Modelling and quantifying mortality and longevity risk

Module C.2 on Statistical and machine learning methods for fine-grained mortality data

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RCLR Research Centre for Longevity Risk

Learning outcomes of this module

In this module you will learn:

- pros and cons of using machine and deep learning methods for modelling mortality data
- how to specify a baseline model (of type Serfling) for short-term mortality
- how to use gradient boosting as a learning method to exlore associations between short term mortality and a high-dimensional set of weather and pollution features.

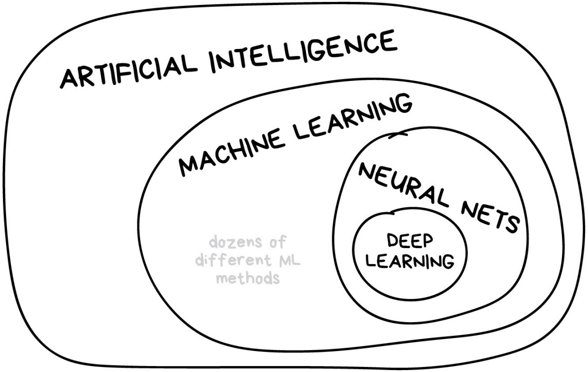
Outline

Use of machine learning techniques in mortality modelling

ML, DL and mortality modelling

In Module 6 on portfolio data we reflected on the use of machine or deep learning methods:

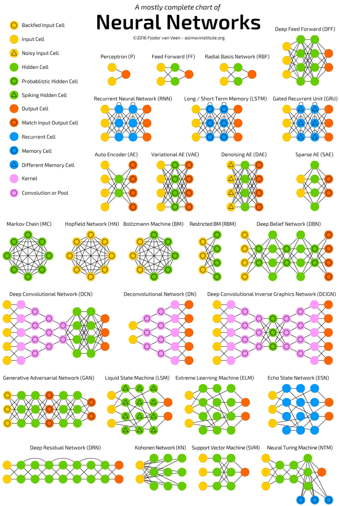
- for death counts $D_{x,t,g,\ell}$ under Poisson assumption, in the presence of risk factors or covariates $\boldsymbol{x}_\ell$
- with techniques such as:
  - Random Forests (RFs)
  - Gradient Boosting Machines (GBMs)
  - Combined Actuarial Neural Networks (CANNs) with Artificial Neural Networks (ANNs) on top of e.g. GLM baseline.



Picture taken from Machine learning for everyone. In simple words. With real-world examples. Yes, again.

(Deep) Feed-forward neural nets:

- as in Richman and Wüthrich [2021]
- with country, gender, age and year as input features
- Lee & Carter and Li & Lee revisited with a network version + its calibration (no hidden layers)
- deep feed-forward neural nets are explored as well
- focus on improving a loss function, residual plots, . . .



(Deep) Recurrent neural nets (RNNs), Long short-term memory nets (LSTMs)

- as in Lindholm and Palmborg [2022], with focus on LSTM for the $\hat{\kappa}_t$'s
- as in Perla et al. [2021] where the $\mu_{x,t}^{(i)}$'s are modelled with (deep) RNNs and LSTMs for different populations (i) (i.e., genders and countries)
- as in Euthum et al. [2024] where the $\mu_{x,t}^{(i)}$'s are modelled with (deep) RNNs and LSTMs using inputs age, gender, year and deprivation group.

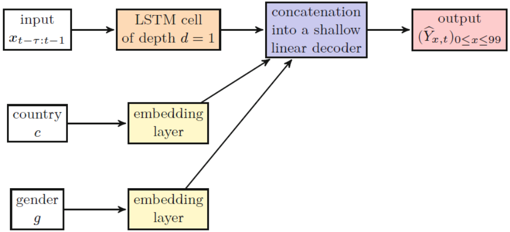


Fig. 8.13 LSTM architecture used to process the raw mortality rates $(M_{x,t})_{x,t}$

Picture from Wüthrich and Merz [2023].

(Deep) Convolutional neural nets (CNNs):

- as in Perla et al. [2021] where the $\mu_{x,t}^{(i)}$'s are modelled with CNN-type layers for the $\mu_{x,t}^{(i)}$s, combined with input features gender and region for population (i)
- as in Wang et al. [2021] who use CNNs with as input a 2-dimensional 'image' around $\mu_{x,t}$ with mortality information for ages in $[x - x_1, x + x_2]$, lagging k years $1 \leq k \leq s$.

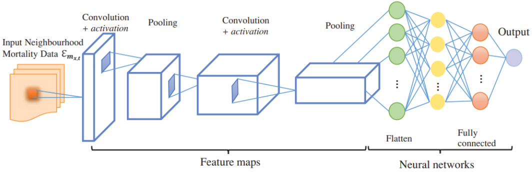


FIGURE 6: An illustrative CNN architecture in constructing neighbouring prediction models.

Today's approach

For details on analyzing mortality data with neural networks, we refer to:

- Lee and Carter go Machine Learning: Recurrent Neural Networks, a tutorial by the Swiss Actuarial Data Science working group
- the chapters on neural nets in Wüthrich and Merz [2023].

Today, we take a deep dive into a recent working paper by Jens, Katrien and Torsten:

- try to explain weekly death counts across European regions
- with a baseline Serfling model (e.g., alike EuroMoMo)
- combined with a (high-dimensional) set of weather and pollution features.

Short-term mortality and environmental variables

I will present today our JRSS A paper:

The association between environmental variables and short-term mortality: evidence from Europe

working paper on arxiv, code on GitHub, by Jens Robben (UvA, RCLR), Katrien Antonio and Torsten Kleinow (UvA, RCLR).

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1. Identify the primary environmental factors contributing to the estimation of mortality deviations from a pre-defined baseline level.

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2. Investigate the marginal impact of an environmental factor on deviations from the mortality baseline level.
3. Investigate how environmental factors interact when modelling mortality rates. Are there harvesting effects present?
4. Quantify the added contribution of environmental factors when aggregating the resulting estimated weekly mortality rates on an annual or country-level basis.

Short-term association between environmental variables and mortality rates

Introduction

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Using fine-grained open data, study the association between environmental factors and weekly mortality rates in European regions.

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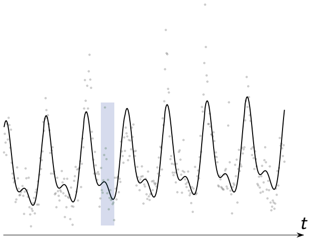

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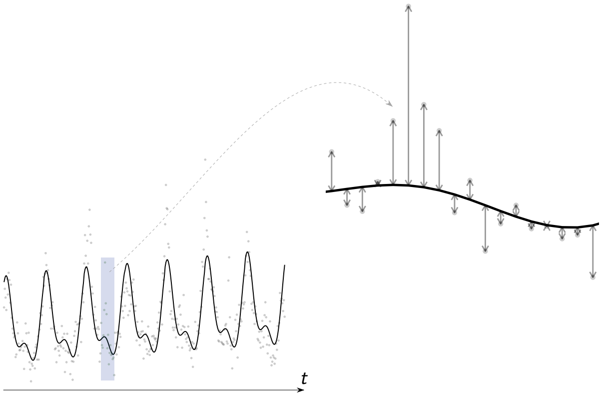

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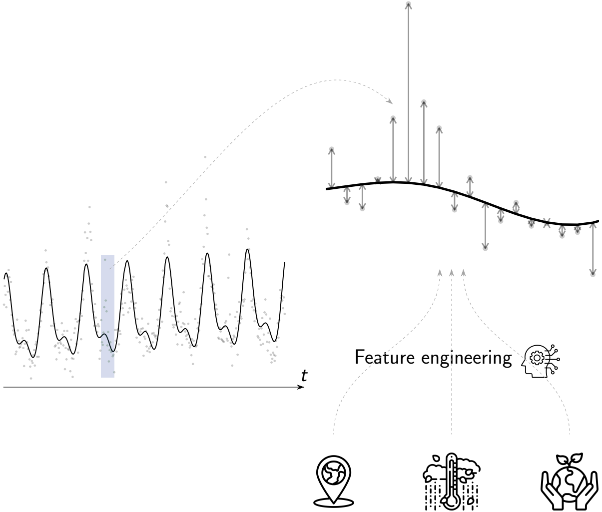

Data

Death counts

Eurostat: deaths by week, gender, 5-year age group and NUTS 3 region from 20 European countries throughout the years 2013-2019 ($>$ 500 regions).

