

Modelling and quantifying mortality and longevity risk

Module D1 : Sensitivities and Redistribution of Risk

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Overview



In this module:

- Quantifying Longevity Risk Sensitivities
- Internal mitigation:
Redistribution of longevity shocks
- External mitigation:
Transfer of longevity shocks

Sensitivities

Effect on different actuarial contracts

We distinguish

- the dynamics of the valuation of actuarial liabilities, and
- the uncertainty in those valuations
- contracts which may help to mitigate this uncertainty

Effects can be quite complicated due to

- Interaction between mortality / longevity risk and other sources of risk such as interest rate risk
- Nonlinear relationships between mortality rates and
 - Present value of premia paid until death
 - Present value of (deferred) annuity
 - Present value of benefits to survivors

Back of an envelope calculation

Let's get a quick feeling for sensitivities.

Consider annuity with

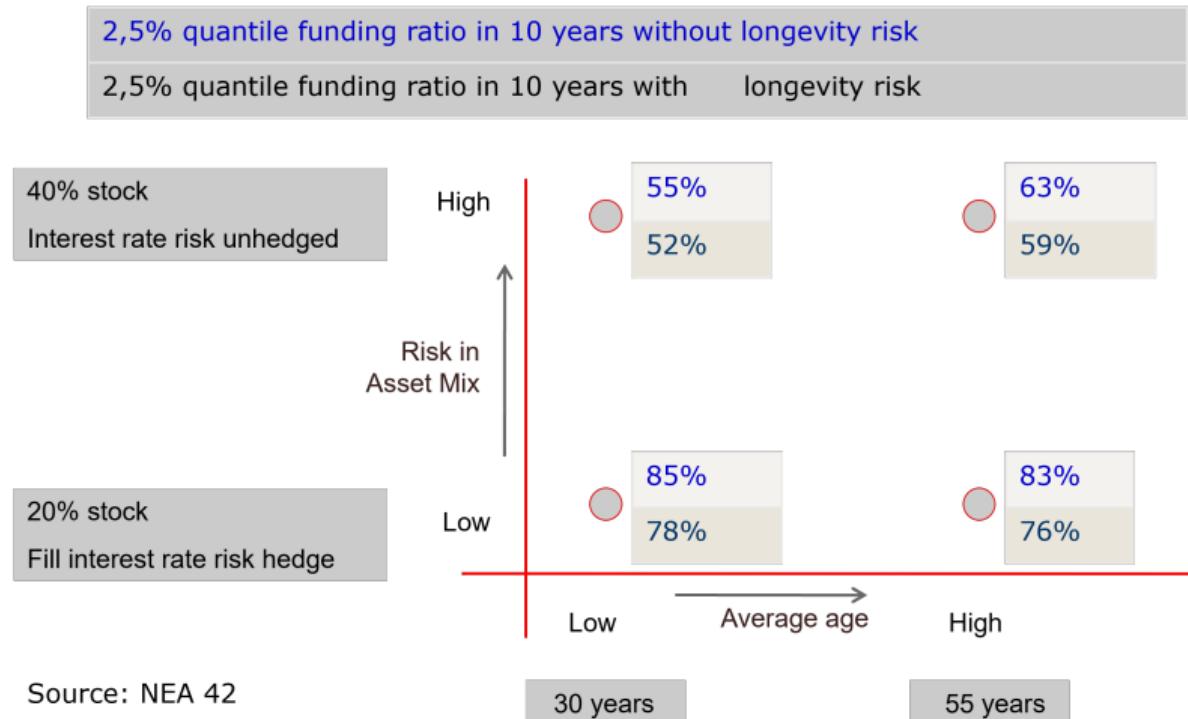
- Fixed payment of one unit per year, starting 1 year from now
- Fixed probability of death per year, $q = 1 - p$
- Fixed interest rate per year, r

$$A = \sum_{k=1}^{\infty} \frac{(1-q)^k}{(1+r)^k} = \frac{1}{1 - \frac{1-q}{1+r}} - 1 = \frac{1+r}{1+r-(1-q)} = \frac{1-q}{r+q}.$$

- Compared to case $q = 0$ we see reduction in nominator and 'extra interest' in denominator.
- If q is interpreted as hazard rate, **duration** seems good sensitivity measure for annuities.

Sensitivities: Pensions

Relative Effect Longevity risk also depends on assets



Longevity and Intergenerational Solidarity

Intergenerational solidarity for investment risk:

- **Bad scenarios**
pension payments unchanged when funding ratio below 100%
(young subsidize old)
- **Good scenarios**
pensioners only receive indexation, not returns above this
(old subsidize young)

This seems 'fair' if
expected present value (i.e. average over *all* scenarios) of future transfers equals zero.

Intergenerational solidarity for **longevity risk**:

- Do we actually expect 'good scenarios' ?
- Should we redistribute internally/ hedge externally ?

