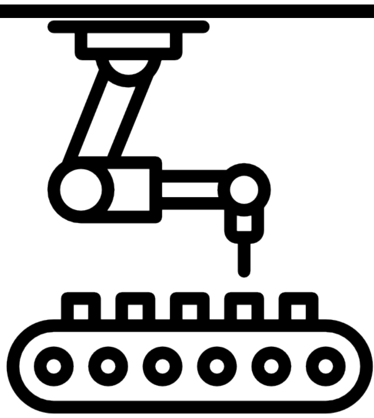
Ingenieria Electromecánica

UNIVERSIDAD AUTÓNOMA GABRIEL RENE MORENO

INGENIERIA ELECTROMECÁNICA



Sistemas de control ELN-360

PRACTICO # 3

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Ejercicio # 1

Sean los siguientes sistemas:

Indicar aplicando el método de Routh-Hurwitzs si el sistema es estable o no y porque, la función de transferencia de un sistema de control es:

a.
$$\frac{C(s)}{R(s)} = \frac{s^2+4s-1}{2s^5+2s^4+s^3+s^2+5s+1}$$

Tomamos el denominador de la FT

$D(s) = 2s^5 + 2s^4 + s^3 + s^2 + 5s + 1$

Por Routh-Hurwitzs:

$$\begin{array}{c|ccc}
s^5 & 2 & 1 & 5 \\
s^4 & 2 & 1 & 1 \\
s^3 & b_1 & b_2 & 0 \\
s^2 & c_1 & c_2 & 0 \\
s & d_1 & 0 & \\
s^0 & e_1 & &
\end{array}$$

$$b_1 = \frac{-\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}}{2} = \frac{-(2-2)}{2} = 0.$$

Este es un caso especial de Routh-Hurwitzs:

hacemos $b_1 = \varepsilon$ *donde* $0 < \varepsilon << 1$

$$b_2 = \frac{-\begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}}{2} = \frac{-(2-10)}{2} = 4.$$

$$c_1 = \frac{-\begin{bmatrix} 2 & 1 \\ \varepsilon & 4 \end{bmatrix}}{\varepsilon} = \frac{-(8-\varepsilon)}{\varepsilon} = \frac{\varepsilon - 8}{\varepsilon}.$$

$$c_2 = \frac{-\begin{bmatrix} 2 & 1 \\ \varepsilon & 0 \end{bmatrix}}{\varepsilon} = \frac{-(0-\varepsilon)}{\varepsilon} = 1$$

$$d_1 = \frac{-\begin{bmatrix} \varepsilon & 4 \\ \dfrac{\varepsilon-8}{\varepsilon} & 1 \end{bmatrix}}{\dfrac{\varepsilon-8}{\varepsilon}} = \frac{-[\varepsilon - 4(\dfrac{\varepsilon-8}{\varepsilon})]}{\dfrac{\varepsilon-8}{\varepsilon}} = 4 - \frac{\varepsilon^2}{\varepsilon-8} = \frac{-\varepsilon^2+4\varepsilon-32}{\varepsilon-3}$$

$$e_1 = \frac{-\begin{bmatrix} \dfrac{\varepsilon-8}{\varepsilon} & 1 \\ d_1 & 0 \end{bmatrix}}{d_1} = \frac{-(0-d_1)}{d_1} = 1$$

Ahora verificamos que signo tienen c_1 y d_1: como $\varepsilon \to 0$

$c_1 = \lim\limits_{\varepsilon\to 0} \dfrac{\cancelto{0}{\varepsilon} - 8}{\varepsilon} = -\dfrac{8}{\varepsilon}$ $\qquad c_1 \approx -\dfrac{8}{\varepsilon} < 0$ $\qquad \therefore \quad c_1$ es un numero negativo.

$d_1 = \lim\limits_{\varepsilon\to 0} \dfrac{\cancelto{0}{\varepsilon^2} - \cancelto{0}{4\varepsilon} - 32}{\cancelto{0}{\varepsilon} - 3} = \dfrac{32}{3}$ $\qquad d_1 = \dfrac{32}{3} > 0$ $\qquad \therefore \quad d_1$ es un numero positivo.

