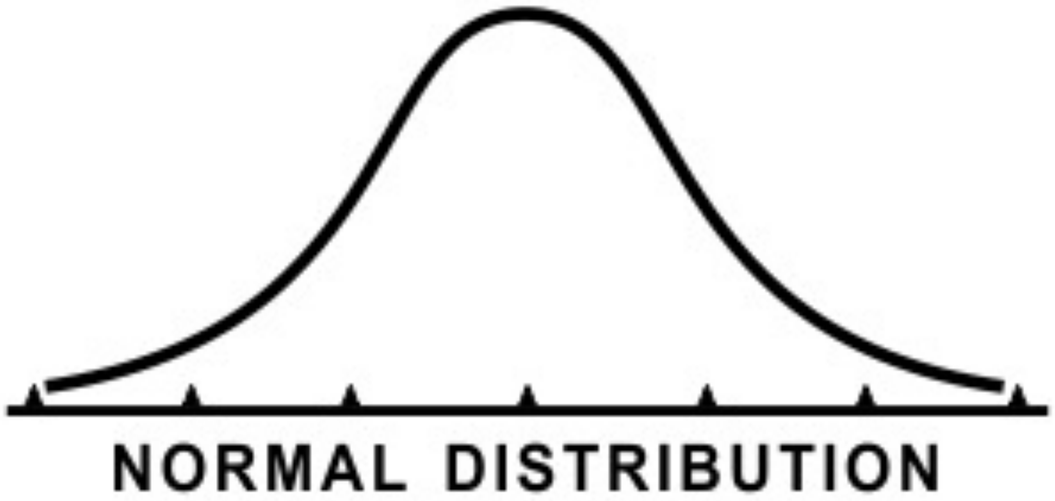




# Lecture 05

10.6.2021

# Refresher Quiz

**Wavy hair in mice is a recessive genetic trait. If mice with wavy hair are mated with straight haired (heterozygous) mice, each offspring has a probability of 0.5 of having wavy hair. Consider a large number of such matings, each producing a litter of 5 offspring. What percentage of the litters will consist of two wavy-haired and three straight-haired offspring?**

**Wavy hair in mice is a recessive genetic trait. If mice with wavy hair are mated with straight haired (heterozygous) mice, each offspring has a probability of 0.5 of having wavy hair. Consider a large number of such matings, each producing a litter of 5 offspring. What percentage of the litters will consist of two wavy-haired and three straight-haired offspring?**

**n = 5**

**Wavy or straight**

**P(wavy) = 0.5**

**B - I - n - S**

Binary - Independent - n - Same p

**Wavy hair in mice is a recessive genetic trait. If mice with wavy hair are mated with straight haired (heterozygous) mice, each offspring has a probability of 0.5 of having wavy hair. Consider a large number of such matings, each producing a litter of 5 offspring. What percentage of the litters will consist of two wavy-haired and three straight-haired offspring?**

**Prob = relative frequency**

$$ {}_nC_j p^j (1-p)^{n-j} $$

**n = 5**

**Wavy or straight**

**P(wavy) = 0.5**

**B - I - n - S**

Binary - Independent - n - Same p

**Wavy hair in mice is a recessive genetic trait. If mice with wavy hair are mated with straight haired (heterozygous) mice, each offspring has a probability of 0.5 of having wavy hair. Consider a large number of such matings, each producing a litter of 5 offspring. What percentage of the litters will consist of two wavy-haired and three straight-haired offspring?**

**Prob = relative frequency**

$$ {}_5C_2(0.5)^2(0.5)^3 = 0.3125 $$

**n = 5**

**Wavy or straight**

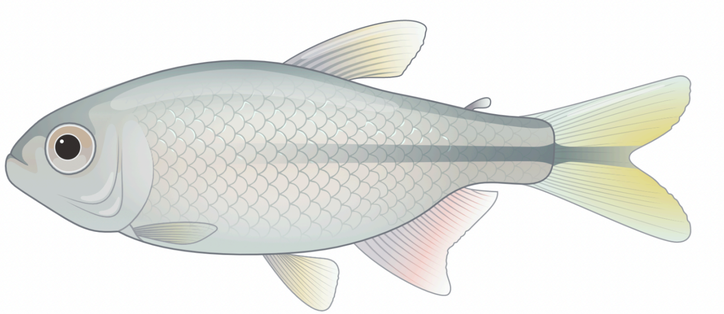
**P(wavy) = 0.5**

**B - I - n - S**

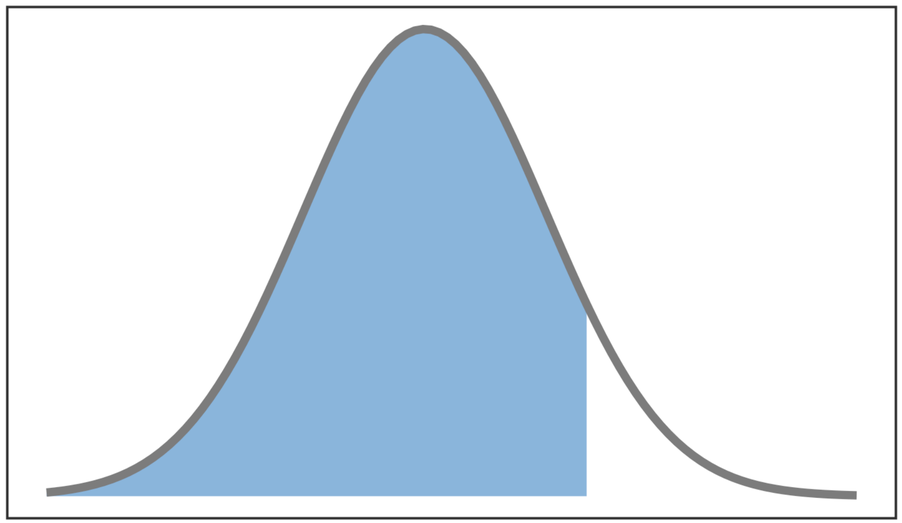
Binary - Independent - n - Same p

**In a certain population of fish, individual lengths follow a normal distribution. The mean length of the fish is 54.0 mm, and the standard deviation is 4.5 mm. <u>What percentage of the fish are shorter than 60 mm?</u>**

*Even if you use R to calculate p-value, ALWAYS smart to make a quick sketch*



$$Z = \frac{y - \mu}{\sigma}$$



**In a certain population of fish, individual lengths follow a normal distribution. The mean length of the fish is 54.0 mm, and the standard deviation is 4.5 mm. <u>What percentage of the fish are shorter than 60 mm?</u>**

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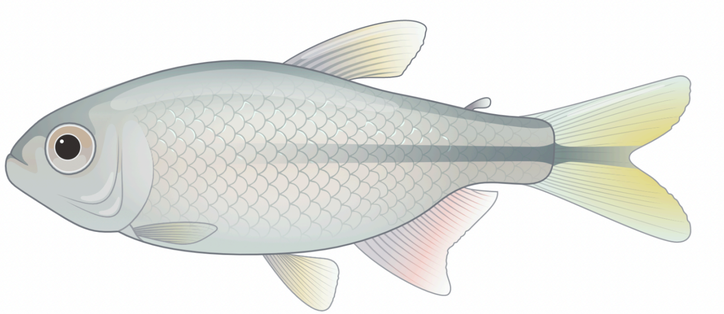
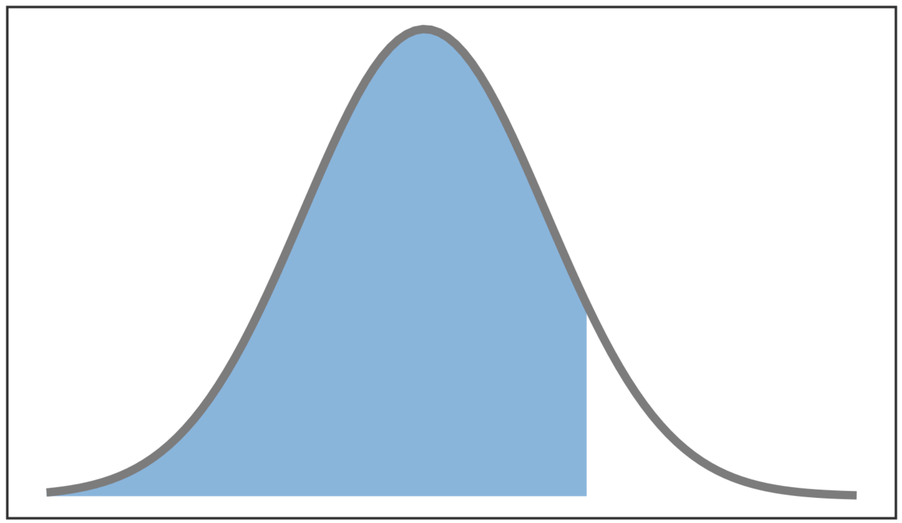


$$Z = \frac{60 - 54}{4.5} = 1.33$$



Z = 1.33

Area = 0.9082

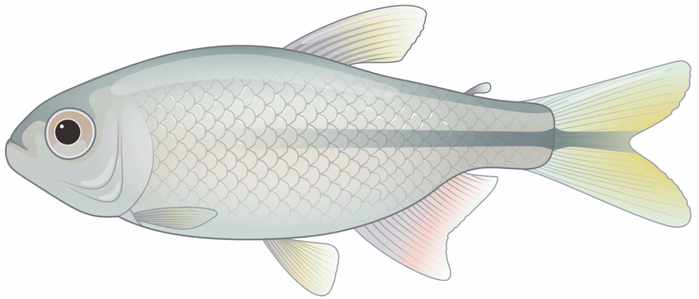
# Tables of the Normal Distribution

## Probability Content from -oo to Z

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | [illegible] | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.906[illegible] | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | [illegible] | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

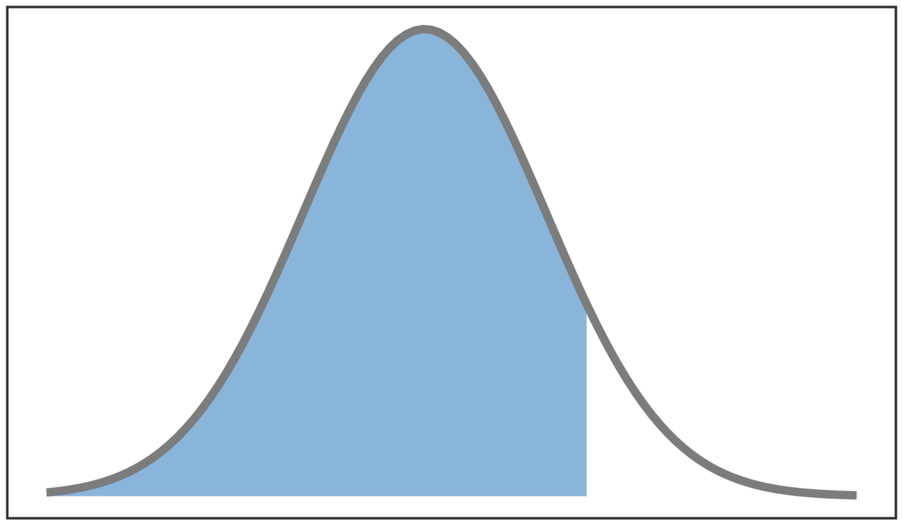
**In a certain population of fish, individual lengths follow a normal distribution. The mean length of the fish is 54.0 mm, and the standard deviation is 4.5 mm. What percentage of the fish are shorter than 60 mm?**

*Even if you use R to calculate p-value, ALWAYS smart to make a quick sketch*



$$Z = \frac{60 - 54}{4.5} = 1.33$$

Area = 0.9082



```
> pnorm(z, mean, sd)

> pnorm(1.33, 0, 1)

> pnorm(1.33)

> 0.908
```

Default is mean = 0, sd = 1

**In a certain population of fish, individual lengths follow a normal distribution. The mean length of the fish is 54.0 mm, and the standard deviation is 4.5 mm. What percentage of the fish are shorter than 60 mm?**

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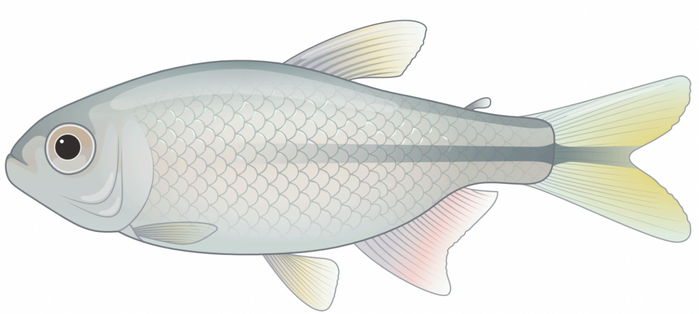

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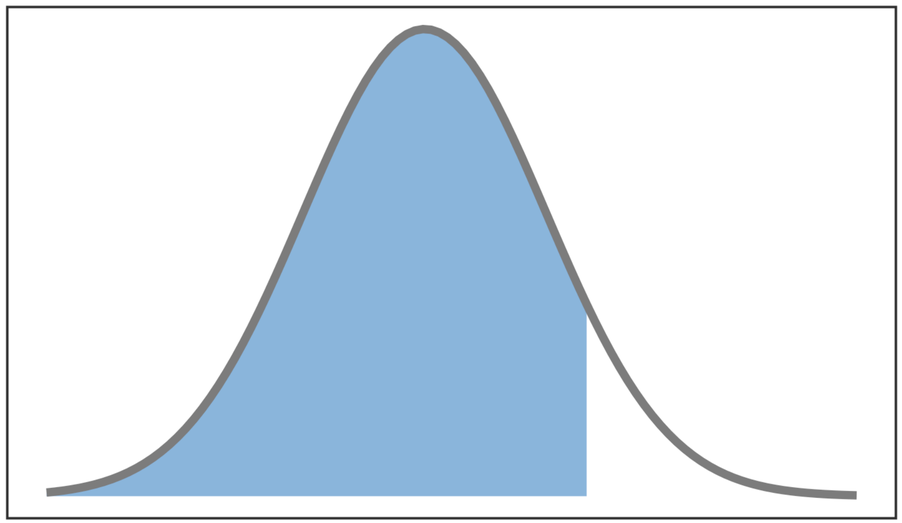

```
> pnorm(z, mean, sd)

> pnorm(60, 54, 4.5)

> 0.908
```

**In a certain population of fish, individual lengths follow a normal distribution. The mean length of the fish is 54.0 mm, and the standard deviation is 4.5 mm. What percentage of the fish are longer than 51 mm?**

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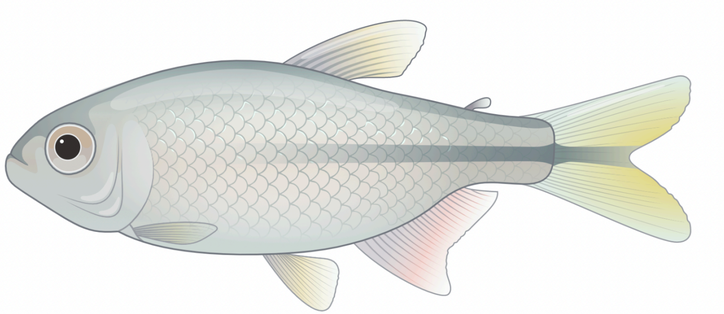
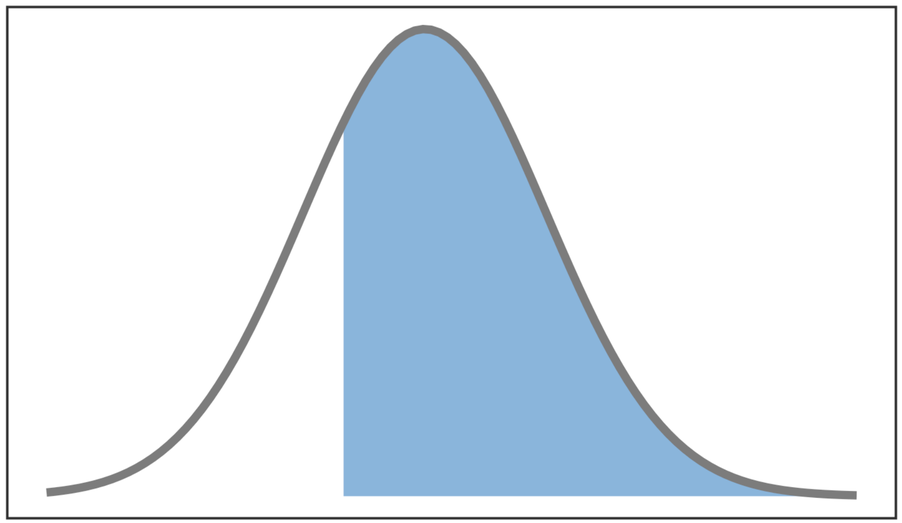




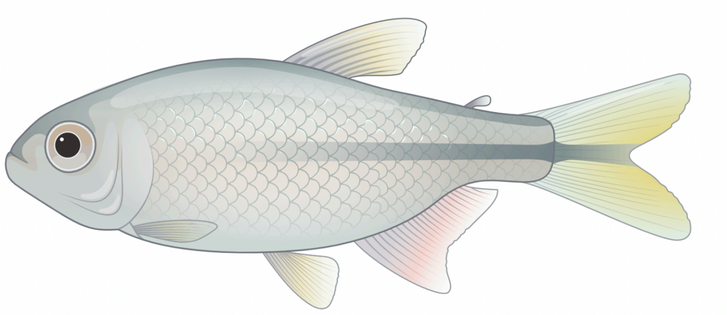
```
> 1 - pnorm(z, mean, sd)
```

```
> 1 - pnorm(51, 54, 4.5)
```

```
> 0.747
```

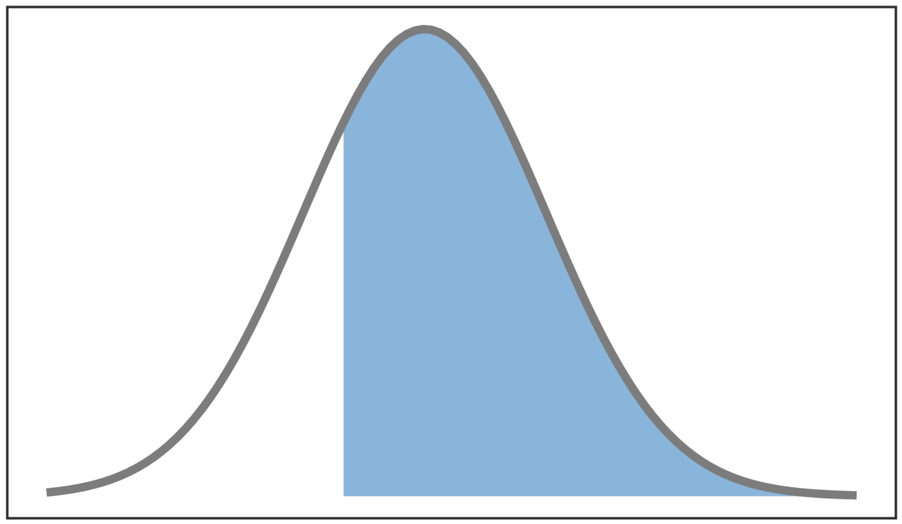
**In a certain population of fish, individual lengths follow a normal distribution. The mean length of the fish is 54.0 mm, and the standard deviation is 4.5 mm. <u>What percentage of the fish are longer than 51 mm?</u>**

*Even if you use R to calculate p-value, ALWAYS smart to make a quick sketch*



$N(54,4.5)$

(How we describe this distribution)



```
> pnorm(z, mean, sd)
```

```
> 1 - pnorm(51, 54, 4.5)
```

```
> pnorm(51, 54, 4.5, lower.tail = F)
```

```
> 0.747
```

**In a certain population of fish, individual lengths follow a normal distribution. The mean length of the fish is 54.0 mm, and the standard deviation is 4.5 mm. What percentage of the fish are longer than 51 mm?**

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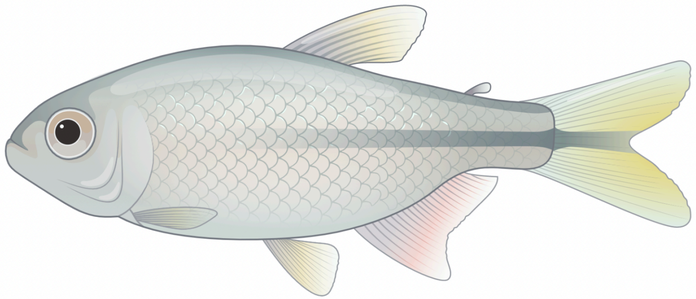
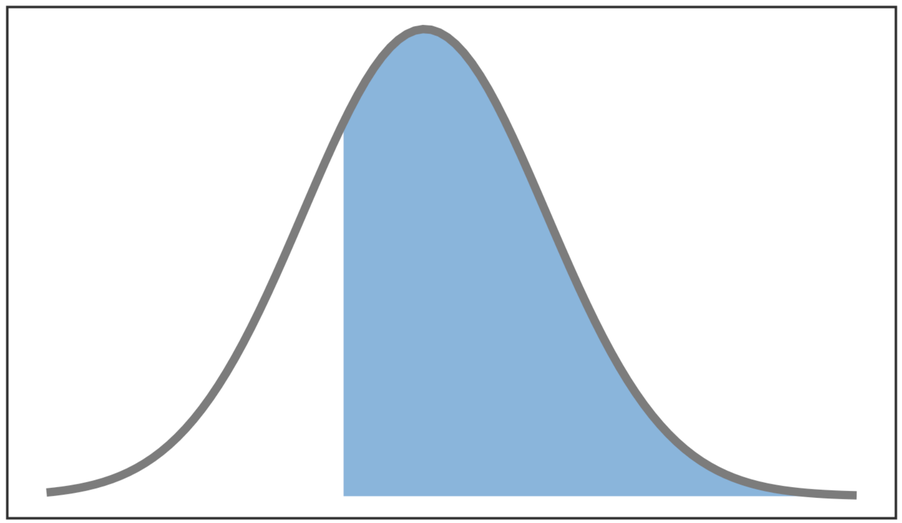


$$x \sim N(54,4.5) > 51$$

How we could write this question



```
> pnorm(z, mean, sd)
```

```
> 1 - pnorm(51, 54, 4.5)
```

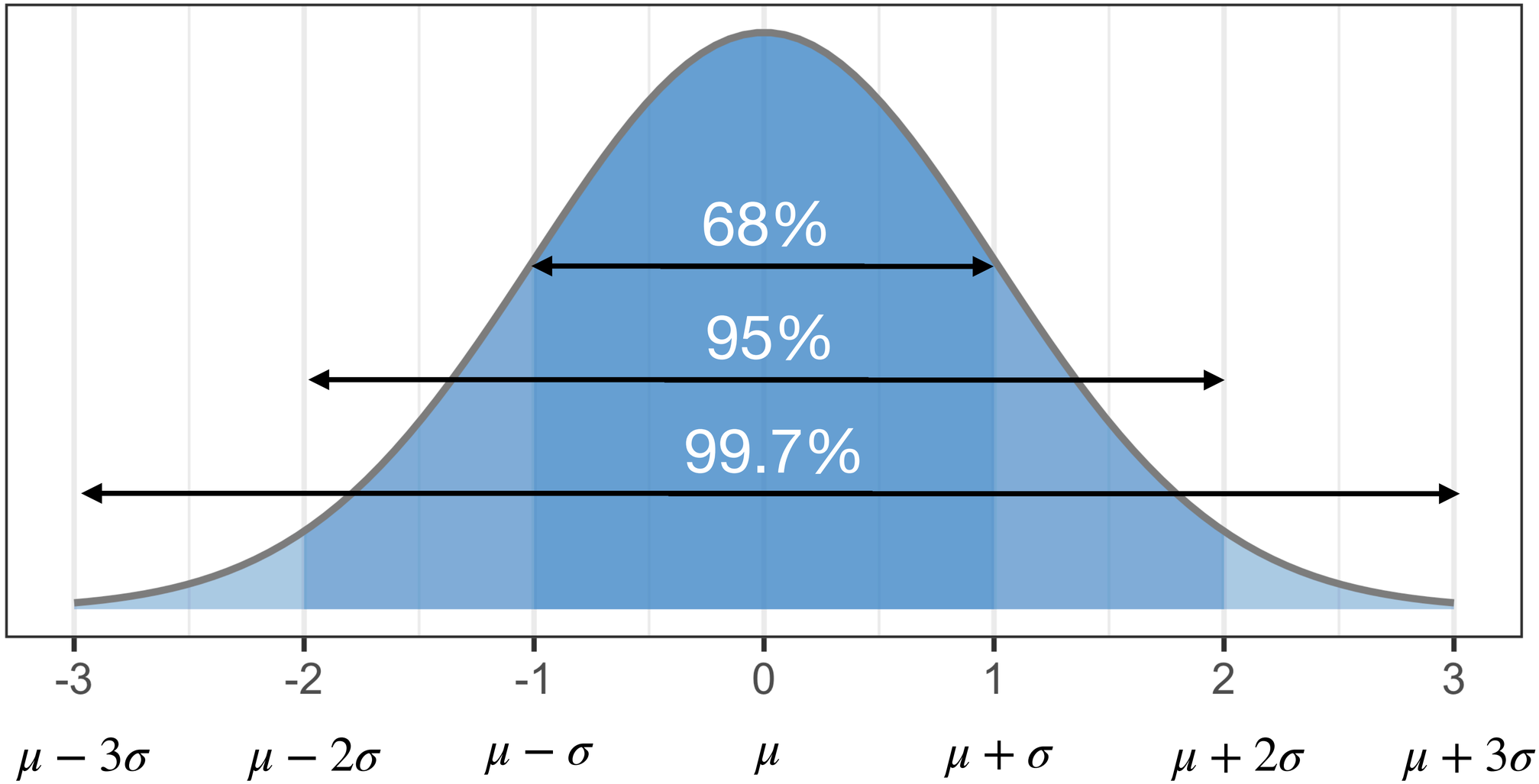
```
> pnorm(51, 54, 4.5, lower.tail = F)
```

```
> 0.747
```

# How can we assess normality?

- As we will see, many statistical tests are based on having data from a normal distribution, but how do you know if your data follows a normal distribution?
    - And what should you do if it doesn’t?

