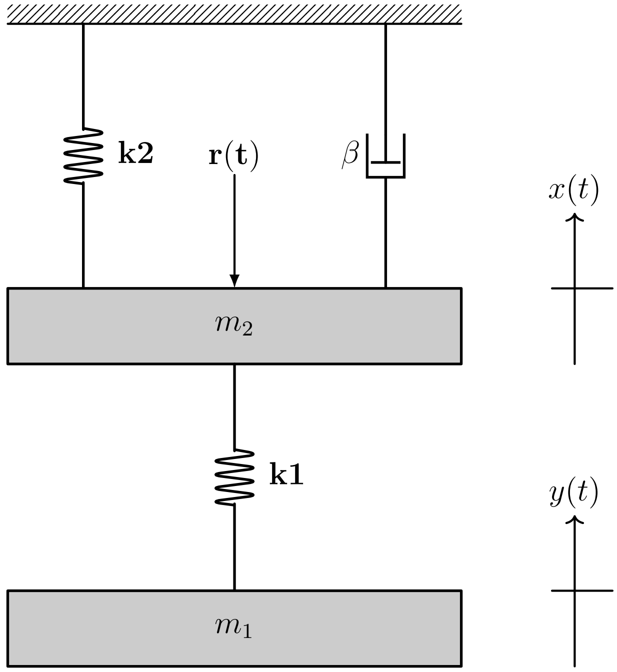
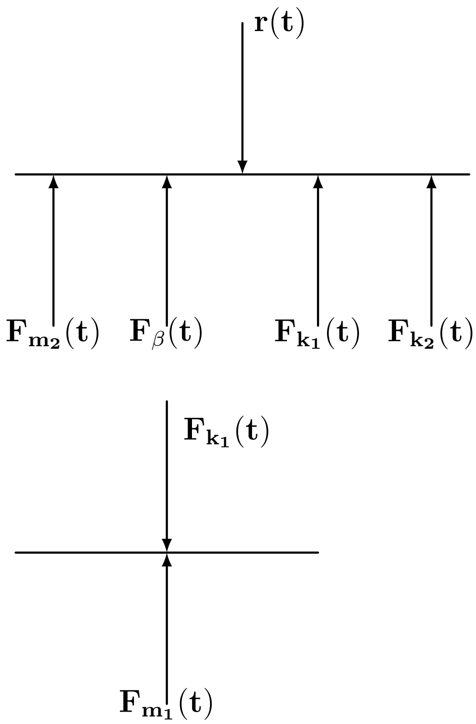
1 Ejercicio # 1

Hallar las ecuaciónes de función de transferencia: $FT_1(s) = \dfrac{X(s)}{R(s)}$ y $FT_2(s) = \dfrac{Y(s)}{R(s)}$.





Solucion:

1. Tenemos las siguientes ecuaciones:

   $F_{m_1} = m_1\dfrac{d^2y(t)}{dt^2}$; $F_{m_2} = m_2\dfrac{d^2x(t)}{dt^2}$; $F_\beta = \beta\dfrac{dx(t)}{dt}$; $F_{k_1} = k_1[x(t) - y(t)]$; $F_{k_2} = k_2x(t)$.

2. Hacemos la sumatoria de las fuerzas (DCL):

   Para la masa 2.

$$F_{k_1} + F_{k_2} + F_\beta + F_{m_2} = r(t)$$

$$[x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2} = r(t)$$

   Para la masa 1.

$$-F_{k_1} + F_{m_1} = 0$$

$$-[x(t) - y(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

$$[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

3. Aplicamos la transformada de Laplace: $y'(0) = y(0) = x'(0) = x(0) = 0$

   Para la primera ecuación:

$$\begin{aligned} [x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2} &= r(t) \\ \mathscr{L}\left\{[x(t) - y(t)]k_1 + x(t)k_2 + \beta\frac{dx(t)}{dt} + m_2\frac{d^2x(t)}{dt^2}\right\} &= \mathscr{L}\{r(t)\} \\ [X(s) - Y(s)]k_1 + X(s)k_2 + S\beta X(s) + m_2S^2X(s) &= R(t) \end{aligned}$$

