

# Assignment 3: Predators, Prey, and Foodwebs

## 1. Community Stability: connections count

With  $S = 250$ ,  $\sigma = 0.3$ , how does stability depend on  $C$ ?

```
library(ggplot2)
```

```
## Warning in register(): Can't find generic 'scale_type' in package ggplot2 to  
## register S3 method.
```

```
library(psych)
```

```
##
```

```
## Attaching package: 'psych'
```

```
## The following objects are masked from 'package:ggplot2':
```

```
##
```

```
##      %+%, alpha
```

```
S <- 250
```

```
sigma <- 0.3
```

```
MayMatrix<-function(S,C,sigma){
```

```
  ## This matrix determines the connections
```

```
  A<-matrix(runif(S*S),S,S)
```

```
  ## Contains the values for the connections
```

```
  B<-matrix(rnorm(S*S,0.0,sigma),S,S)
```

```
  A<-(A<= C)*1 # A matrix contains 1 when A[i,j] <= C
```

```
  M<-A*B
```

```
  diag(M)<- -1
```

```
  return(M)
```

```
}
```

```
PPMatrix<-function(S,C,sigma){
```

```
  ## Determine the signs for the connections
```

```
  MyS<-sign(rnorm(S*(S-1)/2))
```

```
  A<-matrix(0,S,S)
```

```
  A[upper.tri(A,diag=F)]<-MyS
```

```
  D<-matrix(runif(S*S),S,S)
```

```
  D<-(D <= C)*1
```

```
  A<-D*A
```

```
  A<- A-t(A)
```

```
  ## Contains the values for the connections
```

```
  B<-matrix(abs(rnorm(S*S,0.0,sigma)),S,S)
```

```

M<-A*B
diag(M)<- -1
return(M)
}

stableQ<- function(m){
  if(tr(m)<0 & det(m)>0){
    return(1)
  }else{
    return(0)
  }
}

C <-seq(0.01,0.5,by=0.01)

mays <- c()
for (cs in C){
  mays <- append(mays,sum(replicate(100, stableQ(MayMatrix(S,cs,sigma)), simplify = TRUE )))
}

pred <- c()
for (cs in C){
  pred <- append(pred,sum(replicate(100, stableQ(PPMatrix(S,cs,sigma)), simplify = TRUE )))
}

mays <- mays/100
pred <- pred/100

df=data.frame(C = C,
              values=c(mays, pred),
              matrix=c(rep("May",50),rep("Predator-Prey",50))
)

ggplot(df,aes(C,values,col=matrix))+geom_line() +
  ylab("fraction of stable communities")

```



We can see in the plot that both methods of generating matrices result in high stability for low values of  $C$ . The threshold May proves in his paper  $\sqrt{SC} > \frac{1}{\sigma}$ , is  $C > 2/45 (\approx 0.04444)$  when  $S = 250$  and  $\sigma = 0.3$ . This is consistent with what we simulated. We can see the predator-prey matrix undergoes a similar transition at a much higher value of  $C$ , closer to 0.25.

## 2. Examples of particular dynamics

### 2.1 damped oscillations of a population to a stable equilibrium point

An example system that will show damped oscillations to a stable equilibrium is the predator-prey model with logistic growth described by the set of equations.

$$\begin{cases} \frac{dX}{dt} = (b-d)X(1-X/K) - \alpha XY \\ \frac{dY}{dt} = \alpha \epsilon XY - mY \end{cases}$$

Fi

### 2.2 stable cycles

The most famous example is the simple Lotka-Volterra model we started with.

### 2.3 unstable dynamics away from a nontrivial equilibrium

### 2.4 chaotic dynamics

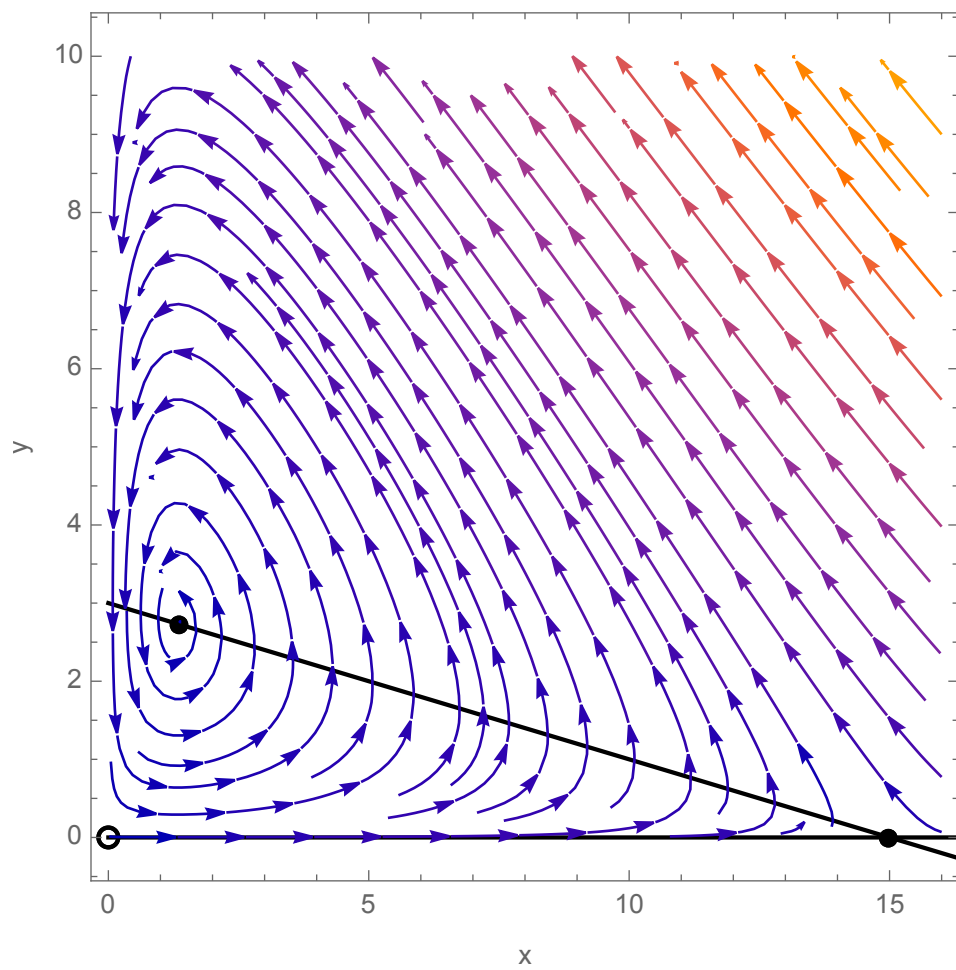


Figure 1: Figure 2: The dynamics of the predator prey model with logistic growth