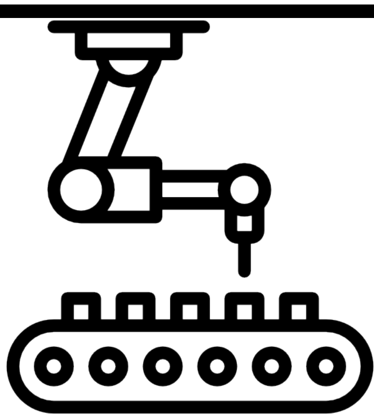
Ingenieria Electromecánica

# UNIVERSIDAD AUTÓNOMA GABRIEL RENE MORENO

**INGENIERIA ELECTROMECÁNICA**



# Sistemas de control ELN-360

***PRACTICO # 1***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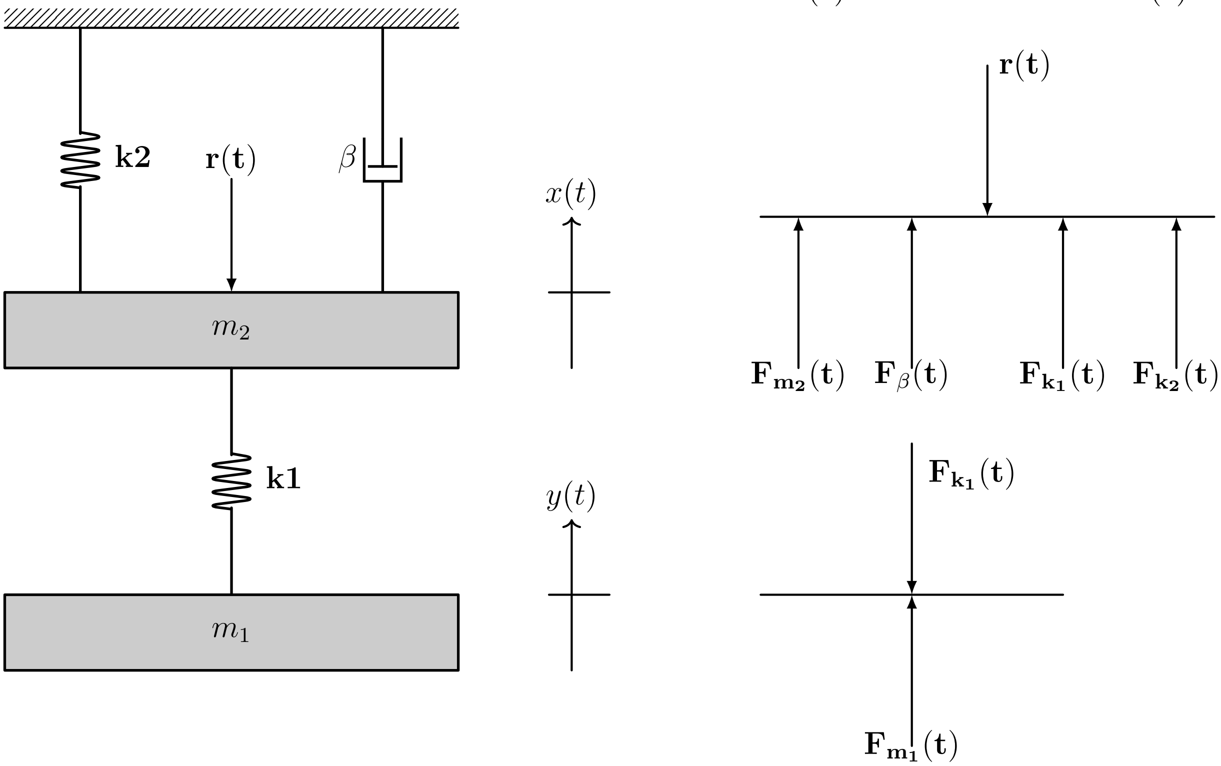
***SANTA CRUZ DE LA SIERRA, 2022***

Universidad Autónoma Gabriel Rene Moreno

# Ejercicio # 1

Hallar las ecuaciónes de función de transferencia: $FT_1(s) = \dfrac{X(s)}{R(s)}$ y $FT_2(s) = \dfrac{Y(s)}{R(s)}$.




Solucion:

1. Tenemos las siguientes ecuaciones:

   $F_{m_1} = m_1\dfrac{d^2y(t)}{dt^2}$ ; $F_{m_2} = m_2\dfrac{d^2x(t)}{dt^2}$ ; $F_\beta = \beta\dfrac{dx(t)}{dt}$ ; $F_{k_1} = k_1[x(t) - y(t)]$ ; $F_{k_2} = k_2x(t)$.

2. Hacemos la sumatoria de las fuerzas (DCL):

   Para la masa 2.

$$F_{k_1} + F_{k_2} + F_\beta + F_{m_2} = r(t)$$

$$[x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2} = r(t)$$

   Para la masa 1.

$$-F_{k_1} + F_{m_1} = 0$$

$$-[x(t) - y(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

$$[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

3. Aplicamos la transformada de Laplace: $y'(0) = y(0) = x'(0) = x(0) = 0$

   Para la primera ecuación:

$$\begin{aligned}
[x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2} &= r(t) \\
\mathscr{L}\left\{[x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2}\right\} &= \mathscr{L}\{r(t)\} \\
[X(s) - Y(s)]k_1 + X(s)k_2 + S\beta X(s) + m_2S^2X(s) &= R(t)
\end{aligned}$$

   Para la segunda ecuación:

$$[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

$$\mathscr{L}\left\{[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2}\right\} = 0$$

$$[Y(s) - X(s)]k_1 + m_1S^2Y(s) = 0$$

4. Ordenamos las ecuaciones:

$$\overbrace{(k_1 + k_2S\beta + m_2S^2)}^{a}X(s) - \overbrace{k_1}^{b}Y(s) = R(s) \tag{1}$$

$$\overbrace{-k_1}^{c}X(s) + \overbrace{(k_1 + m_1S^2)}^{d}Y(s) = 0 \tag{2}$$

5. Resolvemos las ecuaciones por el metodo de cramer:

$$X(s) = \frac{\begin{vmatrix} R(s) & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{R(s)\cdot d - 0\cdot b}{a\cdot d - c\cdot b} = \frac{R(s)d}{ad - cb} = \frac{(k_1 + m_1S^2)R(s)}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1(k_1)}$$

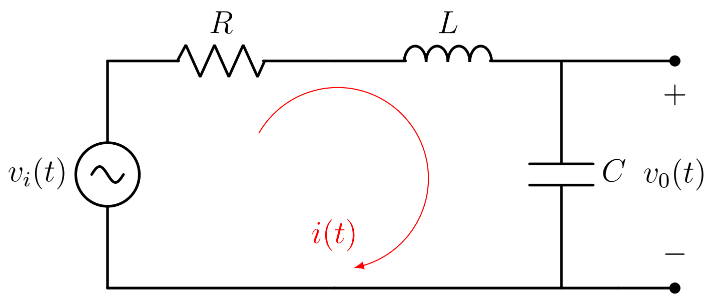
$$\boxed{FT_1(s) = \frac{X(s)}{R(s)} = \frac{k_1 + m_1S^2}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1^2}}$$

$$Y(s) = \frac{\begin{vmatrix} a & R(s) \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{a\cdot 0 - c\cdot R(s)}{a\cdot d - c\cdot b} = \frac{-cR(s)}{ad - cb} = \frac{k_1R(s)}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1(k_1)}$$

$$\boxed{FT_2(s) = \frac{Y(s)}{R(s)} = \frac{k_1}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1^2}}$$

