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CS 225 Section 400, Su 2017

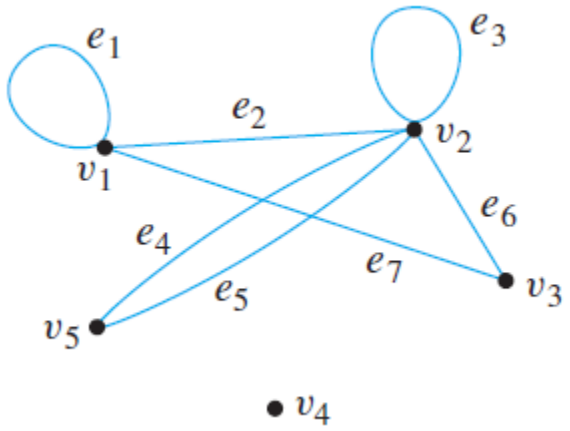
Assignment 7 – Due 8/13/2017

Part I. Exercise Set 10.1 [9, 27b, 44]

9 Q: For each of the graphs in 8 and 9:

- **i) Find all edges that are incident on v_1**
- **ii) Find all vertices that are adjacent to v_3**
- **iii) Find all edges that are adjacent to e_1**
- **iv) Find all loops**
- **v) Find all parallel edges**
- **vi) Find all isolated vertices**
- **vii) Find the degree of v_3**
- **viii) Find the total degree of the graph**

9.



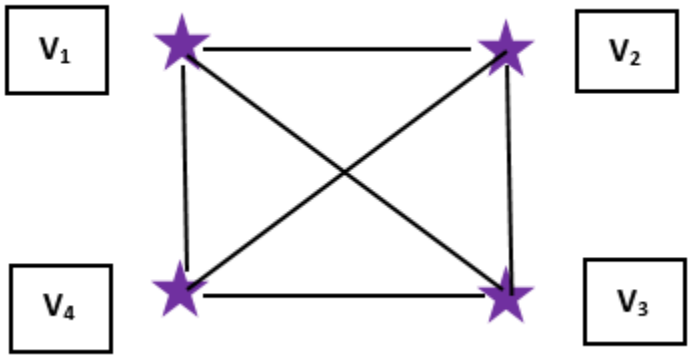
A:

- **i) e_1, e_2, e_7**
- **ii) v_1, v_2**
- **iii) e_2, e_7**
- **iv) e_1, e_3**
- **v) e_4, e_5**
- **vi) v_4**
- **vii) 2**
- **viii) $\deg(v_1) + \deg(v_2) + \deg(v_3) + \deg(v_4) + \deg(v_5)$**
  $= 4+6+2+0+2$
  $=14$

27b Q: In a group of 4 people, is it possible for each person to have exactly 3 friends? Why?

A:

If we picture 4 vertices v_1, v_2, v_3, and v_4 and positioned them as below:



Where v_1 is person 1, v_2 is person 2, v_3 is person 3, and v_4 is person 4.

By this arrangement, each vertex has a deg(v_i)= 3 which means each person is connected to three others.

Therefore it is possible for 4 people to each have exactly three friends.

44Q:
a) In a simple graph, must every vertex have degree that is less than the number of vertices in the graph? Why?

b) Can there be a simple graph that has four vertices all of different degrees?

c) Can there be a simple graph that has n vertices all of different degrees?

A:

a) **By definition, a simple graph "… does not have any loops or parallel edges". This means that each vertex connects to another vertex by just one edge. If we have 2 vertices, they would be connected by 1 edge; 3 vertices are connected by 2 edges, and so forth. The number of edges in a simple graph is then n – 1 for n number of vertices.**
b) **No. A simple graph can have at most n-1 degrees for n vertices. For n = 4 vertices, the maximum degree is 3. If each vertex must have different degrees, then there would can be degrees 1, 2, and 3 among them. (0 would mean an isolated vertex which would not be connected to any other vertices by an edge).**
c) **No. This is similar to the question in b). n vertices have a maximum of n-1 degrees, which is not enough for all vertices to connect to each other with different degrees. (0 isolates a vertex).**