# How can we assess normality?



# How can we assess normality?

```
> summary(fish)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  46.21   52.41   55.13   54.82   57.46   62.83
```

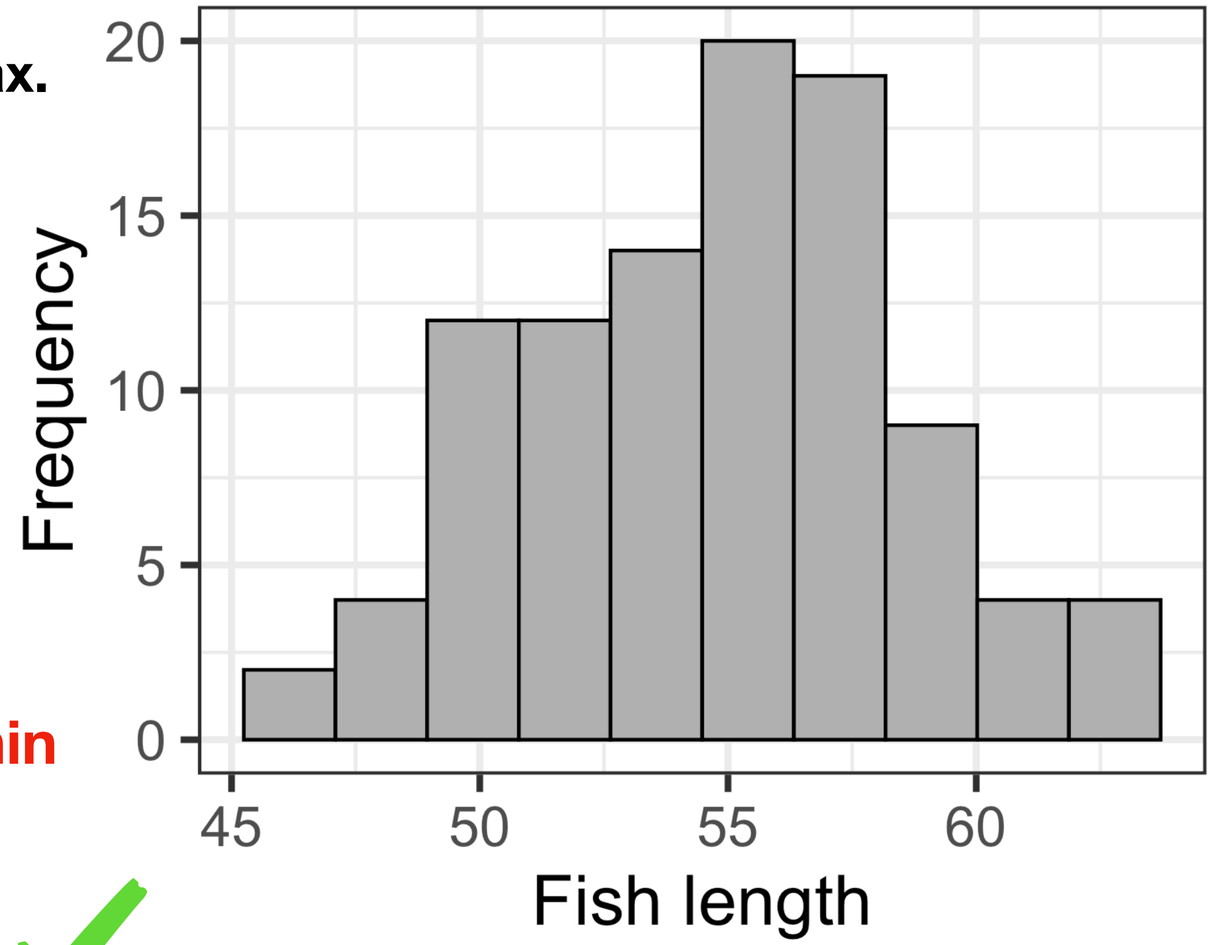
**1. Calculate the mean +/- 1 SD**

```
> mean(fish) + sd(fish)
[1] 58.65478
```

```
> mean(fish) - sd(fish)
[1] 50.97822
```

**2. What percent of the population is within these values?**

```
> sum(fish < 58.65 & fish > 50.97)
[1] 65
```



# How can we assess normality?

```
> summary(fish)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  46.21   52.41   55.13   54.82   57.46   62.83
```

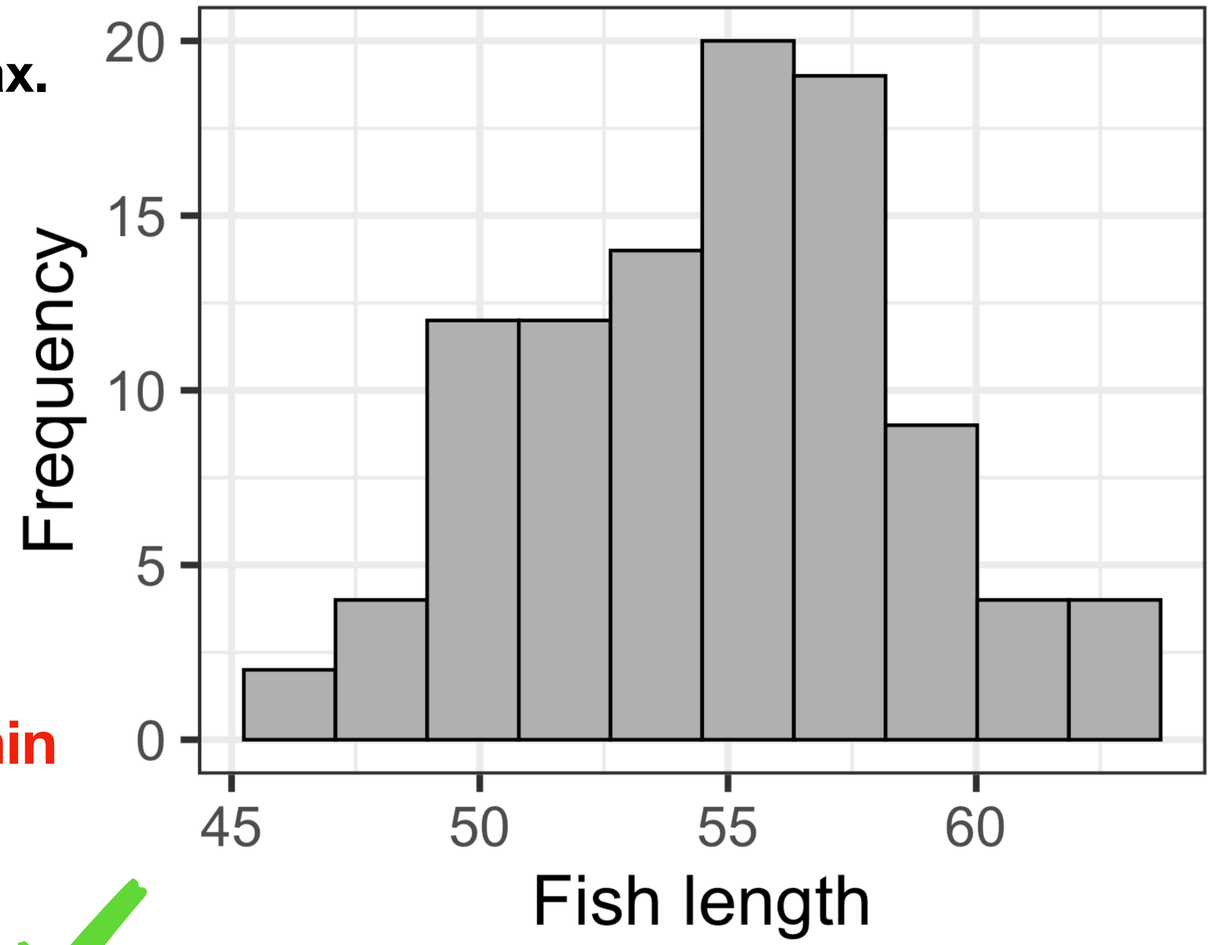
**1. Calculate the mean +/- 2 SD**

```
> mean(fish) + 2*sd(fish)
[1] 62.49306
```

```
> mean(fish) - 2*sd(fish)
[1] 47.13994
```

**2. What percent of the population is within these values?**

```
> sum(fish < 62.49 & fish > 47.13)
[1] 96
```



# How can we assess normality?

```
> summary(fish)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  46.21   52.41   55.13   54.82   57.46   62.83
```

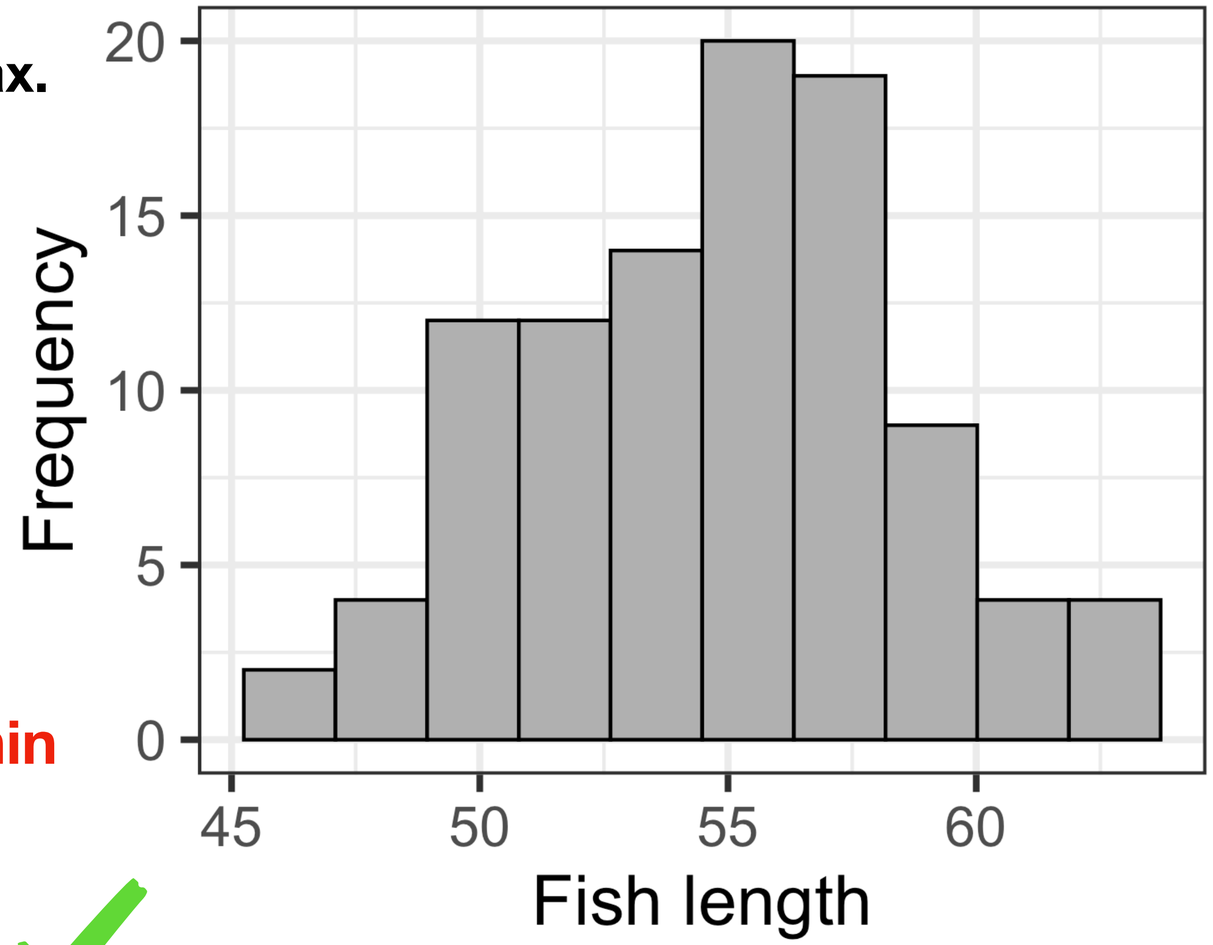
**1. Calculate the mean +/- 3 SD**

```
> mean(fish) + 3*sd(fish)
[1] 66.33134
```

```
> mean(fish) - 3*sd(fish)
[1] 43.30166
```

**2. What percent of the population is within these values?**

```
> sum(fish < 66.33 & fish > 43.30)
[1] 100
```

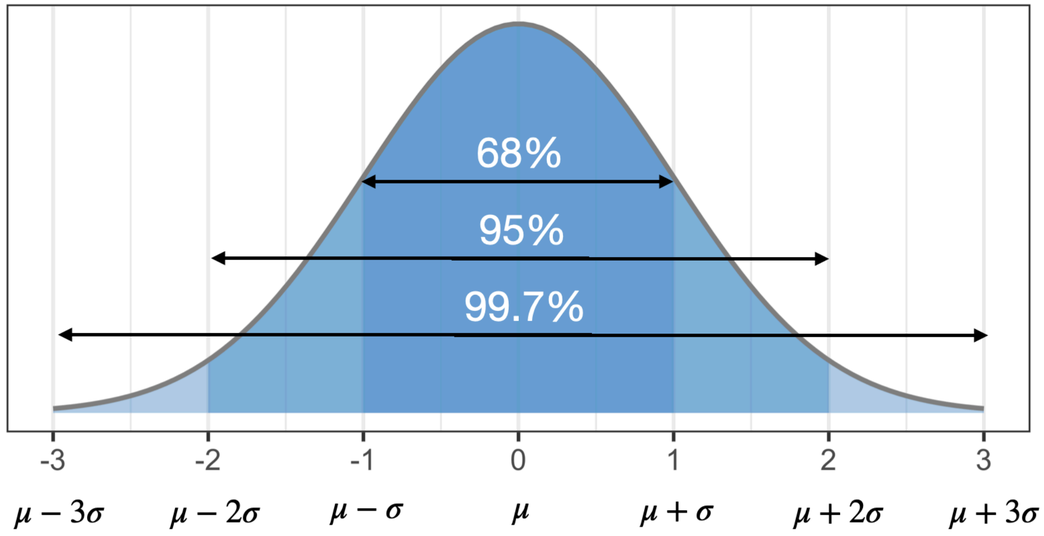


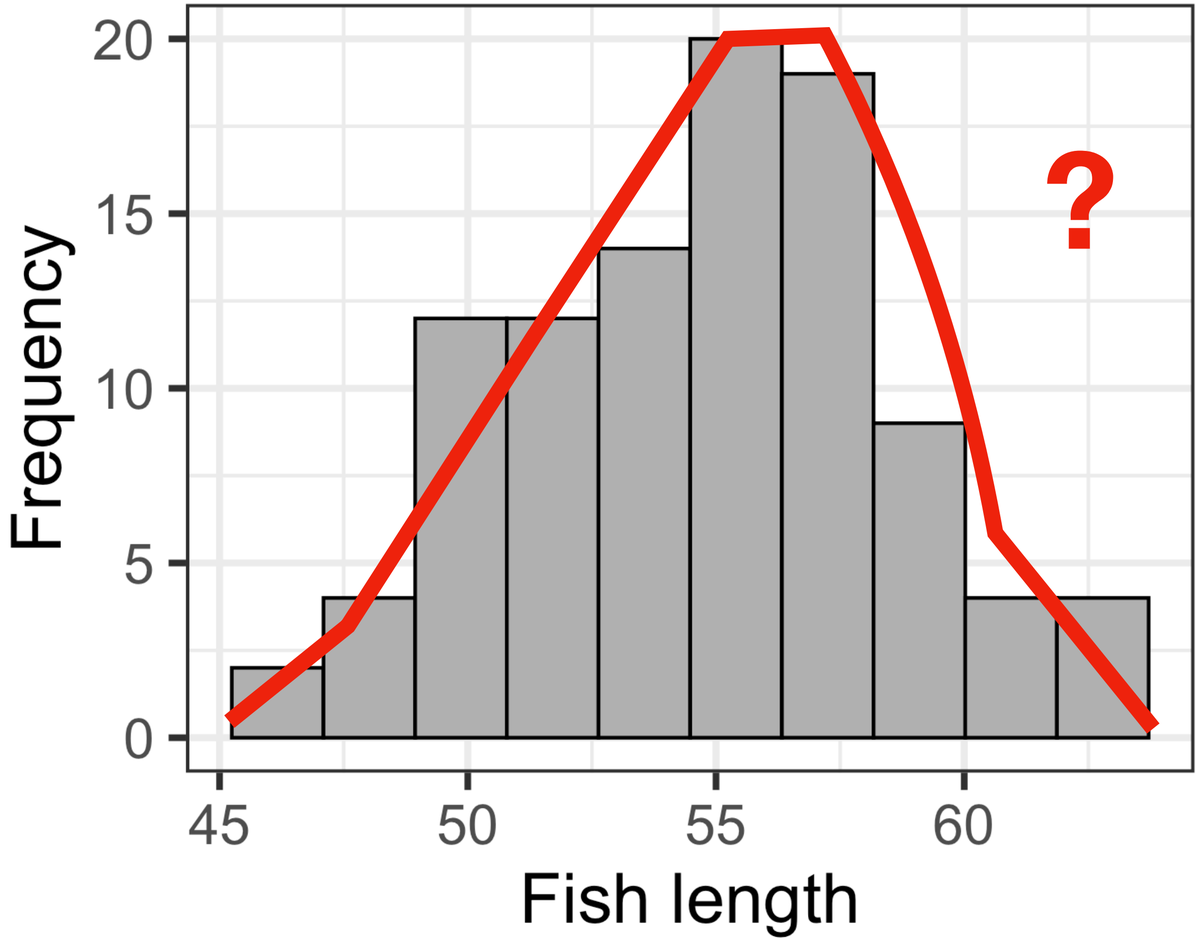
# How can we assess normality?

**+/- 1 SD = 65%**

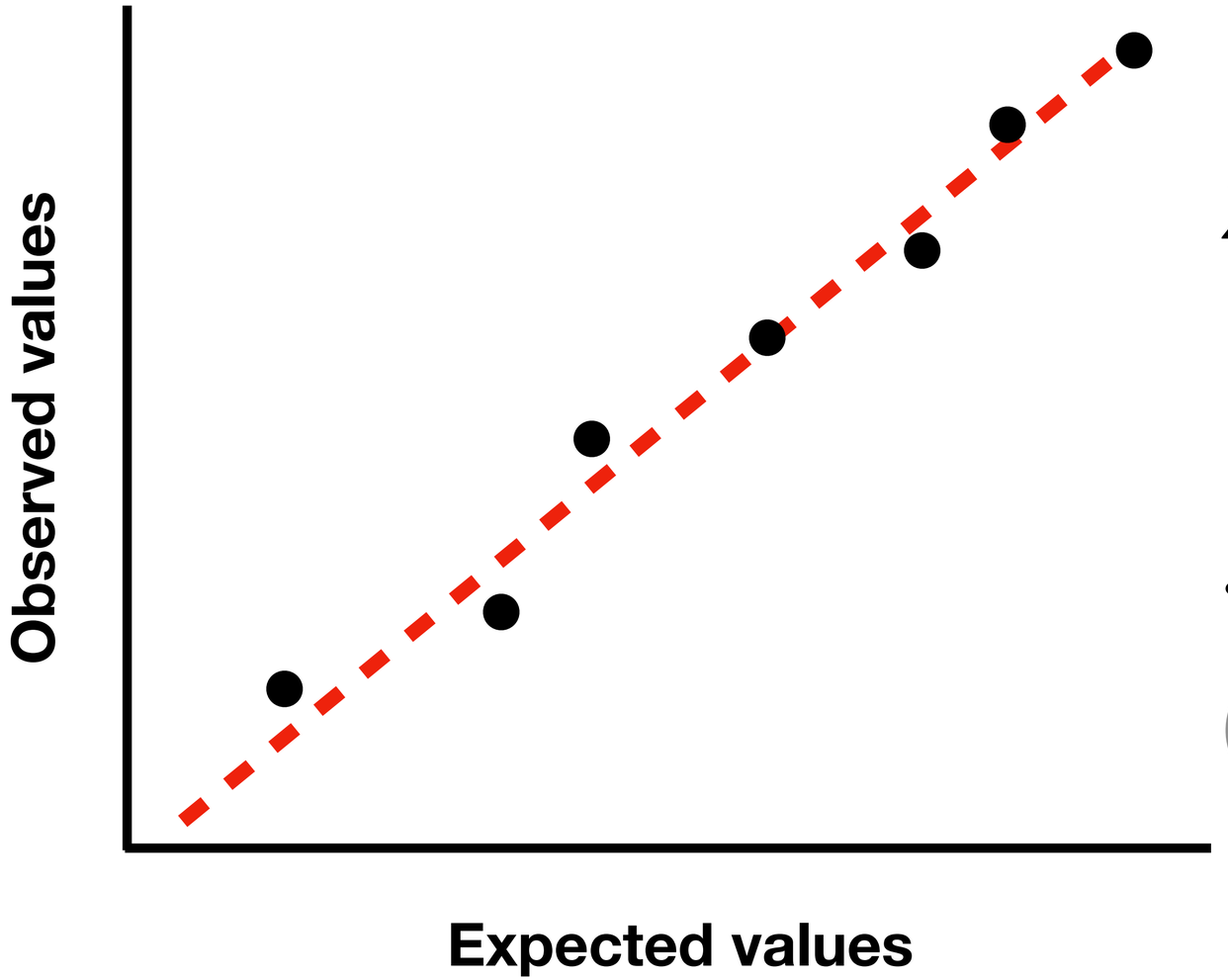
**+/- 2 SD = 95%**

**+/- 3 SD = 100%**



# Normal Quantile Plots

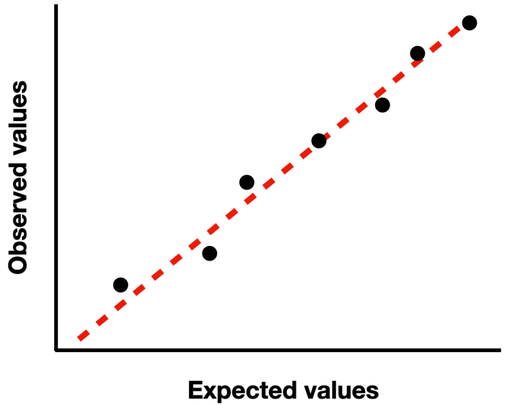



$$z = \frac{y - \mu}{\sigma}$$

$$\downarrow$$

$$y = \sigma z + \mu$$

$$(y = mx + b)$$




# Normal Quantile Plots

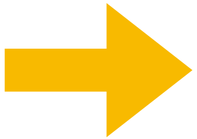
**1. Calculate (adjusted) percentile of each data point**