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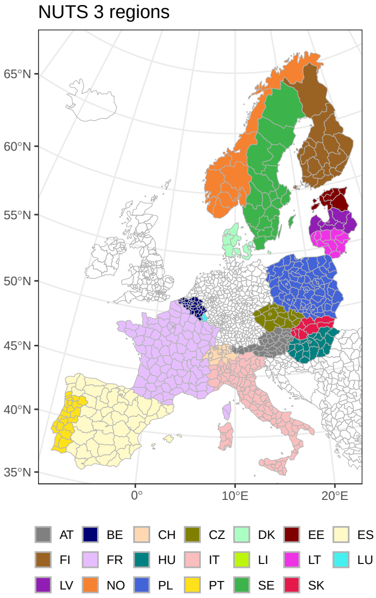

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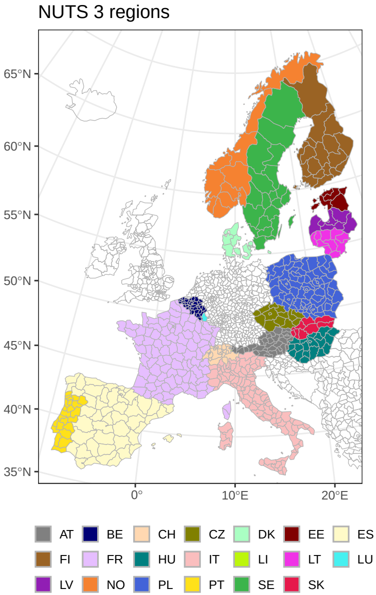

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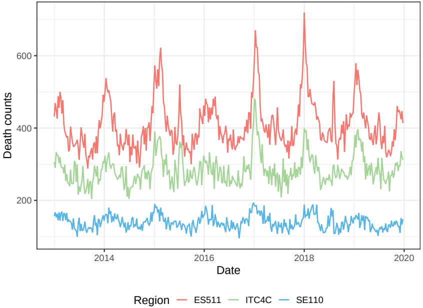

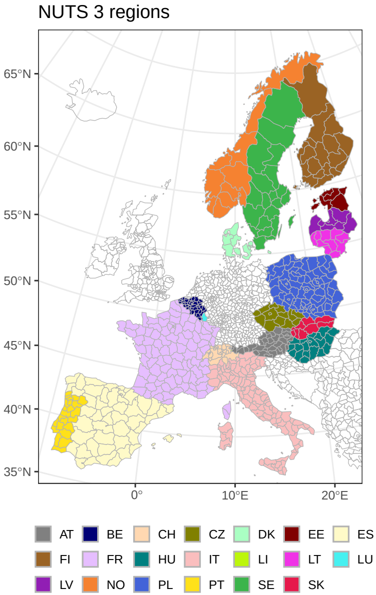

Weather data

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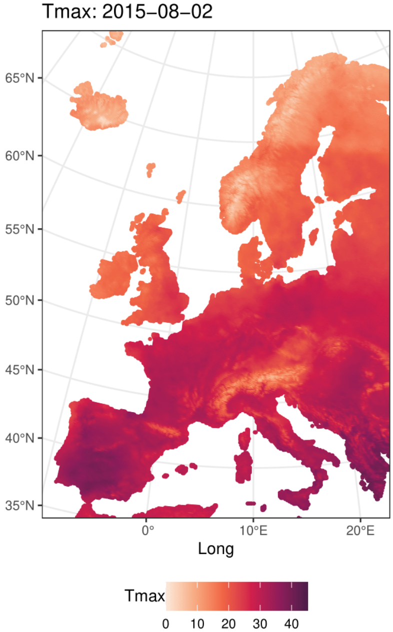

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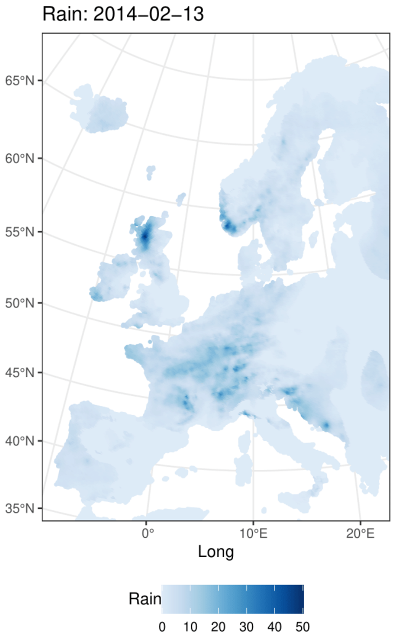

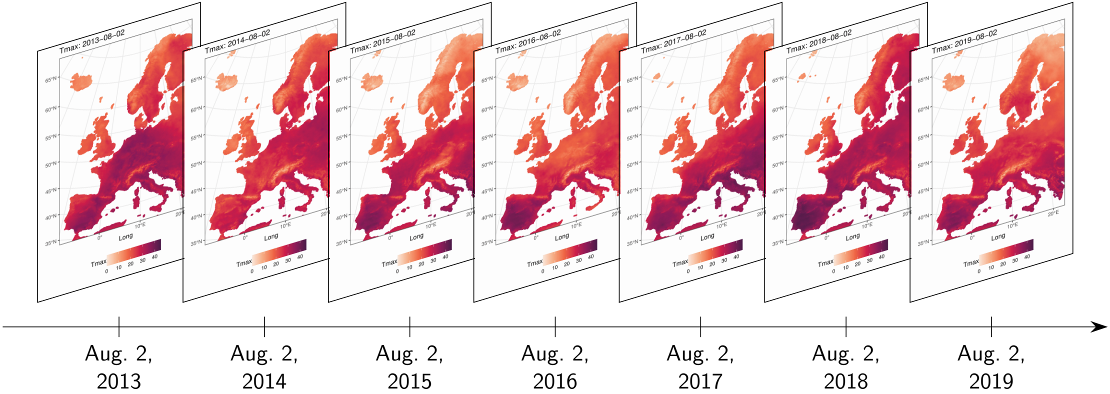
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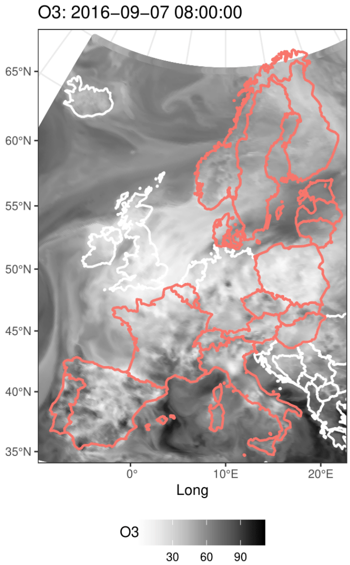

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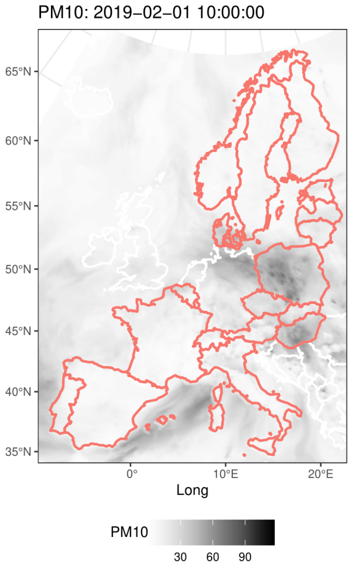
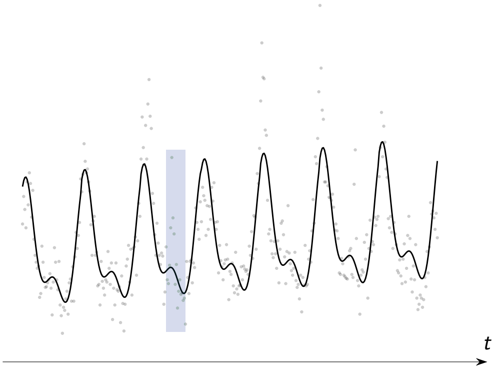


Model specification

Weekly, region-specific baseline mortality model

A weekly, region-specific baseline mortality model to capture the overall seasonal trends in the considered regions.