# Ejercicio # 2

Hallar: $\dfrac{V_o(s)}{V_i(s)}$




aAplicamos la ley de Kirchhoff de las mallas:

$$v_i(t) = v_R(t) + v_L(t) + v_C(t)$$

Donde:

$$v_C(t) = \frac{1}{C}\int i_C(t)\mathrm{d}t$$

$$v_L(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t}$$

$$v_R(t) = Ri(t)$$

Ecuación diferencial del circuito RLC:

$$v_i(t) = Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i_C(t)\mathrm{d}t$$

Aplicamos la transformada de Laplace:

$$\mathscr{L}\{v_i(t)\} = \mathscr{L}\{v_R(t) + v_L(t) + v_C(t)\}$$

$$\mathscr{L}\{v_i(t)\} = \mathscr{L}\left\{Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i_C(t)\mathrm{d}t\right\}$$

$$V_i(s) = RI(s) + LSI(s) + \frac{1}{CS}I(s)$$

$$V_i(s) = I(s)\left(R + LS + \frac{1}{CS}\right)$$

$$V_i(s) = I(s)\left(\frac{S^2LC + 1 + SRC}{SC}\right)$$

$$I(s) = V_i(s)\left(\frac{SC}{S^2LC + 1 + SRC}\right)$$

Aplicamos la Ley de tension de Kirchhoff:

$$-v_o(t) + v_c(t) = 0$$

$$v_o(t) = v_C(t) = \frac{1}{C}\int i_C(t)\mathrm{d}t$$

$$\mathscr{L}\{v_o(t)\} = \mathscr{L}\left\{\frac{1}{C}\int i_C(t)\mathrm{d}t\right\}$$

$$V_o(s) = \frac{1}{CS}I(s)$$

Luego reemplazamos:

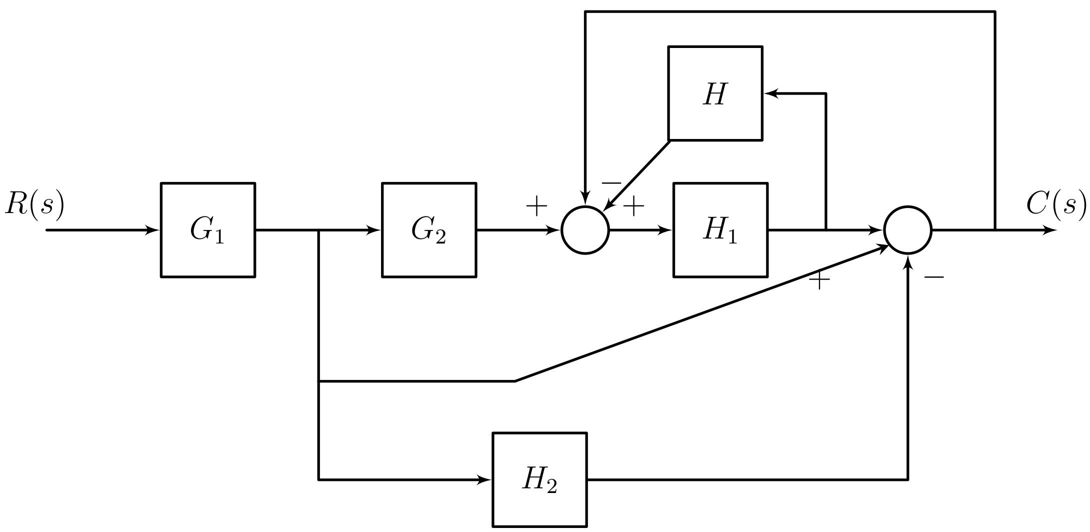
$$V_o(s) = \frac{1}{SC}\left[V_i(s)\left(\frac{SC}{S^2LC + 1 + SRC}\right)\right]$$

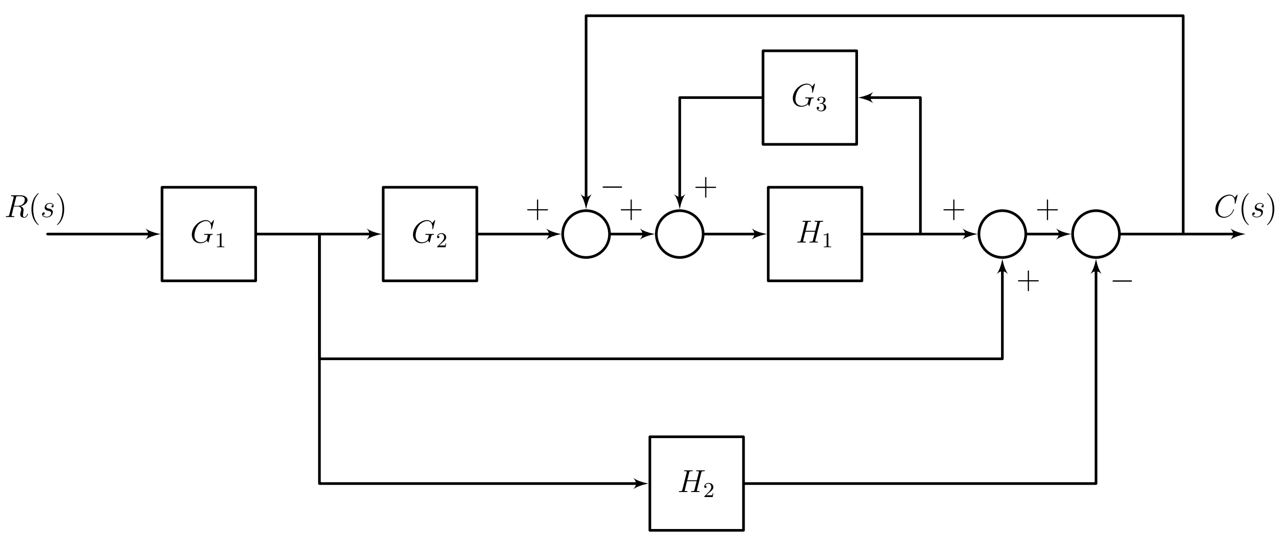
$$V_o(s) = V_i(s)\left(\frac{1}{S^2LC + 1 + SRC}\right)$$