Idiosyncratic and systematic risk

We distinguish macro longevity risk (decrease in mortality probabilities, i.e. risk due to changes in K/κ -processes) and micro longevity risk (individual coin flips by the Grim Reaper, **given** these probabilities).

- Micro longevity risk is **idiosyncratic**.

Risk diversifies (i.e., becomes relatively smaller in large groups) and thus is largest for oldest age groups, which have fewest members.

Often negligible for young pensioners and workers/sleepers.

- Macro longevity risk is **systematic**.

Risk hits youngest most: longevity shocks hits all ages at the same time in Lee Carter model (one factor model !) but youngest have highest exposure in terms of relative change in their deferred annuity.

Idiosyncratic and systematic risk

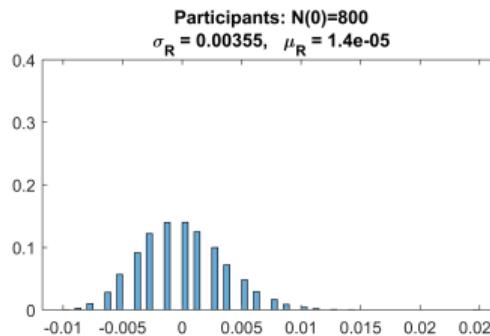
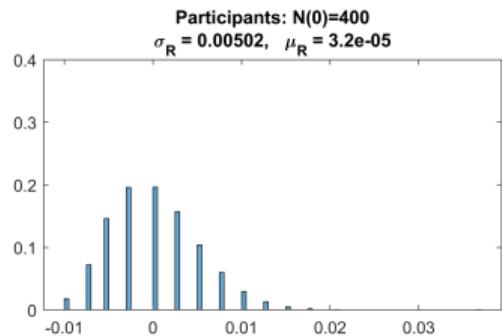
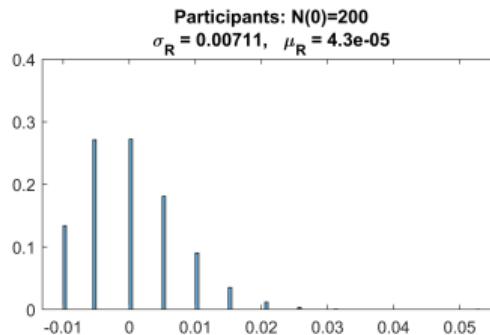
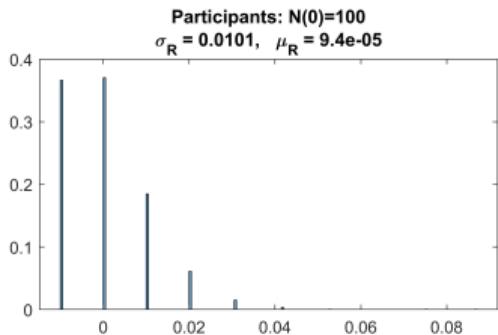
Micro longevity risk is **idiosyncratic**.

- Assume that $N(0)$ participants of the same age x and gender g have accrued an amount of money and that each has an independent probability p of survival for the coming year.
- The number of survivors $N(1)$ one year later has a binomial distribution with parameters $N(0)$ and p .
- We define the micro longevity risk factor as

