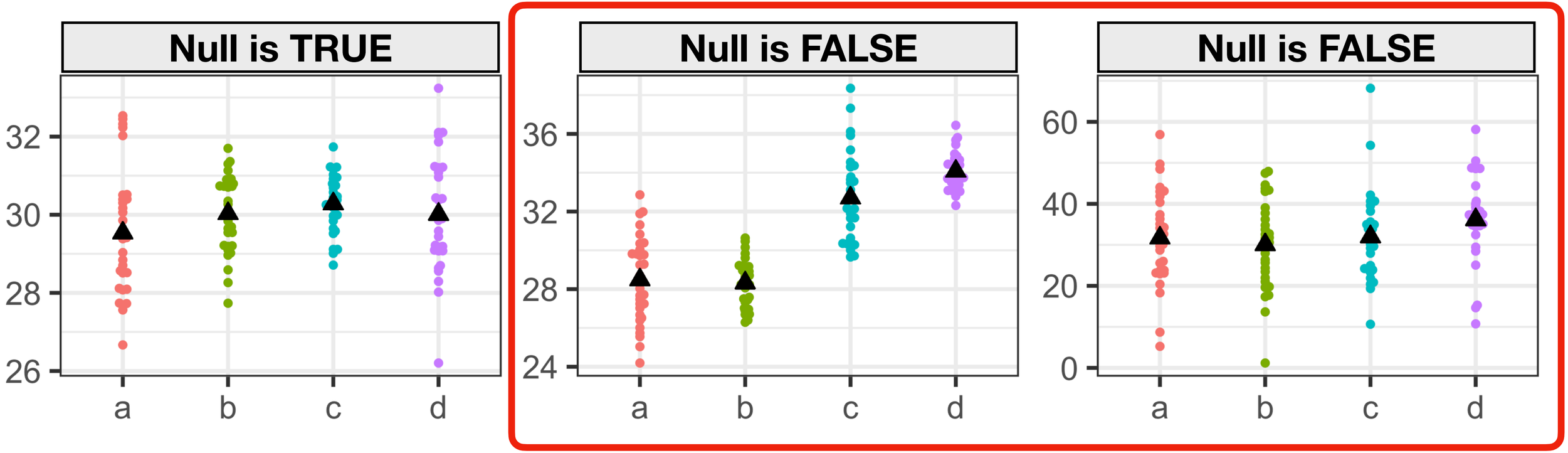
A graphical view of ANOVA



$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_A : Null is false (i.e. at least some of the means are different)

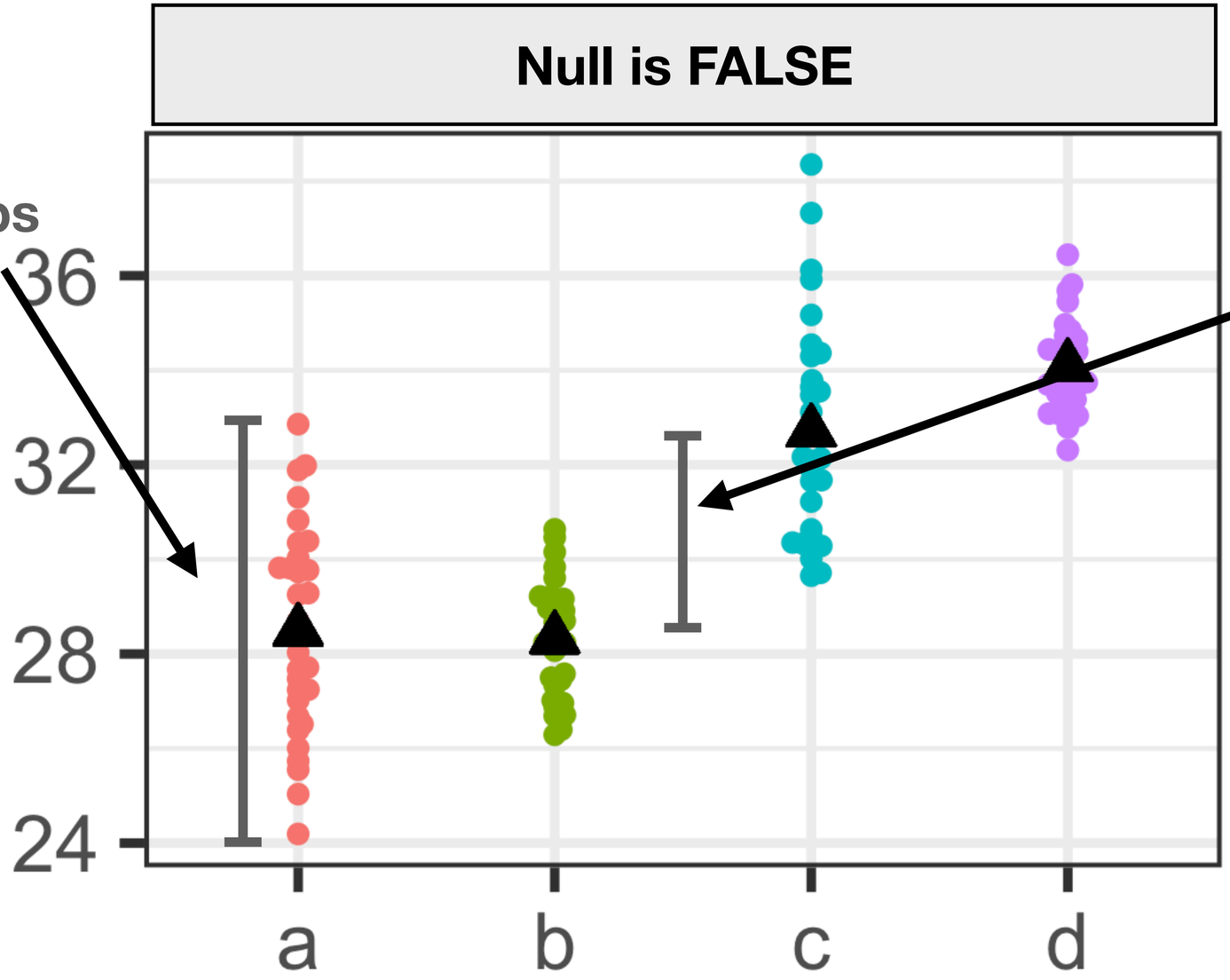
A graphical view of ANOVA



$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

H_A : Null is false (i.e. at least some of the means are different)

“Analysis of variance” (ANOVA)



Variation within groups [SS(within)]

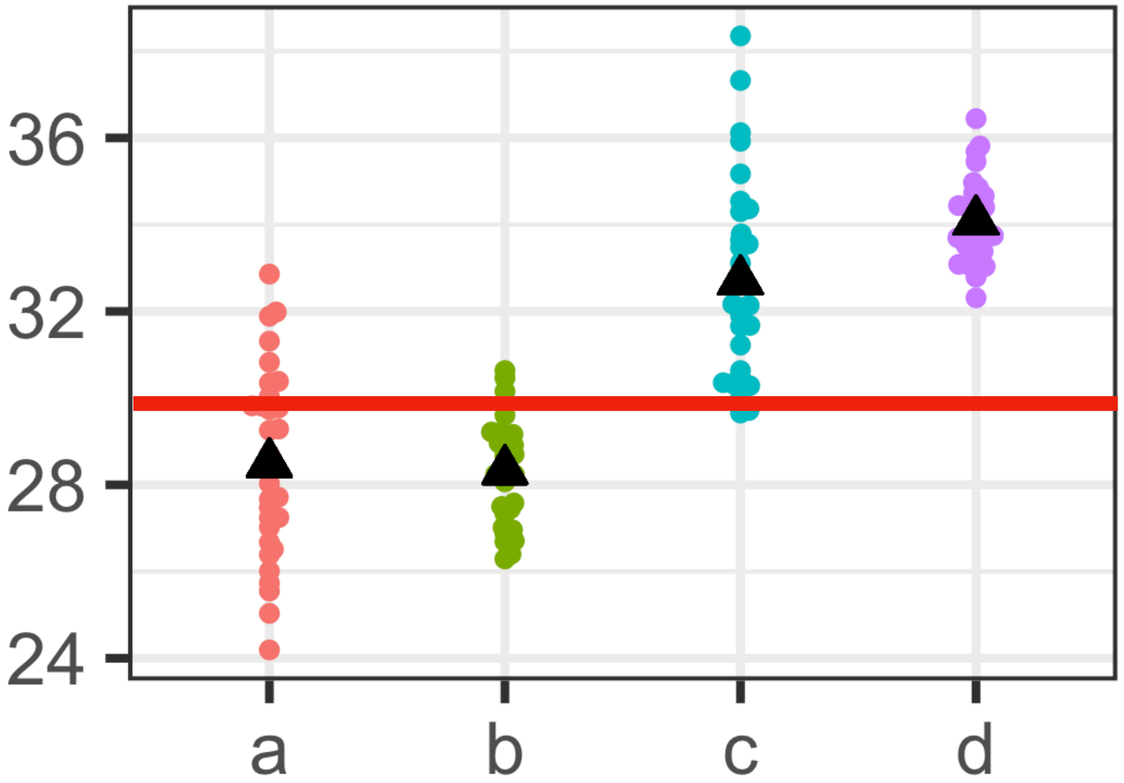
Variation between groups [SS(between)]

H_0 TRUE:
SS(within) >= SS(between)

H_0 FALSE:
SS(within) < SS(between)

But first, some notation

Grand mean $(\bar{\bar{y}})$ **[Mean of all the observations]**



$$= \frac{\textbf{Sum of all values}}{\textbf{Total number of values}}$$

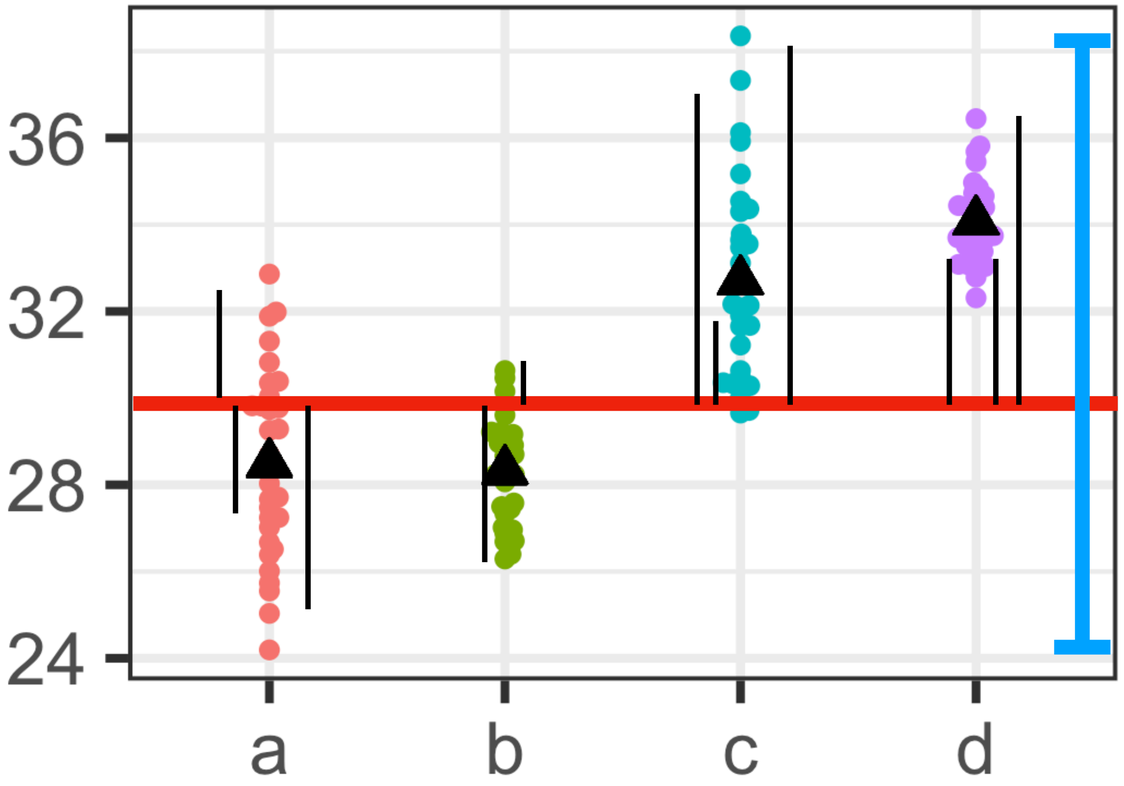
= *Mean of group means*

IF *group n is same…*

But first, some notation

Total sum of squares (SST)

(df = total observations - 1)



