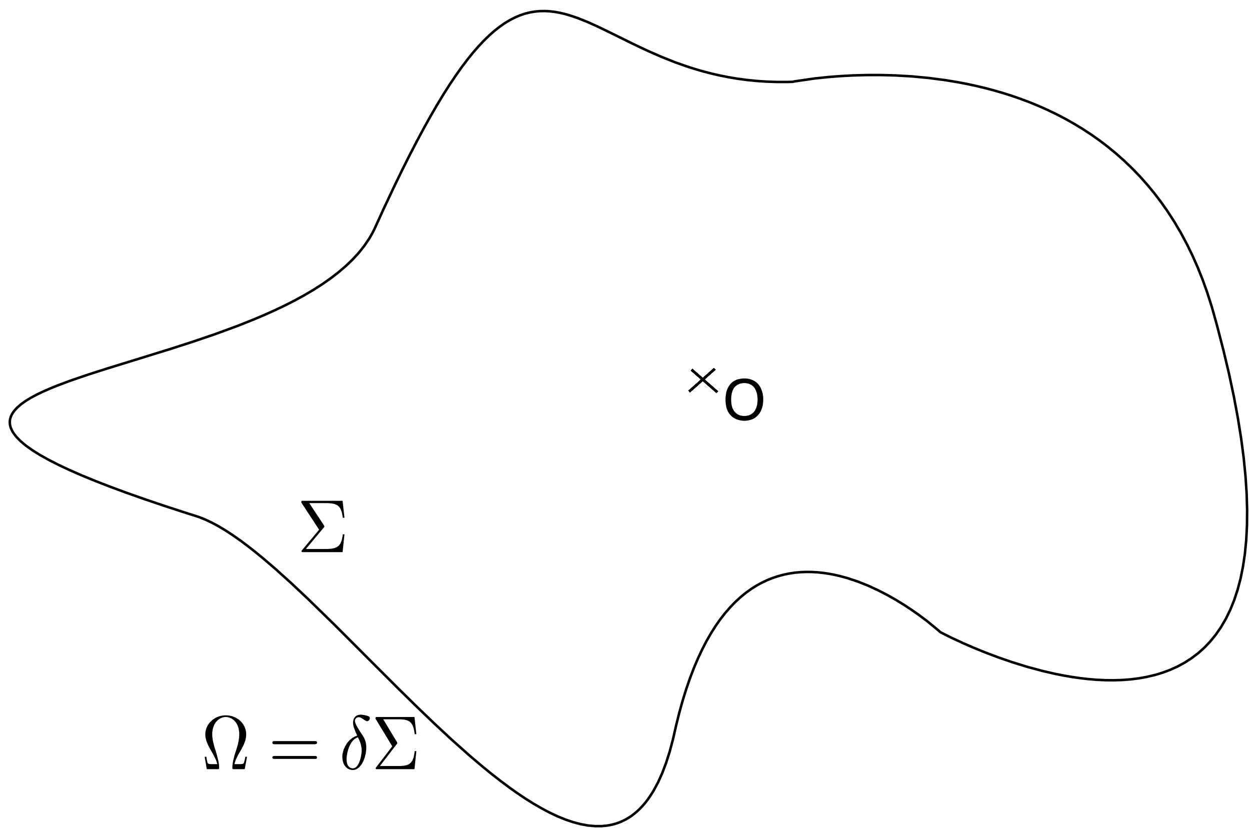
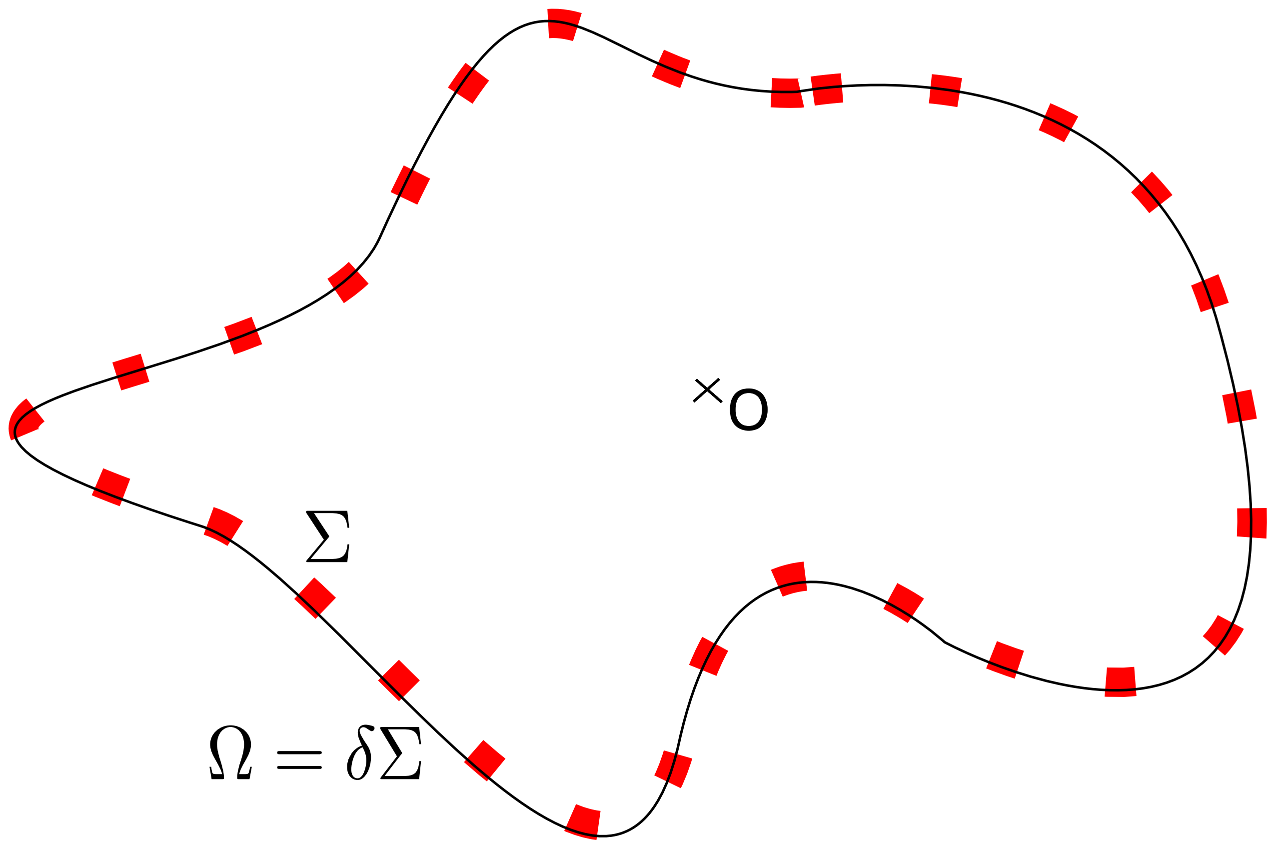
Representations of Depth Maps: Spherical Harmonics and Spherical Haar Wavelets