$$R = \frac{\mathbb{E}[N(1)]}{N(1)} - 1.$$

Idiosyncratic and systematic risk

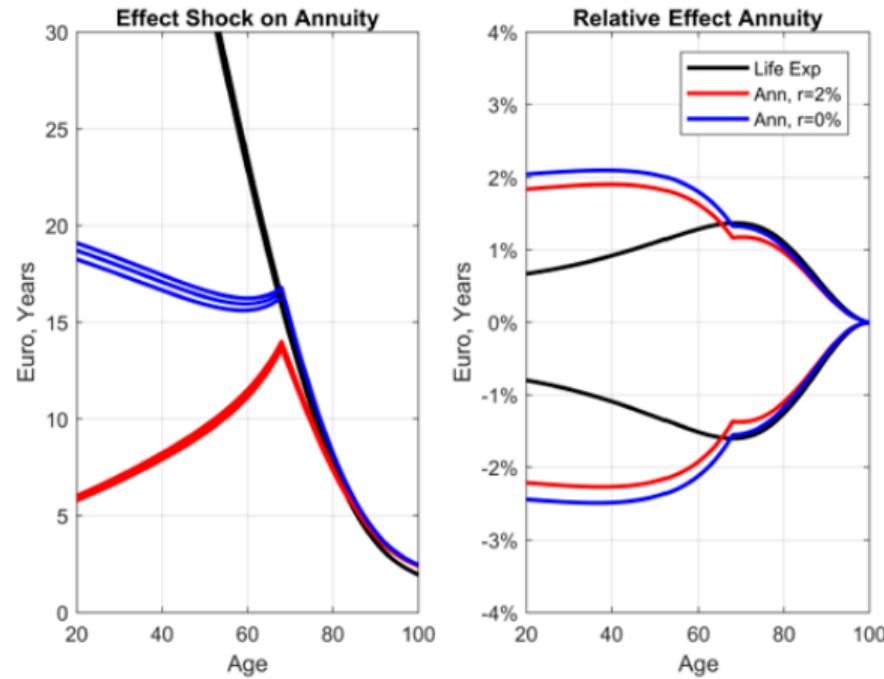
Micro Longevity Risk
p=1%



Internal Mitigation: Redistribution

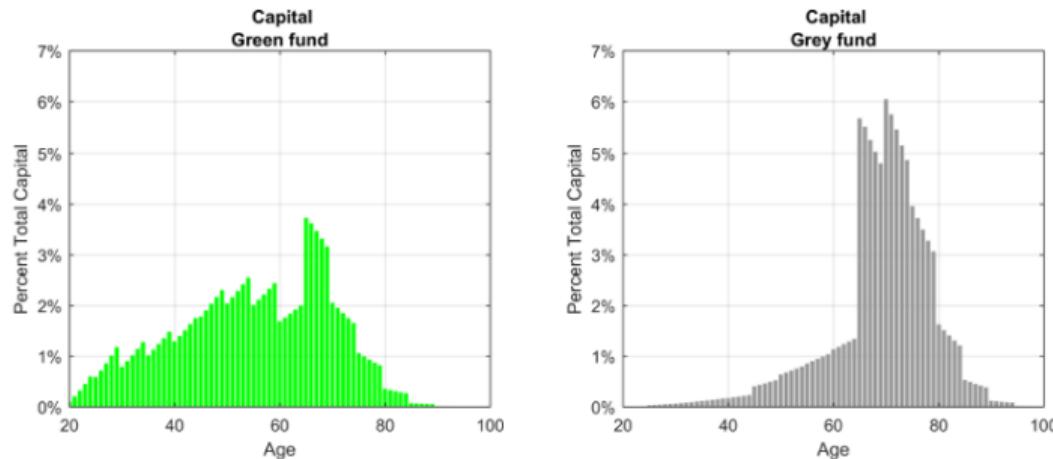
Lee Carter shock

Change in prognosis of future survival probabilities changes best estimate value of annuities for all ages: 2.5% and 97.5% quantiles.



Redistribution

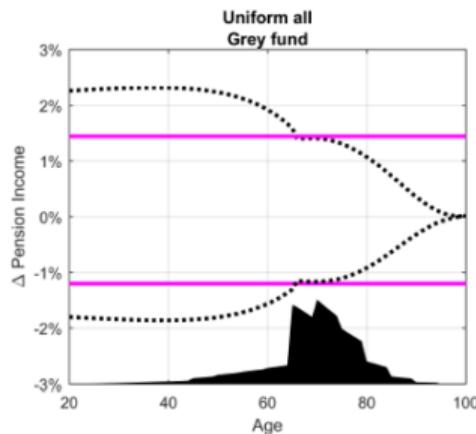
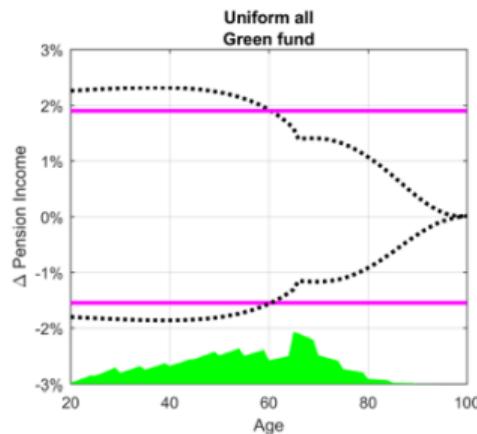
Age composition determines percentage of total pension capital in each cohort. We use two benchmark funds (Green and Grey).



Represent 10% and 90% quantiles in modified duration. (Data: DNB)

Sensitivities: Uniform Redistribution

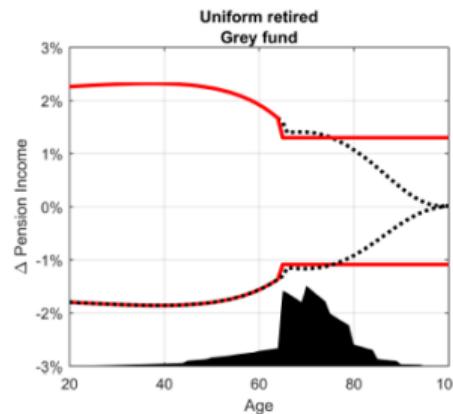
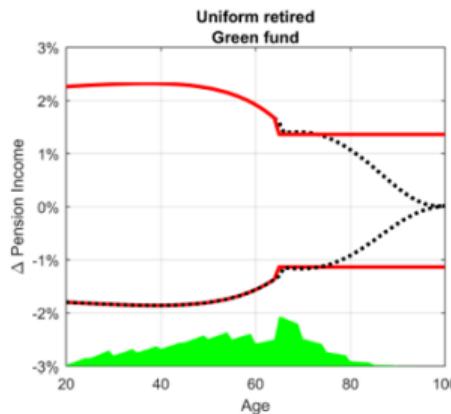
Effect shock (2.5% / 97.% quantiles) on accrued pension income per age group, as a percentage + effect of uniform redistribution.