[Total variability among all groups & observations]

$$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$$

= Sum of squared deviations between each data point and the grand mean

$$SST = SSW + SSB$$

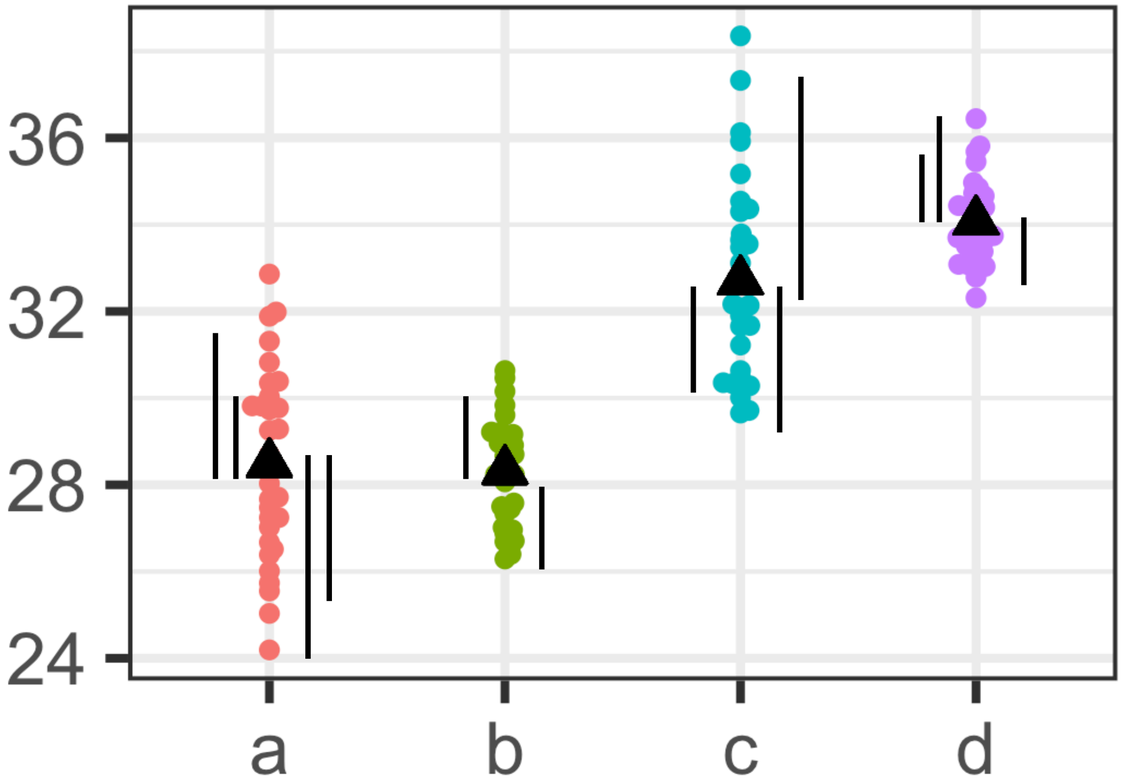
= Sum of squared WITHIN + sum of square BETWEEN

But first, some notation

Sum of squares within (SSW)

[Variability within groups]

(df = total observations - number of groups)



$$SSW = \sum_{i=1}^{I} \sum_{j=1}^{n_j} (y_{ij} - \bar{y}_i)^2$$

= Sum of squared deviations between each data point and their group mean

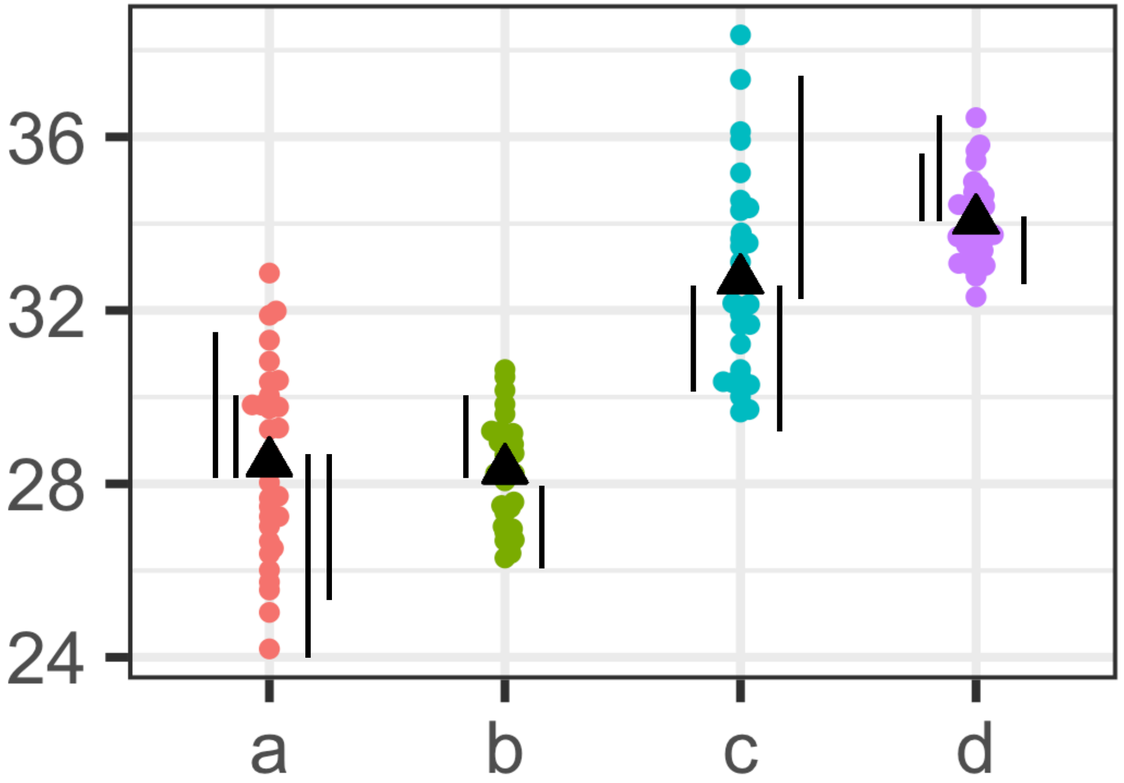
$$SSW = \sum_{i=1}^{I} (n_i - 1) s_i^2 \qquad \frac{(y_i - \bar{y})^2}{n - 1}$$

But first, some notation

Sum of squares within (SSW)

[Variability within groups]

(df = total observations - number of groups)



$$SSW = \sum_{i=1}^{I} \sum_{j=1}^{n_j} (y_{ij} - \bar{y}_i)^2$$

= Sum of squared deviations between each data point and their group mean

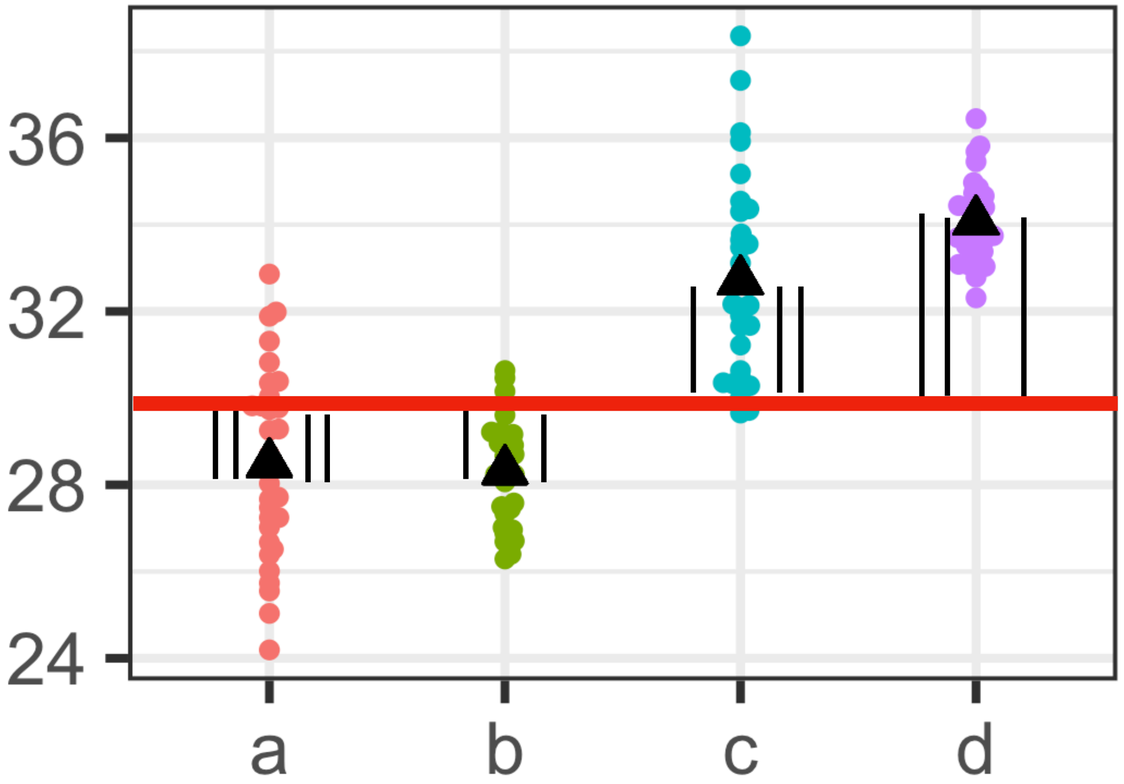
$$SSW = \sum_{i=1}^{I} (n_i - 1) \frac{(y_i - \bar{y})^2}{n - 1}$$

But first, some notation

Sum of squares between (SSB)

[Variability between groups]

(df = total groups - 1)



$$SSB = \sum_{i=1}^{I} \sum_{j=1}^{n_j} (\bar{y}_i - \bar{\bar{y}})^2$$

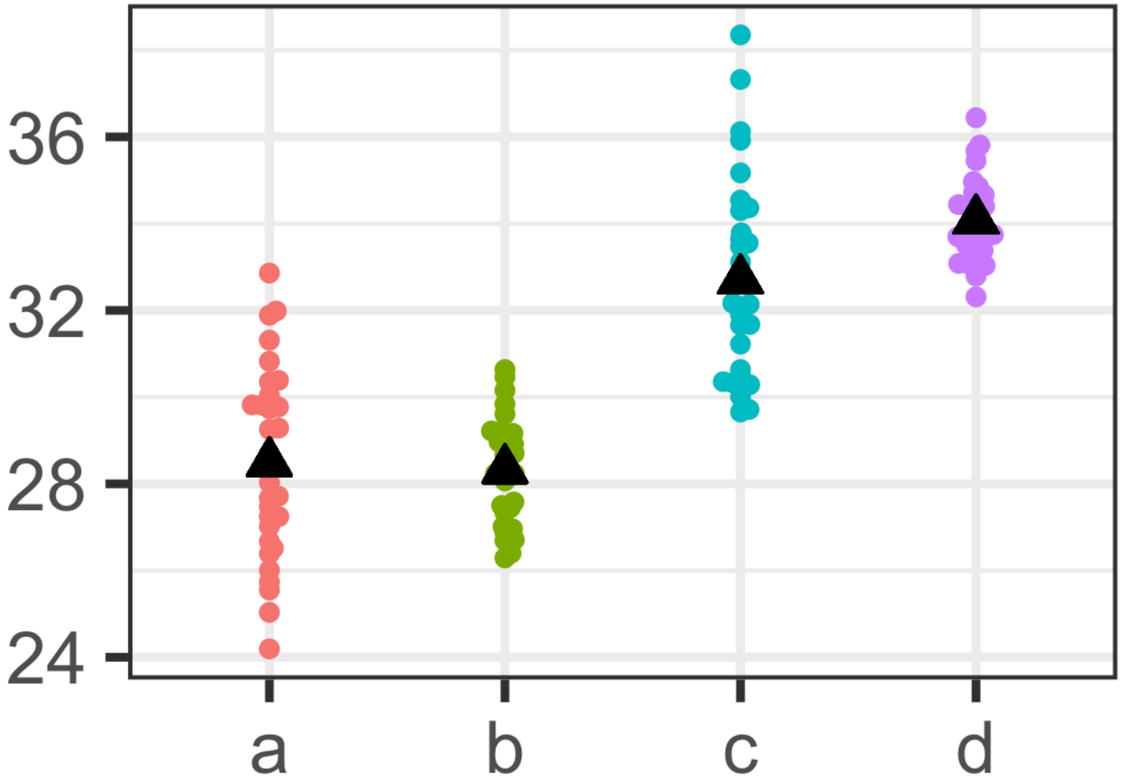
= Sum of squared deviations between each group mean and the grand mean (repeated for each data point)

$$SSB = \sum_{i=1}^{I} n_i (\bar{y}_i - \bar{\bar{y}})^2$$

But first, some notation

Mean square (MS)

[Ratio of SS / degrees of freedom]



$$MS = \frac{SS}{df}$$

= Variation between (MSB) or within (MSW) groups

But first, some notation

Understand the concept but don't need to memorize these equations

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	Groups - 1	$SSB = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2$	SSB/df
Within groups	Observations - Groups	$SSW = \sum_{i=1}^{I} (n_i - 1)s_i^2$	SSW/df
Total	Observations - 1	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

Let’s try out some real data…

Data: weight gain for three different diets

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	

Let's try out some real data…

Data: weight gain for three different diets

	Diet 1	**Diet 2**	**Diet 3**
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$			
Mean = $\bar{y}_i$			
SD = s_i			

Let's try out some real data…

Data: weight gain for three different diets

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$			
Mean = $\bar{y}_i$			
SD = s_i			

Sum:

8 + 16 + 9 = 33

Mean:

33 / 3 = 11

SD:

$$\sqrt{\frac{(8-11)^2+(16-11)^2+(9-11)^2}{3-1}}$$

= 4.359

Let's try out some real data…

Data: weight gain for three different diets

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$	33		
Mean = $\bar{y}_i$	11		
SD = s_i	4.359		

Sum:

8 + 16 + 9 = 33

Mean:

33 / 3 = 11

SD:

$$\sqrt{\frac{(8-11)^2+(16-11)^2+(9-11)^2}{3-1}}$$

= 4.359

Let's try out some real data…

Data: weight gain for three different diets

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$	33	75	48
Mean = $\bar{y}_i$	11	15	12
SD = s_i	4.359	4.950	4.967

Sum:

8 + 16 + 9 = 33

Mean:

33 / 3 = 11

SD:

$$\sqrt{\frac{(8-11)^2+(16-11)^2+(9-11)^2}{3-1}}$$

= 4.359

Let’s try out some real data…

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	3 - 1 = 2	$SSB = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2$	SSB/df
Within groups	12 - 3 = 9	$SSW = \sum_{i=1}^{I} (n_i - 1)s_i^2$	SSW/df
Total	12 - 1 = 11 ✓	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

Let's try out some real data…

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$	33	75	48
Mean = $\bar{y}_i$	11	15	12
SD = s_i	4.359	4.950	4.967

$$SSW = \sum_{i=1}^{I} (n_i - 1)s_i^2$$

$(3 - 1)(4.359)^2 + (5-1)(4.950)^2 + (4-1)(4.967)^2$

$= 210.025$

Let’s try out some real data…

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	3 - 1 = 2	$SSB = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2$	SSB/df
Within groups	12 - 3 = 9	210.025	210.025/9 = 23.336
Total	12 - 1 = 11	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