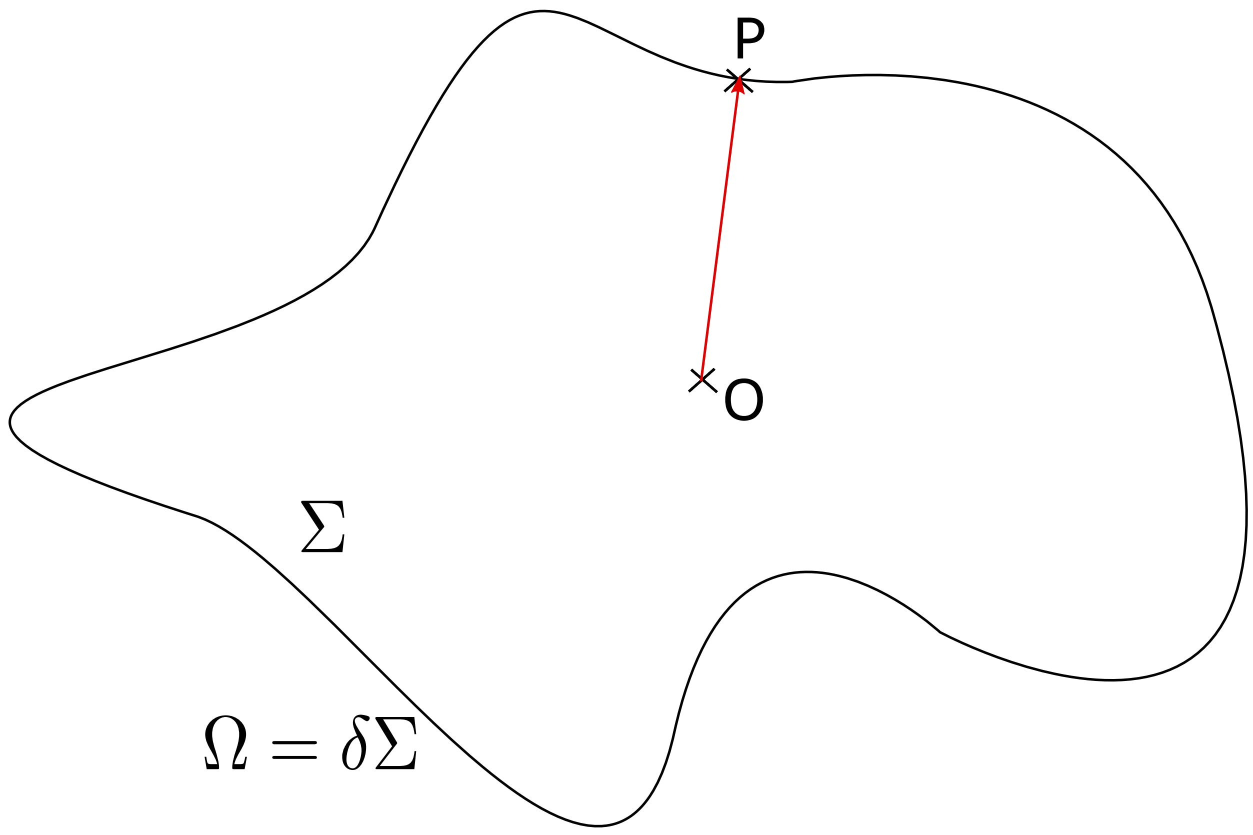
Katrina Ashton: u5586882

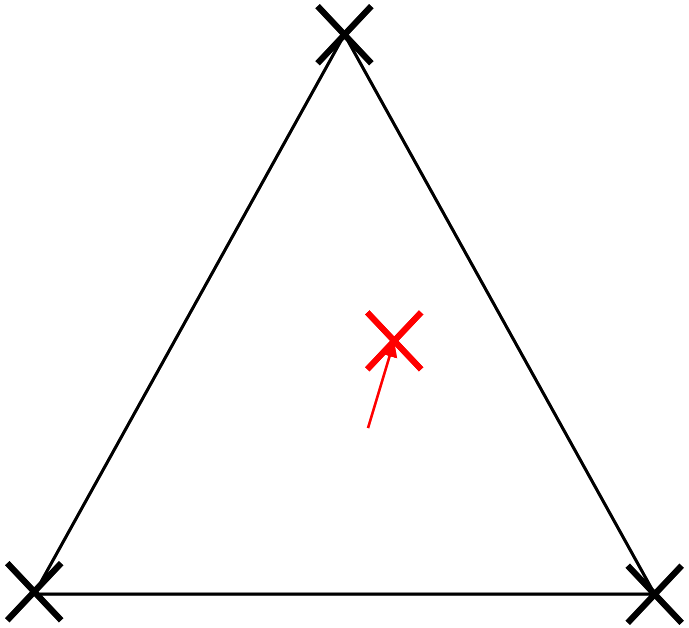
Supervisor: Jochen Trumpf

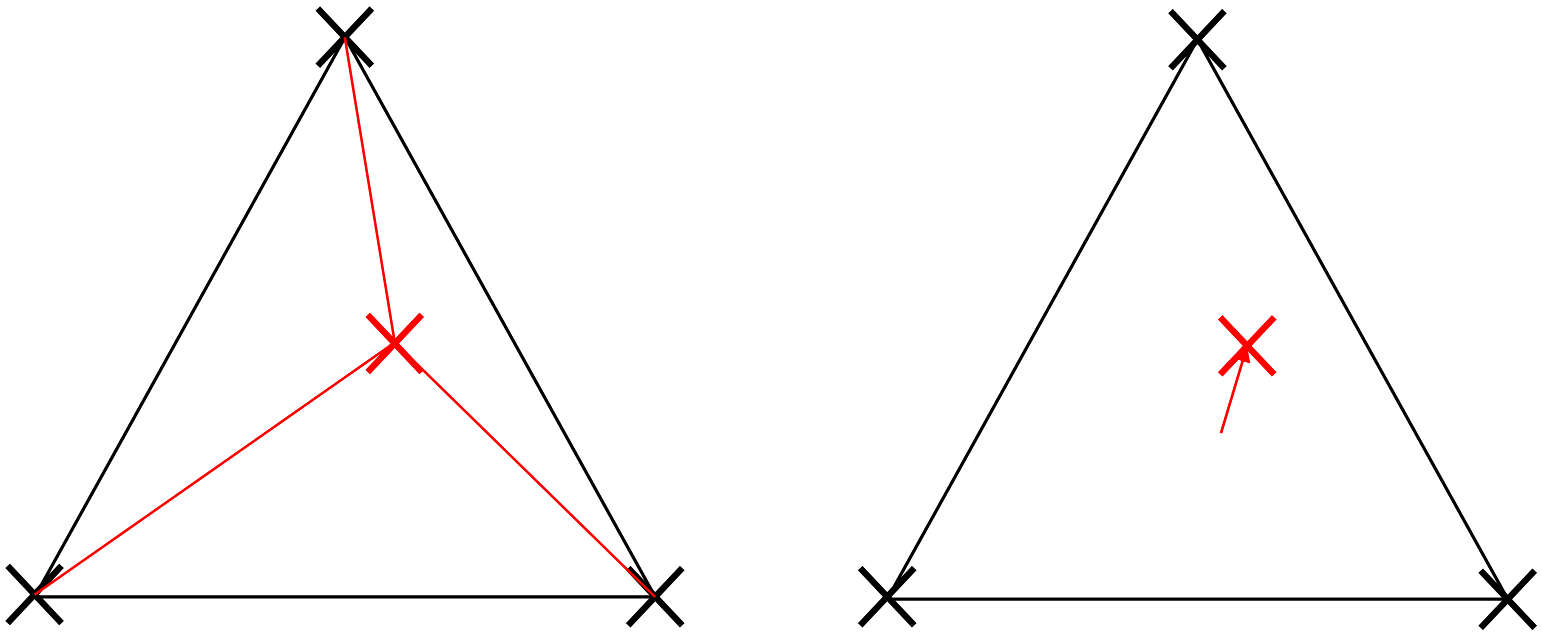
Australian National University

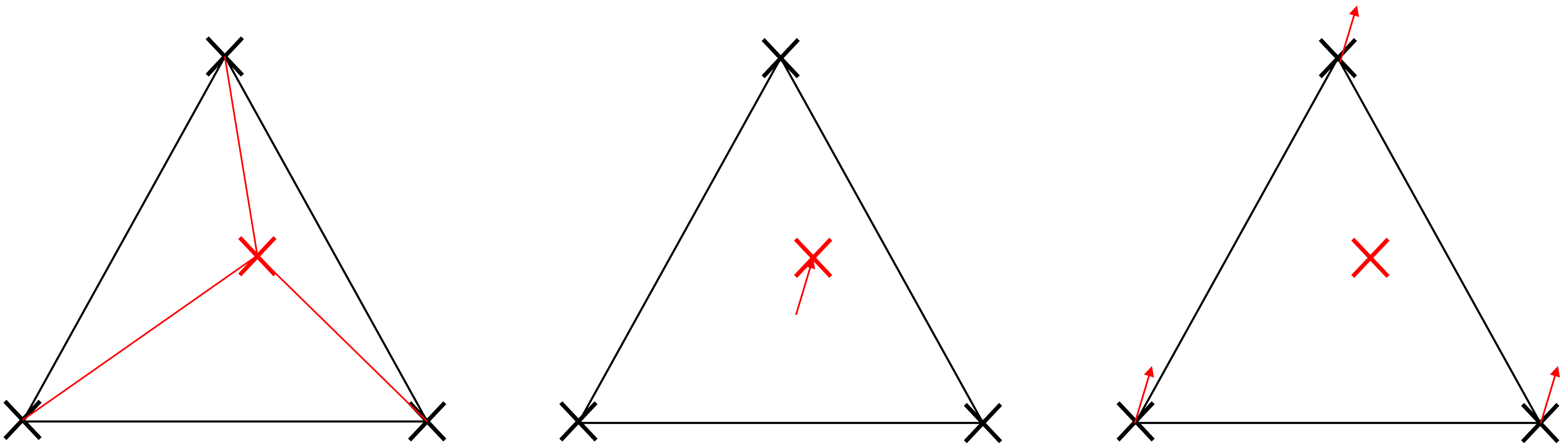

×O
Σ
