

Modelling and quantifying mortality and longevity risk

Module D2 : Pricing Longevity Risk

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“Pricing” of longevity risk (and “derivatives”)

Pricing Longevity Risk

- Most insurers / pension funds are short macro longevity.
- Macro Longevity risk cannot be diversified away in similar products if it is consistent across ages. It is therefore a risk which should be **compensated** in pricing.
- If no (liquid) market for longevity products exist, it is **impossible** to use riskneutral pricing methodology.
- Since macro longevity risk cannot be neutralized, universal fair price cannot be obtained so it must be based on risk appetite:
 - Use explicit model for risk aversion (eg. utility indifference pricing)
 - Use actuarial premium principle (eg. standard deviation)
 - Use asset performance criterion (eg. Sharpe ratio).

Pricing Longevity Risk: Insurance Company

From point of view of protection providing insurance company:

- Longevity swap risk is based on **shock to mortality rates**
- This translates into higher capital requirements (SCR, EC) which are based on mortality shocks, other shocks and their **correlations**
- If more capital needs to be set aside this translates, using a **cost-of- capital** method, into higher risk margins
- Risk premium charged should compensate total costs. Price depends on all elements in red above.

Implicit risk preferences: Solvency II

- Clear example how **NOT** to analyze longevity risk: Solvency II ...

GELDEND

Delegated Regulation (EU) 2015/35 supplementing Directive 2009/138/EC of the European Parliament and of the Council on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II)

Article 138 Longevity risk sub-module

The capital requirement for longevity risk referred to in [Article 105\(3\)\(b\)](#) of Directive 2009/138/EC shall be equal to the loss in basic own funds of insurance and reinsurance undertakings that would result from an instantaneous permanent decrease of 20 % in the mortality rates used for the calculation of technical provisions.

- Defines risk assessment based on
 - **uniform shock over ages**, so structure of improvements ignored,
 - **99.5% Value at Risk**, so effect tail events ignored,
 - **over single year**, so long term trends are ignored.

Pricing Longevity Risk: Investors

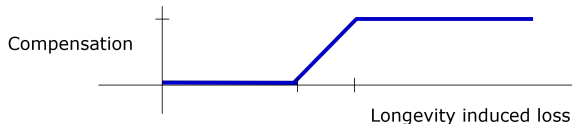
From point of view of protection provider (if not short longevity himself):

- May be interested in earning risk premium on an asset class which is **usually uncorrelated** with other asset classes
- Pricing principles 'supply side':
 - Specify Sharpe ratio:
excess rate of return per standard deviation, often quoted: 25% (Sharpe ratio equity)
 - Specify transformed probabilities
(from best estimates of probabilities), to include risk premium
 - By defining a **Wang transform**
may introduce arbitrage, **strongly discouraged**
 - By defining a **market price of diffusion risk**
can be done in consistent, arbitrage-free way.

Pricing Longevity Risk: Pension Fund

- Two parties (say pension fund and protection provider) may not agree on uncertainty in, or even **best estimates** of, mortality rates. If they don't, longevity swap basically becomes a 'model bet'
- **Diversification effects** mean that pension fund will not have the full exposure to mortality shock, which reduces critical risk premium
- If pension fund just wants protection against extreme longevity scenario's, it may be more natural to look at a **longevity collar**, paying

$$\min(P_{\max}, \max(0, {}_k p_{65} - {}_k \hat{p}_{65} - P_{\min}))$$



Future liquidity of Longevity Swaps

- Most successful derivative structures in the market place
 - are not 'model bets' or 'parameter bets' (short term profit),
 - but are repeatable under similar circumstances (long term reduction of uncertainty).
- Example: commodity futures.
- Important in pension context:
 - How well can off-setting effects of mortality and its hedge be communicated to participants in adverse scenarios?
 - This is extra difficult due to structure of swap product.

Future liquidity of Longevity Swaps

- Is it really a derivative, or a form of (re-)insurance ?
- Compare (classification of) credit default swaps.
- No tradeable underlying so (almost) no asset pricing theory ...

