

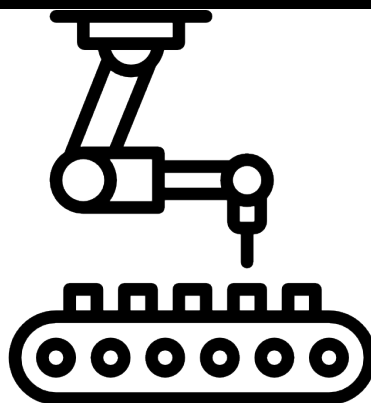
Ingeniería
Electromecánica



Universidad
Autónoma
Gabriel Rene
Moreno

UNIVERSIDAD AUTÓNOMA GABRIEL RENE MORENO

INGENIERIA ELECTROMECAÁNICA



Sistemas de control ELN-360

PRACTICO # 1

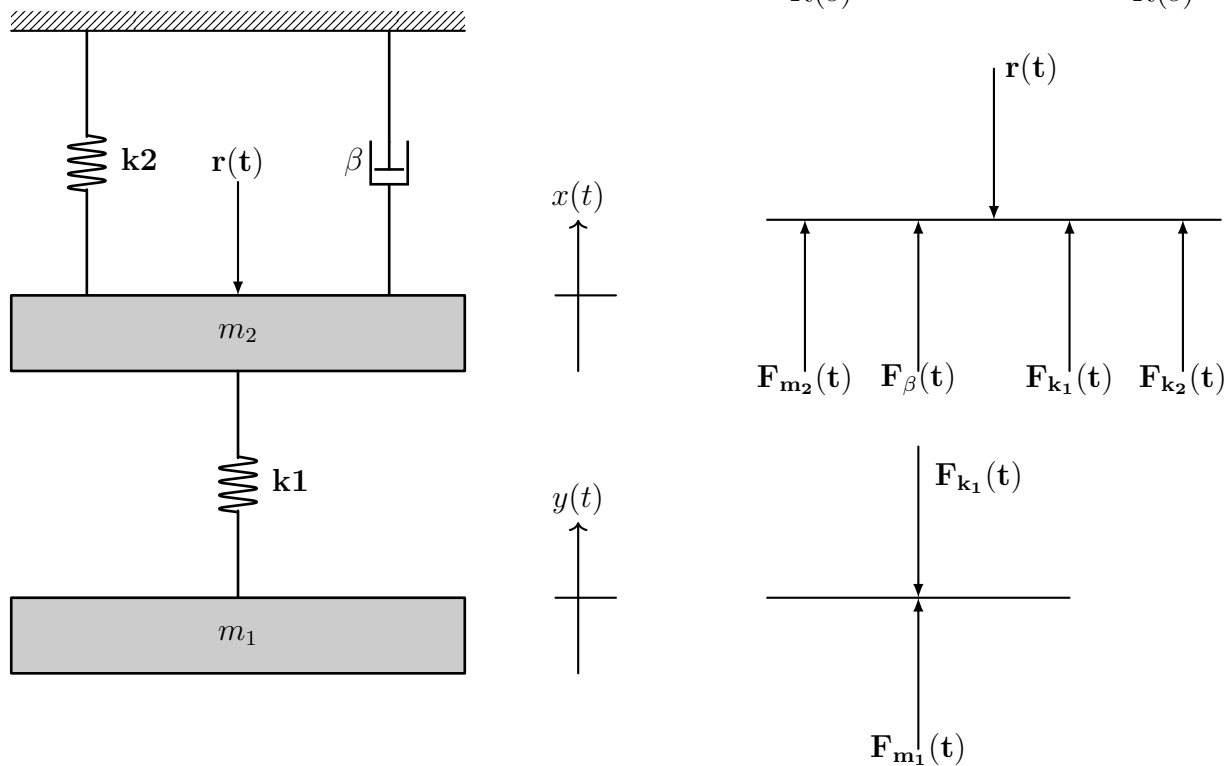
Estudiante:
Ariel Wilson Lima Soto

Docente:
Roy Pastor Piérola Bejarano

SANTA CRUZ DE LA SIERRA, 2022

Ejercicio # 1

Hallar las ecuaciones de función de transferencia: $FT_1(s) = \frac{X(s)}{R(s)}$ y $FT_2(s) = \frac{Y(s)}{R(s)}$.



Solucion:

1. Tenemos las siguientes ecuaciones:

$$F_{m_1} = m_1 \frac{d^2 y(t)}{dt^2} ; F_{m_2} = m_2 \frac{d^2 x(t)}{dt^2} ; F_{\beta} = \beta \frac{dx(t)}{dt} ; F_{k_1} = k_1 [x(t) - y(t)] ; F_{k_2} = k_2 x(t).$$

2. Hacemos la sumatoria de las fuerzas (DCL):

Para la masa 2.

$$F_{k_1} + F_{k_2} + F_{\beta} + F_{m_2} = r(t)$$

$$[x(t) - y(t)]k_1 + x(t)k_2 + \beta \frac{dx(t)}{dt} + m_2 \frac{d^2 x(t)}{dt^2} = r(t)$$

Para la masa 1.

$$-F_{k_1} + F_{m_1} = 0$$

$$-[x(t) - y(t)]k_1 + m_1 \frac{d^2 y(t)}{dt^2} = 0$$

$$[y(t) - x(t)]k_1 + m_1 \frac{d^2 y(t)}{dt^2} = 0$$

3. Aplicamos la transformada de Laplace: $y'(0) = y(0) = x'(0) = x(0) = 0$

Para la primera ecuación:

$$[x(t) - y(t)]k_1 + x(t)k_2 + \beta \frac{dx(t)}{dt} + m_2 \frac{d^2x(t)}{dt^2} = r(t)$$

$$\mathcal{L} \left\{ [x(t) - y(t)]k_1 + x(t)k_2 + \beta \frac{dx(t)}{dt} + m_2 \frac{d^2x(t)}{dt^2} \right\} = \mathcal{L}\{r(t)\}$$

$$[X(s) - Y(s)]k_1 + X(s)k_2 + S\beta X(s) + m_2 S^2 X(s) = R(s)$$

Para la segunda ecuación:

$$[y(t) - x(t)]k_1 + m_1 \frac{d^2y(t)}{dt^2} = 0$$

$$\mathcal{L} \left\{ [y(t) - x(t)]k_1 + m_1 \frac{d^2y(t)}{dt^2} \right\} = 0$$

$$[Y(s) - X(s)]k_1 + m_1 S^2 Y(s) = 0$$

4. Ordenamos las ecuaciones:

$$\overbrace{(k_1 + k_2 S\beta + m_2 S^2)}^a X(s) - \overbrace{k_1}^b Y(s) = R(s) \quad (1)$$

$$\overbrace{-k_1}^c X(s) + \overbrace{(k_1 + m_1 S^2)}^d Y(s) = 0 \quad (2)$$

5. Resolvemos las ecuaciones por el metodo de cramer:

$$X(s) = \frac{\begin{vmatrix} R(s) & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{R(s) \cdot d - 0 \cdot b}{a \cdot d - c \cdot b} = \frac{R(s)d}{ad - cb} = \frac{(k_1 + m_1 S^2)R(s)}{(k_1 + k_2 S\beta + m_2 S^2)(k_1 + m_1 S^2) - k_1(k_1)}$$