   Para la segunda ecuación:

$$[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2} = 0$$

$$\mathscr{L}\left\{[y(t) - x(t)]k_1 + m_1\frac{d^2y(t)}{dt^2}\right\} = 0$$

$$[Y(s) - X(s)]k_1 + m_1S^2Y(s) = 0$$

4. Ordenamos las ecuaciones:

$$\overbrace{(k_1 + k_2S\beta + m_2S^2)}^{a}X(s) - \overbrace{k_1}^{b}Y(s) = R(s) \tag{1}$$

$$\overbrace{-k_1}^{c}X(s) + \overbrace{(k_1 + m_1S^2)}^{d}Y(s) = 0 \tag{2}$$

5. Resolvemos las ecuaciones por el metodo de cramer:

$$X(s) = \frac{\begin{vmatrix} R(s) & b \\ 0 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{R(s)\cdot d - 0\cdot b}{a\cdot d - c\cdot b} = \frac{R(s)d}{ad - cb} = \frac{(k_1 + m_1S^2)R(s)}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1(k_1)}$$

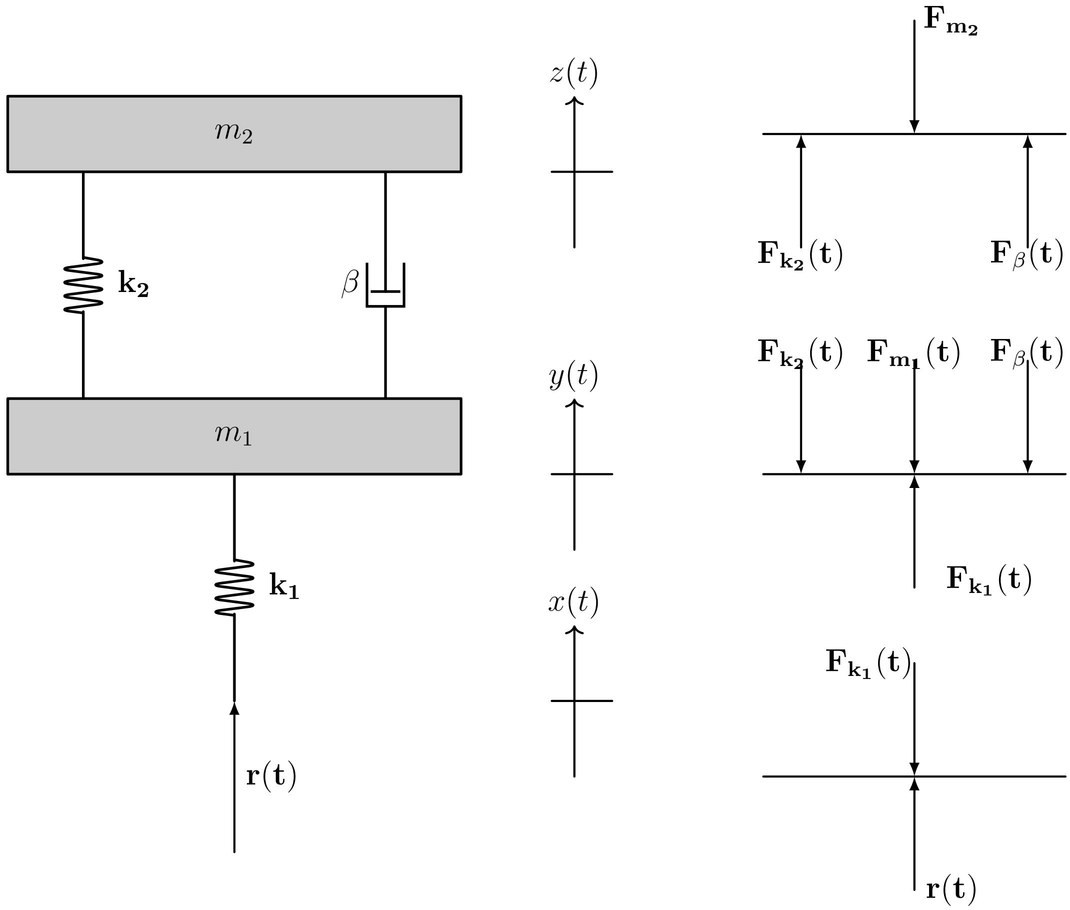
$$\boxed{FT_1(s) = \frac{X(s)}{R(s)} = \frac{k_1 + m_1S^2}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1^2}}$$

$$Y(s) = \frac{\begin{vmatrix} a & R(s) \\ c & 0 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{a\cdot 0 - c\cdot R(s)}{a\cdot d - c\cdot b} = \frac{-cR(s)}{ad - cb} = \frac{k_1R(s)}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1(k_1)}$$

$$\boxed{FT_2(s) = \frac{Y(s)}{R(s)} = \frac{k_1}{(k_1 + k_2S\beta + m_2S^2)(k_1 + m_1S^2) - k_1^2}}$$

