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Assignment 3 – Due 7/16/2017

Part I. Exercise Set 6.1 [3, 7, 13, 18, 33, 34]

3Q: Let sets R, S, and T be defined as follows:

R = {x ϵ Z|x is divisible by 2}
S = {x ϵ Z|y is divisible by 3}
T = {x ϵ Z|z is divisible by 6}

a) Is R⊆T? Explain.
b) Is T⊆R? Explain.
c) Is T⊆S? Explain.

A:

a) No. For **R⊆T**, all elements in R must be in T. There are numbers divisible by 2 but not divisible by 6 (For example, 2, 4, 8, 14).
b) Yes. All elements in T would be in R as well since any integers divisible by 6 would be divisible by 2. (For example, 18 is divisible by 6 and divisible by 2; 6 itself is divisible by 6 and 2).
c) Yes. All elements in T would be in S since any integers divisible by 6 would also be divisible by 3. (For example, 18, 12, 6 are all divisible by 6 and 3).

7Q: Let A = {x ϵ Z|x = 6a + 4 for some integer a}, B = {y ϵ Z|y = 18b – 2 for some integer b}, and C = {z ϵ Z|z = 18c + 16 for some integer c}. Prove or disprove each of the following statements.

a) A⊆B
b) B⊆A
c) B = C

A:

a) A⊆B
Proof:
Suppose x is a particular but arbitrarily chosen element of A. By definition of A, there is an integer a such that x = 6a + 4.
Assume A⊆B is true, then x is also an element of B. By definition of B, there is an integer b such that x = 18b – 2.

x = 6a + 4, and x = 18b – 2

$6a + 4 = 18b – 2$
$6a + 6 = 18b$
$6 (a + 1) = 6 (3b)$
$a + 1 = 3b$

$a = 3b – 1$, and $b = (a + 1) / 3$

Depending on the value of a, b may not be an integer (For example, $a = 1$ results in $b = \frac{2}{3}$ which is not an integer). Since there exists an integer a that does not result in an integer value b, $A \subseteq B$ is false.

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b) $B \subseteq A$
Proof:

Suppose y is an element of B. By definition of B, there is an integer b such that $y = 18b – 2$.
If B is a subset of A, then by definition of A, there is an integer a such that y equals $6a + 4$.

$18b -2 = 6a + 4$
$18b = 6a +6$
$6 (3b) = 6 (a + 1)$
$3b = a + 1$
$b = (a + 1)/3$

Here, the value of b will always be multiplied by 3 to obtain a ($a= 3b – 1$). Since the sums and products of integers are integers, we showed that for each b we can find an integer a. Therefore $B \subseteq A$ is true.

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c) $B = C$
Proof:

We need to show that $B \subseteq C$ and $C \subseteq B$.
Suppose y is an element of B. By definition of B, there is an integer b such that $y = 18b – 2$.
If B is a subset of C, y would also be an element of C. By definition of C, there is an integer c such that $y = 18c + 16$

$18b – 2 = 18c + 16$
$18b = 18c + 18$
$b = c + 1$

For any value of integer b, there will be an integer c such that b = c + 1 (since sums of integers are integers). Therefore $B \subseteq C$ is true.

Next we test $C \subseteq B$.
Suppose z is an element of C. By definition of C, there is an integer z such that z = 18c + 16.
If C is the subset of B, then z is also an element of B. By definition of B, there is an integer b such that z = 18b -2.

18c + 16 = 18b – 2
18c = 18b – 18
c = b – 1

For any value of integer c, there exists an integer b such that c = b – 1. Therefore $C \subseteq B$ Is true.

This shows that B = C

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13Q: Indicate which of the following relationships are true and which are false:

a) **$Z^+ \subseteq Q$ is True because Z are positive integers that make up rational numbers in Q.**
b) **$R^- \subseteq Q$ is False because R^- includes irrational numbers whereas Q only includes rationals.**
c) **$Q \subseteq Z$ is False because Z is a set of integers, whereas Q includes rational numbers (For example, $\frac{1}{2}$ is rational but not considered an integer.**
d) **$Z^- \cup Z^+ = Z$ is False because Z includes 0, which is excluded in both Z^+ and Z^-.**
e) **$Z^- \cap Z^+ = \emptyset$ is True because there is nothing at the intersection of Z^+ and Z^-, which we can represent with $\emptyset$.**
f) **$Q \cap R = Q$ is True because Real numbers contain both rational and irrational numbers. The intersection of Q and R must be the set of rational numbers Q.**
g) **$Q \cup Z = Q$ is True because integers are rational (with denominator 1).**
h) **$Z^+ \cap R = Z^+$ is True because Real numbers include both rational and irrational numbers. All integers are rational (with denominator 1). Therefore the interception of positive integers and Real numbers is the set of positive integers.**
i) **$Z \cup Q = Z$ is False because while integers are rational numbers, some rational numbers are not integers (For example, $\frac{2}{3}$).**

18Q:

a) Is the number 0 in $\emptyset$? Why?
No. $\emptyset$ **has no elements.**

b) Is $\emptyset = \{\emptyset\}$? Why?
No. $\emptyset$ is an empty element while $\{\emptyset\}$ is a set with element $\emptyset$.

c) Is $\emptyset \in \{\emptyset\}$? Why?
Yes because $\{\emptyset\}$ is a set with element $\emptyset$.

d) Is $\emptyset \in \emptyset$? Why?
No because $\emptyset$ has no elements so it cannot take an element of $\emptyset$.

33Q:

a) **Find $\wp(\emptyset)$.**
$\wp(\emptyset) = \{\emptyset\}$

b) **Find $\wp(\wp(\emptyset))$.**
$\wp(\wp(\emptyset)) = \{\emptyset, \{\emptyset\}\}$

c) **Find $\wp(\wp(\wp(\emptyset)))$.**
$\wp(\wp(\wp(\emptyset))) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

34Q: Let $A_1 = \{1, 2, 3\}$, $A_2 = \{u, v\}$, $A_3 = \{m, n\}$. Find each of the following sets:

a) A_1 X (A_2 X A_3)
= {(1, (u, m)), (1, (u, n)), (1, (v, m)), (1, (v, n)), (2, (u, m)), (2, (u, n)), (2, (v, m)), (2, (v, n)), (3, (u, m)), (3, (u, n)), (3, (v, m)), (3, (v, n))}

b) (A_1 X A_2) X A_3
= {((1, u), m), ((1, u), n), ((1, v), m), ((1, v), n), ((2, u), m), ((2, u), n), ((2, v), m), ((2, v), n), ((3, u), m), ((3, u), n), ((3, v), m), ((3, v), n)}

c) A_1 X A_2 X A_3
= {(1, u, m), (1, u, n), (1, v, m), (1, v, n), (2, u, m), (2, u, n), (2, v, m), (2, v, n), (3, u, m), (3, u, n), (3, v, m), (3, v, n)}