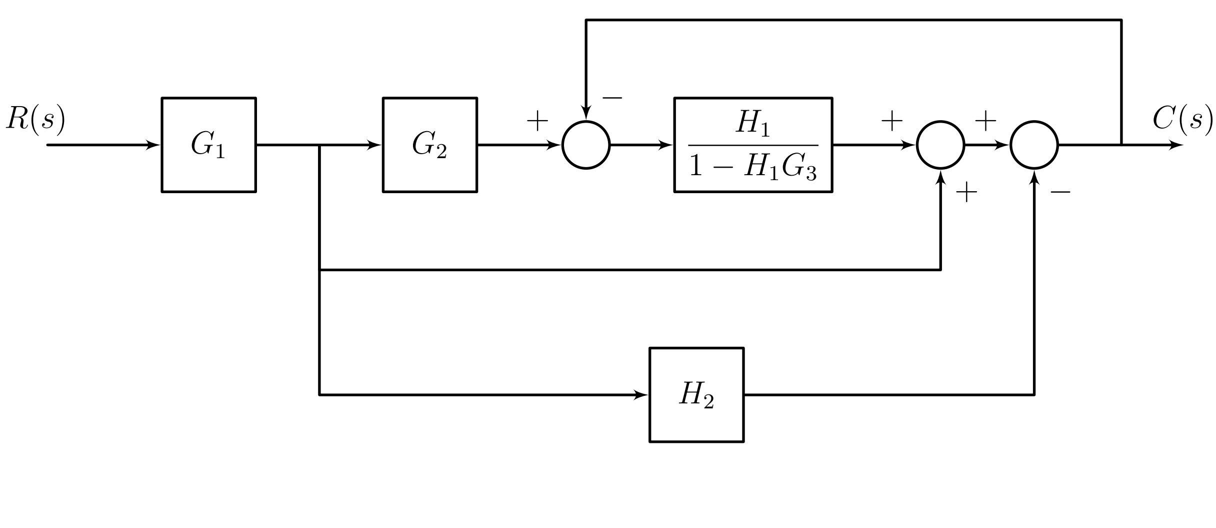
Solución:

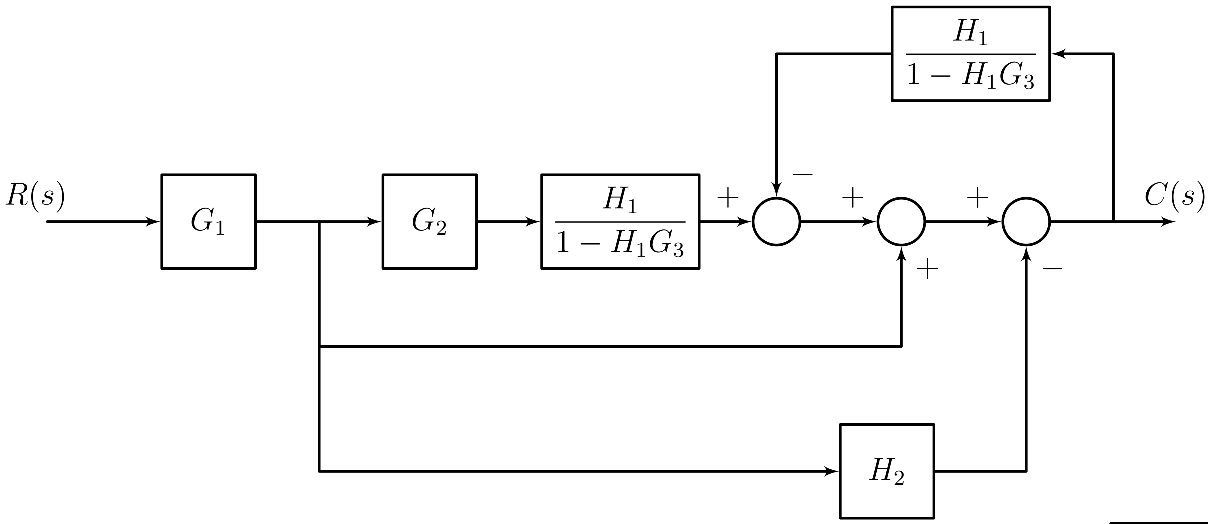
$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1}{S^2LC + 1 + SRC}}$$

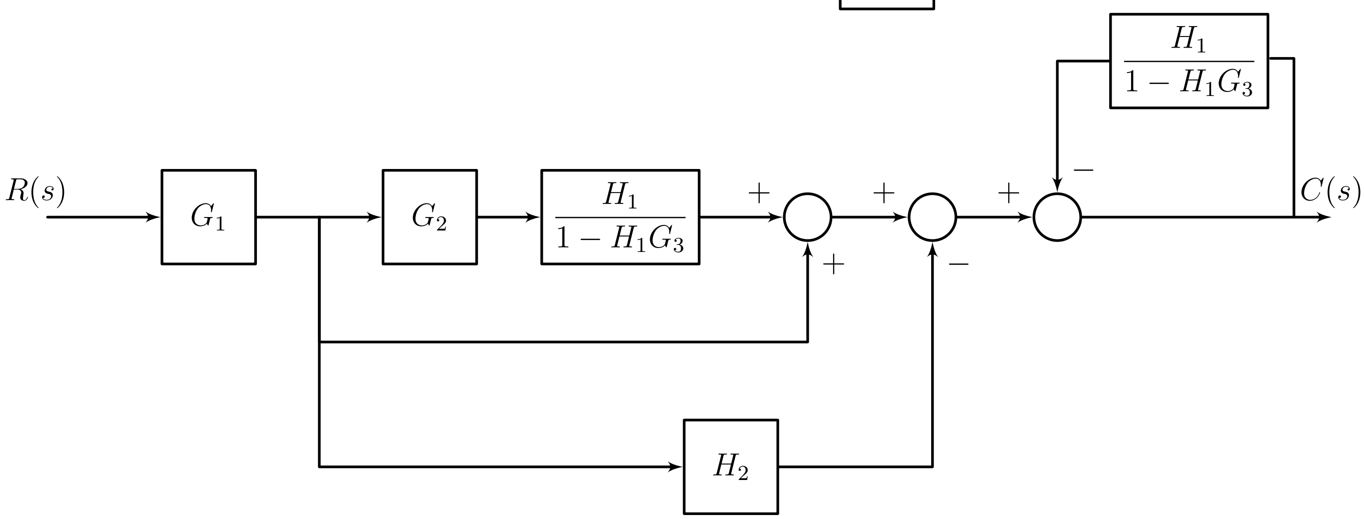
## Ejercicio # 3

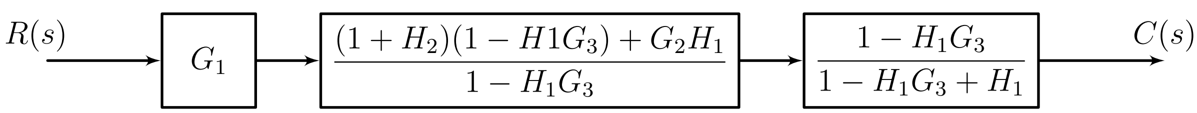


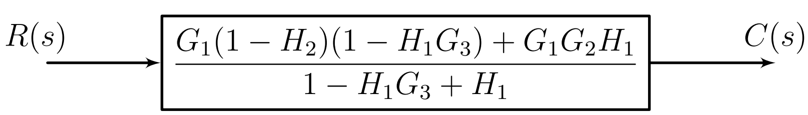




$R(s)$
$G_1$
$G_2$
$\frac{H_1}{1-H_1G_3}$
$\frac{H_1}{1-H_1G_3}$
$H_2$
$C(s)$

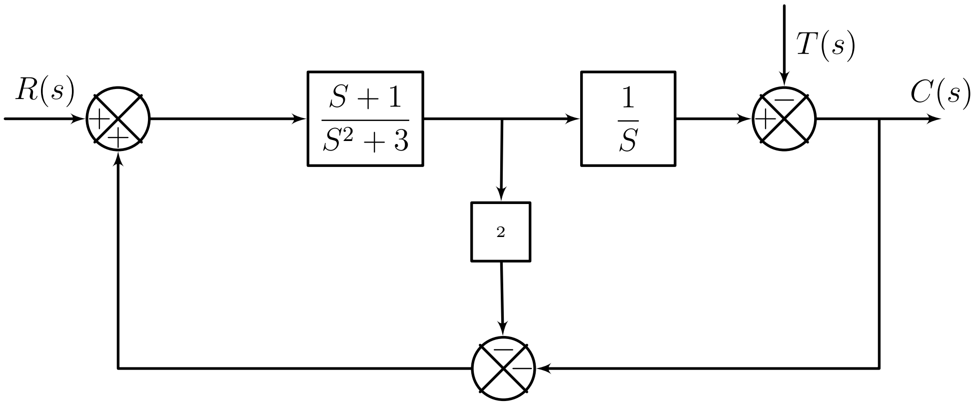

$R(s)$
$G_1$
$G_2$
$\frac{H_1}{1-H_1G_3}$
$\frac{H_1}{1-H_1G_3}$
$H_2$
$C(s)$


$R(s)$
$G_1$
$\frac{(1+H_2)(1-H1G_3)+G_2H_1}{1-H_1G_3}$
$\frac{1-H_1G_3}{1-H_1G_3+H_1}$
$C(s)$

