

Assignment for Week 2. Two competing species

1. In examining the dimensionless Lotka-Volterra competition model in class, we saw that for $u_1^* = 1, u_2^* = 0$ to be stable, we needed $\alpha_{21} > 1$. Interpret and justify this result. Would your conclusions be affected if you learned that α_{12} were extremely large?
2. Another useful tool to understand stability is invasion analysis. The idea is to evaluate the criteria for growth of a vanishingly rare species when its competitor (also called the “resident”) is at equilibrium:

$$\left. \frac{dN_1}{dt} \right|_{N_1=0, N_2=N_2^*} > 0 \quad (1)$$

Perform invasion analysis for the two single-species equilibria of the original (i.e., not dimensionless) Lotka-Volterra competition model. How do the conditions for invasion compare to the stability criteria of the equilibria from the formal stability analysis? Explain the differences in the two approaches.

3. Suppose we have the following two-species system:

$$\frac{dx}{dt} = x(24 - x - 2y) \quad (2)$$

$$\frac{dy}{dt} = y(30 - y - 2x) \quad (3)$$

Analytically solve for the equilibria and determine their stability. Classify them by type (unstable saddle, spiral, etc.). Confirm your results graphically by drawing a rough phase portrait with the isoclines, equilibria, and trajectories (at the quadrant level). What are the possible outcomes? You can do this by hand, if you want, but submit the results electronically.