Ω = δΣ


×O
Σ
Ω = δΣ


P
O
Σ
$\Omega = \delta\Sigma$







Basis Functions

Benefits:

Integrate previous knowledge

Store coefficients, can interpolate

->Efficient

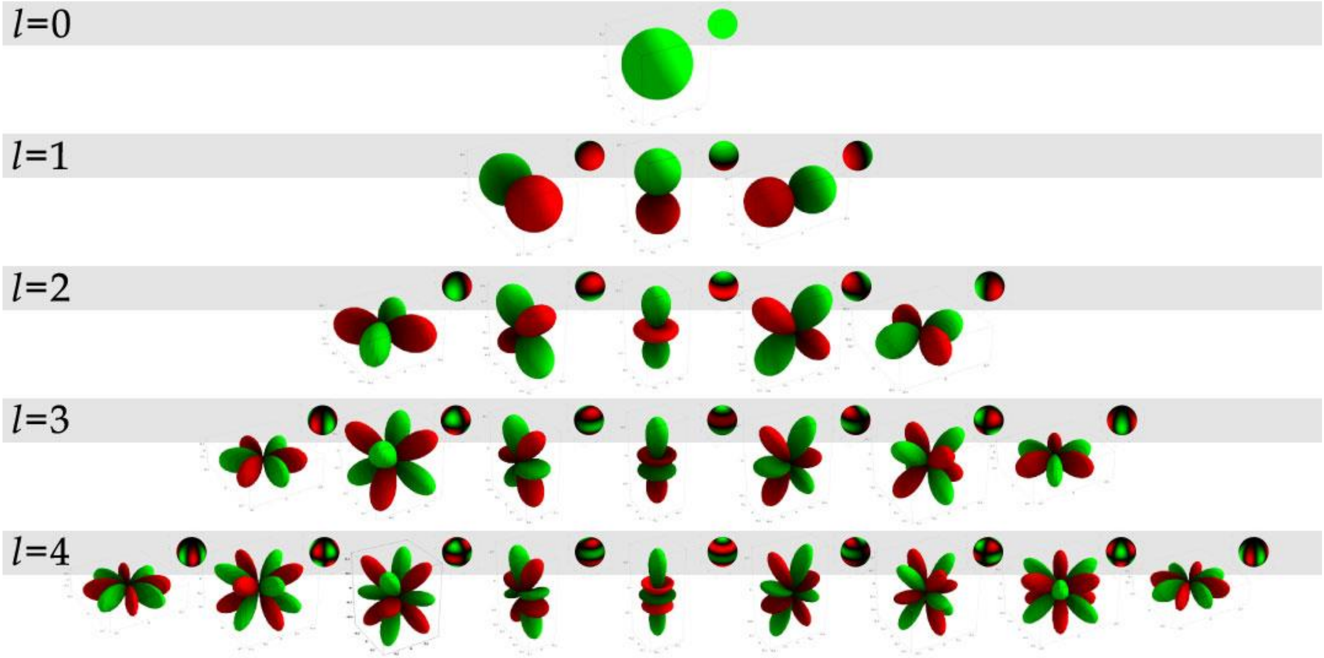
Well-studied update formulae

Bases looked at:

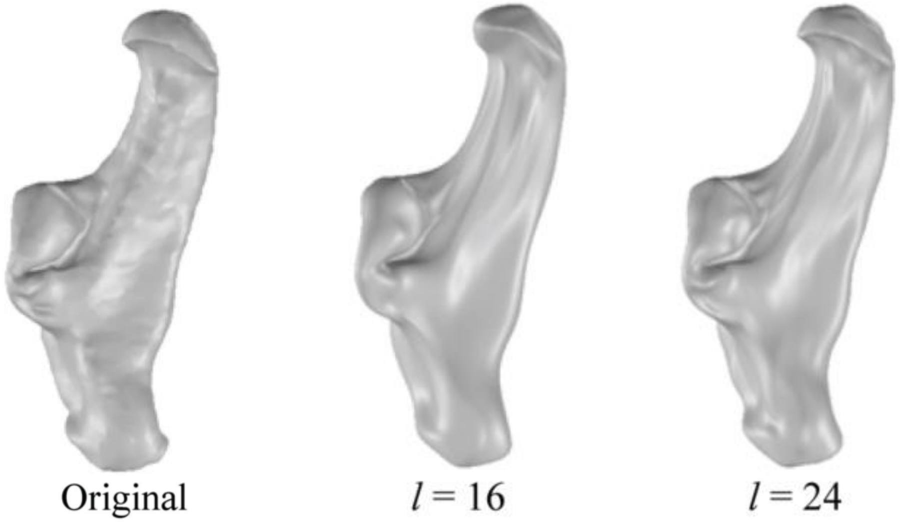
Spherical Harmonics

Spherical Haar Wavelets

Spherical Harmonics

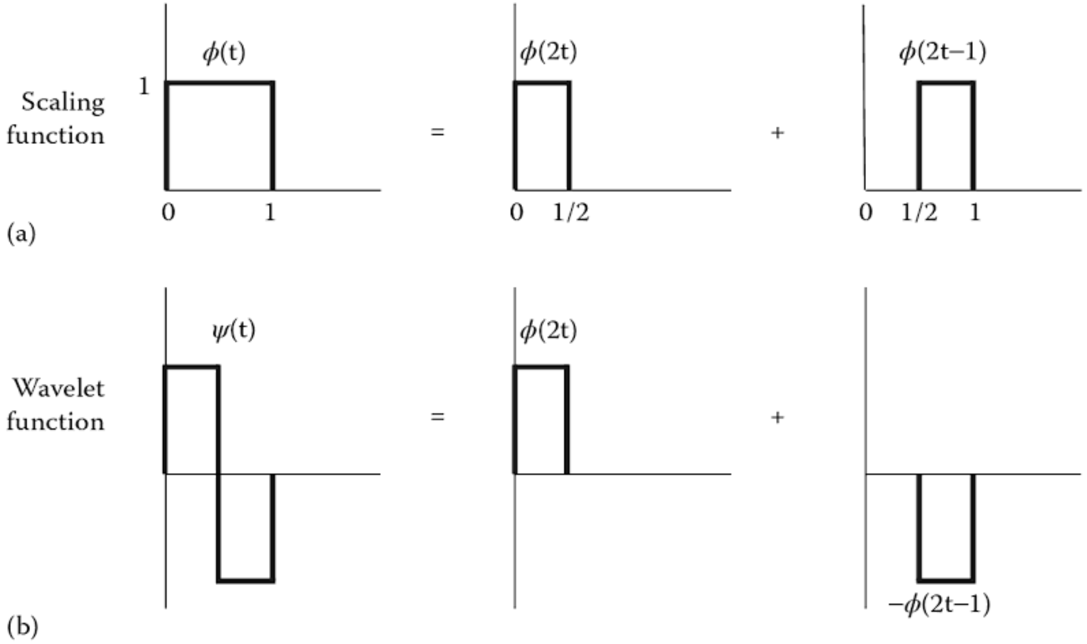


R. Green. Spherical Harmonic Lighting: The Gritty Details. *Archives of the Game Developers Conference*, Vol. 56, 2003.



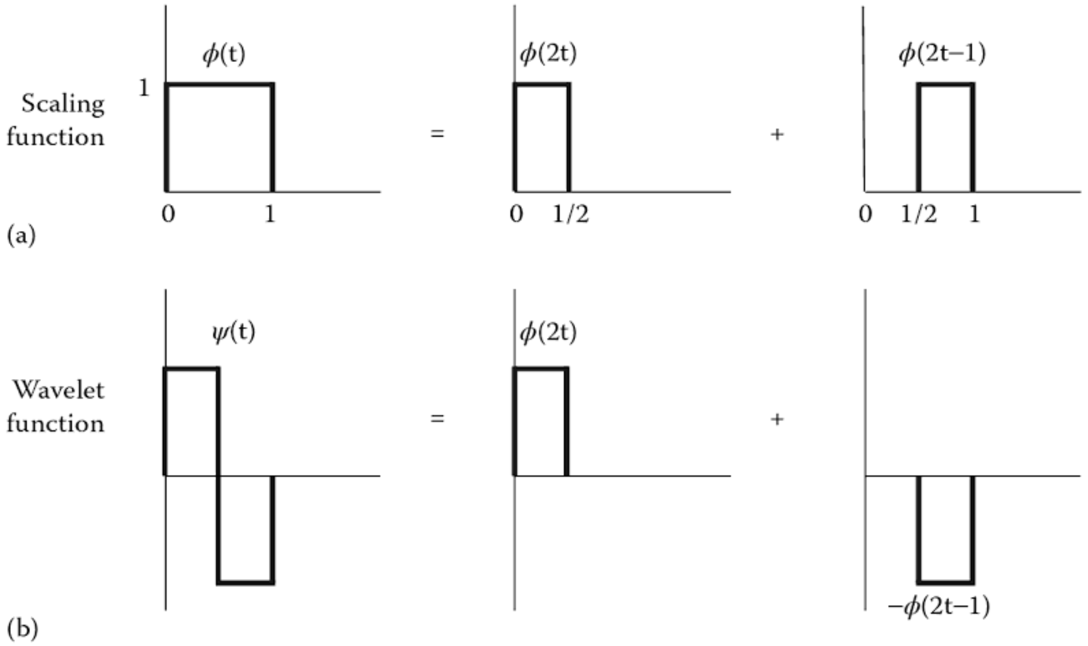
L. Shen, H. Farid, M. McPeek. Modelling three-dimensional morphological structures using spherical harmonics. *Evolution*, 63(4), 2010.