Let's try out some real data…

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$	33	75	48
Mean = $\bar{y}_i$	11	15	12
SD = s_i	4.359	4.950	4.967

$$SSB = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2$$

Grand mean:

(33 + 75 + 48) / (3 + 5 + 4) = 13

(3)(11-13)^2 + (5)(15-13)^2 + (4)(12-13)^2

= 36

Let’s try out some real data…

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	3 - 1 = 2	36	36/2 = 18
Within groups	12 - 3 = 9	210.025	210.025/9 = 23.336
Total	12 - 1 = 11	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

Let's try out some real data…

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$	33	75	48
Mean = $\bar{y}_i$	11	15	12
SD = s_i	4.359	4.950	4.967

$$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$$

$$(SST = SSW + SSB = 246.025)$$

$(8\text{-}13)^2 + (16\text{-}13)^2 + (9\text{-}13)^2$
$(9\text{-}13)^2 + (16\text{-}13)^2 + (21\text{-}13)^2 +$
$(11\text{-}13)^2 + (18\text{-}13)^2 + (15\text{-}13)^2 +$
$(10\text{-}13)^2 + (17\text{-}13)^2 + (6\text{-}13)^2$

= 246 ✓

Let’s try out some real data…

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	3 - 1 = 2	36	36/2 = 18
Within groups	12 - 3 = 9	210.025	210.025/9 = 23.336
Total	12 - 1 = 11	246	

Let's try out some real data…

	Diet 1	Diet 2	Diet 3
	8	9	15
	16	16	10
	9	21	17
		11	6
		18	
n_i	3	5	4
Sum = $\sum y_{ij}$	33	75	48
Mean = $\bar{y}_i$	11	15	12
SD = s_i	4.359	4.950	4.967

Pooled variance:

MS(within)

Pooled standard deviation:

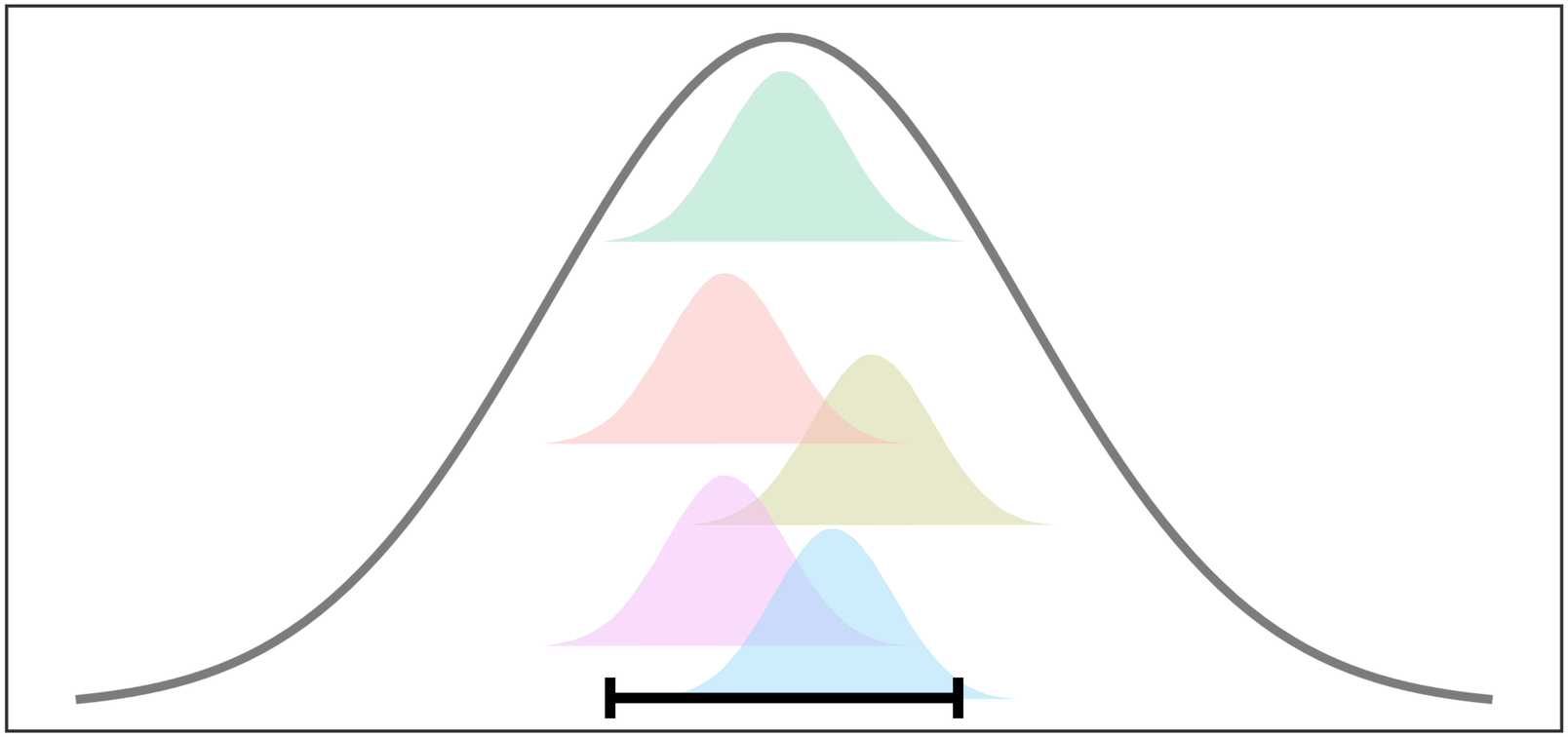
$$\sqrt{MS(within)}$$

$$\sqrt{23.336} = 4.831$$

If we assume the population SD is same for all 3 diets, we would estimate the common value to be 4.831.

Comparing means of many samples

ANOVA (analysis of variance)



★ H_0 : All groups came from same population distribution

H_A : Null is false (i.e. at least some of the means are different)

Example: filling out the ANOVA table

(1) Complete the table

Source	df	SS	MS
Between	3		45
Within	12	337	
Total			

Bonus: How many groups were there in the study?
How many total observations?

Example: filling out the ANOVA table

(1) Complete the table

Source	df	SS	MS
Between	3		45
Within	12	337	
Total	15		

Total df = df(between) + df(within)

Total df = 3 + 12 = 15

Bonus: How many groups were there in the study?
How many total observations?

Example: filling out the ANOVA table

(1) Complete the table

Source	df	SS	MS
Between	3	135	45
Within	12	337	28.08
Total	15		

MS = SS / df

Between: 45 = SS / 3

SS = 45 * 3 = 135

Within: MS = 337 / 12

MS = 28.08

Bonus: How many groups were there in the study?
How many total observations?

Example: filling out the ANOVA table

(1) Complete the table

Source	df	SS	MS
Between	3	135	45
Within	12	337	28.08
Total	15	472	

SST = SSB + SSW

SST = 135 + 337 = 472

Bonus: How many groups were there in the study?
How many total observations?

Example: filling out the ANOVA table

(a) Complete the table

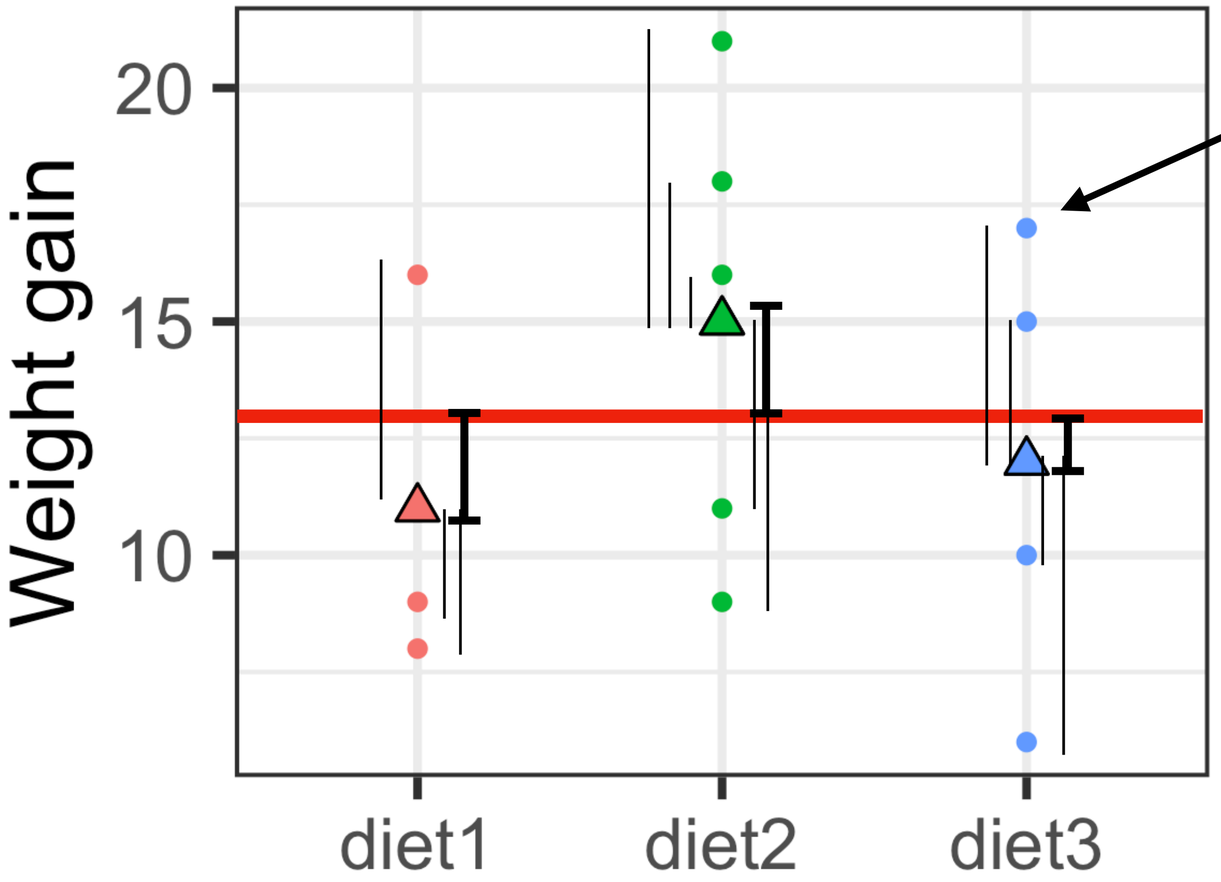
Source	df	SS	MS
Between	3	135	45
Within	12	337	28.08
Total	15	472	

Bonus: How many groups were there in the study?
How many total observations?

df(between) = Groups - 1 ⟶ 4 groups

df(total) = Obs. - 1 ⟶ 16 obs.

The Analysis of Variance Model



$$y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

Grand population mean

Effect of group *"i"*

$(\mu_i - \mu)$

Random error

$(y_{ij} - \mu_i)$

But first, some notation

$$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i)$$

$$y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	Groups - 1	$SSB = \sum_{i=1}^{I} n_i(\bar{y}_i - \bar{\bar{y}})^2$	SSB/df
Within groups	Observations - Groups	$SSW = \sum_{i=1}^{I} (n_i - 1)s_i^2$	SSW/df
Total	Observations - 1	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

But first, some notation

$$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i)$$

$$y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	Groups - 1	$SSB = \sum_{i=1}^{I} n_i \tau_i^2$	SSB/df
Within groups	Observations - Groups	$SSW = \sum_{i=1}^{I} (n_i - 1) s_i^2$	SSW/df
Total	Observations - 1	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

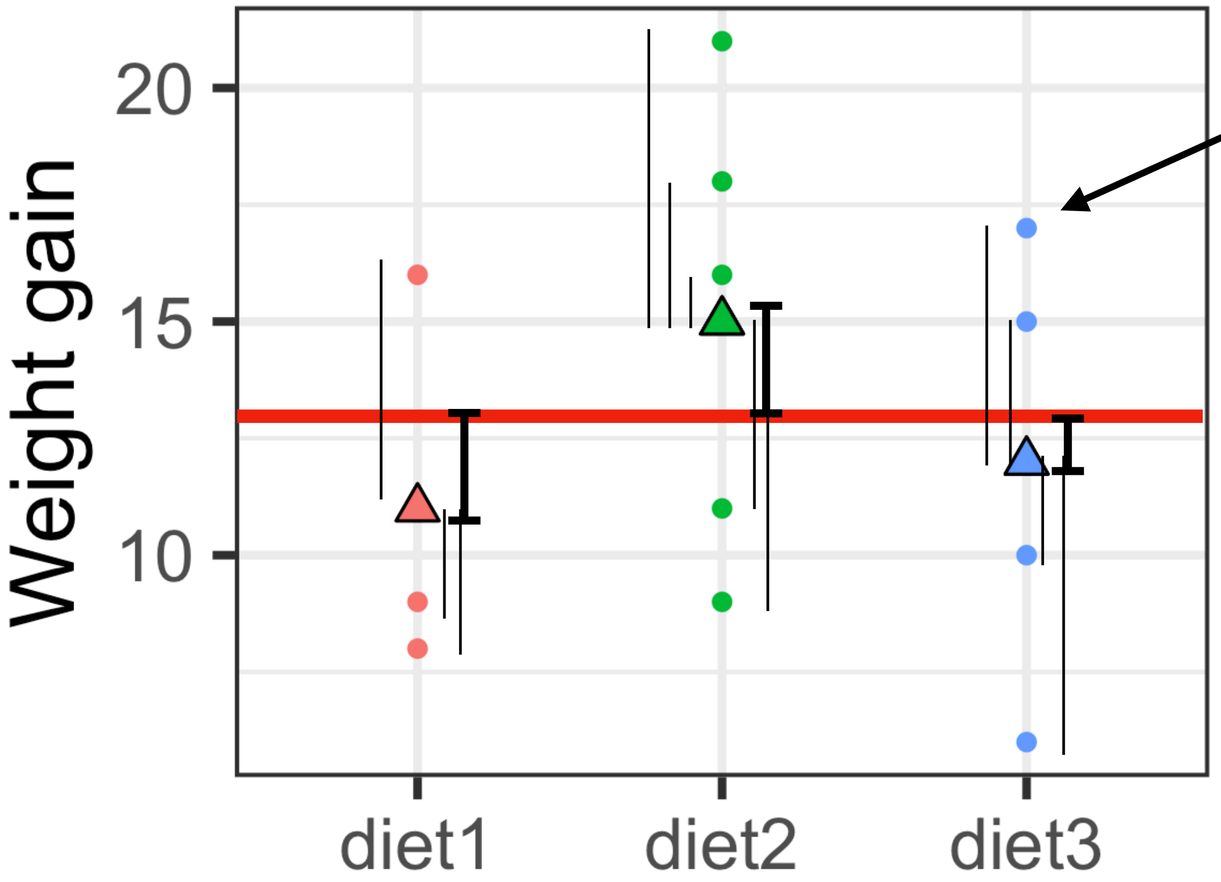
But first, some notation…

$$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i)$$

$$y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

Source	df	SS (Sum of squares)	MS (Mean square)
Between groups	Groups - 1	$SSB = \sum_{i=1}^{I} n_i \tau_i^2$	SSB/df
Within groups	Observations - Groups	$SSW = \sum_{i=1}^{I} \epsilon_{ij}^2$	SSW/df
Total	Observations - 1	$SST = \sum_{i=1}^{I} \sum_{j=1}^{n_i} (y_{ij} - \bar{\bar{y}})^2$	