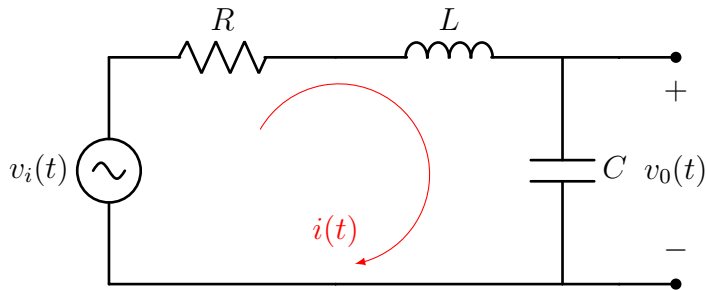
$$FT_1(s) = \frac{X(s)}{R(s)} = \frac{k_1 + m_1 S^2}{(k_1 + k_2 S\beta + m_2 S^2)(k_1 + m_1 S^2) - k_1^2}$$

$$Y(s) = \frac{\begin{vmatrix} a & R(s) \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{a \cdot 0 - c \cdot R(s)}{a \cdot d - c \cdot b} = \frac{-cR(s)}{ad - cb} = \frac{k_1 R(s)}{(k_1 + k_2 S\beta + m_2 S^2)(k_1 + m_1 S^2) - k_1(k_1)}$$

$$FT_2(s) = \frac{Y(s)}{R(s)} = \frac{k_1}{(k_1 + k_2 S\beta + m_2 S^2)(k_1 + m_1 S^2) - k_1^2}$$

Ejercicio # 2

Hallar: $\frac{V_o(s)}{V_i(s)}$



a) Aplicamos la ley de Kirchhoff de las mallas:

$$v_i(t) = v_R(t) + v_L(t) + v_C(t)$$

Donde:

$$v_C(t) = \frac{1}{C} \int i_C(t) dt$$

$$v_L(t) = L \frac{di(t)}{dt}$$

$$v_R(t) = Ri(t)$$

Ecuación diferencial del circuito RLC:

$$v_i(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i_C(t) dt$$

Aplicamos la transformada de Laplace:

$$\mathcal{L}\{v_i(t)\} = \mathcal{L}\{v_R(t) + v_L(t) + v_C(t)\}$$

$$\mathcal{L}\{v_i(t)\} = \mathcal{L}\left\{Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i_C(t) dt\right\}$$

$$V_i(s) = RI(s) + LSI(s) + \frac{1}{CS}I(s)$$

$$V_i(s) = I(s) \left(R + LS + \frac{1}{CS} \right)$$

$$V_i(s) = I(s) \left(\frac{S^2LC + 1 + SRC}{SC} \right)$$

$$I(s) = V_i(s) \left(\frac{SC}{S^2LC + 1 + SRC} \right)$$

Aplicamos la Ley de tension de Kirchhoff:

$$-v_o(t) + v_c(t) = 0$$

$$v_o(t) = v_c(t) = \frac{1}{C} \int i_C(t) dt$$

$$\mathcal{L}\{v_o(t)\} = \mathcal{L}\left\{\frac{1}{C} \int i_C(t) dt\right\}$$

$$V_o(s) = \frac{1}{CS} I(s)$$

Luego reemplazamos:

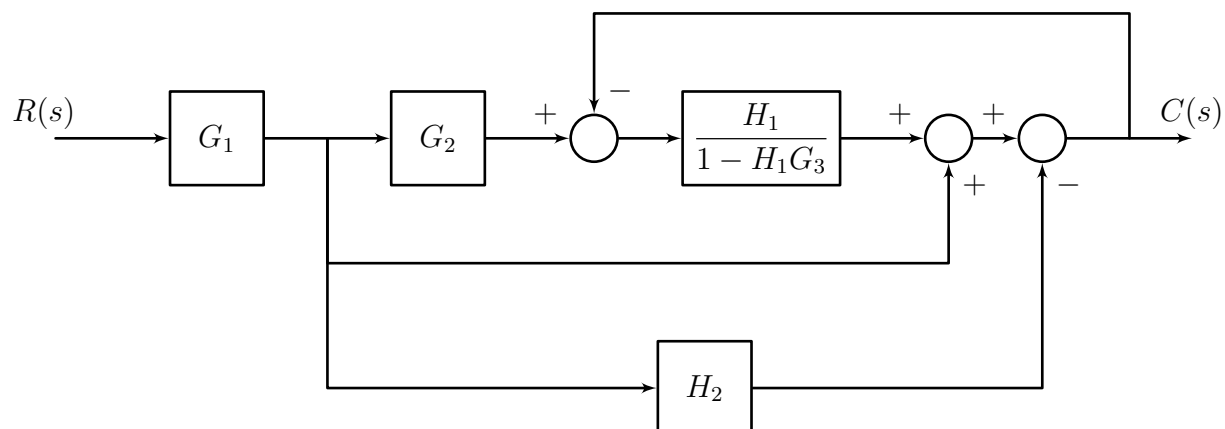
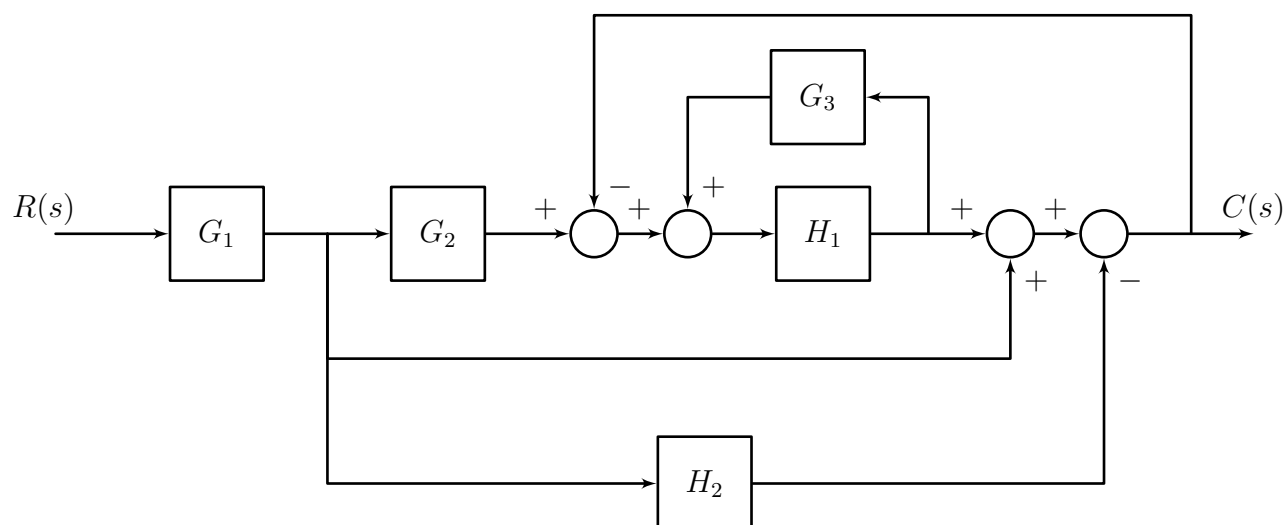
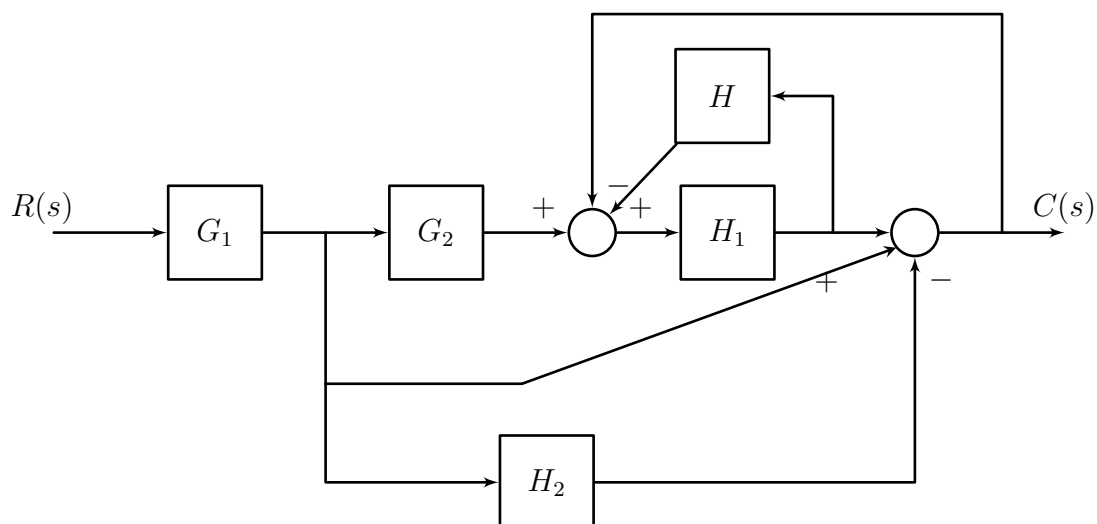
$$V_o(s) = \frac{1}{SC} \left[V_i(s) \left(\frac{SC}{S^2LC + 1 + SRC} \right) \right]$$