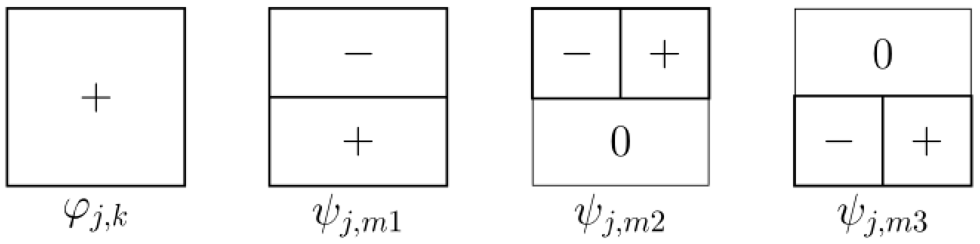
Spherical Haar Wavelets



P. Addison. *The Illustrated Wavelet Transform Handbook.* IOP Publishing, 2002. p 74.

Spherical Haar Wavelets

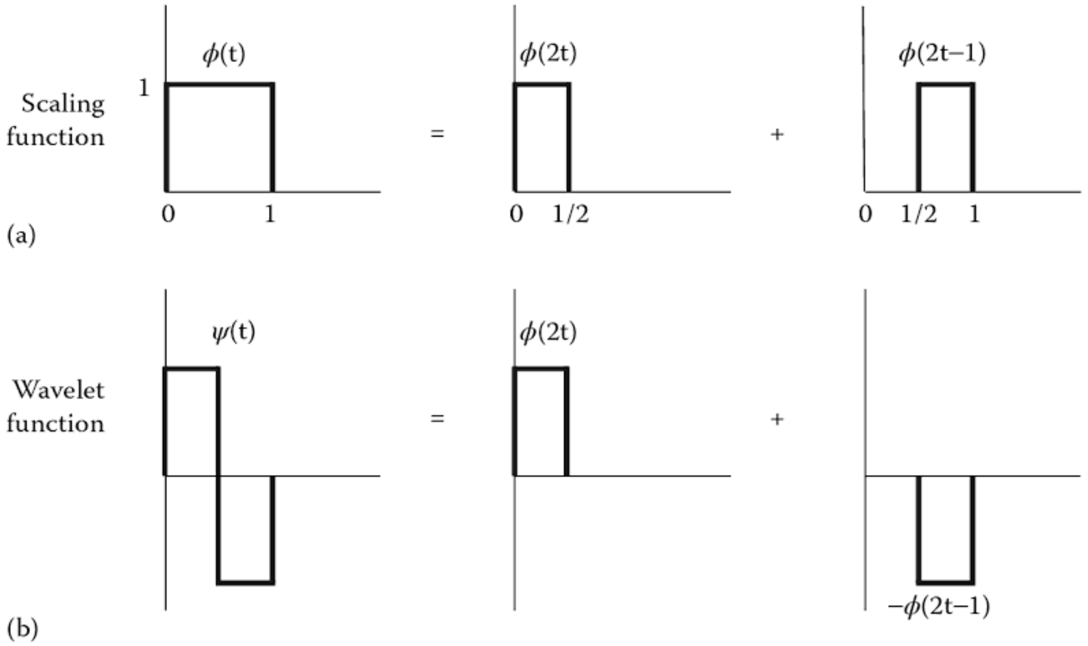


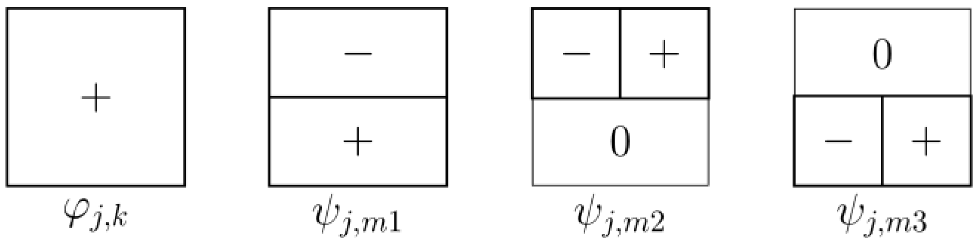


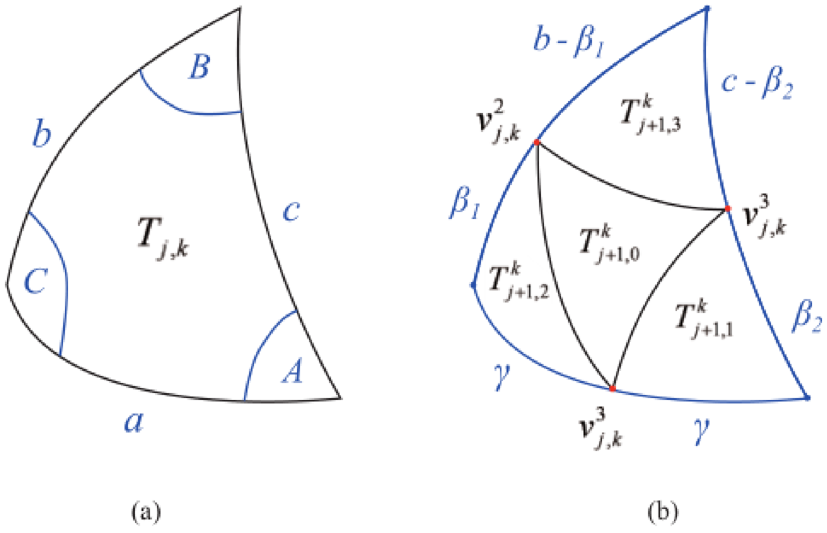
P. Addison. *The Illustrated Wavelet Transform Handbook.* IOP Publishing, 2002. p 74.

W. Sweldens. The lifting scheme: A construction of second generation wavelets.
SLAM journal on mathematical analysis, 29(2):511-546, 1998.

Spherical Haar Wavelets





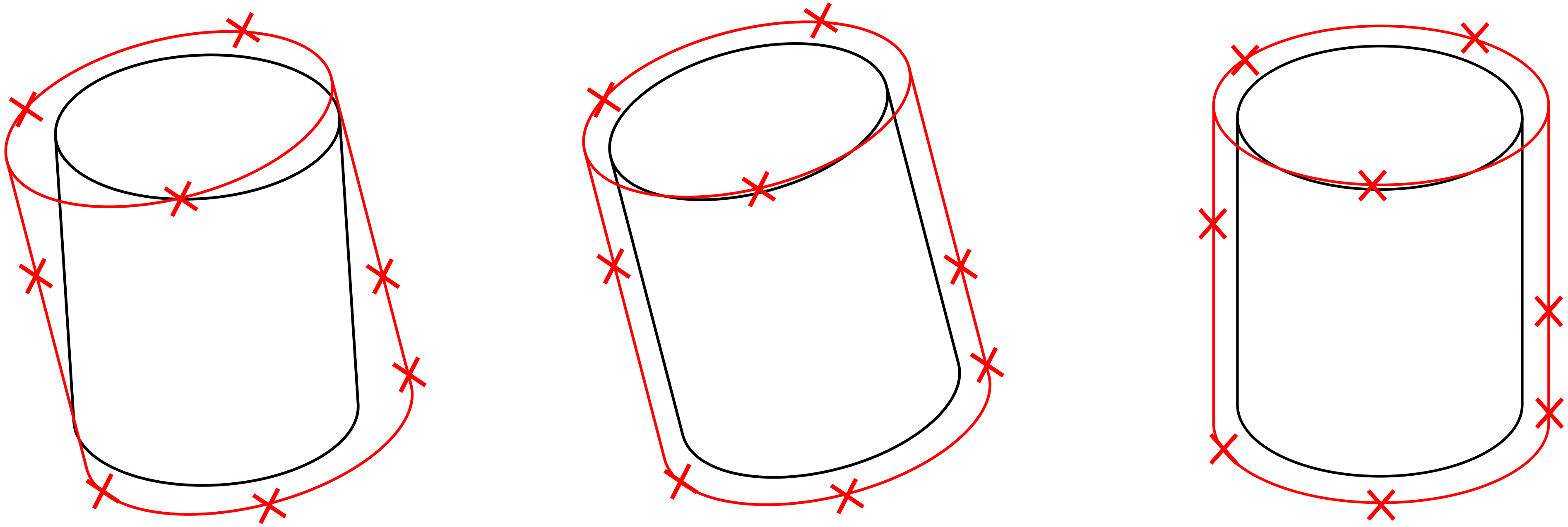


P. Addison. *The Illustrated Wavelet Transform Handbook.* IOP Publishing, 2002. p 74.

W. Sweldens. The lifting scheme: A construction of second generation wavelets.
SLAM journal on mathematical analysis, 29(2):511-546, 1998.