The Analysis of Variance Model



$$y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

Grand population mean

Effect of group *"i"*

$(\mu_i - \mu)$

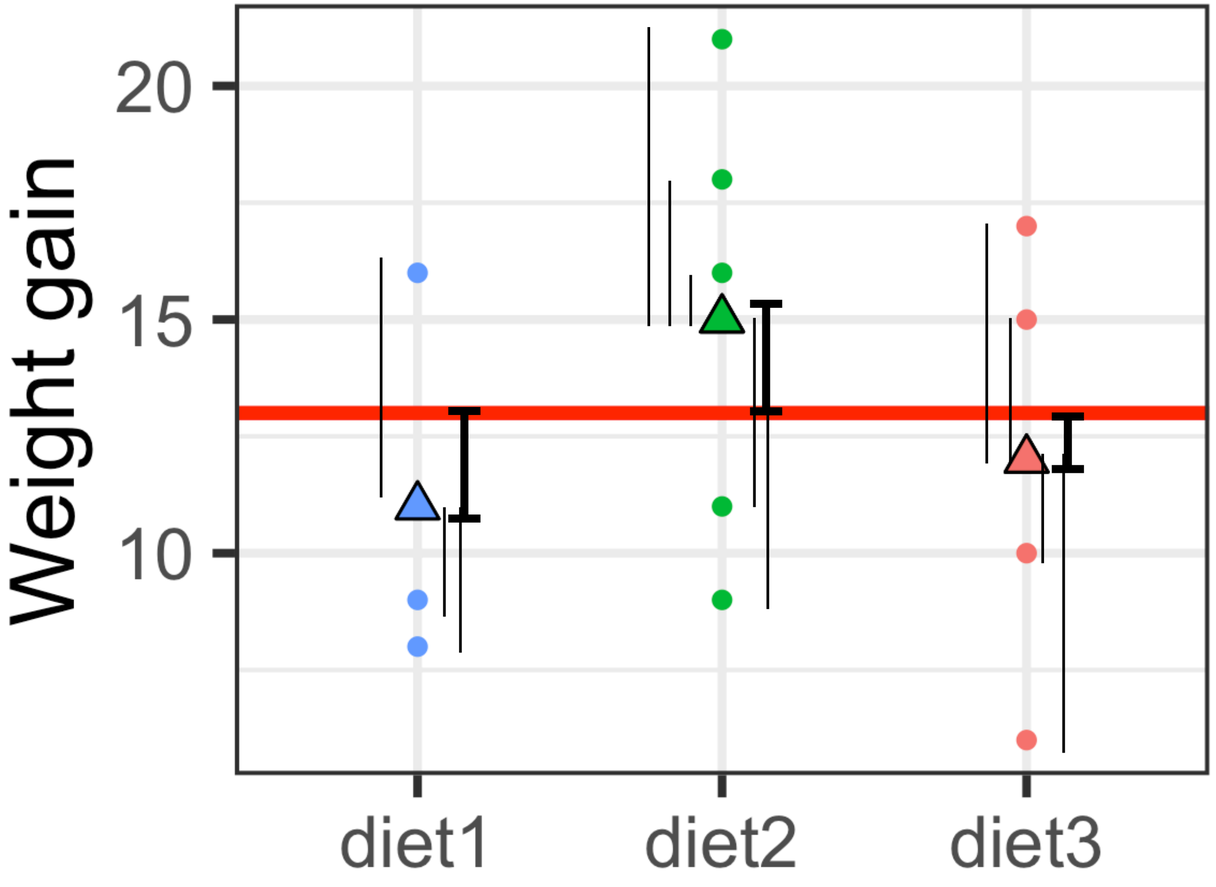
SS(between)

Random error

$(y_{ij} - \mu_i)$

SS(within)

The Analysis of Variance Model



$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_0 : \tau_1 = \tau_2 = \tau_3$$

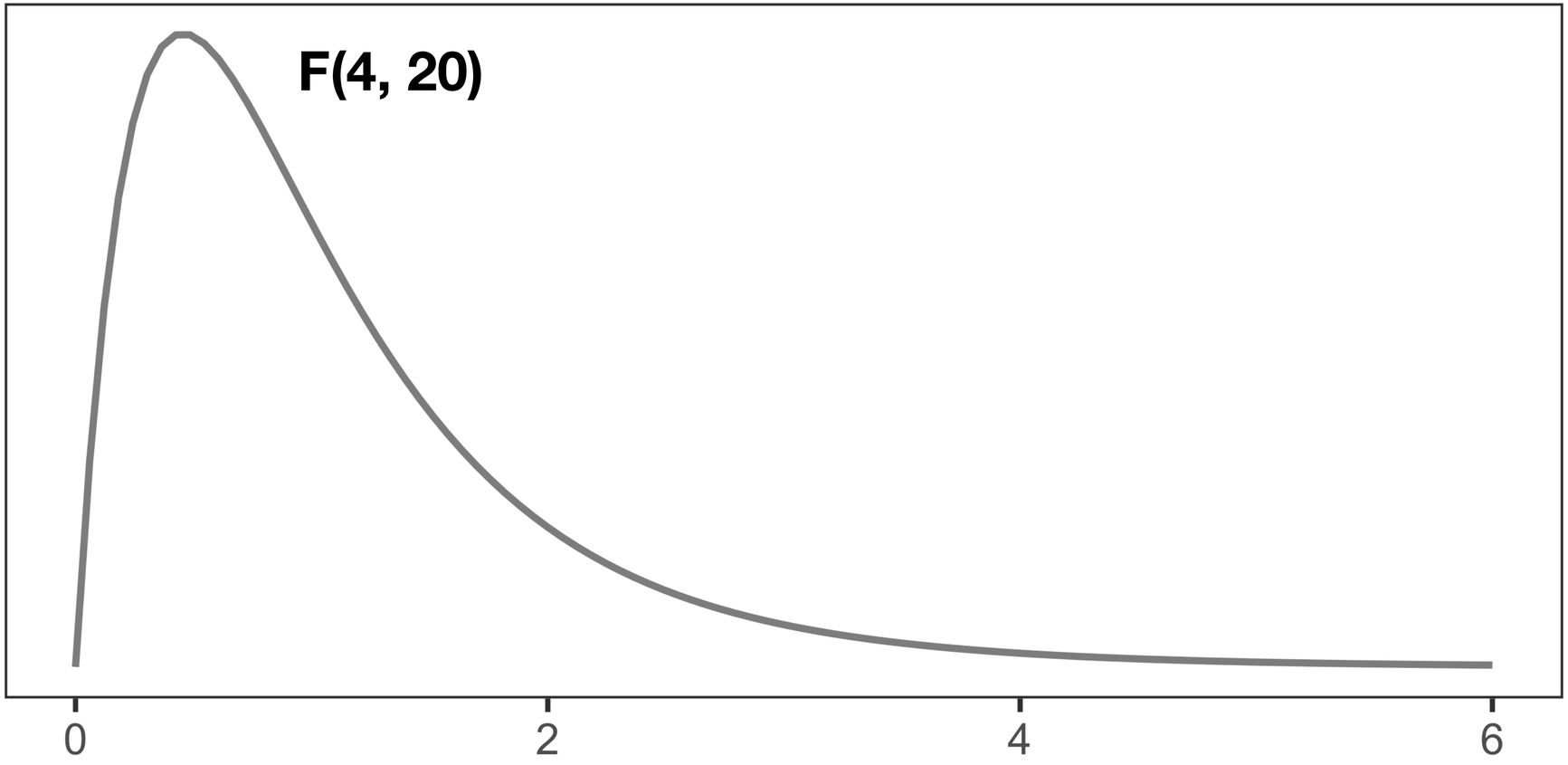
H_A : Null is false (i.e. at least some of the means are different)

“Compound null hypothesis”

Rejection of H_0 does not specify WHICH means are different…

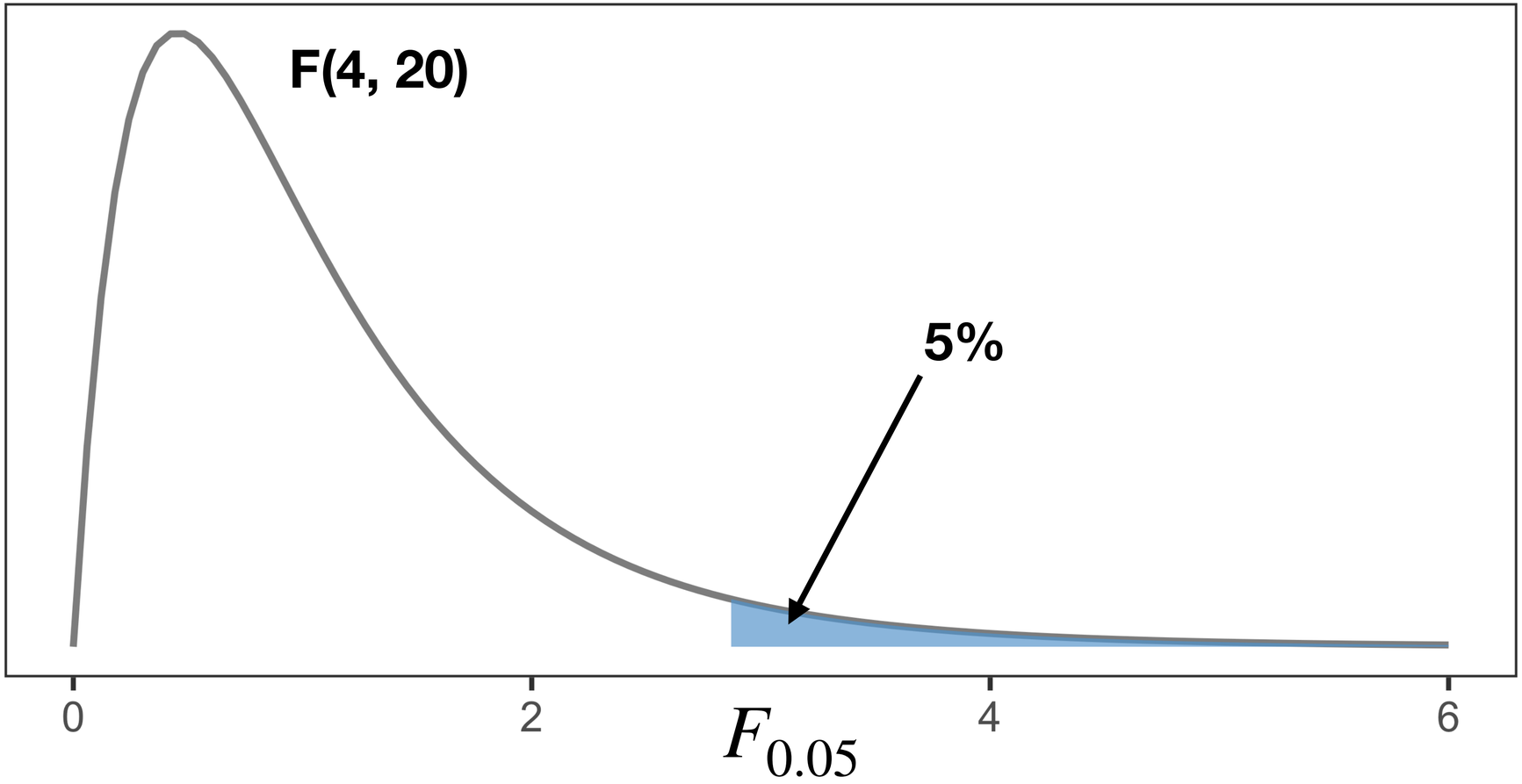
Introduction to the F distribution

Distribution used in several statistical analyses and depends on the numerator degrees of freedom and the denominator degrees of freedom