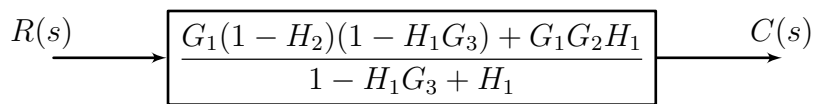
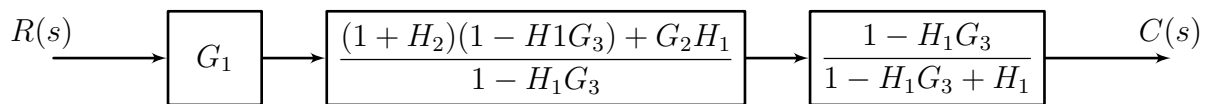
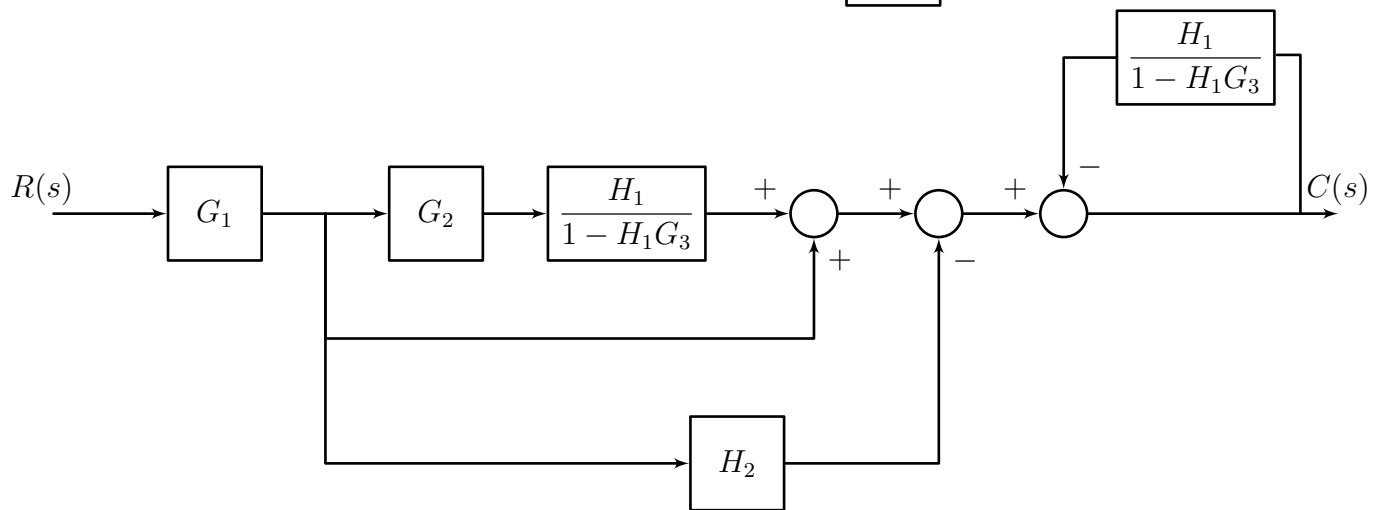
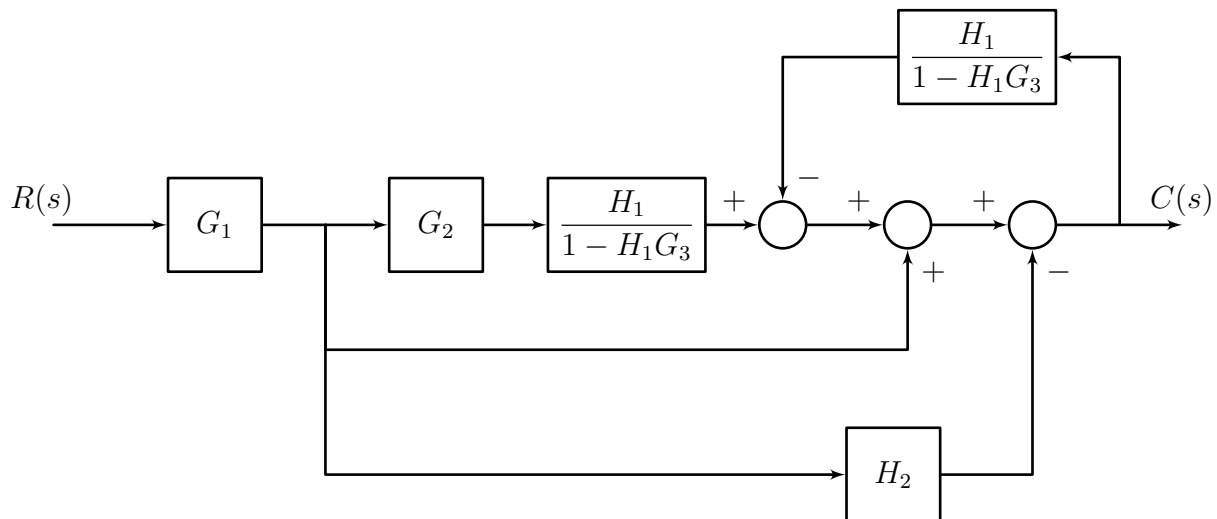
$$V_o(s) = V_i(s) \left(\frac{1}{S^2LC + 1 + SRC} \right)$$