2 Ejercicio # 2

Hallar las ecuaciónes de función de transferencia: $FT_1(s) = \dfrac{X(s)}{R(s)}$ y $FT_2(s) = \dfrac{Y(s)}{R(s)}$.



Solucion:

1. Tenemos las siguientes ecuaciones:

   $F_{m_1} = m_1 \dfrac{d^2y(t)}{dt^2}$; $F_{m_2} = m_2 \dfrac{d^2z(t)}{dt^2}$;

   $F_{k_1} = k_1[x(t) - y(t)]$ visto desde $x(t)$; $F_{k_2} = k_2[y(t) - z(t)]$ visto desde $y(t)$;

   $F_\beta = \beta \left[\dfrac{dy(t)}{dt} - \dfrac{dz(t)}{dt}\right]$ visto desde $y(t)$;

2. Hacemos la sumatoria de las fuerzas (DCL):

   Para el punto donde se encuentra la perturbación:

$$-[x(t) - y(t)]k_1 + r(t) = 0$$
$$[x(t) - y(t)]k_1 = r(t)$$

Para la masa 1.

$$F_{k_1} - F_{k_2} - F_\beta - F_{m_1} = 0$$

$$[x(t) - y(t)]k_1 - [y(t) - z(t)]k_2 - \beta \left[\frac{dy(t)}{dt} - \frac{dz(t)}{dt}\right] - m_1 \frac{d^2y(t)}{dt^2} = 0$$

Para la masa 2.

$$F_{k_2} + F_\beta - F_{m_2} = 0$$

$$[y(t) - z(t)]k_2 + \beta \left[\frac{dy(t)}{dt} - \frac{dz(t)}{dt}\right] - m_2 \frac{d^2z(t)}{dt^2} = 0$$

3. Aplicamos la transformada de Laplace: $z'(0) = z(0) = y'(0) = y(0) = x'(0) = x(0) = 0$

   Para la primera ecuación:

$$[x(t) - y(t)]k_1 = r(t)$$

$$\mathscr{L}\{[x(t) - y(t)]k_1\} = \mathscr{L}\{r(t)\}$$

$$[X(s) - Y(s)]k_1 = R(s)$$

   Para la segunda ecuación:

$$[x(t) - y(t)]k_1 - [y(t) - z(t)]k_2 - \beta \left[\frac{dy(t)}{dt} - \frac{dz(t)}{dt}\right] - m_1 \frac{d^2y(t)}{dt^2} = 0$$

$$\mathscr{L}\left\{[x(t) - y(t)]k_1 - [y(t) - z(t)]k_2 - \beta \left[\frac{dy(t)}{dt} - \frac{z(t)}{dt}\right] - m_1 \frac{d^2y(t)}{dt^2}\right\} = 0$$

$$[X(s) - Y(s)]k_1 - [Y(s) - Z(s)]k_2 - \beta S[Y(s) - Z(s)] - m_1 S^2 Y(s) = 0$$

   Para la tercera ecuación:

$$[y(t) - z(t)]k_2 + \beta \left[\frac{dy(t)}{dt} - \frac{dz(t)}{dt}\right] - m_2 \frac{d^2z(t)}{dt^2} = 0$$

$$\mathscr{L}\left\{[y(t) - z(t)]k_2 + \beta \left[\frac{dy(t)}{dt} - \frac{dz(t)}{dt}\right] - m_2 \frac{d^2z(t)}{dt^2}\right\} = 0$$

$$[Y(s) - Z(s)]k_2 + \beta S[Y(s) - Z(s)] - m_2 S^2 Z(s) = 0$$

4. Ordenamos las ecuaciones:

$$\overbrace{k_1}^{a} X(s) - \overbrace{k_1}^{b} Y(s) + \overbrace{0}^{c} Z(s) = R(s) \tag{1}$$

$$\overbrace{k_1}^{d} X(s) - \overbrace{(k_1 + k_2 + \beta S + m_2 S^2)}^{e} Y(s) + \overbrace{(k_2 + \beta S)}^{f} Z(s) = 0 \tag{2}$$

$$\overbrace{0}^{g} X(s) + \overbrace{(k_2 + \beta S)}^{h} Y(s) - \overbrace{(\beta S + m_2 S^2)}^{i} Z(s) = 0 \tag{3}$$

