

Assignment 5 – Due 7/30/2017

Part II. Exercise Set 9.4 [6, 7, 16, 27]

6Q:

- a) Given any set of seven integers, must there be two that have the same remainder when divided by 6? Why?
- b) Given any set of seven integers, must there be two that have the same remainder when divided by 8? Why?

A:

- a) Yes. There are 6 different remainders for a set of seven integers divided by 6. Since there are 7 items to fit into 6 containers, by the Pigeonhole Principle, there must be two integers that have the same remainder when divided by 6.
- b) No. There are 8 different remainders when divided by 8, or 8 containers. 7 items can each fit into one of the 8 containers so this is false by the Pigeonhole Principle.

7Q: Let $S = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Suppose six integers are chosen from S . Must there be two integers whose sum is 15? Why?

A: Yes.

There are 5 pairs of numbers in this set that would have a sum of 15, or 5 subset/containers.

By the Pigeonhole Principle, $6 > 5$ so two of the integers must fit into the same subset/container.

16Q: How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

A: There are 20 possible integers between 1 and 100 that are divisible by 5. There are a total of 100 integers.

In order to select an integer that is sure to be divisible by 5 then, we must choose $(100-20) + 1 = 81$ integers.

27Q: In a group of 2000 people, must at least 5 have the same birthday? Why?

A: Working with 365 days a year (not assuming leap years). There are 365 subsets or containers that the 2000 people can fit into.

$2000/365 = 5.48$ which is greater than 5. By the Pigeonhole Principle, at least 5 people have the same birthday.

Note: The same holds true if we consider leap year: $2000/366 = 5.46$, which is also greater than 5.