Problem Set 1:
The Hodgkin-Huxley Model

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Computational Neuroscience

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The Hodgkin-Huxley equations model the interaction between the membrane potential and the voltage-gated channels of the squid axon. It is by using an *active* model of the membrane, in that the electrical properties of the membrane *change* with membrane potential, that the model is able to produce the characteristic spike waveform and other, less obvious neuronal phenomena. We will look at **NEST Dekstop** simulations of the HH-model to understand what is happening microscopically during spiking, refraction, excitation, and inhibition. For reference, the Hodkin-Huxley equations are:

$$\begin{aligned}
C\dot{U} &= -\bar{g}_{Na}\, m^3 h\,(U - E_{Na}) - \bar{g}_K\, n^4\,(U - E_K) - \bar{g}_L\,(U - E_L) + I \\
\tau_m \dot{m} &= m_\infty(U) - m \\
\tau_h \dot{h} &= h_\infty(U) - h \\
\tau_n \dot{n} &= n_\infty(U) - n.
\end{aligned} \tag{1}$$

Please use the default parameter values in Nest Desktop to simplify marking.

Problem 1: Spiking threshold — 4 points

Determine (to the nearest picoampere) the height of impulse (spike) input to a HH-neuron at rest required to bring the neuron to fire. Which additional parameters of the input contribute to its impact? Plot the evolution of the membrane potential $U(t)$ and the gating variables $m(t), n(t), h(t)$ caused by below and above threshold impulse input. Use these plots to explain how the all-or-none behaviour arises from the channel and membrane potential dynamics in response to the input.

Problem 2: Refractory period of the neuron — 4 points

Show that an input impulse above the threshold found in Problem 1 is not always guaranteed to induce a spike by focusing on the refractory period of the neuron. Use plots of the gating variables and membrane potential evolutions to explain the cause of neuronal refractoriness. How long (to the nearest millisecond) is the refractory period of the Hodgkin-Huxley neuron? Determine this by finding the shortest time two spikes which are elicited by threshold impulse input can succeed each other.

Problem 3: Current input — 4 points

We have seen that a threshold input impulse height cannot be determined as the state of the full system $U(t_0), h(t_0), m(t_0), n(t_0)$ as well as the input I_0 determines whether the system will consequently evolve through a spike excursion. The same holds for time-dependent inputs $I(t)$. Show that this is the case by demonstrating that even a continuous constant input $I(t) = I_0$ will, for standard parameters, elicit only a single spike approximately at the start of the input, after which the system relaxes into a steady state. Find (to the nearest 5 pA) the amplitude range $I_0 \in (I_i, I_f)$ for which this phenomena occurs. By plotting the evolution of the gating variables and membrane potential during the system's evolution, use the ionic channels to explain why no further spikes occur. How does the neuron respond to current amplitudes either side of the range (I_i, I_f)?