Youngest will see their macro longevity risk reduced by sharing risk.
Oldest pensioners may witness substantial increase.

Sensitivities: Redistribution among pensioners

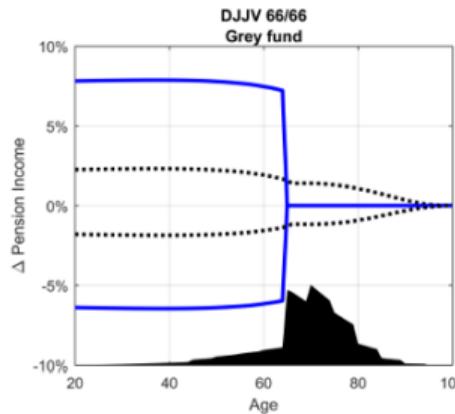
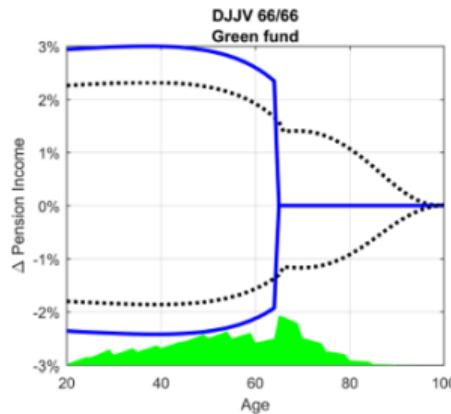
Effect shock (2.5% / 97.% quantiles) on accrued pension income per age group, as a percentage + effect of uniform redistribution.



Youngest pensioners will see their macro longevity risk reduced (a bit).
Oldest pensioners may witness substantial increase.

Sensitivities: Redistribution among active participants

Effect shock (2.5% / 97.% quantiles) on accrued pension income per age group, as a percentage + effect of uniform redistribution.



Effect for active population may be huge!

Redistribution

Notice that:

- Redistribution affects fluctuations (“volatility”) of macro longevity risk but **mean is almost zero**: both advantageous and disadvantageous shocks are redistributed.
- When group takes over all risk for another group (i.e. reduces their fluctuations in pension income due to this risk to zero), they would usually charge a **market price of risk** for it. In pension fund one can choose not to charge this price, but this price will always be charged by external parties.
- In this sense, pension funds provide a possibility to implement an **internal risk sharing mechanism** that cannot be found in the markets.

Redistribution

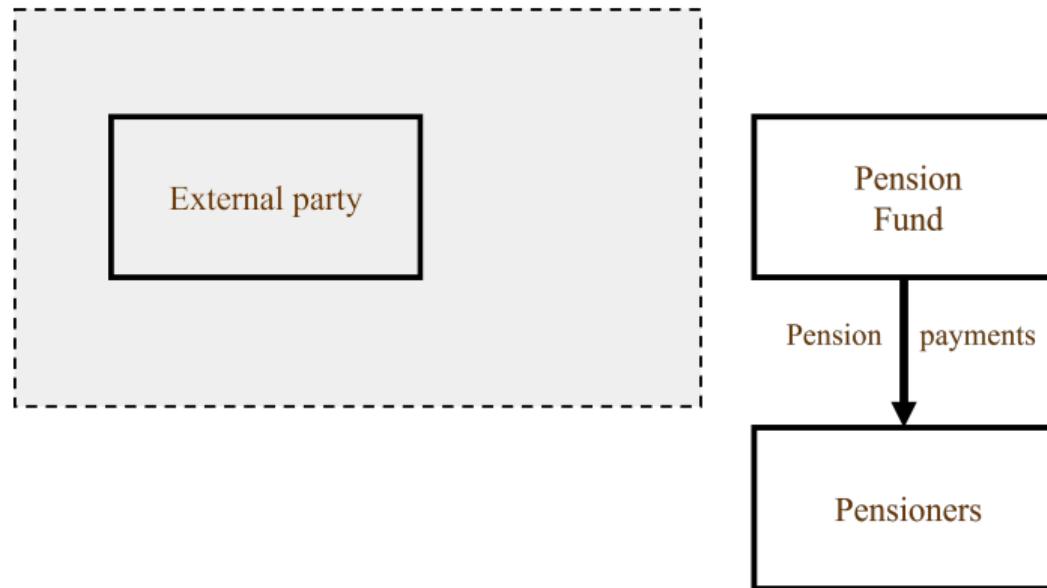
Not taken into consideration in these results:

- Survivor pension for partners.
- Effect on future accrual of pension income (on top of what has been accrued so far).
 - Relative effect of shock to expected pension income at retirement is very similar to relative effect on currently accrued pension income,
 - but risk sharing only affects accrued entitlements!
- Postponing retirement can also be used to mitigate effect of the macro longevity shock corresponding to 2.5% quantile:
 - 2 months (around age 65), up to
 - 3 months (around age 20).

