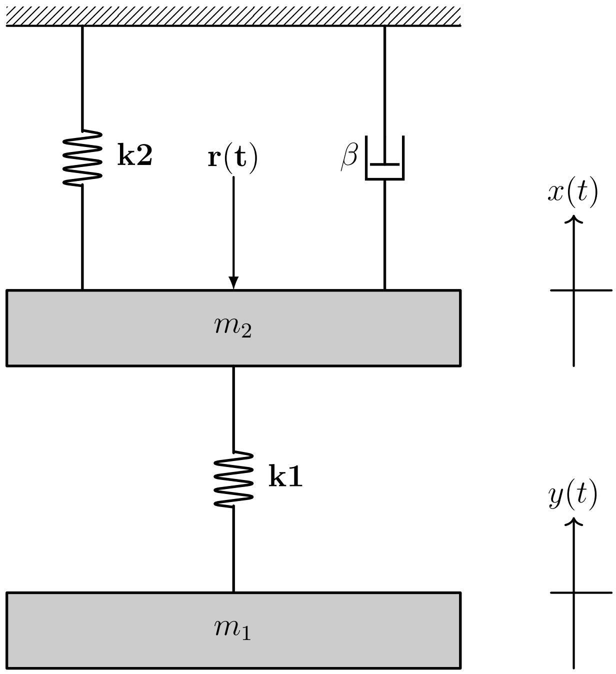
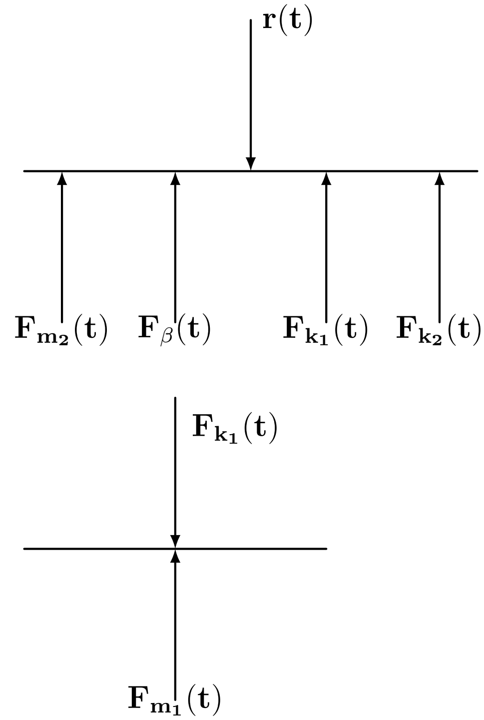
Ejercicio # 1

Hallar las ecuaciónes de función de transferencia: $FT_1(s) = \dfrac{X(s)}{R(s)}$ y $FT_2(s) = \dfrac{Y(s)}{R(s)}$.





Solucion:

1. Tenemos las siguientes ecuaciones:

   $F_{m_1} = m_1\dfrac{d^2y(t)}{dt^2}$; $F_{m_2} = m_2\dfrac{d^2x(t)}{dt^2}$; $F_\beta = \beta\dfrac{dx(t)}{dt}$; $F_{k_1} = k_1[x(t) - y(t)]$; $F_{k_2} = k_2x(t)$.

2. Hacemos la sumatoria de las fuerzas (DCL):

   Para la masa 2.

$$F_{k_1} + F_{k_2} + F_\beta + F_{m_2} = r(t)$$

$$[x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2} = r(t)$$

   Para la masa 1.

$$-F_{k_1} + F_{m_1} = 0$$

$$-[x(t) - y(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

$$[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

3. Aplicamos la transformada de Laplace: $y'(0) = y(0) = x'(0) = x(0) = 0$

   Para la primera ecuación:

$$\begin{aligned} [x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2} &= r(t) \\ \mathscr{L}\left\{[x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2}\right\} &= \mathscr{L}\{r(t)\} \\ [X(s) - Y(s)]k_1 + X(s)k_2 + S\beta X(s) + m_2S^2X(s) &= R(t) \end{aligned}$$

   Para la segunda ecuación:

$$[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

$$\mathscr{L}\left\{[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2}\right\} = 0$$

$$[Y(s) - X(s)]k_1 + m_1S^2Y(s) = 0$$

4. Ordenamos las ecuaciones:

$$\overbrace{(k_1 + k_2S\beta + m_2S^2)}^{a}X(s) - \overbrace{k_1}^{b}Y(s) = R(s) \tag{1}$$

$$\overbrace{-k_1}^{c}X(s) + \overbrace{(k_1 + m_1S^2)}^{d}Y(s) = 0 \tag{2}$$

5. Resolvemos las ecuaciones por el metodo de cramer:

$$X(s) = \frac{\begin{vmatrix} R(s) & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{R(s)\cdot d - 0\cdot b}{a\cdot d - c\cdot b} = \frac{R(s)d}{ad - cb} = \frac{(k_1 + m_1S^2)R(s)}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1(k_1)}$$