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Incorporate seasonality through Fourier terms Serfling [1963]:

$$D_{t,w}^{(r)} \sim \text{Poisson}\left(E_{t,w}^{(r)} \cdot \mu_{t,w}^{(r)}\right),$$

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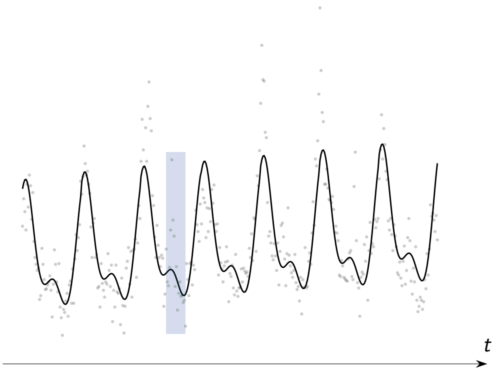

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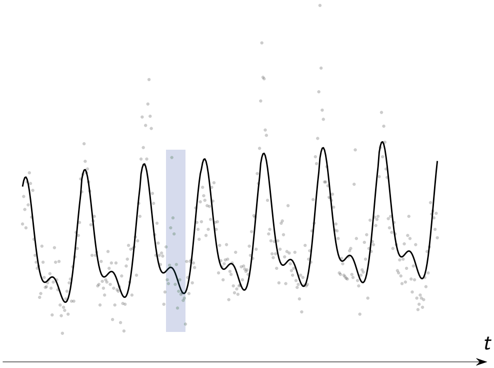

Region-specific population exposures $E_{t,w}^{(r)}$ from Eurostat.

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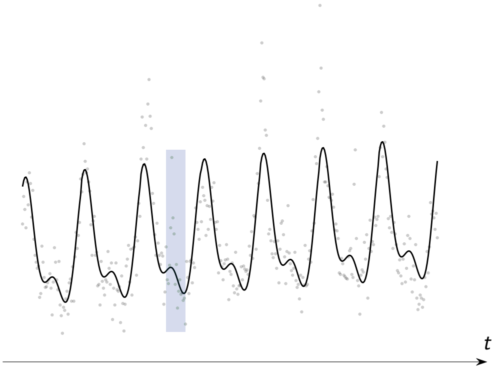
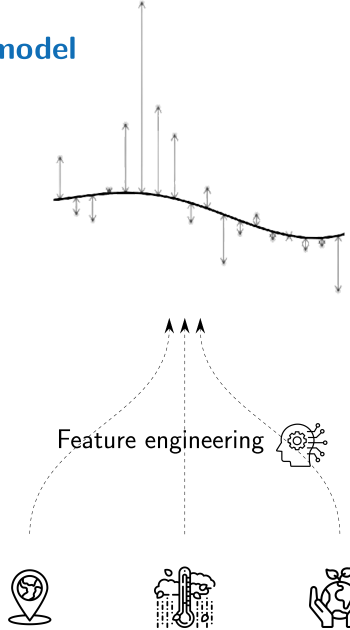


Region-specific population exposures $E_{t,w}^{(r)}$ from Eurostat.

Estimated baseline death counts: $\hat{b}_{t,w}^{(r)} := E_{t,w}^{(r)} \cdot \hat{\mu}_{t,w}^{(r)}$.

Modelling mortality deviations from the baseline model

Explain observed deviations from the baseline deaths using region-specific environmental features.



Modelling mortality deviations from the baseline model

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Fix estimated baseline deaths and impose distributional assumption:

$$D_{t,w}^{(r)} \sim \text{Poisson}\left(\hat{b}_{t,w}^{(r)}\,\phi_{t,w}^{(r)}\right),$$

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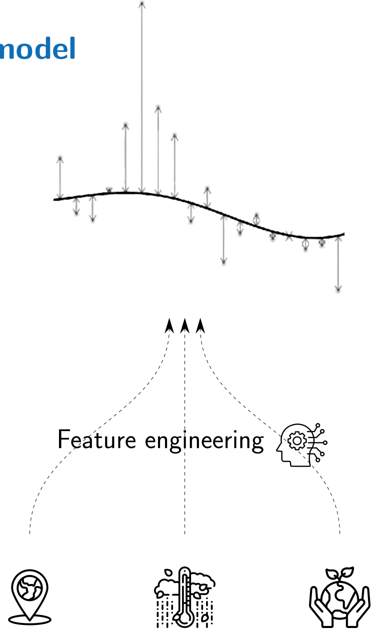
$$l^1\left(\boldsymbol{c}_{t,w}^{(r)}\right),\ l^1\left(\boldsymbol{e}_{t,w}^{(r)}\right),\ \dots,\ l^s\left(\boldsymbol{c}_{t,w}^{(r)}\right),\ l^s\left(\boldsymbol{e}_{t,w}^{(r)}\right)\Big).$$



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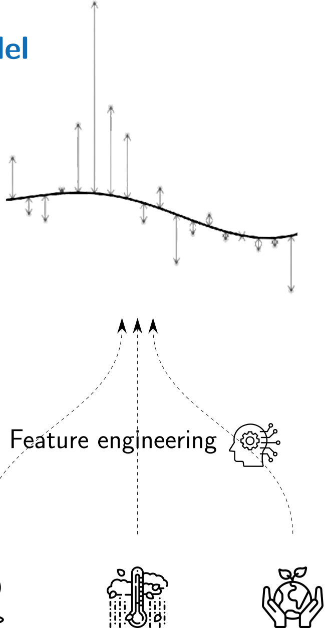
$f(\cdot)$ is a selected predictive modelling technique.



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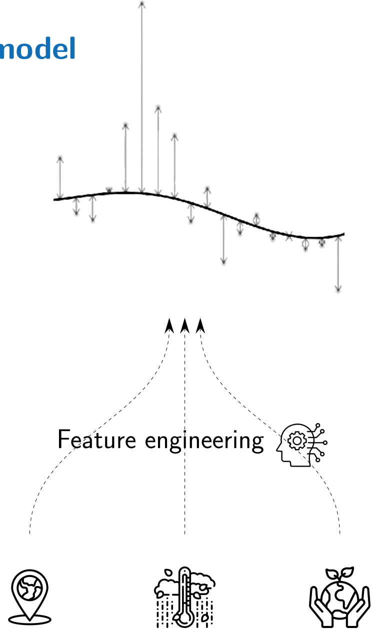
We opt for a machine learning model.



Model calibration

Fit a Poisson GLM jointly across all regions, add a smoothness penalty:

$$\hat{\boldsymbol{\beta}} = \underset{\boldsymbol{\beta}}{\operatorname{argmin}} \left(-\log \mathcal{L}_P(\boldsymbol{\beta}) + \sum_{p=0}^{5} \lambda_p \boldsymbol{\beta}_p^T \boldsymbol{S} \boldsymbol{\beta}_p \right),$$

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where:

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$\log \mathcal{L}_P(\boldsymbol{\beta})$: Poisson log-likelihood

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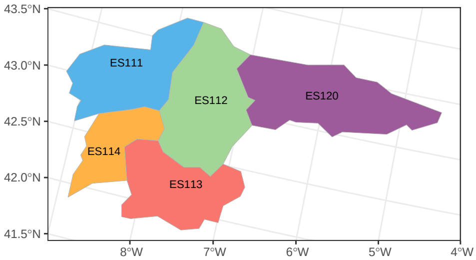
Example (5 Spanish NUTS 3 regions):



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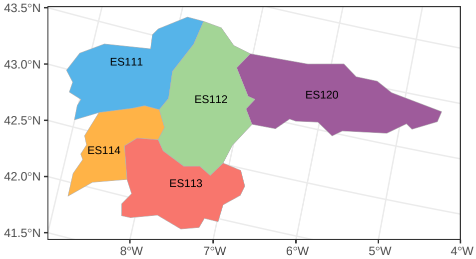

Penalty matrix $\boldsymbol{S}$:

$$\begin{array}{c} \\ ES111 \\ ES112 \\ ES113 \\ ES114 \\ ES120 \end{array} \begin{array}{c} \begin{array}{ccccc} ES111 & ES112 & ES113 & ES114 & ES120 \end{array} \\ \begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 4 & -1 & -1 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & -1 & -1 & 3 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{pmatrix} \end{array}.$$

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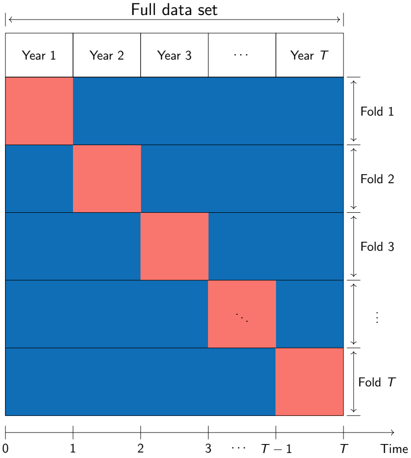
Parameter tuning with T-fold cross-validation.