5. Resolvemos las ecuaciones por el metodo de cramer:

$$X(s) = \frac{\begin{vmatrix} R(s) & -b & c \\ 0 & -e & f \\ 0 & h & -i \end{vmatrix}}{\begin{vmatrix} a & -b & c \\ d & -e & f \\ g & h & -i \end{vmatrix}} = \frac{R(s)ei - bf0 + 0hc - [-0ec + 0bi + hfR(s)]}{aei - \cancel{bfg} + \cancel{dhc} - [-\cancel{gec} + dbi + hfa]} = \frac{(ei - hf)R(s)}{aei - dbi - hfa}$$

$$X(s) = \frac{(k_1 + k_2 + \beta S + m_2 S^2)(\beta S + m_2 S^2)R(s)}{k_1(k_1 + k_2 + \beta S + m_2 S^2)(\beta S + m_2 S^2) - k_1^2(\beta S + m_2 S^2) - (k_2 + \beta S)(k_2 + \beta S)k_1}$$

$$\boxed{FT_1(s) = \frac{X(s)}{R(s)} = \frac{(k_1 + k_2 + \beta S + m_2 S^2)(\beta S + m_2 S^2)}{k_1(k_1 + k_2 + \beta S + m_2 S^2)(\beta S + m_2 S^2) - k_1^2(\beta S + m_2 S^2) - (k_2 + \beta S)(k_2 + \beta S)k_1}}$$

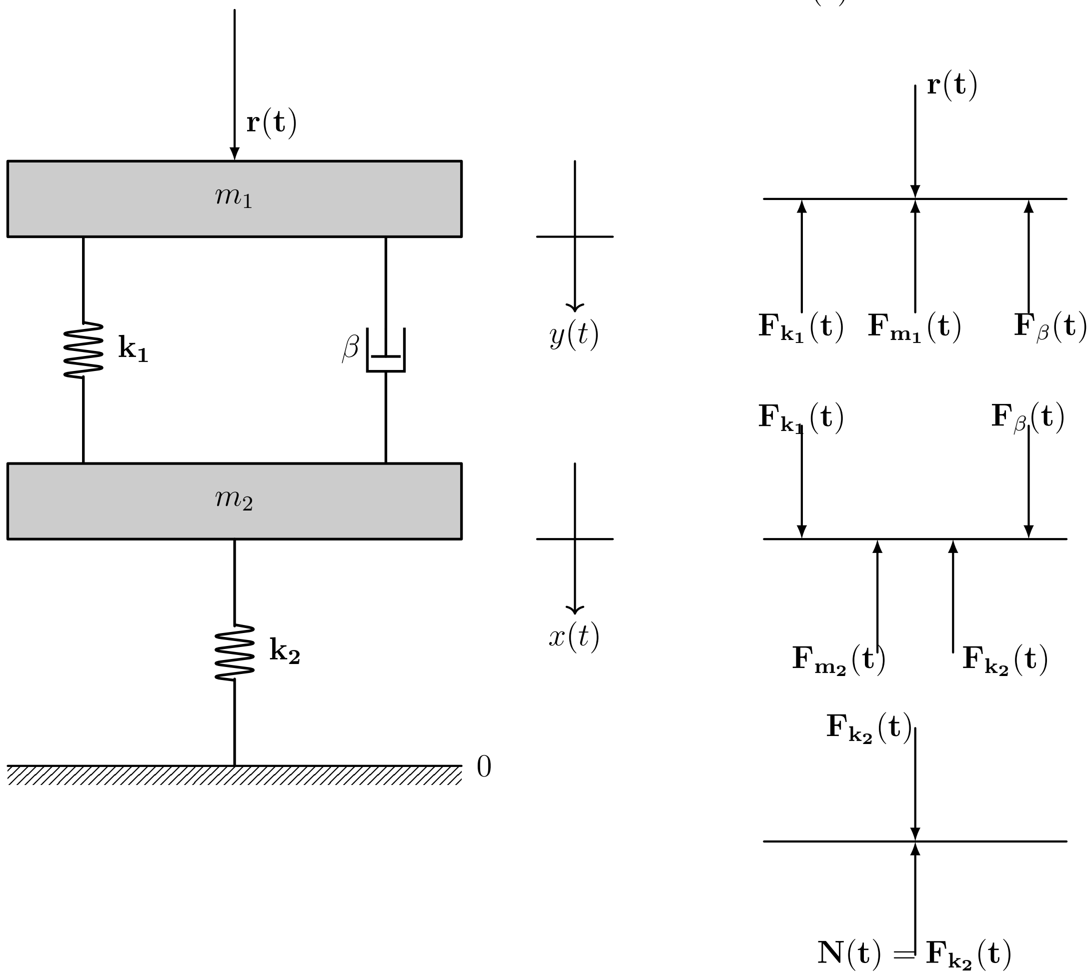
$$Y(s) = \frac{\begin{vmatrix} a & R(s) & 0 \\ d & 0 & f \\ 0 & 0 & -i \end{vmatrix}}{\begin{vmatrix} a & -b & 0 \\ d & -e & f \\ 0 & h & -i \end{vmatrix}} = \frac{dR(s)i}{aei - dbi - hfa}$$

