Interpreting the interval, we are 95% confident that the proportion of live births among Washington women seeking *in vitro* fertilization is between 0.100 and 0.184. It is interesting to note that these values are lower than the U.S. rate of 0.224 suggesting that older women are less likely to have a successful live birth from *in vitro* fertilization than the general population of all women.

Unit III Summary Exercises

III.1 Randomly chosen college students were asked if they could touch their nose with their tongue. Six out of 53 men said yes, compared to 15 out of 56 women.

(a) Use these data to construct a 95% confidence interval (CI) for the difference in population proportions.

(b) Use your CI from part (a) to test, using $\alpha = 0.05$, H_0: $p_{men} = p_{women}$ against H_a: $p_{men} \neq p_{women}$. Do you reject H_0? Why or why not?

(c) State the conclusion from your hypothesis test in part (b) in everyday language *in the context of this problem.*

III.2 A random sample of 46 college students were asked if they were doing community service work during a particular semester. The following table gives the data, comparing men and women. Do the data provide sufficient evidence to conclude that men are less likely than women to be involved in community service?

Community service

Sex	Yes	No	Total
Male	5	18	23
Female	11	12	23
Total	16	30	46

(a) Consider a chi-square test against a directional alternative with $\alpha = 0.05$. The value of the test statistic is 3.45. Give bounds for the *P*-value and state the conclusion that you reach in the context of the problem.

(b) Now consider Fisher's Exact Test, with a directional alternative, which involves calculating the probabilities of several possible 2×2 tables. Don't calculate any probabilities. Instead, state the number of tables for which the probability would need to be calculated if you were to carry out the test.

III.3 A researcher planted various kinds of seeds and recorded whether or not they germinated within 5 weeks. The following table gives the data.

Type of seed

Germinate?	Okra	Sunflower	Eggplant	Total
Yes	9	12	11	32
No	11	6	8	25
Total	20	18	19	57

(a) Consider the null hypothesis that type of seed is independent germination. The value of the expected cell count for the "Okra/Yes" cell is $32 \times 20/57 = 11.23$. Why is this correct? That is, explain the statistical reasoning that leads to (row total) $\times$ (column total)/(grand total) being the formula for an expected cell count.

(b) The value of the test statistic is 1.84. Conduct a test of the null hypothesis that type of seed is independent germination. Use $\alpha = 0.05$. Provide all steps, including degrees of freedom, *P*-value, and conclusion.

III.4 In a study of the effect of oral contraceptives on thromboembolic disease (blood clots) 175 pairs of women were studied.[3] The pairs were matched on age, race, and other variables. Within each pair one woman (the "case") had the disease but the other woman (the "control") did not. The following table shows the data on oral contraceptive use.

Oral contraceptives used by	Number of pairs
Both women in the pair	10
Only the case	57
Only the control	13
Neither woman in the pair	95
Total	175

Conduct a test of the hypothesis that presence of the disease is independent of oral contraceptive use. Use a nondirectional alternative and let $\alpha = 0.01$. (*Hint*: You might first want to display the data in a different format.)

(a) State the null hypothesis in symbols.

(b) Do you reject H_0? Why or why not?

(c) State your conclusion from part (b) in the context of this setting.

III.5 You think that approximately 25% of the members of a certain population smoke, but you want to take a sample in order to make an estimate. You want the standard error of $\tilde{p}$ to be no greater than 0.06.

(a) Using 0.25 as your guess of $\tilde{p}$, how large does your sample need to be?

(b) Suppose you have no guess for $\tilde{p}$. How large does your sample need to be so that the SE is no greater than 0.06 no matter what $\tilde{p}$ is?

Background for III.6–III.9

The following questions are motivated by the February 28, 2013 article: "A Brain-to-Brain Interface for Real-Time Sharing of Sensorimotor Information" (*Scientific Reports* **3**, 1319). The primary research question was to determine if the brain could assimilate signals from sensors from a different body. Here is a description of the study found on the Duke University Website (www.dukehealth.org/health_library/news/brain-to-brain-interface-allows-transmission-of-tactile-and-motor-information-between-rats).[4]

> To test this hypothesis, the researchers first trained pairs of rats to solve a simple problem: to press the correct lever when an indicator light above the lever switched on, which rewarded the rats with a sip of water. They next connected the two animals' brains via arrays of microelectrodes inserted into the area of the cortex that processes motor information.
>
> One of the two rodents was designated as the "encoder" animal. This animal received a visual cue that showed it which lever to press in exchange for a water reward. Once this "encoder" rat pressed the right lever, a sample of its brain activity that coded its behavioral decision was translated into a pattern of electrical stimulation that was delivered directly into the brain of the second rat, known as the "decoder" animal.
>
> The decoder rat had the same types of levers in its chamber, but it did not receive any visual cue indicating which lever it should press to obtain a reward. Therefore, to press the correct lever and receive the reward it craved, the decoder rat would have to rely on the cue transmitted from the encoder via the brain-to-brain interface.

