The Neural Code

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Problem Set 4 **—Model Identification—** **25.04.2019**

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1. (White noise)

Consider a random process $X(t)$ where X at any time point t follows a Gaussian distribution $\mathcal{N}(0, \sigma^2)$.

(a) The autocorrelation of the above signal can be calculcated as $\varphi_{XX}(\tau) = \mathrm{E}[X(t)X(t+\tau)]$. Show analytically that

$$\varphi_{XX}(\tau) = \sigma^2 \delta(\tau)$$

(b) Show analytically that the Fourier transform of the autocorrelation of a signal $y(t)$ is its spectral power density

$$\mathcal{F}\left[\varphi_{yy}(\tau)\right] = |y(w)|^2$$

(c) Using the result above, derive the spectral power density $|X(w)|^2$ of the random process $X(t)$. What does it tell us about "white" noise?

2. (Model identification)

(Python exercise) Consider a kernel function:

$$H(t) = \sin\left(\frac{2\pi t}{0.1\ \mathrm{s}}\right) \exp\left(\frac{-t}{0.06\ \mathrm{s}}\right)$$

(a) Write a python function that generates a white noise stimulus $x(t)$ with mean 0 and variance $\sigma^2 = 10$ for a duration of 10s. You can do this by sampling $X \sim \mathcal{N}(0, \sigma^2)$ for every time bin (use `numpy.random.normal`).

(b) Convolve the white noise stimulus $x(t)$ with the kernel $H(t)$ to obtain the output firing rate $y(t)$ (use `numpy.convolve`).

(c) Approximate the kernel function by reverse correlation (use `numpy.correlate`)

$$\hat{H}(t) = \frac{1}{|x(w)|^2} \int_{\infty}^{\infty} ds\ y(s)x(s-t)$$

Plot both $\hat{H}(t)$ and $H(t)$ and compare them.

3. (Spike-triggered average)

(Python exercise) This is an exercise from Dayan & Abbott Theoretical Neuroscience, chapter 2, exercise 3. Load the data file by `data = scipy.io.loadmat('c2p3.mat')`. `data['counts']` contains responses of a cat LGN cell to 2D image stimuli. It is a vector containing a spike count of this cell in each 15.6ms time bin. `data['stim']` is an array of size `(16, 16, 32767)` containing 16 by 16 image stimuli for every time bin. Compute the spike-triggered average for the 12 time steps before a spike occurs.