Reemplazamos los valores en la tabla de Routh-Hurwitzs:

s^5	2	1	5
s^4	2	1	1
s^3	ε	4	0
s^2	$-\dfrac{8}{\varepsilon}$	1	0
s	$\dfrac{32}{3}$	0	
s^0	1		

$\therefore$ es un sistema inestable con dos polos en el semiplano derecho.

b.

$$\frac{Y(s)}{R(s)} = \frac{s+3}{s^5+3s^4+2s^3+6s^2+3s+3}$$

Tomamos el denominador de la FT

$D(s) = s^5 + 3s^4 + 2s^3 + 6s^2 + 3s + 3$

Por Routh-Hurwitzs:

$$
\begin{array}{c|ccc}
s^5 & 1 & 2 & 3 \\
s^4 & 3 & 6 & 3 \\
s^3 & b_1 & b_2 & 0 \\
s^2 & c_1 & c_2 & 0 \\
s & d_1 & 0 & \\
s^0 & e_1 & &
\end{array}
$$

$$b_1 = \frac{-\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}}{3} = \frac{-(6-6)}{3} = 0.$$

Este es un caso especial de Routh-Hurwitzs:

hacemos $b_1 = \varepsilon$ *donde* $0 < \varepsilon << 1$

$$b_2 = \frac{-\begin{bmatrix} 1 & 3 \\ 3 & 3 \end{bmatrix}}{3} = \frac{-(3-9)}{3} = 2$$

$$c_1 = \frac{-\begin{bmatrix} 3 & 6 \\ \varepsilon & 2 \end{bmatrix}}{\varepsilon} = \frac{-(6-6\varepsilon)}{\varepsilon} = \frac{6\varepsilon-6}{\varepsilon}$$

$$c_2 = \frac{-\begin{bmatrix} 3 & 3 \\ \varepsilon & 0 \end{bmatrix}}{\varepsilon} = \frac{-(0-3\varepsilon)}{\varepsilon} = 3$$

$$d_1 = \frac{-\begin{bmatrix} \varepsilon & 2 \\ \frac{6\varepsilon-6}{\varepsilon} & 3 \end{bmatrix}}{\frac{6\varepsilon-6}{\varepsilon}} = \frac{-[3\varepsilon - 2(\frac{6\varepsilon-6}{\varepsilon})]}{\frac{6\varepsilon-6}{\varepsilon}} = 2 - \frac{3\varepsilon^2}{6\varepsilon-6} = \frac{-3\varepsilon^2+12\varepsilon-12}{6\varepsilon-6}$$

$$e_1 = \frac{-\begin{bmatrix} \frac{6\varepsilon-6}{\varepsilon} & 3 \\ d_1 & 0 \end{bmatrix}}{d_1} = \frac{-(0-3d_1)}{d_1} = 3$$

Ahora verificamos que signo tienen c_1 y d_1: como $\varepsilon \to 0$

$c_1 = \lim\limits_{\varepsilon \to 0} \frac{\cancelto{0}{6\varepsilon}-6}{\varepsilon} = -\frac{6}{\varepsilon}$ $\quad c_1 \approx -\frac{6}{\varepsilon} < 0$ $\quad \therefore$ c_1 es un numero negativo.

$d_1 = \lim\limits_{\varepsilon \to 0} \frac{\cancelto{0}{3\varepsilon^2} - \cancelto{0}{12\varepsilon} - 12}{\cancelto{0}{6\varepsilon}-6} = \frac{12}{6}$ $\quad d_1 = 2 > 0$ $\quad \therefore$ d_1 es un numero positivo.

Reemplazamos los valores en la tabla de Routh-Hurwitzs:

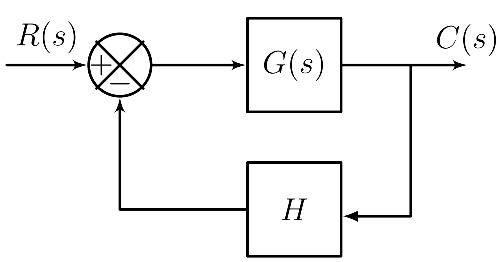
$$
\begin{array}{c|ccc}
s^5 & 1 & 2 & 3 \\
s^4 & 3 & 6 & 3 \\
s^3 & \varepsilon & 2 & 0 \\
s^2 & -\dfrac{6}{\varepsilon} & 3 & 0 \\
s & 2 & 0 & \\
s^0 & 3 & &
\end{array}
$$

$\therefore$ es un sistema inestable con dos polos en el semiplano derecho.