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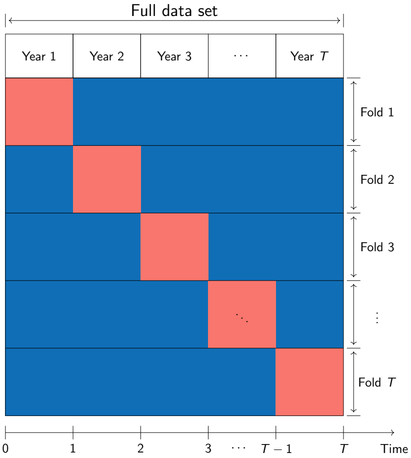
Parameter tuning with T-fold cross-validation.

Calibrate XGBoost model on entire training data with optimal parameter configuration.



XGBoost: flexible and efficient implementation of gradient boosting.

Tuning parameters:

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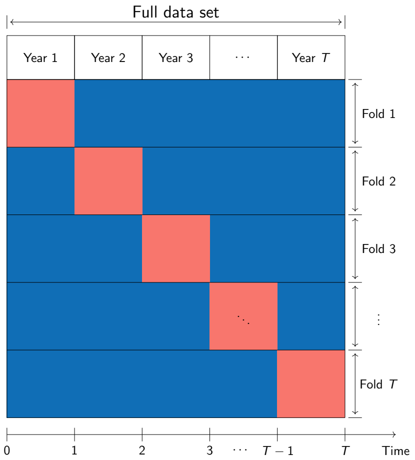
Interpretation tools to gain insights: VIP, ALE.



Feature engineering

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Differences in spatial and temporal dimension across the data sources:

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- environmental data: hourly or daily time scale, spatial grid.

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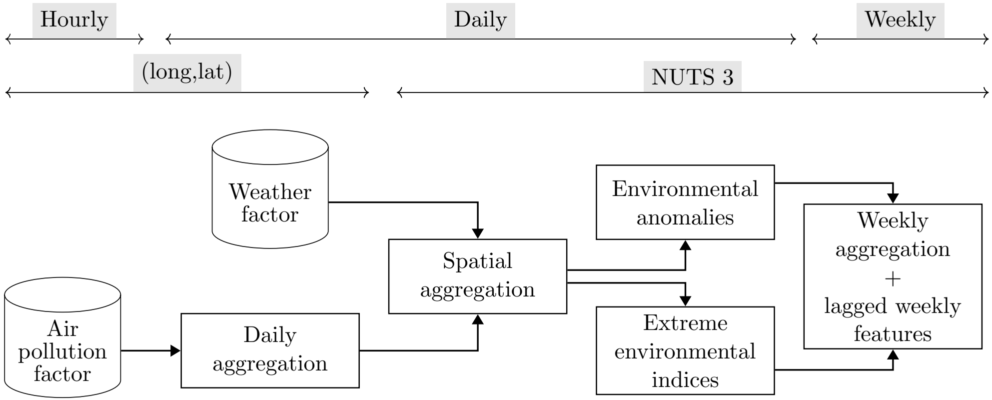
- create features that measure deviations from baseline conditions from environmental data to explain excess or deficit mortality (other ideas welcome!).

Flow chart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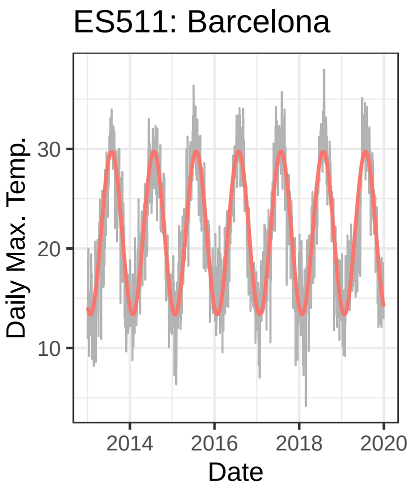

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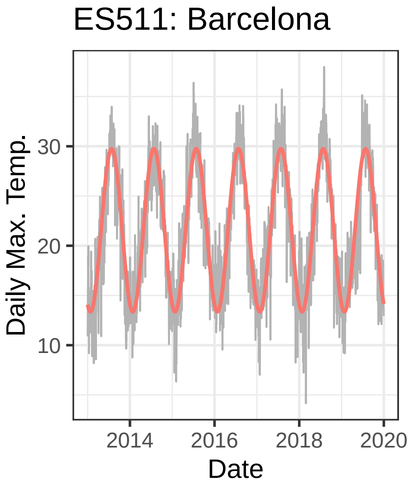

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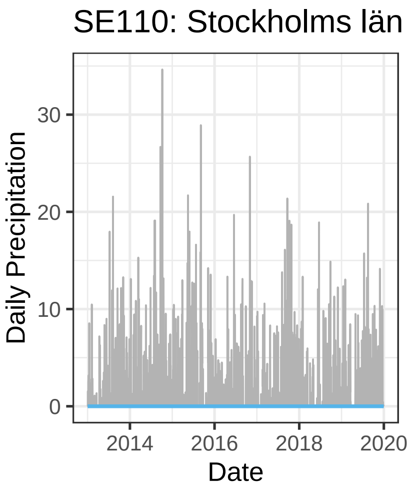
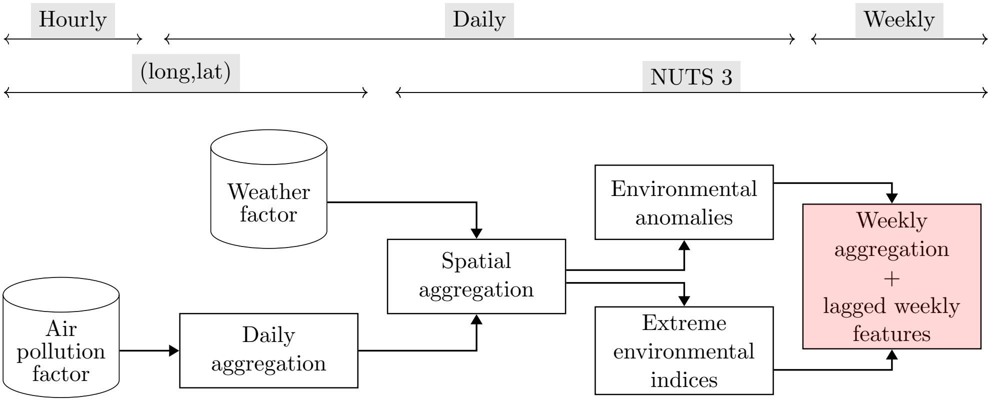