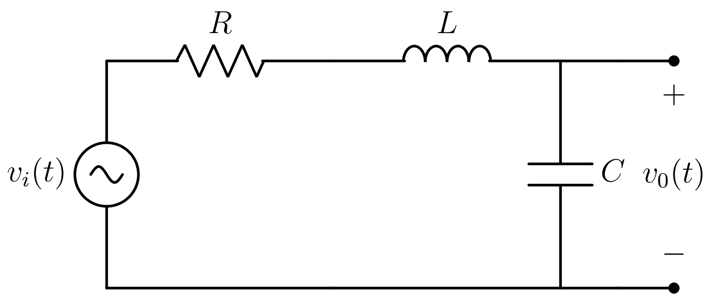
$$\boxed{FT_1(s) = \frac{X(s)}{R(s)} = \frac{k_1 + m_1S^2}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1^2}}$$

$$Y(s) = \frac{\begin{vmatrix} a & R(s) \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{a\cdot 0 - c\cdot R(s)}{a\cdot d - c\cdot b} = \frac{-cR(s)}{ad - cb} = \frac{k_1R(s)}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1(k_1)}$$

$$\boxed{FT_2(s) = \frac{Y(s)}{R(s)} = \frac{k_1}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1^2}}$$

Ejercicio # 2

Hallar: $\dfrac{V_o(s)}{V_i(s)}$



aAplicamos la ley de Kirchhoff de las mallas:

$$v_i(t) = v_R(t) + v_L(t) + v_C(t)$$

Donde:

$$v_C(t) = \frac{1}{C}\int i_C(t)\mathrm{d}t$$

$$v_L(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$v_R(t) = Ri(t)$$

Ecuación diferencial del circuito RLC:

$$v_i(t) = Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i_C(t)\mathrm{d}t$$

Aplicamos la transformada de Laplace:

$$\mathscr{L}\{v_i(t)\} = \mathscr{L}\{v_R(t) + v_L(t) + v_C(t)\}$$

$$\mathscr{L}\{v_i(t)\} = \mathscr{L}\left\{Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i_C(t)\mathrm{d}t\right\}$$

$$V_i(s) = RI(s) + LSI(s) + \frac{1}{CS}I(s)$$

$$V_i(s) = I(s)\left(R + LS + \frac{1}{CS}\right)$$

$$V_i(s) = I(s)\left(\frac{S^2LC + 1 + SRC}{SC}\right)$$

$$I(s) = V_i(s)\left(\frac{SC}{S^2LC + 1 + SRC}\right)$$

Aplicamos la Ley de tension de Kirchhoff:

$$-v_o(t) + v_c(t) = 0$$

$$v_o(t) = v_C(t) = \frac{1}{C}\int i_C(t)\mathrm{d}t$$

$$\mathscr{L}\{v_o(t)\} = \mathscr{L}\left\{\frac{1}{C}\int i_C(t)\mathrm{d}t\right\}$$

$$V_o(s) = \frac{1}{CS}I(s)$$

Luego reemplazamos:

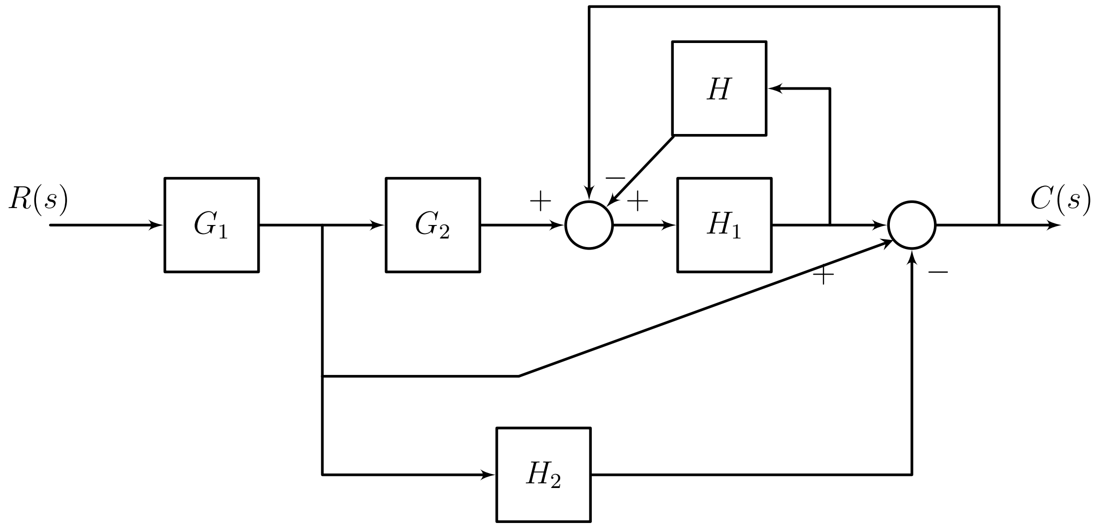
$$V_o(s) = \frac{1}{SC}\left[V_i(s)\left(\frac{SC}{S^2LC + 1 + SRC}\right)\right]$$