External Mitigation: transfer

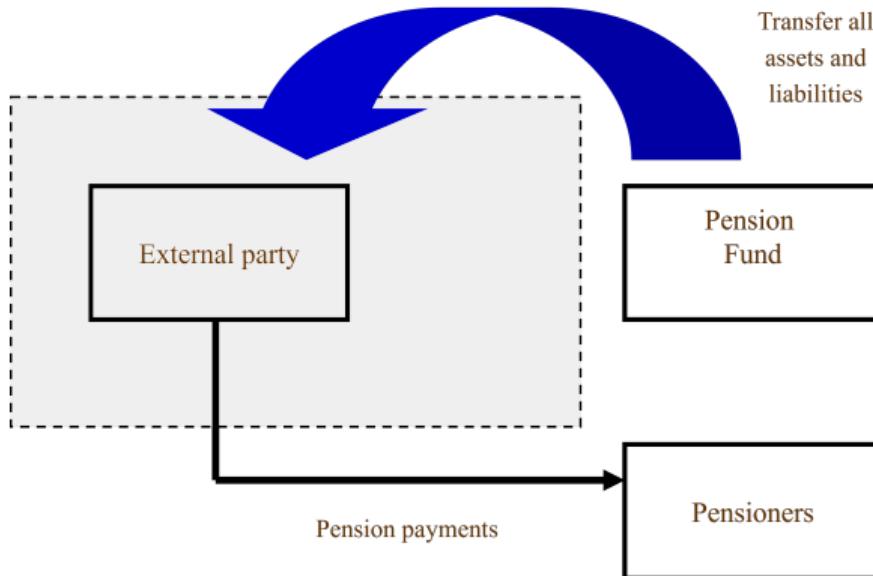
Transfer of Longevity Risk

Initial Situation



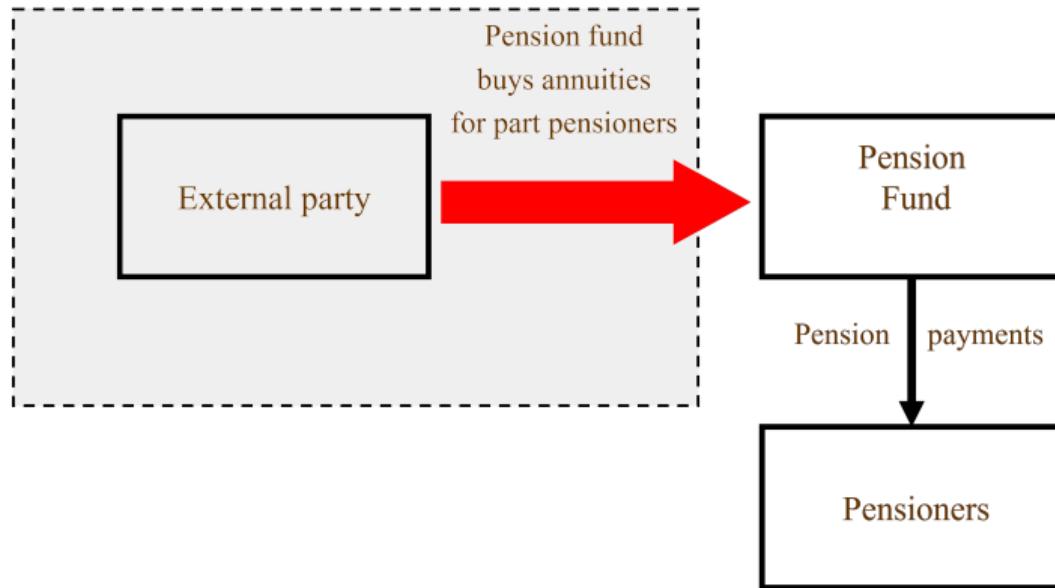
Transfer of Longevity Risk

Buy Out



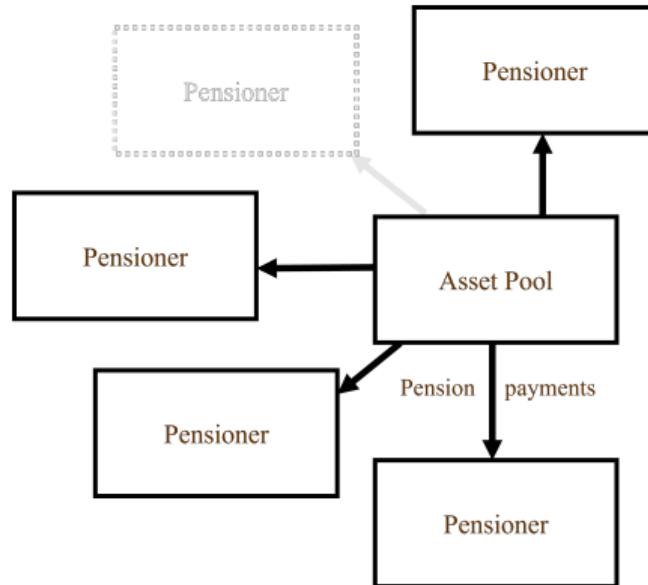
Transfer of Longevity Risk

Buy In



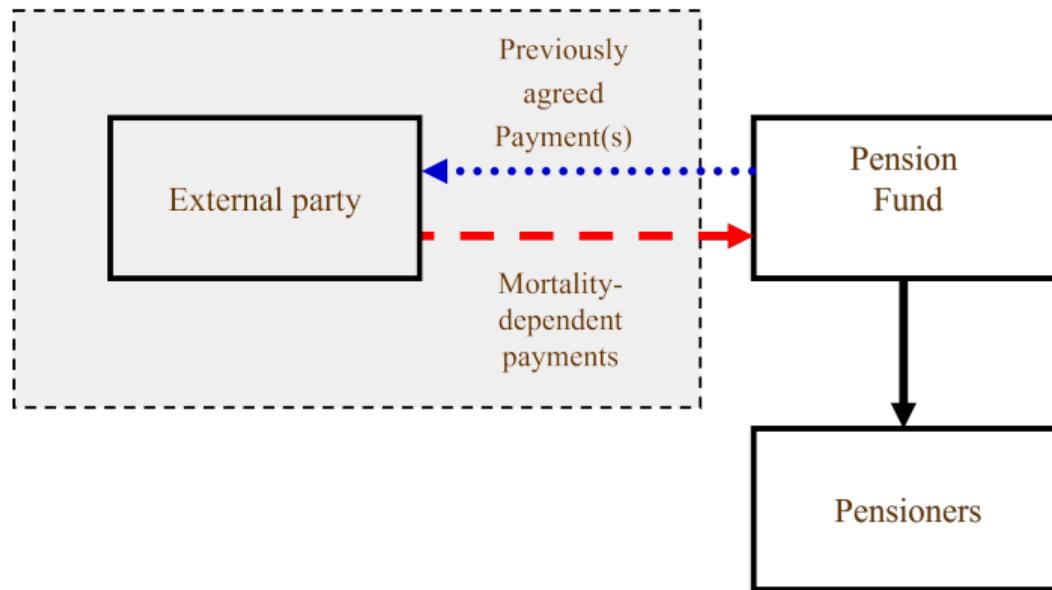
Transfer of Longevity Risk

Tontine



Transfer of Longevity Risk

Longevity products : General Structure



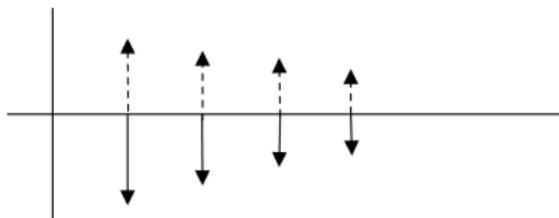
Period versus Cohort Tables