**2. Calculate the z score from the percentile**

**3. Calculate the theoretical y (expected value) from z score**

**4. Plot expected vs. observed**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |



**1. Calculate (adjusted) percentile of each data point**

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| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |

**1. Calculate (adjusted) percentile of each data point**

$$1/11 = 9.09\,\%$$

$$11/11 = 100\,\%$$

**2. Calculate the z score from the percentile**

**3. Calculate the theoretical y (expected value) from z score**

**4. Plot expected vs. observed**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |

**1. Calculate (adjusted) percentile of each data point**

$$100(i - \frac{1}{2})/n = 100(1 - \frac{1}{2})/11 = 4.55\,\%$$

$(11 - 0.5)/11 = 95.45\,\%$

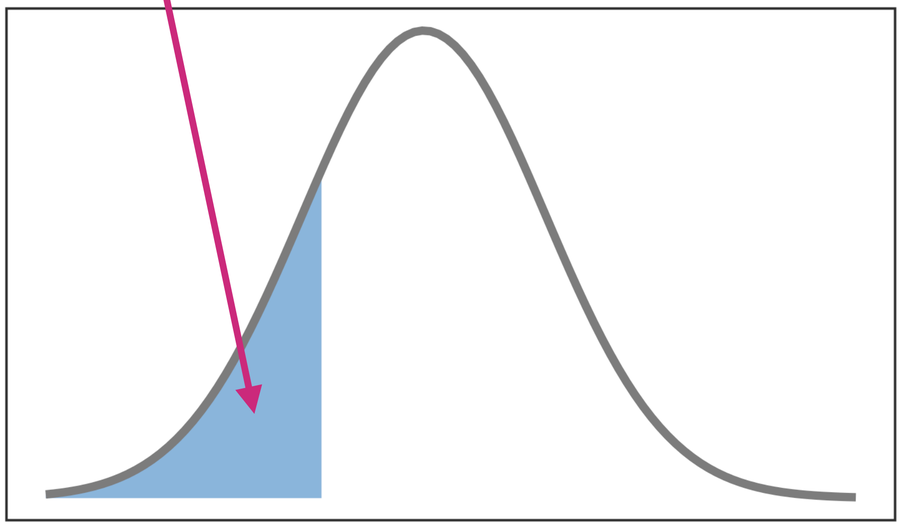
**2. Calculate the z score from the percentile**

**3. Calculate the theoretical y (expected value) from z score**

**4. Plot expected vs. observed**

4.55%

(Area under curve)



z = -1.69

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.6 | .0002 | .0002 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 | .0001 |
| −3.5 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 | .0002 |
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |

**1. Calculate (adjusted) percentile of each data point**

$$100(i - \frac{1}{2})/n = 100(1 - \frac{1}{2})/11 = 4.55\,\%$$

$(11 - 0.5)/11 = 95.45\,\%$

**2. Calculate the z score from the percentile**

```
> qnorm(0.0455)
```

**= -1.69**

**3. Calculate the theoretical y (expected value) from z score**

**4. Plot expected vs. observed**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |

**1. Calculate (adjusted) percentile of each data point**

$$100(i - \frac{1}{2})/n = 100(1 - \frac{1}{2})/11 = 4.55\,\%$$

$(11 - 0.5)/11 = 95.45\,\%$

**2. Calculate the z score from the percentile**

```
> qnorm(0.0455)
```

**= -1.69**

**3. Calculate the theoretical y (expected value) from z score**

$$Z = \frac{y - \mu}{\sigma}$$

**4. Plot expected vs. observed**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |

**1. Calculate (adjusted) percentile of each data point**

$$100(i - \frac{1}{2})/n = 100(1 - \frac{1}{2})/11 = 4.55\,\%$$

$(11 - 0.5)/11 = 95.45\,\%$

**2. Calculate the z score from the percentile**

```
> qnorm(0.0455)
```

= -1.69

**3. Calculate the theoretical y (expected value) from z score**

$$Z = \frac{y - \mu}{\sigma} \longrightarrow y = z\sigma + \mu$$

**4. Plot expected vs. observed**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |

**1. Calculate (adjusted) percentile of each data point**

$$100(i - \frac{1}{2})/n = 100(1 - \frac{1}{2})/11 = 4.55\,\%$$

$(11 - 0.5)/11 = 95.45\,\%$

**2. Calculate the z score from the percentile**

```
> qnorm(0.0455)
```

**= -1.69**

**3. Calculate the theoretical y (expected value) from z score**

$$Z = \frac{y - \mu}{\sigma} \longrightarrow y = -1.69 * 2.87 + 65.5 = 60.6$$

**4. Plot expected vs. observed**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |

**1. Calculate (adjusted) percentile of each data point**

$$100(i - \frac{1}{2})/n = 100(1 - \frac{1}{2})/11 = 4.55\,\%$$

$(11 - 0.5)/11 = 95.45\,\%$

**2. Calculate the theoretical y (expected value) from percentile**

```
> qnorm(0.0455, mean(height), sd(height))
```

```
> qnorm(0.0455, 65.5, 2.87)
```

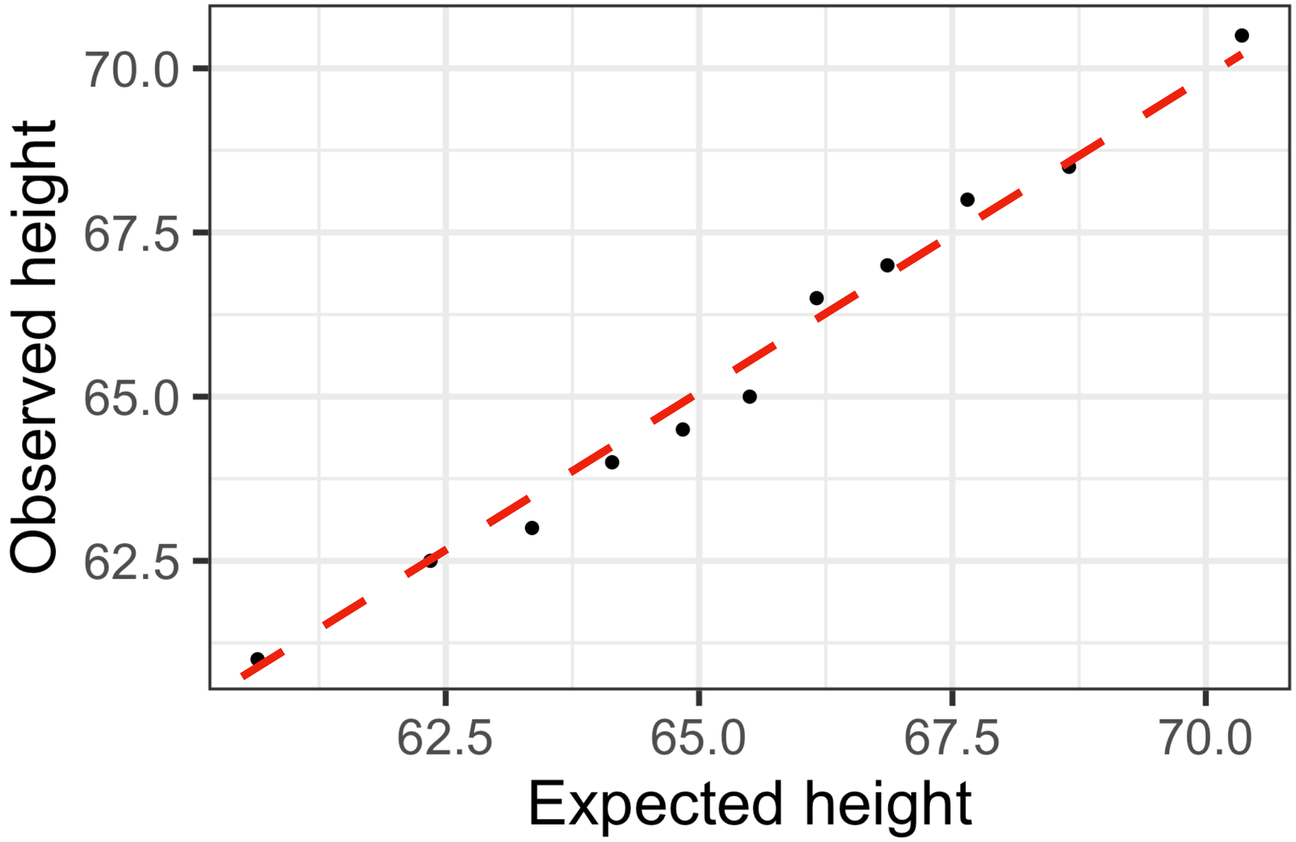
**= 60.06**

**4. Plot expected vs. observed**

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |
| **Percentile** | 2.29 | | | | | | | | | | |
| **Adjusted Percentile** | 4.55 | | | | | | | | | | |
| **Z score** | -1.69 | | | | | | | | | | |
| **Theoretical height** | 60.6 | | | | | | | | | | |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |
| **Percentile** | 0.00 | 18.18 | 27.27 | 36.36 | 45.45 | 54.55 | 63.64 | 72.73 | 81.82 | 90.91 | 100 |
| **Adjusted Percentile** | 4.55 | 13.64 | 22.73 | 31.82 | 40.91 | 50 | 59.09 | 68.18 | 77.27 | 86.36 | 95.45 |
| **Z score** | -1.69 | -1.10 | -0.75 | -0.47 | -0.23 | 0 | 0.23 | 0.47 | 0.75 | 1.10 | 1.69 |
| **Theoretical height** | 60.6 | 62.3 | 63.4 | 64.1 | 64.8 | 65.5 | 66.2 | 66.9 | 67.6 | 68.7 | 70.4 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Observed height** | 61.0 | 62.5 | 63 | 64 | 64.5 | 65 | 66.5 | 67 | 68 | 68.5 | 70.5 |
| **Theoretical height** | 60.6 | 62.3 | 63.4 | 64.1 | 64.8 | 65.5 | 66.2 | 66.9 | 67.6 | 68.7 | 70.4 |

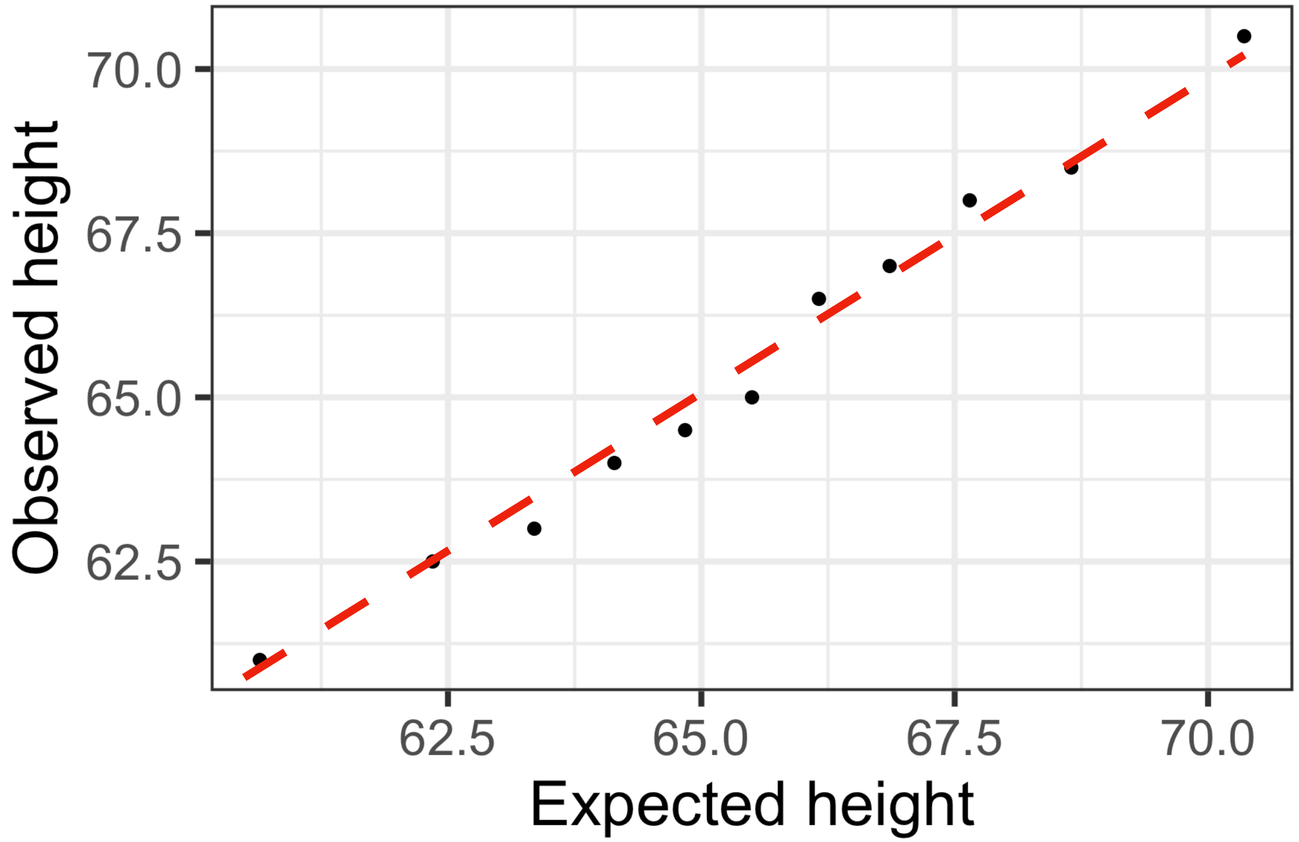


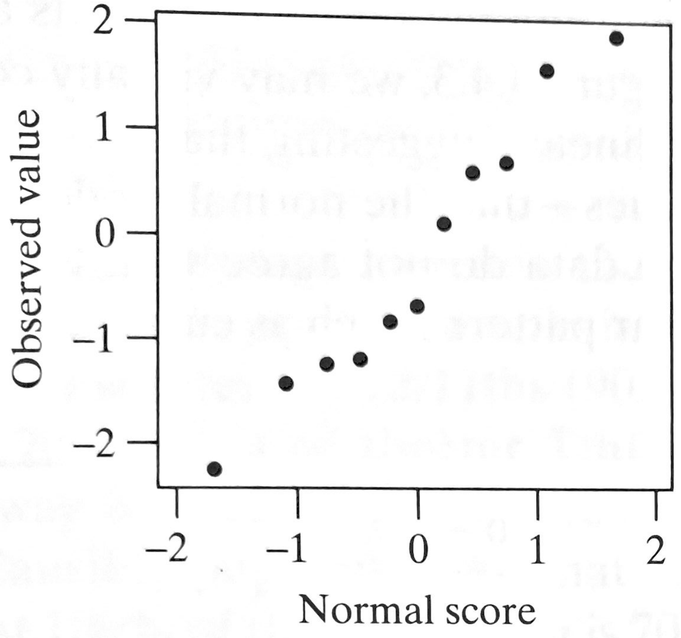
# Shapiro-Wilk test to assess non-normality

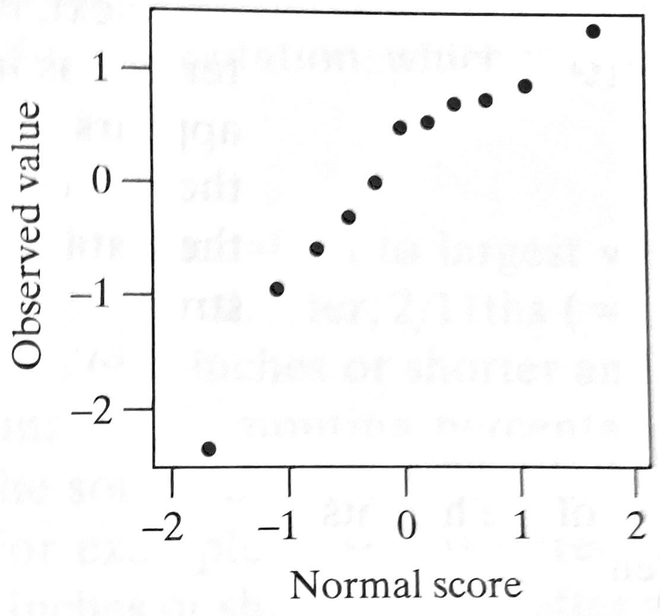
```
> shapiro.test(height)

        Shapiro-Wilk normality test

data:  height
W = 0.98371, p-value = 0.9833
```

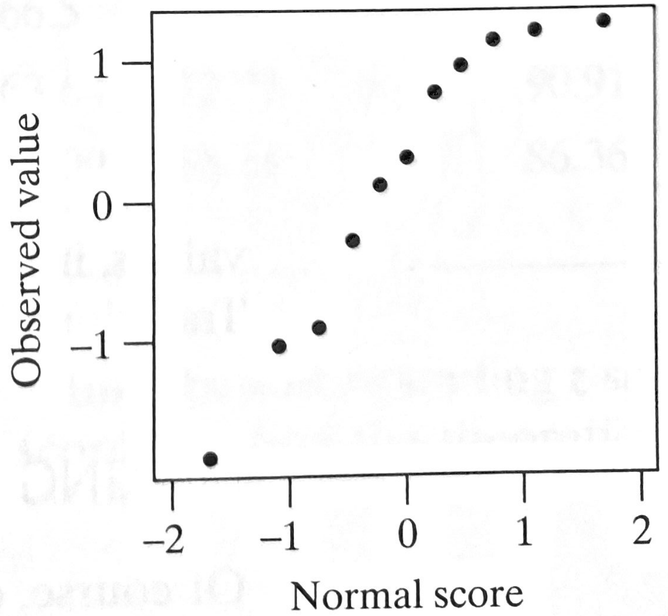
**Smaller the p-value = stronger evidence for NON-normality**

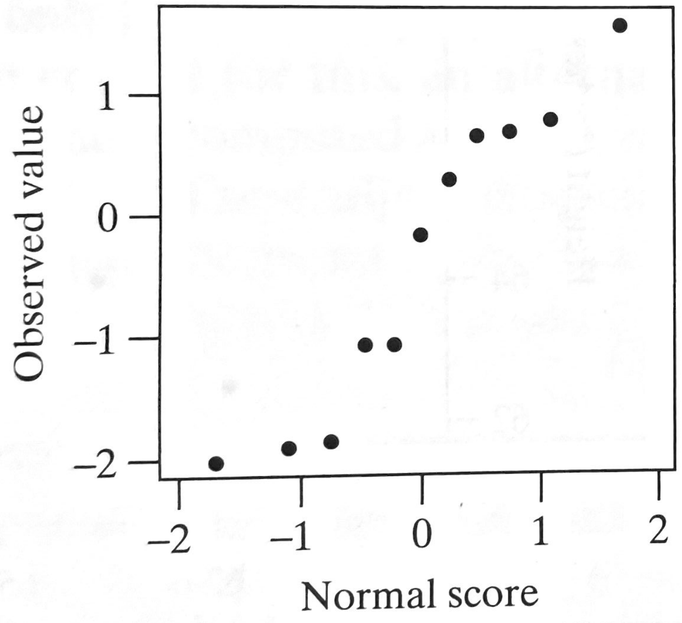
**Larger the p-value, stronger evidence for normality**