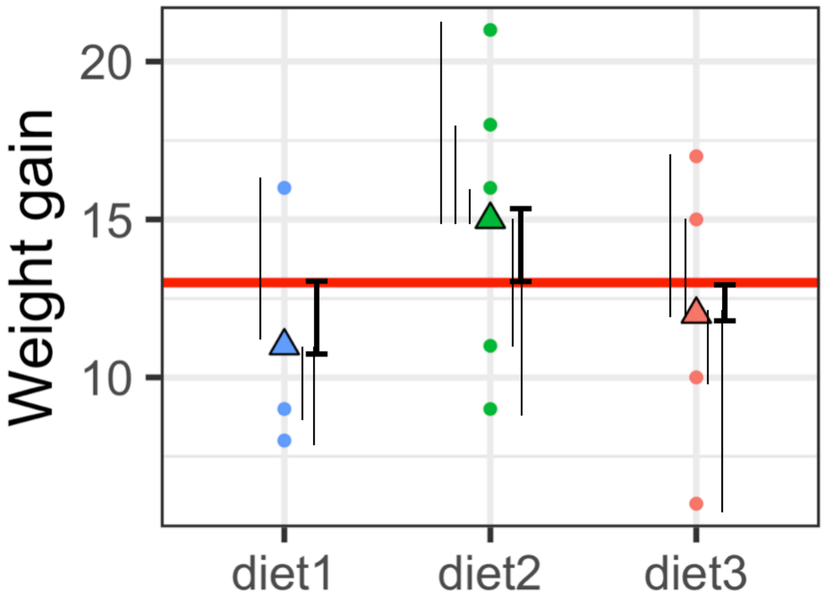
Introduction to the F distribution

Distribution used in several statistical analyses and depends on the numerator degrees of freedom and the denominator degrees of freedom



Introduction to the F distribution and test

Distribution used in several statistical analyses and depends on the numerator degrees of freedom and the denominator degrees of freedom



$$F_s = \frac{MS(between)}{MS(within)}$$

F_s is LARGE if discrepancies among the groups are large relative to variability within the groups

Large F_s -> small p-value (evidence against null)

Numerator df = df(between)
Denominator df = df(within)

Introduction to the F distribution and test

Source	df	SS	MS
Between	2	36	18
Within	9	210.025	23.336
Total	11	246	

$$F_s = \frac{MS(between)}{MS(within)}$$

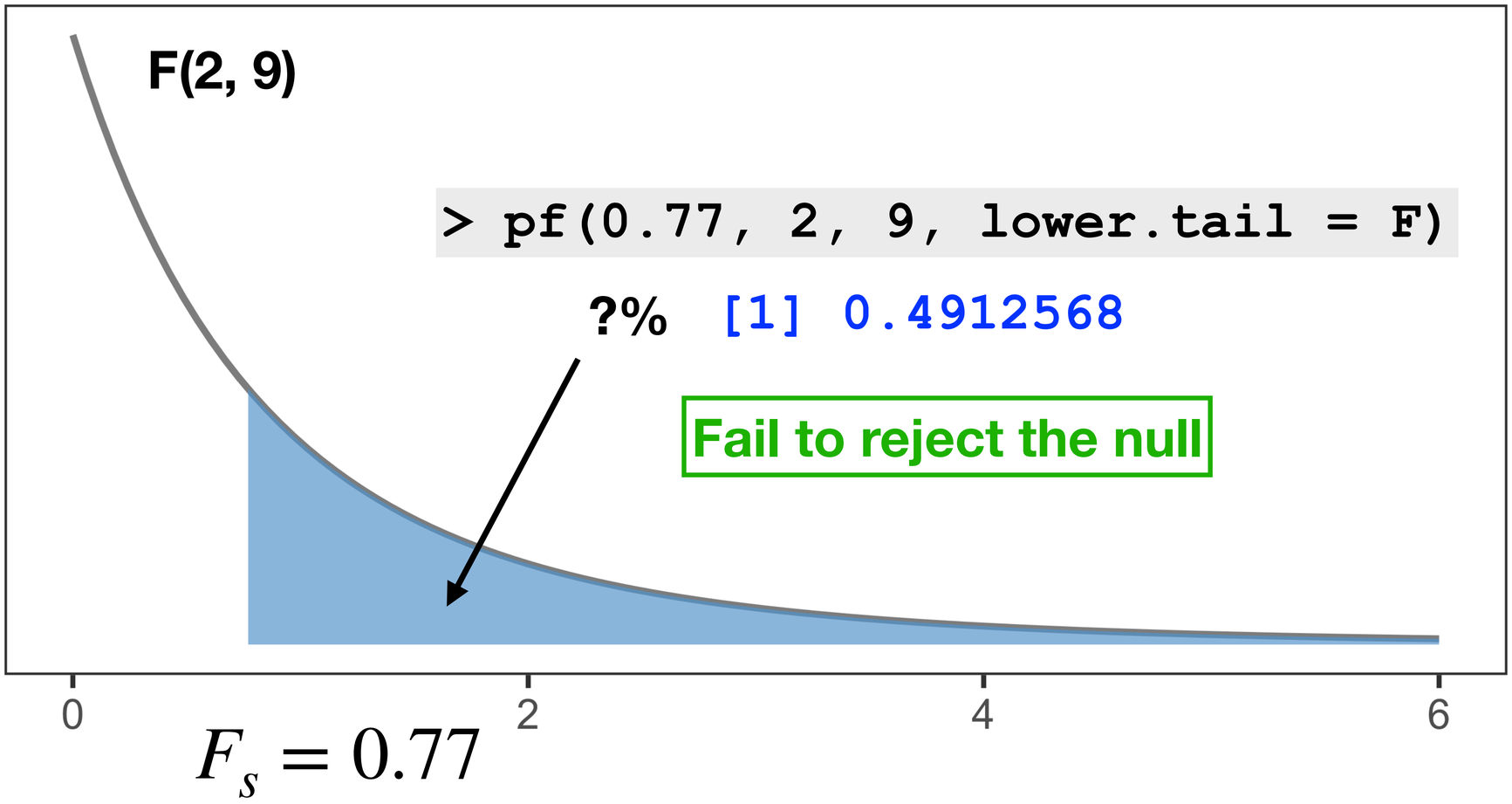
$$F_s = \frac{18}{23.336} = 0.77$$

H_0 : Mean weight gain is the same on all three diets

H_A : Mean weight gain is not the same on all three diets

$$F(2,9)$$

Introduction to the F distribution and test



$F_{0.01} = 8.0215$

F > 8.0215:

Significant at $\alpha = 0.01$

F < 8.0215:

Not Significant at $\alpha = 0.01$

F - Distribution ($\alpha = 0.01$ in the Right Tail)

df_2 \ df_1	Numerator Degrees of Freedom								
Denominator Degrees of Freedom	1	2	3	4	5	6	7	8	9
1	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
2	98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158
6	13.745	10.925	9.7795	9.1483	8.7459	8.4661	8.2600	8.1017	7.9761
7	12.246	9.5466	8.4513	7.8466	7.4604	7.1914	6.9928	6.8400	6.7188
8	11.259	8.6491	7.5910	7.0061	6.6318	6.3707	6.1776	6.0289	5.9106
9	10.561	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10	10.044	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11	9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12	9.3302	6.9266	5.9525	5.4120	5.0643	4.8206	4.6395	4.4994	4.3875
13	9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14	8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15	8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16	8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17	8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822
18	8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19	8.1849	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20	8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21	8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22	7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23	7.8811	5.6637	4.7649	4.2636	3.9392	3.7102	3.5390	3.4057	3.2986
24	7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25	7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
26	7.7213	5.5263	4.6366	4.1400	3.8183	3.5911	3.4210	3.2884	3.1818
27	7.6767	5.4881	4.6009	4.1056	3.7848	3.5580	3.3882	3.2558	3.1494
28	7.6356	5.4529	4.5681	4.0740	3.7539	3.5276	3.3581	3.2259	3.1195
29	7.5977	5.4204	4.5378	4.0449	3.7254	3.4995	3.3303	3.1982	3.0920
30	7.5625	5.3903	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40	7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60	7.0771	4.9774	4.1259	3.6490	3.3389	3.1187	2.9530	2.8233	2.7185
120	6.8509	4.7865	3.9491	3.4795	3.1735	2.9559	2.7918	2.6629	2.5586
∞	6.6349	4.6052	3.7816	3.3192	3.0173	2.8020	2.6393	2.5113	2.4073

Example: filling out the ANOVA table

(1) Complete the table then (2) calculate the F statistic

Source	df	SS	MS
Between	3	135	45
Within	12	337	28.08
Total	15	472	

$$F_s = \frac{MS(between)}{MS(within)}$$

$$F_s = \frac{45}{28.08} = 1.602$$

```
> pf(1.602, 3, 12, lower.tail = F)
```

[1] 0.2406565

Fail to reject the null

Bonus: How many groups were there in the study?
How many total observations?

df(between) = Groups - 1 ⟶ 4 groups

df(total) = Obs. - 1 ⟶ 16 obs.

Relationship between F test and regression

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
    Min      1Q  Median      3Q     Max
-3.5033 -0.8873  0.0459  0.9218  3.1279

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.34291    0.16505   8.136  3.3e-15 ***
income       0.22765    0.01727  13.182  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.234 on 496 degrees of freedom
Multiple R-squared:  0.2595,    Adjusted R-squared:  0.258
F-statistic: 173.8 on 1 and 496 DF,  p-value: < 2.2e-16
```

Relationship between F test and regression

```
> summary(lm(happiness ~ income, data = income_data))

Call:
lm(formula = happiness ~ income, data = income_data)

Residuals:
    Min      1Q  Median      3Q     Max
-3.5037 -0.8873  0.0459  0.9210  3.1270

