The Neural Code

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Problem Set 1 **—Point Processes—** **25.04.2019**
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1. (Exponential distribtion) Consider

$$p(x) = \lambda\, \mathrm{e}^{-\lambda x}$$

(a) Show that $p(x)$ is normalized!

(b) For $a > 0$, show that

$$\langle x \rangle = -\frac{\mathrm{d}}{\mathrm{d}a} \int_0^\infty \mathrm{d}x \lambda\, \mathrm{e}^{-a\,x}|_{a=\lambda}$$

(c) Show that

$$\langle x^n \rangle = (-1)^n \frac{\mathrm{d}^n}{\mathrm{d}a^n} \int_0^\infty \mathrm{d}x \lambda\, \mathrm{e}^{-a\,x}|_{a=\lambda}$$

(d) Compute the moment-generating function $\int_0^\infty \mathrm{d}x \lambda\, \mathrm{e}^{-a\,x}$.

(e) Compute $\langle x^n \rangle$!

2. (Binomial Statistics)

Consider the binomial distribution

$$b_N(c) = q^c\,(1-q)^{N-c} \binom{N}{c}$$

(a) Show that $b_N(c)$ is normalized!

(b) Show that

$$\langle c \rangle = q \frac{\mathrm{d}}{\mathrm{d}q} (q+a)^N|_{a=1-q}$$

(c) Show that

$$\langle c^n \rangle = (q \frac{\mathrm{d}}{\mathrm{d}q})^n (q+a)^N|_{a=1-q}$$

3. (Simulating Poisson Processes)

(a) Write a python function that simultaneously generates N realizations of homogeneous Poisson Processes with rates $\lambda_1, ..., \lambda_N$ within a duration T.

(b) Write a python function that simultaneously generates N realizations of inhomogeneous Poisson Processes with rates $\lambda_1(t), ..., \lambda_N(t)$ within a duration T and test it on

$$\lambda_m(t) = \lambda_0\,[1 + \cos(2\pi m\, t/T)]/2$$

with $N = 10$, $T = 100$s and $\lambda_0 = 1$Hz.

(c) Write a python function that simultaneously generates N realizations of homogeneous Poisson Processes with refractory periods $\tau_1, ..., \tau_N$ within a duration T. Test the function for a $N = 5$ processes with homogenous rate of 10 Hz within $T = 50$s and

$$\tau_m = 2^{m-1} \text{ ms} .$$

Plot the autocorrelation functions (use numpy `correlate`) of the $N = 5$ processes.