Flow chart



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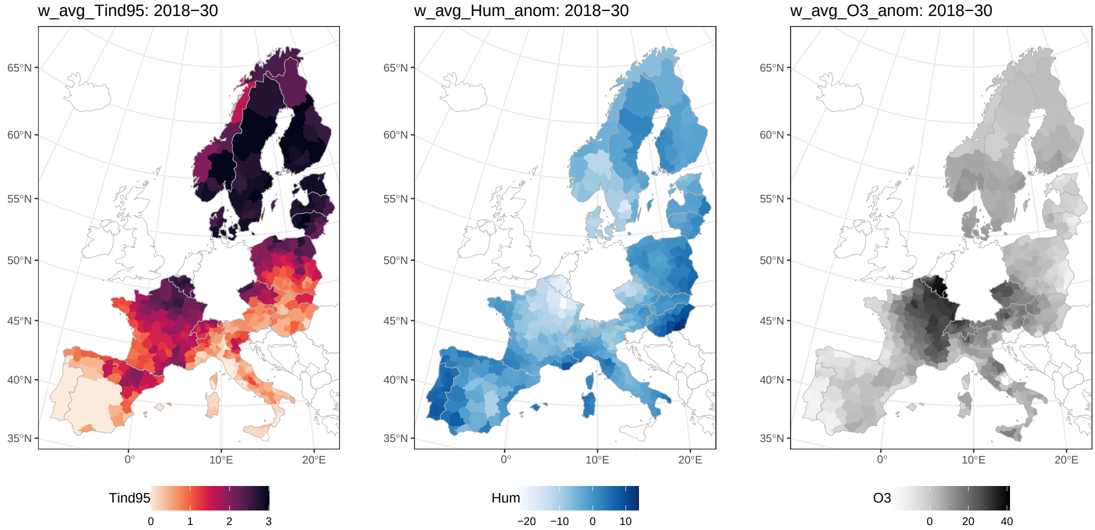
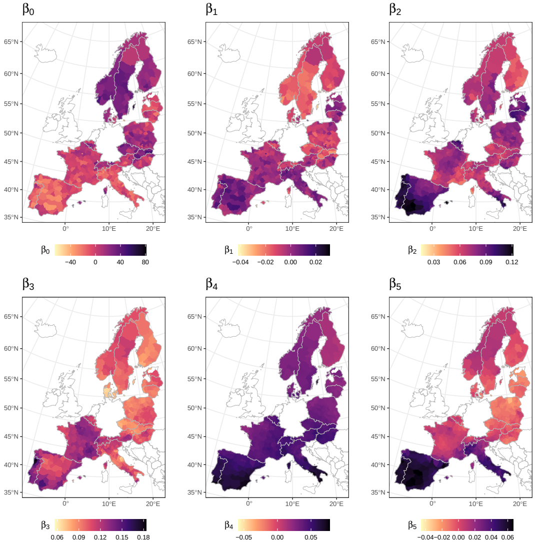


Calibration results

Baseline model



Machine learning model

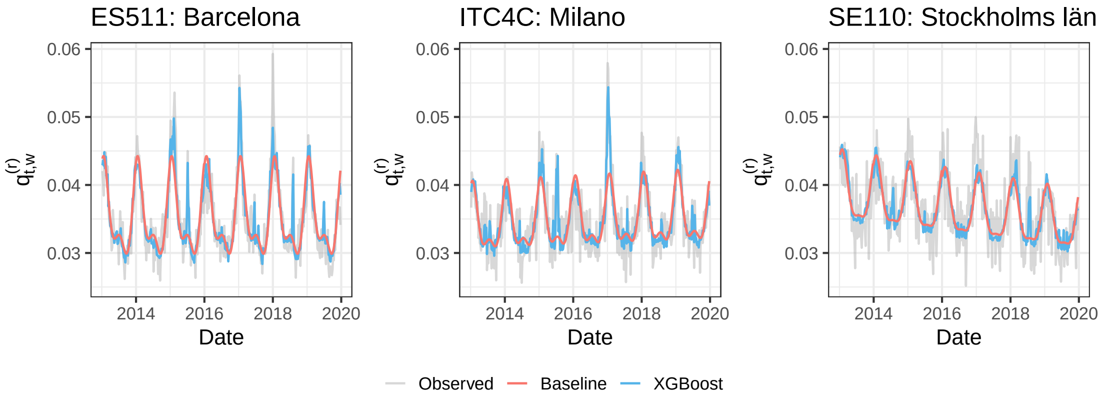
Input features: longitude-latitude coordinates, season, (one-week lagged) environmental anomalies and extreme indices.

Tuning by 7-fold cross validation over the years 2013-2019 using an extensive tuning grid.

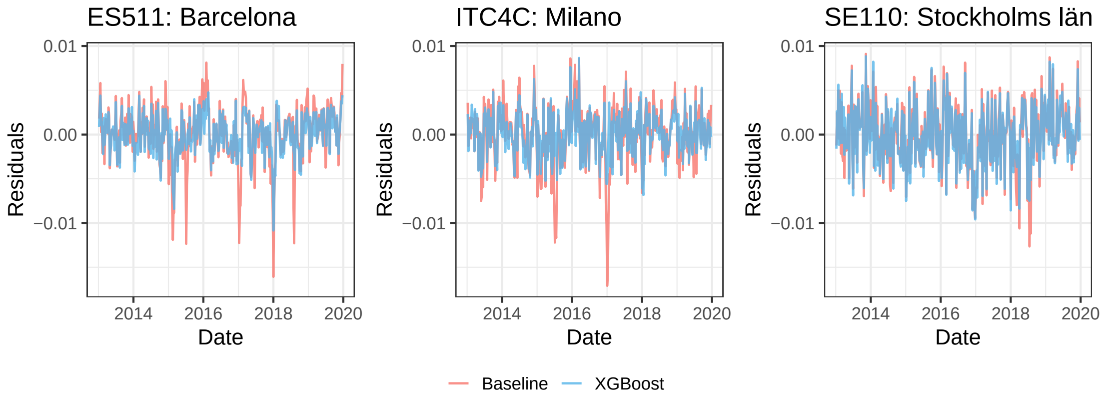
Tuning parameters: `nrounds` (490), `eta` (0.01), `min_child_weight` (1000), `max.depth` (7), `subsample` (0.75), `colsample_bytree` (0.50).

Insights in the machine-learning model

Observed and estimated mortality rates (baseline + XGBoost):



Residuals of the estimated weekly mortality rates (baseline + XGBoost):



Feature importance

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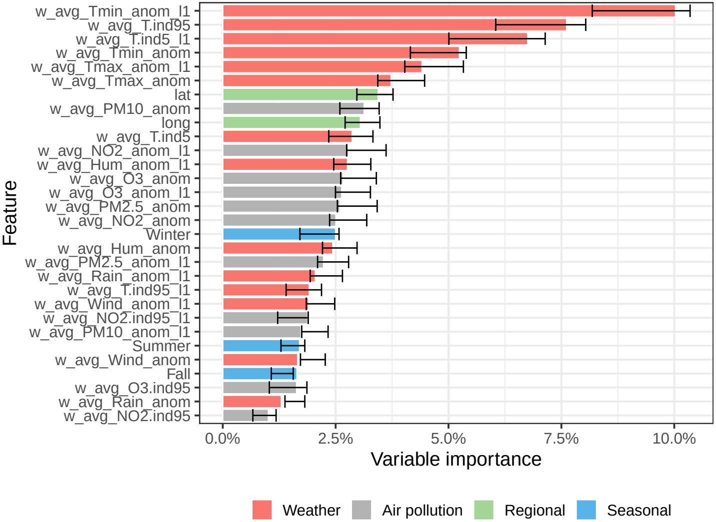
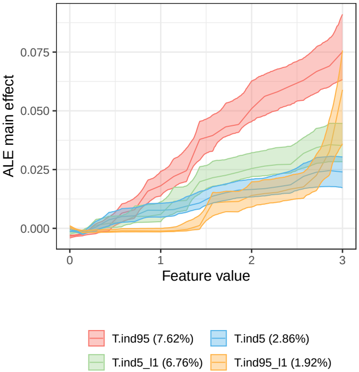
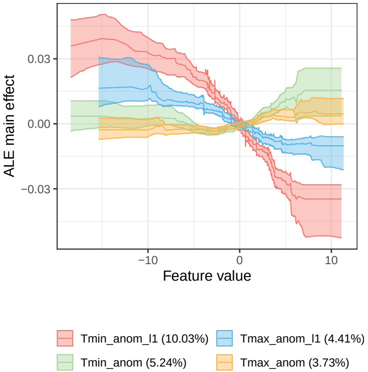
Features with a high importance appear often and high in the tree.