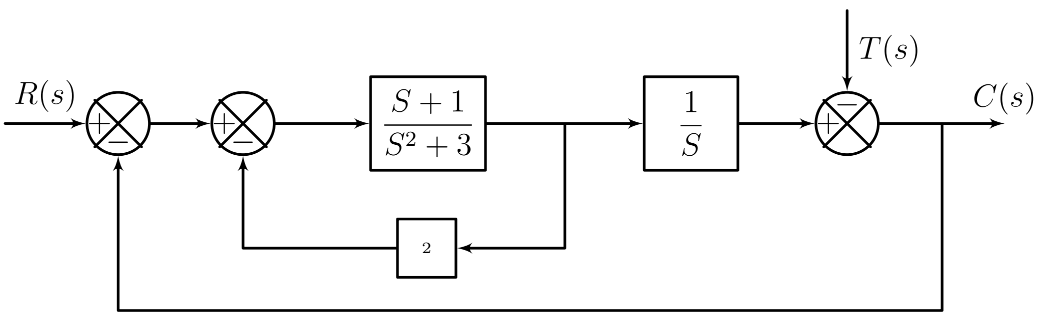

$R(s)$
$\frac{G_1(1-H_2)(1-H_1G_3)+G_1G_2H_1}{1-H_1G_3+H_1}$
$C(s)$

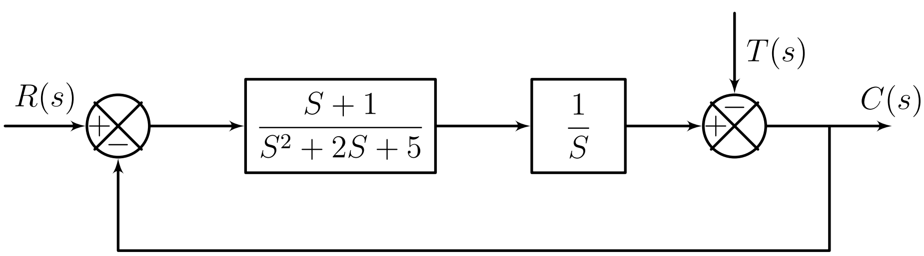
# Ejercicio # 4

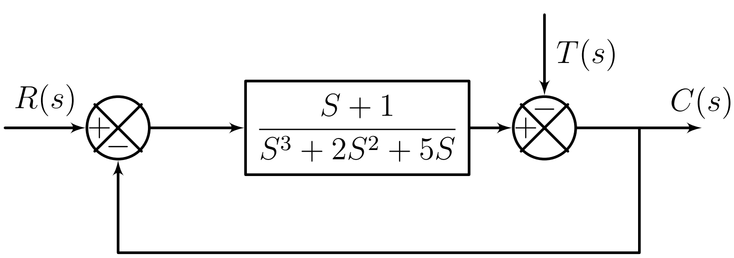
Hallar: $C(s) = ?$



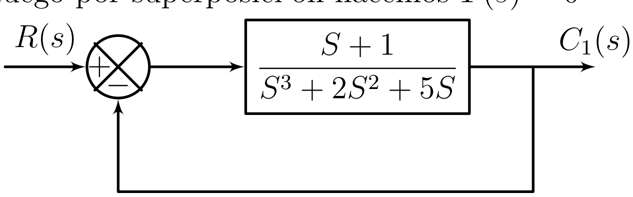

primero ordenamos un poco.

