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Assignment 2 – Due 7/9/2017

Part II. Exercise Set 4.6 [12, 16, 28]

Prove each statement in 10-17 by contradiction

12. Q: If a and b are rational numbers, b $\neq$ 0, and r is an irrational number, then a + br is irrational.

A:

Negation of the statement: If a and b are rational numbers, b≠0, and r is an irrational number, then a + br is rational.

Proof:

By definition of rational, a = $\frac{c}{d}$, b = $\frac{e}{f}$, a+br = $\frac{m}{n}$ for some integers c,d,e,f,m,n where d≠0, f ≠0, n≠ 0

a+br = $\frac{m}{n}$

$\frac{c}{d}$ + ($\frac{e}{f}$)r = $\frac{m}{n}$ //By substitution

($\frac{e}{f}$)r = $\frac{m}{n}$ - $\frac{c}{d}$

($\frac{e}{f}$)r = $\frac{md-cn}{nd}$

r = $\frac{md-cn}{nd}$($\frac{f}{e}$)

r = $\frac{(md-cn)f}{nde}$

We know that the product and difference of rational numbers are rational, therefore r is a rational number. This contradicts the supposition that r is irrational.

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16 Q: the proof, use the properties of even and odd integers that are listed on Example 4.2.3.)

Negation of statement: For all odd integers a, b, and c, if z is a solution to $ax^2 + bx + c = 0$ then z is rational.

Proof: For all odd integers a, b, and c, if z is a solution of $ax^2 + bx + c = 0$ then z is irrational. (In

Suppose a, b, and c, are odd integers, z is a solution to $ax^2 + bx + c = 0$ and z is rational.

By definition of rational, z = $\frac{p}{q}$ for some integers p and q where $q \neq 0$. p and q have no common factors and thus they are not both even.

a$(\frac{p}{q})^2$ + b$(\frac{p}{q})$ + c = 0 //By substitution

$(\frac{ap^2}{q^2})$ + $(\frac{bp}{q})$ + c = 0

$ap^2 + bpq + cq^2 = 0$ // Multiplied by q^2 on both sides

Case 1: Show that the assumption that p is even leads to a contradiction.

If p is even, q is odd

ap^2 is even given that product of even integers is even.

bpq is even given that product of even integers is even.

cq^2 is odd given that product of odd integers is odd.

$ap^2 + bpq$ is even given that the sum of even integers is even.

$(ap^2 + bpq) + cq^2$ is odd given that the sum of odd and even integers is odd.

This contradicts $ap^2 + bpq + cq^2 = 0$ which is even.

Case 2: Show that the assumption that q is even leads to a contradiction.

If q is even, p is odd

ap^2 is odd given that product of odd integers is odd.

bpq is even given that product of even integer and odd integer is even.

cq^2 is even given that product of even integers is even.

$ap^2 + bpq$ is odd given that sum of odd and even integers is odd.

$(ap^2 + bpq) + cq^2$ is odd given that the sum of odd and even integers is odd.

This contradicts $ap^2 + bpq + cq^2 = 0$ which is even.

Case 3: Show that the assumption that both p and q are odd leads to a contradiction.

If p is odd, q is odd

ap^2 is odd given that product of odd integers is odd.

bpq is odd given that product of odd integers is odd.

cq^2 is odd given that product of odd integers is odd.

$ap^2 + bpq$ is even given that sum of odd integers is even.

$(ap^2 + bpq) + cq^2$ is odd given that the sum of even and odd integers is even.

This contradicts $ap^2 + bpq + cq^2 = 0$ which is even.

All three cases lead to contradictions, thus proving the original statement (by contradiction) that for all odd integers a, b, and c, if z is a solution of $ax^2 + bx + c = 0$ then z is irrational.

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28. Q: For all integers m and n, if mn is even then m is even or n is even. (Proof by contradiction).

Negation of statement: There exist integers m and n such that if mn is even, m is odd or n is odd.

Proof:

By definition of odd, m = 2k + 1 and n = 2p + 1 for some integers k and p.

mn = (2k + 1)(2p + 1) //By substitution

= 4kp + 2k + 2p + 1

= 2(2kp + k + p) + 1 //By factoring out 2

Let t = 2kp + k + p, mn = 2t + 1 which is odd by definition.

This contradicts the supposition that mn is even.

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