Thermal plume turbulence

A. Celani[a)]
CNRS, INLN, 1361 Route des Lucioles, 06560 Valbonne, France

A. Mazzino
CNR-ISIAtA, Università di Lecce, I-73100 Lecce, Italy and INFM-Dipartimento di Fisica, Università di Genova, I-16146 Genova, Italy

M. Vergassola
CNRS, Observatoire de la Côte d'Azur, B.P. 4229, 06304 Nice Cedex 4, France

(Received 12 December 2000; accepted 4 April 2001)

A two-dimensional Boussinesq model of thermal convection is investigated by numerical simulations. The turbulent heat transport is mainly due to thermal plumes. It is shown that their thin, wrinkled interfaces control the statistics of large temperature excursions. Away from the interfaces, the temperature fluctuations are strongly suppressed and the field in those regions is well mixed. Their statistical signature is in the low-order moments of the temperature increments, whose scaling exponents depend linearly on the order.
[DOI: 10.1063/1.1375145]

The role of thermal plumes in convection at very high Rayleigh numbers has been investigated in a variety of settings (see, e.g., Ref. 1 for a review). Plumes are well-organized structures of warm rising (cold descending) air that appear when the buoyancy effects are important. In the atmosphere, their generation and interaction strongly affect the heat transport properties and the temperature statistics in the convective boundary layer.[2] In that region, at intermediate heights, the mean temperature is well approximated by a linear decreasing profile (see, e.g., Ref. 3) and its fluctuations are driven locally by down-gradient turbulent diffusion. This situation is modeled by accounting for a linear mean temperature profile in the two-dimensional Boussinesq equations:

$$\begin{aligned} &\partial_t T + \mathbf{v}\cdot\nabla T = \kappa \Delta T, \\ &\partial_t \omega + \mathbf{v}\cdot\nabla \omega = \nu \Delta \omega - \beta \nabla T \times \mathbf{g}. \end{aligned} \tag{1}$$

Here, T is the temperature field, $\omega = \nabla \times \mathbf{v}$ is the vorticity, $\mathbf{g}$ is the gravitational acceleration, β is the thermal expansion coefficient and κ, ν are the molecular diffusivity and viscosity. Since the fluid is being heated from below, a mean profile $\langle T(\mathbf{r},t)\rangle = \mathbf{G}\cdot\mathbf{r}$ is assumed, with a large-scale gradient $\mathbf{G}$ pointing downward as the gravity field. In a similar model, studied in Refs. 4 and 5, no mean gradient is present and a forcing term is added to the equation for the temperature field. That model mimics the convective boundary layer at larger heights. There, the average temperature becomes constant and turbulence is not excited locally, but emitted from the underlying layer. In Eq. (1), the temperature field affects the vorticity through the buoyancy forces, thus providing a simple example of active scalar turbulence. At large enough values of β, the buoyancy forces can equilibrate the inertial terms in the velocity dynamics, while the temperature fluctuations cascade toward the small scales at a rate ϵ. Dimensional arguments based on this phenomenological picture lead to the Bolgiano–Obukhov scaling $\delta_r T \sim r^{1/5}$ (see, e.g., Ref. 1, and references therein) for the temperature increments $\delta_r T = T(\mathbf{r},t) - T(\mathbf{0},t)$ in the inertial range of scales. Actually, due to the presence of the coherent structures, the statistics of the temperature increments exhibits a nontrivial scale dependence. Large and small excursions depend on the separation r in two completely different ways, both differing from the dimensional Bolgiano–Obukhov predictions.

In Fig. 1 we show a snapshot of the temperature field obtained in our numerical simulations.[6] Cold and warm regions of the fluid are coded in black and white, respectively. In the central upper part of the image, an uprising warm plume and a descending cold one right below are recognized. The whole pattern is rotating counterclockwise. This picture illustrates the main features of the convective thermal plumes. A warm plume is, for example, formed by a well-mixed hot region, where the temperature fluctuations are weak, bounded by a thin interface separating warm fluid from the colder surrounding background. The thickness of the boundary is of the order of the diffusive length scale. The temperature excursions across the interface are as large as the rms value $T_{\mathrm{rms}} \equiv (\langle T^2\rangle - \langle T\rangle^2)^{1/2}$. Elsewhere, efficient mixing is taking place and the temperature fluctuations are depleted. From this sketch we infer that the temperature differences $\delta_r T$ across a scale $\mathbf{r}$ will be either small or $O(T_{\mathrm{rms}})$ depending on whether the separation $\mathbf{r}$ is inside a plume or crosses its boundary.[7] Therefore, the plume boundaries will control the tails ($\delta_r T \gg T_{\mathrm{rms}}$) of the probability density function (pdf) of the temperature increments, whereas the bulk and the background fluctuations will determine its core ($\delta_r T \ll T_{\mathrm{rms}}$).