Solución:

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{S^2LC + 1 + SRC}$$

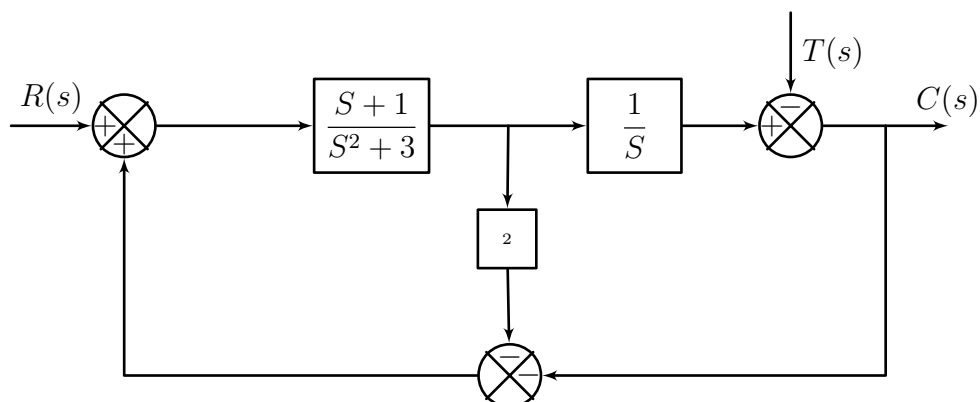
Ejercicio # 3



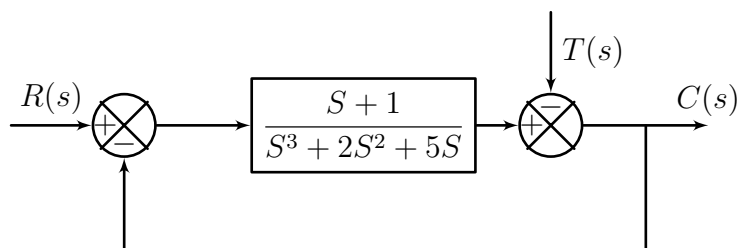
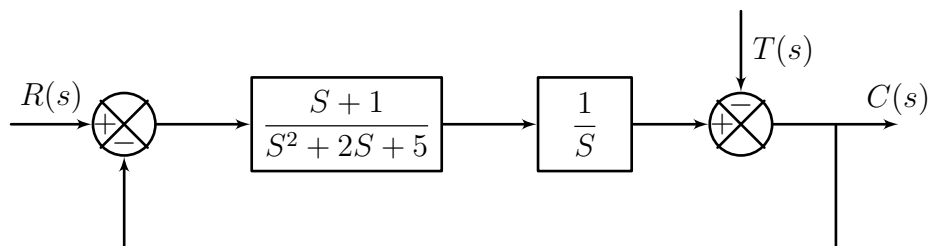
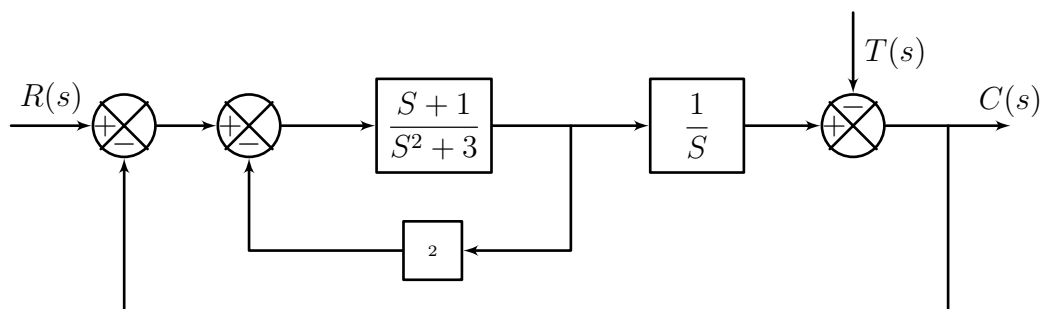


Ejercicio # 4

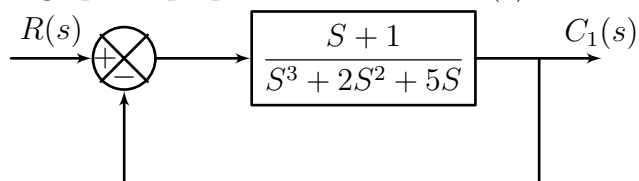
Hallar: $C(s) = ?$

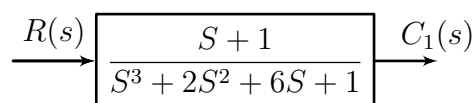


primero ordenamos un poco.

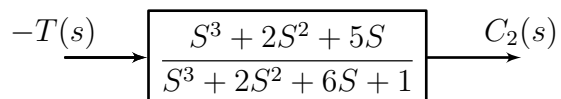
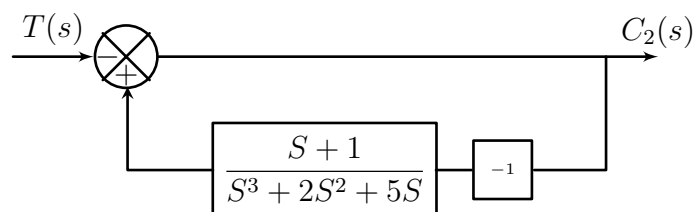
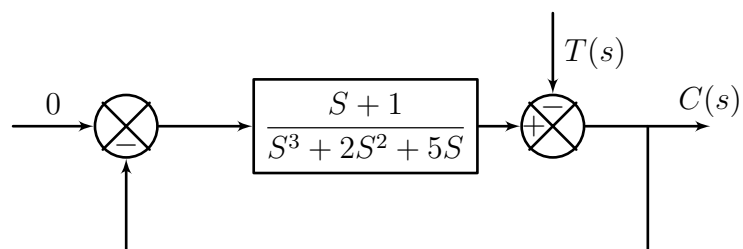
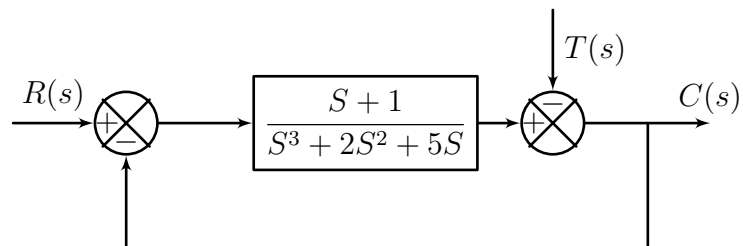