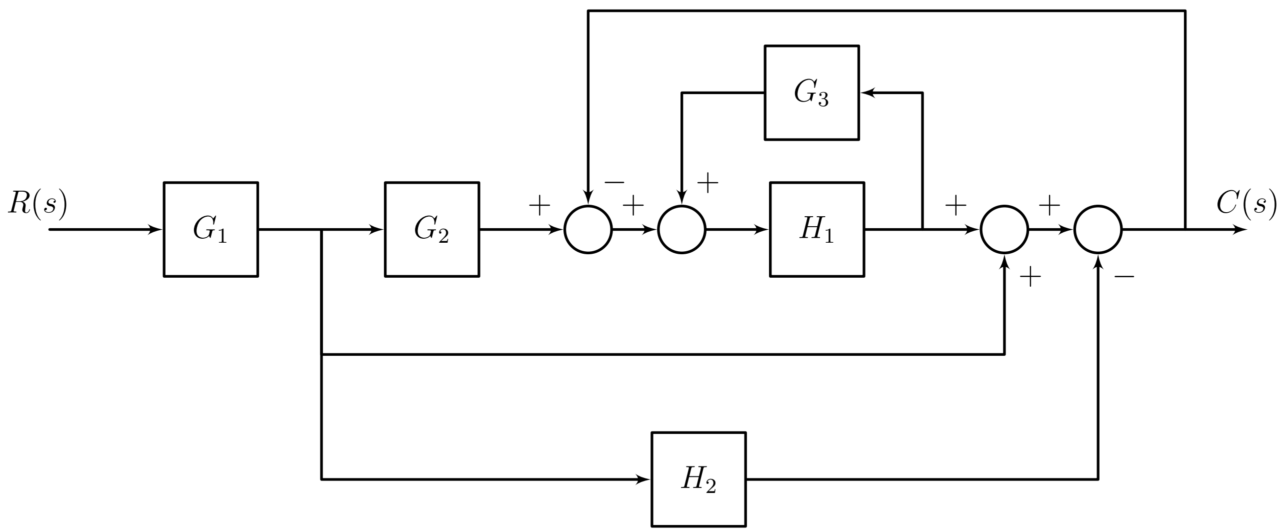
$$V_o(s) = V_i(s)\left(\frac{1}{S^2LC + 1 + SRC}\right)$$

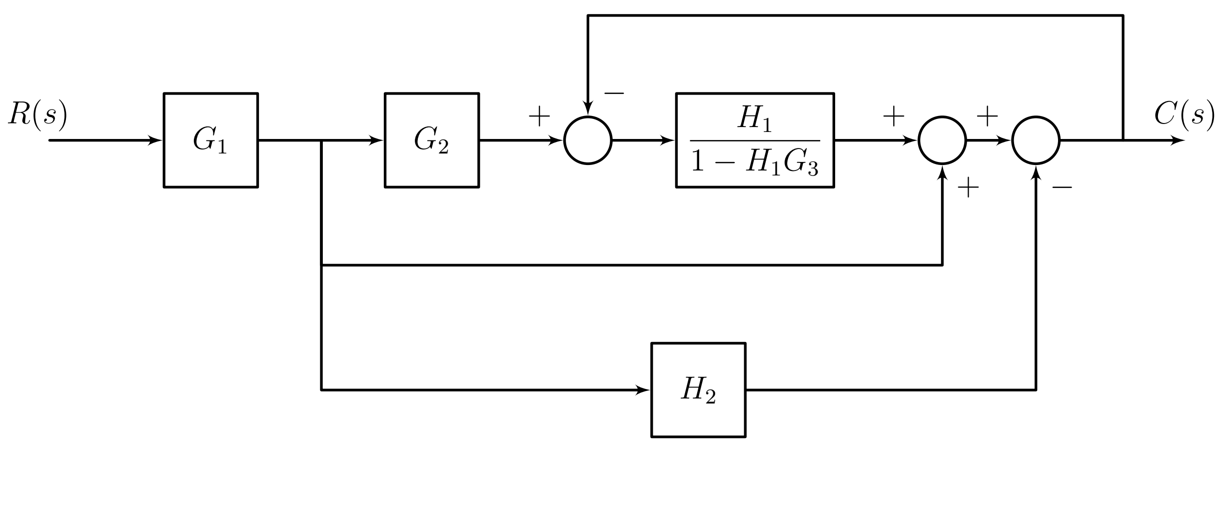
Solución:

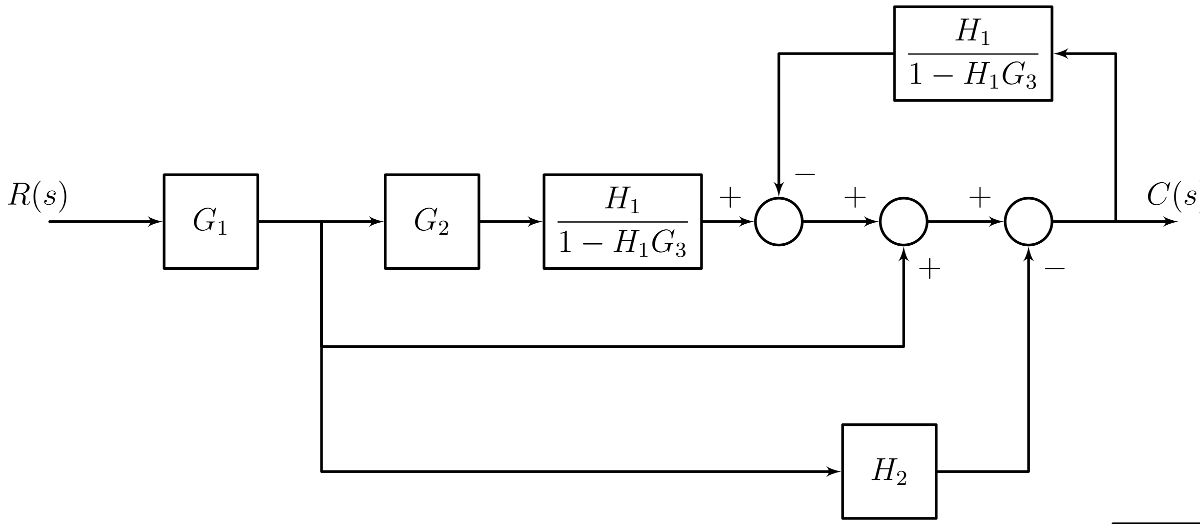
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{S^2LC + 1 + SRC}}$$

Ejercicio # 3

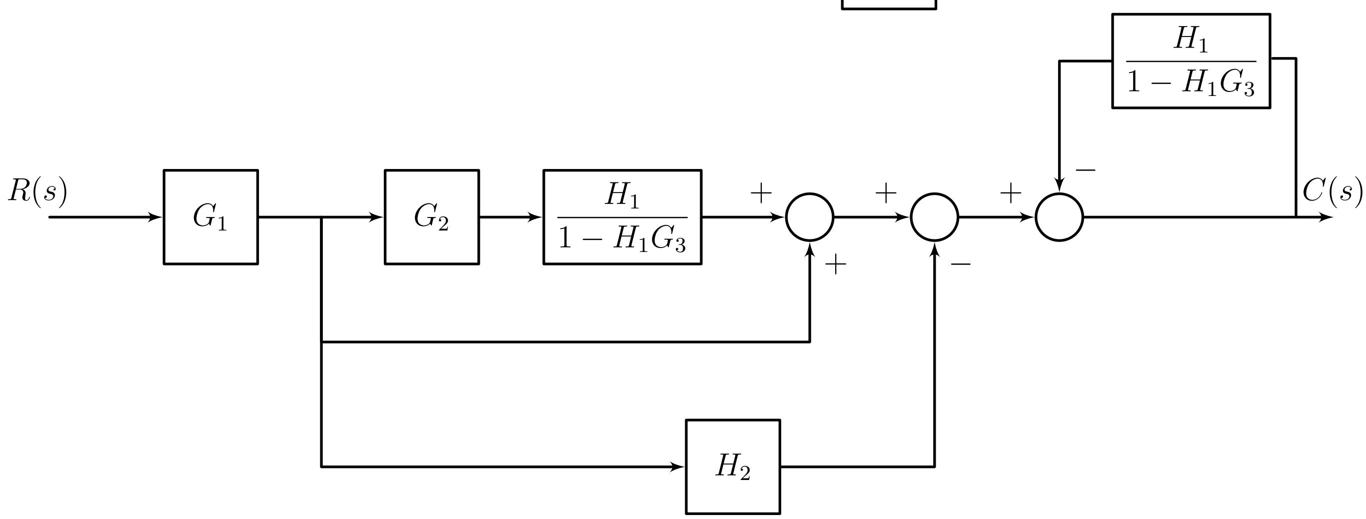




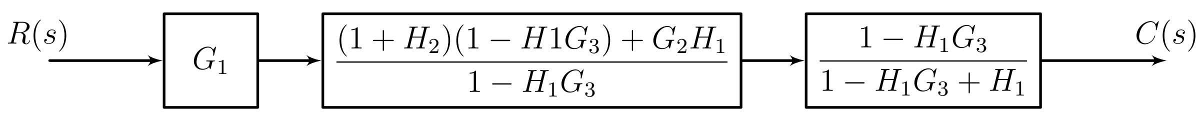




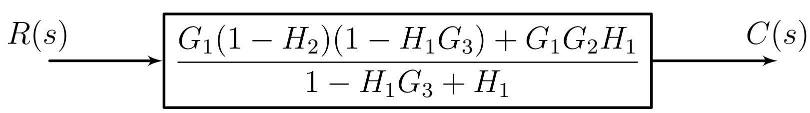
$R(s)$
G_1
G_2
$\frac{H_1}{1 - H_1G_3}$
$\frac{H_1}{1 - H_1G_3}$
H_2
$C(s)$