Let us first consider the large temperature excursions. As shown in Fig. 2, the tails of the pdfs $P(\delta_r T)$ at various r within the inertial range of scales can be collapsed onto a

[a)]Electronic mail: celani@inln.cnrs.fr

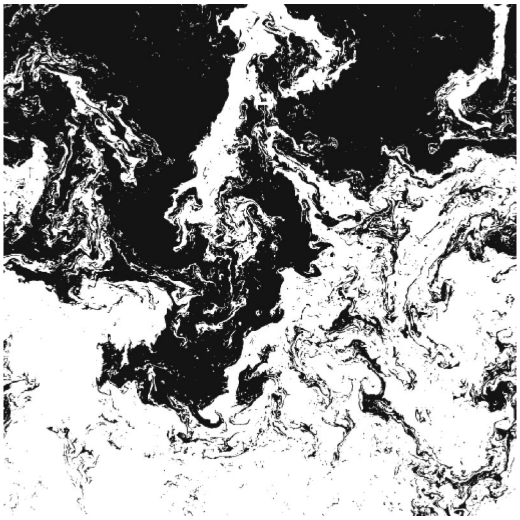

FIG. 1. Two-color coded image of the temperature field. Black (white) regions represent cold (warm) fluid.

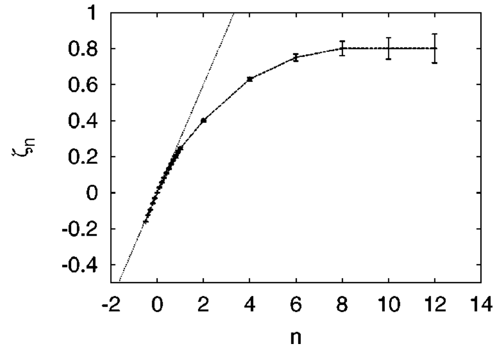

FIG. 3. Scaling exponents ζ_n of temperature structure functions $\langle|\delta_r T|^n\rangle \sim r^{\zeta_n}$. The dotted line is αn, with $\alpha=0.3$.

single curve by a simple multiplicative factor $r^{-\zeta_\infty}$. In other terms,

$$P(\delta_r T)\simeq r^{\zeta_\infty}Q(\delta_r T/T_{\rm rms}) \quad \text{for} \quad \delta_r T \gg T_{\rm rms}. \tag{2}$$

In our simulations, we measured an exponent $\zeta_\infty \simeq 0.8$ and a shape of Q which decays roughly as an exponential. As a direct consequence of (2), the structure functions $S_n(r)=\langle(\delta_r T)^n\rangle$ behave as r^{ζ_∞} for large enough orders, that is with an exponent independent of n. In our case, this saturation of the scaling exponents is taking place for the orders larger than the eighth, whose exponents are all coinciding with ζ_∞ (see Fig. 3). What is most remarkable in expression (2) is the factorization of the pdf in a scaling factor, r^{ζ_∞}, and a term that depends solely on the intensity of the fluctuation, $Q(\delta_r T/T_{\rm rms})$. The physical interpretation is that the occurrence of a large excursion $\delta_r T$ is due to two events: first, the separation $\mathbf{r}$ must intercept an interface, and this happens with a probability $\propto r^{\zeta_\infty}$; second, the temperature jump $\delta_r T$ across the interface occurs with a probability $\propto Q(\delta_r T/T_{\rm rms})$. This is confirmed by measuring the box covering dimension D_F of the interface. The probability to intercept an interface across $\mathbf{r}$ scales as r^{2-D_F} (see, e.g., Ref. 8). In our simulations we find indeed by straightforward box counting that the set of points $\mathbf{x}$ such that $|T(\mathbf{x}+\mathbf{r})-T(\mathbf{x})|$ exceeds three times $T_{\rm rms}$ has a dimension $D_F\simeq 1.2$, in agreement with the expectation that $D_F=2-\zeta_\infty$. The dimension larger than unity is the quantitative counterpart of the visual impression of the interfaces as wrinkled lines. In short, the scale dependence of the probability of large temperature excursions is governed by the geometrical properties of the plume boundaries. A similar picture holds for the passive ($\beta=0$) scalar turbulence.[9,10] There also, the rescaled pdfs of the scalar increments are collapsing as in Fig. 2 and, in the so-called fronts or cliffs, the scalar excursions are comparable to the rms scalar value. However, it must be remarked that for the passive scalar, the tails of the pdfs are not dominated by the same structures at various separations r. The majority of the events in the tails are indeed associated with nonmature objects, whose thickness is comparable to r. In other words, their thickness has not reached the diffusive scale yet, the cliffs are still in their steepening process, and they have relatively small increments across separations much smaller than r. These physical arguments are confirmed by analyzing the conditional probabilities of the increments at two different scales.[10] The same analysis for the convective case suggests that the scenario is qualitatively similar to the one drawn previously for the passive scalar.

