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Assignment 4 – Due 7/23/2017

## Part I. Exercise Set 5.2 [9], Set 5.3 [10, 18, 23b] Set 5.4 [2, 10]

**Set 5.2**

**Prove each statement using mathematical induction. Do not derive them from Theorem 5.2.2 or Theorem 5.2.3**

**9Q: For all integers $n \geq 3$,**

$\mathbf{4^3 + 4^4 + 4^5 + \dots + 4^n = [4(4^n - 16)] / 3}$

**A:**

Basis Step:

P(n) = $\mathbf{4^3 + 4^4 + 4^5 + \dots + 4^n = [4(4^n - 16)] / 3}$

P(3) = $\mathbf{4^3 = [4(4^3 - 16)] / 3}$

**LHS: $\mathbf{4^3 = 64}$**

**RHS:** = [4(64 – 16)] / 3

= [4(48)]/3

= **64**

Hence P(n) is true for n = 3

Induction: Assume P(k) is true, show that P(k+1) is true

P(k) = $\mathbf{4^3 + 4^4 + 4^5 + \dots + 4^k = [4(4^k - 16)] / 3}$

P(k+1) = $\mathbf{4^3 + 4^4 + 4^5 + \dots + 4^{k+1} = [4(4^{k+1} - 16)] / 3}$

This can be re-written as:

$\mathbf{4^3 + 4^4 + 4^5 + \dots + 4^k} + \mathbf{4^{k+1}} = \mathbf{[4(4^{k+1} - 16)] / 3}$

**Since we assume P(k) is true, we can re-write P(k+1) again as:**

$\mathbf{[4(4^k - 16)] / 3} + \mathbf{4^{k+1}} = \mathbf{[4(4^{k+1} - 16)] / 3}$

**LHS:**

**$[4(4^k - 16)] / 3$** + **$4^{k+1}$**

**$[4(4^k - 16) + 3(4^{k+1})] / 3$** //Simplifying by finding common denominator

**$4[(4^k - 16) + 3(4^k)] / 3$** //Factored out a 4

**$4[4^k((1 - 16) + 3)] / 3$** //Factored out a $4^k$

**$4[4^k((1 + 3) - 16)] / 3$** //Rearranged to see if it will match up with RHS.

**$4[4^k(4) - 16)] / 3$**

**$4[4^{k+1} - 16)] / 3$** //Same as RHS

Hence, P(k+1) is true. P(n) is true for all integers n ≥ 3.

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**Set 5.3 [10, 18, 23b]**

Prove each statement by mathematical induction.

**10Q: $n^3 - 7n + 3$ is divisible by 3, for each integer $n \geq 0$.**

**A:**

**$P(n) = n^3 - 7n + 3$ is divisible by 3**

**$P(0) = 0^3 - 7(0) + 3 = 3$ //Which is divisible by 3, hence P(0) is true.**

**By Induction:**

**Assume $P(k) = k^3 - 7k + 3$ is divisible by 3 is true.**

**Show that $P(k+1) = (k+1)^3 - 7(k+1) + 3$ is divisible by 3**

**$P(k+1) = (k+1)^3 - 7(k+1) + 3$**

**$= (k^3 + k^2 + 2k^2 + 2k + k + 1) - (7k + 7) + 3$ //Expanded by algebra**

**$= (k^3 - 7k + 3) + (3k^2 + 3k + 1 - 7)$ //Rearranging to see if it will match P(k)**

**$= (k^3 - 7k + 3) + (3k^2 + 3k - 6)$**

**$= (k^3 - 7k + 3) + 3(k^2 + k - 2)$ //Factored out 3**

**Let $t = (k^2 + k - 2)$. t is an integer as sums and products of integers are integers.**

**$= (k^3 - 7k + 3) + 3t$ //substituted t**

**($k^3 - 7k + 3$) is divisible by 3, as assumed. 3t is divisible by 3 ($3t = 3 \times t$)**

**This shows that P(k+1) is divisible by 3.**

**Therefore P(n) is divisible by 3 for all integers $n \geq 0$.**

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**18Q: $5^n + 9 < 6^n$ for all integers $n \geq 2$.**

**Basis Step:**

**$P(n) = 5^n + 9 < 6^n$**

**$P(2) = 5^2 + 9 < 6^2$**

**LHS = 34; RHS = 36**

**Hence $5^n + 9 < 6^n$ is indeed true for n = 2**

**Induction:**

**Assume $P(k) = 5^k + 9 < 6^k$ is true, show that P(k+1) is true.**

**$P(k+1) = 5^{k+1} + 9 < 6^{k+1}$**

**$6(5^k + 9) < 6(6^k)$ //Getting to k+1 form**

**$(6)(5^k + 9) < 6^{k+1}$ //RHS is in the k+1 form**

**$(5+1)(5^k + 9) < 6^{k+1}$**

**$5^{k+1} + 45 + 5^k + 9 < 6^{k+1}$**

**$(5^{k+1} + 9) + (5^k + 45) < 6^{k+1}$ //Rearranging to get as close to P(n) form as possible**

**Let $a = (5^{k+1} + 9)$, $b = (5^k + 45)$, and $c = 6^{k+1}$**

**Then we get $a + b < c$**

**If that is true, then $a < c$ and $b < c$**

**$(5^{k+1} + 9) < 6^{k+1}$ and $(5^k + 45) < 6^{k+1}$**

**Since $(5^{k+1} + 9) < 6^{k+1}$, P(k+1) is true. Then by induction, P(n) is true for all $n \geq 2$.**

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**23B: $n! > n^2$, for all integers $n \geq 4$.**

**Basis Step:**

**$P(n) = n! > n^2$**

**$P(4) = 4! > 4^2 = (4x3x2x1) > 16 = 24 > 16$ which is True.**

**By Induction:**

**Assume $P(k) = k! > k^2$ is true.**

**$P(k+1) = (k+1)! > (k+1)^2$**

**Which can be rewritten as:**

**$(k+1)! - (k+1)^2 > 0$**

**LHS: $(k+1)\, k! – (k+1)^2$**

**$= (k+1)\, (k! – (k+1))$ //Factored out (k+1)**

**$> (k+1)\, (k^2 – k - 1)$ //Since we assume $k! > k^2$, we can substitute $k^2$ and make the equation less thank k!**

**$> (k+1)\, (k^2 -k + ¼ - ¼ -1)$ //Completing the square.**

**$> (k+1)\, ((k- ½ )^2 – 5/4)$**

**Because $k \geq 4$, $k- ½ \geq 3 ½$**

**$(k – ½ )^2 \geq 12 ¼$**

**$(k – ½ )^2 - 5/4 \geq 11$ //This means that it is also greater than 0**

**$(k+1)\, ((k- ½ )^2 – 5/4)$ would also be greater than 0.**

**Hence,**

**$(k+1)! - (k+1)^2 > (k+1)\, ((k- ½ )^2 – 5/4) > 0$**

**$(k+1)! - (k+1)^2 > 0$ //Since if a + b > c then a >c and b > c**

**Therefore P(k+1) is true, and P(k) is true for all integers where $k \geq 4$.**

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