$R(s)$
G_1
G_2
$\frac{H_1}{1 - H_1G_3}$
$\frac{H_1}{1 - H_1G_3}$
H_2
$C(s)$



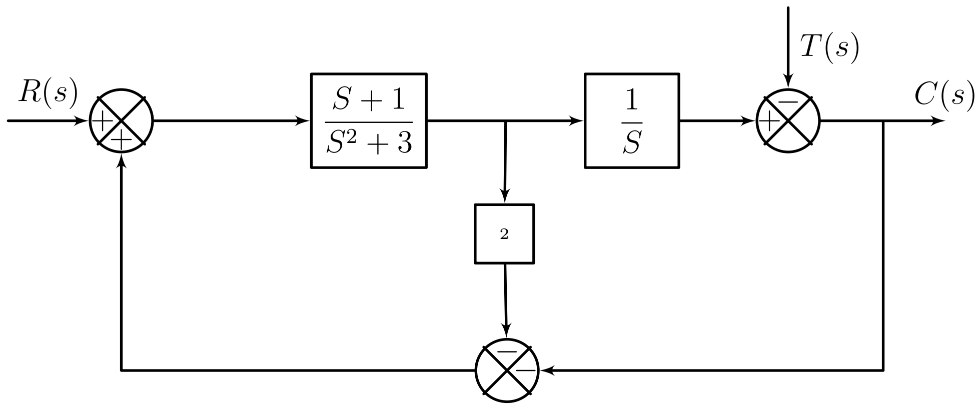
$R(s)$
G_1
$\frac{(1 + H_2)(1 - H1G_3) + G_2H_1}{1 - H_1G_3}$
$\frac{1 - H_1G_3}{1 - H_1G_3 + H_1}$
$C(s)$



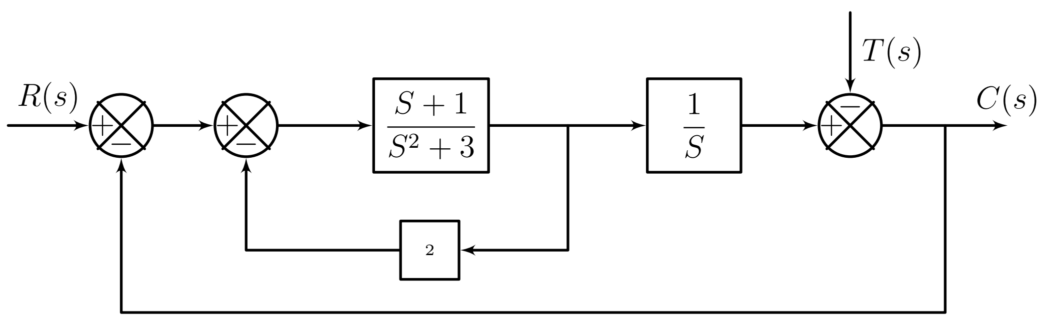
$R(s)$
$\frac{G_1(1 - H_2)(1 - H_1G_3) + G_1G_2H_1}{1 - H_1G_3 + H_1}$
$C(s)$

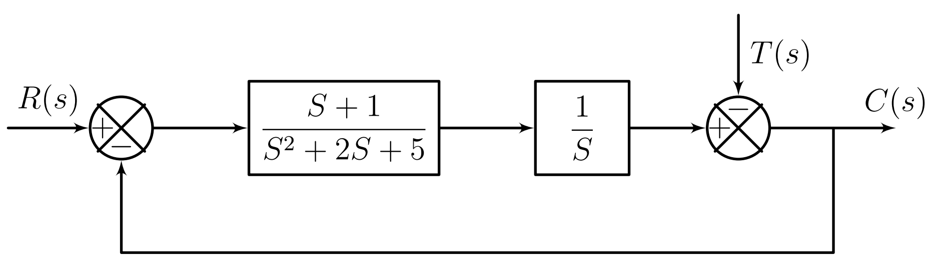
Ejercicio # 4

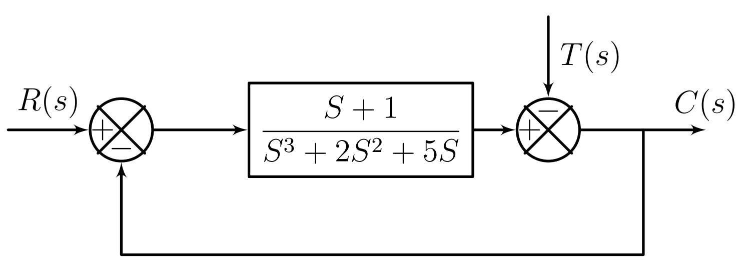
Hallar: $C(s) = ?$



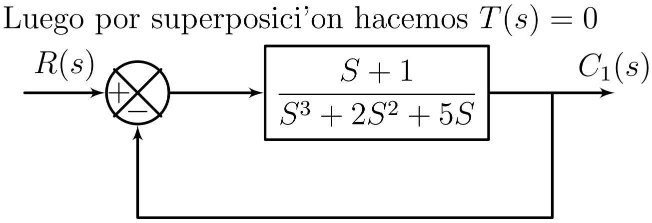
primero ordenamos un poco.