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.34291    0.16505   8.136  3.3e-15 ***
income       0.22765    0.01727  13.182  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.234 on 496 degrees of freedom
Multiple R-squared:  0.2595,    Adjusted R-squared:  0.258
F-statistic: 173.8 on 1 and 496 DF,  p-value: < 2.2e-16
```

$H_0 : y = \beta_0$ model fits data best (i.e. all independent variables = 0)

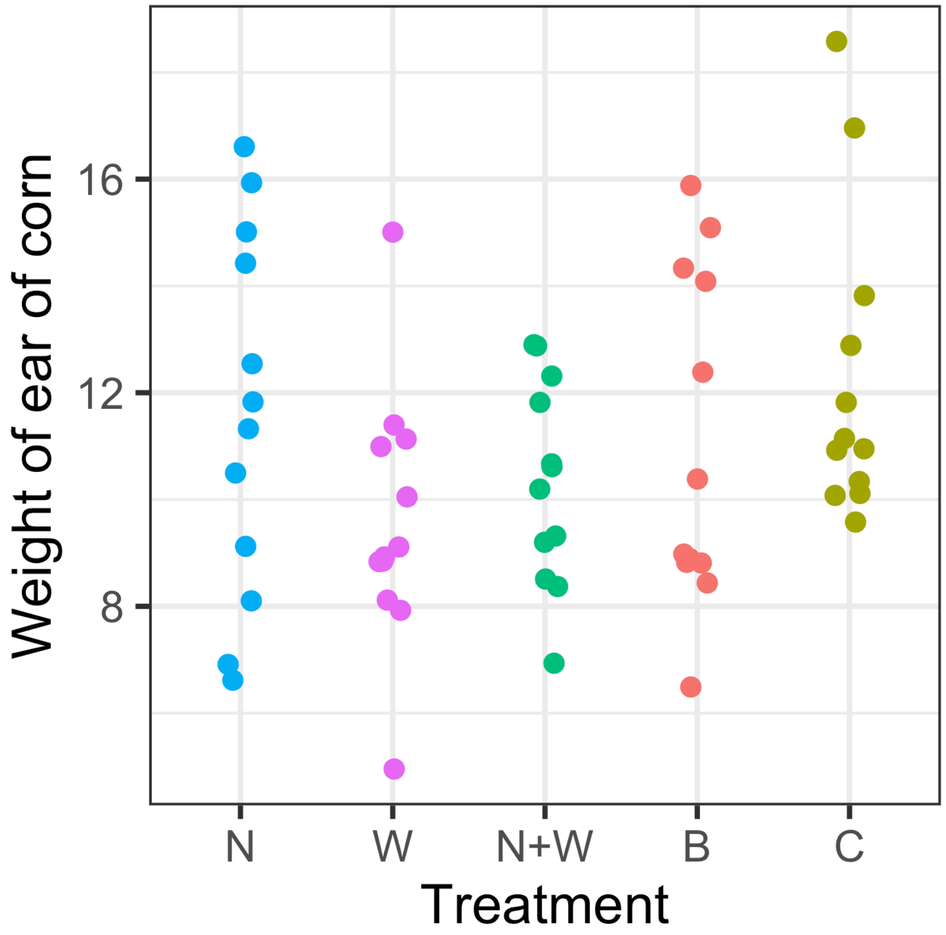
$H_A : y = \beta_0 + \beta_1 X$ model fits data best

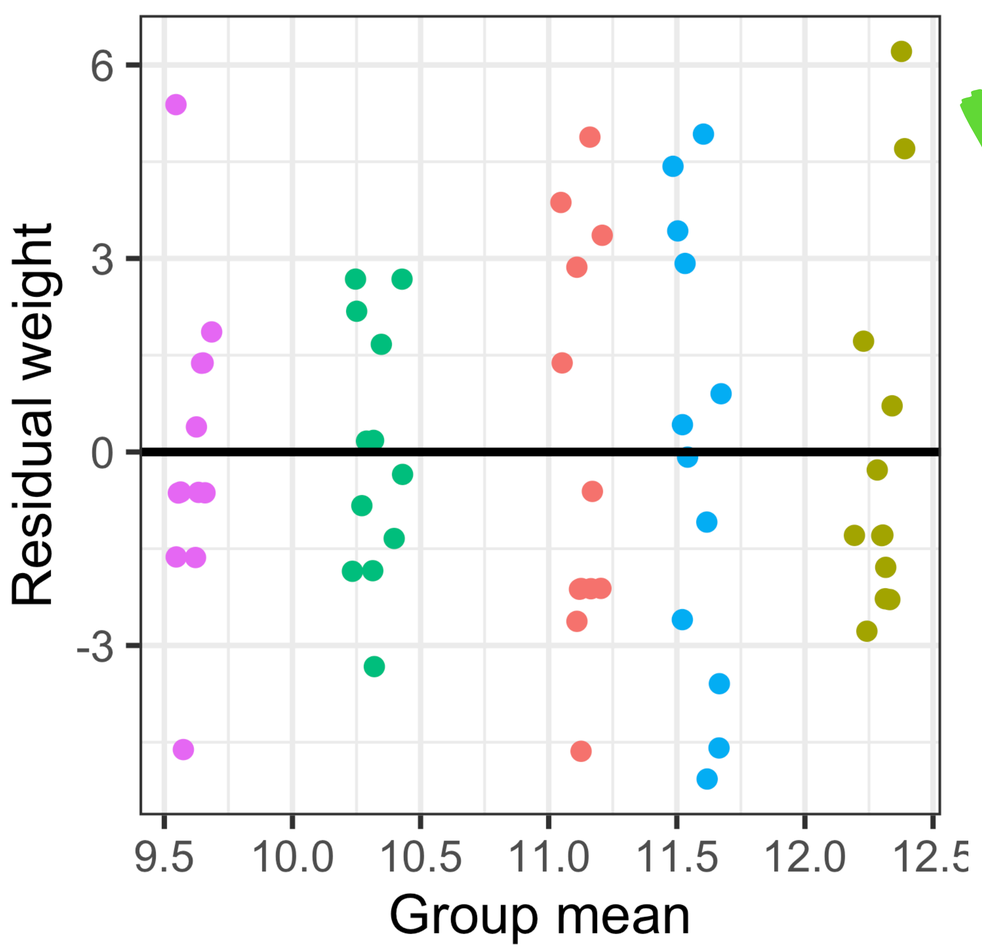
Here, we reject the null and conclude that there is a relationship between happiness and income.

Conditions for using an ANOVA

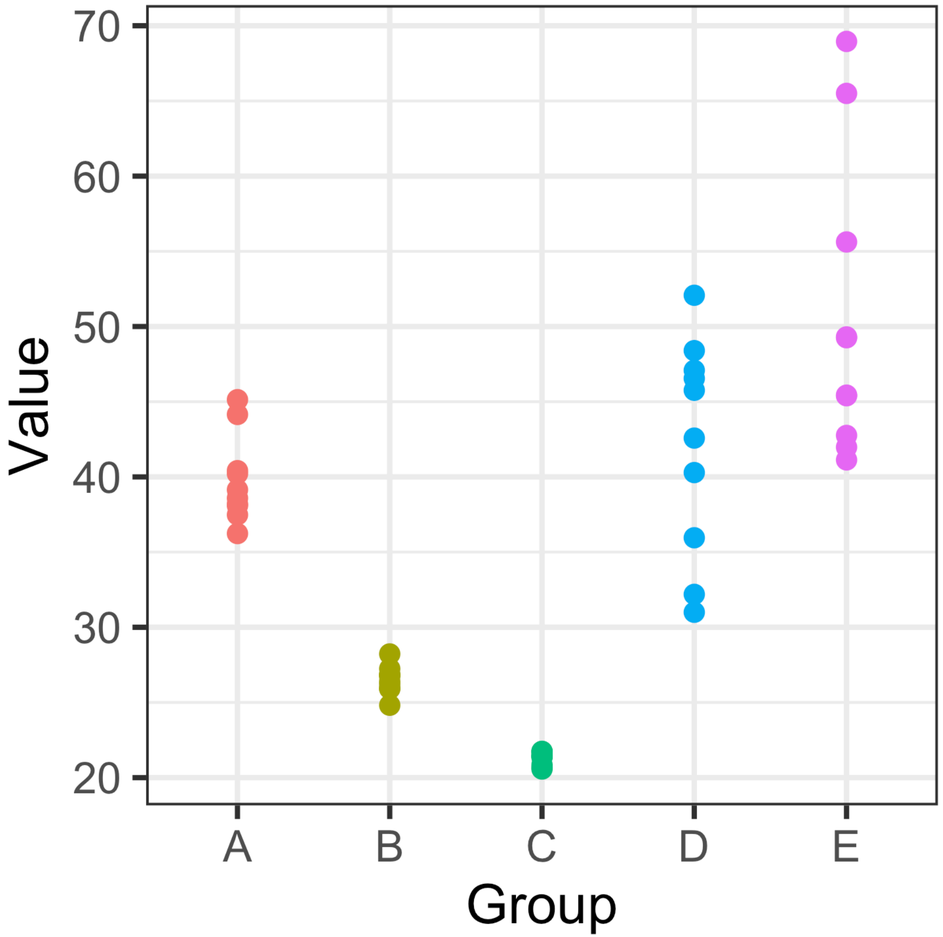
- **Design conditions**
    - Groups of observations are random samples from their respective populations
    - The groups (I) must be independent of each other
- **Population conditions**
    - The group (I) population distributions must be approximately normal with equal standard deviations
        - *This is less crucial for larger sample size (if $n_1 = n_2 = n_3$)*

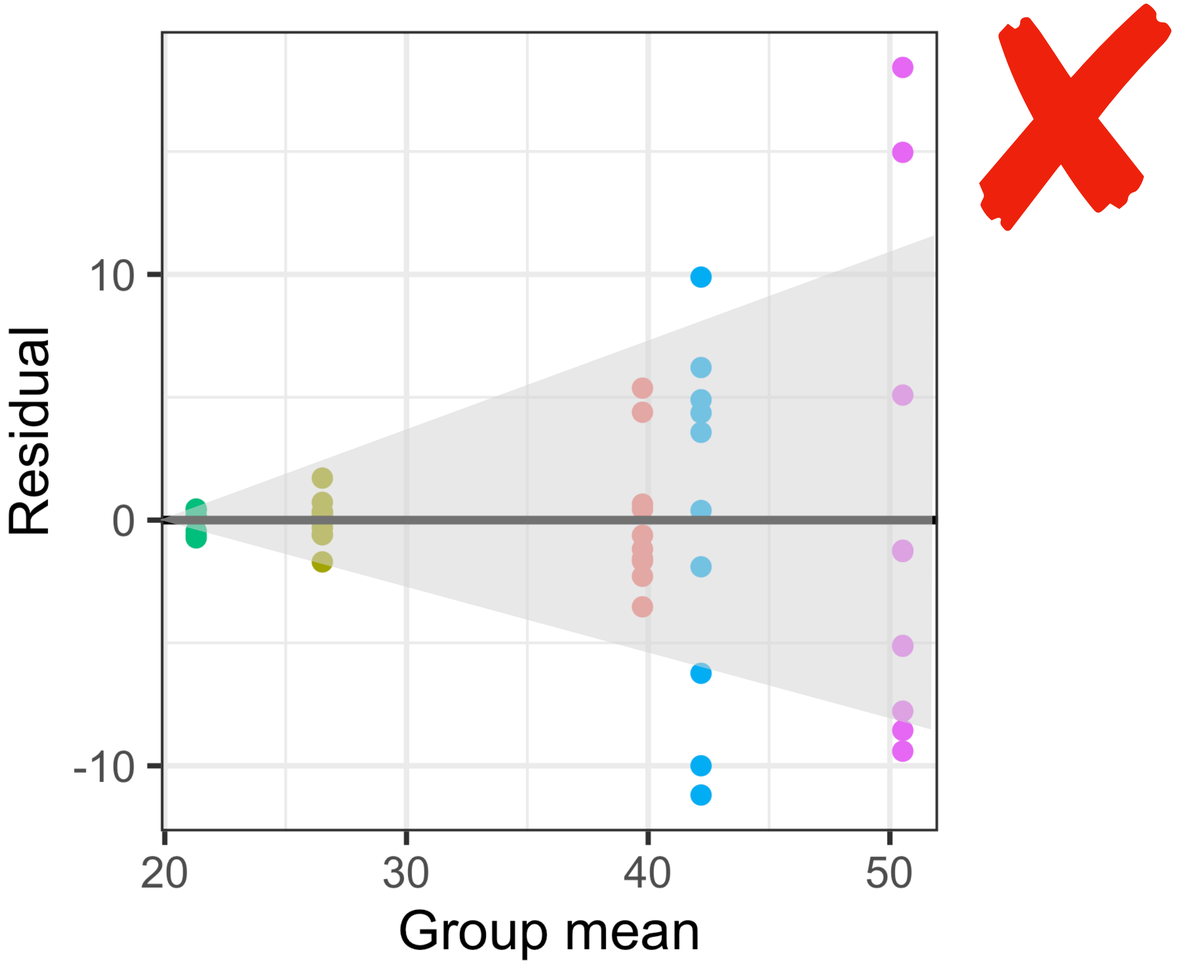
Conditions for using an ANOVA





Conditions for using an ANOVA





~Largest SD / smallest SD <= 2

Calculating ANOVA using R

diet	weight
diet1	8
diet2	9
diet3	15
diet1	16
diet2	16
diet3	10
diet1	9
diet2	21
diet3	17

Data: weight gain for three different diets

Source	df	SS	MS
Between	2	36	18
Within	9	210.025	23.336
Total	11	246	

```
> anova(lm(weight ~ diet, data = weight))
```

```
Analysis of Variance Table

Response: weight
          Df Sum Sq Mean Sq F value Pr(>F)
diet       2     36  18.000  0.7714 0.4907
Residuals  9    210  23.333
```

Calculating ANOVA using R

LONG

diet	weight
diet1	8
diet2	9
diet3	15
diet1	16
diet2	16
diet3	10
diet1	9
diet2	21
diet3	17

Data: weight gain for three different diets

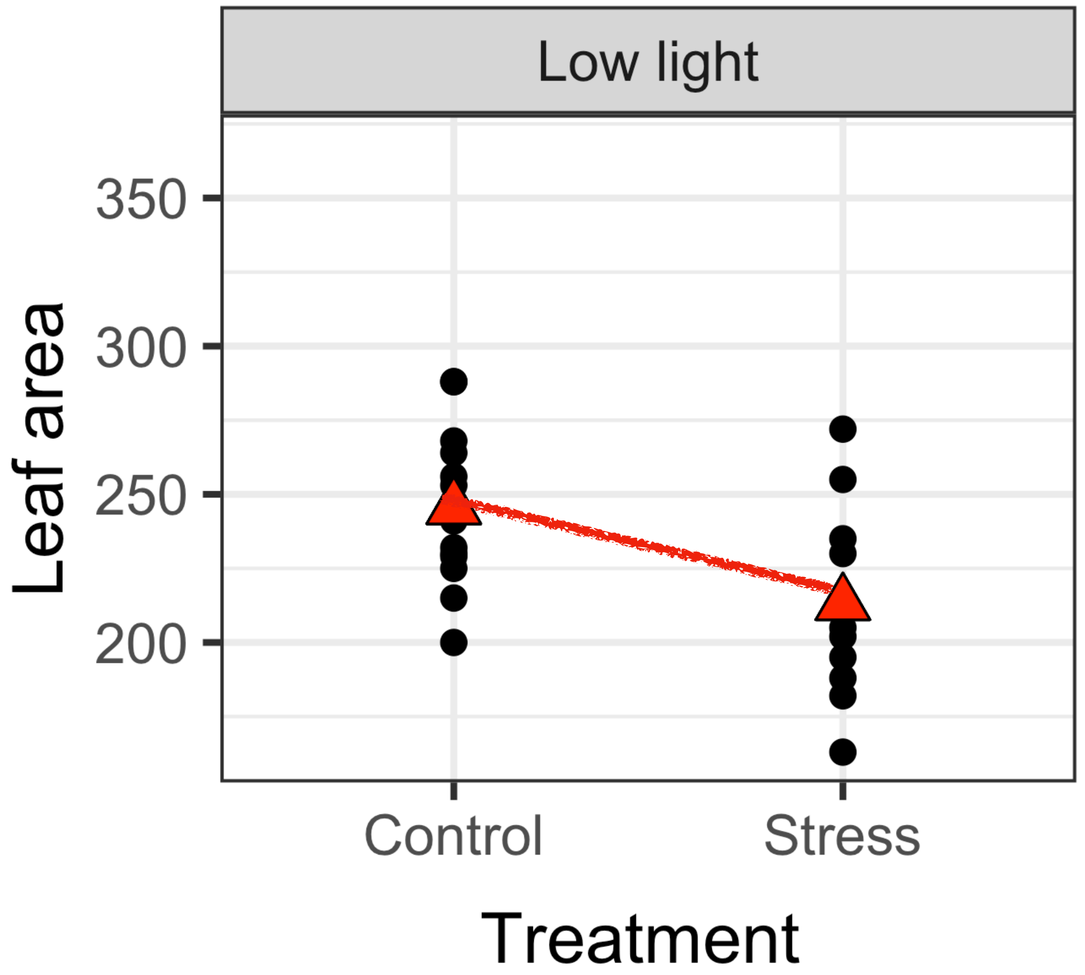
WIDE

diet1	diet2	diet3
8	9	15
16	16	10
9	21	17
NA		
NA		

