Assignment 3: Predators, Prey, and Foodwebs

1. Community Stability: connections count

With $S = 250, \sigma = 0.3$, how does stability depend on C?

```
library(ggplot2)
```

```
## Warning in register(): Can't find generic `scale_type` in package ggplot2 to
## register S3 method.
```

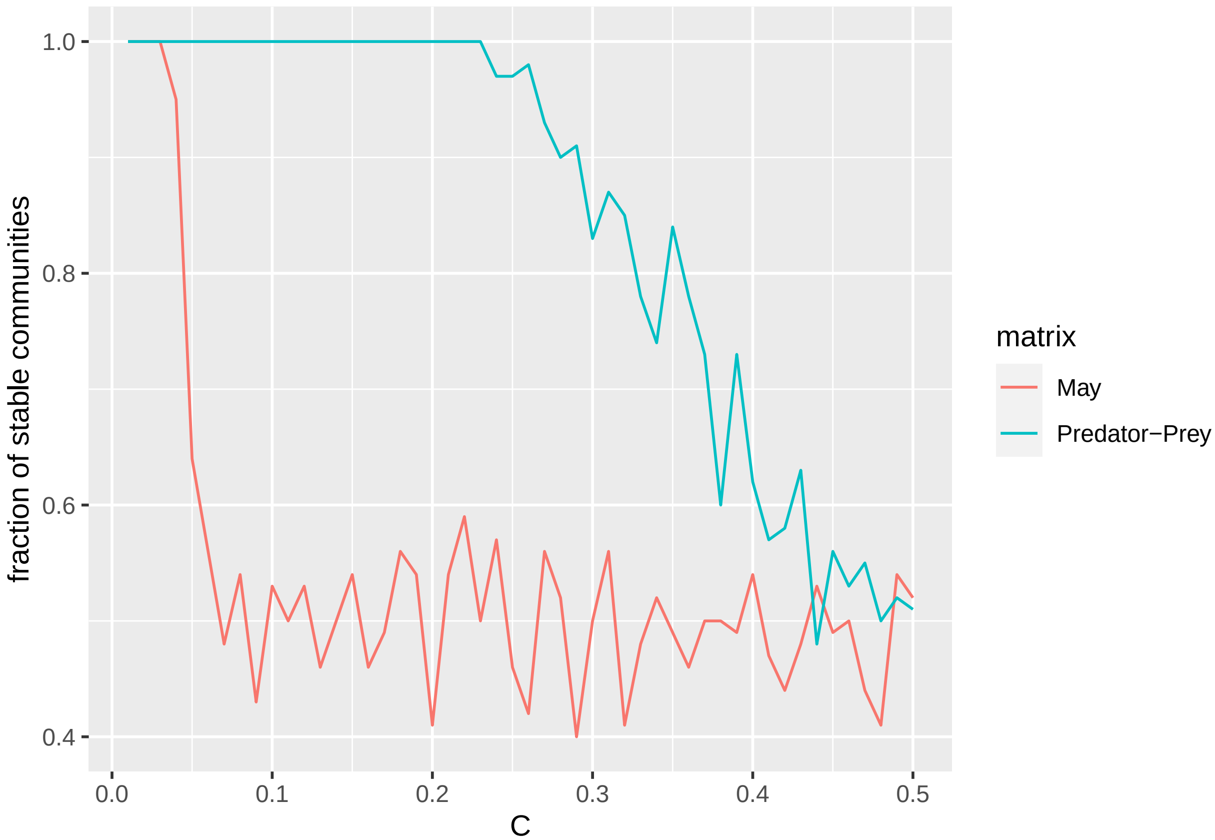
```
library(psych)
```

```
##
## Attaching package: 'psych'
```

```
## The following objects are masked from 'package:ggplot2':
##
##     %+%, alpha
```

```
S <- 250
sigma <- 0.3

MayMatrix<-function(S,C,sigma){
  ## This matrix determines the connections
  A<-matrix(runif(S*S),S,S)
  ## Contains the values for the connections
  B<-matrix(rnorm(S*S,0.0,sigma),S,S)
  A<-(A<= C)*1 # A matrix contains 1 when A[i,j] <= C
  M<-A*B
  diag(M)<- -1
  return(M)
}

PPMatrix<-function(S,C,sigma){
  ## Determine the signs for the connections
  MyS<-sign(rnorm(S*(S-1)/2))
  A<-matrix(0,S,S)
  A[upper.tri(A,diag=F)]<-MyS
  D<-matrix(runif(S*S),S,S)
  D<-(D <= C)*1
  A<-D*A
  A<- A-t(A)
  ## Contains the  values for the connections
  B<-matrix(abs(rnorm(S*S,0.0,sigma)),S,S)
```

```r
  M<-A*B
  diag(M)<- -1
  return(M)
}

stableQ<- function(m){
  if(tr(m)<0 & det(m)>0){
    return(1)
  }else{
    return(0)
  }
}


C <-seq(0.01,0.5,by=0.01)

mays <- c()
for (cs in C){
  mays <- append(mays,sum(replicate(100, stableQ(MayMatrix(S,cs,sigma)), simplify = TRUE )))
}

pred <- c()
for (cs in C){
  pred <- append(pred,sum(replicate(100, stableQ(PPMatrix(S,cs,sigma)), simplify = TRUE )))
}

mays <- mays/100
pred <- pred/100


df=data.frame(C = C,
              values=c(mays, pred),
              matrix=c(rep("May",50),rep("Predator-Prey",50))
)


ggplot(df,aes(C,values,col=matrix))+geom_line() +
  ylab("fraction of stable communities")
```