Ejercicio #2

Determinar el rango de k para el cual el sistema es estable, ganancia limite y frecuencia de oscilación.

a. $G(s) = \dfrac{100}{(s-0.5)(s+4)}$; $H(s) = \dfrac{k(s+3)}{s^2+4s+5}$



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)} = \frac{\dfrac{100}{(s-0.5)(s+4)}}{1+\dfrac{100k(s+3)}{(s-0.5)(s+4)(s^2+4s+5)}} = \frac{\dfrac{100}{\cancel{(s-0.5)(s+4)}}}{\dfrac{(s+0.5)(s+4)(s^2+4s+5)+100k(s+3)}{\cancel{(s-0.5)(s+4)}(s^2+4s+5)}}$$

$$\frac{C(s)}{R(s)} = \frac{100(s^2+4s+5)}{(s+0.5)(s+4)(s^2+4s+5)+100k(s+3)} = \frac{100(s^2+4s+5)}{s^4+7.5s^3+17s^2+9.5s+100ks+290}$$

Tomamos el denominador de la FT:

$s^4+7.5s^3+17s^2+(9.5+100k)s+290$

Por metodo de Roouth-Hurwitz, armamos la tabla:

$$\begin{array}{c|lll} s^4 & 1 & 17 & 290 \\ s^3 & 7.5 & (9.5+100k) & 0 \\ s^2 & b_1 & b_2 & 0 \\ s & c_1 & 0 & \\ s^0 & d_1 & & \end{array}$$

$$b_1 = \frac{-\begin{bmatrix} 1 & 17 \\ 7.5 & (9.5+100k) \end{bmatrix}}{7.5} = \frac{-(9.5+100k)+127.5}{7.5} = \frac{118-100k}{7.5}$$

$$b_2 = \frac{-\begin{bmatrix} 1 & 290 \\ 7.5 & 0 \end{bmatrix}}{7.5} = \frac{-(0-2175)}{7.5} = 290$$

$$c_1 = \frac{-\begin{bmatrix} 7.5 & (9.5+100k) \\ \frac{118-100k}{7.5} & 290 \end{bmatrix}}{\frac{118-100k}{7.5}} = -2175 + (9.5+100k) = 100k - 2165.5$$

$$c_2 = \frac{-\begin{bmatrix} \frac{118-100k}{7.5} & 290 \\ 100k-2165.5 & 0 \end{bmatrix}}{100k-2165.5} = 290$$

Reemplazando los valores en la tabla:

$$\begin{array}{c|lll} s^4 & 1 & 17 & 290 \\ s^3 & 7.5 & (9.5+100k) & 0 \\ s^2 & \frac{118-100k}{7.5} & 290 & 0 \\ s & 100k-2165.5 & 0 & \\ s^0 & 290 & & \end{array}$$

Rango de valores de k:

b_1 debe ser mayor a cero para que sea estable $\dfrac{118-100k}{7.5} > 0$

$118 - 100k > 0$

$k < \dfrac{118}{100} \quad \rightarrow \quad k < 1.18$

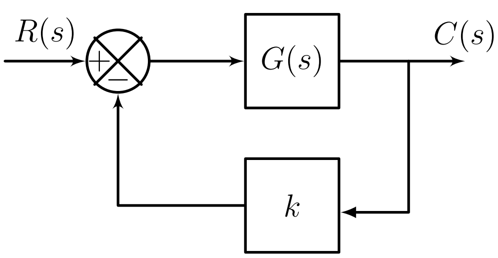
c_1 debe ser mayor a cero para que sea estable $100k - 2165.5 > 0$

$100k > 2165.5 \quad \rightarrow \quad k > 21.655$

$\therefore k_{lim} = 1.18$

Ecuacion auxiliar: $\dfrac{118-100k_{lim}}{7.5}s^2 + 290 = 0$

b. $G(s) = \dfrac{s+1}{s(s-1)(s+6)}$



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)k} = \frac{\dfrac{s+1}{s(s-1)(s+6)}}{1+\dfrac{(s+1)k}{s(s-1)(s+6)}} = \frac{\dfrac{s+1}{\cancel{s(s-1)(s+6)}}}{\dfrac{s(s-1)(s+6)+(s+1)k}{\cancel{s(s-1)(s+6)}}} = \frac{s+1}{s(s-1)(s+6)+(s+1)k}$$

$$\frac{C(s)}{R(s)} = \frac{s+1}{s^3+5s^2+(k-6)s+k}$$

$$\begin{array}{c|cc} s^3 & 1 & (k-6) \\ s^2 & 5 & k \\ s & b_1 & 0 \\ s^0 & c_1 & \end{array}$$

$$b_1 = \frac{-\begin{bmatrix} 1 & (k-6) \\ 5 & k \end{bmatrix}}{5} = \frac{-[k-5(k-6)]}{5} = \frac{4k-30}{5}$$

$$c_1 = \frac{-\begin{bmatrix} 5 & k \\ \dfrac{4k-30}{5} & 0 \end{bmatrix}}{\dfrac{4k-30}{5}} = k$$