Luego por superposici'on hacemos $T(s) = 0$





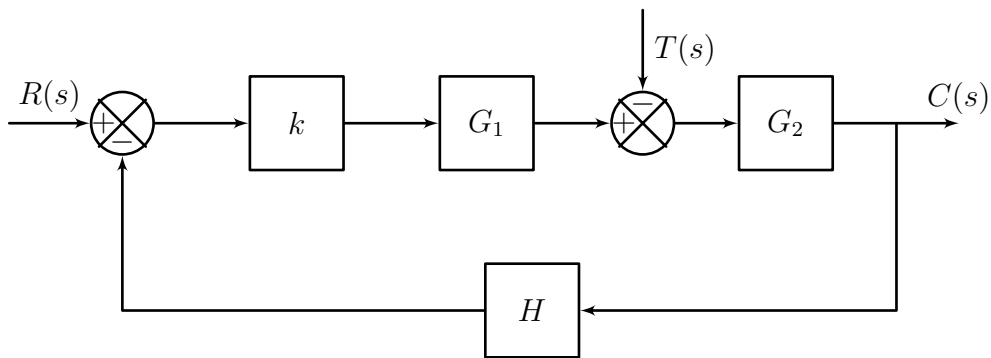
Luego por superposici'on hacemos $R(s) = 0$



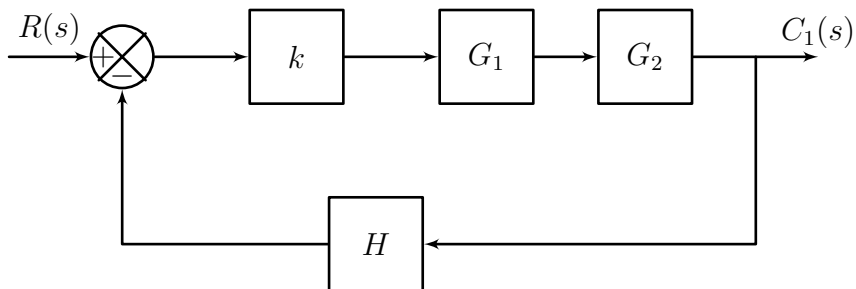
Superposici'on de las salidas $C(s) = C_1(s) + C_2(s)$

$$C(s) = R(s) \frac{S+1}{S^3 + 2S^2 + 6S + 1} - T(s) \frac{S^3 + 2S^2 + 5S}{S^3 + 2S^2 + 6S + 1}$$

Ejercicio # 5

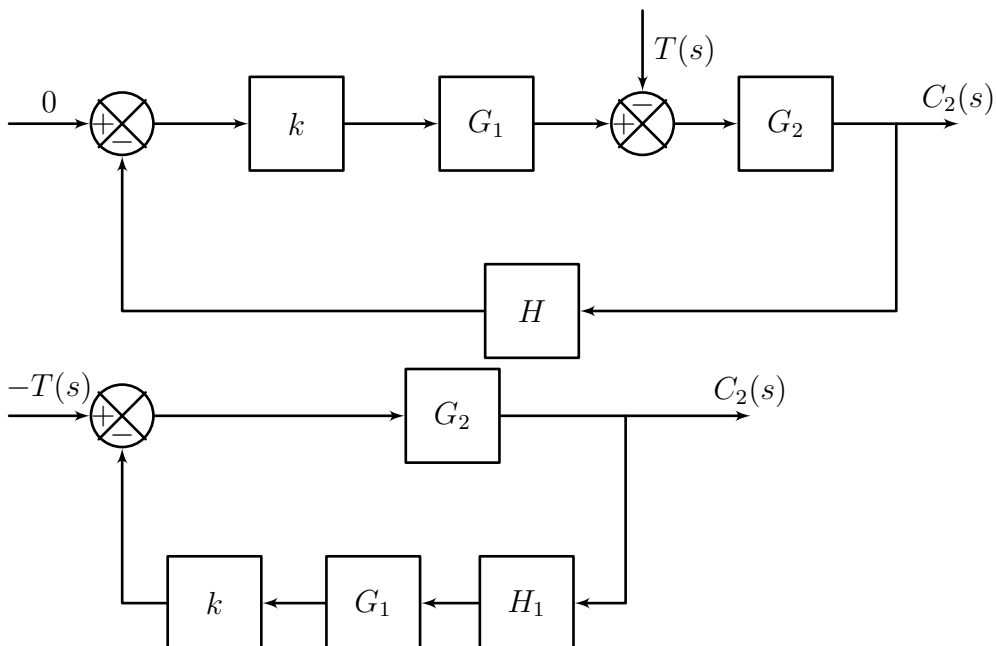


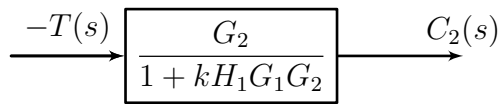
Por superposici'on hacemos $T(s) = 0$.



$$R(s) \rightarrow \boxed{\frac{kG_1G_2}{1 + kG_1G_2H_1}} \rightarrow C_1(s)$$

Por superposici'on hacemos $R(s) = 0$.



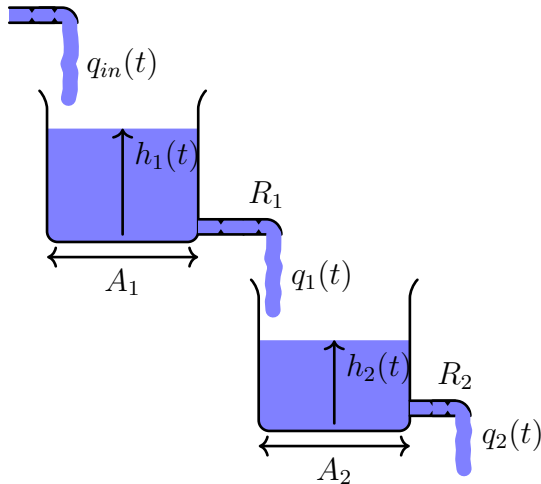


Superposici' on de las salidas $C(s) = C_1(s) + C_2(s)$

$$C(s) = R(s) \frac{kG_1G_2}{1 + kH_1G_1G_2} - T(s) \frac{G_2}{1 + kH_1G_1G_2}$$

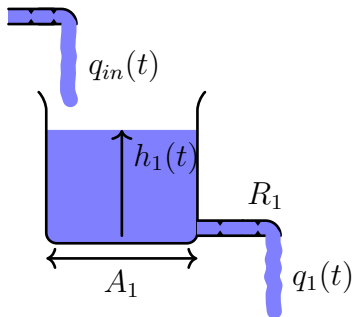
Ejercicio # 6

Hallar: $FT = \frac{H_2(s)}{Q_{in}(s)}$; $FT = \frac{Q_2(s)}{Q_{in}(s)}$



Solución:

1. Primero analizamos el primer tanque de agua.



En el tanque 1 tenemos:

$$V_1(t) = A_1 h_1(t)$$

$$\frac{dV_1(t)}{dt} = A_1 \frac{dh_1(t)}{dt}$$

$$q_{in}(t) - q_1(t) = A_1 \frac{dh_1(t)}{dt}$$

Luego tenemos que el caudal de salida del primer tanque $q_1(t)$ es inversamente proporcional a la resistencia de la valvula R_1 y directamente proporcional a $h_1(t)$.

$$q_1(t) = \frac{1}{R_1} h_1(t)$$

Entonces tenemos el siguiente sistema de ecuaciones:

$$q_{in}(t) - q_1(t) = A_1 \frac{dh_1(t)}{dt} \quad (1)$$

$$q_1(t) = \frac{1}{R_1} h_1(t) \quad (2)$$

Luego aplicamos la transformada de Laplace:

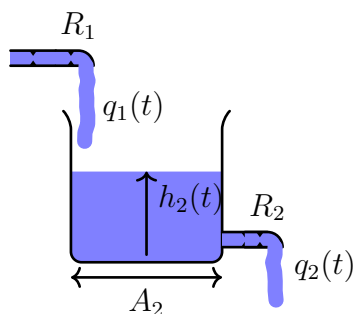
$$\mathcal{L}\{q_{in}(t) - q_1(t)\} = \mathcal{L}\left\{A_1 \frac{dh_1(t)}{dt}\right\}$$

$$\mathcal{L}\{q_1(t)\} = \mathcal{L}\left\{\frac{1}{R_1} h_1(t)\right\}$$

$$Q_{in}(s) - Q_1(s) = A_1 s H_1(s) \quad (1)$$

$$Q_1(s) = \frac{1}{R_1} H_1(s) \quad (2)$$

2. Ahora analizamos el segundo tanque.



En el tanque 2 tenemos:

$$V_2(t) = A_2 h_2(t)$$

$$\frac{dV_2(t)}{dt} = A_2 \frac{dh_2(t)}{dt}$$

$$q_1(t) - q_2(t) = A_2 \frac{dh_2(t)}{dt}$$

Luego tenemos que el caudal de salida del segundo tanque $q_2(t)$ es inversamente proporcional a la resistencia de la valvula R_2 y directamente proporcional a $h_2(t)$.

$$q_2(t) = \frac{1}{R_2} h_2(t)$$

Entonces tenemos el siguiente sistema de ecuaciones:

$$q_1(t) - q_2(t) = A_2 \frac{dh_2(t)}{dt}$$

$$q_2(t) = \frac{1}{R_2} h_2(t)$$

Luego aplicamos la transformada de Laplace:

$$\mathcal{L}\{q_1(t) - q_2(t)\} = \mathcal{L}\left\{A_2 \frac{dh_2(t)}{dt}\right\}$$

$$\mathcal{L}\{q_2(t)\} = \mathcal{L}\left\{\frac{1}{R_2} h_2(t)\right\}$$

$$Q_1(s) - Q_2(s) = A_2 S H_2(s) \quad (3)$$

$$Q_2(s) = \frac{1}{R_2} H_2(s) \quad (4)$$

3. Luego hacemos (2) en (1) y (4) en (3)

$$\text{De (1)} \quad Q_{in}(s) = A_1 S H_1(s) + Q_1(s)$$

reemplazar (2) en (1)

$$Q_{in}(s) = A_1 S Q_1(s) R_1 + Q_1(s)$$

$$Q_{in}(s) = Q_1(s) [A_1 S R_1 + 1] \rightarrow (a)$$

$$\text{De (3)} \quad Q_1(s) = Q_2(s) + A_2 S H_2(s)$$

reemplazar (4) en (3)

$$Q_1(s) = \frac{1}{R_2} H_2 + A_2 S H_2$$

$$Q_1(s) = H_2(s) \left[\frac{1}{R_2} + A_2 S \right] \rightarrow (b)$$

reemplazar (b) en (a)

$$Q_{in}(s) = H_2(s) \left[\frac{1}{R_2} + A_2 S \right] (A_1 S R_1 + 1)$$

$$Q_{in}(s) = H_2(s) \left(A_1 A_2 S^2 R_1 + \frac{A_1 R_1 S}{R_2} + A_2 S + \frac{1}{R_2} \right)$$

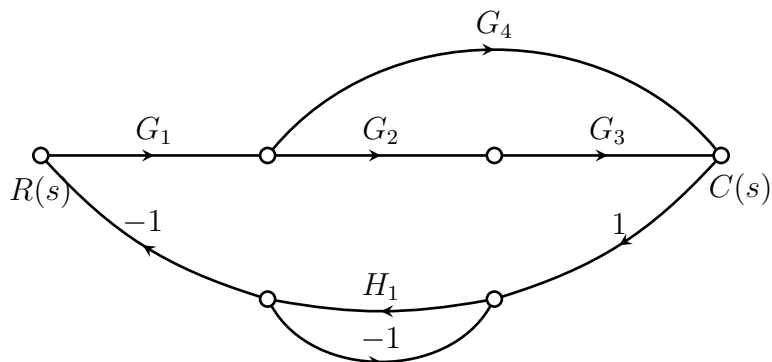
$$FT = \frac{H_2(s)}{Q_{in}(s)} = \frac{R_2}{A_1 A_2 R_1 R_2 S^2 + (A_1 R_1 + A_2 R_2) S + 1}$$

4. Ahora para hallar $\frac{Q_2(s)}{Q_{in}(s)}$ tenemos que: $H_2 = Q_2(s) R_2$ y reemplazamos en la anterior ecuación.

$$FT = \frac{Q_2(s)}{Q_{in}(s)} = \frac{1}{A_1 A_2 R_1 R_2 S^2 + (A_1 R_1 + A_2 R_2) S + 1}$$

Ejercicio # 7

Hallar: $FT = \frac{C(s)}{R(s)}$



Solucion:

1. Número de lazos del sistema:

$$\begin{aligned} L_1 &= G_1 G_2 G_3 (1) H_1 (-1) = -G_1 G_2 G_3 H_1 \\ L_2 &= G_1 G_4 (1) H_1 (-1) = -G_1 G_4 H_1 \\ L_3 &= H_1 (-1) = -H_1 \end{aligned}$$

2. Número de trayectorias del sistema:

$$\begin{aligned} T_1 &= G_1 G_2 G_3 \quad ; \quad \Delta_1 = 1 - L_3 + [0] = 1 + H_1 \\ T_2 &= G_1 G_4 \quad ; \quad \Delta_2 = 1 - L_3 + [0] = 1 + H_1 \end{aligned}$$

3. Ecuación característica

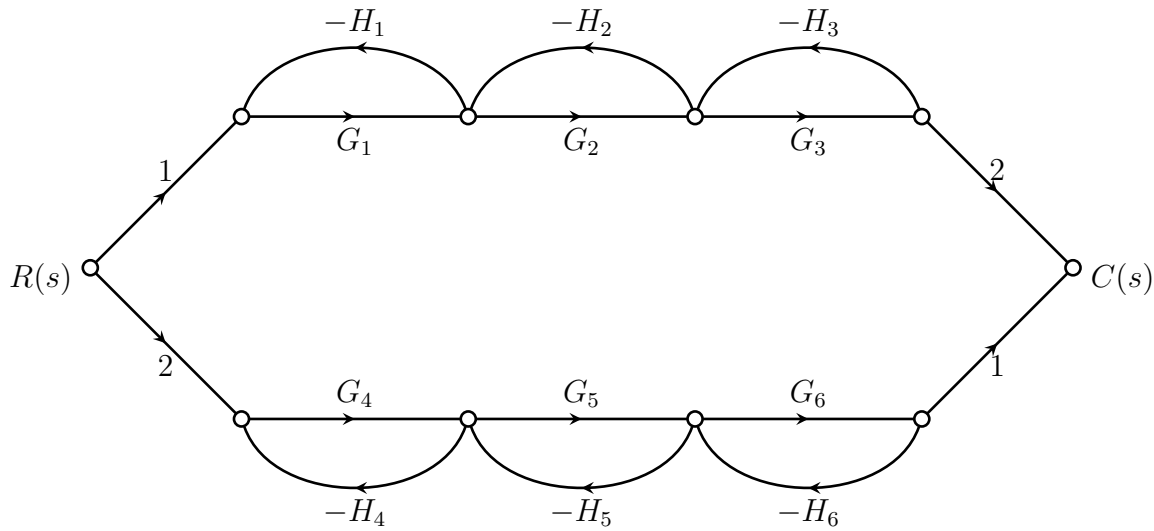
$$\begin{aligned} \Delta &= 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k + \dots + (-1)^m \sum \dots + \dots \\ \Delta &= 1 - [L_1 + L_2 + L_3 + [0]] \\ \Delta &= 1 + G_1 G_2 G_3 H_1 + G_1 G_4 H_1 + H_1 \end{aligned}$$