We can see in the plot that both methods of generating matrices result in high stability for low values of C. The threshold May proves in his paper $\sqrt{SC} > \frac{1}{\sigma}$, is $C > 2/45 (\approx 0.04444)$ when $S = 250$ and $\sigma = 0.3$. This is consistent with what we simulated. We can see the predator-prey matrix undergoes a similar transition at a much higher value of C, closer to 0.25.

2. Examples of particular dynamics

2.1 damped oscillations of a population to a stable equilibrium point

An example system that will show damped oscillations to a stable equilibrium is the predator-prey model with logistic growth described by the set of equations.

$$\begin{cases} \frac{dX}{dt} = (b-d)X(1-X/K) - \alpha XY \\ \frac{dY}{dt} = \alpha\epsilon XY - mY \end{cases}$$

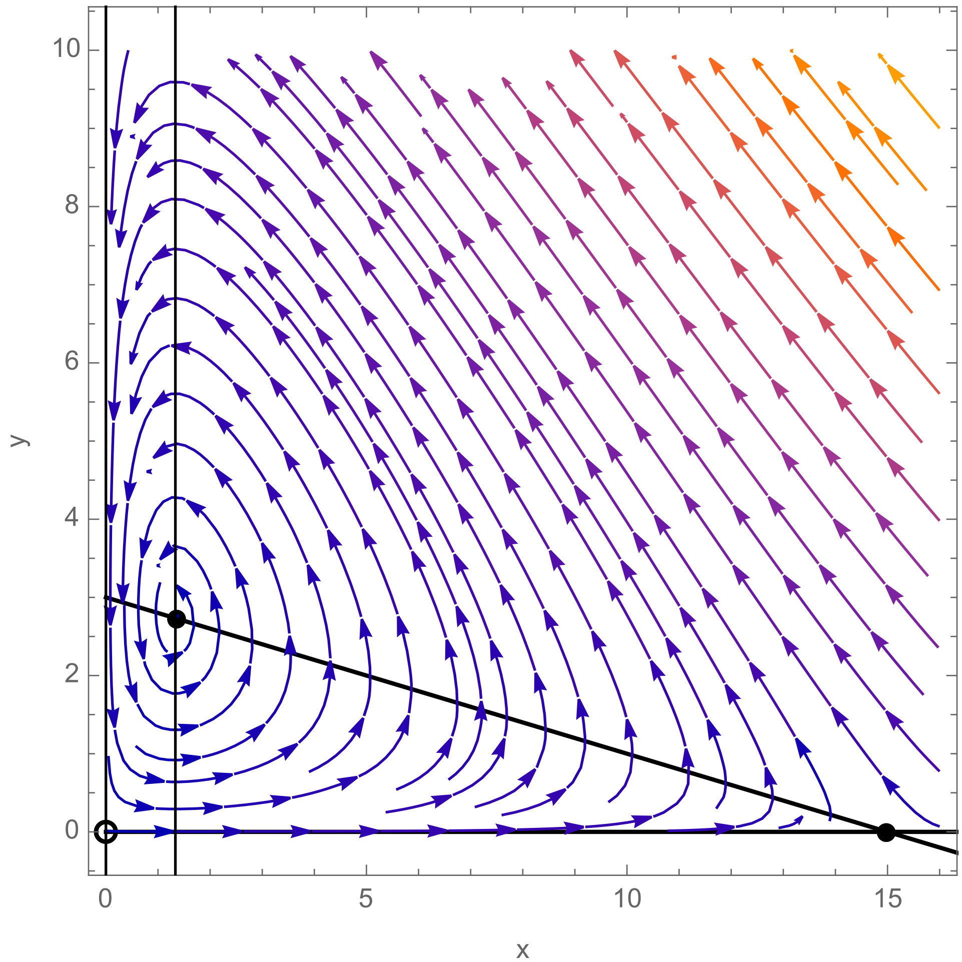


Figure 1 shows the isoclines of this system when the birth rate of the prey $b = 2$, the death rate of the prey $d = 0.5$, the prey carrying capacity $K = 15$, , the encounter rate $\alpha = 0.5$, the conversion efficiency of the predator $\epsilon = 0.9$, and the death rate for the predator $m = 0.6$. We can see in this plot that whenever the system starts with $X > 0$ and $Y > 0$, we expect a spiral to the equilibrium. For example, let's start with $X = 10$ and $Y = 10$. The resulting oscillation can be seen in figure 2.

2.2 stable cycles

The most famous example is the simple Lotka-Volterra model we started with described by the following system of equations:

$$\begin{cases} \frac{dN}{dt} = N(a - bP) \\ \frac{dP}{dt} = P(cN - d) \end{cases}$$

Here a is the growth rate of the prey N independent of the predator P (this means exponential growth without the predator). b is the predation rate of the prey by the predator, c is the conversion rate of eaten prey into predator (this is $b * \epsilon = c$, where ϵ is the conversion efficiency of killed prey into predator. Lastly, d is the decay rate of the predator without any prey.

Figure 3 shows the isoclines and figure 4 the dynamics of this system when $a = 2, b = 1, c = 0.9, d = 0.5, N[0] = 5, P[0] = 2$