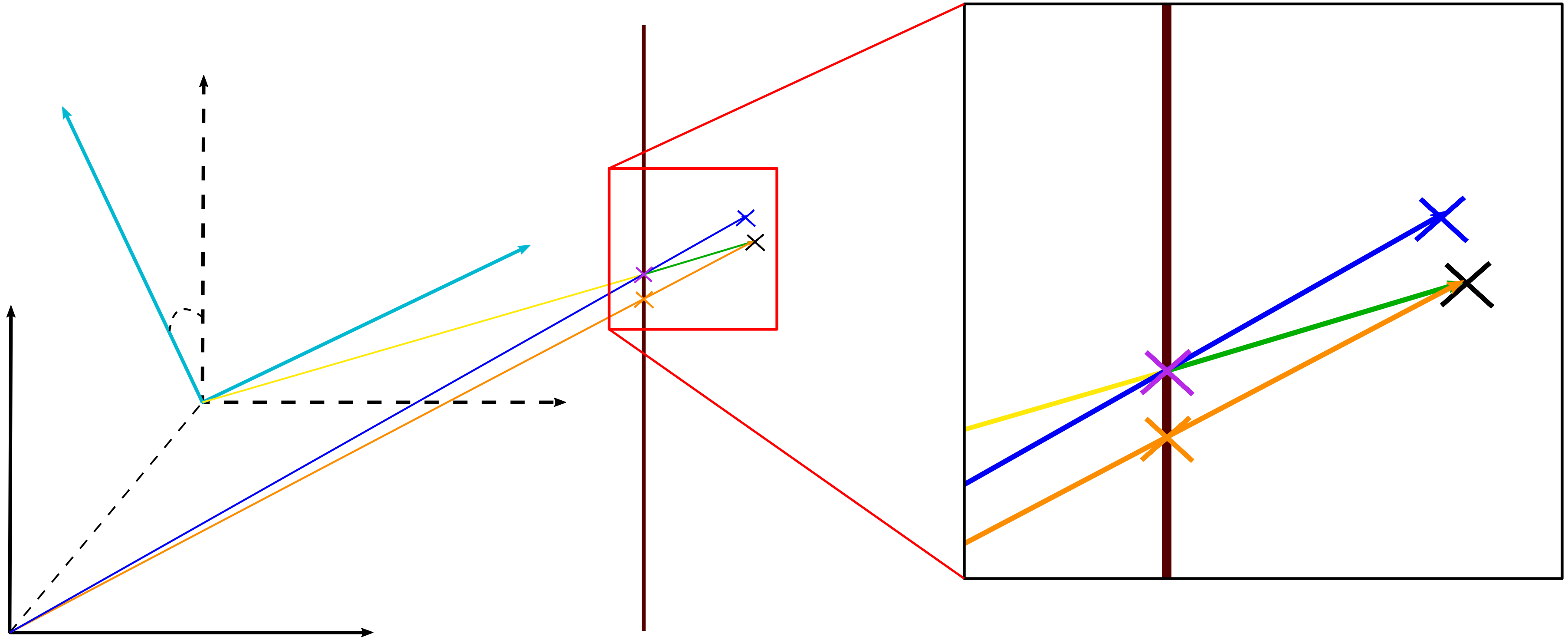
C. Lessig and E. Fiume. SOHO: orthogonal and symmetric Haar wavelets on the sphere.
ACM Transactions on Graphics (TOG), 27(1):4, 2008

Updating Models



Unaligned measurements 1. Transformed model 2. Transformed measurements

Transforming measurements

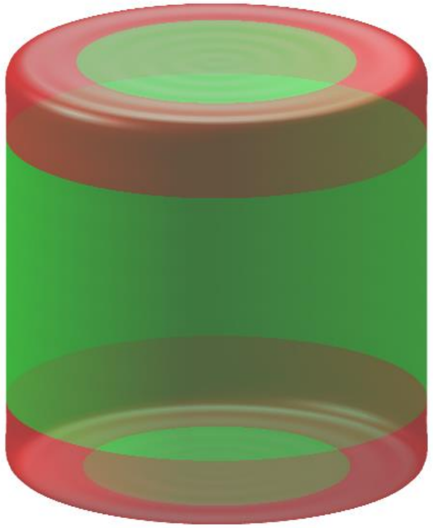


Results

<table>
<tr><th></th><th colspan="2">Spherical Harmonics</th><th colspan="2">Spherical Haar Wavelets</th></tr>
<tr><td>Moved Model Method</td><td rowspan="2">Need to choose good step size

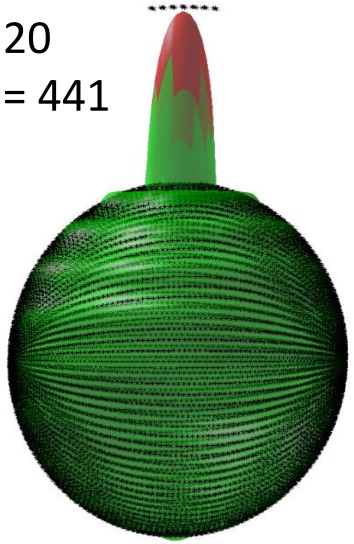
Negligible difference in noise floor, robustness and convergence</td><td>No translation</td><td rowspan="2">Choice of data structure is important
Need to choose good step sizes

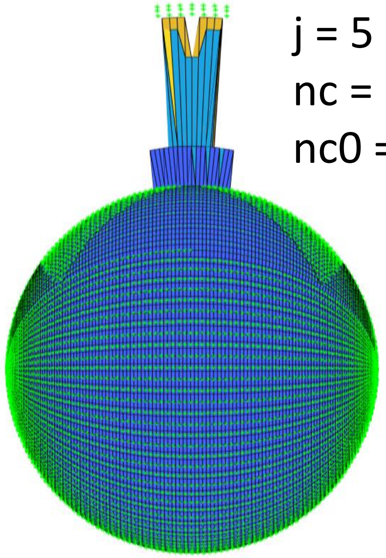
Only have preliminary results</td><td>No translation</td></tr>
<tr><td>Moved Measurements Method</td><td>Translation
Fast</td><td>Translation</td></tr>
</table>