III.6 Before connecting the encoder and decoder rats, the researchers wanted to verify that the decoder rats were sufficiently trained to choose the correct lever when seeing a visual cue. After training, one of the decoder rats was tested 25 times on his ability to select the "correct" lever. He selected the correct lever 19 times.

(a) Compute a 95% Wilson-adjusted confidence interval for the accuracy rate for this rat *and* interpret the interval in the context of the question.

(b) Using your interval computed in part (a) to support your answer, are you convinced that this rat's accuracy rate is better than simple guessing (i.e., 50% accuracy)?

(c) Using your interval computed in part (a) to support your answer, are you convinced that this rat's accuracy rate is better than 70%?

(d) Use a χ^2 goodness-of-fit test, to test whether or not the accuracy rate exceeds 70%. Use $\alpha = 0.05$.

III.7 Once a brain-to-brain connection was established, a pair of rats would undergo 50 lever-pressing trials. At each trial, the researchers would record whether or not the encoder rat pressed the "correct" lever and whether or not the decoder rat also pressed the "correct" lever. Following is a summary of the data.

	Decoder correct	Decoder wrong	Total
Encoder correct	27	11	38
Encoder wrong	6	6	12
Total	33	17	50

(a) Consider the question: Is decoder accuracy related to encoder accuracy? That is, does the decoder accuracy depend on whether or not the encoder pressed the right lever? Express the null hypothesis related to this research question symbolically. You may use the notation EC, EW, DC, and DW as needed to describe the groups (e.g., EC = Encoder correct).

(b) Complete the table of expected counts used for the χ^2 test of independence.

(c) The value of $\chi^2_s = 1.801$ for these data. Is there statistically significant evidence ($\alpha = 0.05$) that the decoder is *more* likely to press the correct lever if the encoder also presses the correct lever than if the encoder does not? Justify your answer.

(d) There is one condition for the χ^2-test of independence that is clearly violated. Which condition is violated?

III.8 Consider the research question: Does the decoder rat do what the encoder rat does? That is, when the encoder is correct, is the decoder also correct, and when the encoder is wrong, is the decoder also wrong? Thinking about the data in this way, the data in Exercise III.7 can be reduced as shown below.

	Agreement in response	Disagreement in response	Total
Count	33	17	50

If the rats do not have a "mental link," then the probability of agreeing would equal the probability of disagreeing or H_0: Pr[agree] = Pr[disagree].

(a) What is the alternative hypothesis for this research question?

(b) Use a χ^2 goodness of fit test to test the hypothesis in (a).

III.9 The study described above was actually carried out on four pairs of connected rats. The table below summarizes the results from the 200 (50 × 4) trials.

	Decoder correct	Decoder wrong	Total
Encoder correct	112	42	154
Encoder wrong	21	25	46
Total	133	67	200

The value of $\chi^2_s = 11.66$ and the corresponding P-value is 0.001, suggesting there is very strong evidence that the accuracy of the decoder is linked to the accuracy of the encoder. Briefly explain why the validity of this χ^2 test is questionable.

III.10 State whether the following statements are true or false and explain why.

(a) It is more difficult to estimate a population proportion precisely if its value is near 0.50.

(b) A χ^2 goodness-of-fit test has the null hypothesis that category proportions are all equal.

(c) A χ^2 goodness-of-fit test is used for observational studies whereas a χ^2 test of independence is used for experiments.

(d) $\chi_s^2 = 0$ when the observed data are in perfect agreement with the null hypothesis.

III.11 Consider the study of behavioral asymmetries in Exercise 10.3.4. Suppose we want to test the null hypothesis that righthanded women are equally likely to be right-footed or left-footed.

(a) Calculate the chi-square statistic for this hypothesis.

(b) What conclusions can be drawn? Let $\alpha = 0.05$.