MLE for $\hat{\beta}_0$

$$\mathcal{L} = -\frac{n}{2} \ln(2\pi) - \frac{n}{2} \ln(\sigma^2) - \frac{1}{2\sigma} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

To maximize: $\frac{\partial \mathcal{L}}{\partial (\hat{\beta}_0)} = 0$

$$-\frac{1}{2\sigma} \sum_{i=1}^{n} (-2y_i + 2\beta_0 + 2\beta_1 x_i) = 0$$

$$-\frac{1}{2\sigma} \left(2\beta_0 n + \sum_{i=1}^{n} (-2y_i + 2\beta_1 x_i)\right) = 0$$

$$-\frac{\beta_0 n}{\sigma} - \frac{\sum_{i=1}^{n} (-2y_i + 2\beta_1 x_i)}{2\sigma} = 0$$

$$\frac{2\sum_{i=1}^{n} y_i}{2\sigma} - \frac{2\beta_1 \sum_{i=1}^{n} x_i}{2\sigma} = \frac{\beta_0 n}{\sigma}$$

$$\hat{\beta}_0 = \frac{1}{n} \sum_{i=1}^{n} y_i - \frac{1}{n} \beta_1 \sum_{i=1}^{n} x_i$$

MLE for β_1

$$\frac{\partial L}{\partial \beta_1} = -\frac{1}{\sigma^2}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)x_i = 0$$

$$\sum_{i=1}^{n}(y_i x_i) - \sum_i \beta_0 x_i - \sum_i \beta_1 x_i^2 = 0$$

$$n\,\overline{y*x} - \beta_0 n \bar{x} - n\beta_1 (\overline{x^2}) = 0$$

$$\overline{y*x} - (\bar{y} - \beta_1 \bar{x})\bar{x} - \beta_1 \overline{x^2} = 0$$

$$\mathrm{cov}(x,y) + \bar{x}\cdot\bar{y} - \bar{x}\bar{y} - (\beta_1 \bar{x}^2) - \beta_1(\overline{x^2}) = 0$$

$$\mathrm{cov}(x,y) + \beta_1(-\bar{x}^2 + \overline{x^2}) = 0$$

$$\hat{\beta}_1 = \frac{-\mathrm{cov}(x,y)}{\overline{x^2} - \bar{x}^2} = \frac{\mathrm{cov}(x,y)}{\mathrm{Var}(x)} = \hat{\beta}$$

$$\mathrm{var}(x) = \overline{x^2} - \bar{x}^2$$

$y, x \rightarrow$ dependent thus $\overline{xy} = \mathrm{cov}(x,y) + \bar{x}\bar{y}$