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Assignment 2 – Due 7/9/2017

Part I. Exercise Set 4.1 [32, 61], Set 4.2 [20, 25], Set 4.6 [28]

Set 4.1 [32, 61]

Prove the statements in 24-34. In each case use only the definitions of the terms and the Assumptions listed on page 146, not any previously established properties of odd and even integers. Follow the directions given in this section for writing proofs of universal statements.

32. Q: If a is any odd integer and b is any even integer, then, 2a + 3b is even.

A:

Proof:

Suppose a is any odd integer and b is any even integer. By definitions of odd and even, a = 2s + 1 and b = 2r for some integers s and r. Then,

2a + 3b = 2 (2s + 1) + 3 (2r) //By substitution
= 4s + 2 + 6r
= 2 (2s + 1 + 3r) //By factoring out 2

Because product of integers is an integer, and sum of integers is an integer, 2a + 3b results in the form 2t where t is an integer representing 2s + 1 + 3r. This shows that 2a + 3b is even.

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61. Q: Suppose that integers m and n are perfect squares. Then m + n + 2$\sqrt{mn}$ is also a perfect square. Why?

Proof:

Suppose integers m and n are perfect squares. Then by definition of a perfect square, m = a^2 and n = b^2 for some nonnegative integers a and b.

m + n + 2$\sqrt{mn}$ = a^2 + b^2 + 2$\sqrt{a^2b^2}$ //By substitution
= a^2 + b^2 + 2($\sqrt{a^2}$)($\sqrt{b^2}$) //Per Q#59, $\sqrt{ab} = \sqrt{a}\sqrt{b}$ for all nonnegative real numbers a and b
= a^2 + b^2 + 2ab
= $(a + b)^2$ //By factoring

Let a + b = t where t is an integer because the sum of integers is an integer. Then m + n + 2$\sqrt{mn}$ = t^2, which shows that m + n + 2$\sqrt{mn}$ is a perfect square by definition.

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Set 4.2 [20, 25]

Determine which of the statements in 15-20 are true and which are false. Prove each true statement directly from the definitions, and give a counterexample for each false statement. In case statement is false, determine whether a small change would make it true. If so, make the change and prove the new statement. Follow the directions for writing proofs on page 154.

20 Q: Given any two rational numbers r and s with r < s, there is another rational number between r and s. (Hint: Use the results of exercises 18 and 19).

Proof in 2 parts

I. **Show that for real numbers r and s, if r < s, then r < $\frac{r+s}{2}$ < s.**

Proof I:

Let r and s be any real numbers and suppose that r < s.
Given that if r < s, then r + k < b + k where k is any real number //Per T19 in Appendix A

i) r < s → r + r < s + r
→ 2r < s + r
→ r < $\frac{s+r}{2}$

Similarly,

ii) r < s → r + s < s + s
→ r + s < 2s
→ $\frac{r+s}{2} < s$

Per i) and ii), r < $\frac{r+s}{2} < s.$

II. **Show that for any two rational numbers r and s, that $\frac{r+s}{2}$ is rational.**

Proof II:

Suppose r and s are 2 rational numbers. By definition of rational,
r = $\frac{a}{b}$ and s = $\frac{c}{d}$ for some integers a, b, c, d where b ≠ 0, and d ≠ 0. Then,

$\frac{r+s}{2}$ = $\frac{\frac{a}{b}+\frac{c}{d}}{2}$
= $\frac{\frac{ad}{bd}+\frac{cb}{db}}{2}$ //Finding common denominator
= $\frac{ad+cb}{2bd}$ //By algebra

Let m = ad + cb, and n = 2bd. Then,

$\frac{r+s}{2} = \frac{m}{n}$ where m and n are integers as sum and product of integers are integers. n $\neq$ 0 per zero product property.

Therefore, $\frac{r+s}{2}$ is rational by definition of rational numbers.

Per Proof I, we learn that r < $\frac{r+s}{2} < s$. Per Proof II, we learn that $\frac{r+s}{2}$ is rational.

Therefore, it is true that when given any 2 rational numbers r and s, with r < s, there is another rational number between r and s.

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Derive the statements in 24-26 as corollaries of Theorems 4.2.1 4.2.2, and the results of exercises 12, 13, 14, 15, and 17.

25 Q: If r is any rational number, then $3r^2 - 2r + 4$ is rational.

A:

We know that:

Theorem 4.2.1: every integer is a rational number
Theorem 4.2.2.: The sum of any 2 rational numbers is rational.
Exercise 12: Square of any rational number is rational.
Exercise 13: The negative of any rational number is rational.
Exercise 14: The square of any rational number is rational.
Exercise 15: The product of any 2 rational numbers is rational.
Exercise 17: The difference between 2 rational numbers is rational.

Given that r is a rational number. By definition of rational numbers,
r = $\frac{a}{b}$ for some integers a and b, where b $\neq$ 0. Per Theorem 4.2.1, integers a and b are rational. Then,

$3r^2 - 2r + 4 = 3\left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right) + 4$ // By substitution

Per exercise 12, $\left(\frac{a}{b}\right)^2$ is rational.
Per exercise 15, $3\left(\frac{a}{b}\right)^2$ and $2\left(\frac{a}{b}\right)$ are rational.
Per exercise 17, $3\left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)$ is rational.
Per Theorem 4.2.2, $\left(3\left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\right) + 4$ is rational.

Therefore, $3r^2 - 2r + 4$ is rational.

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Set 4.6 [28]

Prove statement 28 by contraposition.

28 Q: For all integers m and n, if mn is even then m is even or n is even.

A:

To proof by contraposition:

Contrapositive of statement 28: For all integers m and n, if m is not even and n is not even, then mn is not even.

Restated:

For all integers m and n, if m is odd and n is odd, then mn is odd.

Proof:

Suppose m and n are any odd integers. By definition of odd:

m = 2k + 1, n = 2p + 1 for some integers k and p.

mn = (2k + 1)(2p + 1) //By substitution

= 4kp + 2k + 2p + 1

= 2(2kp + k + p) + 1 //Factoring out 2

We know that the sum and product of 2 integers are integers. Let integer a represent 2kp + k +p

= 2a + 1

Therefore, mn is odd by the definition of odd. The original statement is then proved by contraposition.

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