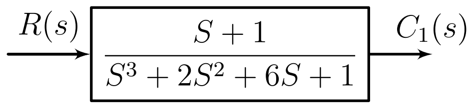




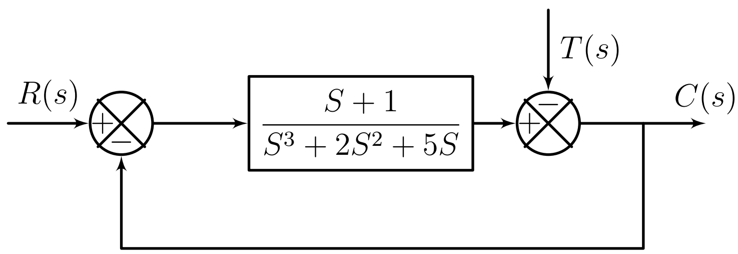


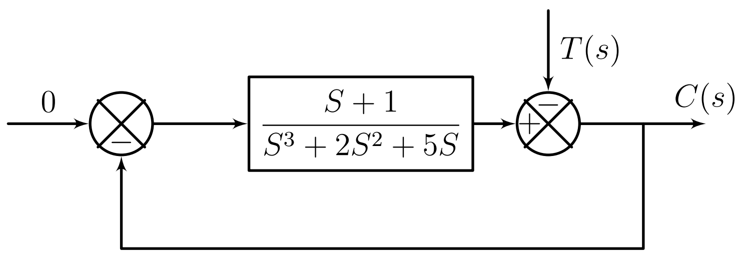
Luego por superposici'on hacemos $T(s) = 0$

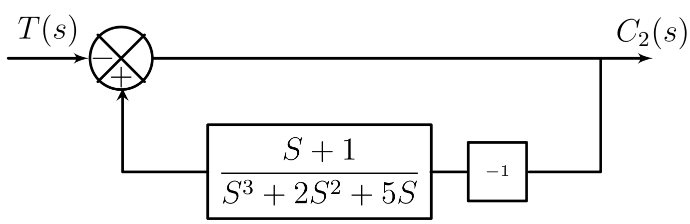


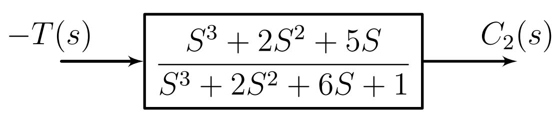


Luego por superposici'on hacemos $R(s) = 0$





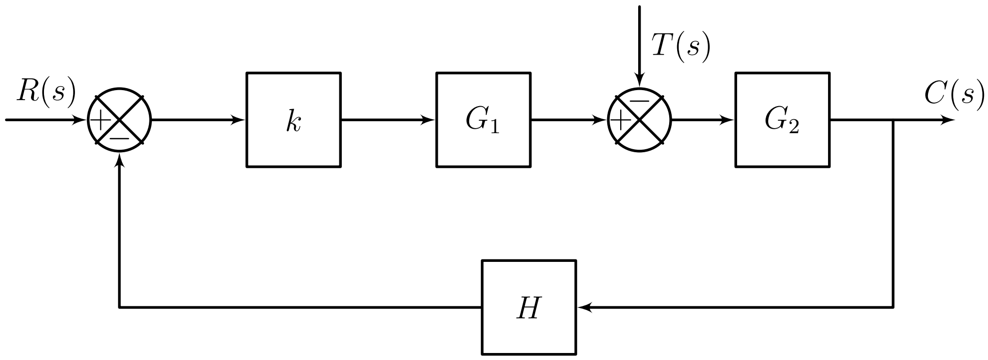




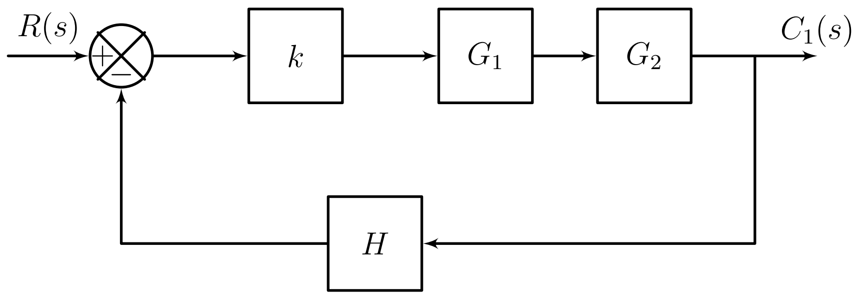
Superposici'on de las salidas $C(s) = C_1(s) + C_2(s)$