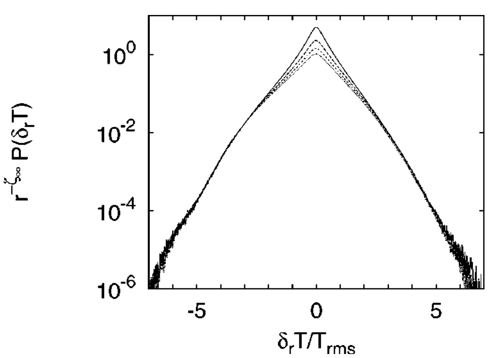

FIG. 2. The probability density functions of the temperature increments, for four different separations $r=0.04$, 0.08, 0.12, 0.16. The curves are collapsed by the rescaling factor r^{ζ_∞}, with $\zeta_\infty=0.8$.

It is worth remarking that the statistics of the velocity fluctuations does not show the same strongly intermittent behavior as the temperature. The velocity increments follow quite closely the dimensional law $\delta_r v \sim r^{3/5}$. Some deviations associated with the presence of structures in the form of thin vorticity filaments can be detected, but they are small. It is hardly conceivable that such objects could play a role in the formation of the plumes. Those are rather built up by the constructive interference between the large-scale compressive velocity components aligned in the vertical direction and the buoyancy forces.

Let us now move to the weak temperature fluctuations. As shown in Fig. 4, for small arguments the pdf $P(\delta_r T)$ takes the form

$$P(\delta_r T)\simeq r^{-\alpha}F(\delta_r T/r^{\alpha}) \quad \text{for} \quad \delta_r T \ll T_{\rm rms}. \tag{3}$$

This rescaling means that small excursions reduce self-similarly as the scale decreases. We measured an exponent $\alpha\simeq 0.3$, to be compared with the dimensional expectation 1/5. Such a relatively large exponent reveals that, inside the

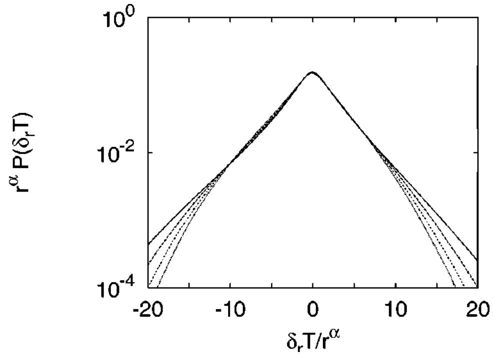

FIG. 4. The core of the probability density functions of temperature increments rescaled by r^α, for the same four scales r as in Fig. 2, with $\alpha=0.3$.

plumes, the temperature variability is strongly suppressed and an efficient mixing is taking place. The counterpart is in the formation of the interfaces, where the excursions are concentrated and the mixing is reduced. A similar situation holds for the statistics of the weak fluctuations in passive scalar turbulence.[10] Let us recall that the rescaling (3) entails a linear behavior for the scaling exponents of the low-order moments of the temperature fluctuations, as shown in Fig. 3.