- Payer (pension fund)
pays fixed cashflows, agreed at start of the contact
- Receiver (counterparty)
pays cashflows based on realized mortality

If fixed cashflows correspond to **expected** realized cashflows, fund

- receives compensation for extra payments due to **worse** realized mortality rates and
- pays the counterparty when realized mortality rates are **better** than expected.

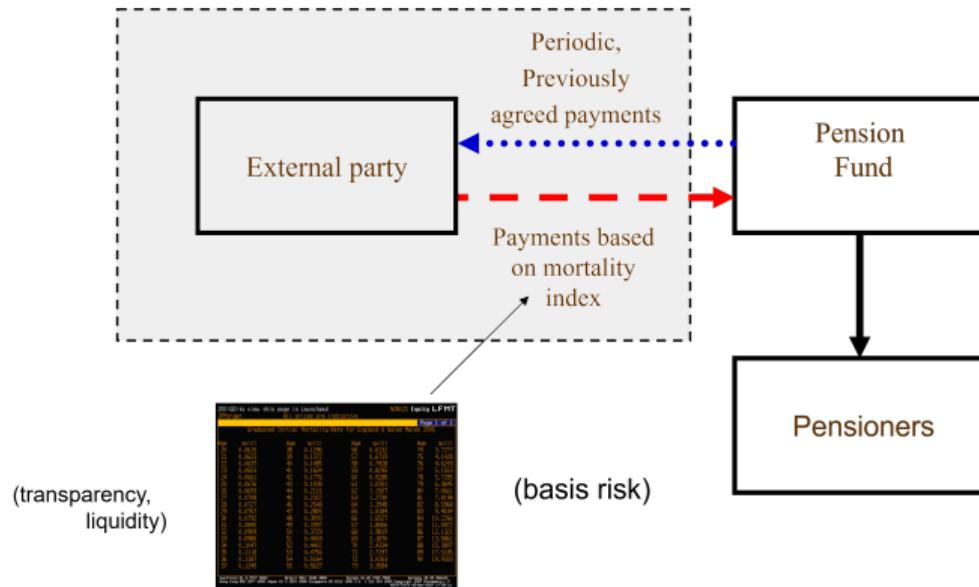
Receiver will charge risk premium on top.