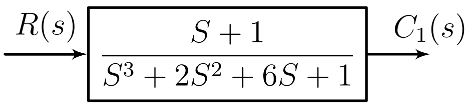


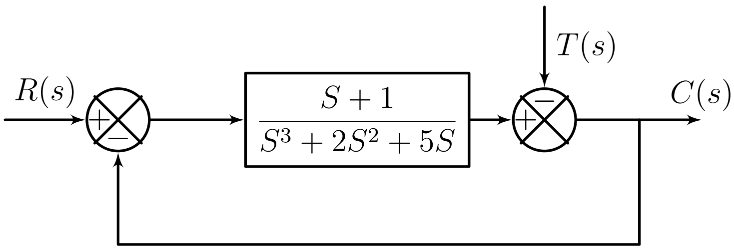



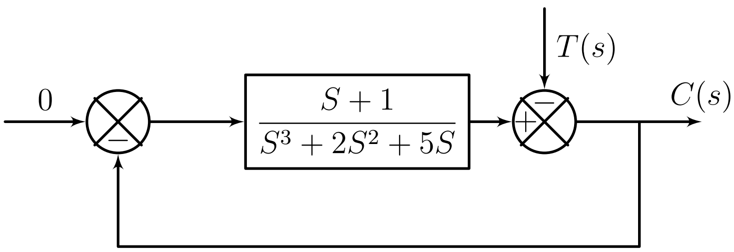

Luego por superposici'on hacemos $T(s) = 0$

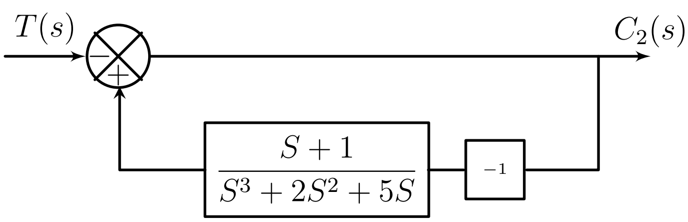


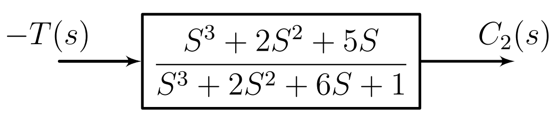



Luego por superposici'on hacemos $R(s) = 0$



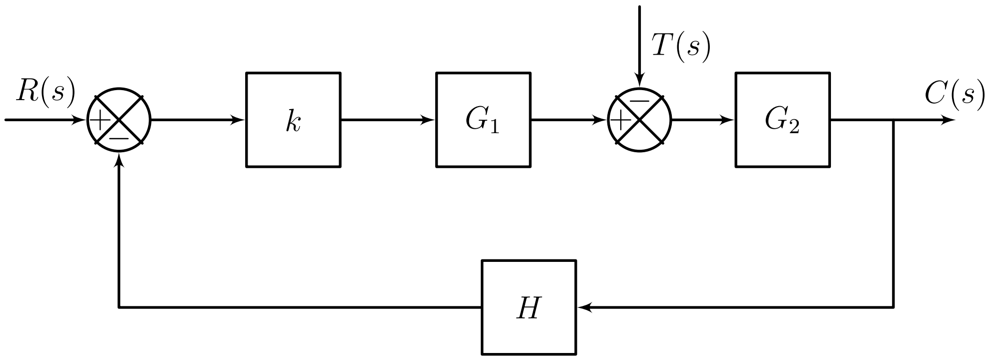



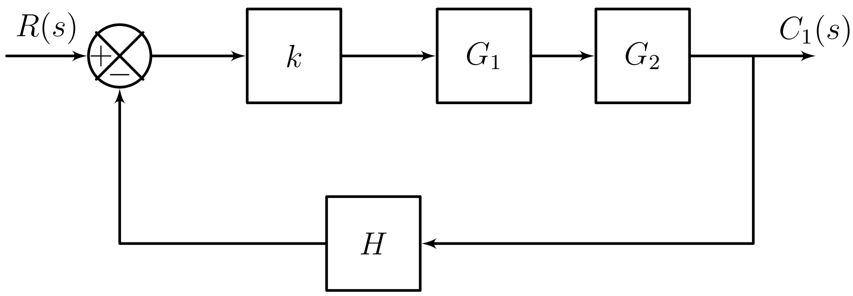

Superposici'on de las salidas $C(s) = C_1(s) + C_2(s)$

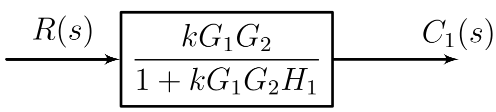
$$C(s) = R(s)\frac{S+1}{S^3+2S^2+6S+1} - T(s)\frac{S^3+2S^2+5S}{S^3+2S^2+6S+1}$$

# Ejercicio # 5

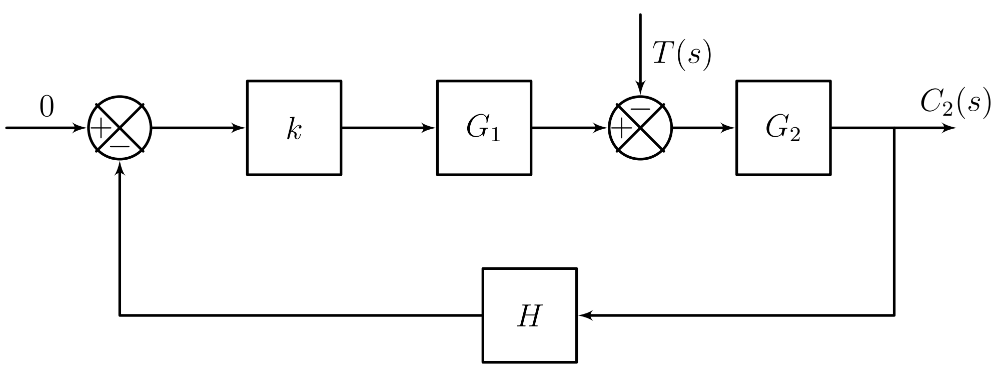


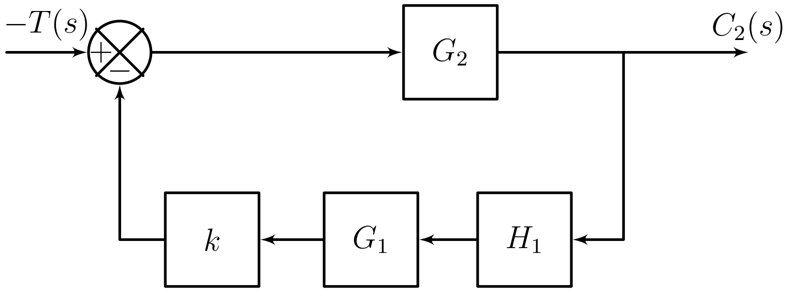

Por superposici'on hacemos $T(s) = 0$.

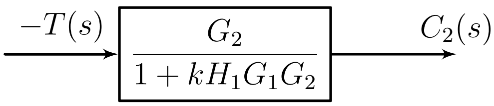



Por superposici'on hacemos $R(s) = 0$.



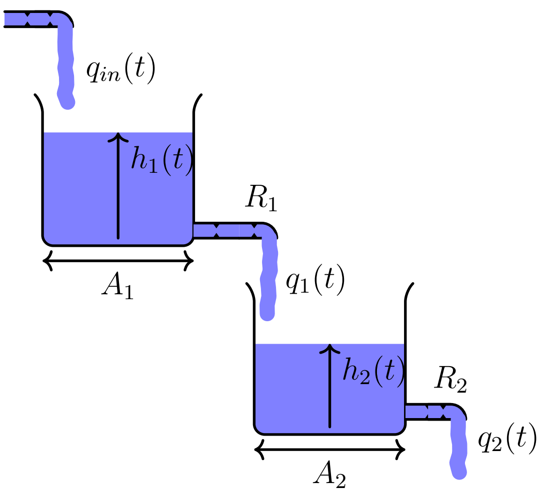



Superposici'on de las salidas $C(s) = C_1(s) + C_2(s)$

$$C(s) \quad = \quad R(s)\frac{kG_1G_2}{1+kH_1G_1G_2} - T(s)\frac{G_2}{1+kH_1G_1G_2}$$

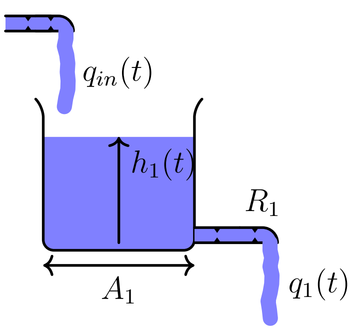
# Ejercicio # 6

Hallar: $FT = \dfrac{H_2(s)}{Q_{in}(s)}$ ; $FT = \dfrac{Q_2(s)}{Q_{in}(s)}$




Solución:

1. Primero analisamos el primer tanque de agua.




En el tanque 1 tenemos:

$V_1(t) = A_1 h_1(t)$

$\dfrac{dV_1(t)}{dt} = A_1 \dfrac{dh_1(t)}{dt}$

$q_{in}(t) - q_1(t) = A_1 \dfrac{dh_1(t)}{dt}$

Luego tenemos que el caudal de salida del primer tanque $q_1(t)$ es inverzamente proporcional a la resitencia de la valvula $R_1$ y directamente proporcional a $h_1(t)$.

