02_exercises_katja

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1 Problem Set 2: Two-State Ionic Channels in Python

[1]:
```python
import numpy as np
from matplotlib import pyplot as plt
from scipy import stats
```

1.1 Problem 1: Probability of transition

[2]:
```python
ts = 0.1
alpha = 1
beta = 2

# compute stochastic matrix
A = np.array([[1 - (ts*alpha), (ts*beta)],
             [(ts*alpha), 1 - (ts*beta)]])

print("Stochastic matrix:\n", A)
```

```
Stochastic matrix:
 [[0.9 0.2]
 [0.1 0.8]]
```

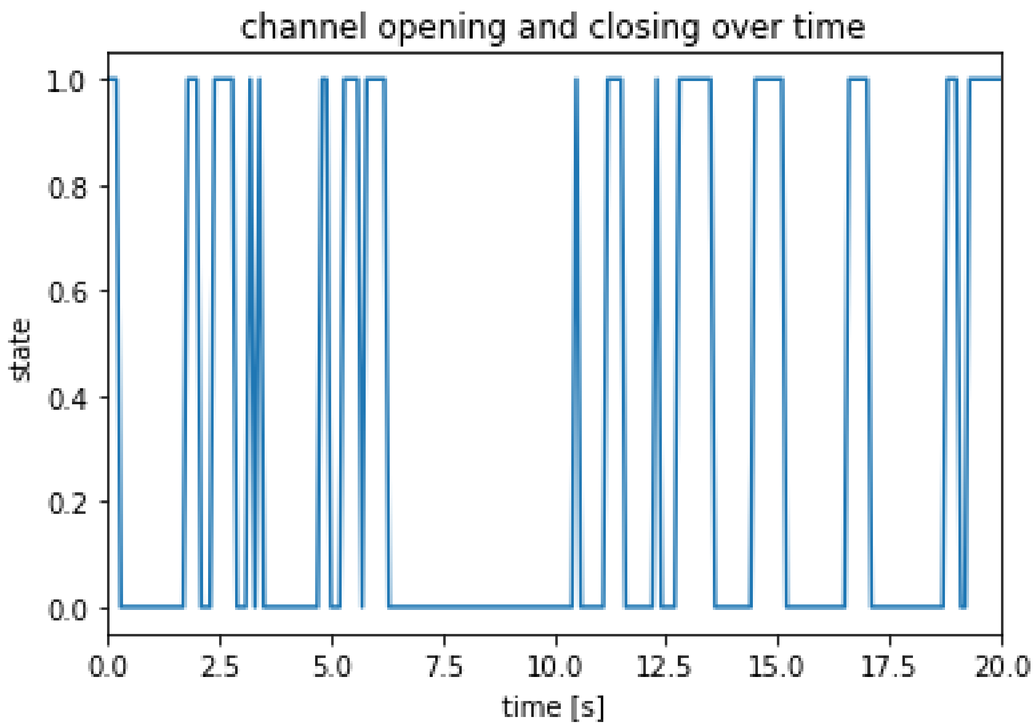
1.2 Problem 2: Simulation of a single channel

[3]:
```python
ts = 0.1
A = np.array([[1 - (ts*alpha), (ts*beta)],
             [(ts*alpha), 1 - (ts*beta)]])
x = np.array([0,1]).T
T = 500

t = np.arange(0, T, ts)
X = np.zeros((2, int(T//ts)+1))

X[:, 0] = x
channel = [1]
for i in range(1, int(T//ts)+1):
    p = A @ X[:, i-1]
```

```
    u = np.random.rand()
    if u <= p[1]:
        channel.append(1)
        X[:, i] = np.array([0, 1])
    if u > p[1]:
        channel.append(0)
        X[:, i] = np.array([1, 0])
```

[4]:
```
# plot simulation of single channel for t = 20s
plt.plot(t, channel)
plt.xlabel("time [s]")
plt.ylabel("state")
plt.title("channel opening and closing over time")
plt.xlim(0, 20)
```

[4]: (0.0, 20.0)



[5]:
```
print("Mean open rate:", np.mean(channel)) # mean open rate
```

```
Mean open rate: 0.3452
```

The channel is open at around $\frac{1}{3}$ of the time.