As always:
**give up upside potential
to avoid downside risk**

Transfer of Longevity Risk

Longevity products : Index Swap



Longevity Markets

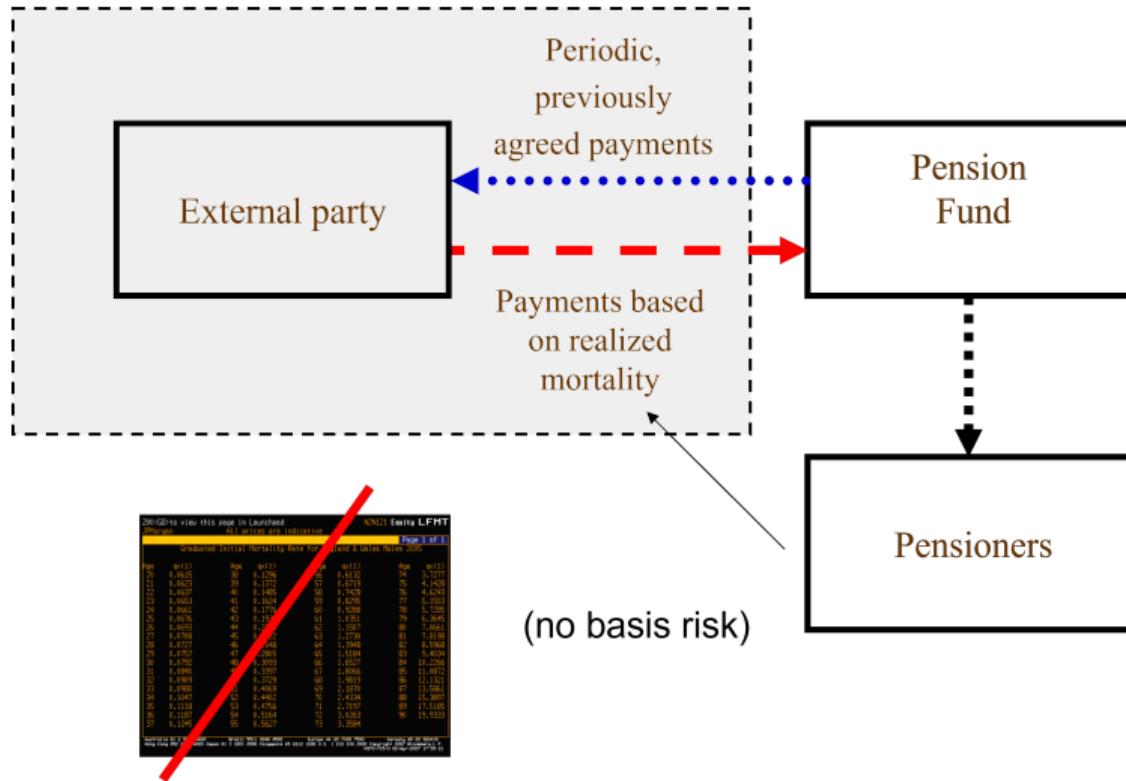
Trading screen for (British) mortality rates

200<GO>to view this page in Launchpad		N2N121 Equity LFMT					
JPMorgan		All prices are indicative					
Graduated Initial Mortality Rate for England & Wales Males 2005							
Age	qx(%)	Age	qx(%)	Age	qx(%)	Age	qx(%)
20	0.0615	38	0.1296	56	0.6132	74	3.7277
21	0.0623	39	0.1372	57	0.6719	75	4.1428
22	0.0637	40	0.1485	58	0.7428	76	4.6243
23	0.0653	41	0.1624	59	0.8295	77	5.1553
24	0.0661	42	0.1776	60	0.9288	78	5.7285
25	0.0676	43	0.1938	61	1.0351	79	6.3645
26	0.0693	44	0.2116	62	1.1507	80	7.0661
27	0.0708	45	0.2322	63	1.2730	81	7.8198
28	0.0727	46	0.2548	64	1.3948	82	8.5968
29	0.0757	47	0.2805	65	1.5184	83	9.4034
30	0.0792	48	0.3093	66	1.6527	84	10.2266
31	0.0840	49	0.3397	67	1.8066	85	11.0872
32	0.0909	50	0.3729	68	1.9819	86	12.1321
33	0.0980	51	0.4069	69	2.1870	87	13.5861
34	0.1047	52	0.4402	70	2.4334	88	15.3897
35	0.1118	53	0.4756	71	2.7197	89	17.5185
36	0.1187	54	0.5164	72	3.0263	90	19.9333
37	0.1245	55	0.5627	73	3.3584		