The final point that we briefly address is the persistence at the small scales of the anisotropies induced by the large-scale mean temperature profile. By inspecting Fig. 2, it is evident that the pdfs are skewed. This asymmetry is due to the fact that hotter fluid most probably lies on the lower side of the interface. In other words, the temperature gradients preferentially point downwards rather than upwards. The anisotropy may be quantified by the odd-order temperature structure functions (vanishing in isotropic situations). We found a skewness $S_3(r)/(S_2(r))^{3/2}$ practically independent of r and a hyperskewness $S_5(r)/(S_2(r))^{5/2}$ that grows going toward the small scales as $r^{-0.2}$. Dimensional predictions would give $r^{4/5}$ for both quantities. Those behaviors signal a strong persistence of the anisotropies, even more noticeable than for the passive scalar case in two dimensions, where the skewness scales as $r^{0.25}$,[9] and comparable to the three-dimensional case.[11]

ACKNOWLEDGMENTS

We are grateful to A. Lanotte for stimulating discussions. The work was partially supported by the European Union under Contract Nos. HPRN-CT-2000-00162 and FMRX-CT-98-0175. A.M. has been partially supported by INFM Project No. GEPAIGG01. Simulations were performed at IDRIS (Contract No. 991226) and CINECA (INFM Parallel Computing Initiative).

[1] E. D. Siggia, ''High Rayleigh number convection,'' Annu. Rev. Fluid Mech. **26**, 137 (1994).
[2] G. S. Young, ''Convection in the atmospheric boundary layer,'' Earth-Sci. Rev. **25**, 179 (1988).
[3] Y.-B. Du and P. Tong, ''Turbulent thermal convection in a cell with ordered rough boundaries,'' J. Fluid Mech. **407**, 57 (1999).
[4] S. Toh and T. Matsumoto, ''The dynamics of structures of T-vorticity in 2D free convection turbulence,'' *Proceedings of IUTAM Symposium on Geometry and Statistics of Turbulence*, Hayama, November 1999 (Kluwer Academic, Dordrecht, 1999).
[5] D. Biskamp and E. Schwarz, ''Scaling properties of turbulent convection in two-dimensional periodic systems,'' Europhys. Lett. **40**, 637 (1997); D. Biskamp, K. Hallatschek, and E. Schwarz, ''Scaling laws in two-dimensional turbulent convection'' (preprint).
[6] The integration of both equations in (1) is performed by a standard 2/3-dealiased pseudospectral method on a doubly periodic square domain of length 2π. The resolution is 1024^2. The dissipative terms have been substituted, as customary, by hyperviscous terms (of order eight for the viscosity and of order four for the diffusivity). A linear friction term is added to the vorticity equation to prevent the energy from accumulating at the lowest accessible modes. The time evolution is implemented by a standard second-order Adams–Bashforth scheme. The integration is carried out for 200 eddy turn-over times after the system has reached the stationary state.
[7] Temperature increments are evaluated along a separation $\mathbf{r}$ at 45° with the vertical axis. This choice is motivated by the anisotropy of the problem: in two dimensions, the pdf of increments $P(\mathbf{r},\delta T)$ can be developed in a trigonometric series in the angle φ between the vertical axis and the separation $\mathbf{r}$. The first term in the development is φ independent and therefore represents the isotropic contribution to the pdf. The second one is proportional to $\cos(\varphi)$ and accounts for the skewness of the pdf. The third one is proportional to $\cos(2\varphi)$ and it is eliminated by the choice $\varphi=\pi/4$. The higher-order terms give subdominant contributions to the pdf, as has been checked numerically. For more details on the extraction of isotropic and anisotropic contributions see Ref. 10, and references therein.
[8] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, 1995), pp. 136–137.
[9] A. Celani, A. Lanotte, A. Mazzino, and M. Vergassola, ''Universality and saturation of intermittency in passive scalar turbulence,'' Phys. Rev. Lett. **84**, 2385 (2000).
[10] A. Celani, A. Lanotte, A. Mazzino, and M. Vergassola, ''Fronts in passive scalar turbulence,'' Phys. Fluids **13**, 1768 (2001).
[11] L. Mydlarski and Z. Warhaft, ''Passive scalar statistics in high-Péclet-number grid turbulence,'' J. Fluid Mech. **358**, 135 (1998).