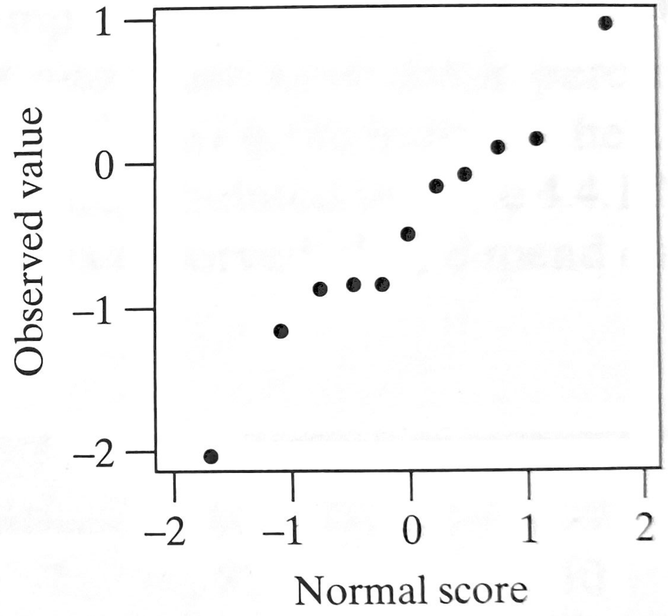


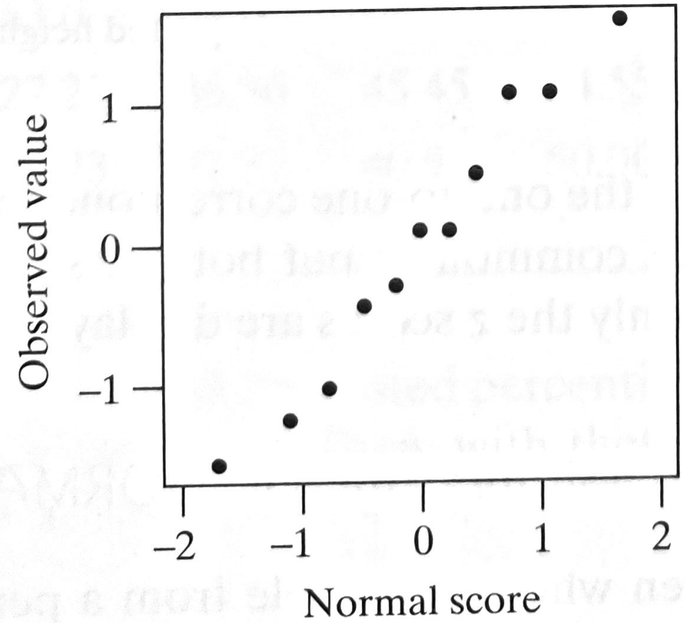

Observed value
Normal score
2
1
0
−1
−2
−2
−1
0
1
2


Observed value
Normal score
1
0
−1
−2
−2
−1
0
1
2

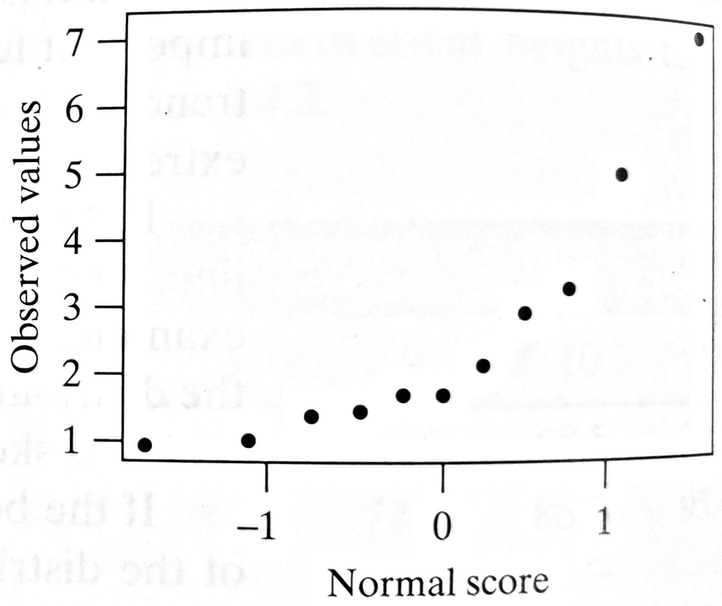

Observed value
Normal score
1
0
−1
−2
−1
0
1
2

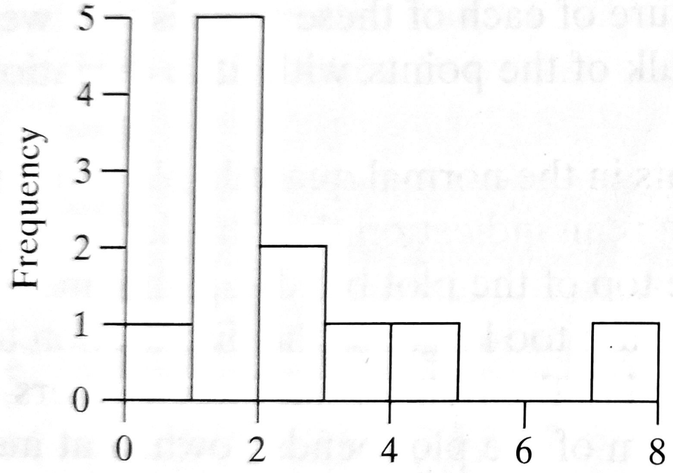

Observed value
Normal score
1
0
−1
−2
−2
−1
0
1
2


Observed value
Normal score
1
0
−1
−2
−2
−1
0
1
2

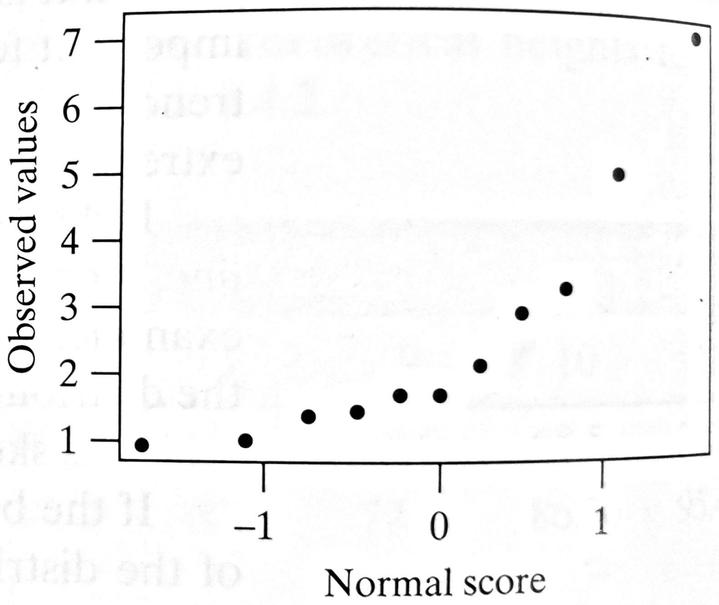

Observed value
Normal score
1
0
−1
−2
−1
0
1
2

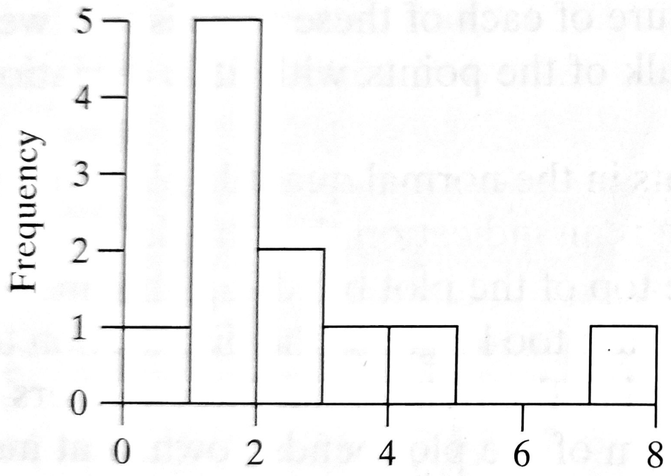
# Right skew


Observed values
7
6
5
4
3
2
1
−1
0
1
Normal score

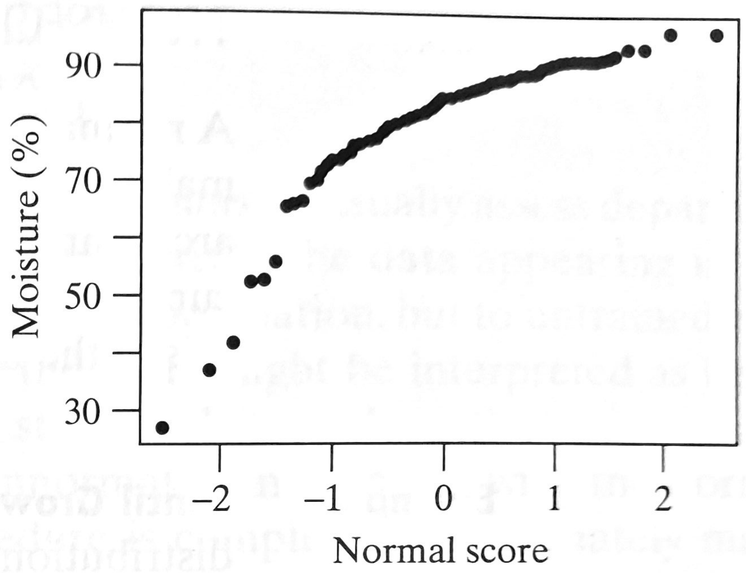

Frequency
5
4
3
2
1
0
0
2
4
6
8

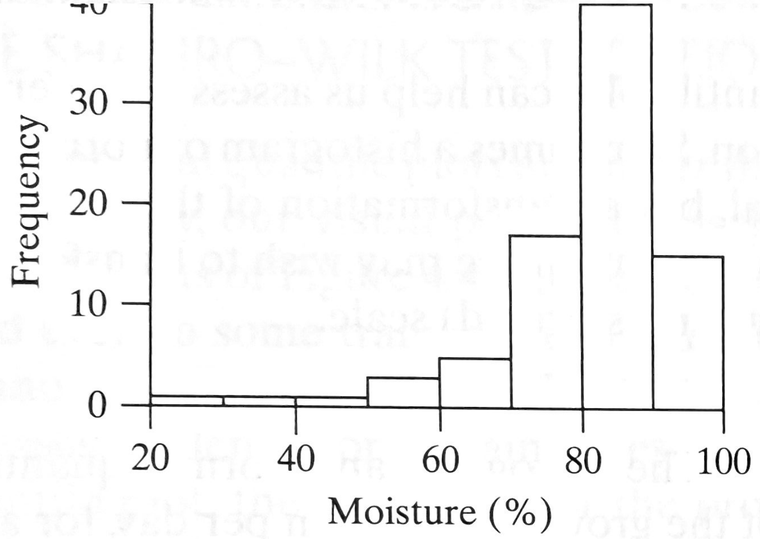
## Right skew



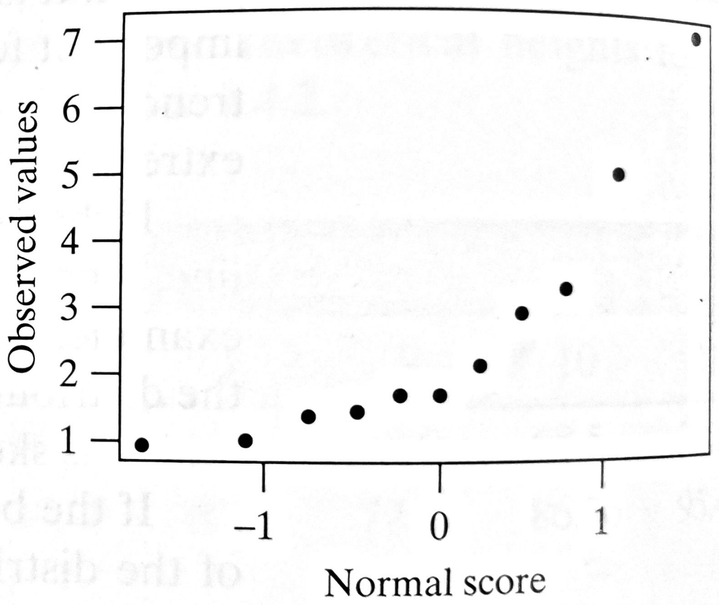



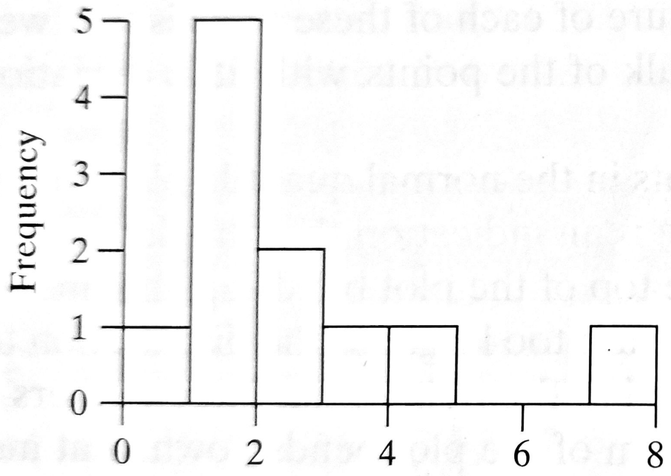

## Left skew



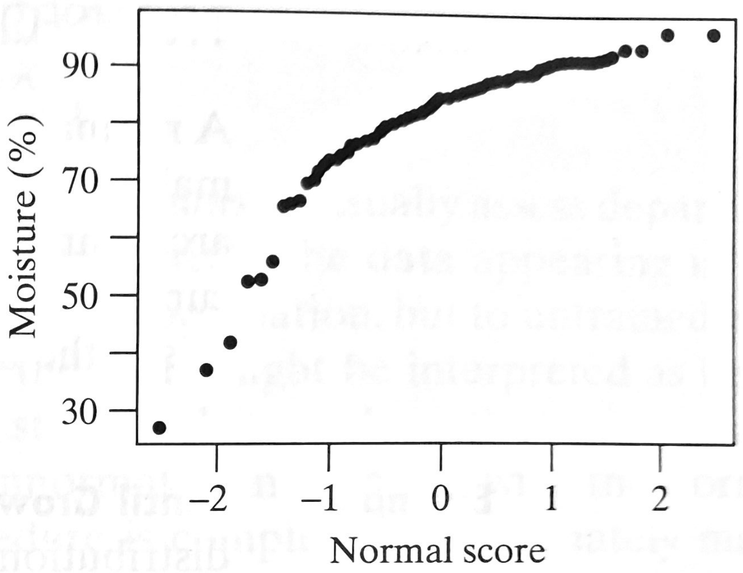



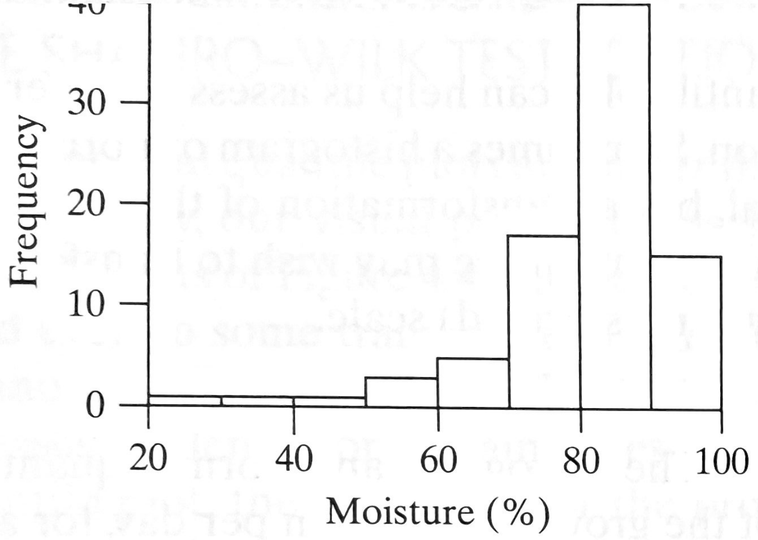
## Right skew



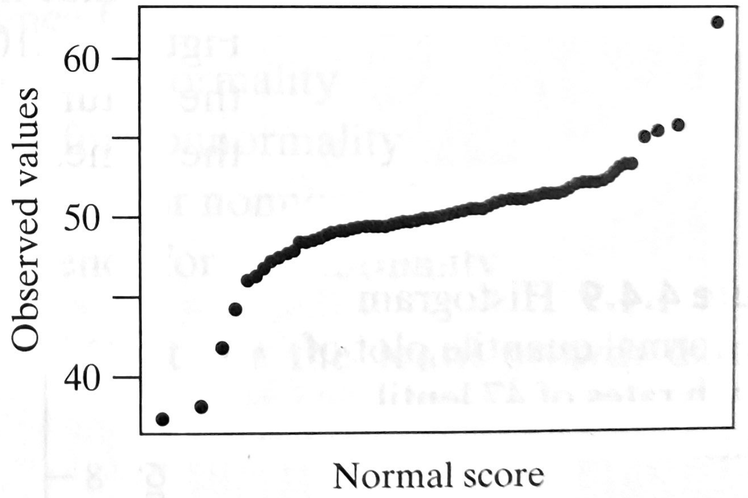



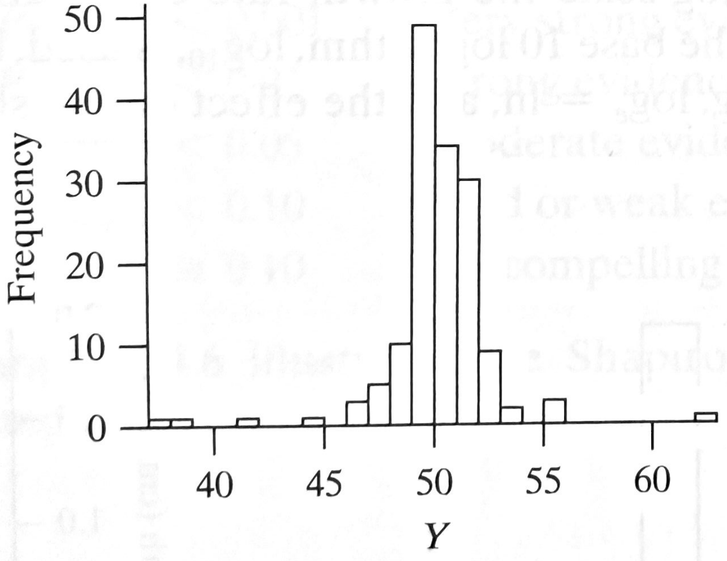

## Left skew



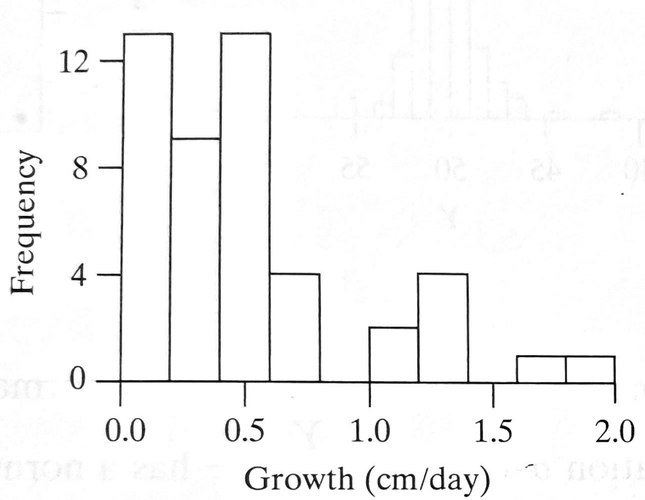



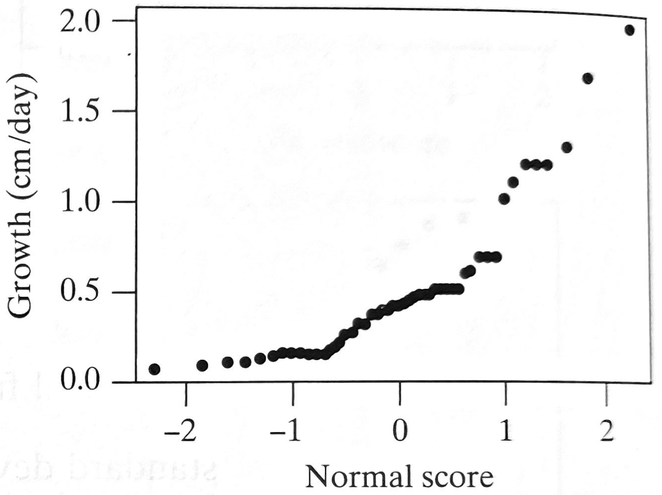

## Normal, long tails



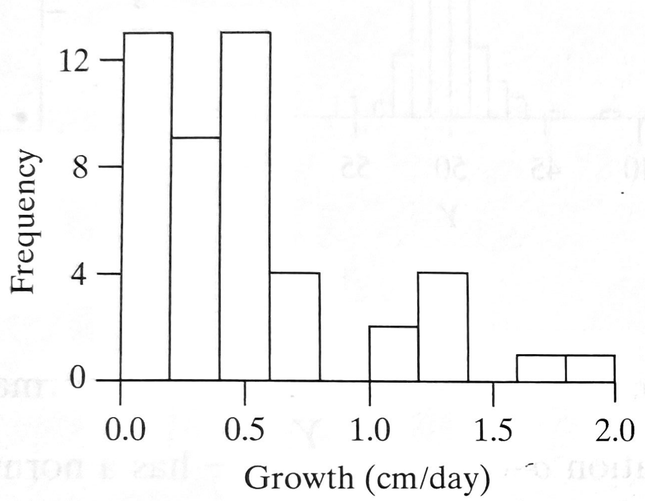



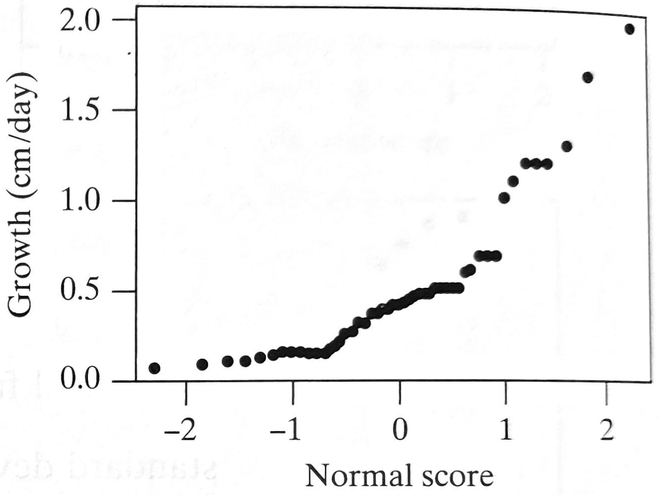
# Transformation for non-normal data

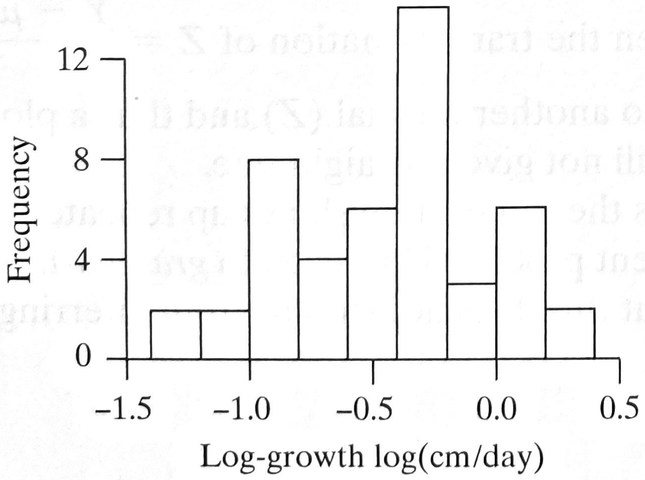


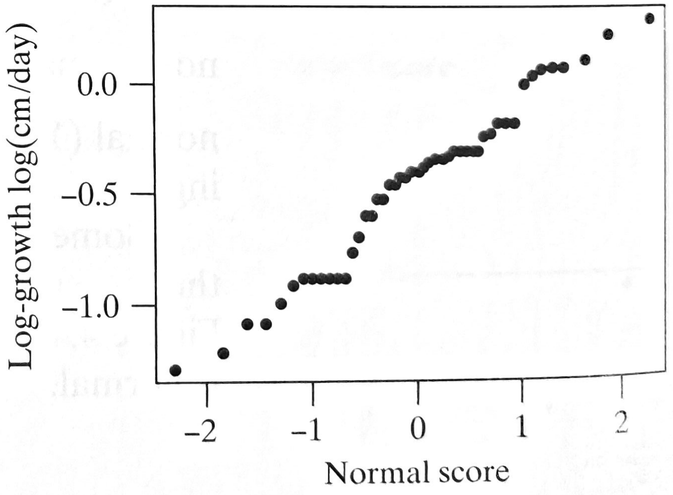



# Transformation for non-normal data



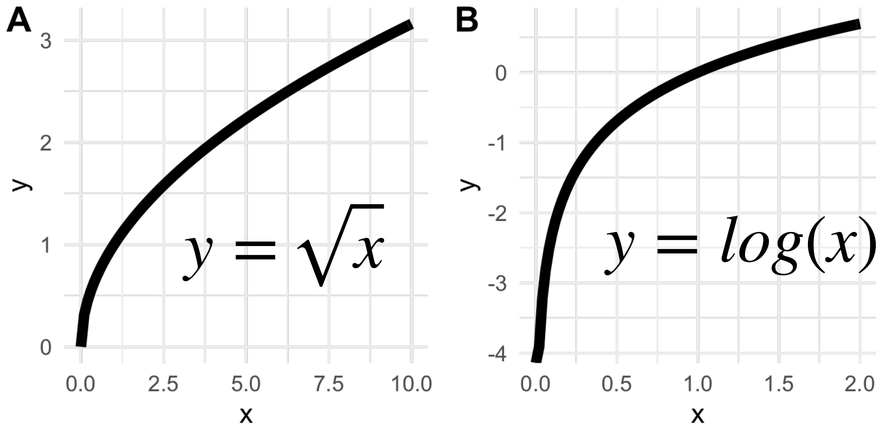



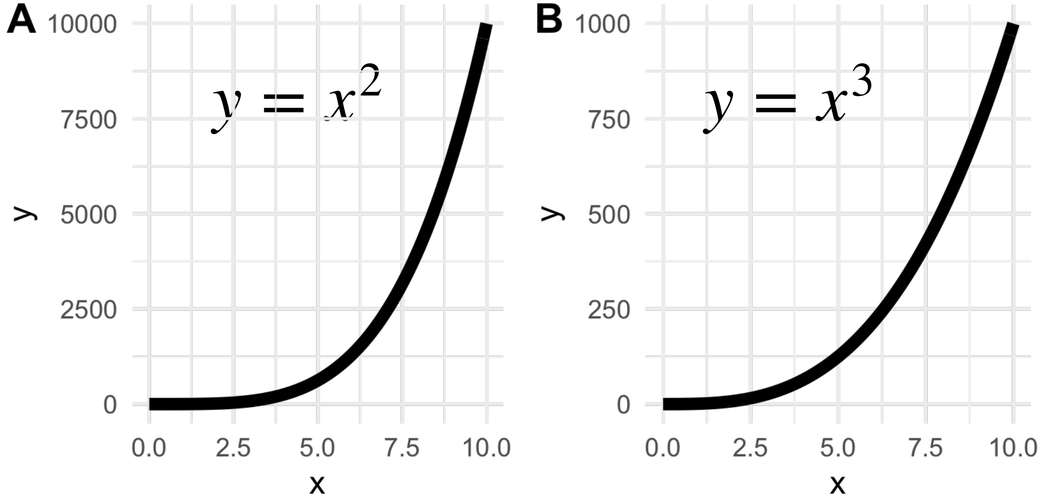