Utility Indifference Pricing

- Economic agent with wealth W needs to consider whether he/she would like to avoid an **immediate** stochastic payoff $-X$ with $\mathbb{E}X = 0$ for a certain price P . Since X adds risk without return, a risk averse agent is willing to pay to avoid this risk. The maximal price P the agent would be willing to pay should satisfy

$$U(W - P) = \mathbb{E}U(W - X).$$

for a utility function U .

Utility Indifference Pricing

- Taylor Expansion of $U(W - P) = \mathbb{E}U(W - X)$ gives

$$U(W) - PU'(W) + o(P) = \mathbb{E}[U(W) - XU'(W) + \frac{1}{2}X^2U''(W) + o(X^2)]$$

so

$$P \approx -\frac{1}{2} \frac{U''(W)}{U'(W)} \mathbb{E}X^2 = \frac{1}{2} r_U(W) \text{Var}X$$

with $r_U(x) = -U''(x)/U'(x)$ the [Arrow-Pratt measure of absolute risk aversion](#).

- For HARA class it has the form

$$r_U(x) = \frac{1}{a + bx}$$

for certain a and b .

- Not utility function but risk aversion function is natural representation of risk attitude of an agent.

Utility Indifference Pricing

- For **exponential utility** function $U(x) = -e^{-\gamma x}/\gamma$ (with $\gamma > 0$), we find for current wealth W , risky payoff X and indifference premium P

$$U(W - P) = \mathbb{E}U(W - X) \quad \Rightarrow \quad P = \frac{\ln \mathbb{E}[e^{\gamma X}]}{\gamma},$$

so premium (and risk aversion) do not depend on wealth W .

- For γ very close to zero we can use a second order moment approximation for exponential utility:

$$P \approx \mathbb{E}[X] + \frac{1}{2}\gamma \text{Var}[X].$$

- For **general utility functions** we find:

$$P = W - U^{-1}(\mathbb{E}U(W - X)).$$

A Portfolio of Annuities

- As an example, we use the exponential utility second-order moment approximation to analyze longevity risk in a portfolio.
- Take M policyholders indicated by $i \in \{1, 2, \dots, M\}$ with ages $x(i)$ who are alive today and die at the stochastic time T_i . Then discounted value of all annual annuity payments (of 1 euro) to policyholder i equals

$$A(i) := \sum_{k=1}^{\omega} d_{0,k} \mathbf{1}_{T_i > k}$$

where $\omega = 120$ is enough to make sure no one survives after ω yearly periods.

- Value of whole annuity portfolio is thus

$$A := \sum_{i=1}^M A(i) = \sum_{i=1}^M \sum_{k=1}^{\omega} d_{0,k} \mathbf{1}_{T_i > k}.$$

We assume that discount rates $d_{0,k}$ are deterministic here.

Systematic and Non-Systematic Longevity Risk

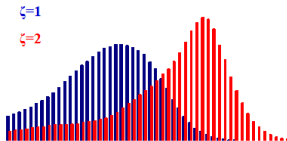
- Now assume that survival probabilities are stochastic themselves: they depend on a stochastic scenario ζ , so

$$\mathbb{P}(T_i > k | \zeta) := {}_k p_{x(i)}(\zeta).$$

- We do not yet know which mortality scenario will materialize, only that there is a distribution

$$\mathbb{P}(\zeta = j) = \tilde{p}_j$$

for possible scenarios $j \in \{1, 2, \dots, J\}$ with probabilities \tilde{p}_j that sum to one.



Implied distribution of age at death, under two scenarios.