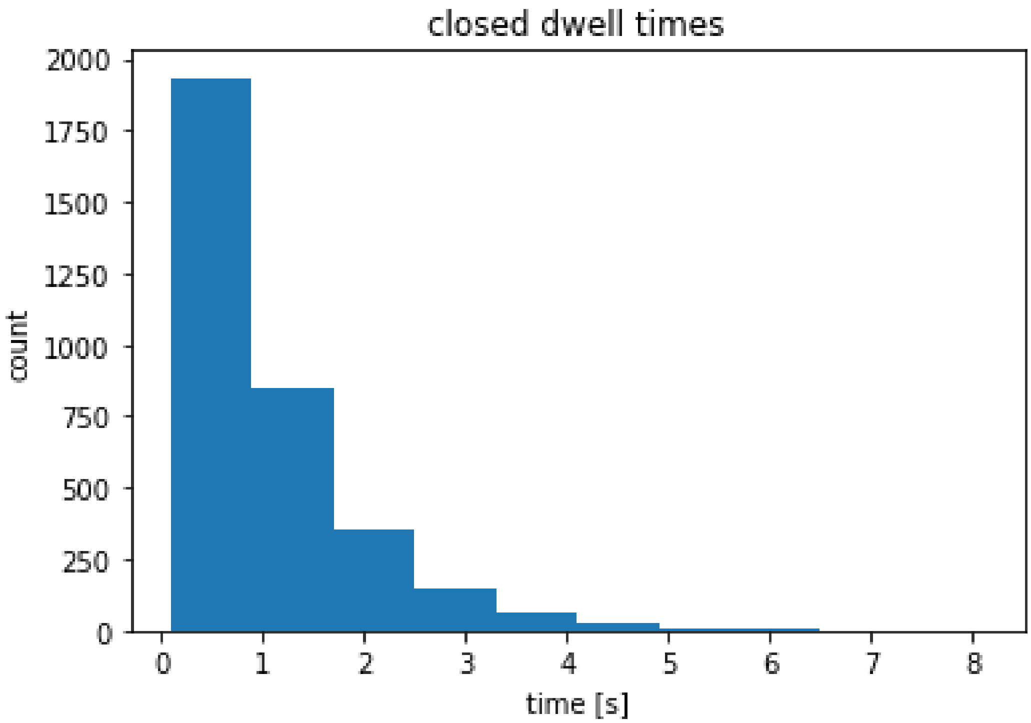
1.3 Problem 3: Channel dwell times

[6]:
```
ts = 0.1
A = np.array([[1 - (ts*alpha), (ts*beta)],
              [(ts*alpha), 1 - (ts*beta)]])
x = np.array([0,1]).T
T = 5000

t = np.arange(0, T, ts)
X = np.zeros((2, int(T//ts)+1))

X[:, 0] = x
channel = [1]
for i in range(1, int(T//ts)+1):
    p = A @ X[:, i-1]
    u = np.random.rand()
    if u <= p[1]:
        channel.append(1)
        X[:, i] = np.array([0, 1])
    if u > p[1]:
        channel.append(0)
        X[:, i] = np.array([1, 0])
```

[7]:
```
previous_channel = 1
previous_t = 0

t_open = []
t_closed = []

for i in range(1, int(T//ts)+1):
    current_channel = channel[i]
    current_t = t[i]
    if current_channel != previous_channel:
        if previous_channel == 0:
            t_closed.append(current_t - previous_t)
        elif previous_channel == 1:
            t_open.append(current_t - previous_t)
        previous_channel = current_channel
        previous_t = current_t
```