$$Y(s) = \frac{K_1(\beta S + m_2 S^2)R(s)}{k_1(k_1 + k_2 + \beta S + m_2 S^2)(\beta S + m_2 S^2) - k_1^2(\beta S + m_2 S^2) - (k_2 + \beta S)(k_2 + \beta S)k_1}$$

$$\boxed{FT_2(s) = \frac{Y(s)}{R(s)} = \frac{K_1(\beta S + m_2 S^2)}{k_1(k_1 + k_2 + \beta S + m_2 S^2)(\beta S + m_2 S^2) - k_1^2(\beta S + m_2 S^2) - (k_2 + \beta S)(k_2 + \beta S)k_1}}$$

3 Ejercicio # 3

Hallar las ecuaciónes de función de transferencia: $FT_1(s) = \dfrac{X(s)}{R(s)}$ y $FT_2(s) = \dfrac{Y(s)}{R(s)}$.



Solucion:

1. Tenemos las siguientes ecuaciones:

   $F_{m_1} = m_1 \dfrac{d^2y(t)}{dt^2}$; $F_{m_2} = m_2 \dfrac{d^2x(t)}{dt^2}$;

   $F_\beta = \beta \left[\dfrac{dy(t)}{dt} - \dfrac{dz(t)}{dt} \right]$ visto desde $y(t)$;

   $F_{k_1} = k_1[y(t) - x(t)]$ visto desde $y(t)$; $F_{k_2} = k_2 x(t)$ visto desde $x(t)$;

2. Hacemos la sumatoria de las fuerzas (DCL):

   Para el punto donde se encuentra la perturbación:

$$r(t) - F_{k_1}(t) - F_{m_1}(t) - F_\beta(t) = 0$$

$$[y(t) - x(t)]k_1 + m_1 \frac{d^2y}{dt^2} + \beta \left[\frac{dy}{dt} - \frac{dx}{dt} \right] = r(t)$$

Para la masa 2.

$$F_{k_1} - F_{m_2} - F_{k_2} + F_\beta = 0$$

$$[y(t) - x(t)]k_1 - m_2\frac{d^2x}{dt^2} - k_2x(t) + \beta\left[\frac{dy(t)}{dt} - \frac{dx(t)}{dt}\right] = 0 \quad (-1)$$

$$m_2\frac{d^2x}{dt^2} + \beta\left[\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right] + k_1[x(t) - y(t)] + k_2x(t) = 0$$