l = 40
nc = 1,681

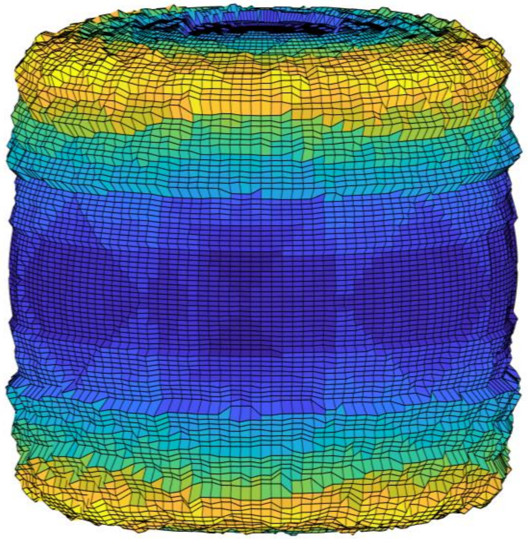
l = 20
nc = 441



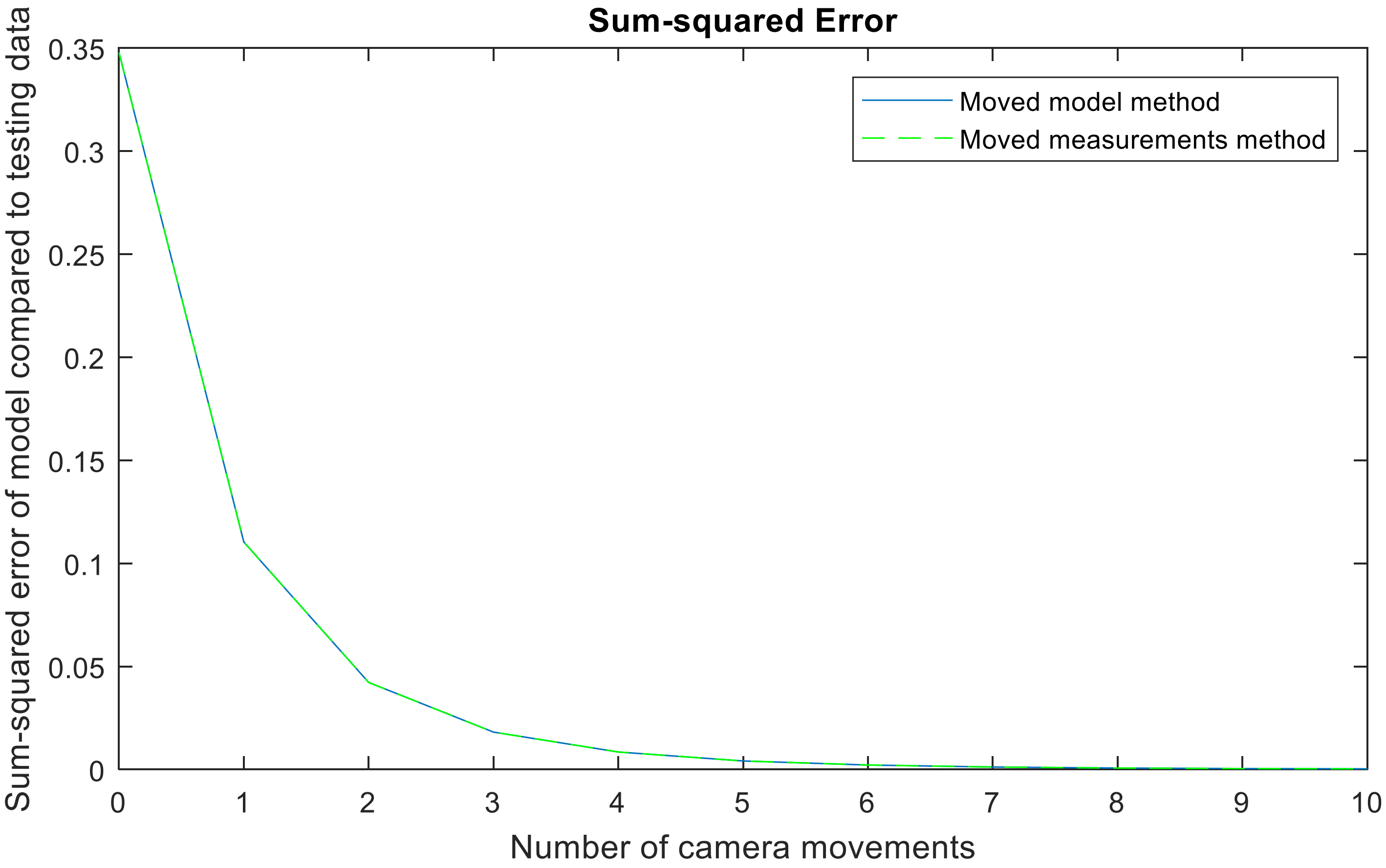


j = 5
nc = 20
nc0 =8,192

j = 5
nc = 1,681
nc0 =8,192



Results


Sum-squared Error
Sum-squared error of model compared to testing data
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7 8 9 10
Number of camera movements
Moved model method
Moved measurements method

Conclusion and Future Work

Main outcomes

Spherical Harmonic Basis Functions
- Implementation of creating a base model
- Implementation of update method

Spherical Haar Wavelet Basis Functions
- Implementation of creating a base model
- Implementation of update method

