

Assignment 6 – Due 8/6/2017

Part II. Exercise Set 9.6 [4, 12, 18]

4Q: A camera shop stocks eight different types of batteries, one of which is type A7b. Assume there are at least 30 batteries of each type.

- a) How many ways can a total inventory of 30 batteries be distributed among the eight different types?
- b) How many ways can a total inventory of 30 batteries be distributed among the eight different types if the inventory must include at least four A7b batteries?

A:

- a) By Theorem 9.6.1:

$$r = 30, n = 8$$

$$\binom{30 + 8 - 1}{30}$$

$$= \binom{37}{30}$$

$$= 37! / 30! 7!$$

$$= 46376$$

- b) $r=26, n=8$

$$\binom{26 + 8 - 1}{26}$$

$$= \binom{33}{26}$$

$$= 33! / 26! 7!$$

$$= 27405$$

12Q: Find how many solutions there are to the given equation that satisfy the given condition.

$y_1 + y_2 + y_3 + y_4 = 30$, each y_i is a nonnegative integer.

A:

$$\binom{30 + 4 - 1}{30}$$

$$= \binom{33}{30}$$

$$= 33! / 30! 3!$$

$$= 5456$$

18Q: A large pile of coins consists of pennies, nickels, dimes, and quarters.

- a) How many different collections of 30 coins can be chosen if there are at least 30 of each kind of coin?
- b) If the pile contains only 15 quarters but at least 30 of each other kind of coin. How many collections of 30 coins can be chosen?
- c) If the pile contains only 20 dimes but at least 30 of each other kind of coin. How many collections of 30 coins can be chosen?
- d) If the pile contains only 15 quarters and only 20 dimes but at least 30 of each other kind of coin. How many collections of 30 coins can be chosen?

A:

$$\begin{aligned} \text{a) } & \binom{30+4-1}{30} \\ &= \binom{33}{30} \\ &= 5456 \end{aligned}$$

$$\begin{aligned} \text{b) } T &= Q_{\leq 15} \cup Q_{\geq 16} \\ Q_{\leq 15} \cap Q_{\geq 16} &= \emptyset \end{aligned}$$

$$\begin{aligned} T &= 5456 \\ Q_{\geq 16} &= \binom{15+4-1}{15} \\ &= \binom{18}{15} \\ &= 816 \end{aligned}$$

$$\begin{aligned} Q_{\leq 15} &= T - Q_{\geq 16} = 5456 - 816 \\ &= 4640 \end{aligned}$$

$$\begin{aligned} \text{c) } T &= D_{\leq 20} \cup D_{\geq 21} \\ D_{\leq 20} \cap D_{\geq 21} &= \emptyset \end{aligned}$$

$$\begin{aligned} T &= 5456 \\ D_{\geq 21} &= \binom{10+4-1}{10} \\ &= \binom{13}{10} \\ &= 286 \end{aligned}$$

$$\begin{aligned} R_{\leq 20} &= T - R_{\geq 21} = 5456 - 286 \\ &= 5170 \end{aligned}$$

$$\begin{aligned} \text{d) } T - (Q_{\geq 16} + D_{\geq 21}) \\ &= 5456 - (816 + 286) \\ &= 4354 \end{aligned}$$