$q_1(t) = \dfrac{1}{R_1} h_1(t)$

Entonces tenemos el siguiente sistema de ecuaciones:

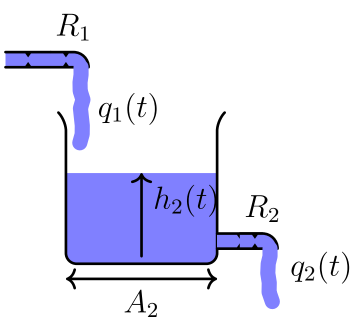
$$
\begin{aligned}
q_{in}(t) - q_1(t) &= A_1 \frac{dh_1(t)}{dt} && (1)\\
q_1(t) &= \frac{1}{R_1} h_1(t) && (2)
\end{aligned}
$$

Luego aplicamos la transformada de Laplace:

$$
\begin{aligned}
\mathscr{L}\{q_{in}(t) - q_1(t)\} &= \mathscr{L}\left\{A_1 \frac{dh_1(t)}{dt}\right\}\\
\mathscr{L}\{q_1(t)\} &= \mathscr{L}\left\{\frac{1}{R_1} h_1(t)\right\}
\end{aligned}
$$

$$
\begin{aligned}
Q_{in}(s) - Q_1(s) &= A_1 S H_1(s) && (1)\\
Q_1(s) &= \frac{1}{R_1} H_1(s) && (2)
\end{aligned}
$$

2. Ahora nalizamos el segundo tanque.




En el tanque 2 tenemos:

$V_2(t) = A_2 h_2(t)$

$\frac{dV_2(t)}{dt} = A_2 \frac{dh_2(t)}{dt}$

$q_1(t) - q_2(t) = A_2 \frac{dh_2(t)}{dt}$

Luego tenemos que el caudal de salida del segundo tanque $q_2(t)$ es inverzamente proporcional a la resitencia de la valvula $R_2$ y directamente proporcional a $h_2(t)$.

$q_2(t) = \frac{1}{R_2} h_2(t)$

Entonces tenemos el siguiente sistema de ecuaciones:

$$
\begin{aligned}
q_1(t) - q_2(t) &= A_2 \frac{dh_2(t)}{dt}\\
q_2(t) &= \frac{1}{R_2} h_2(t)
\end{aligned}
$$

Luego aplicamos la transformada de Laplace:

$$\begin{aligned}\mathscr{L}\{q_1(t)-q_2(t)\} &= \mathscr{L}\left\{A_2\frac{dh_2(t)}{dt}\right\}\\ \mathscr{L}\{q_2(t)\} &= \mathscr{L}\left\{\frac{1}{R_2}h_2(t)\right\}\end{aligned}$$

$$Q_1(s)-Q_2(s) = A_2SH_2(s) \tag{3}$$

$$Q_2(s) = \frac{1}{R_2}H_2(s) \tag{4}$$

3. Luego hacemos (2) en (1) y (4) en (3)

De (1) $\quad Q_{in}(s) = A_1SH_1(s) + Q_1(s)$
remplazamos (2) en (1)
$Q_{in}(s) = A_1SQ_1(s)R_1 + Q_1(s)$

$Q_{in}(s) = Q_1(s)[A_1SR_1 + 1] \rightarrow (a)$

De (3) $\quad Q_1(s) = Q_2(s) + A_2SH_2(s)$
remplazamos (4) en (3)
$Q_1(s) = \dfrac{1}{R_2}H_2 + A_2SH_2$

$Q_1(s) = H_2(s)\left[\dfrac{1}{R_2} + A_2S\right] \rightarrow (b)$
remplazamos (b) en (a)
$Q_{in}(s) = H_2(s)\left[\dfrac{1}{R_2} + A_2S\right](A_1SR_1 + 1)$
$Q_{in}(s) = H_2(s)\left(A_1A_2S^2R_1 + \dfrac{A_1R_1S}{R_2} + A_2S + \dfrac{1}{R_2}\right)$

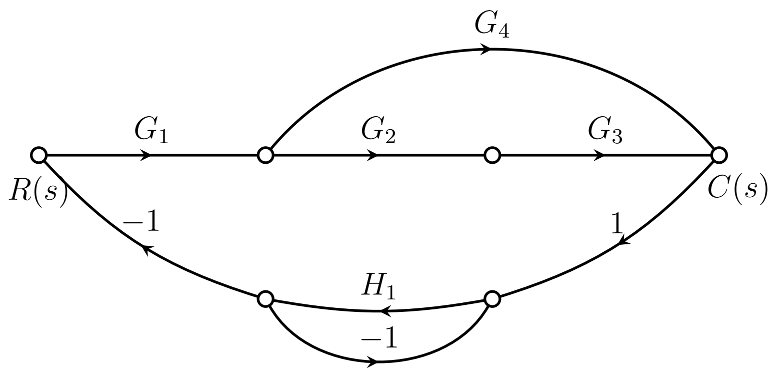
$$\boxed{FT = \frac{H_2(s)}{Q_{in}(s)} = \frac{R_2}{A_1A_2R_1R_2S^2 + (A_1R_1 + A_2R_2)S + 1}}$$

4. Ahora para hallar $\frac{Q_2(s)}{Q_{in}(s)}$ tenemos que: $H_2 = Q_2(s)R_2$ y reemplazamos en la anterior ecuación.

$$\boxed{FT = \frac{Q_2(s)}{Q_{in}(s)} = \frac{1}{A_1A_2R_1R_2S^2 + (A_1R_1 + A_2R_2)S + 1}}$$

# Ejercicio # 7

Hallar: $FT = \dfrac{C(s)}{R(s)}$




Solucion:

1. **Número de lazos del sistema:**

$$
\begin{aligned}
L_1 &= G_1G_2G_3(1)H_1(-1) = -G_1G_2G_3H_1 \\
L_2 &= G_1G_4(1)H_1(-1) = -G_1G_4H_1 \\
L_3 &= H_1(-1) = -H_1
\end{aligned}
$$

2. **Número de trayectorias del sistema:**

$$
\begin{aligned}
T_1 &= G_1G_2G_3 \quad ; \quad \Delta_1 = 1 - L_3 + [0] = 1 + H_1 \\
T_2 &= G_1G_4 \quad ; \quad \Delta_2 = 1 - L_3 + [0] = 1 + H_1
\end{aligned}
$$

3. **Ecuación caracteristica**

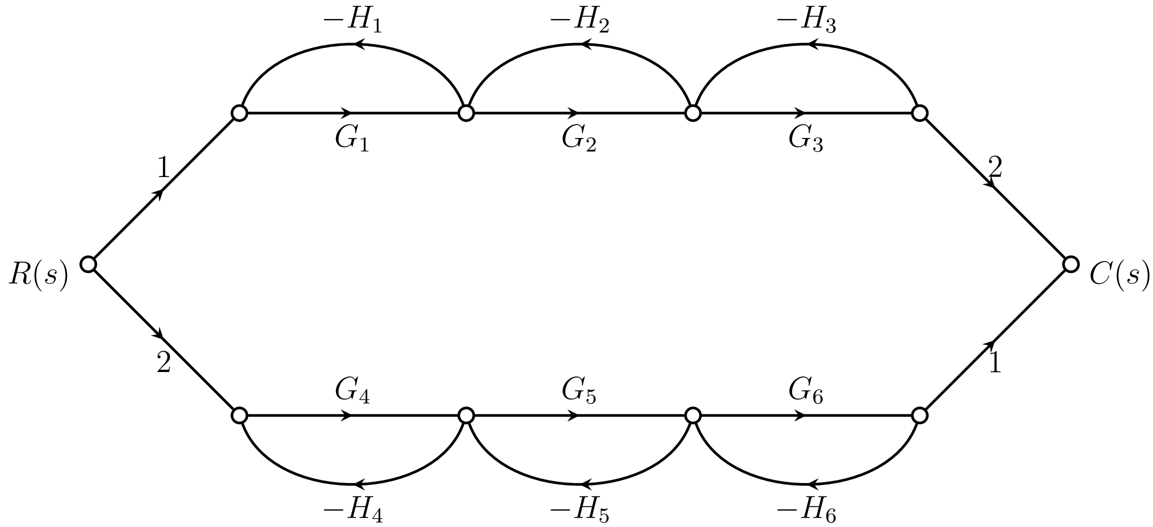
$$
\begin{aligned}
\Delta &= 1 - \sum L_i + \sum L_iL_j - \sum L_iL_jL_k + \cdots + (-1)^m \sum \cdots + \cdots \\
\Delta &= 1 - [L_1 + L_2 + L_3 + [0]] \\
\Delta &= 1 + G_1G_2G_3H_1 + G_1G_4H_1 + H_1
\end{aligned}
$$