Para que sea estable, $k > 0 \quad y \quad \dfrac{4k-30}{5} > 0$

$\dfrac{4k-30}{5} > 0$

$4k - 30 > 0 \quad \rightarrow \quad k > 7.5$

$\therefore k_{lim} = 7.5$

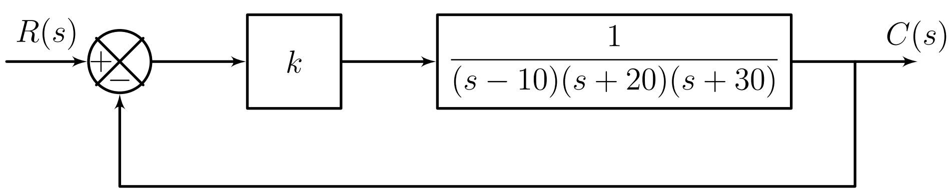
Ecuación auxiliar: $5s^2 + k_{lim} = 0$

$s = j\sqrt{\dfrac{k_{lim}}{5}} = j\omega \qquad \omega = 2\pi f$

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k_{lim}}{5}} = \dfrac{1}{2\pi}\sqrt{\dfrac{7.5}{5}} = 0.195[Hz]$

$T_{osc} = \dfrac{1}{f} = 5.13[s]$

c. Determiar el rango de k para el cual el sistema es estable, ganancia limite y frecuencia de oscilación.



$$\frac{C(s)}{R(s)} = \frac{\dfrac{k}{(s-10)(s+20)(s+30)}}{1+\dfrac{k}{(s-10)(s+20)(s+30)}} = \frac{\dfrac{k}{\cancel{(s-10)(s+20)(s+30)}}}{\dfrac{(s-10)(s+20)(s+30)+k}{\cancel{(s-10)(s+20)(s+30)}}} = \frac{k}{(s-10)(s+20)(s+30)+k}$$

$$\frac{C(s)}{R(s)} = \frac{k}{s^3 + 40s^2 + 100s + (k-6000)}$$

Metodo de Routh-Hurwitz

$$\begin{array}{c|ll} s^3 & 1 & 100 \\ s^2 & 40 & k-6000 \\ s & b_1 & 0 \\ s^0 & c_1 & \end{array}$$

$$b_1 = \frac{-\begin{bmatrix} 1 & 100 \\ 40 & k-6000 \end{bmatrix}}{40} = \frac{-[(k-6000)-4000]}{40} = \frac{10000-k}{40}$$

$$c_1 = \frac{-\begin{bmatrix} 40 & k-6000 \\ \frac{10000-k}{40} & 0 \end{bmatrix}}{\frac{4k-30}{5}} = k-6000$$

Reemplazando los valores en la tabla.

$$\begin{array}{c|ll} s^3 & 1 & 100 \\ s^2 & 40 & k-6000 \\ s & \frac{1000-k}{40} & 0 \\ s^0 & k-6000 & \end{array}$$

Para que sea estable, $k - 6000 > 0 \quad y \quad \dfrac{10000-k}{40} > 0$

$k > 6000$

$$\frac{10000-k}{40} > 0$$

$10000 - k > 0 \quad \rightarrow \quad k < 10000$

El rango de valores de k es: $6000 < k < 10000$

$\therefore k_{lim} = 10000$

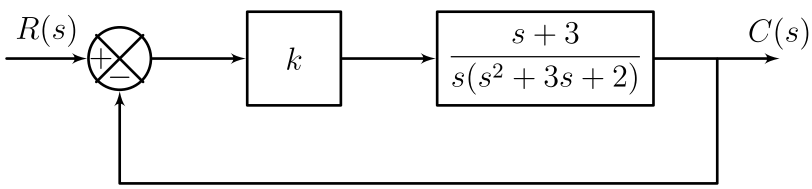
Ecuación auxiliar: $40s^2 + k_{lim} - 6000 = 0$

$$s = j\sqrt{\frac{k_{lim} - 6000}{40}} = j\omega \qquad \omega = 2\pi f$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{lim} - 6000}{40}} = \frac{1}{2\pi}\sqrt{\frac{10000 - 6000}{40}} = \frac{5}{\pi}[Hz]$$

$$T_{osc} = \frac{1}{f} = \frac{\pi}{5}[s]$$

d. Determiar el rango de k para el cual el sistema es estable, ganancia limite y frecuencia de oscilación.