3. Aplicamos la transformada de Laplace: $y'(0) = y(0) = x'(0) = x(0) = 0$

   Para la primera ecuación:

$$[y(t) - x(t)]k_1 + m_1\frac{d^2y}{dt^2} + \beta\left[\frac{dy}{dt} - \frac{dx}{dt}\right] = r(t)$$

$$\mathscr{L}\left\{[y(t) - x(t)]k_1 + m_1\frac{d^2y}{dt^2} + \beta\left[\frac{dy}{dt} - \frac{dx}{dt}\right]\right\} = \mathscr{L}\{r(t)\}$$

$$k_1[Y(s) - X(s)] + m_1S^2Y(s) + \beta S[Y(s) - X(s)] = R(s)$$

   Para la segunda ecuación:

$$m_2\frac{d^2x}{dt^2} + \beta\left[\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right] + k_1[x(t) - y(t)] + k_2x(t) = 0$$

$$\mathscr{L}\left\{m_2\frac{d^2x}{dt^2} + \beta\left[\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right] + k_1[x(t) - y(t)] + k_2x(t)\right\} = 0$$

$$m_2S^2X(s) + \beta S[X(s) - Y(s)] + k_1[X(s) - Y(s)] + k_2X(s) = 0$$

4. Ordenamos las ecuaciones:

$$\overbrace{-(k_1 + \beta S)}^{a}X(s) + \overbrace{(k_1 + \beta S + m_1S^2)}^{b}Y(s) = R(s) \tag{1}$$

$$\overbrace{(k_1 + k_2 + \beta S + m_2S^2)}^{c}X(s) - \overbrace{(k_1 + \beta S)}^{d}Y(s) = 0 \tag{2}$$

5. Resolvemos las ecuaciones por el metodo de cramer:

$$X(s) = \frac{\begin{vmatrix} R(s) & b \\ 0 & -d \end{vmatrix}}{\begin{vmatrix} -a & b \\ c & -d \end{vmatrix}} = \frac{-R(s)d - 0b}{ad - cb} = \frac{-dR(s)}{ad - cb}$$

$$X(s) = \frac{-(k_1 + \beta S)R(s)}{(k_1 + \beta S)(k_1 + \beta S) - (k_1 + k_2 + \beta S + m_2S^2)(k_1 + \beta S + m_1S^2)}$$

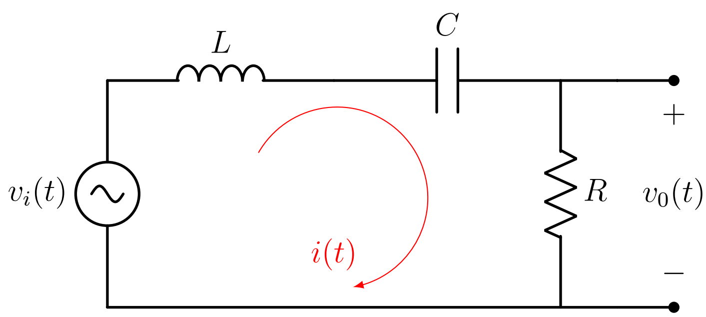
$$\boxed{FT_1(s) = \frac{X(s)}{R(s)} = \frac{-(k_1+\beta S)}{(k_1+\beta S)(k_1+\beta S)-(k_1+k_2+\beta S+m_2S^2)(k_1+\beta S+m_1S^2)}}$$

$$\begin{aligned}
Y(s) &= \frac{\begin{vmatrix} -a & R(s) \\ c & 0 \end{vmatrix}}{\begin{vmatrix} -a & b \\ c & -d \end{vmatrix}} = \frac{-a0-cR(s)}{ad-cb} = \frac{-cR(s)}{ad-cb} \\
Y(s) &= \frac{-(k_1+k_2+\beta S+m_2S^2)R(s)}{(k_1+\beta S)(k_1+\beta S)-(k_1+k_2+\beta S+m_2S^2)(k_1+\beta S+m_1S^2)}
\end{aligned}$$

$$\boxed{FT_2(s) = \frac{Y(s)}{R(s)} = \frac{-(k_1+k_2+\beta S+m_2S^2)}{(k_1+\beta S)(k_1+\beta S)-(k_1+k_2+\beta S+m_2S^2)(k_1+\beta S+m_1S^2)}}$$

4 Ejercicio # 4

Hallar: $\frac{V_o(s)}{V_i(s)}; \frac{I(s)}{V_i(s)}$



Aplicamos la ley de tensión Kirchhoff(LTK):

$$v_i(t) = v_L(t) + v_C(t) + v_R(t)$$

Donde:

$$\begin{aligned} v_L(t) &= L\frac{\mathrm{d}i(t)}{\mathrm{d}t} \\ v_C(t) &= \frac{1}{C}\int i_C(t)\mathrm{d}t \\ v_R(t) &= Ri(t) \end{aligned}$$