4. **Funcion de transferencia:**

$$
\begin{aligned}
FT &= \frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \frac{G_1G_2G_3(1+H_1) + G_1G_4(1+H_1)}{1 + G_1G_2G_3H_1 + G_1G_4H_1 + H_1} \\
FT &= \frac{G_1G_2G_3(1+H_1) + G_1G_4(1+H_1)}{1 + G_1G_2G_3H_1 + G_1G_4H_1 + H_1}
\end{aligned}
$$

# Ejercicio # 8

Hallar: $FT = \dfrac{C(s)}{R(s)}$




Solucion:

1. **Número de lazos del sistema:**

$$\begin{aligned} L_1 &= -G_1H_1 \\ L_2 &= -G_2H_2 \\ L_3 &= -G_3H_3 \\ L_4 &= -G_4H_4 \\ L_5 &= -G_5H_5 \\ L_6 &= -G_6H_6 \end{aligned}$$

2. **Número de trayectorias del sistema:**

$$\begin{aligned} T_1 &= 2G_1G_2G_3 \\ T_2 &= 2G_4G_5G_6 \\ \Delta_1 &= 1 - [L_4 + L_5 + L_6] + [L_4L_6] = 1 + G_4H_4 + G_5H_5 + G_6H_6 + (G_4H_4G_6H_6) \\ \Delta_2 &= 1 - [L_1 + L_2 + L_3] + [L_1L_3] = 1 + G_1H_1 + G_2H_2 + G_3H_3 + (G_1H_1G_3H_3) \end{aligned}$$

3. **Ecuación caracteristica**

$$\Delta = 1 - \sum L_i + \sum L_iL_j - \sum L_iL_jL_k + \cdots$$

$$\begin{aligned} \Delta = 1 - [&L_1 + L_2 + L_3 + L_4 + L_5 + L_6] + \\ &+ [L_1L_3 + L_1L_4 + L_1L_5 + L_1L_6 + L_2L_4 + L_2L_5 + L_2L_6 + L_3L_4 + L_3L_5 + L_3L_6 + L_4L_6] \\ &- [L_1L_3L_4 + L_1L_3L_5 + L_1L_3L_6 + L_4L_6L_1 + L_4L_6L_2 + L_4L_6L_3] + [L_1L_3L_4L_6] \end{aligned}$$

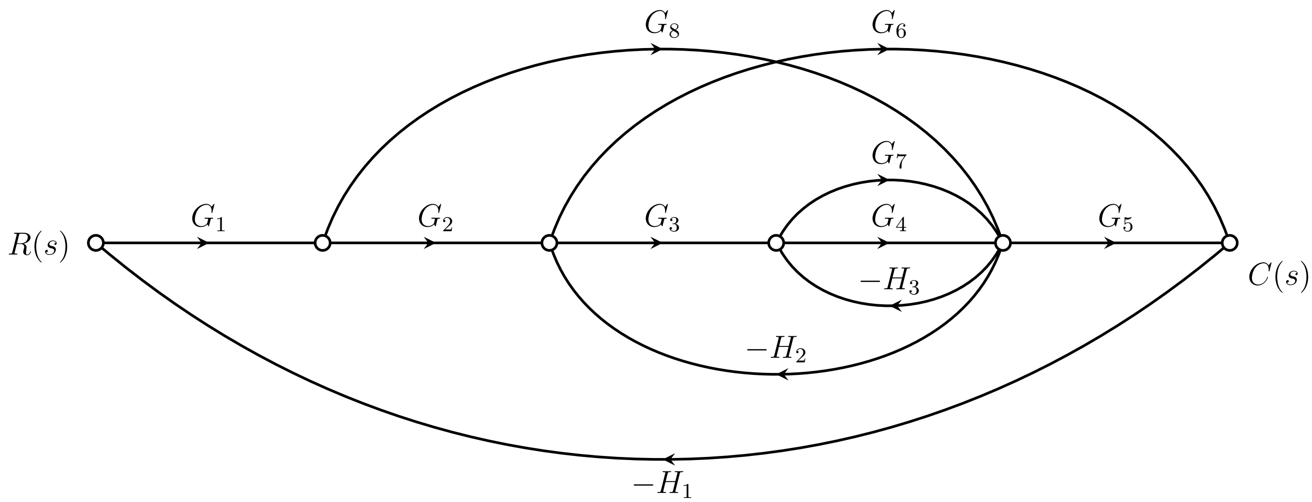
$$\Delta = 1 - [G_1H_1 + G_2H_2 + G_3H_3 + G_4H_4 + G_5H_5 + G_6H_6]+$$
$$+[G_1H_1(G_3H_3+G_4H_4+G_5H_5+G_6H_6)+G_2H_2(G_4H_4+G_5H_5+G_6H_6)+G_3H_3(G_4H_4+G_5H_5+G_6H_6)+G_4H_4G_6H_6]$$
$$+ [G_1H_1G_3G_3(G_4H_4 + G_5H_5 + G_6H_6) + G_4H_4G_6H_6(G_1H_1 + G_2H_2 + G_3H_3)] + [G_1H_1G_3H_3G_4H_4G_6H_6]$$

4. **Funcion de transferencia:**

$$\begin{aligned} FT &= \frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} \\ FT &= \frac{2G_1G_2G_3(1 + G_4H_4 + G_5H_5 + G_6H_6 + G_4H_4G_6H_6)}{\Delta} \\ &+ \frac{2G_4G_5G_6(1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1H_1G_3H_3)}{\Delta} \end{aligned}$$

# Ejercicio # 9

Hallar: $FT = \dfrac{C(s)}{R(s)}$




Solucion:

1. **Número de lazos del sistema:**

$$
\begin{aligned}
L_1 &= -G_1G_2G_3G_4G_5H_1 \\
L_2 &= -G_1G_2G_3G_7G_5H_1 \\
L_3 &= -G_1G_8G_5H_1 \\
L_4 &= -G_1G_2G_6H_1 \\
L_5 &= -G_3G_4H_2 \\
L_6 &= -G_3G_7H_2 \\
L_7 &= -G_4H_3 \\
L_8 &= -G_7H_3
\end{aligned}
$$

2. **Número de trayectorias del sistema:**

$$
\begin{aligned}
T_1 &= G_1G_2G_3G_4G_5 \\
T_2 &= G_1G_2G_3G_7G_5 \\
T_3 &= G_1G_8G_5 \\
T_4 &= G_1G_2G_6 \\
\Delta_1 &= 1-[0]+[0]=1 \\
\Delta_2 &= 1-[0]+[0]=1 \\
\Delta_3 &= 1-[0]+[0]=1 \\
\Delta_4 &= 1-[L_7+L_8]+[0]=1+G_4H_3+G_7H_3
\end{aligned}
$$