4. Funcion de transferencia:

$$\begin{aligned} FT &= \frac{C(s)}{R(s)} = \frac{T_1 \Delta_1 + T_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 (1 + H_1) + G_1 G_4 (1 + H_1)}{1 + G_1 G_2 G_3 H_1 + G_1 G_4 H_1 + H_1} \\ FT &= \frac{G_1 G_2 G_3 (1 + H_1) + G_1 G_4 (1 + H_1)}{1 + G_1 G_2 G_3 H_1 + G_1 G_4 H_1 + H_1} \end{aligned}$$

Ejercicio # 8

Hallar: $FT = \frac{C(s)}{R(s)}$



Solucion:

1. Número de lazos del sistema:

$$L_1 = -G_1 H_1$$

$$L_2 = -G_2 H_2$$

$$L_3 = -G_3 H_3$$

$$L_4 = -G_4 H_4$$

$$L_5 = -G_5 H_5$$

$$L_6 = -G_6 H_6$$

2. Número de trayectorias del sistema:

$$T_1 = 2G_1 G_2 G_3$$

$$T_2 = 2G_4 G_5 G_6$$

$$\Delta_1 = 1 - [L_4 + L_5 + L_6] + [L_4 L_6] = 1 + G_4 H_4 + G_5 H_5 + G_6 H_6 + (G_4 H_4 G_6 H_6)$$

$$\Delta_2 = 1 - [L_1 + L_2 + L_3] + [L_1 L_3] = 1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + (G_1 H_1 G_3 H_3)$$

3. Ecuación característica

$$\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k + \dots$$

$$\begin{aligned} \Delta = & 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6] + \\ & + [L_1 L_3 + L_1 L_4 + L_1 L_5 + L_1 L_6 + L_2 L_4 + L_2 L_5 + L_2 L_6 + L_3 L_4 + L_3 L_5 + L_3 L_6 + L_4 L_6] \\ & - [L_1 L_3 L_4 + L_1 L_3 L_5 + L_1 L_3 L_6 + L_4 L_6 L_1 + L_4 L_6 L_2 + L_4 L_6 L_3] + [L_1 L_3 L_4 L_6] \end{aligned}$$

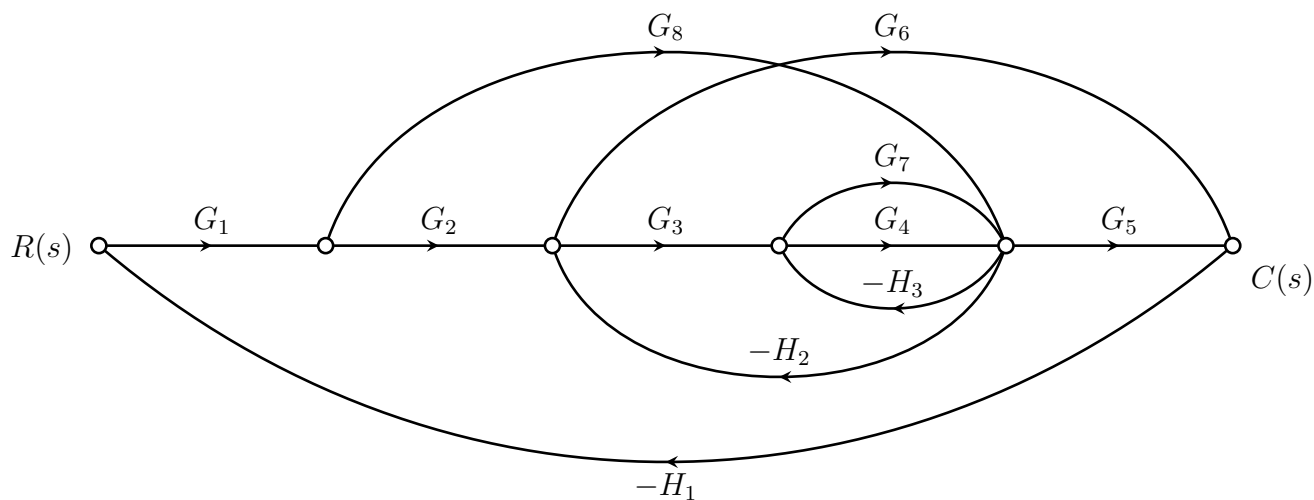
$$\begin{aligned}\Delta = & 1 - [G_1H_1 + G_2H_2 + G_3H_3 + G_4H_4 + G_5H_5 + G_6H_6] + \\ & + [G_1H_1(G_3H_3 + G_4H_4 + G_5H_5 + G_6H_6) + G_2H_2(G_4H_4 + G_5H_5 + G_6H_6) + G_3H_3(G_4H_4 + G_5H_5 + G_6H_6) + G_4H_4G_6H_6] \\ & + [G_1H_1G_3G_3(G_4H_4 + G_5H_5 + G_6H_6) + G_4H_4G_6H_6(G_1H_1 + G_2H_2 + G_3H_3)] + [G_1H_1G_3H_3G_4H_4G_6H_6]\end{aligned}$$

4. **Funcion de transferencia:**

$$\begin{aligned}FT &= \frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} \\ FT &= \frac{2G_1G_2G_3(1 + G_4H_4 + G_5H_5 + G_6H_6 + G_4H_4G_6H_6)}{\Delta} \\ &+ \frac{2G_4G_5G_6(1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1H_1G_3H_3)}{\Delta}\end{aligned}$$

Ejercicio # 9

Hallar: $FT = \frac{C(s)}{R(s)}$



Solucion:

1. Número de lazos del sistema:

$$L_1 = -G_1G_2G_3G_4G_5H_1$$

$$L_2 = -G_1G_2G_3G_7G_5H_1$$

$$L_3 = -G_1G_8G_5H_1$$

$$L_4 = -G_1G_2G_6H_1$$

$$L_5 = -G_3G_4H_2$$

$$L_6 = -G_3G_7H_2$$

$$L_7 = -G_4H_3$$

$$L_8 = -G_7H_3$$

2. Número de trayectorias del sistema:

$$T_1 = G_1G_2G_3G_4G_5$$

$$T_2 = G_1G_2G_3G_7G_5$$

$$T_3 = G_1G_8G_5$$

$$T_4 = G_1G_2G_6$$

$$\Delta_1 = 1 - [0] + [0] = 1$$

$$\Delta_2 = 1 - [0] + [0] = 1$$

$$\Delta_3 = 1 - [0] + [0] = 1$$

$$\Delta_4 = 1 - [L_7 + L_8] + [0] = 1 + G_4H_3 + G_7H_3$$

3. Ecuación característica

$$\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k + \cdots +$$

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8] + [L_4 L_7 + L_4 L_8]$$

$$\Delta = 1 + [G_1 G_2 G_3 G_4 G_5 H_1 + G_1 G_2 G_3 G_7 G_5 H_1 + G_1 G_8 G_5 H_1 + G_1 G_2 G_6 H_1 + G_3 G_4 H_2 + G_3 G_7 H_2 + G_4 H_3 + G_7 H_3] \\ + [G_1 G_2 G_6 H_1 G_4 H_3 + G_1 G_2 G_6 H_1 G_7 H_3]$$

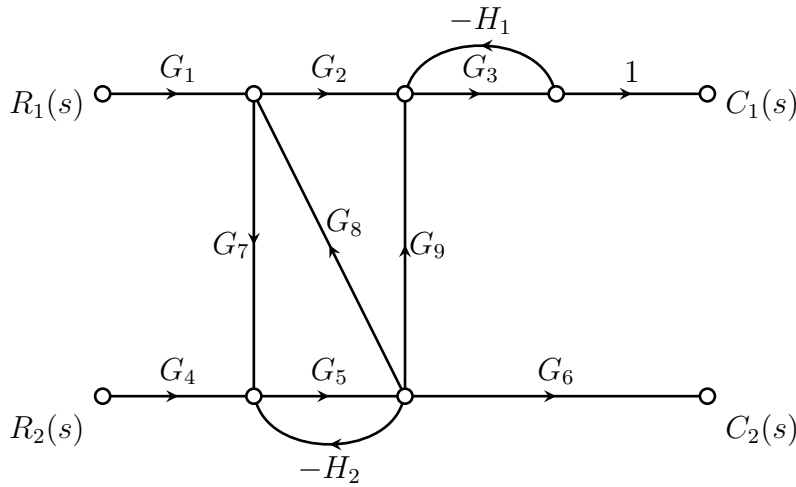
4. Funcion de transferencia:

$$FT = \frac{C(s)}{R(s)} = \frac{T_1 \Delta_1 + T_2 \Delta_2 + T_3 \Delta_3 + T_4 \Delta_4}{\Delta}$$

$$FT = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_2 G_3 G_7 G_5 + G_1 G_8 G_5 + G_1 G_2 G_6 (1 + G_4 H_3 + G_7 H_3)}{\Delta}$$

Ejercicio # 10

Hallar: $FT_1 = \frac{C_1(s)}{R_2(s)}$; $FT_2 = \frac{C_2(s)}{R_1(s)}$; $FT_3 = \frac{C_2(s)}{R_2(s)}$; $FT_4 = \frac{C_1(s)}{R_1(s)}$



Solución para hallar $FT_1 = \frac{C_1(s)}{R_2(s)}$

1. Lazos del sistema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sistema ($R_2(s)$ a $C_1(s)$):

$$\begin{aligned} T_1 &= G_4G_5G_9G_3 \\ T_2 &= G_4G_5G_8G_2G_3 \\ \Delta_1 &= 1 - [0] = 1 \\ \Delta_2 &= 1 - [0] = 1 \end{aligned}$$

3. Ecuación característica:

$$\begin{aligned} \Delta &= 1 - [L_1 + L_2 + L_3] + [L_1L_3 + L_2L_3] \\ \Delta &= 1 - [-G_5H_2 - G_3H_1 + G_5G_8G_7] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7] \end{aligned}$$

4. Función de transferencia:

$$FT_1 = \frac{C_1(s)}{R_2(s)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

$$FT_1 = \frac{C_1(s)}{R_2(s)} = \frac{G_4G_5G_9G_3 + G_4G_5G_8G_2G_3}{1 - [-G_5H_2 - G_3H_1 + G_5G_8G_7] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]}$$

Solución para hallar $FT_2 = \frac{C_2(s)}{R_1(s)}$

1. Lazos del sistema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sistema ($R_1(s)$ a $C_2(s)$):

$$\begin{aligned} T_1 &= G_1G_7G_5G_6 \\ \Delta_1 &= 1 - [L_2] = 1 + G_3H_1 \end{aligned}$$

3. Ecuación característica:

$$\begin{aligned} \Delta &= 1 - [L_1 + L_2 + L_3] + [L_1L_3 + L_2L_3] \\ \Delta &= 1 - [-G_5H_2 - G_3H_1 + G_5G_8G_7] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7] \end{aligned}$$

4. Función de transferencia:

$$\begin{aligned} FT_2 &= \frac{C_2(s)}{R_1(s)} = \frac{T_1\Delta_1}{\Delta} \\ FT_2 &= \frac{C_2(s)}{R_1(s)} = \frac{G_1G_7G_5G_6(1 + G_3H_1)}{1 - [G_5G_8G_7 - G_5H_2 - G_3H_1] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]} \end{aligned}$$

Solución para hallar $FT_3 = \frac{C_2(s)}{R_2(s)}$

1. Lazos del sistema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sistema ($R_2(s)$ a $C_2(s)$):

$$\begin{aligned} T_1 &= G_4G_5G_6 \\ \Delta_1 &= 1 - [L_2] = 1 + G_3H_1 \end{aligned}$$

3. Ecuación característica:

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1L_3 + L_2L_3]$$

$$\Delta = 1 - [-G_5H_2 - G_3H_1 + G_5G_8G_7] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]$$

4. Función de transferencia:

$$FT_3 = \frac{C_2(s)}{R_2(s)} = \frac{T_1\Delta_1}{\Delta}$$

$$FT_3 = \frac{C_2(s)}{R_2(s)} = \frac{G_4G_5G_6(1 + G_3H_1)}{1 - [G_5G_8G_7 - G_5H_2 - G_3H_1] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]}$$

Solución para hallar $FT_2 = \frac{C_1(s)}{R_1(s)}$

1. Lazos del sistema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sistema ($R_1(s)$ a $C_2(s)$):

$$\begin{aligned} T_1 &= G_1G_2G_3 \\ T_2 &= G_1G_7G_5G_9G_3 \\ T_3 &= G_1G_7G_5G_8G_2G_3 \\ \Delta_1 &= 1 - [L_1] = 1 + G_5H_2 \\ \Delta_2 &= 1 - [0] = 1 \\ \Delta_3 &= 1 - [0] = 1 \end{aligned}$$

3. Ecuación característica:

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1L_3 + L_2L_3]$$

$$\Delta = 1 - [-G_5H_2 - G_3H_1 + G_5G_8G_7] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]$$

4. Función de transferencia:

$$FT_4 = \frac{C_1(s)}{R_1(s)} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3}{\Delta}$$

$$FT_4 = \frac{C_1(s)}{R_1(s)} = \frac{G_1G_2G_3(1 + G_5H_2) + G_1G_7G_5G_9G_3 + G_1G_7G_5G_8G_2G_3}{1 - [G_5G_8G_7 - G_5H_2 - G_3H_1] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]}$$