# Transformation for non-normal data

## RIGHT SKEW




## LEFT SKEW

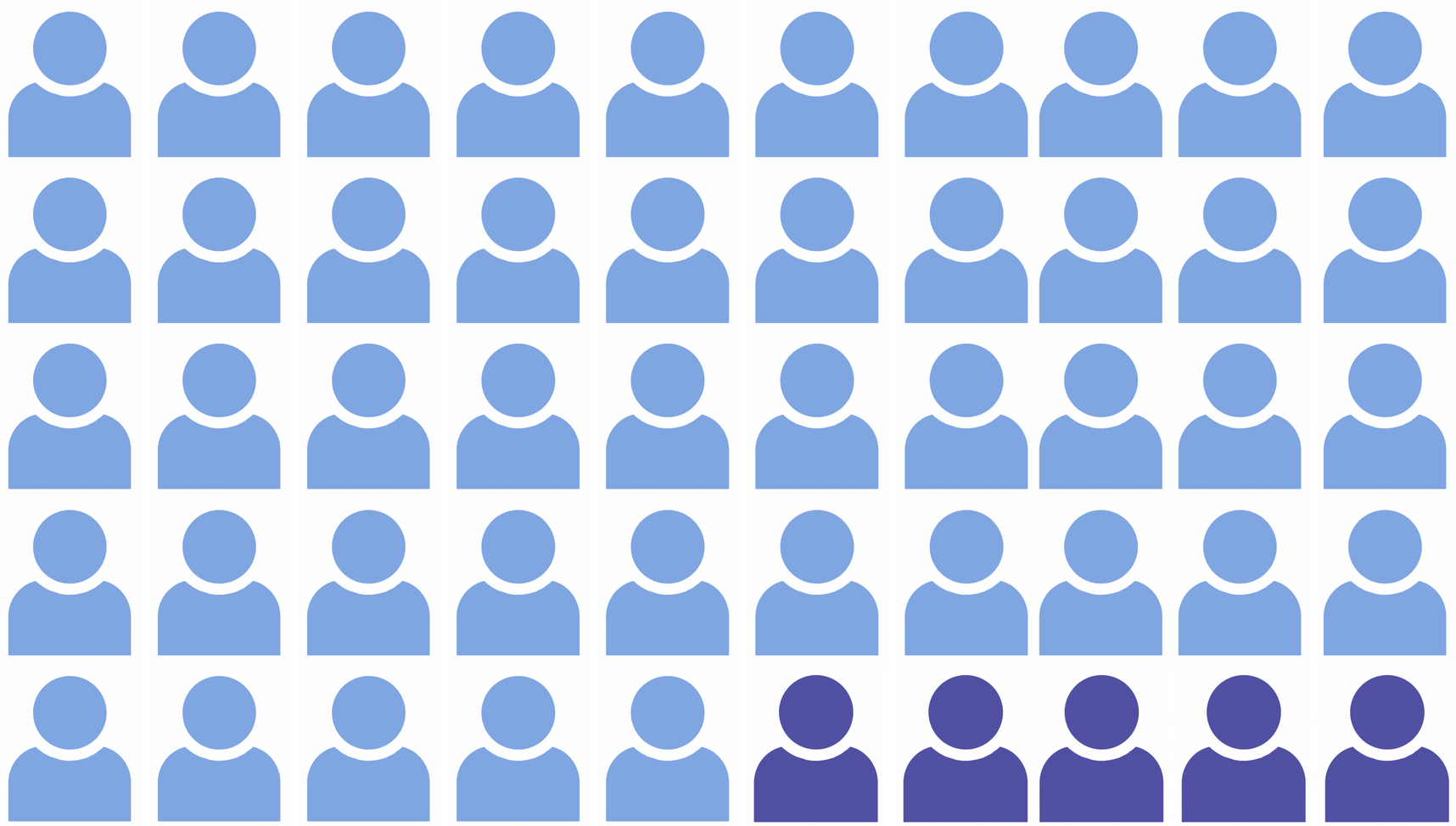


# How can we assess ~~normality~~?

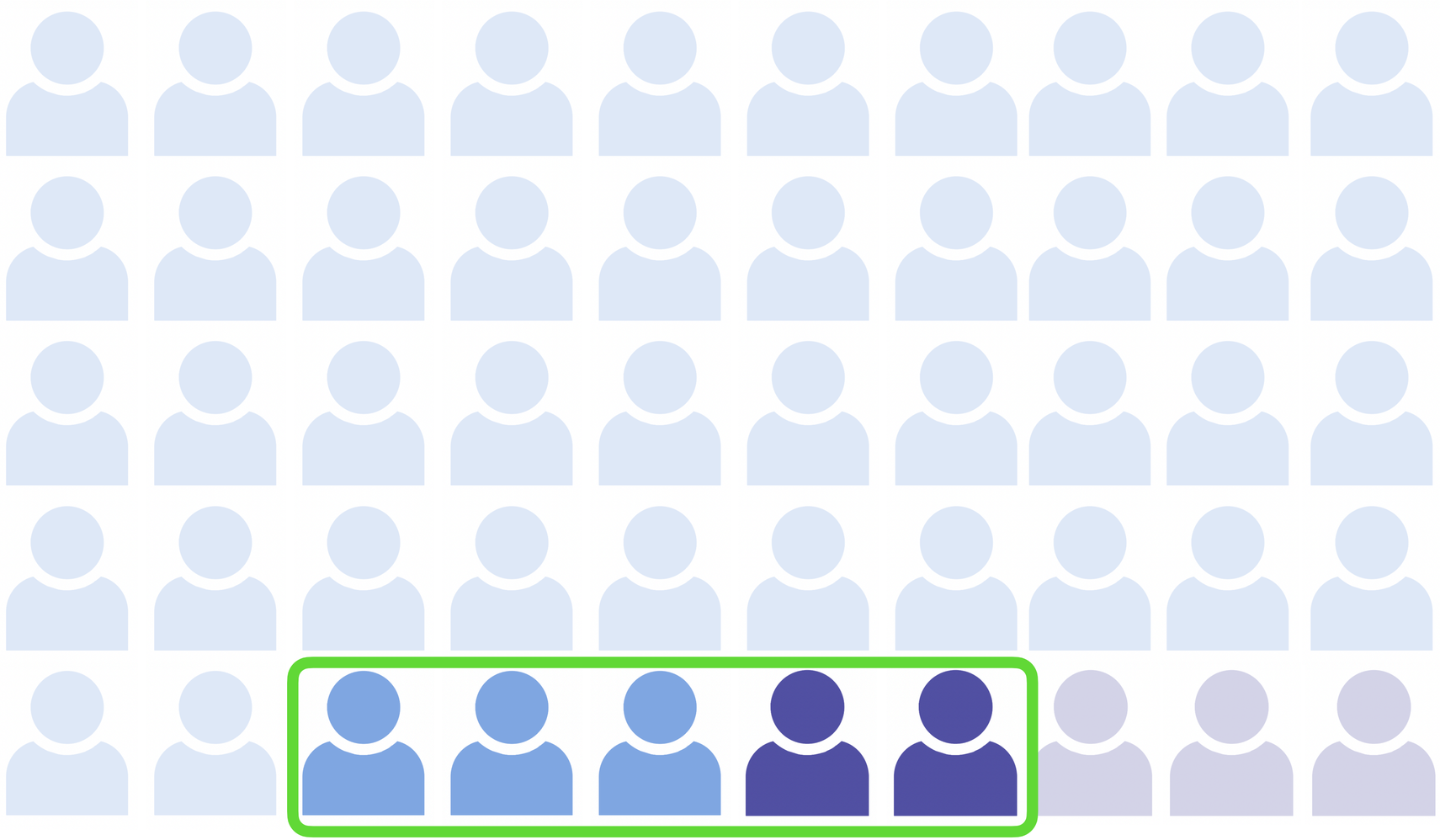
*Gaussian distribution*

- As we will see, many statistical tests are based on having data from a normal distribution, but how do you know if your data follows a normal distribution?
  - You can make a quantile plot (Q-Q plot) comparing expected and observed values (*straight line = normally distributed*)
  - You can also do a Shapiro-Wilk test of non-normality (*higher p-value = more evidence of a normal distribution*)
- And what should you do if it doesn't?
  - You can transform your data (*i.e. log(x) for right skewed data and $x^2$ for left skewed data*)
  - Don't freak out! Common for biological data to not have a normal distribution… (*and we will learn soon that sampling distributions are often normal…)*

# Population vs. Sample

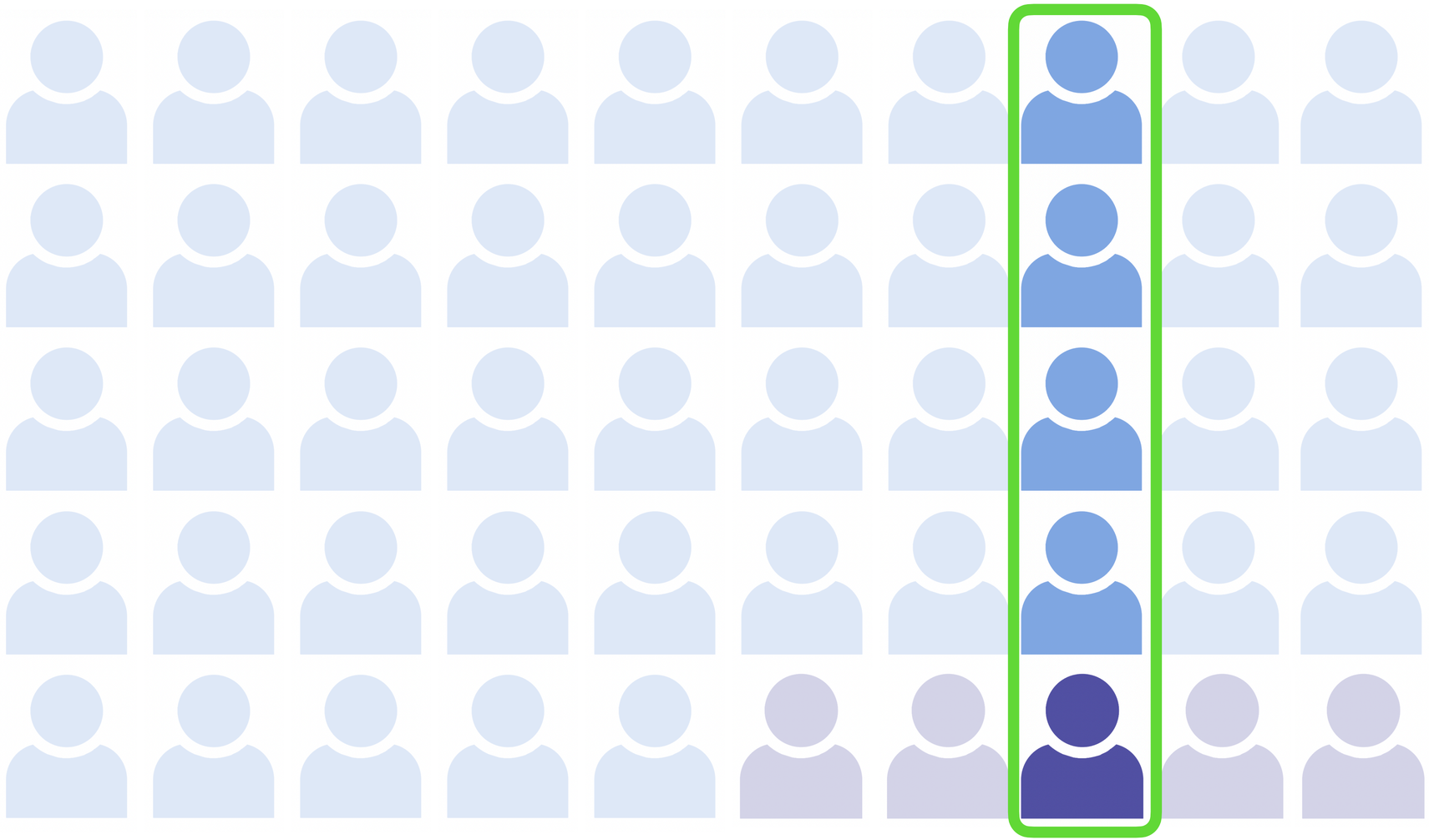


# Population vs. Sample



**Sample 1:** 40%

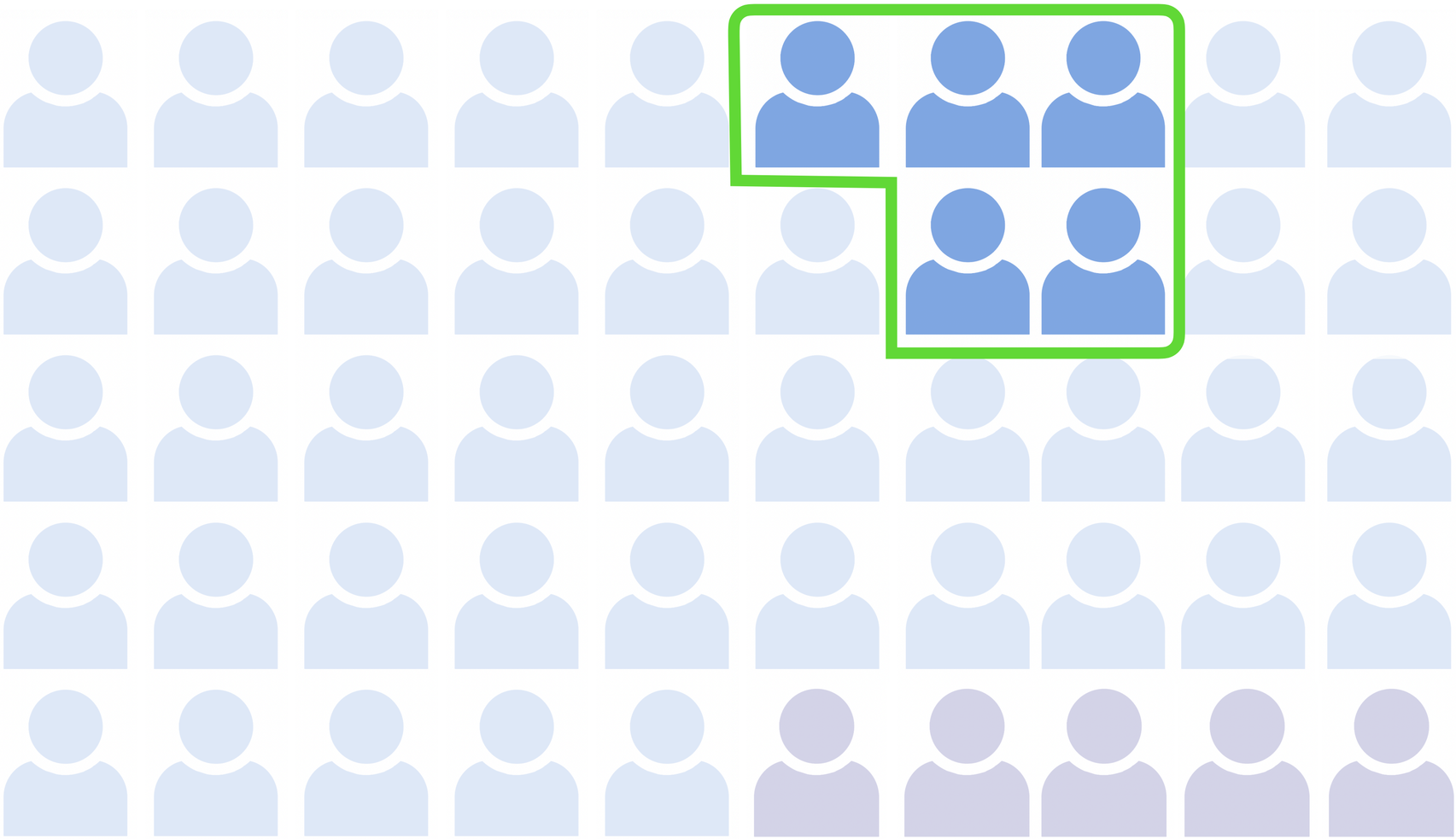
# Population vs. Sample



**Sample 1:** 40%

**Sample 2:** 10%

# Population vs. Sample

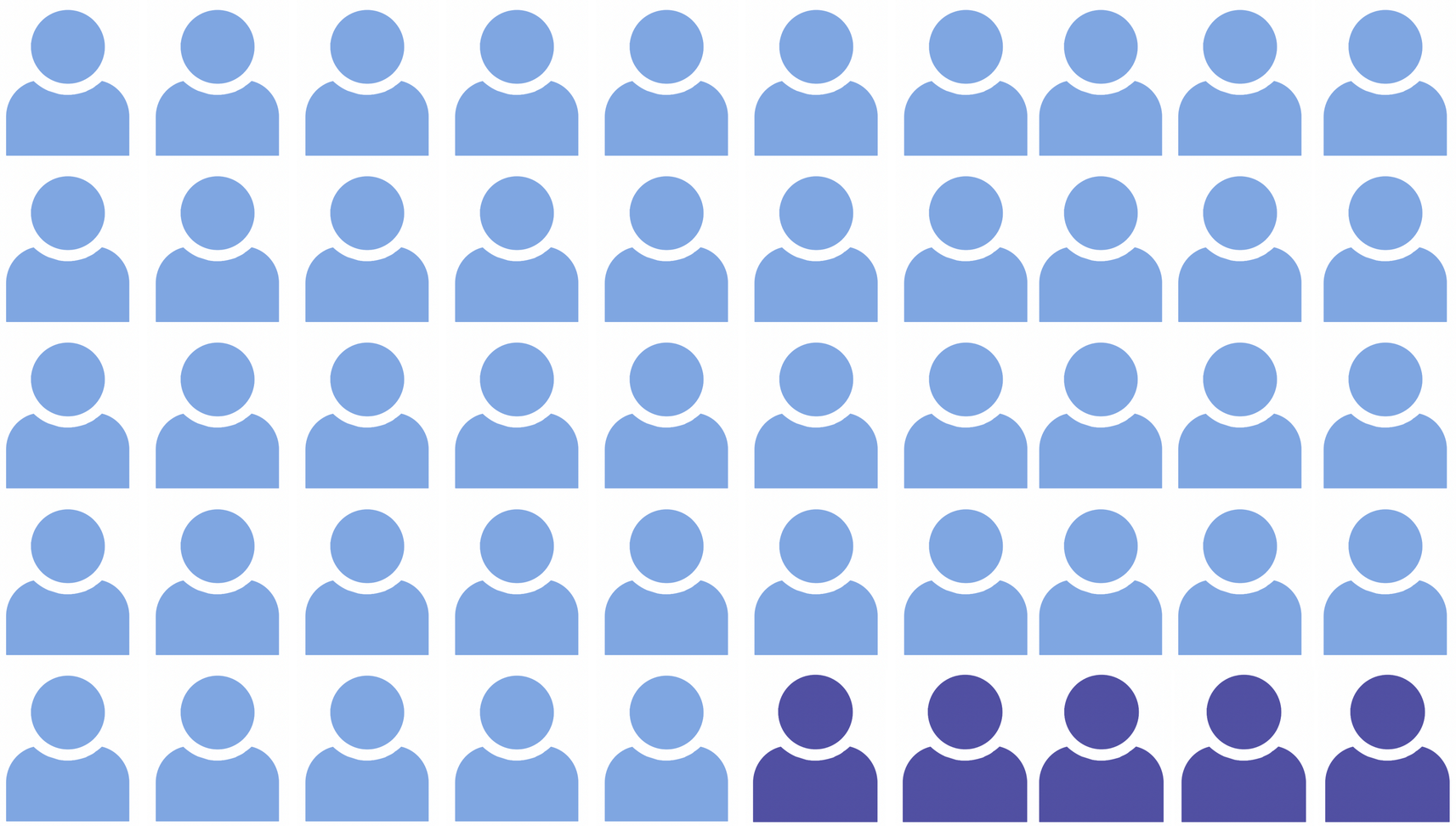


**Sample 1:** 40%

**Sample 2:** 10%

**Sample 3:** 0%

# Population vs. Sample



**Sample 1:** 40%

**Sample 2:** 10%

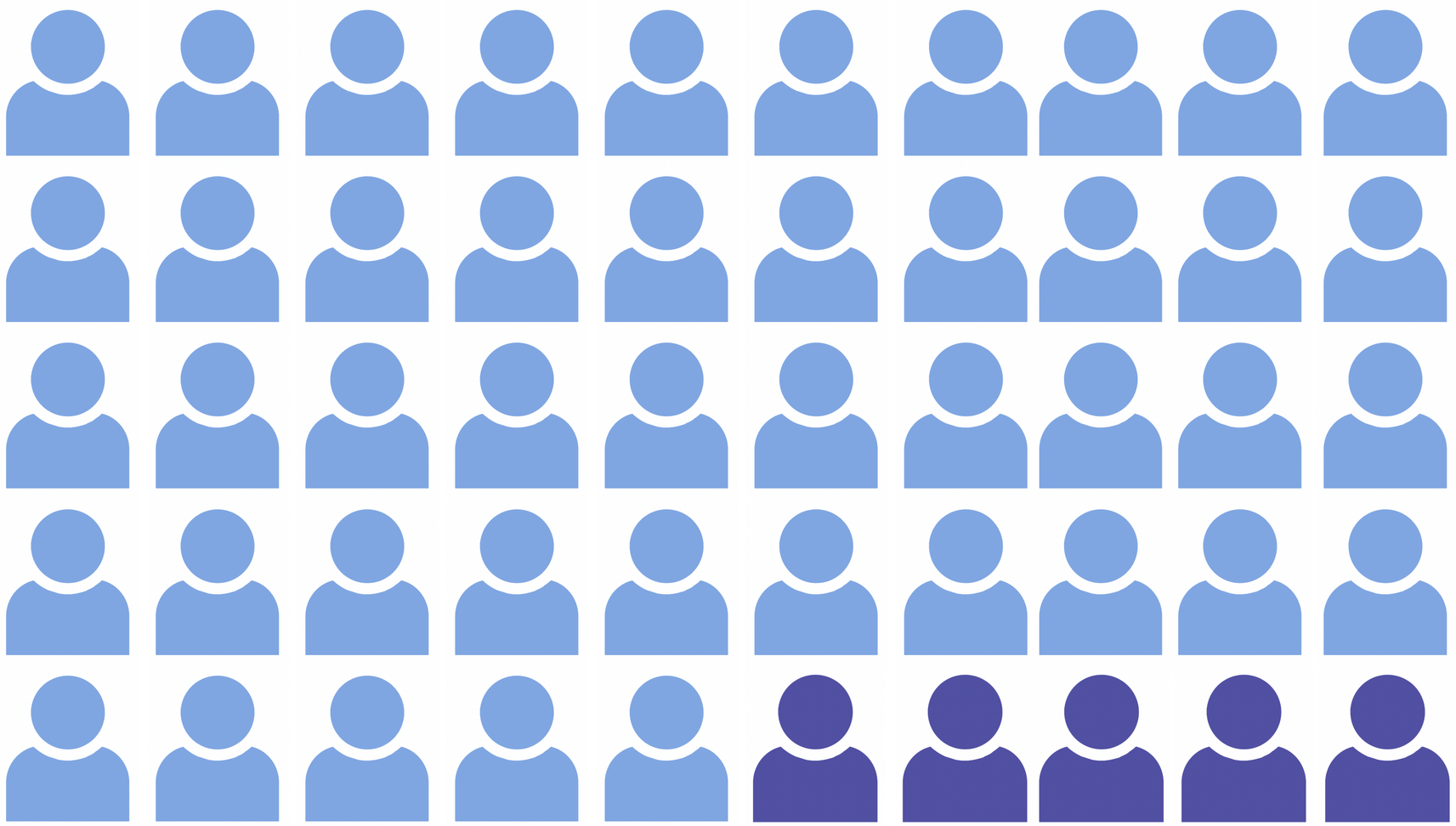
**Sample 3:** 0%

---

**Population:** 10%

***“Sampling variability”***

# Population vs. Sample



***"Meta-study"***

**Sample 1:** 40%

**Sample 2:** 10%

**Sample 3:** 0%

...

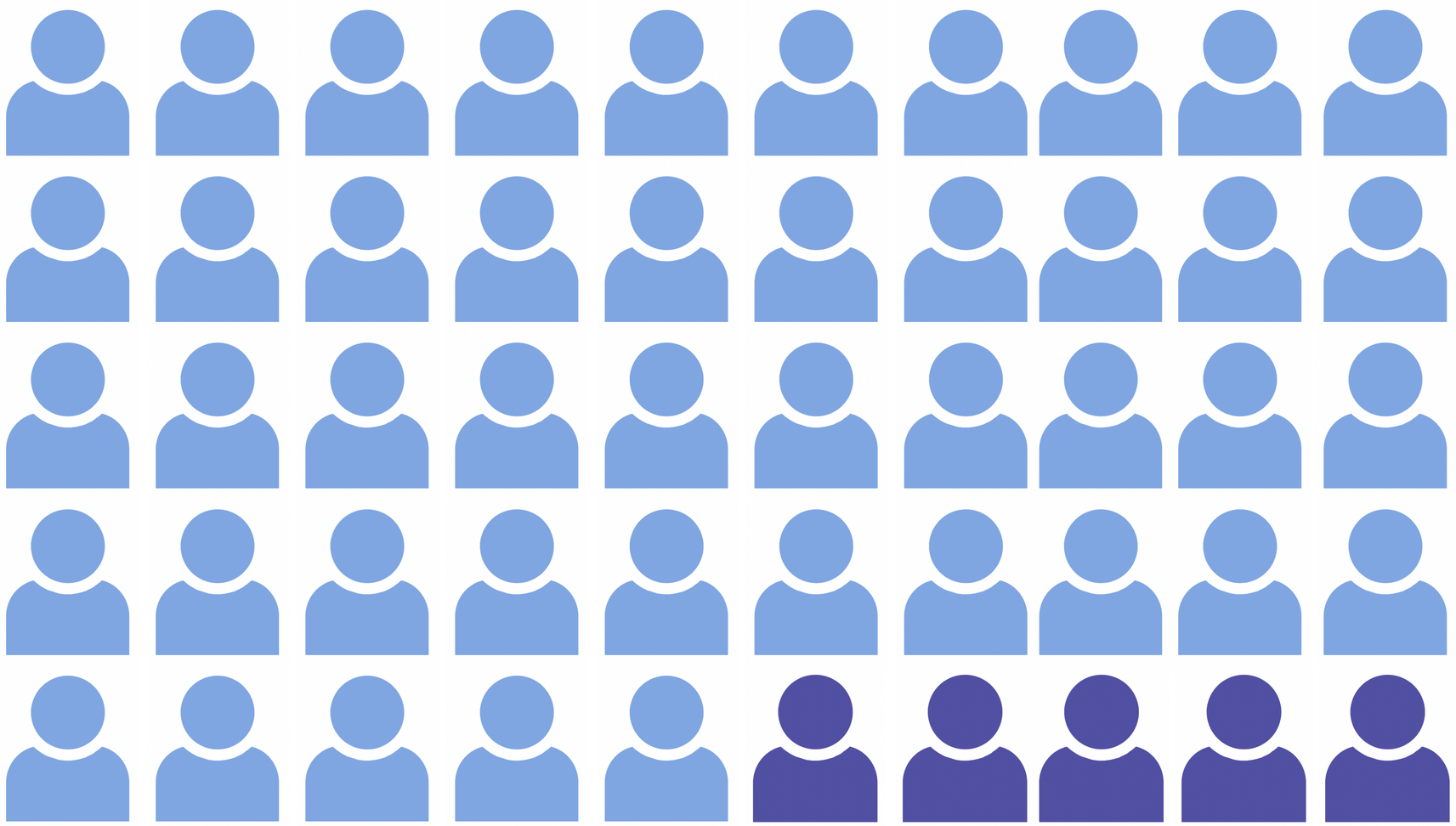
**Sample n:** 10%

?