Ecuación diferencial del circuito RLC:

$$v_i(t) = L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i_C(t)\mathrm{d}t + Ri(t)$$

Aplicamos la transformada de Laplace:

$$\mathscr{L}\{v_i(t)\} = \mathscr{L}\{v_L(t) + v_C(t) + v_R(t)\}$$

$$\mathscr{L}\{v_i(t)\} = \mathscr{L}\left\{L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i_C(t)\mathrm{d}t + Ri(t)\right\}$$

$$V_i(s) = LSI(s) + \frac{1}{CS}I(s) + RI(s)$$

$$V_i(s) = I(s)\left(LS + \frac{1}{CS} + R\right)$$

$$V_i(s) = I(s)\left(\frac{S^2LC + 1 + SRC}{SC}\right)$$

$$I(s) = V_i(s)\left(\frac{SC}{S^2LC + 1 + SRC}\right)$$

Aplicamos la Ley de tension de Kirchhoff:

$$-v_o(t) + v_R(t) = 0$$

$$v_o(t) = v_R(t) = Ri(t)$$

$$\mathscr{L}\{v_o(t)\} = \mathscr{L}\{Ri(t)\}$$

$$V_o(s) = RI(s)$$

Luego reemplazamos:

$$V_o(s) = R\left[V_i(s)\left(\frac{SC}{S^2LC + 1 + SRC}\right)\right]$$

$$V_o(s) = V_i(s)\left(\frac{SRC}{S^2LC + 1 + SRC}\right)$$

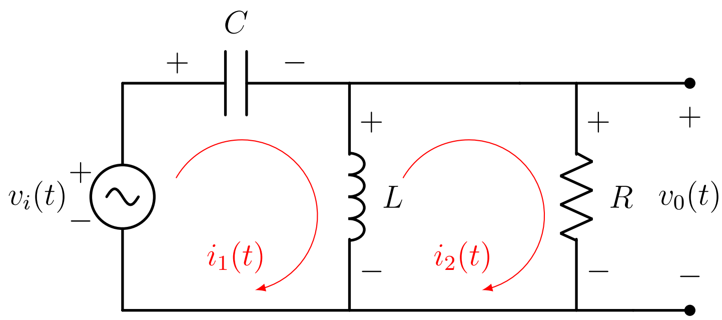
Solución:

$$\boxed{\frac{I(s)}{V_i(s)} = \frac{SC}{S^2LC + 1 + SRC}}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{SRC}{S^2LC + 1 + SRC}}$$

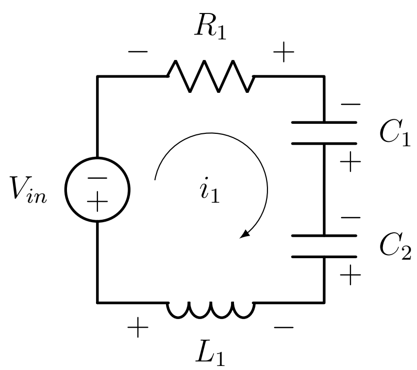
5 Ejercicio # 5

Hallar: $\dfrac{V_o(s)}{V_i(s)}$



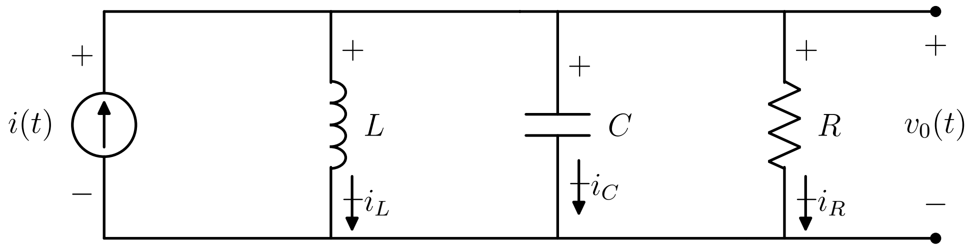
Aplicamos la ley de tensión Kirchhoff(LTK):

aAplicamos la ley de Kirchhoff de las mallas:



6 Ejercicio # 6

Hallar: $\dfrac{V_o(s)}{V_i(s)}$



7 Ejercicio # 7

Hallar: $\dfrac{V_o(s)}{V_i(s)}; \dfrac{I(s)}{V_i(s)}$