Future work

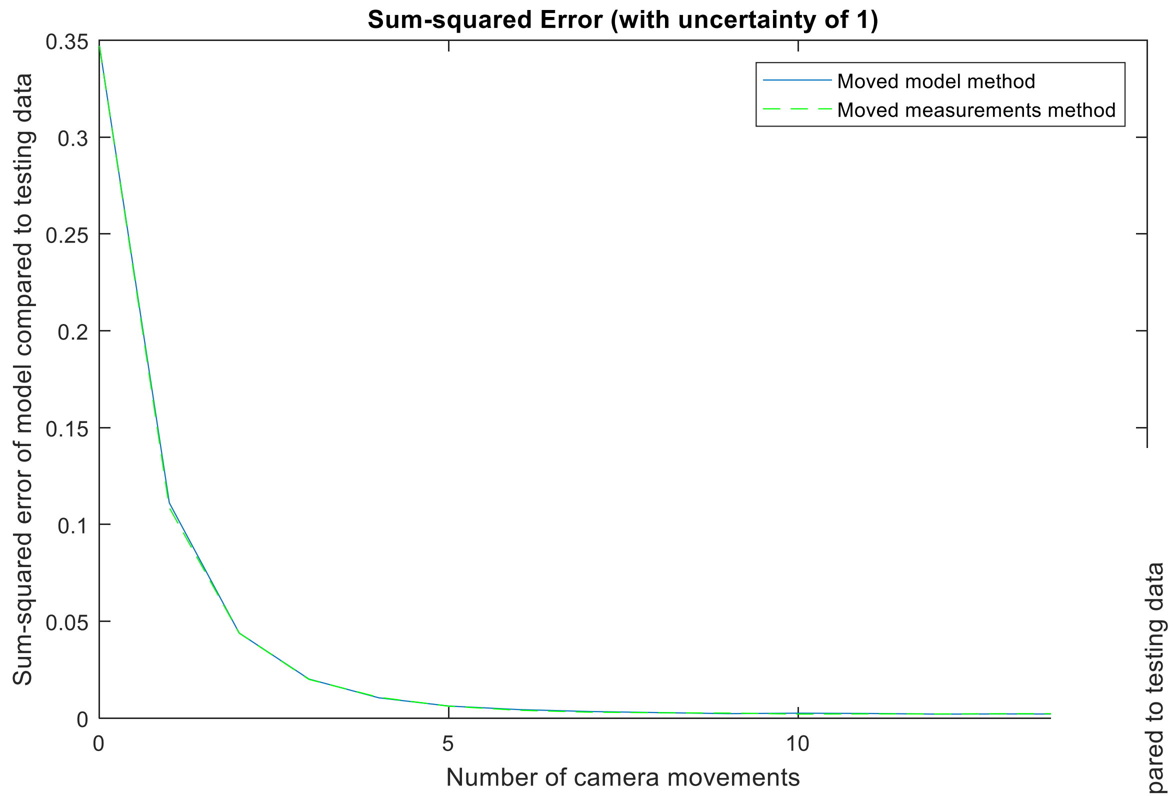
Modelling different shapes

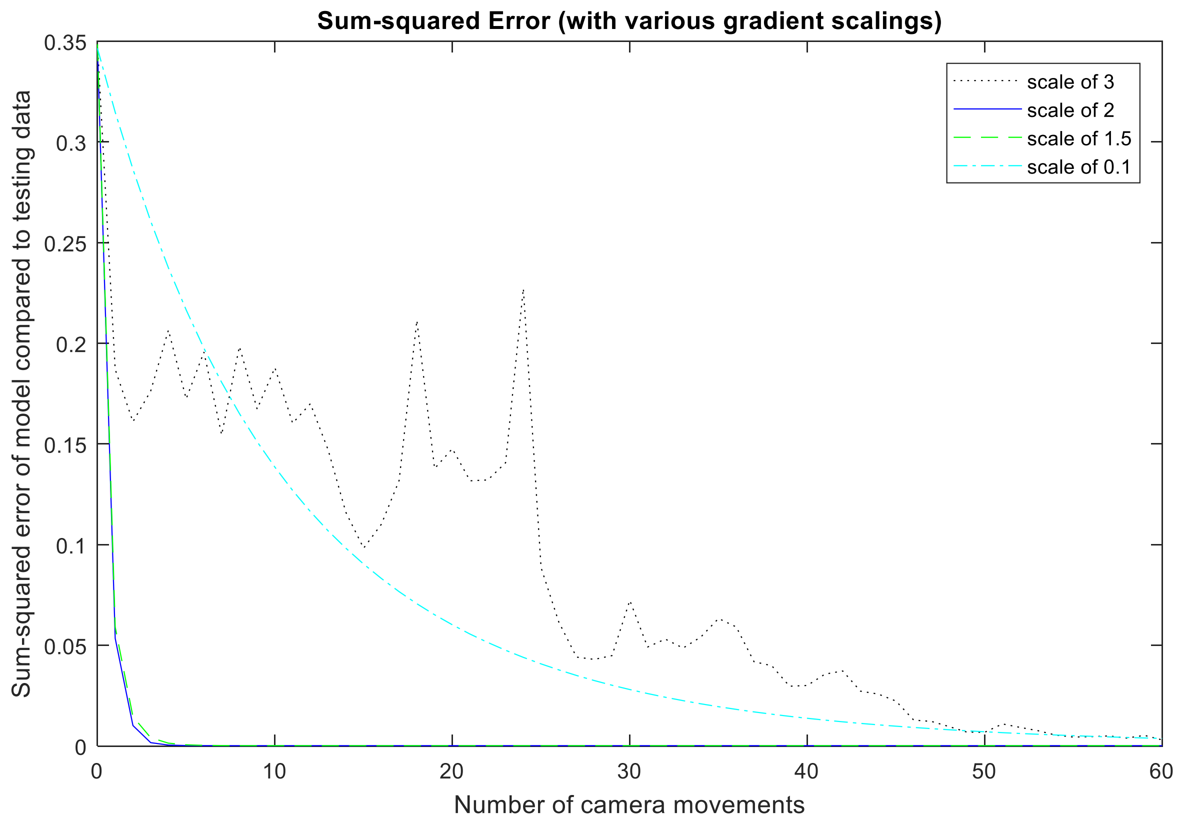
Investigate which way to implement the moved measurements method

Investigate different data structures

Investigate different spherical wavelet bases

Questions?


Sum-squared Error (with uncertainty of 1)
Moved model method
Moved measurements method
Sum-squared error of model compared to testing data
Number of camera movements
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
5
10


Sum-squared Error (with various gradient scalings)
scale of 3
scale of 2
scale of 1.5
scale of 0.1
Sum-squared error of model compared to testing data
Number of camera movements
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
10
20
30
40
50
60

Formulae

Spherical Harmonics

The complex spherical harmonic Y_{lm} can be written as

$$Y_{lm}(\theta, \phi) = (-1)^m N_{lm} P_l^m(cos(\theta)) e^{im\phi},$$

where $l \in \{0, 1, 2, ..., n, ...\}$ and $m \in \{-l, -l+1, ..., 0, ...l-1, l\}$, N_{lm} is the normalisation factor

$$N_{lm} = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}}$$

and $P_l^m(x)$ are the Legendre associated polynomials, that is

$$P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x),$$

where $P_l(x)$ is the usual Legendre polynomial of degree l,

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l.$$

The real spherical harmonics can be found by combining complex conjugates, corresponding to opposite values of m. Then the real spherical harmonics S_{lm} can be defined as

$$S_{lm}(\theta, \phi) = \begin{cases} N_{lm} P_l^m(\cos\theta) \sqrt{2} \cos(m\phi) & m > 0 \\ N_{lm} P_l^m(\cos\theta) & m = 0 \\ N_{l|m|} P_l^{|m|}(\cos\theta) \sqrt{2} \sin(|m|\theta) & m < 0 \end{cases}$$

Spherical Haar Wavelets

The scaling basis functions $\phi_{j,k}$ are constant over their support $T_{j,k}$, with their value given by a normalisation constant $\eta_{j,k}$:

$$\phi_{j,k} = \eta_{j,k} \tau_{j,k}.$$

We choose $\eta_{j,k} = 1/\sqrt{\alpha_{j,k}}$, as this combined with the disjoint nature of the $T_{j,k}$ for fixed j gives that the $\phi_{j,k}$ on the same level are orthogonal

The wavelet basis functions $\psi_{j,k}^l$ are as follows

$$\begin{aligned} \psi_{j,k}^0 &= \frac{\Lambda_1}{\Lambda_0}\tau_0 + \frac{1}{\Lambda_1}((-2a+1)\tau_1 + a\tau_2 + a\tau_3) \\ \psi_{j,k}^1 &= \frac{\Lambda_1}{\Lambda_0}\tau_0 + \frac{1}{\Lambda_1}(a\tau_1 + (-2a+1)\tau_2 + a\tau_3) \\ \psi_{j,k}^2 &= \frac{\Lambda_1}{\Lambda_0}\tau_0 + \frac{1}{\Lambda_1}(a\tau_1 + a\tau_2 + (-2a+1)\tau_3), \end{aligned}$$

where

$$a = \frac{\alpha_0 \pm \sqrt{\alpha_0^2 + 3\alpha_0\alpha_1}}{2\alpha_0}$$

$$\Lambda_l = \sqrt{\alpha_{j+1,l}^k}.$$