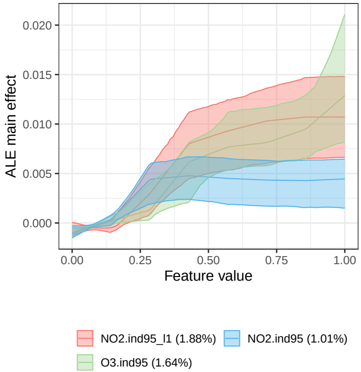
Feature importance



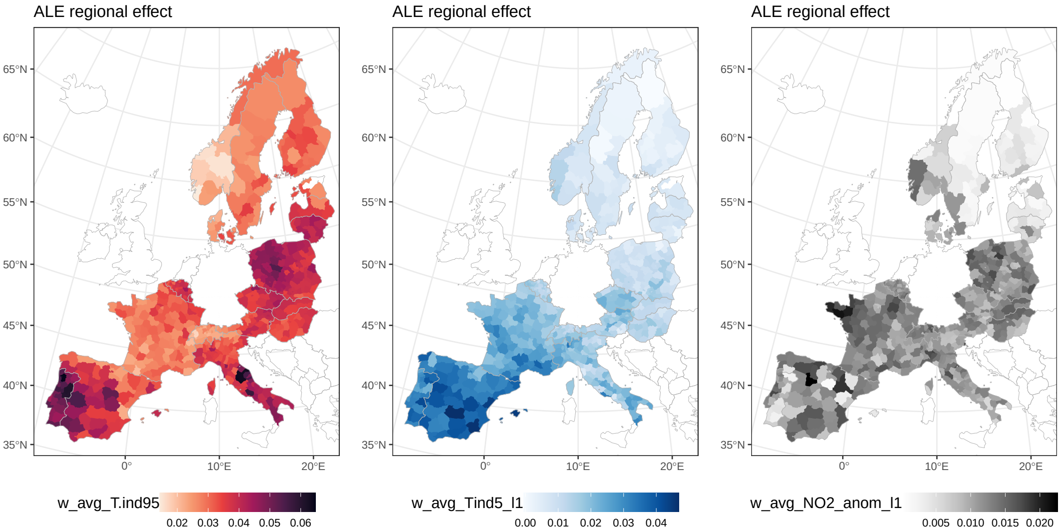
ALE main effects

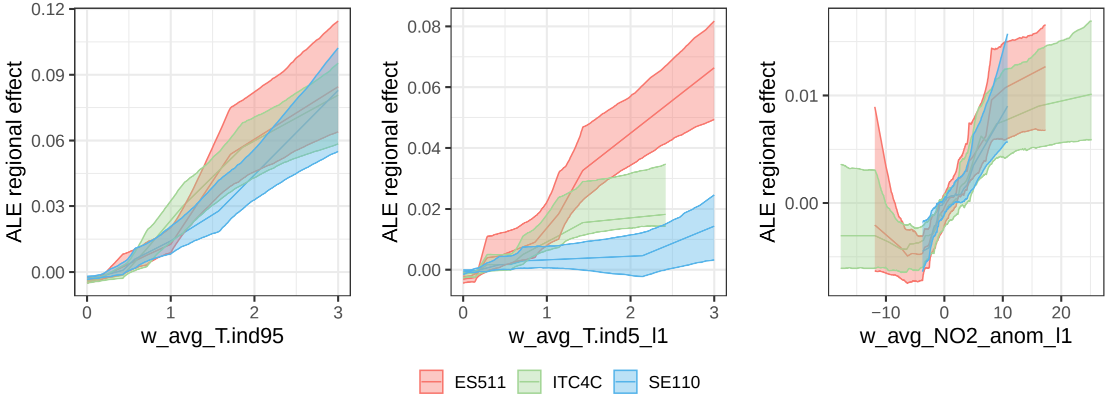






ALE regional effects




ALE regional effect
0.12
0.09
0.06
0.03
0.00
0 1 2 3
w_avg_T.ind95
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0.02
0.00
w_avg_T.ind5_l1
0.01
0.00
−10 0 10 20
w_avg_NO2_anom_l1
ES511
ITC4C
SE110

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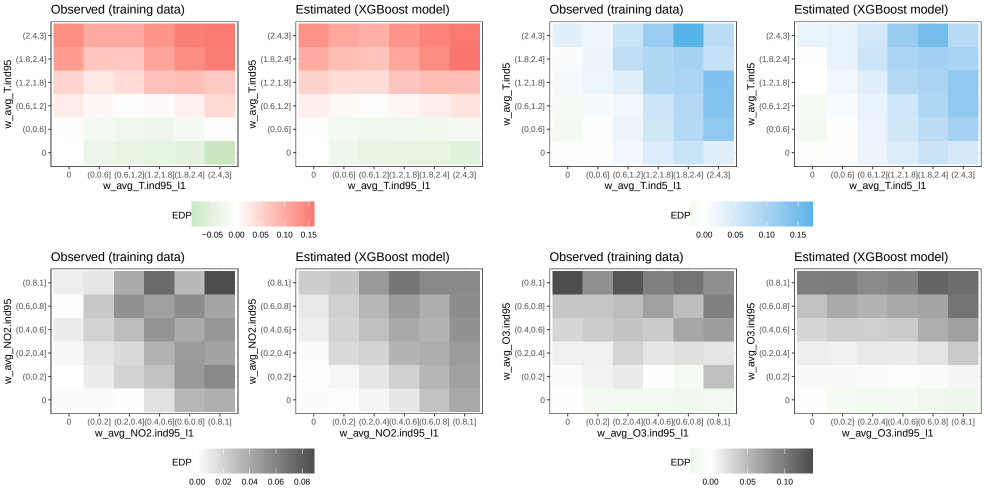

Wrap up

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However, feature engineering, (proper) interpretation tools, interplay of more traditional actuarial + statistical learning and sophisticated machine learning methods are key!

M. Euthum, M. Scherer, and F. Ungolo. A neural network approach for the mortality analysis of multiple populations: a case study on data of the italian population. *European Actuarial Journal*, 2024.

Mathias Lindholm and Lisa Palmborg. Efficient use of data for lstm mortality forecasting. *European Actuarial Journal*, 12:749–778, 2022.

Francesca Perla, Ronald Richman, Salvatore Scognamiglio, and Mario V. Wüthrich. Time-series forecasting of mortality rates using deep learning. *Scandinanivan Actuarial Journal*, 7(2): 572–598, 2021.

Ronald Richman and Mario V. Wüthrich. A neural network extension of the lee–carter model to multiple populations. *Annals of Actuarial Science*, 15(2):346–366, 2021.

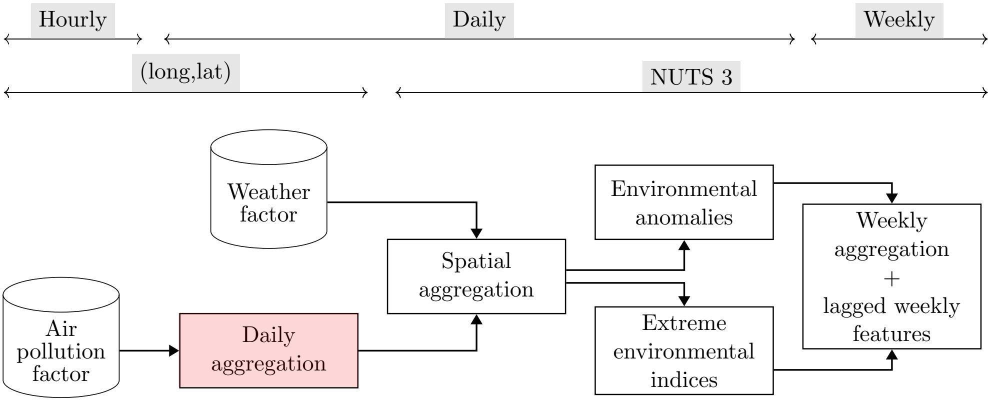
Robert E Serfling. Methods for current statistical analysis of excess pneumonia-influenza deaths. *Public health reports*, 78(6):494, 1963.

Chou-Wen Wang, Jinggong Zhang, and Wenjun Zhu. Neighbouring prediction for mortality. *ASTIN Bulletin*, 51(3):689–718, 2021.

References II

Mario Wüthrich and Michael Merz. *Statistical foundations of actuarial learning and its applications*. Springer, 2023.

