The Neural Code

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Problem Set 2 **—Tuning Curves—** **25.04.2019**

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1. (MMSE) Consider the mean squre error of an estimator

$$\mathrm{MSE}(\hat{x}) = \langle (\hat{x}(y) - x)^2 \rangle = \sum_{i,j} p_{ij} \left(\hat{x}(y_i) - x_j \right)^2$$

in a discrete input-output space (y_i, x_j) with probabilities p_{ij}.

(a) Compute the derivative $\partial_{\hat{x}(y_k)}\mathrm{MSE}$ and show that it equals

$$2\, p_k (\hat{x}(y_k) - \langle x \rangle|_k)$$

with $p_k = \sum_j p_{kj} = \sum_k p_{j|k}\, p_k$.

(b) Interpret the extremal solution $\hat{x}(y_k)$ and show it is a mimimum!

2. (Fisher information of Poisson Process)

Show that the Fisher information of tuning curve with Poisson statistics is

$$\frac{f'(x)^2}{f(x)}$$

3. (MASE)

(a) Compute the Fisher Information matrix for a Gaussian process

$$p(y|x_1, x_2) = \frac{1}{\sqrt{2\pi C}} \exp[-(y - f)^2/(2C)]$$

on a 2-d stimulus space with tuning curve $f(x_1, x_2)$ and and Covariance $C(x_1, x_2)$.

(b) Apply the above result to a Gaussian processes with tuning curves on a cylinder ($\phi \in [0, 2\pi), z \in [0, 1]$)

$$f(\phi, z) = f_0 \exp\left[\cos\left(\phi - 2\pi z\right)\right]$$

and covariance matrix

$$C(\phi, z) = \sigma_0^2 + \sigma_1^2\, z$$

(c) Write a python function that computes the Fisher information matrix $\mathcal{I}$ for given parameters $\sigma_{0/1}$ at given ϕ, z. Plot the MASE $\mathrm{tr}\mathcal{I}^{-1}$ for the given example as a function of ϕ and z for varying choices of $\sigma_{0/1}$.