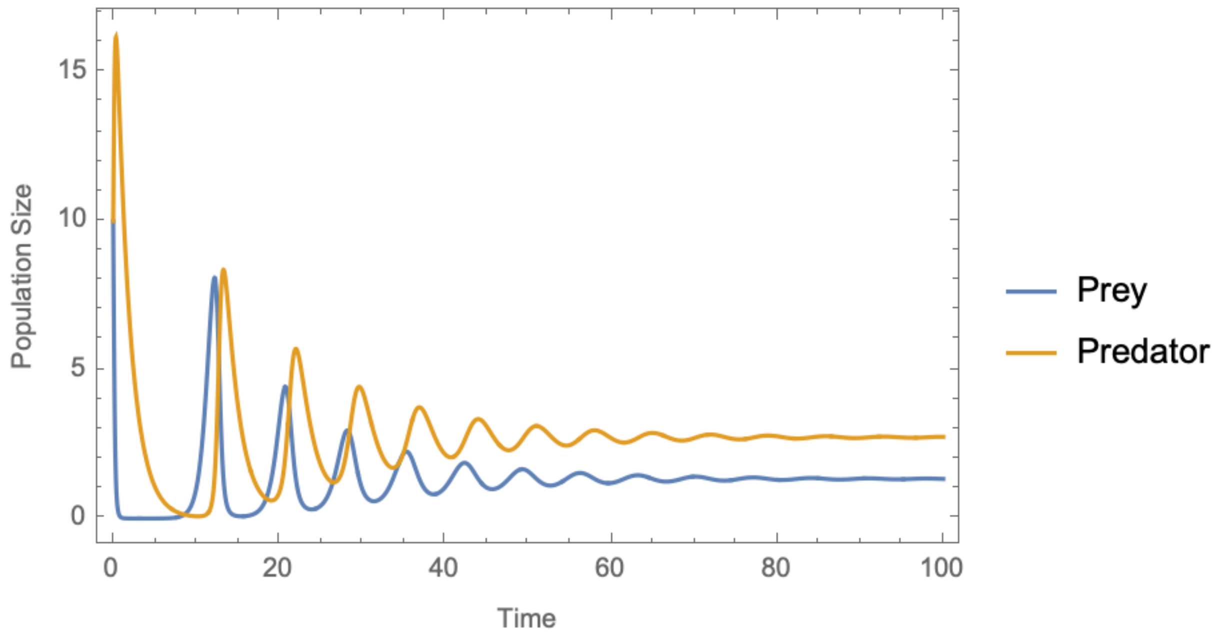


Figure 1: The dynamics of the predator prey model with logistic growth

2.3 unstable dynamics away from a nontrivial equilibrium

2.4 chaotic dynamics