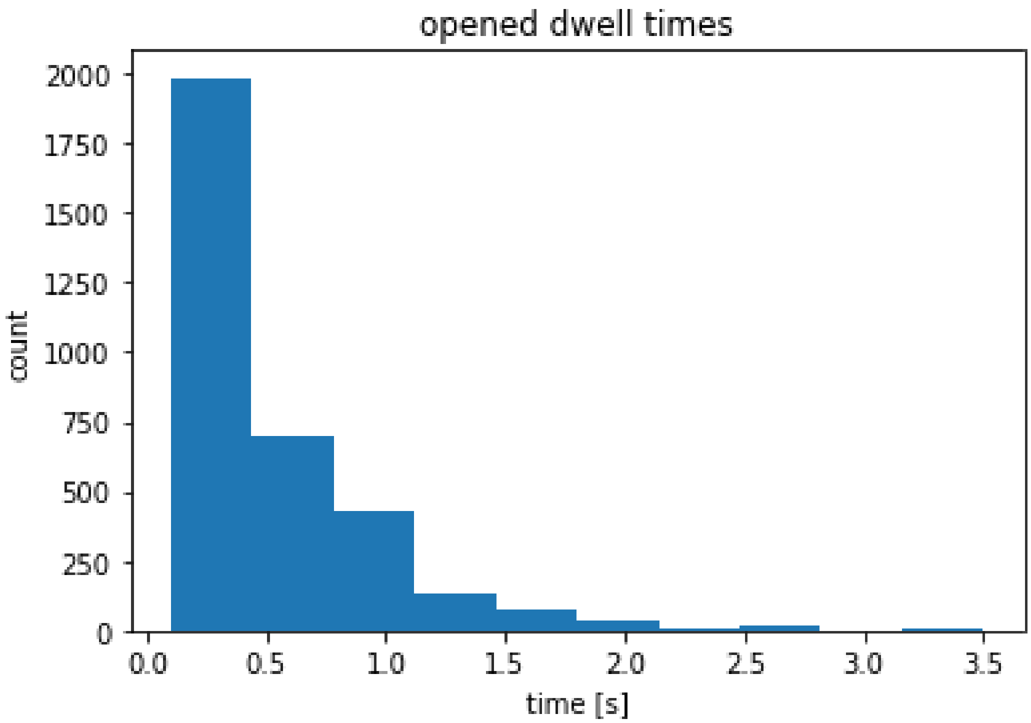
[8]:
```
plt.hist(t_closed)
plt.xlabel("time [s]")
plt.ylabel("count")
plt.title("closed dwell times")
```

[8]:
```
Text(0.5, 1.0, 'closed dwell times')
```



[9]:
```
plt.hist(t_open)
plt.xlabel("time [s]")
plt.ylabel("count")
plt.title("opened dwell times")
```

[9]:
```
Text(0.5, 1.0, 'opened dwell times')
```



[10]:
```
print("Mean closed time:", np.mean(t_closed))
print("Mean opened time:", np.mean(t_open))
```

```
Mean closed time: 0.9782737215489167
Mean opened time: 0.4995861661247459
```

My theoretical expectations would be that (1) there are many short closed and opened times with only a few very long times, (2) the mean closed time is twice as big as the mean opened times (because the channel is open at a rate of $\frac{1}{3}$. This fits with the simulation.

1.4 Problem 4: Propagation of state probabilities

[11]:
```
ts = 0.1
A = np.array([[1 - (ts*alpha), (ts*beta)],
             [(ts*alpha), 1 - (ts*beta)]])
x = np.array([0,1]).T
T = 500
```

[12]:
```
t = np.arange(0, T, ts)
X = np.zeros((2, int(T//ts)+1))

X[:, 0] = x
channel = [1]
for i in range(1, int(T//ts)+1):
```

```
    X[:, i] = A @ X[:, i-1]
    u = np.random.rand()
    if u <= X[1, i]:
        channel.append(1)
    if u > X[1, i]:
        channel.append(0)
```

[13]:
```
print("Asymptotc probabilitites:", X[:, -1])
```

```
Asymptotc probabilitites: [0.66666667 0.33333333]
```

The asymptotic probabilities agree with mean channel opening from problem 1.

[14]:
```
v, w = np.linalg.eig(A) # obtain eigenvalues and -vectors
print("Normalized eigenvector with eigenvalue = 1:", w[:, 0] / sum(w[:, 0]))
```

```
Normalized eigenvector with eigenvalue = 1: [0.66666667 0.33333333]
```

The eigenvector with eigenvalue $\lambda = 1$ normalized to probabilities corresponds to the asymptotic probabilities.

Also the equilibrium value fits with the asymptoic probabilities:

$$x_\infty = \frac{\alpha}{\alpha + \beta} = \frac{1}{1+2} = \frac{1}{3}$$

Time constant:

$$\tau = \frac{1}{\alpha + \beta} = \frac{1}{1+2} = \frac{1}{3}$$

I don't know what to say about the time constant.