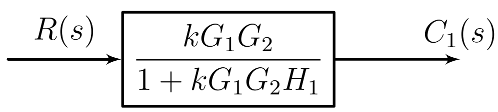
$$C(s) \quad = \quad R(s)\frac{S+1}{S^3+2S^2+6S+1} - T(s)\frac{S^3+2S^2+5S}{S^3+2S^2+6S+1}$$

Ejercicio # 5

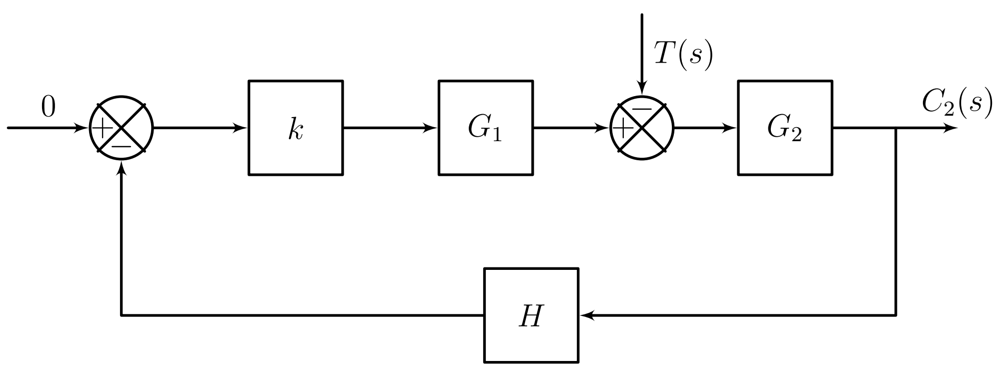


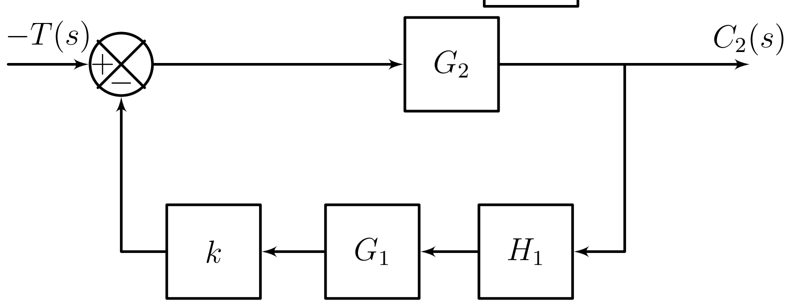
Por superposici'on hacemos $T(s) = 0$.





Por superposici'on hacemos $R(s) = 0$.





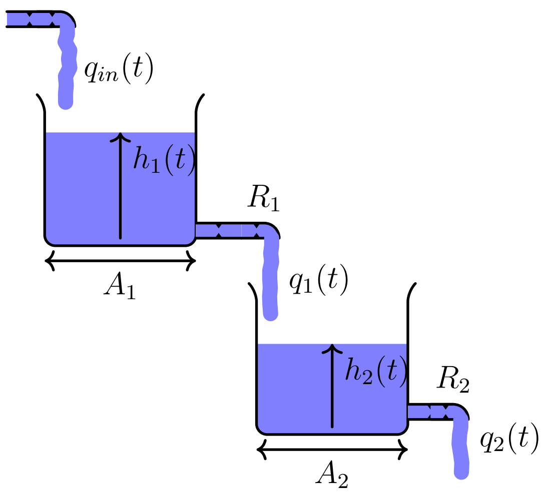
$$-T(s) \longrightarrow \boxed{\frac{G_2}{1+kH_1G_1G_2}} \longrightarrow C_2(s)$$

Superposici'on de las salidas $C(s) = C_1(s) + C_2(s)$

$$C(s) \quad = \quad R(s)\frac{kG_1G_2}{1+kH_1G_1G_2} - T(s)\frac{G_2}{1+kH_1G_1G_2}$$