$$\frac{C(s)}{R(s)} = \frac{ks + 3k}{s^3 + 3s^2 + (2+k)s + 3k}$$

Metodo de Routh-Hurwitz:

$$\begin{array}{c|ll} s^3 & 1 & (2+k) \\ s^2 & 3 & 3k \\ s & b_1 & 0 \\ s^0 & c_1 & \end{array}$$

$$b_1 = \frac{-\begin{bmatrix} 1 & (2+k) \\ 3 & 3k \end{bmatrix}}{3} = \frac{-[3k - (6+3k)]}{3} = \frac{6 - 6k}{3} = 2 - 2k$$

$$c_1 = \frac{-\begin{bmatrix} 3 & 3k \\ 2-2k & 0 \end{bmatrix}}{2 - 2k} = 3k$$

Reemplazando los valores en la tabla.

$$\begin{array}{c|ll} s^3 & 1 & (2+k) \\ s^2 & 3 & 3k \\ s & 2-2k & 0 \\ s^0 & 3k & \end{array}$$

Para que sea estable, $3k > 0 \quad y \quad 2-2k > 0$

$k > 0$

$2 - 2k > 0$

$-2k > -2 \quad \rightarrow \quad k < 1$

El rango de valores de k es: $0 < k < 1$

$\therefore k_{lim} = 1$

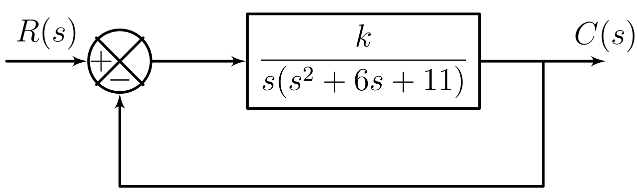
Ecuación auxiliar: $3s^2 + 3k_{lim} = 0$

$s = j\sqrt{k_{lim}} = j\omega \qquad \omega = 2\pi f$

$f = \dfrac{1}{2\pi}\sqrt{k_{lim}} = \dfrac{1}{2\pi}\sqrt{1} = \dfrac{1}{2\pi}[Hz]$

$T_{osc} = \dfrac{1}{f} = 2\pi[s]$

e. Determiar el rango de k para el cual el sistema es estable, ganancia limite y frecuencia de oscilación.



$$\frac{C(s)}{R(s)} = \frac{k}{s^3 + 6s^2 + 11s + k}$$

Metodo de Routh-Hurwitz:

$$\begin{array}{c|ll} s^3 & 1 & 11 \\ s^2 & 6 & k \\ s & b_1 & 0 \\ s^0 & c_1 & \end{array}$$

$$b_1 = \frac{-\begin{bmatrix} 1 & 11 \\ 6 & k \end{bmatrix}}{6} = \frac{-(k-66)}{6} = \frac{66-k}{6}$$

$$c_1 = \frac{-\begin{bmatrix} 6 & k \\ \frac{66-k}{6} & 0 \end{bmatrix}}{\frac{66-k}{6}} = k$$

Reemplazando los valores en la tabla.

$$\begin{array}{c|ll} s^3 & 1 & 11 \\ s^2 & 6 & k \\ s & \frac{66-k}{6} & 0 \\ s^0 & k & \end{array}$$

Para que sea estable, $k > 0 \quad y \quad \frac{66-k}{6} > 0$

$k > 0$

$\frac{66-k}{6} > 0$

$-k > -66 \quad \rightarrow \quad k < 66$

El rango de valores de k es: $0 < k < 66$

$\therefore k_{lim} = 66$

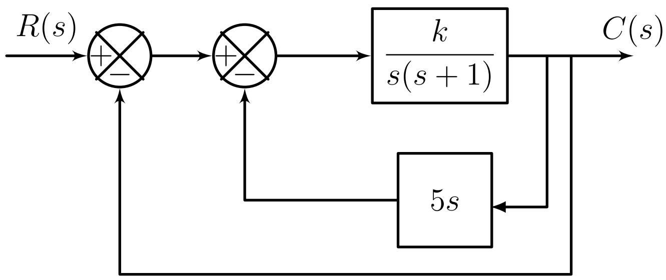
Ecuación auxiliar: $s^2 + k_{lim} = 0$

$s = j\sqrt{\frac{k_{lim}}{6}} = j\omega \qquad \omega = 2\pi f$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{lim}}{6}} = \frac{1}{2\pi}\sqrt{\frac{66}{6}} = 0.53[Hz]$$

$$T_{osc} = \frac{1}{f} = 1.89[s]$$

f. Determiar el rango de k para el cual el sistema es estable, ganancia limite y frecuencia de oscilación.



g. Determiar el rango de k para el cual el sistema es estable, ganancia limite y frecuencia de oscilación.