3. **Ecuación caracteristica**

$$\begin{aligned}\Delta &= 1-\sum L_i+\sum L_iL_j-\sum L_iL_jL_k+\cdots+\\ \Delta &= 1-[L_1+L_2+L_3+L_4+L_5+L_6+L_7+L_8]+[L_4L_7+L_4L_8]\end{aligned}$$

$$\Delta = 1+[G_1G_2G_3G_4G_5H_1+G_1G_2G_3G_7G_5H_1+G_1G_8G_5H_1+G_1G_2G_6H_1+G_3G_4H_2+G_3G_7H_2+G_4H_3+G_7H_3] \\ +[G_1G_2G_6H_1G_4H_3+G_1G_2G_6H_1G_7H_3]$$

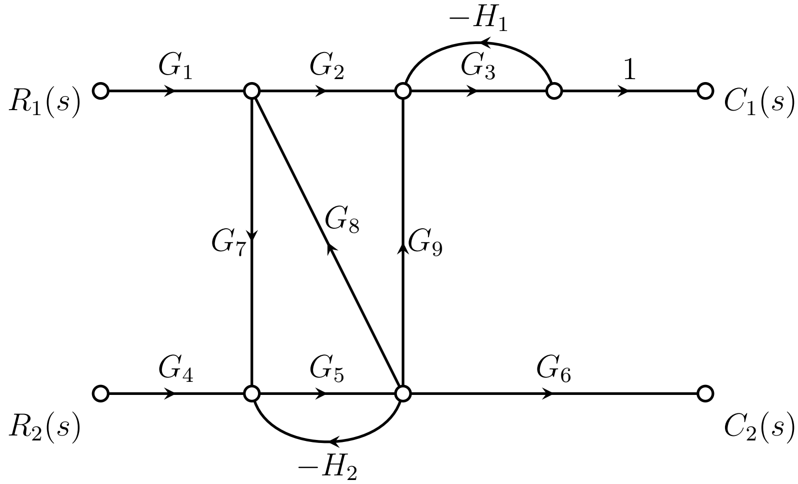
4. **Funcion de transferencia:**

$$FT = \frac{C(s)}{R(s)} = \frac{T_1\Delta_1+T_2\Delta_2+T_3\Delta_3+T_4\Delta_4}{\Delta}$$

$$FT = \frac{G_1G_2G_3G_4G_5+G_1G_2G_3G_7G_5+G_1G_8G_5+G_1G_2G_6(1+G_4H_3+G_7H_3)}{\Delta}$$

# Ejercicio # 10

Hallar: $FT_1 = \frac{C_1(s)}{R_2(s)}; FT_2 = \frac{C_2(s)}{R_1(s)}; FT_3 = \frac{C_2(s)}{R_2(s)}; FT_4 = \frac{C_1(s)}{R_1(s)}$




Sulucion para hallar $FT_1 = \frac{C_1(s)}{R_2(s)}$

1. Lazos del sitema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sitema ($R_2(s)$ a $C_1(s)$):

$$\begin{aligned} T_1 &= G_4G_5G_9G_3 \\ T_2 &= G_4G_5G_8G_2G_3 \\ \Delta_1 &= 1-[0]=1 \\ \Delta_2 &= 1-[0]=1 \end{aligned}$$

3. Ecuación caracteristica:

$$\Delta = 1-[L_1+L_2+L_3]+[L_1L_3+L_2L_3]$$

$$\Delta = 1-[-G_5H_2-G_3H_1+G_5G_8G_7]+[G_5H_2G_3H_1-G_3H_1G_5G_8G_7]$$

4. Función de transferencia:

$$FT_1 = \frac{C_1(s)}{R_2(s)} = \frac{T_1\Delta_1+T_2\Delta_2}{\Delta}$$

$$FT_1 = \frac{C_1(s)}{R_2(s)} = \frac{G_4G_5G_9G_3+G_4G_5G_8G_2G_3}{1-[-G_5H_2-G_3H_1+G_5G_8G_7]+[G_5H_2G_3H_1-G_3H_1G_5G_8G_7]}$$

Sulucion para hallar $FT_2 = \dfrac{C_2(s)}{R_1(s)}$

1. Lazos del sitema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sitema ($R_1(s)$ a $C_2(s)$):

$$\begin{aligned} T_1 &= G_1G_7G_5G_6 \\ \Delta_1 &= 1-[L_2] = 1+G_3H_1 \end{aligned}$$

3. Ecuación caracteristica:

$$\Delta = 1-[L_1+L_2+L_3]+[L_1L_3+L_2L_3]$$

$$\Delta = 1-[-G_5H_2-G_3H_1+G_5G_8G_7]+[G_5H_2G_3H_1-G_3H_1G_5G_8G_7]$$

4. Función de transferencia:

$$\begin{aligned} FT_2 &= \frac{C_2(s)}{R_1(s)} = \frac{T_1\Delta_1}{\Delta} \\ FT_2 &= \frac{C_2(s)}{R_1(s)} = \frac{G_1G_7G_5G_6(1+G_3H_1)}{1-[G_5G_8G_7-G_5H_2-G_3H_1]+[G_5H_2G_3H_1-G_3H_1G_5G_8G_7]} \end{aligned}$$

Sulucion para hallar $FT_3 = \dfrac{C_2(s)}{R_2(s)}$

1. Lazos del sitema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sitema ($R_2(s)$ a $C_2(s)$):

$$\begin{aligned} T_1 &= G_4G_5G_6 \\ \Delta_1 &= 1-[L_2] = 1+G_3H_1 \end{aligned}$$

3. Ecuación caracteristica:

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1L_3 + L_2L_3]$$

$$\Delta = 1 - [-G_5H_2 - G_3H_1 + G_5G_8G_7] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]$$

4. Función de transferencia:

$$FT_3 = \frac{C_2(s)}{R_2(s)} = \frac{T_1\Delta_1}{\Delta}$$

$$FT_3 = \frac{C_2(s)}{R_2(s)} = \frac{G_4G_5G_6(1 + G_3H_1)}{1 - [G_5G_8G_7 - G_5H_2 - G_3H_1] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]}$$

Sulucion para hallar $FT_2 = \dfrac{C_1(s)}{R_1(s)}$

1. Lazos del sitema:

$$\begin{aligned} L_1 &= -G_5H_2 \\ L_2 &= -G_3H_1 \\ L_3 &= G_5G_8G_7 \end{aligned}$$

2. Trayectorias del sitema ($R_1(s)$ a $C_2(s)$):

$$\begin{aligned} T_1 &= G_1G_2G_3 \\ T_2 &= G_1G_7G_5G_9G_3 \\ T_3 &= G_1G_7G_5G_8G_2G_3 \\ \Delta_1 &= 1 - [L_1] = 1 + G_5H_2 \\ \Delta_2 &= 1 - [0] = 1 \\ \Delta_3 &= 1 - [0] = 1 \end{aligned}$$

3. Ecuación caracteristica:

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1L_3 + L_2L_3]$$

$$\Delta = 1 - [-G_5H_2 - G_3H_1 + G_5G_8G_7] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]$$

4. Función de transferencia:

$$FT_4 = \frac{C_1(s)}{R_1(s)} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3}{\Delta}$$

$$FT_4 = \frac{C_1(s)}{R_1(s)} = \frac{G_1G_2G_3(1 + G_5H_2) + G_1G_7G_5G_9G_3 + G_1G_7G_5G_8G_2G_3}{1 - [G_5G_8G_7 - G_5H_2 - G_3H_1] + [G_5H_2G_3H_1 - G_3H_1G_5G_8G_7]}$$