Flow chart



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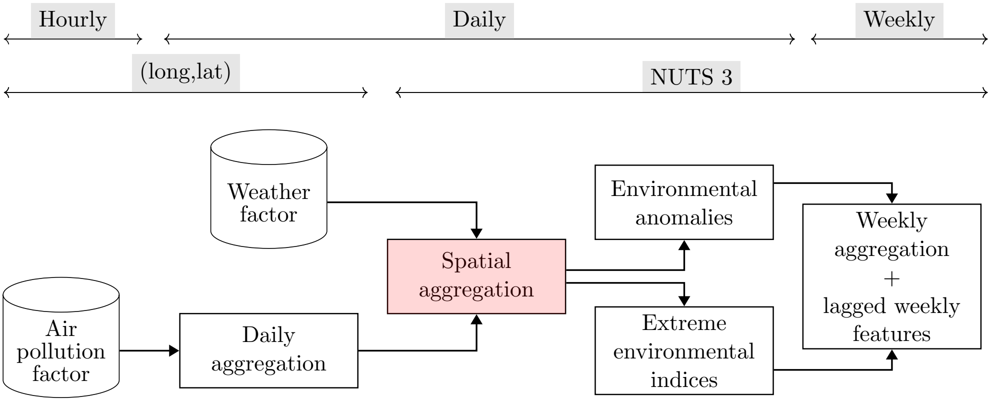
\end{aligned}$$

Weather factors already available at the daily level (no need for daily aggregation).

Flow chart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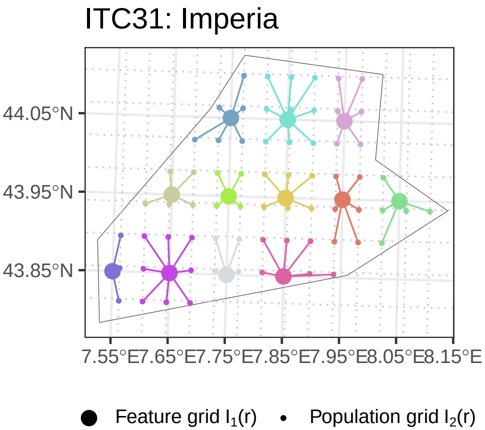

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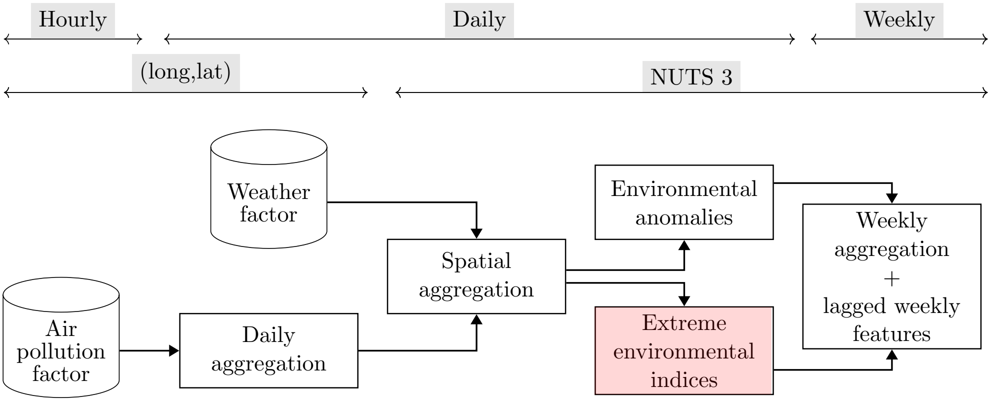
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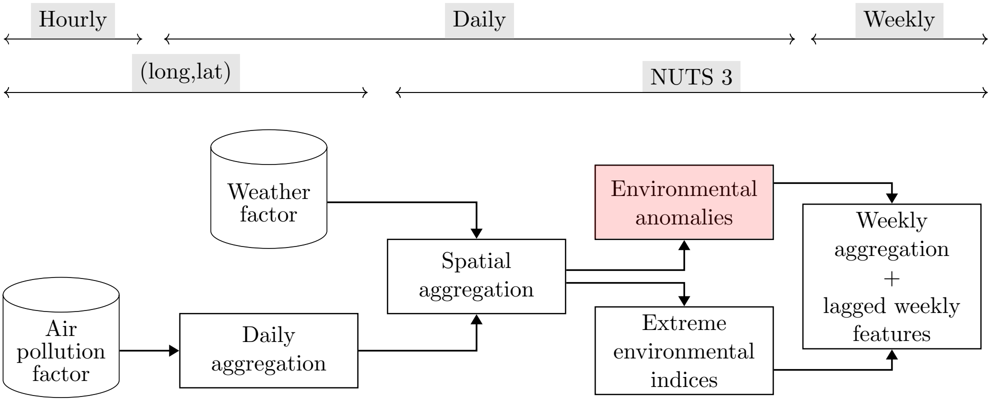

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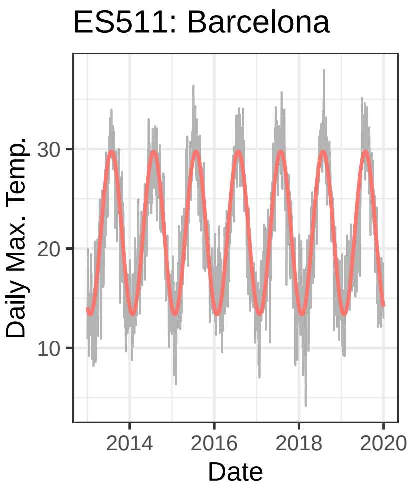

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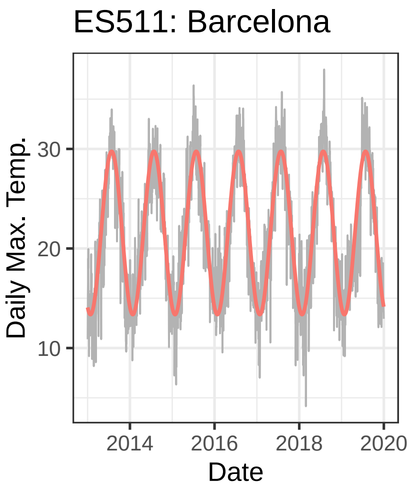

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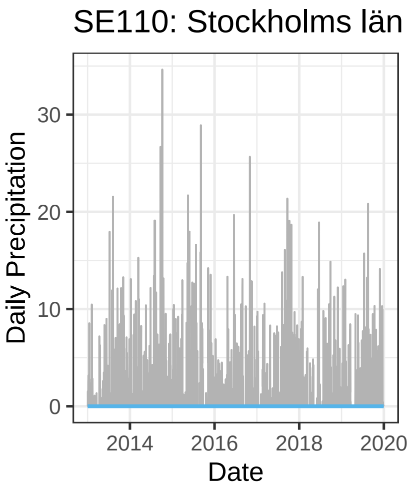
In the paper we work with excesses or deviations from the baseline (anomalies):

$$\tilde{x}^{(r)}_{t,w,d} - \hat{\tilde{x}}^{(r)}_{t,w,d}$$



How does each feature impact the predictions produced by the machine learning model?

Partial dependence plots interpret the marginal effect of a feature X_l on the model predictions as:

$$f_{l,\mathrm{PDP}}(x) = \frac{1}{n}\sum_{j=1}^{n} \hat{f}(x, \boldsymbol{x}_j^{(-l)}),$$

where $\boldsymbol{x}_j^{(-l)}$ is the j-th training observation where the l-th feature is omitted.

Problem: correlated features may lead to unrealistic data points.

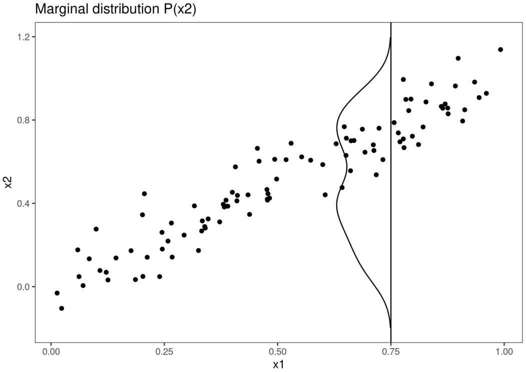


Figure: Visualisation taken from Molnar (2019).

Accumulated local effects - motivation

How does each feature impact the predictions generated by a machine learning model?

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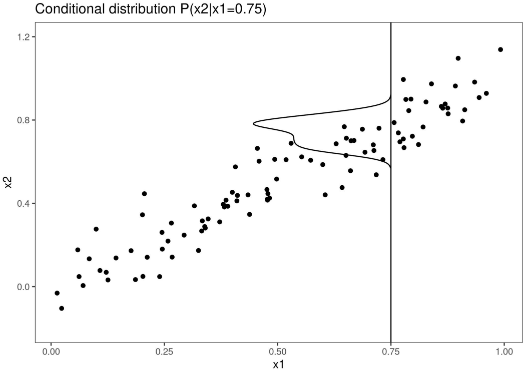

Figure: Visualisation taken from Molnar (2019).