Australia 61 2 9277 9600 Brazil 5511 3048 4500 Europe 44 20 7230 7500 Germany 49 69 920410
Hong Kong 852 2977 6000 Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2007 Bloomberg L.P.
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Transfer of Longevity Risk

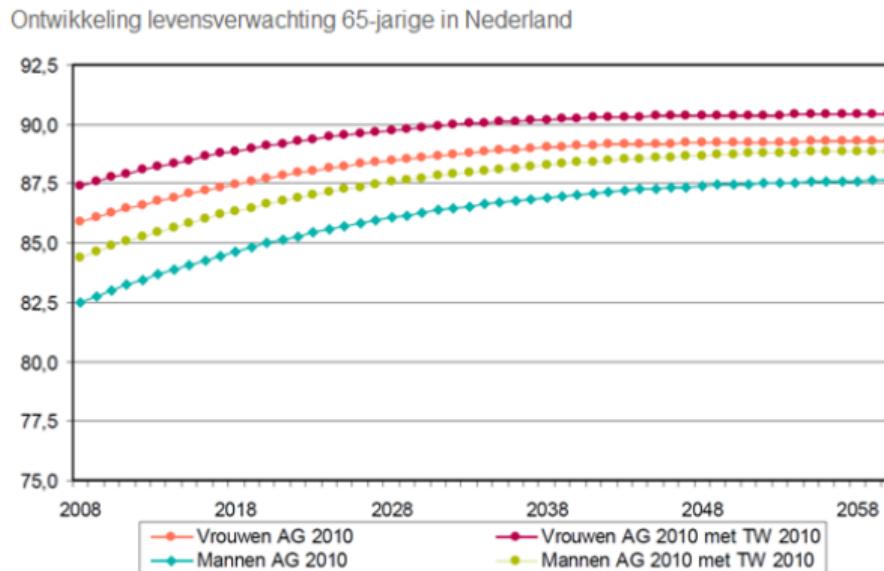
Longevity products : Indemnity Swap



Age	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113
20	0.0025	36	0.1526	52	0.4532	54	0.5222
21	0.0025	39	0.1372	57	0.4519	75	0.4248
22	0.0025	42	0.1228	59	0.4506	78	0.3978
23	0.0025	41	0.1084	59	0.4506	77	0.3963
24	0.0025	45	0.0940	61	0.4506	78	0.3958
25	0.0025	46	0.1178	62	0.4506	79	0.3953
26	0.0025	44	0.1334	61	0.4506	80	0.3948
27	0.0025	46	0.1490	62	0.4506	81	0.3943
28	0.0025	46	0.1646	64	0.4506	82	0.3938
29	0.0025	47	0.1702	65	0.4506	83	0.3934
30	0.0025	48	0.1758	66	0.4506	84	0.3930
31	0.0025	48	0.1814	67	0.4506	85	0.3926
32	0.0025	49	0.1870	68	0.4506	86	0.3922
33	0.0025	50	0.1926	69	0.4506	87	0.3918
34	0.0025	52	0.4462	70	0.4506	88	0.3905
35	0.0025	54	0.4792	71	0.4506	89	0.3891
36	0.0025	54	0.5124	72	0.4506	90	0.3878
37	0.0025	55	0.5627	73	0.4506		

Basis Risk

Difference between third pillar pension participants and general population. Only unexpected part is basis risk !



Source: Towers Watson

Longevity Markets

Coronavirus deaths may trigger longevity swap margin calls for pensions

14TH APRIL 2020 - AUTHOR: STEVE EVANS

Share: 

The sudden rise in deaths in the United Kingdom due to the Covid-19 coronavirus pandemic could result in some pensions having to post more collateral against their longevity swaps, as report suggest.

£20bn of longevity swaps forecast for 2023 by WTW

23RD JANUARY 2023 - AUTHOR: STEVE EVANS

Share: 

Brokerage and consultancy WTW is forecasting a busier year for the longevity swap market in 2023, with increasing reinsurance capacity and market participants available and attracted to longevity risk transfer a driver.

NN Group taps Prudential & Swiss Re for €13bn longevity risk transfer

20TH DECEMBER 2023 - AUTHOR: STEVE EVANS

Share: 

NN Group has announced the successful transfer of €13 billion in longevity risk associated with pension liabilities in the Netherlands to an insurance subsidiary of Prudential Financial, Inc. and global reinsurance specialist Swiss Re.

Source: www.artemis.bm

Key References

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