**Population:** 10%

$({}_{50}C_5)$

# Population vs. Sample



***"Meta-study"***

**Sample 1:** 40%

**Sample 2:** 10%

**Sample 3:** 0%

**...**

**Sample n:** 10%

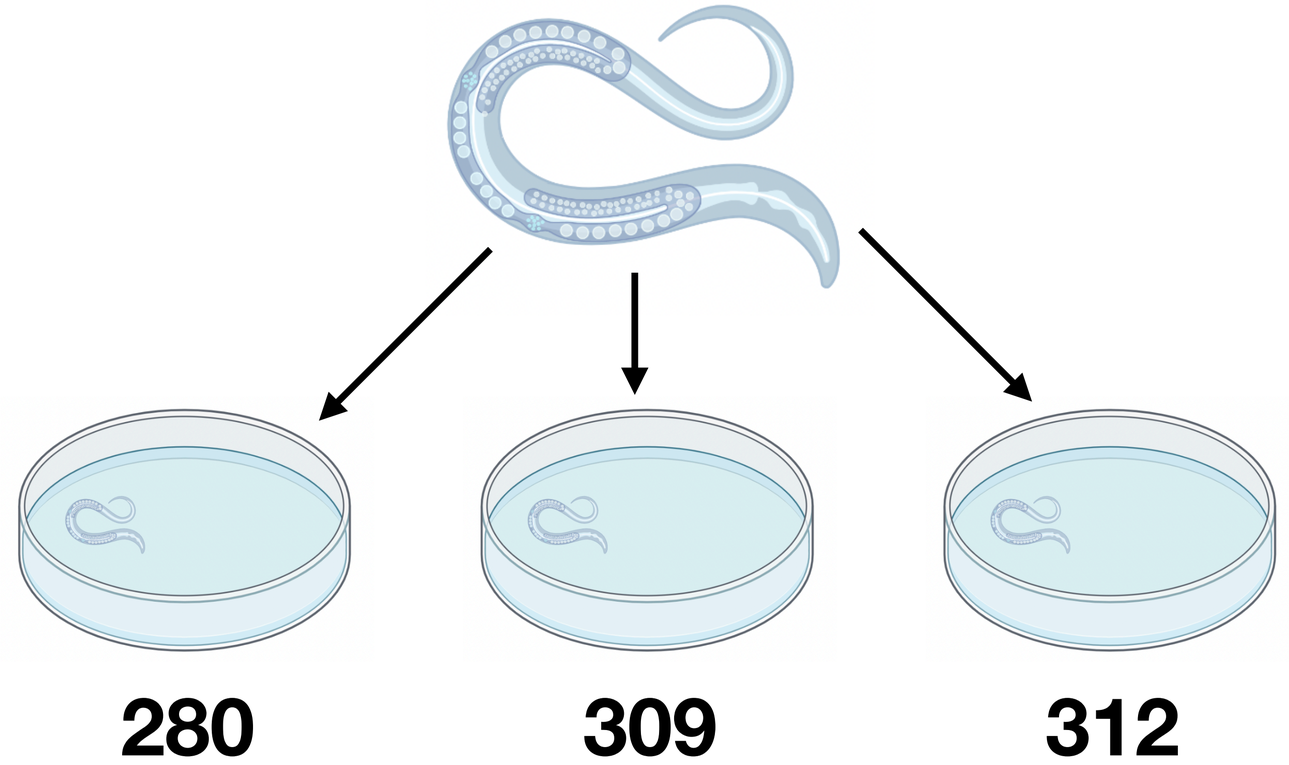
$({}_{50}C_5)$

**Sample mean:** 10%

***Probabilities of random sample = relative frequencies in meta-study***

# Population vs. Sample

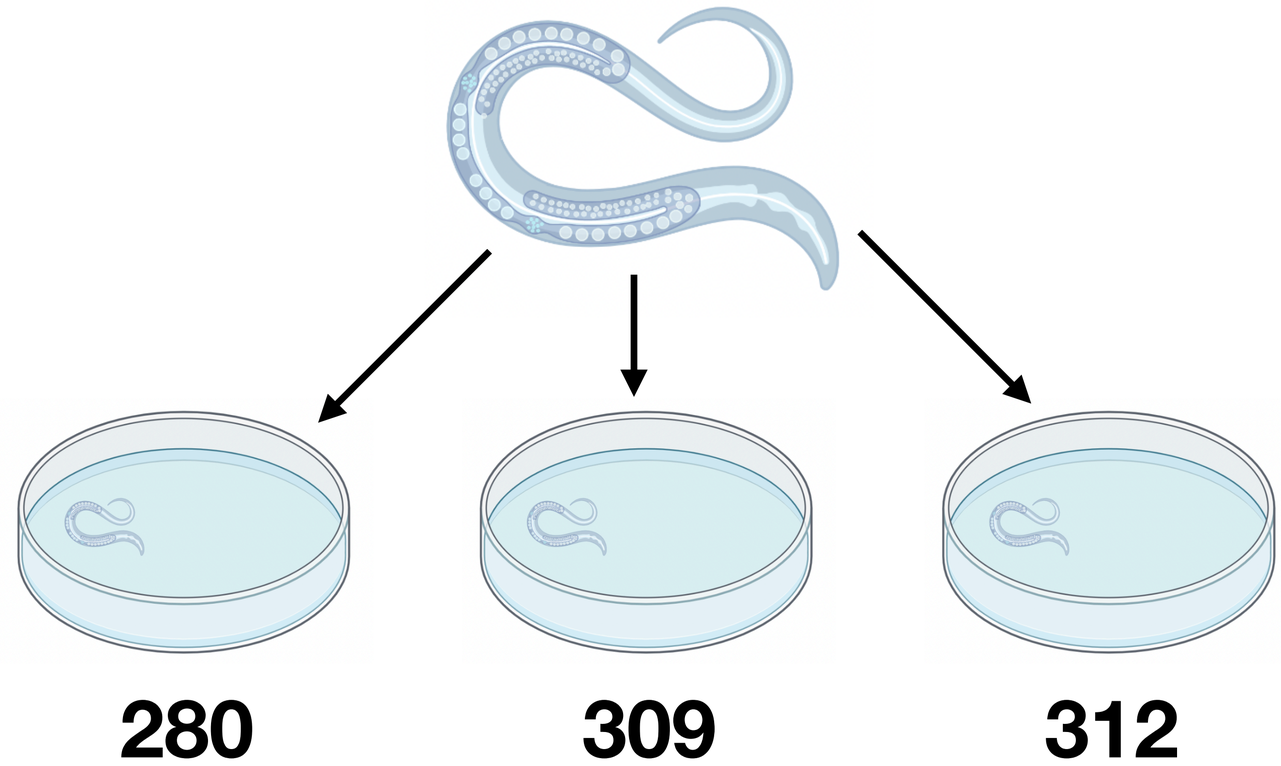
**On average, the laboratory strain of *C. elegans* has 300 progeny. You pick three individuals each to their own plate and choose two plates to count broods. What is the sampling distribution?**



*(1/3)(1/2)*2 ways = 1/3*

# Population vs. Sample

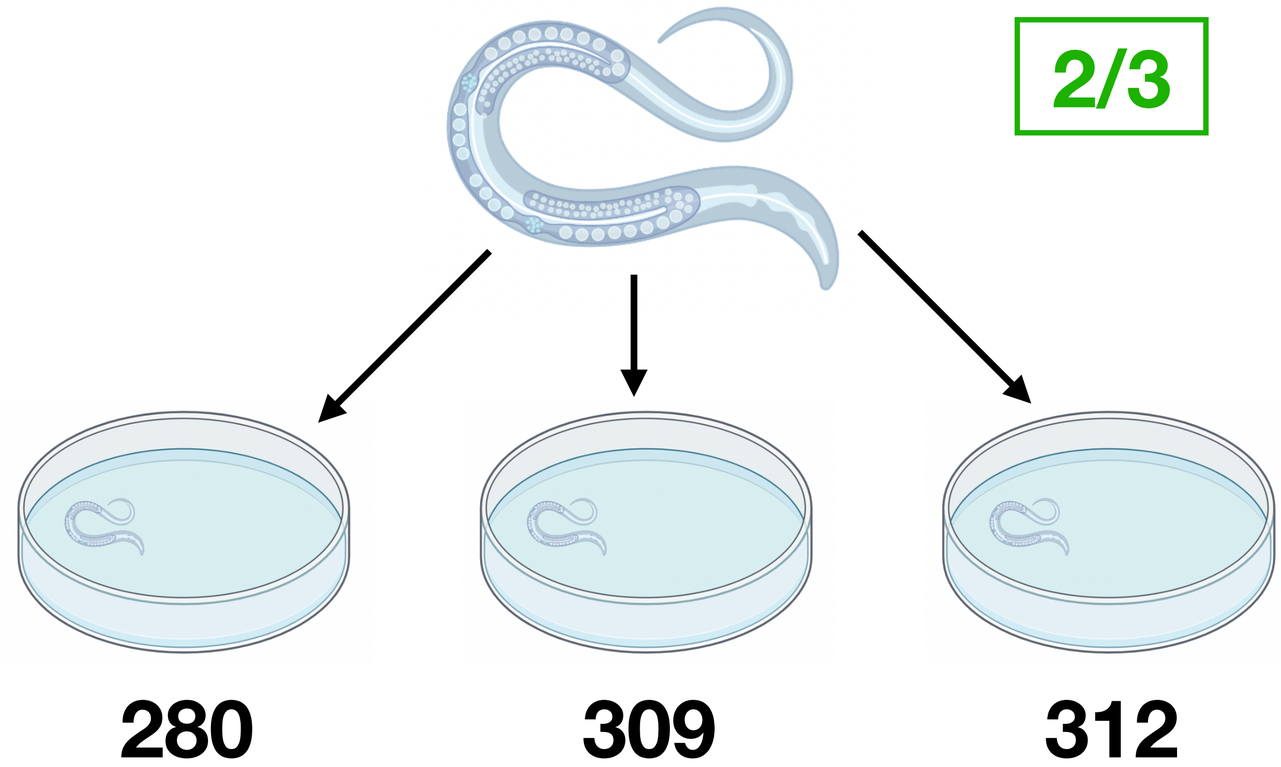
**On average, the laboratory strain of *C. elegans* has 300 progeny. You pick three individuals each to their own plate and choose two plates to count broods. What is the sampling distribution?**




| SAMPLE | MEAN | PROB |
|---|---|---|
| 280, 309 | **(294.5)** | 1/3 |
| 280, 312 | **(296)** | 1/3 |
| 309, 312 | **(310.5)** | 1/3 |

# Population vs. Sample

**On average, the laboratory strain of *C. elegans* has 300 progeny. You pick three individuals each to their own plate and choose two plates to count broods. <u>What is the probability the average sample brood < 300?</u>**



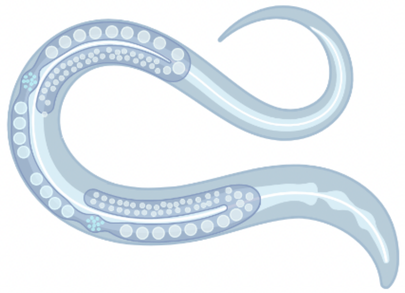

**2/3**

| **SAMPLE** | **MEAN** | **PROB** |
|---|---|---|
| 280, 309 | **(294.5)** | 1/3 |
| 280, 312 | **(296)** | 1/3 |
| ~~309, 312~~ | ~~**(310.5)**~~ | ~~1/3~~ |

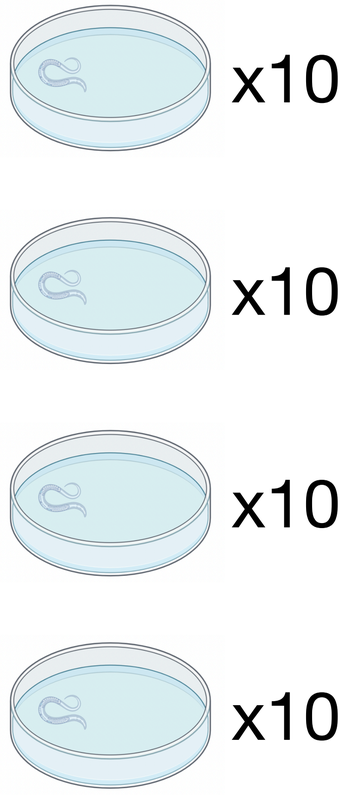
# Sampling distribution of the sample mean

**Let's take our *C. elegans* example and expand our sampling:**

- Count broods for 10 individuals
- Repeat this experiment on four different days



Mean: 300
Sd: 20



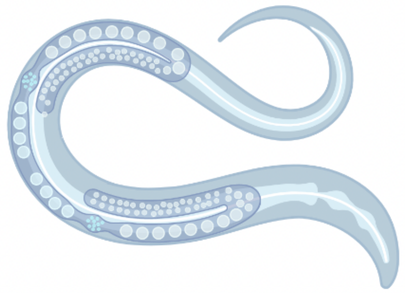

*290, 304, 293, 315, 304, 298, 287, 302, 267, 291*

**mean: 295.4, sd: 12.7**

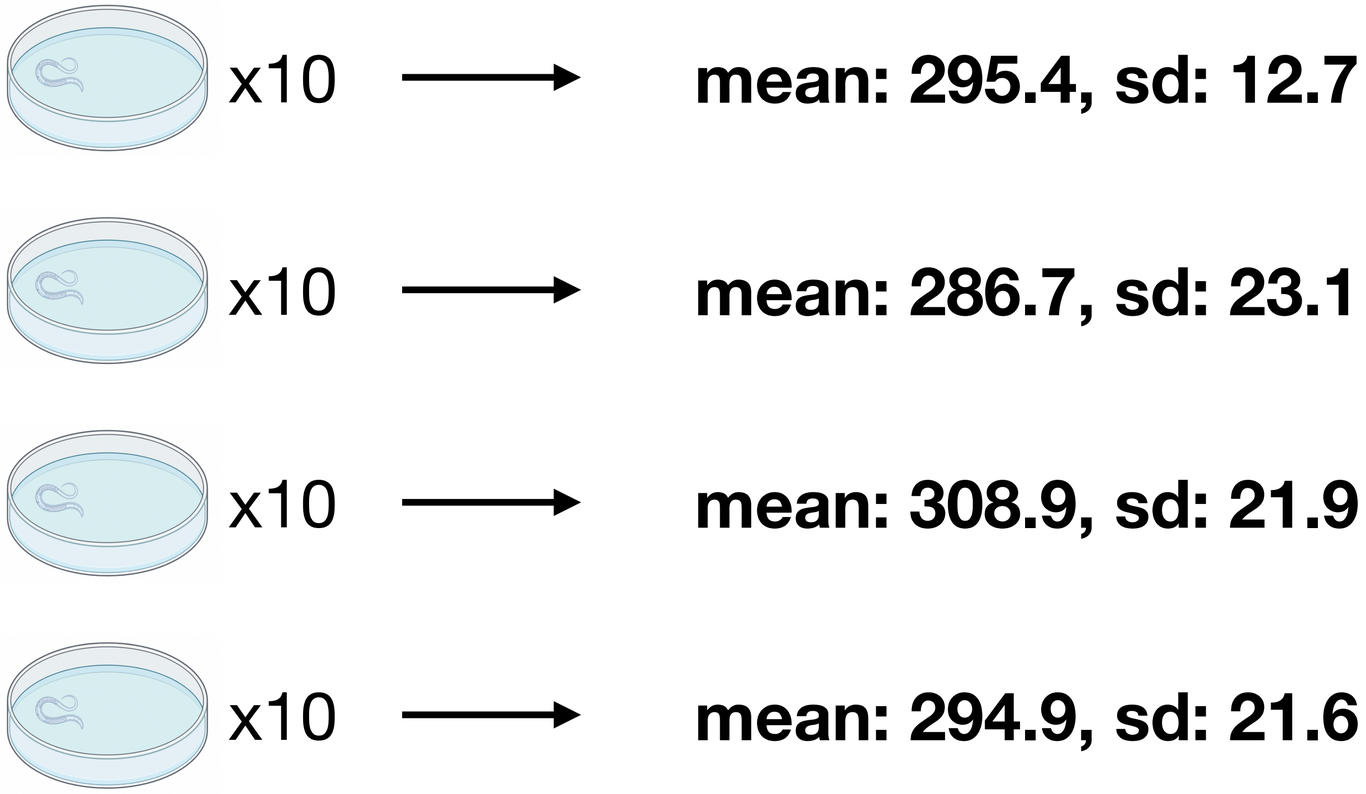
# Sampling distribution of the sample mean

**Let's take our *C. elegans* example and expand our sampling:**

- Count broods for 10 individuals
- Repeat this experiment on four different days



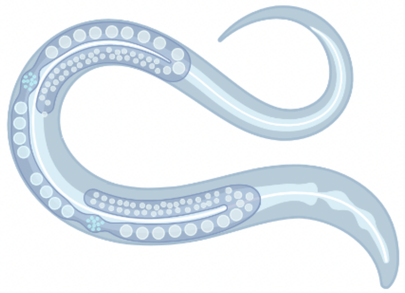
Mean: 300
Sd: 20



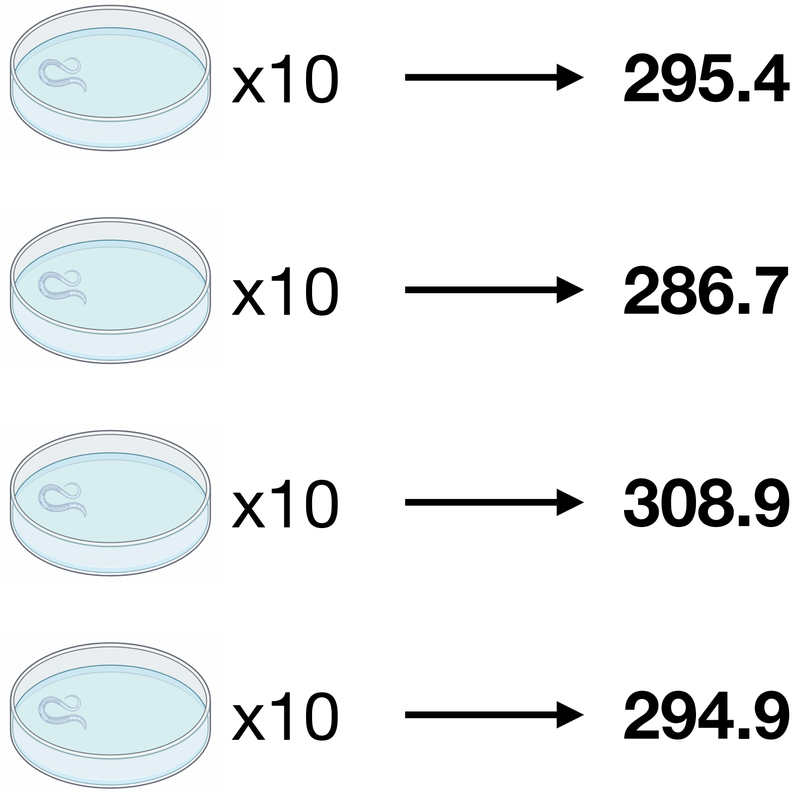
# Sampling distribution of the sample mean

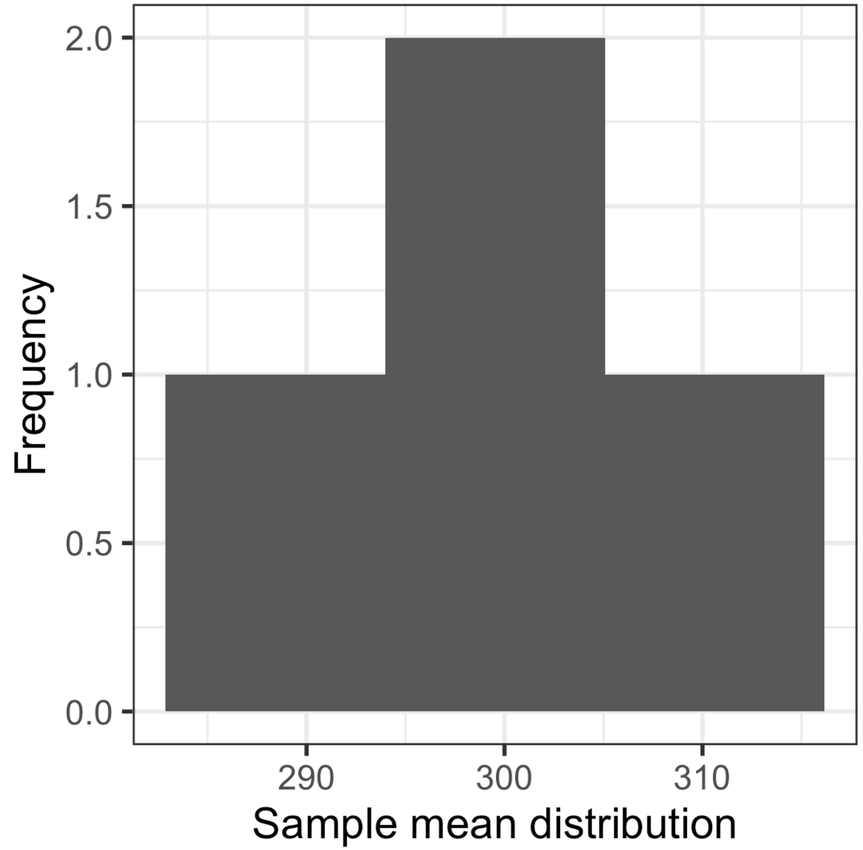
**Let's take our *C. elegans* example and expand our sampling:**

- Count broods for 10 individuals
- Repeat this experiment on four different days

