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Assignment 3 – Due 7/16/2017

**Part II. Exercise Set 6.1 [12, 16]; Set 6.2 [4, 10, 14]; Set 6.3 [12, 37, 42]**

## Set 6.1 [12, 16]

**12Q: Let the universal set be the set R of all real numbers and let** $A = [x \in \mathbb{R} \mid -3 \leq x \leq 0]$, $B = [x \in \mathbb{R} \mid -1 < x < 2]$, and $C = [x \in \mathbb{R} \mid 6 < x \leq 8]$. Find each of the following:

a) $A \cup B = [-3, 2)$
b) $A \cap B = (-1, 0]$
c) $A^c = (-\infty, -3) \cup (0, \infty)$
d) $A \cup C = [-3, 0] \cup (6, 8]$
e) $A \cap C = \emptyset$ //The two sets don't intersect
f) $B^c = (-\infty, -1] \cup [2, \infty)$
g) $A^c \cap B^c = (-\infty, -3) \cup [2, \infty)$
h) $A^c \cup B^c = (-\infty, -1] \cup (0, \infty)$
i) $(A \cap B)^c = (-\infty, -1] \cup (0, \infty)$
j) $(A \cup B)^c = (-\infty, -3] \cup (2, \infty)$

**16Q: Let A= [a, b, c], B = [b, c, d], and C = [b, c, e]**

**a) Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?**
$A \cup (B \cap C) = [a, b, c]$
$(A \cup B) \cap C = [b, c]$
$(A \cup B) \cap (A \cup C) = [a, b, c]$

**Therefore $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$ are equal.**

**b) Find $A \cap (B \cup C)$, $(A \cap B) \cup C$, and $(A \cap B) \cup (A \cap C)$. Which of these sets are equal?**
$A \cap (B \cup C) = [b, c]$
$(A \cap B) \cup C = [b, c, e]$
$(A \cap B) \cup (A \cap C) = [b, c]$

**Therefore $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$ are equal.**

**c) Find $(A - B) - C$ and $A - (B - C)$. Are these sets equal?**
$(A - B) - C = \{a\}$
$A - (B - C) = \{a, b, c\}$

**Therefore $(A - B) - C \neq A - (B - C)$**

## Set 6.2 [4, 10, 14]

**4Q: The following is a proof that for all sets A and B, if $A \subseteq B$, then $A \cup B \subseteq B$. Fill in the blanks.**

**A: Proof: Suppose A and B are any sets and $A \subseteq B$. [We must show that $A \cup B \subseteq B$.] Let $x \in A \cup B$. [We must show that $x \in B$. By definition of unions, $x \in A$ or $x \in B$. In case $x \in A$, then since $A \subseteq B$, $x \in B$. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in B$ [as was to be shown].**

**Use an element argument to prove each statement in 7-19. Assume that all sets are subsets of a universal set U.**

**10Q: For all sets A, B, and C,**

**$(A - B) \cap (C - B) = (A \cap C) - B$**

**A:**

First we must show that $(A - B) \cap (C - B)$ is a subset of $(A \cap C) - B$.

Suppose there is an element x in $(A - B) \cap (C - B)$. By definition of intersection, $x \in (A - B)$ and $x \in (C - B)$.

$x \in (A - B) \leftrightarrow x \in A$ and $x \notin B$ //By set definition
$x \in (C - B) \leftrightarrow x \in C$ and $x \notin B$ // By set definition

$x \in A$ and $x \in C$ is $x \in (A \cap C)$//By definition of intersection
We get $x \in (A \cap C)$ and $x \notin B$ which shows $x \in (A \cap C) - B$ by definition of set difference.

Because we showed that $(A - B) \cap (C - B) \rightarrow x \in (A \cap C) - B$, we proved that $(A - B) \cap (C - B) \subseteq (A \cap C) - B$.

**14Q: For all sets A, B, and C, if $A \subseteq B$ then $A \cup C \subseteq B \cup C$**

**A:**

Suppose A, B, and C are sets and $\mathbf{A \subseteq B}$. We must show that $\mathbf{A \cup C \subseteq B \cup C}$.

Let $x \in A \cup C$. By definition of union, $x \in A$ or $x \in C$.

Examining $x \in A$ and $x \in C$, we can assume that $x \in B$ because $A \subseteq B$.

Hence, $x \in B$ or $x \in C$, which is $x \in B \cup C$ by definition of union.

Therefore, if $A \subseteq B$ then $A \cup C \subseteq B \cup C$

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## Set 6.3 [12, 37, 42]

**For each of 5-21 prove each statement that is true and find a counterexample for each statement that is false. Assume all sets are subsets of a universal set U.**

**12Q: For all sets A, B, and C,**

**$A \cap (B - C) = (A \cap B) - (A \cap C)$**

**A:** Show that $A \cap (B - C) \subseteq (A \cap B) - (A \cap C)$ and $(A \cap B) - (A \cap C) \subseteq A \cap (B - C)$

Let $x \in A \cap (B - C)$, $x \in A$, $x \in (B - C)$ // By def. of intersection

Which means $x \in A$, $x \in B$, and $x \notin C$ //by set difference.

= $x \in (A \cap B)$ and $x \notin (A \cap C)$ //by intersection

= $x \in (A \cap B) - (A \cap C)$

Let $x \in (A \cap B) - (A \cap C)$

$x \in (A \cap B)$ and $x \notin (A \cap C)$ // By set difference

Which means $x \in A$, $x \in B$, and $x \notin C$

$x \in A$ and $x \in (B - C)$

$x \in A \cap (B - C)$.

Therefore, **$A \cap (B - C) = (A \cap B) - (A \cap C)$**

■

**37Q: For all sets A and B, $(B^c \cup (B^c - A))^c = B$**

A:

$(B^c \cup (B^c - A))^c = (B^c \cup (B^c \cap A^c))^c$ //By Difference
$= (B^c)^c \cap (B^c \cap A^c)^c$ //By DeMorgan's
$= (B^c)^c \cap (B^c)^c \cup (A^c)^c$ //By DeMorgan's
$= B \cap (B \cup A)$ //By double complement
$= B$ //By Absorption

■

**42Q: Simplify: $(A - (A \cap B)) \cap (B - (A \cap B))$**

A:

$(A - (A \cap B)) \cap (B - (A \cap B)) = (A \cap (A \cap B)^c) \cap (B \cap (A \cap B)^c)$ //By difference

$= (A \cap (A^c \cup B^c) \cap (B \cap (A^c \cup B^c)$ // By DeMorgan's

$= ((A \cap A^c) \cup (A \cap B^c) \cap ((B \cap A^c) \cup (B \cap B^c)$ // By Distributive

$= (\emptyset \cup (A \cap B^c)) \cap ((B \cap A^c) \cup \emptyset)$ // By Complementation

$= (A \cap B^c) \cap (B \cap A^c)$ //By Identity

$= (A \cap A^c) \cap (B \cap B^c)$ //By Associative

$= \emptyset \cap \emptyset$ // By Complementation

$= \emptyset$ // By Universal Bound Law