```
tidyr::pivot_longer(diet1:diet3,
                    names_to = "diet",
                    values_to = "weight")
```

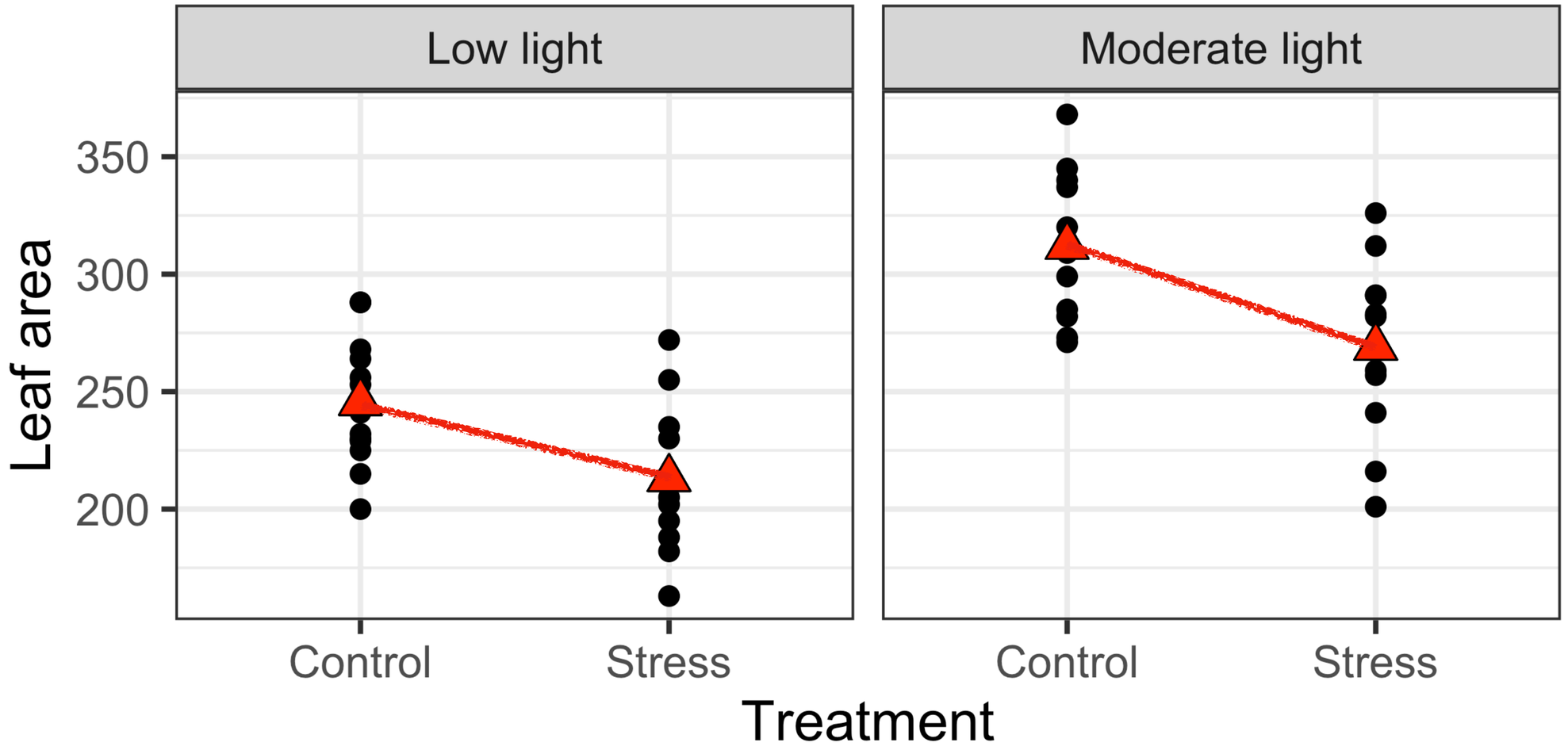
```
> anova(lm(weight ~ diet, data = weight))
```

Two-way (Factorial) ANOVA

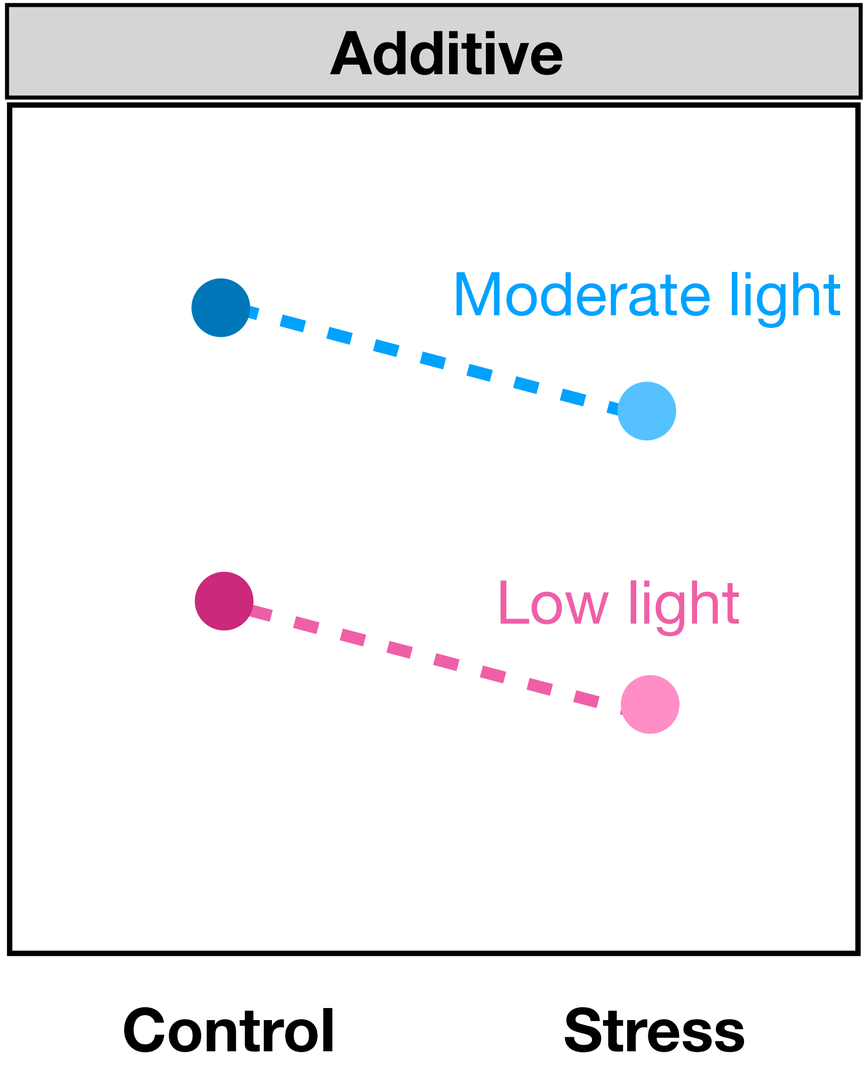


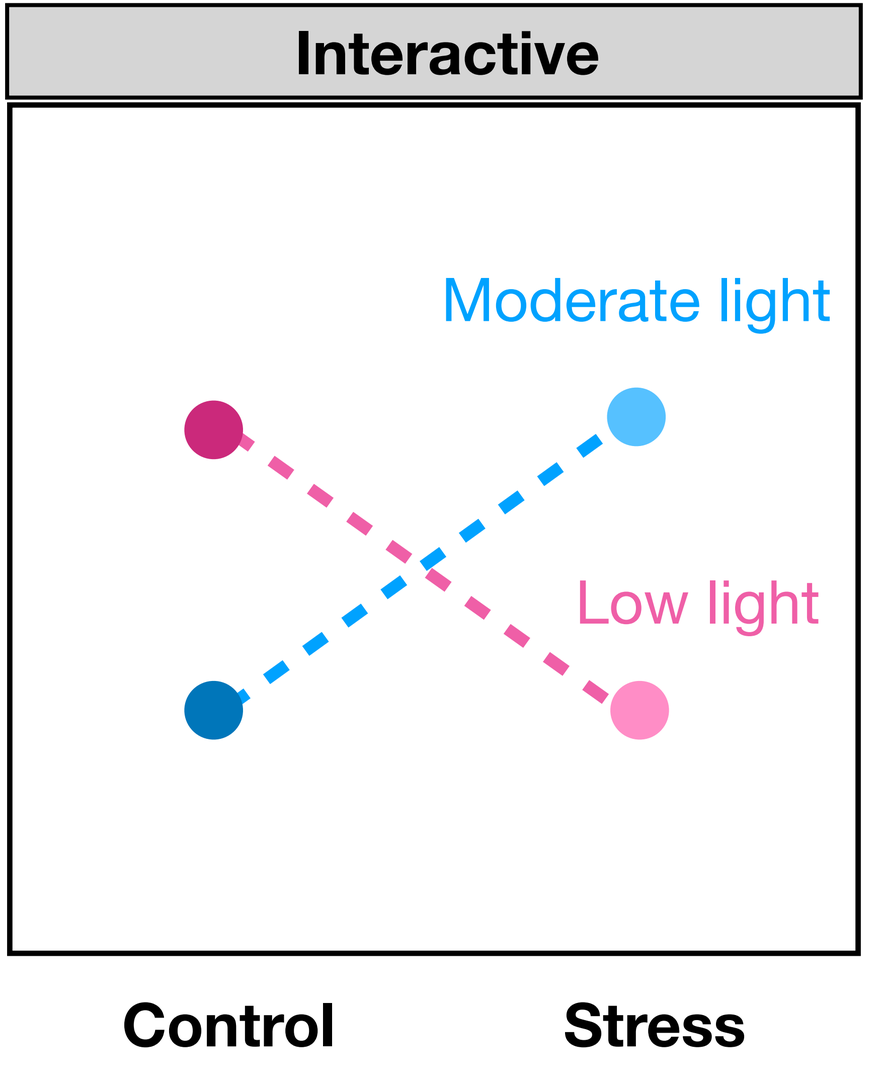
Two-way (Factorial) ANOVA

$$y_{ijk} = \mu + \tau_i + \beta_j + \epsilon_{ijk}$$

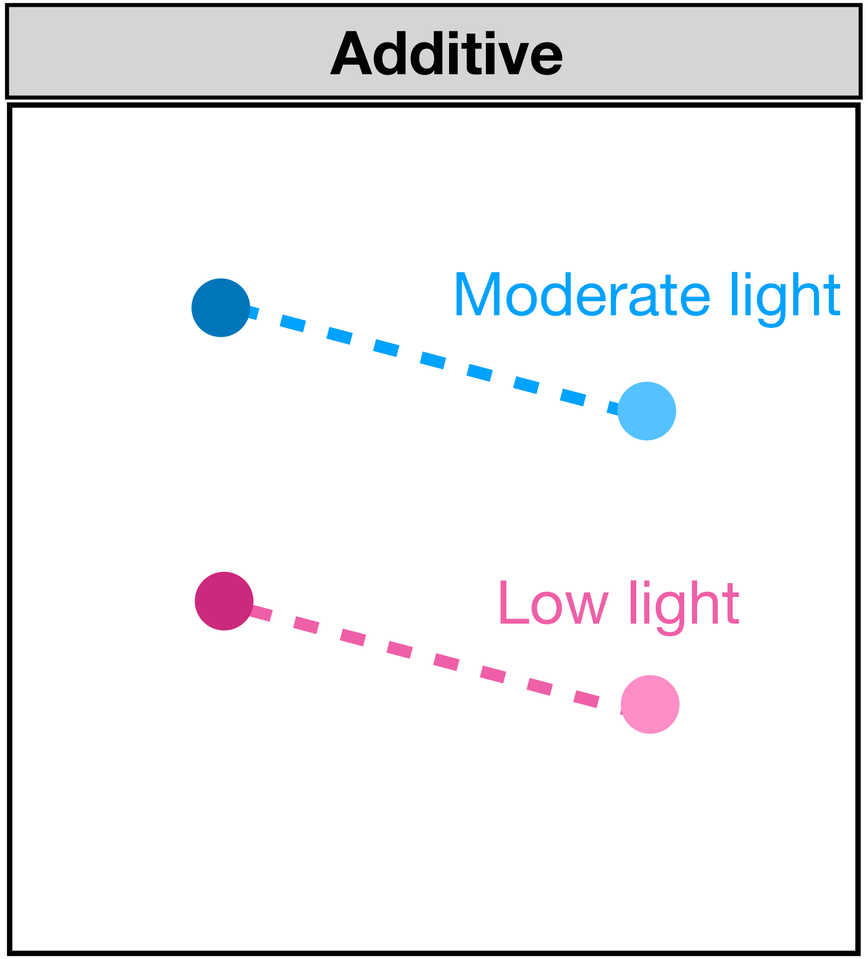


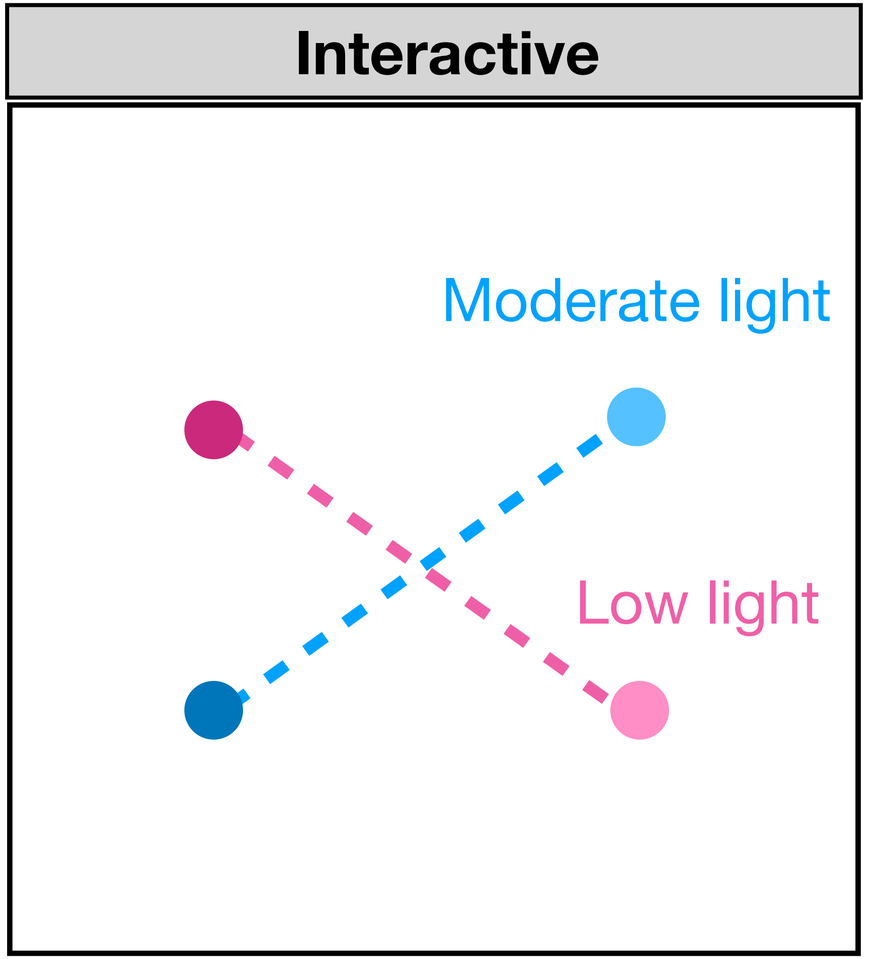
Additive or interactive factors?





Additive or interactive factors?





$$y_{ijk} = \mu + \tau_i + \beta_j + \epsilon_{ijk}$$

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$

Two-way (Factorial) ANOVA

Check for interactions

Yes → **Done! Reject H_0.**

No → **Test each factor for additive effects**