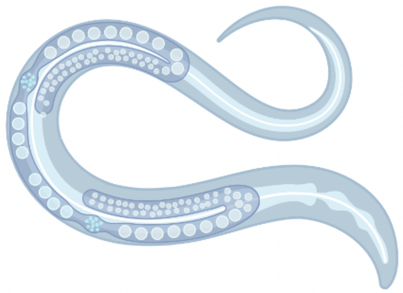


Mean: 300
Sd: 20



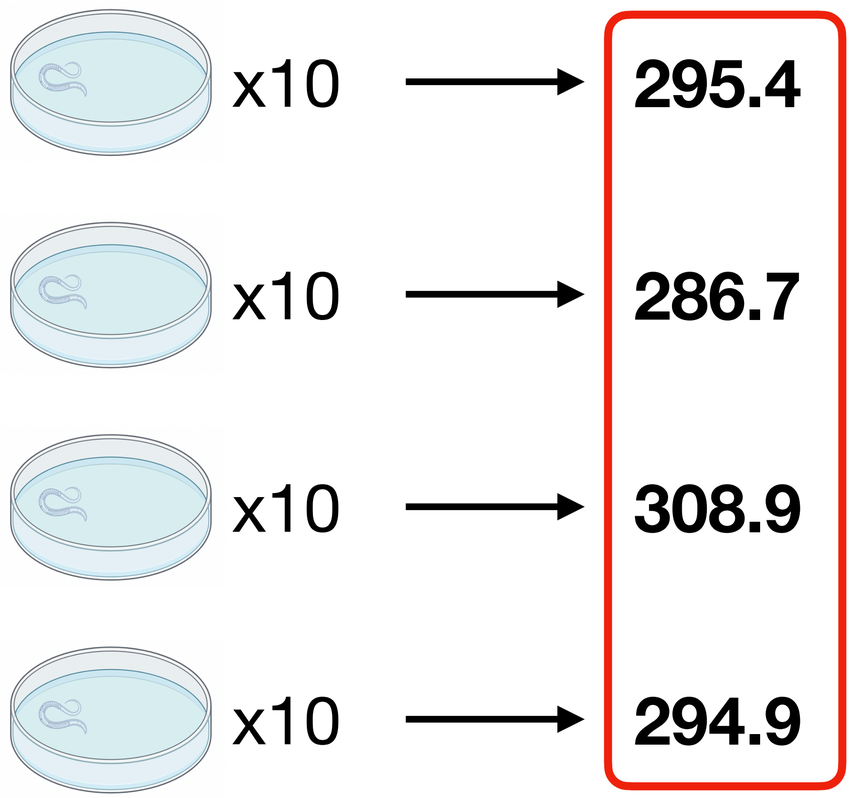



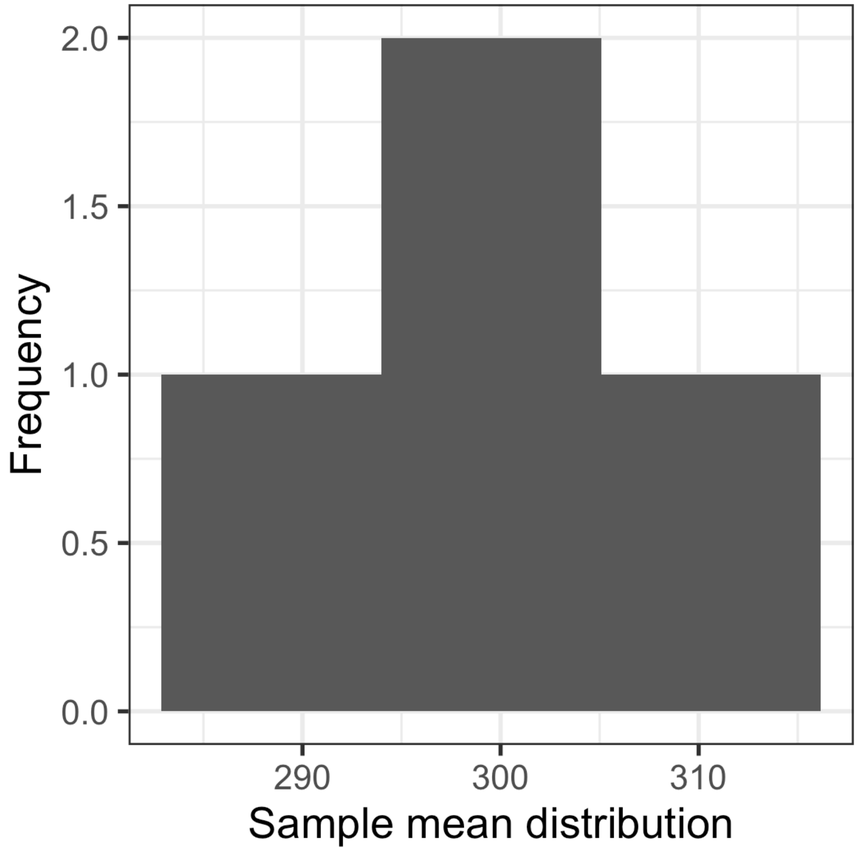
# Sampling distribution of the sample mean



**Mean: 300**
Sd: 20

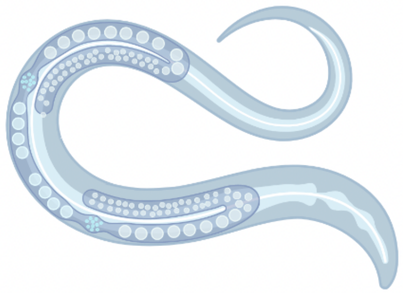
**Sample mean: 296.5**



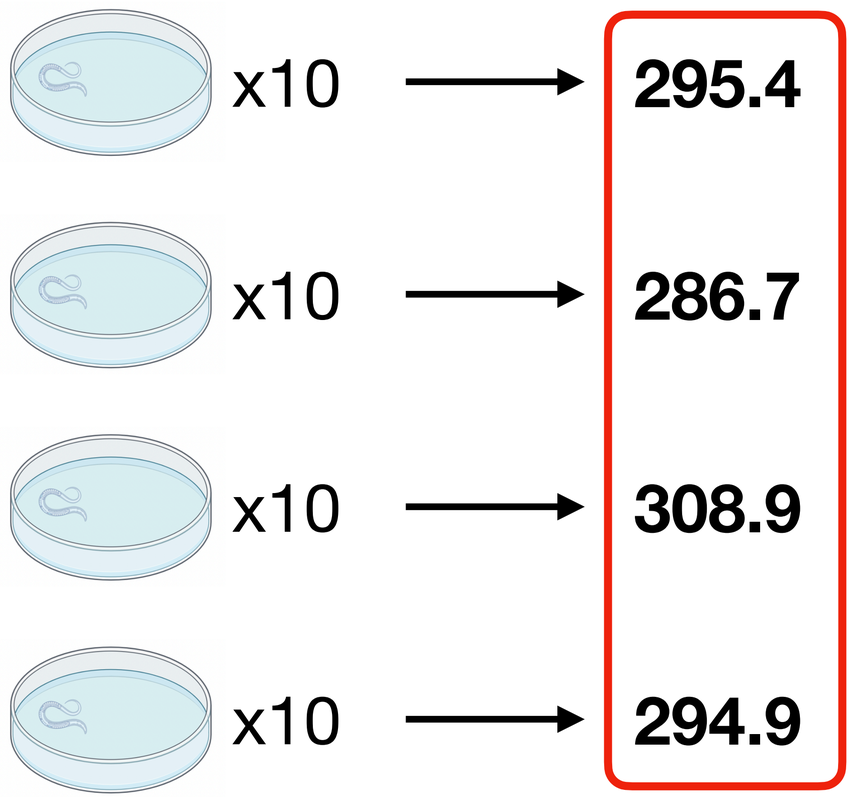



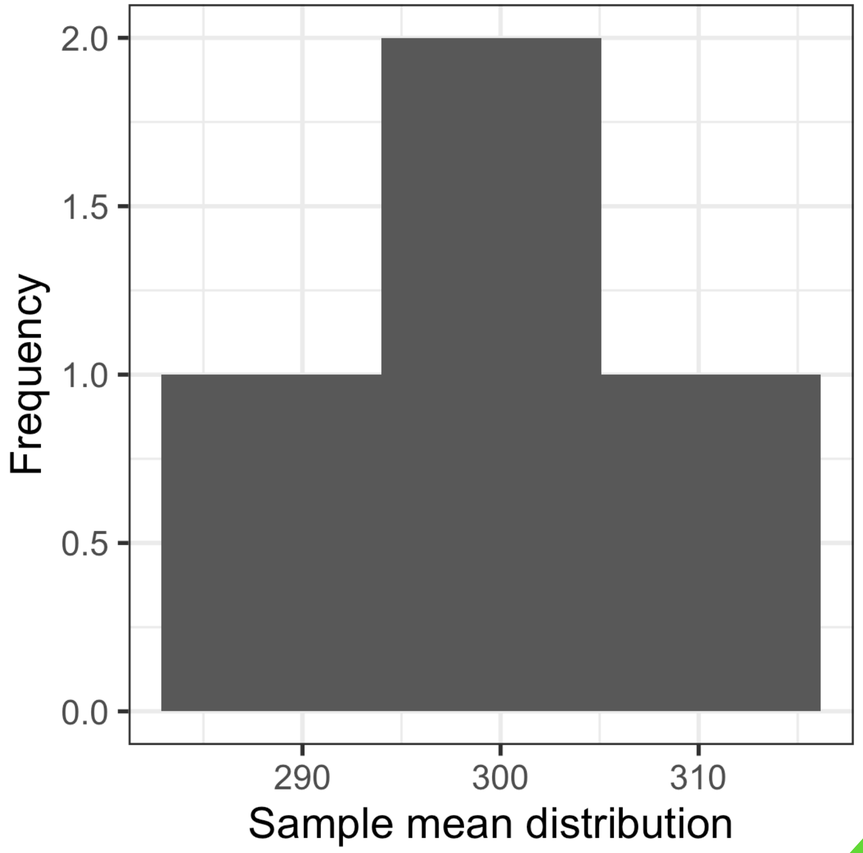

**1. The sample mean = the population mean**

# The sample standard variation



**Mean: 300**
Sd: 20




**Sample mean: 296.5**

**Sample sd: 9.19**

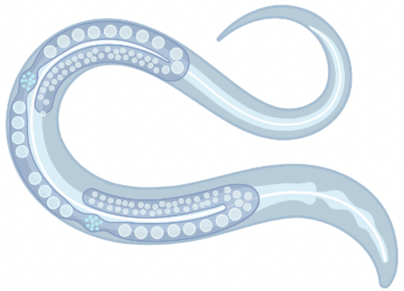
**1. The sample mean = the population mean**

**2. The sample sd = population sd / sqrt(n)** $20/\sqrt{4} = 10$

# The sample standard variation

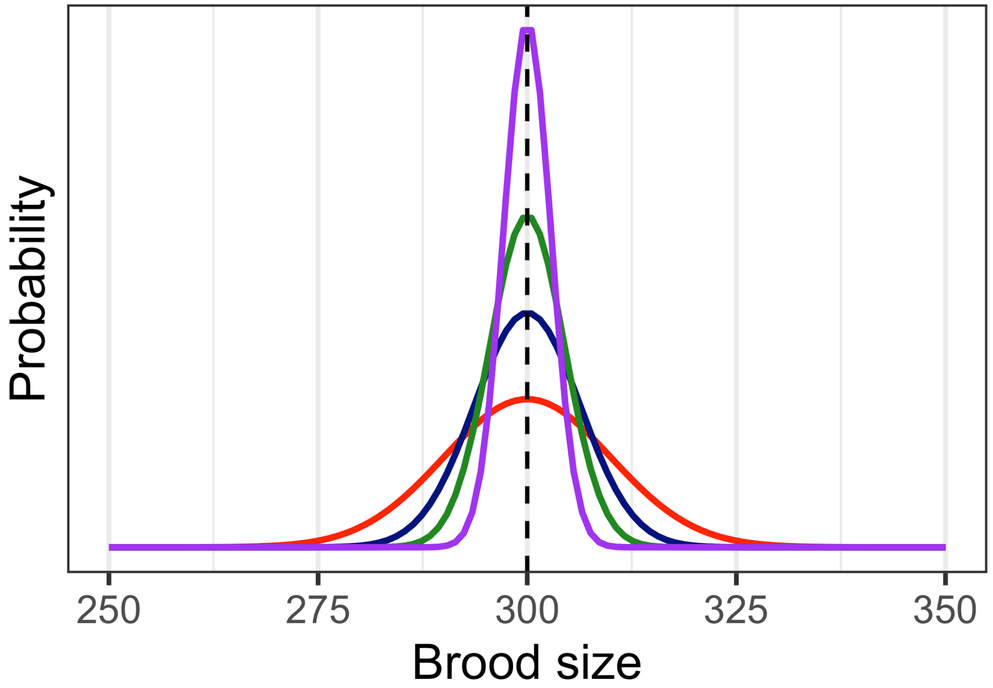
**Let's take our *C. elegans* example and expand our sampling:**

- Count broods for 10 individuals
- Repeat this experiment on **10** different days



Mean: 300
Sd: 20

| n | mean | s |
|---|---|---|
| 4 | 300 | 10 |
| 10 | 300 | 6.32 |
| 20 | 300 | 4.47 |
| 50 | 300 | 2.82 |

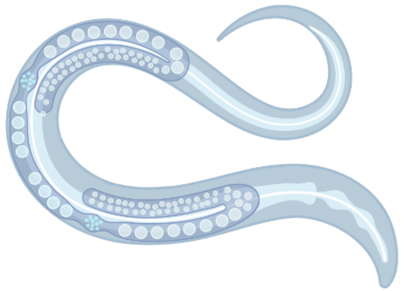



**2. The sample sd = population sd / sqrt(n)**

# Shape of sampling distribution

**Let's take our *C. elegans* example and expand our sampling:**

- Count broods for 10 individuals
- Repeat this experiment on **10** different days



Mean: 300
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| n | mean | s |
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| 4 | 300 | 10 |
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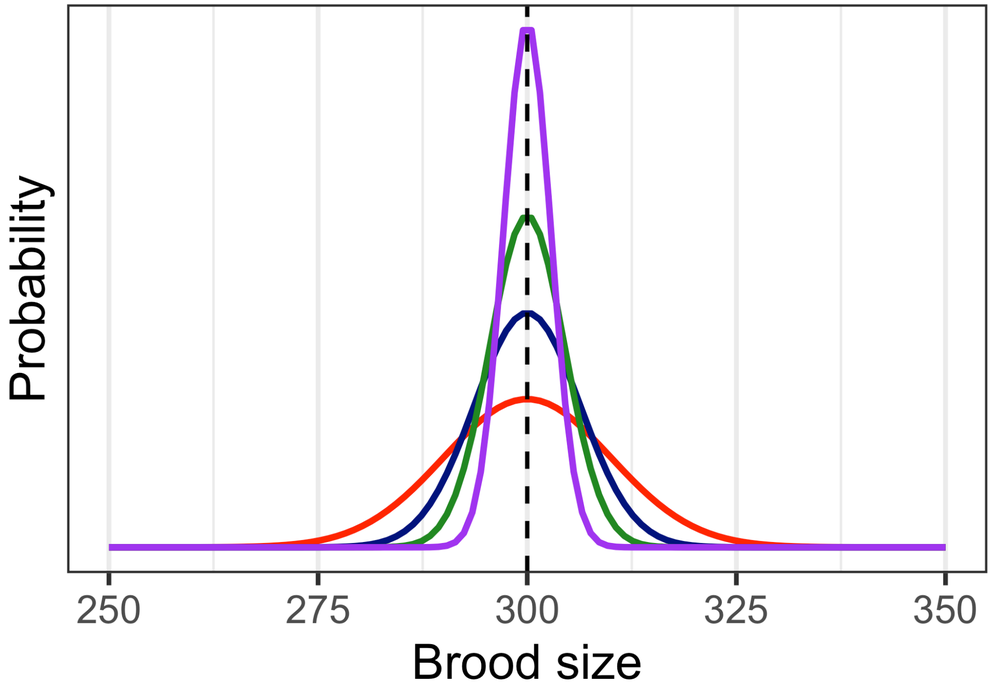



**3. If the population distribution is normal, the sampling distribution will be normal**

# Summary: sampling distribution

**Population**

$\mu, \sigma$

**Samples**

$\bar{y}, s$

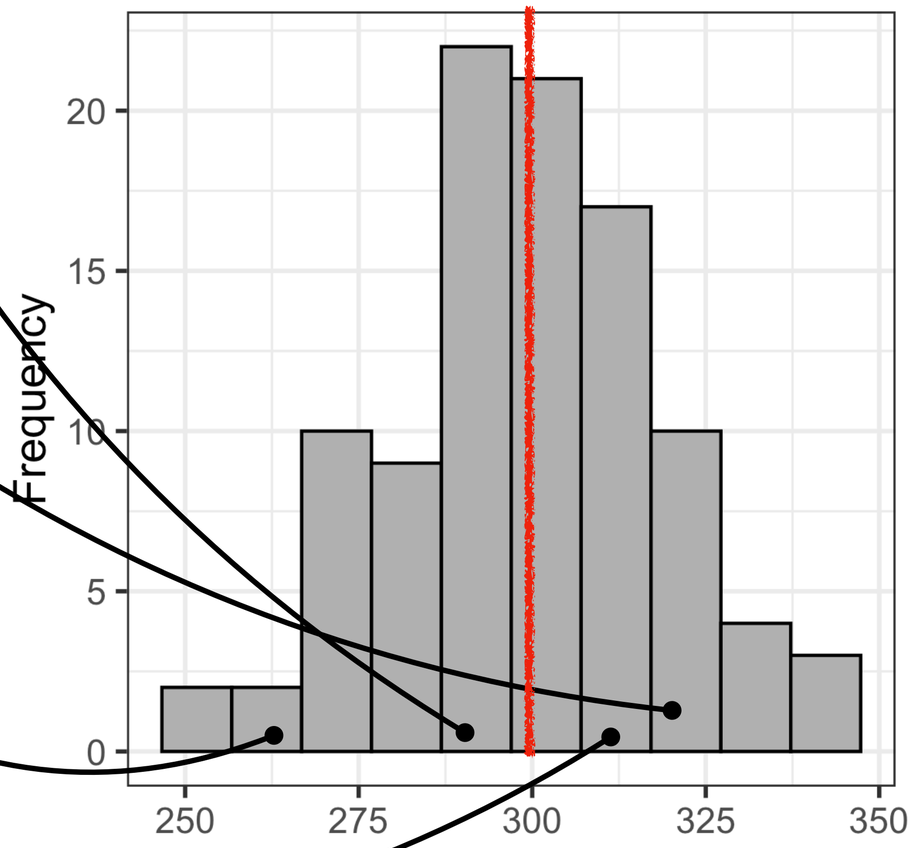
$\bar{y}, s$

$\bar{y}, s$

$\bar{y}, s$

$$\mu_{\bar{Y}} = \mu$$

$$\sigma_{\bar{Y}} = \sigma/\sqrt{n}$$




**(Distribution of $\bar{Y}$ )**

# Example

**A large population of seeds ($\mu = 500mg; \sigma = 120mg^2$) are weighed. If you take a random sample of four seeds, what is the probability that the mean weight of the four seeds will be greater than 550 mg?**

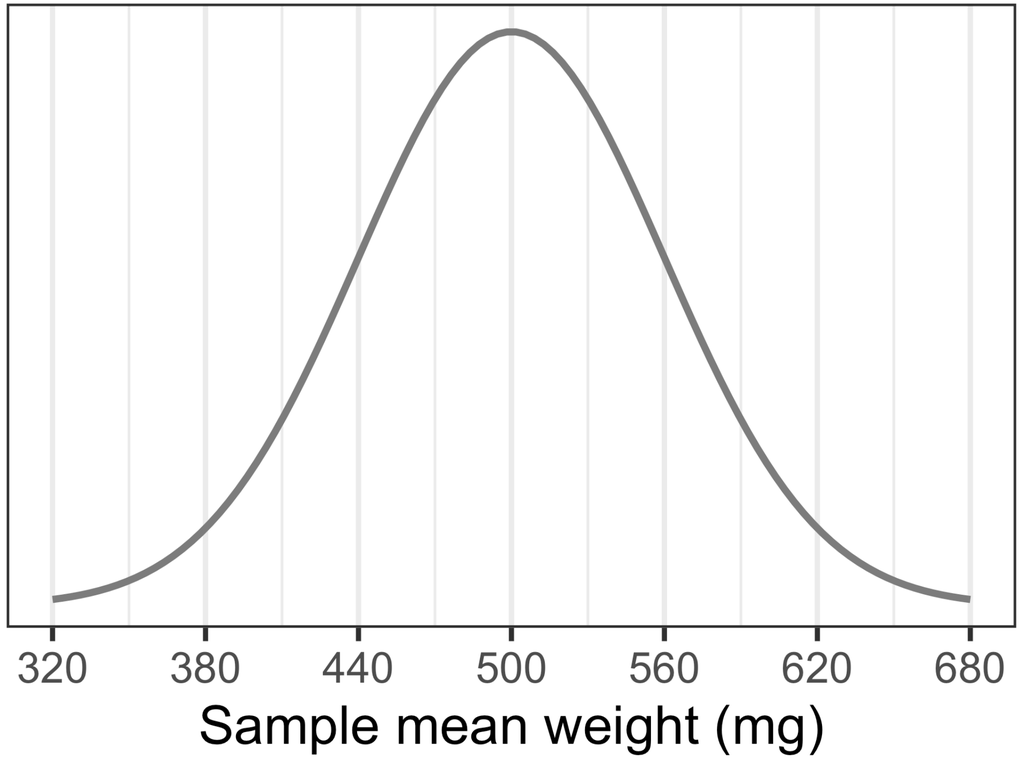
$$\mu_{\bar{Y}} = \mu$$ 500 mg

$$\sigma_{\bar{Y}} = \sigma/\sqrt{n}$$ 120 /$\sqrt{4}$ = 60 mg

# Example

**A large population of seeds ($\mu = 500mg; \sigma = 120mg^2$) are weighed. If you take a random sample of four seeds, what is the probability that the mean weight of the four seeds will be greater than 550 mg?**

$$\sigma_{\bar{Y}} = 60mg$$



# Example

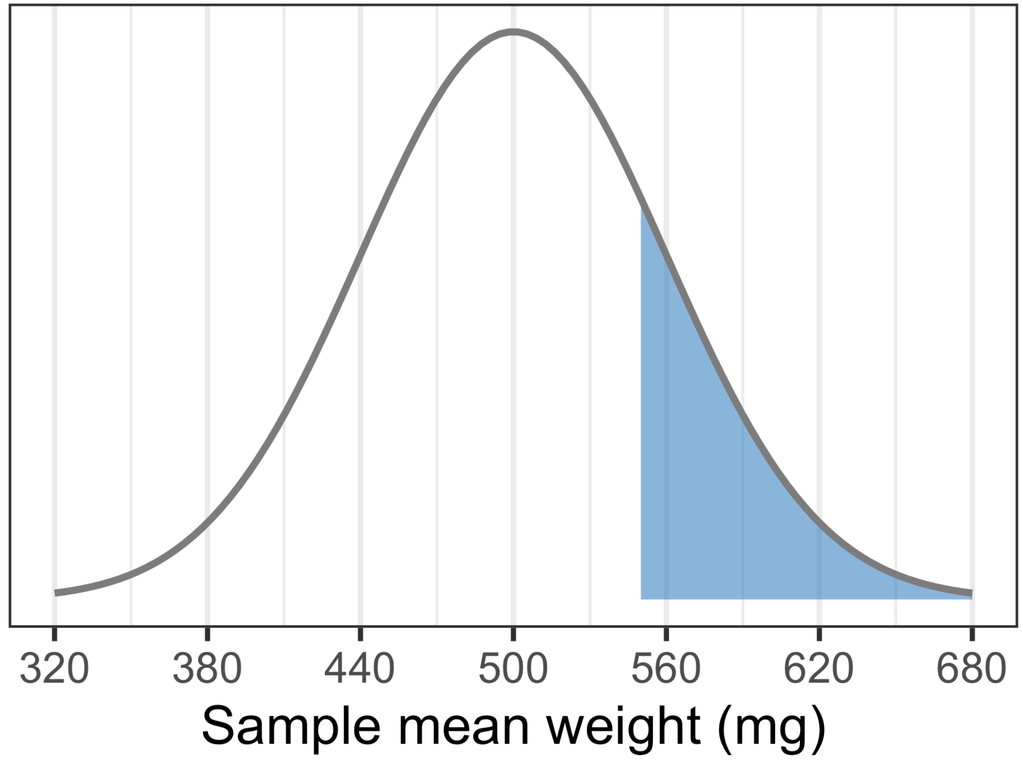
**A large population of seeds ($\mu = 500mg; \sigma = 120mg^2$) are weighed. If you take a random sample of four seeds, what is the probability that the mean weight of the four seeds will be greater than 550 mg?**

$$\mu = 500mg$$
$$\sigma_{\bar{Y}} = 60mg$$

$$z = \frac{\bar{y} - \mu_{\bar{Y}}}{\sigma_{\bar{Y}}}$$

$$z = \frac{550 - 500}{60}$$

$$z = 0.83$$



# Example

**A large population of seeds ($\mu = 500mg; \sigma = 120mg^2$) are weighed. If you take a random sample of four seeds, what is the probability that the mean weight of the four seeds will be greater than 550 mg?**