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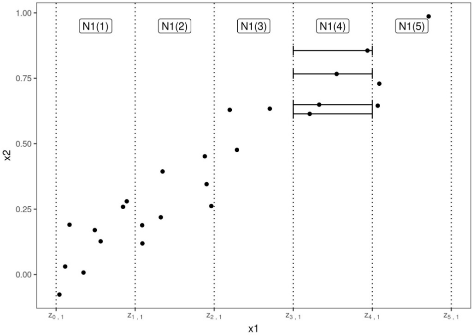


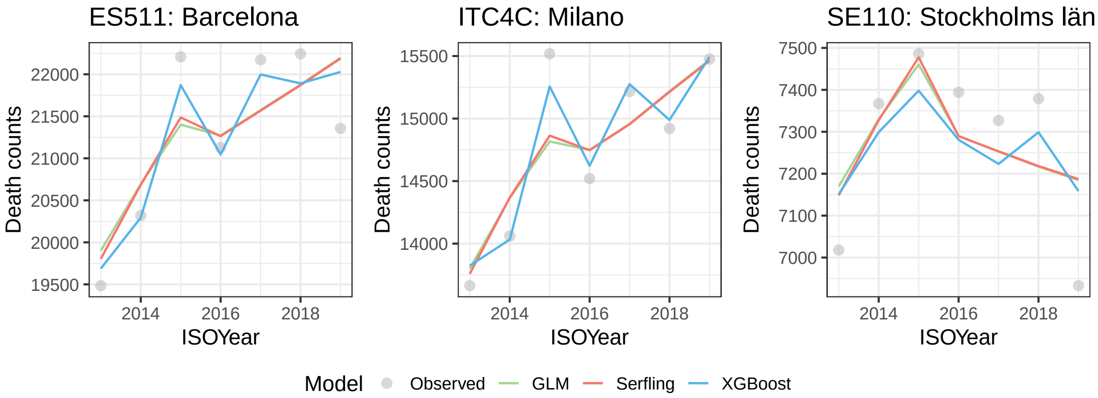
Figure: Visualisation taken from Molnar (2019).

Accumulated local effects

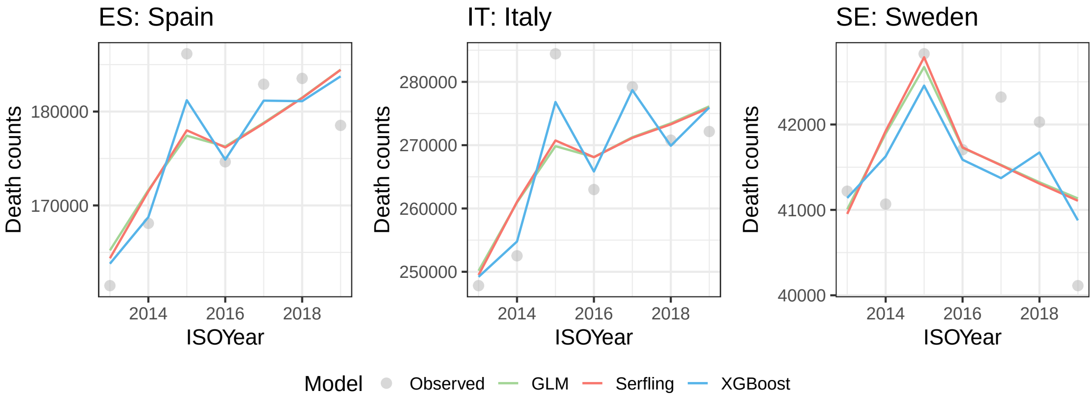
Accumulated Local Effects avoid the use of unrealistic data instances, average changes in predictions and accumulate them. For a feature X_l, the ALE effect equals:

$$f_{l,\text{ALE}}(x) = \int_{z_{0,l}}^{x} \mathbb{E}\left[\frac{\partial f(X_1, X_2, ..., X_p)}{\partial X_l}\middle| X_l = z_l\right] dz_l - c_l,$$

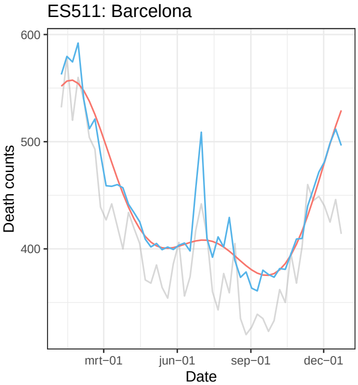
Instead of averaging predictions, ALE average changes of predictions (via the partial derivatives). The integral over z_l accumulates the differences in the predictions when moving over the range of X_l. A constant c_l is subtracted to center the ALE plot and make sure the mean effect is zero.



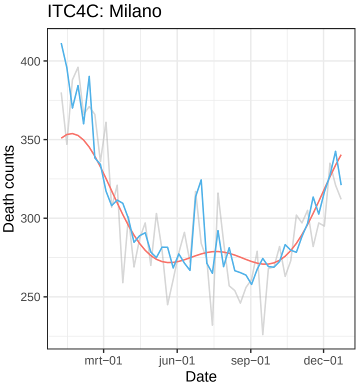
ES511: Barcelona
ITC4C: Milano
SE110: Stockholms län
Death counts
ISOYear
22000
21500
21000
20500
20000
19500
15500
15000
14500
14000
7500
7400
7300
7200
7100
7000
2014
2016
2018
Model
Observed
GLM
Serfling
XGBoost



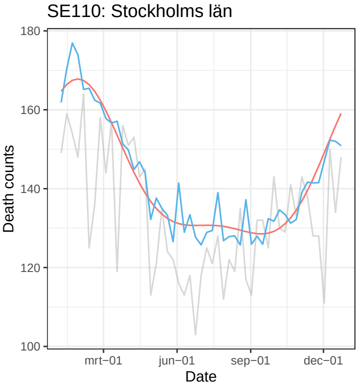
ES: Spain
IT: Italy
SE: Sweden
Death counts
180000
170000
280000
270000
260000
250000
42000
41000
40000
2014
2016
2018
ISOYear
Model
Observed
GLM
Serfling
XGBoost



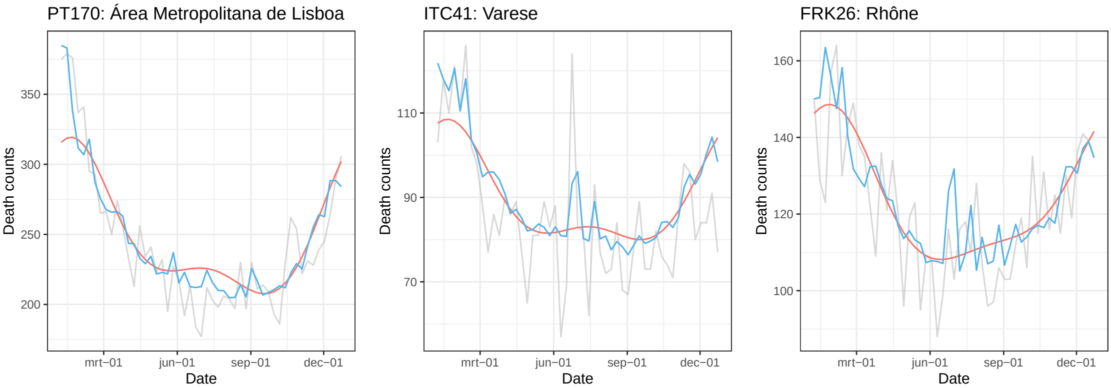
ES511: Barcelona
600
500
400
Death counts
mrt−01
jun−01
sep−01
dec−01
Date



ITC4C: Milano
400
350
300
250
Death counts
mrt−01
jun−01
sep−01
dec−01
Date



SE110: Stockholms län
180
160
140
120
100
Death counts
mrt−01
jun−01
sep−01
dec−01
Date



PT170: Área Metropolitana de Lisboa
ITC41: Varese
FRK26: Rhône
Death counts
Date
mrt−01
jun−01
sep−01
dec−01
350
300
250
200
110
90
70
160
140
120
100