```
> anova(lm(area ~ light * stress))
```

```
Response: leaf_area
             Df Sum Sq Mean Sq F value    Pr(>F)
stress        1  18432   18432 19.6640 5.364e-05 ***
light         1  48678   48678 51.9333 3.550e-09 ***
stress:light  1    361     361  0.3851    0.5378
Residuals    48  44992     937
```

Interaction ✗

Two-way (Factorial) ANOVA

Check for interactions

Yes → **Done! Reject H_0.**

No → **Test each factor for additive effects**

```
> anova(lm(area ~ light * stress))
```

```
Response: leaf_area
              Df Sum Sq Mean Sq F value    Pr(>F)
stress         1  18432   18432 19.6640 5.364e-05 ***
light          1  48678   48678 51.9333 3.550e-09 ***
stress:light   1    361     361  0.3851    0.5378
Residuals     48  44992     937
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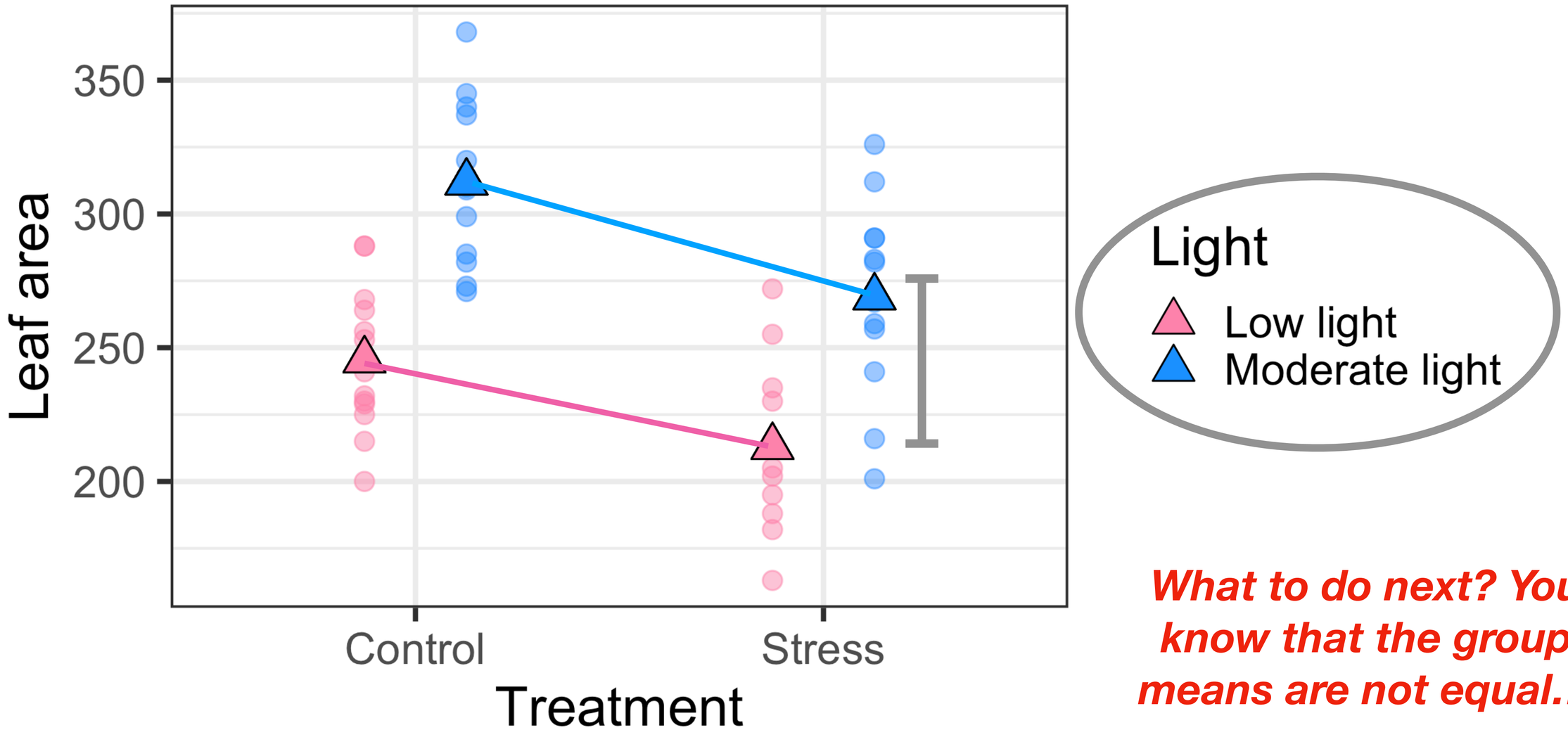
Two-way (Factorial) ANOVA

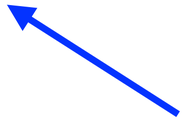


What to do next? You know that the group means are not equal…

Next steps for significant ANOVAs

- Pairwise t-tests to see which groups are significantly different
    - ***Need to make sure you adjust for multiple testing!***
- Many options: one of my favorites is **`TukeyHSD(model)`**

```
> TukeyHSD(aov(leaf_area ~ light + stress, data = leaf_area))
```



`anova(lm(. . .))`

Next steps for significant ANOVAs

```
> TukeyHSD(aov(leaf_area ~ light + stress, data = leaf_area))
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

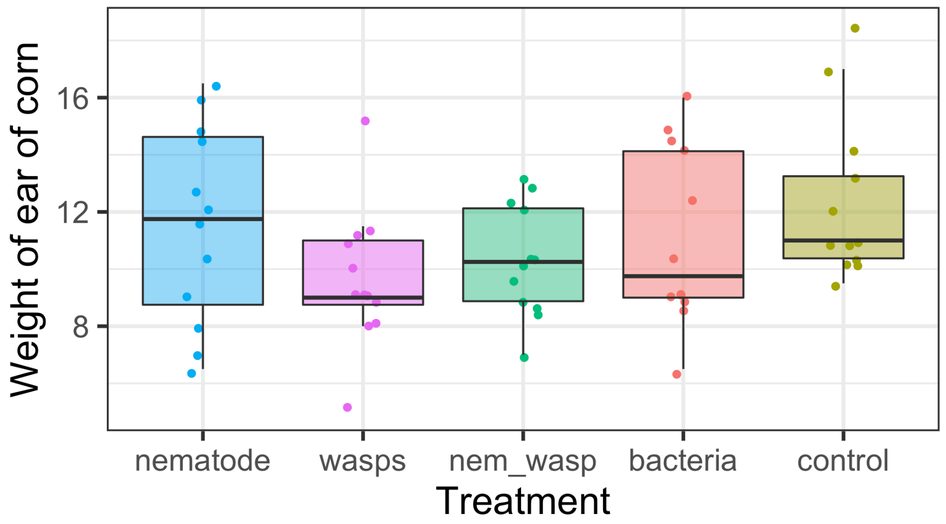
Fit: aov(formula = leaf_area ~ light + stress, data = leaf_area)

$light
                                diff       lwr       upr p adj
Moderate light-Low light 61.19231 44.23582 78.14879     0

$stress
                      diff       lwr       upr    p adj
Stress-Control -37.65385 -54.61033 -20.69736 4.75e-05
```

Next steps for significant ANOVAs

```
> TukeyHSD(aov(weight ~ treatment, data = corn))
```




```
  Tukey multiple comparisons of means
    95% family-wise confidence level

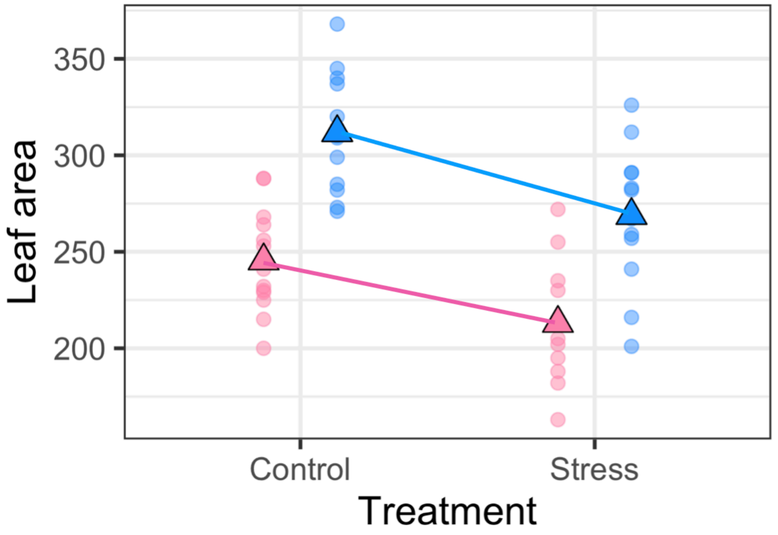
Fit: aov(formula = weight ~ treatment, data = long_corn)

$treatment

                          diff       lwr       upr     p adj
control-bacteria     1.1666667 -2.078614 4.4119471 0.8478963
nem_wasp-bacteria   -0.7916667 -4.036947 2.4536138 0.9582644
nematode-bacteria    0.4583333 -2.786947 3.7036138 0.9945376
wasps-bacteria      -1.5000000 -4.745280 1.7452805 0.6901281
nem_wasp-control    -1.9583333 -5.203614 1.2869471 0.4412802
nematode-control    -0.7083333 -3.953614 2.5369471 0.9720121
wasps-control       -2.6666667 -5.911947 0.5786138 0.1548970
nematode-nem_wasp    1.2500000 -1.995280 4.4952805 0.8127831
wasps-nem_wasp      -0.7083333 -3.953614 2.5369471 0.9720121
wasps-nematode      -1.9583333 -5.203614 1.2869471 0.4412802
```

Shows pairwise t-test results

Aside: using ANOVA to compare models



Which variables (light, stress, or both) best predict leaf area?

```
# light as a predictor
> model1 <- lm(leaf_area ~ light)

# stress as a predictor
> model2 <- lm(leaf_area ~ stress)

# light and stress as predictors (additive)
> model3 <- lm(leaf_area ~ light + stress)

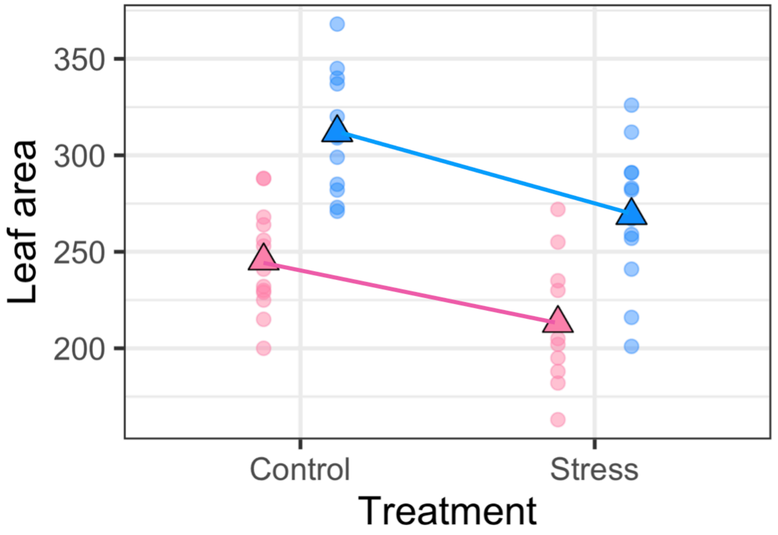
# light and stress as predictors (interacting)
> model4 <- lm(leaf_area ~ light * stress)
```

```
> anova(model1, model3)
```

```
Model 1: leaf_area ~ light
Model 2: leaf_area ~ stress + light
  Res.Df   RSS Df Sum of Sq      F    Pr(>F)
1     50 63784
2     49 45353  1     18432 19.914 4.748e-05 ***
---
```

Stress AND light > only light

Aside: using ANOVA to compare models



Which variables (light, stress, or both) best predict leaf area?

```
# light as a predictor
> model1 <- lm(leaf_area ~ light)

# stress as a predictor
> model2 <- lm(leaf_area ~ stress)

# light and stress as predictors (additive)
> model3 <- lm(leaf_area ~ light + stress)

# light and stress as predictors (interacting)
> model4 <- lm(leaf_area ~ light * stress)
```

```
> anova(model3, model4)
```

```
Model 1: leaf_area ~ stress + light
Model 2: leaf_area ~ stress * light
  Res.Df   RSS Df Sum of Sq      F Pr(>F)
1     49 45353
2     48 44992  1    360.94 0.3851 0.5378
```

Interaction is not better than additive

Announcements

No class Thursday (Happy Thanksgiving!)

No homework this week

Next Tuesday: Practicum 3 (bring your laptops!)

Final Review (TA) - Mon. Dec. 6 (AM)

Remember to look at the data for your project! (And if you want to change projects that's okay! No need for a new proposal)