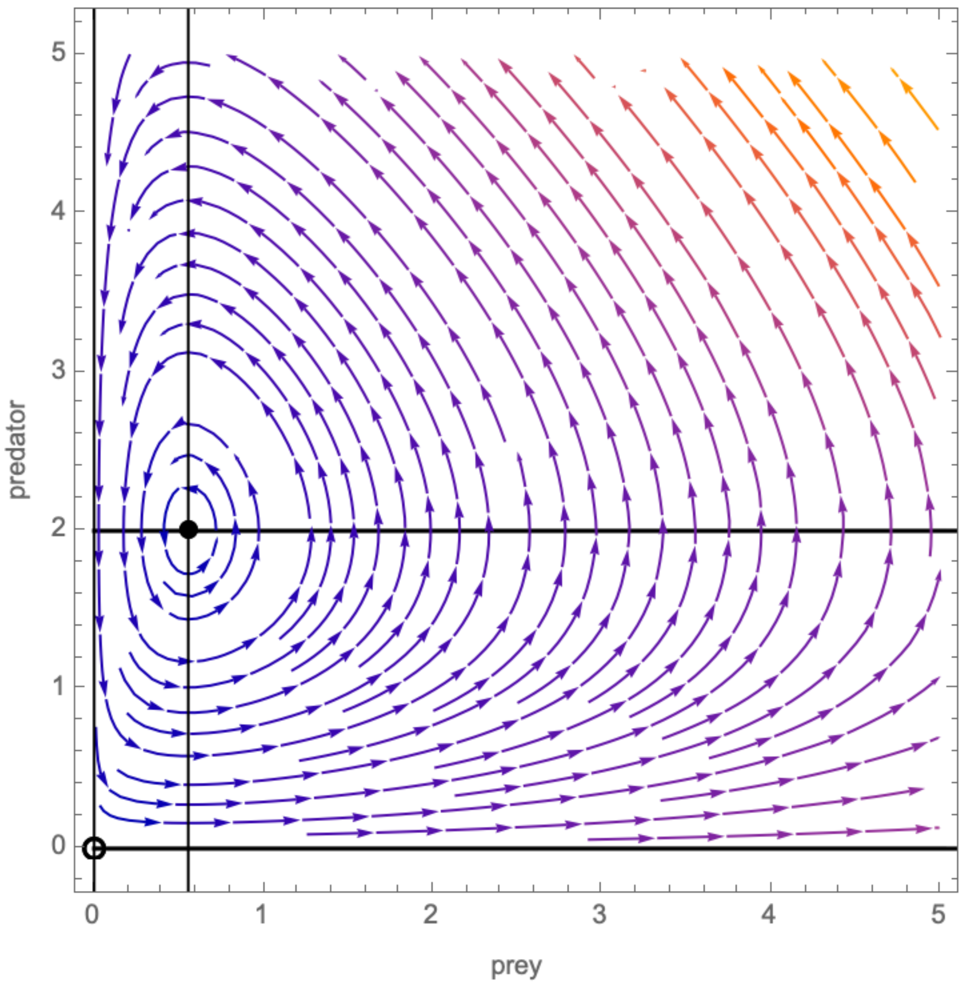


Figure 2: The dynamics of the basic, Lotka-Volterra predator-prey model

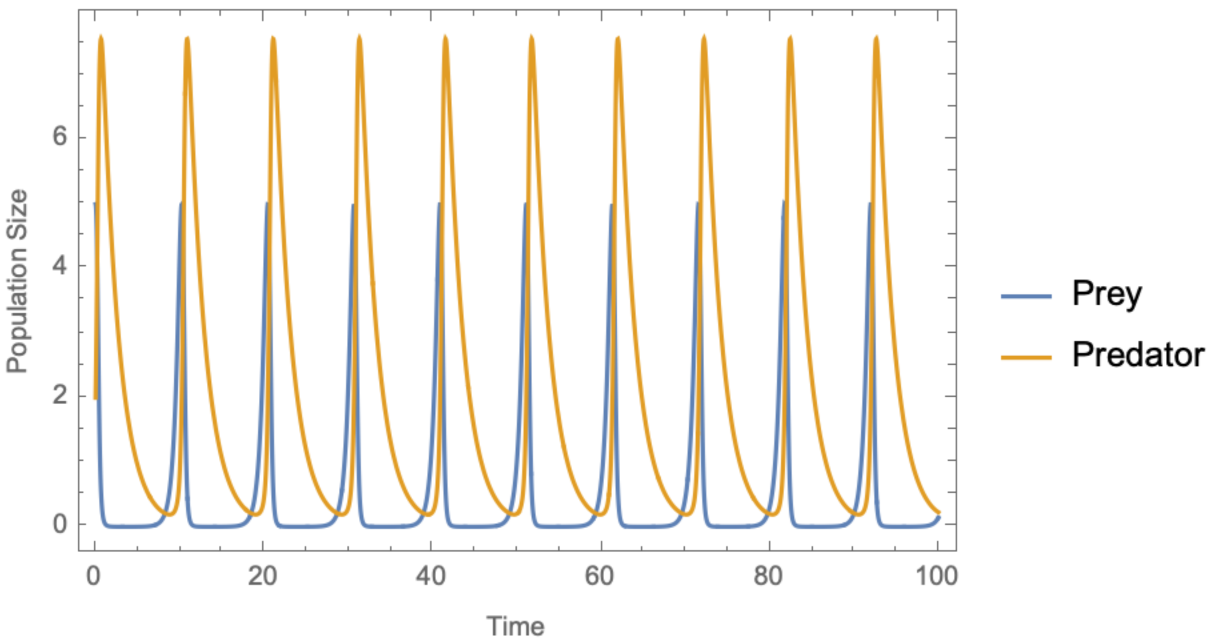


Figure 3: The dynamics of the basic, Lotka-Volterra predator-prey model