$$\mu = 500mg$$

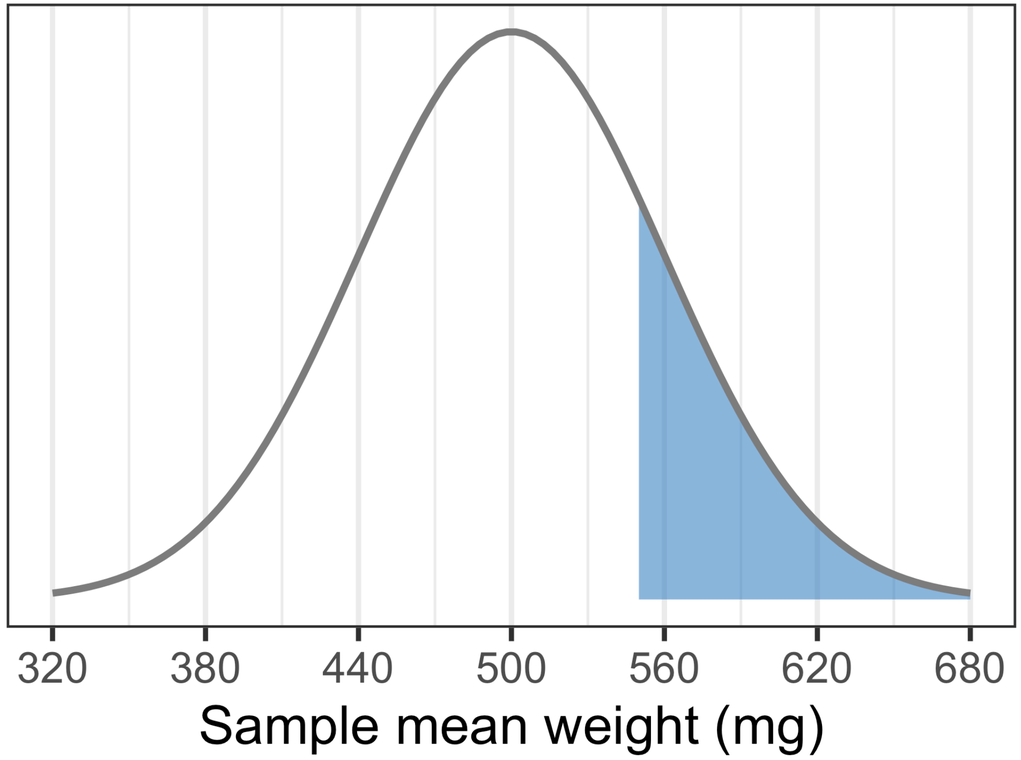
$$\sigma_{\bar{Y}} = 60mg$$

$$z = 0.83$$

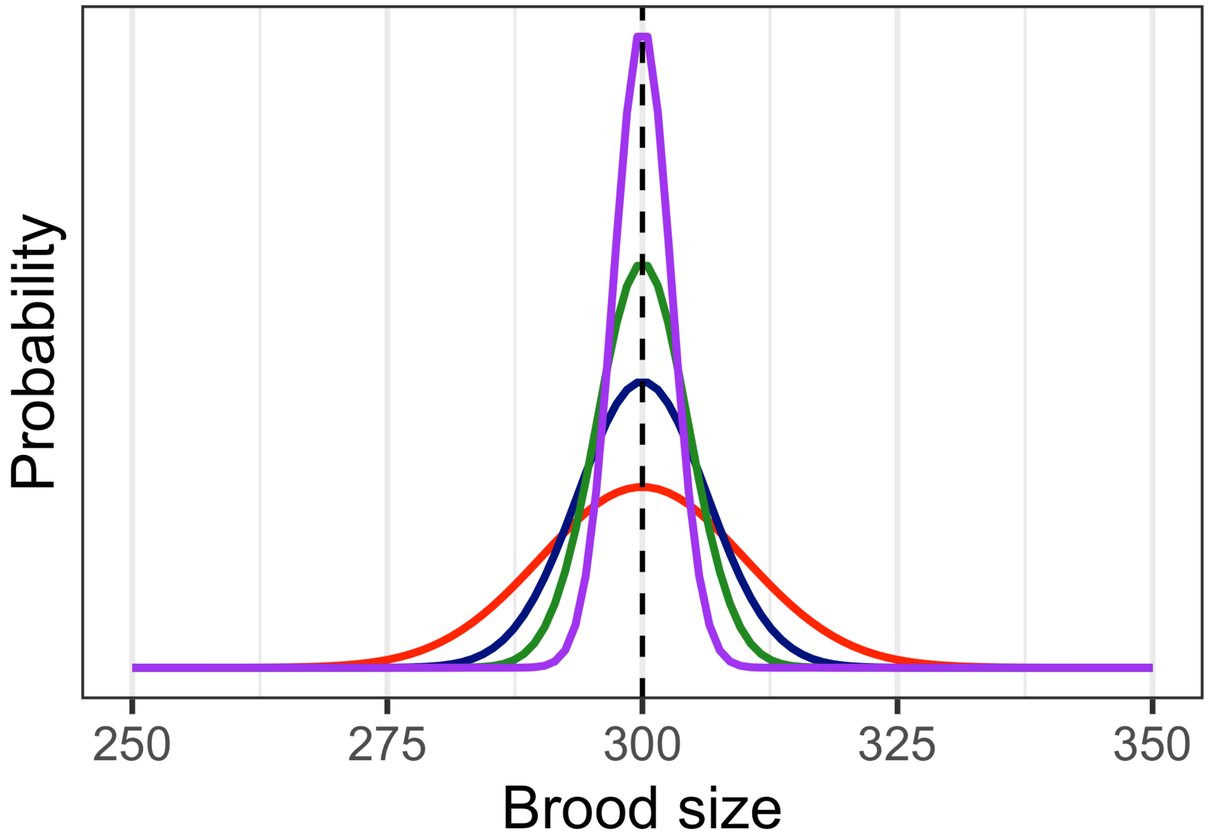
```
1 - pnorm(0.83)
```

**= 0.20**

```
1 - pnorm(550, 500, 60)
```



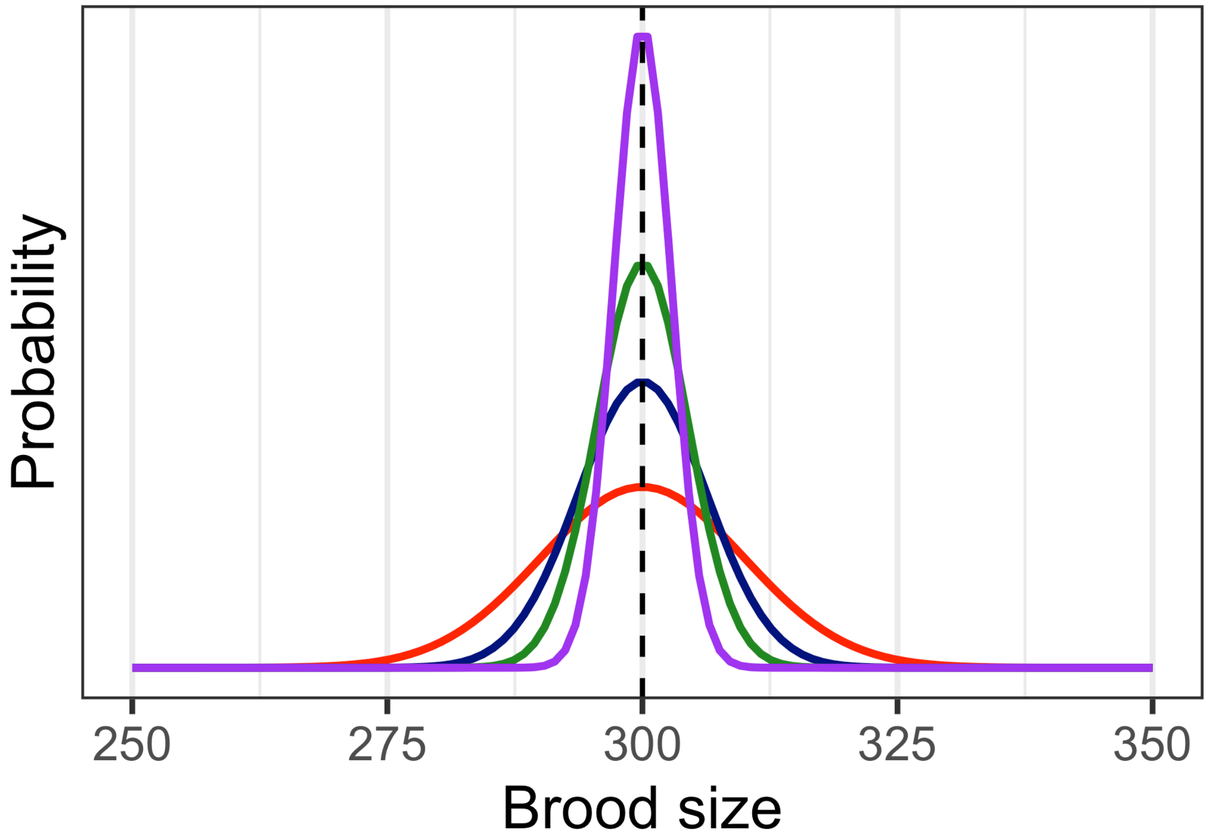
# Central limit theorem




**3. If the population distribution is normal, the sampling distribution will be normal**

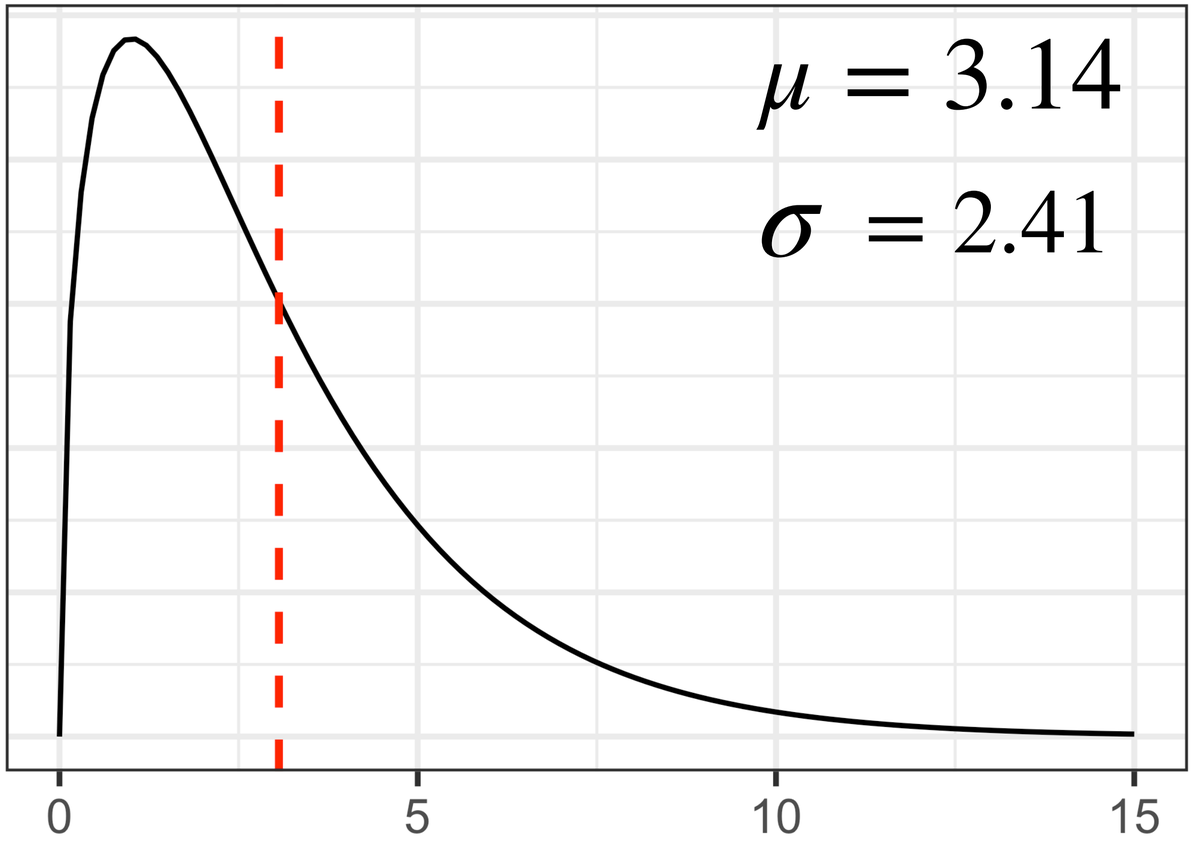
# Central limit theorem

**How large?**
*Large enough…*



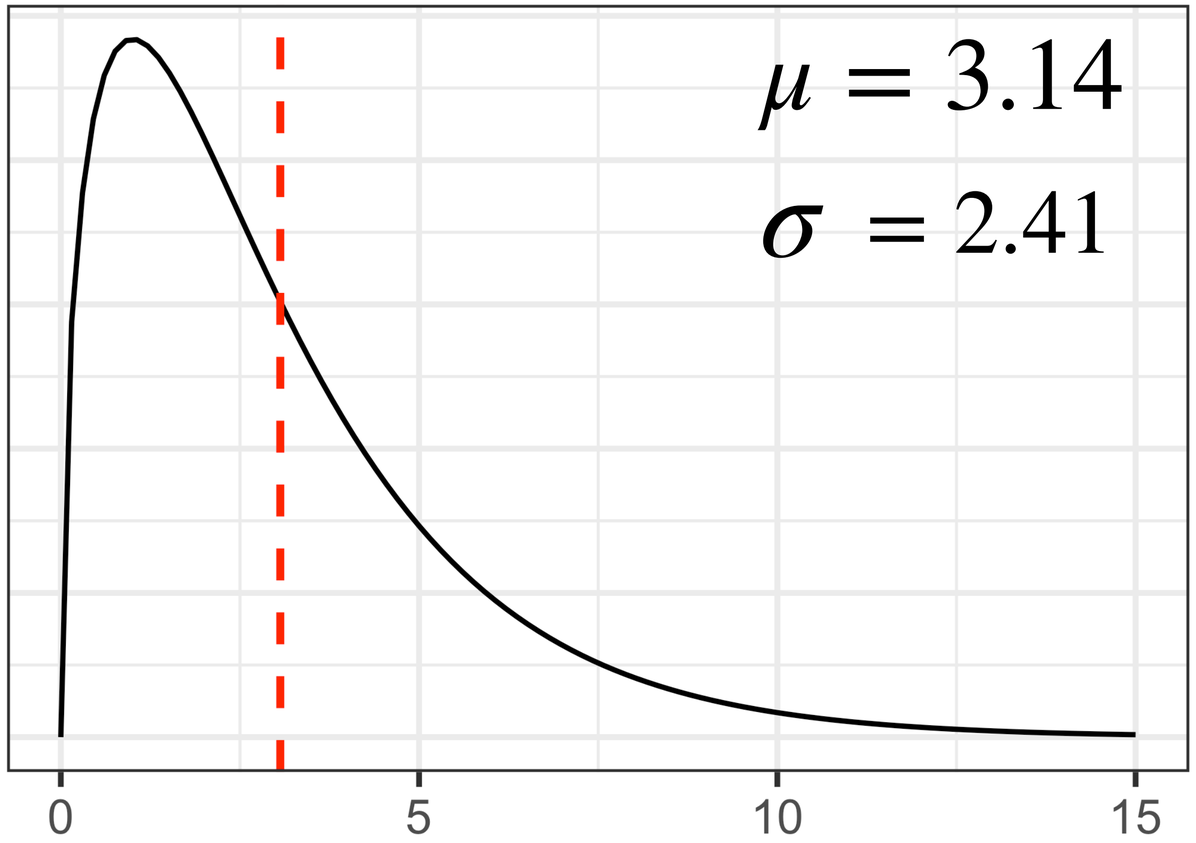

**4. If the population distribution is NOT normal, but *n* is LARGE, the sampling distribution will be normal**

# Central limit theorem




**4. If the population distribution is NOT normal, but *n* is LARGE, the sampling distribution will be normal**

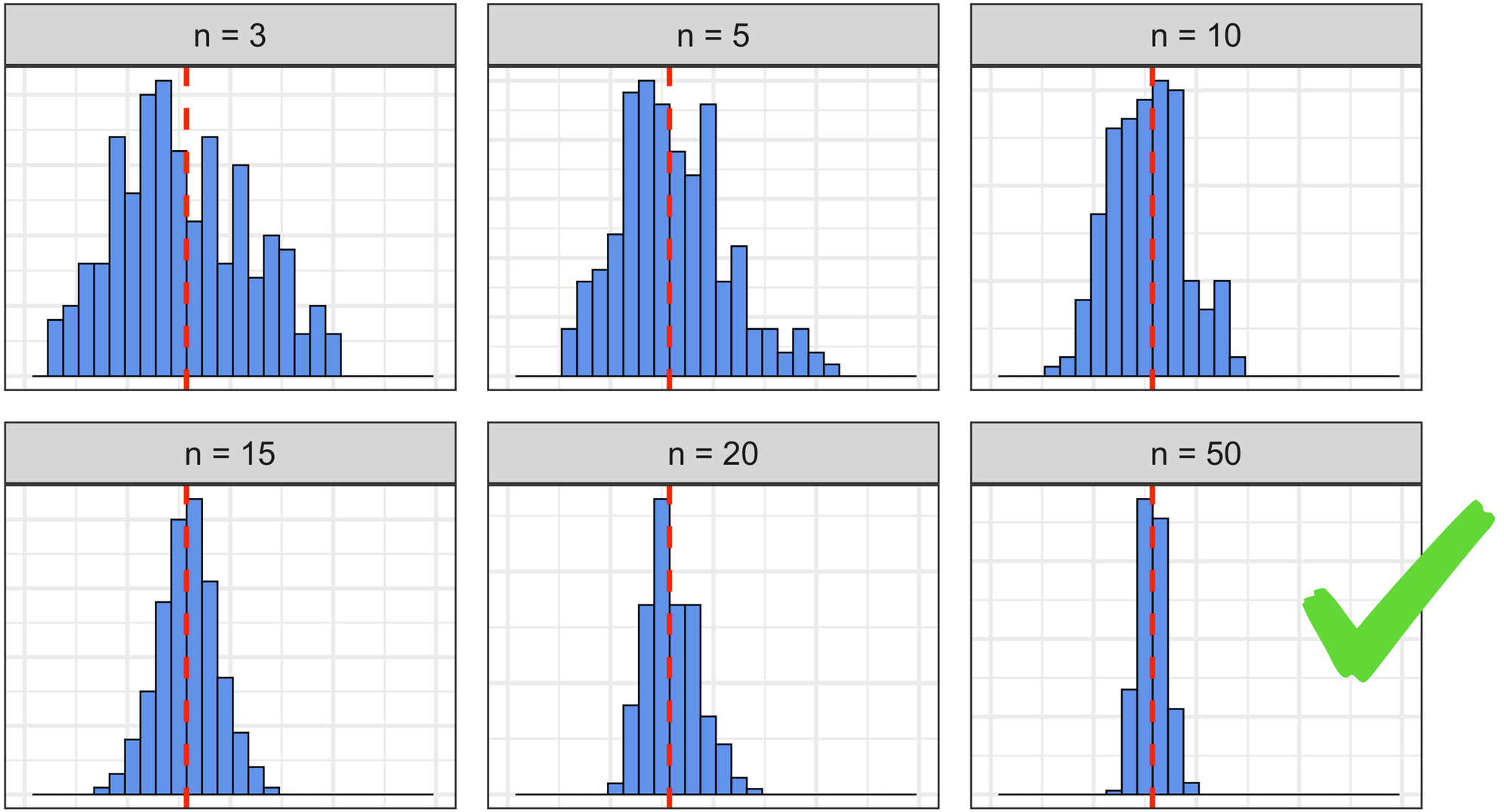
# Central limit theorem




- Take a sample of 20 individuals
- Calculate the mean of the sample
- Repeat this sampling 3, 10, 20, 50, 100, or 1000 times
- Plot the distribution of the means across samples

**4. If the population distribution is NOT normal, but *n* is LARGE, the sampling distribution will be normal**

# Central limit theorem: 100 samples of size n

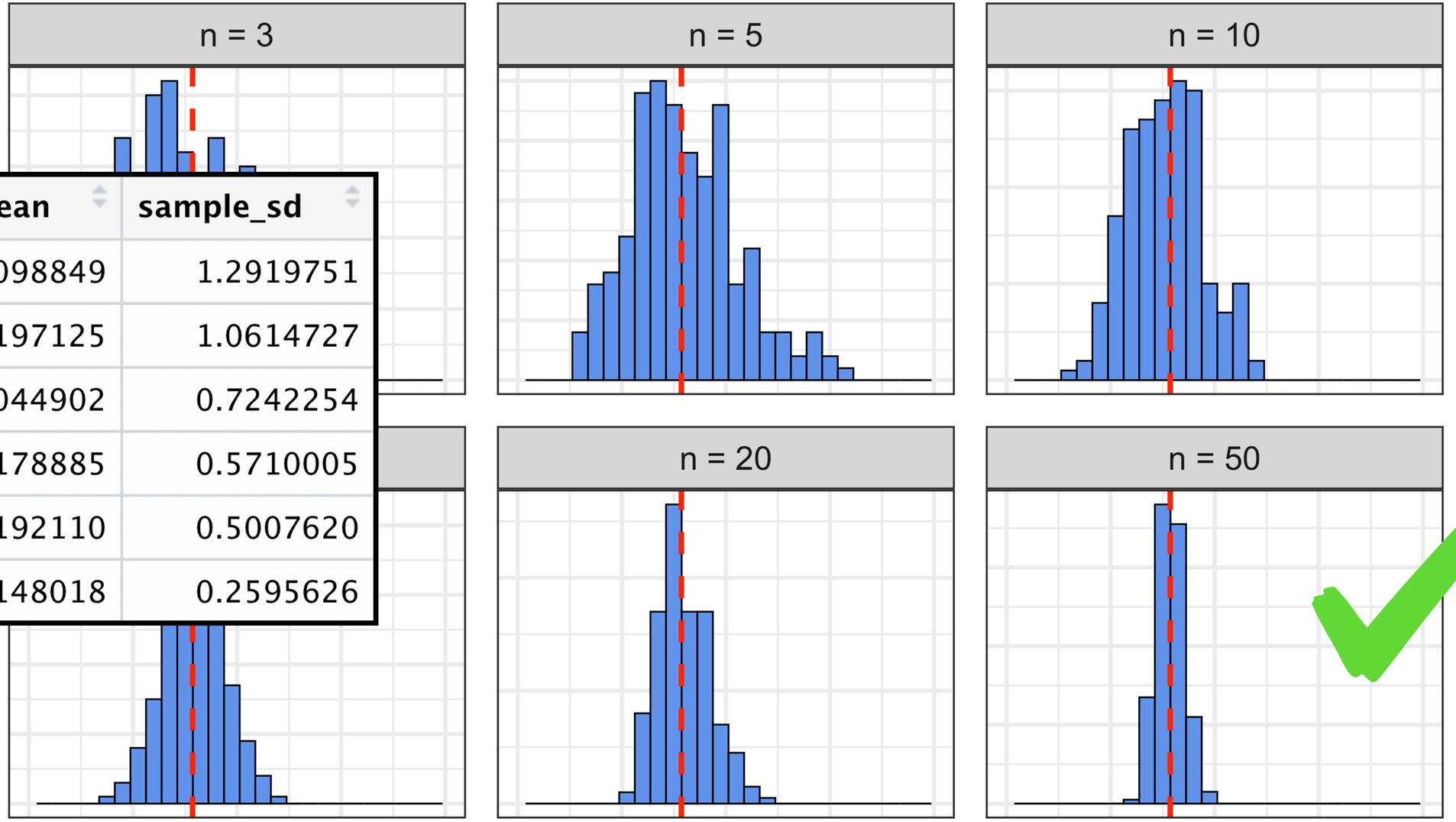


# Central limit theorem: 100 samples of size n

$\mu = 3.14$

$\sigma = 2.41$

| sample_n | sample_mean | sample_sd |
|---|---|---|
| n = 3 | 3.098849 | 1.2919751 |
| n = 5 | 3.197125 | 1.0614727 |
| n = 10 | 3.044902 | 0.7242254 |
| n = 15 | 3.178885 | 0.5710005 |
| n = 20 | 3.192110 | 0.5007620 |
| n = 50 | 3.148018 | 0.2595626 |



# Central limit theorem: 100 samples of size n

```
qqnorm(data)
qqline(data)
```

*data = means of samples*

```
ggplot2::ggplot(data) +
  ggplot2::aes(sample = mean) +
  ggplot2::stat_qq() +
  ggplot2::stat_qline()
```

***R does the calc + plot at same time!***