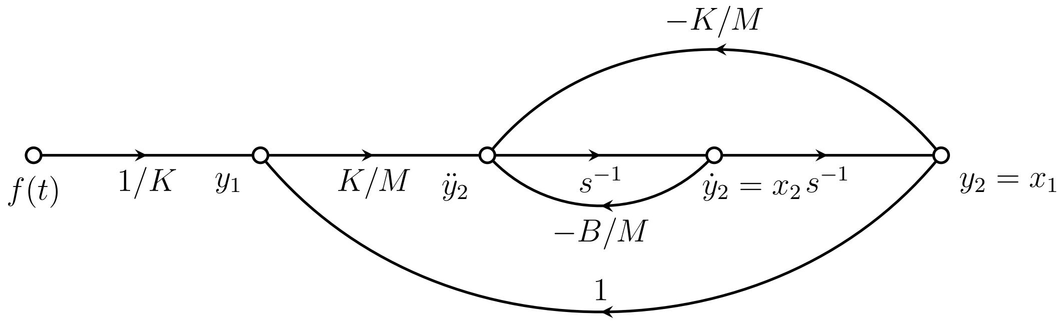
Ejercicio # 6

Hallar: $FT = \dfrac{H_2(s)}{Q_{in}(s)}$; $FT = \dfrac{Q_2(s)}{Q_{in}(s)}$



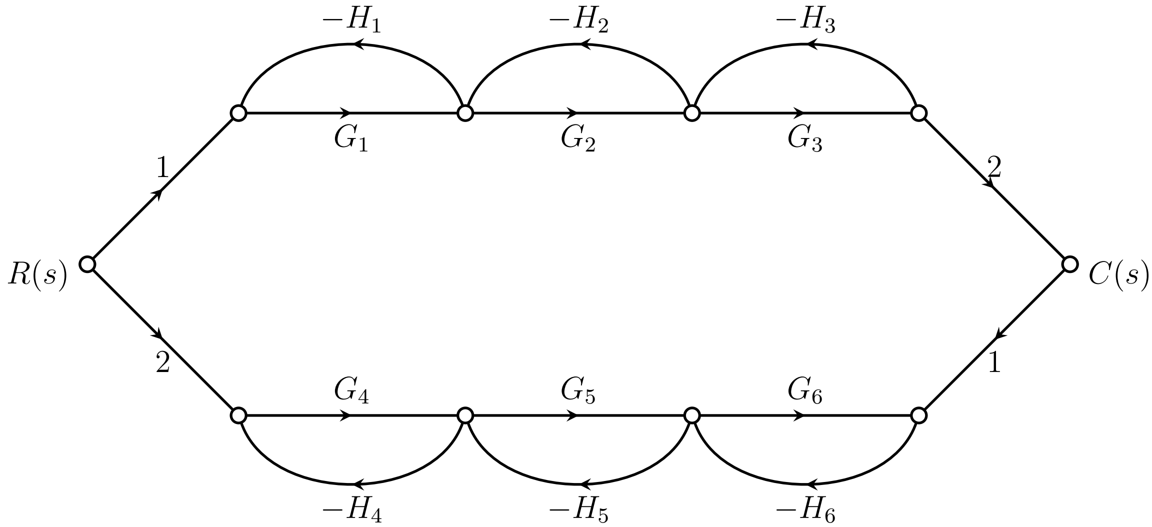
Ejercicio # 7

Hallar: $FT = \dfrac{C(s)}{R(s)}$



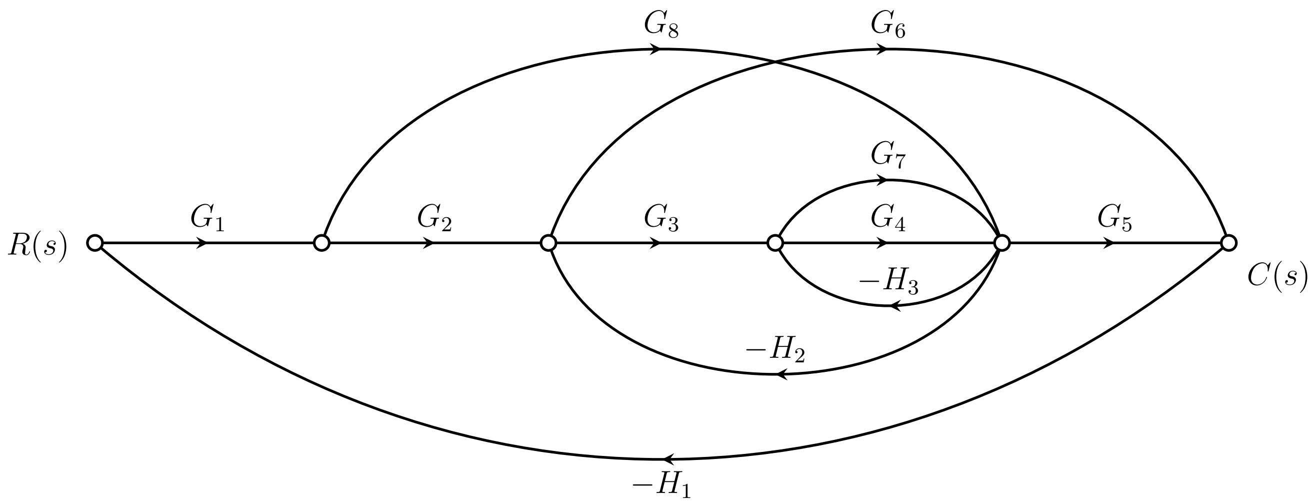
Ejercicio # 8

Hallar: $FT = \dfrac{C(s)}{R(s)}$



Ejercicio # 9

Hallar: $FT = \dfrac{C(s)}{R(s)}$



Ejercicio # 10

Hallar: $FT_1 = \dfrac{C_1(s)}{R_2(s)}; FT_2 = \dfrac{C_2(s)}{R_1(s)}; FT_3 = \dfrac{C_2(s)}{R_2(s)}; FT_4 = \dfrac{C_1(s)}{R_1(s)}$