

Assignment 5 – Due 7/30/2017

Part I. Exercise Set 9.2 [11c, 14c, e, 17] Set 9.3 [5,24, 33e, f]

Set 9.2 [11c, 14c, e, 17]

11c Q: How many bit strings of length 8 begin and end with a 1?

A:

$$2^6 = 64$$

14c Q: Suppose that in a certain state, all automobile license plates have four letters followed by three digits.

c. How many license plates could begin with TGIF?

A:

$$10^3 = 1000$$

e. How many license plates could begin with AB and have all letters and digits distinct?

A:

$$\text{For all letters and digits distinct: } 24 \times 23 \times 10 \times 9 \times 8 = 397,440$$

17Q:

- a) How many integers are there from 1000 through 9999?
- b) How many odd integers are there from 1000 through 9999?
- c) How many integers from 1000 through 9999 have distinct digits?
- d) How many odd integers from 1000 through 9999 have distinct digits?
- e) What is the probability that a randomly chosen four-digit integer has distinct digits? Has distinct digits and is odd?

A:

- a) $m = 1000, n = 9999; n - m + 1 = (9999 - 1000) + 1 = 9000$ total integers
- b) $9000 / 2 = 4500$ odd integers (or 4500 even integers).
- c) $9 \times 9 \times 8 \times 7 = 4536$ integers with distinct digits.
- d) $8 \times 8 \times 7 \times 5 = 2240$

- e) Four-digit integer with distinct digits: $4536/9000 = 0.504$ or 50.4%
 Four-digit integer with distinct digits and is odd: $2240/9000 = 0.2489$ or 24.89%

Set 9.3 [5, 24, 33e, f]

5Q:

- a) How many five-digit integers (integers from 10000 through 99999) are divisible by 5?
 b) What is the probability that a five-digit integer chosen at random is divisible by 5?

A:

- a) Let A_1 be the set of five-digit integers that end with a 5 and A_2 be the set that end with a 0 (to show divisibility by 5).
 $A_1 = 9 \times 10 \times 10 \times 10 = 9000$; $A_2 = 9 \times 10 \times 10 \times 10 = 9000$
 Answer is $A_1 + A_2 = 18000$
- b) Total number of five-digit integers is: $9 \times 10 \times 10 \times 10 \times 10 = 90000$
 Probability of a five-digit integer divisible by 5 is: $18000/90000 = 0.2$ or 20%

24Q:

- a) How many integers from 1 through 1000 are multiples of 2 or multiples of 9?
 b) Suppose the integer from 1 through 1000 is chosen at random. Use the result from part a to find the probability that the integer is a multiple of 2 or multiple of 9.
 c) How many integers from 1 through 1000 are neither multiples of 2 nor multiples of 9?

A:

- a) Let A = the set of all integers from 1 through 1000 that are multiples of 2
 Let B = the set of all integers from 1 through 1000 that are multiples of 9

$A \cup B$ = the set of all integers from 1 through 1000 that are multiples of 2 or multiples of 9

$A \cap B$ = the set of all integers from 1 through 1000 that are multiples of both 2 and 9, or 18.

Need to find $N(A \cup B) = N(A) + N(B) - N(A \cap B)$

$$N(A) = 2k = 500$$

$$N(B) = 9k = 111$$

$$N(A \cap B) = 18k = 55$$

$$N(A) + N(B) - N(A \cap B)$$

$$= (500 + 111) - 55$$

$$= 556$$

- b) Total integers from 1 through 1000 = 1000
 Probability that the integer is a multiple of 2 or multiple of 9
 $556 / 1000$
 $= 0.556$ or 55.6%
- c) $1000 - 556 = 444$

33 Q: A college conducted a survey to explore the academic interests and achievements of its students. It asked students to place checks beside the numbers of all the statements that were true of them. Statement #1 was "I was on the honor roll last term," statement #2 was "I belong to an academic club, such as the math club or the Spanish club," and statement #3 was "I am majoring in at least two subjects." Out of a sample of 100 students, 28 checked #1, 26 checked #2, and 14 checked #3, 8 checked both #1 and #2, 4 checked both #1 and #3, 3 checked both #2 and #3, and 2 checked all three statements.

33Q: How many students checked #2 and #3 but not #1?

Let A = Students who chose #1

Let B = Students who chose #2

Let C = Students who chose #3

Let $A \cap B$ = Students who chose #1 and #2

Let $A \cap C$ = Students who chose #1 and #3

Let $B \cap C$ = Students who chose #2 and #3

Let $A \cap B \cap C$ = Students who chose #1, 2, and 3

e. How many students checked #2 and #3 but not #1?

$$A: (B \cap C) - (A \cap B \cap C) = 3 - 2 = 1$$

f. How many students checked #2 but neither of the other two?

$$A: B - (A \cap B) - (B \cap C) + (A \cap B \cap C) = 26 - 8 - 3 + 2 = 17$$