Systematic and Non-Systematic Longevity Risk

- If we use our second order moment approximation for exponential indifference premiums, we should charge policyholder i a premium $P(i)$ such that

$$\begin{aligned}P &= \sum_{i=1}^M P(i) = \mathbb{E}\left[\sum_{i=1}^M A(i)\right] + \frac{1}{2}\gamma \operatorname{Var}\left[\sum_{i=1}^M A(i)\right] \\&= \mathbb{E}\left[\sum_{i=1}^M \sum_{k=1}^{\omega} d_{0,k} \mathbf{1}_{T_i > k}\right] + \frac{1}{2}\gamma \operatorname{Var}\left[\sum_{i=1}^M \sum_{k=1}^{\omega} d_{0,k} \mathbf{1}_{T_i > k}\right] \\&= \sum_{k=1}^{\omega} d_{0,k} \sum_{i=1}^M \mathbb{E}[{}_k p_{x(i)}(\zeta)] + \frac{1}{2}\gamma \operatorname{Var}\left[\sum_{k=1}^{\omega} d_{0,k} \sum_{i=1}^M \mathbf{1}_{T_i > k}\right]\end{aligned}$$

- First term is simply **expectation over all longevity scenarios**.
- In second term we have interaction of **systematic longevity risk** (variance due to unknown scenario ζ , 'macro') and **idiosyncratic longevity risk** (variance due to individual deaths given the scenario, 'micro').

Variance Decomposition Rule

- We can decompose the risk premium component in the variance using the following general rule for any stochastic variables X and Y :

$$\text{Var}[X] = \mathbb{E}[\text{Var}[X|Y]] + \text{Var}[\mathbb{E}[X|Y]]$$

- This rule follows easily when we write

$$\begin{aligned}\mathbb{E}[\text{Var}[X|Y]] &= \mathbb{E}[\mathbb{E}[X^2|Y] - \mathbb{E}[X|Y]^2] \\ \text{Var}[\mathbb{E}[X|Y]] &= \mathbb{E}[\mathbb{E}[X|Y]^2] - (\mathbb{E}[\mathbb{E}[X|Y]])^2\end{aligned}$$

since summation then gives

$$\mathbb{E}(\mathbb{E}[X^2|Y]) - (\mathbb{E}[X])^2 = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \text{Var}[X].$$

Decomposition of Longevity Risk Premia

- We decompose the variance term for the premium, using this rule for $Y = \zeta$:

$$\begin{aligned} \frac{1}{2}\gamma \text{Var}\left[\sum_{k=1}^{\omega} d_{0,k} \sum_{i=1}^M \mathbf{1}_{T_i > k}\right] &= \frac{1}{2}\gamma \mathbb{E}\left[\text{Var}\left[\sum_{k=1}^{\omega} d_{0,k} \sum_{i=1}^M \mathbf{1}_{T_i > k} \mid \zeta\right]\right] \\ &\quad + \frac{1}{2}\gamma \text{Var}\left[\mathbb{E}\left[\sum_{k=1}^{\omega} d_{0,k} \sum_{i=1}^M \mathbf{1}_{T_i > k} \mid \zeta\right]\right] \end{aligned}$$

- If policyholder deaths are independent for a given survival table ζ , we find that this equals

$$\sum_{i=1}^M \frac{1}{2}\gamma \mathbb{E}\left[\text{Var}\left[\sum_{k=1}^{\omega} d_{0,k} \mathbf{1}_{T_i > k} \mid \zeta\right]\right] + \frac{1}{2}\gamma \text{Var}\left[\sum_{k=1}^{\omega} d_{0,k} \sum_{i=1}^M k p_{x(i)}(\zeta)\right].$$

Decomposition of Longevity Risk Premia

$$\sum_{i=1}^M \frac{1}{2} \gamma \mathbb{E}[\text{Var}[\sum_{k=1}^{\omega} d_{0,k} \mathbf{1}_{T_i > k} \mid \zeta]] + \frac{1}{2} \gamma \text{Var}[\sum_{k=1}^{\omega} d_{0,k} \sum_{i=1}^M k p_{x(i)}(\zeta)]$$

Note:

- First part of premium (average over all scenarios for variance in payoff of policyholder i) can be attributed to individual policyholder.
- Second part (extra variance due to joint dependence on longevity trends) does not diversify and must be spread over individuals in the collective!

Key References

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